The Intention of Intention

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For Putnam in Representation and Reality*, there cannot be any intentional science, thus dooming cognitive science. His argument is that intentional concepts are functional, and that functionalism cannot explain anything because "everything has every functional organization", providing a proof. Analyzing his proof, we find that Putnam is assuming an ideal interpreting subject who can compute effortlessly and who is not intentional. But the subject doing science is a human being, and we are not that way. Therefore, in order to save cognitive science, we propose to replace the ideal subject with a real and intentional human subject, and we propose to model intentionality by using a problem theory which is an intuitionist set theory where the resolving subject is a computing device. We are intentional because we are living beings, where life is the intention of not to die, so we are embodied intentions designed by evolution. We are real and then we have to compute our resolutions to the survival problem, and fortuitously we are computationally Turing complete, so our language is complete and then full and self referable. In summary, evolutionary subjectivism modeled as problem solving by computing should save cognitive science. Or, in other words, we are proposing to update Kant with Darwin and Turing.*

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The Intention of Intention

§1 The book

¶1 · In the Introduction to *Representation and Reality*, Putnam (1988) defines functionalism and states the aim of his book. For Putnam (page xi) functionalism is "the thesis that the computer is the right model for the mind." This "computational view was itself a reaction against the idea that our matter is more important than our function, that our *what* is more important than our *how*. [...] This much [...] still seems to me [Putnam] to be as true and important as ever did" (page xii). However, he now (1988) thinks that functionalism fails to explain intentionality, and therefore his aim in this book is to show "that the computer analogy, call it the 'computational view of the mind,' or 'functionalism,' or what you will, does not after all answer the question [...] 'What is the nature of mental states?'" (page xi).

¶2 · Then, the first six out of seven chapters are full of smart arguments showing the many issues that are found when trying to explain three intentional concepts —truth, reference, and meaning— from the computational point of view. His own explanations of these concepts are in the last chapter: "truth is idealized rational acceptability" (page 115), "[reference] is a matter of *interpretation*", where "interpretation is an essentially holistic matter" (page 118), and "meaning is use" but "use is holistic" (page 119). In page 109, Putnam summarizes his position very clearly:

> *I do not see any possibility of a scientific theory of the "nature" of the intentional realm, and the very assumption that such a theory* must *be possible if there is something "to" intentional phenomena at all is one that I regard as wholly wrong.*

Therefore, for Putnam (1988), cognitive science is doomed. And to save cognitive science, in this paper we will investigate the possibility of nullifying Putnam's conclusion.

§2 Meanings are not in the head

¶1 · In reconstructing Putnam's argument in this book, we have to follow the trail of his trials to explain the meaning of 'meaning'. Very early, he realized that mental states, as for example 'meaning', cannot be identified with physical states of the brain. As Putnam famously wrote in 1975, page 144: "'meaning' just ain't in the head!" However, he still use to think that, according to functionalism, a meaning was a computational state in the brain, and this book was written to explain that he was wrong about this. Therefore, again but now extending his mantra to functionalism, Putnam writes in 1988, page 73: "Meanings aren't 'in the head'."

¶2 · In dealing with the mind and its mental states and properties, which is usually a fuzzy business, we will use the original mind-body split as defined by Descartes. For Descartes, minds can think and speak freely, while bodies are completely unfree, like machines. Then the intentional realm is entirely inside the mental realm, and by implication truth, reference, meaning, intention, and any other intentional concept, are mental concepts. Conversely, the physical realm, where nothing is free of the laws of nature, is completely outside the mental realm.

¶3 · Still under the Cartesian mind-body split, the computational realm is also completely outside the mental realm, because there is not any freedom in it. In retrospect, this shows us that Putnam's movement from physical states to computational states could not help him to explain the meaning of 'meaning'. Intentionality is as excluded from computing as it is excluded from physics.

§3 The requirement of objectivity

¶1 · As we saw above, Putnam's conclusion is that mental states and intentionality cannot be explained scientifically. If this were true, then biology, for example, could not define its object 'life' as the intention of 'not to die', which would be the natural way of doing it because biological organisms are intentional. And, if mental states could not be explained scientifically, then the status of 'cognitive science' as science would be doomed. But, is it true? Could we save cognitive science?

¶2 · We will call the subject-object split, where the subject is the agent observing and the object is the patient being observed, the epistemological split. Now, the prohibition of using intentional concepts in science is a consequence of using the Cartesian mind-body split for the epistemological split. Then the mindful subject is free, while the mechanical object is not free. Using the epistemological split, we can reformulate the ban positively, *science has to be objective*, or negatively, *science cannot be subjective*.

¶3 · Its limitation to objective reality was the cause of the huge success of science from Descartes time, because from then the final judge of any scientific theory is measurement instead of authority, where measurements are objective and authorities are subjective.

¶4 · Then, Putnam statement about the impossibility of a science of intentions seems to be a consequence of one requirement put on science, namely, that science has to be objective. We will call this requirement the *requirement of objectivity*.

§4 The requirement of physicalism

¶1 · Physics is the paradigm of objective science. Consequently, the aim of every other branch of science is to be reduced to physics. However, as the reduced branch is not physics, or otherwise the reduction would not be needed, then the reduction has to make some trick, some magical refactoring, to achieve its purpose. In fact, every reduction needs to interpret some physical data as something else in the theory that is being reduced. Thus, reducing and interpreting are always suspicious.

¶2 · We will call the requirement that any scientific theory has to be reduced to physics, or else it is not scientific, the *requirement of physicalism*. This requirement is stronger than the requirement of objectivity, and for example computing fulfills the requirement of objectivity, but it does not fulfill the requirement of physicalism. In fact, the main task of Putnam in the book is to explain why computing cannot be reduced to physics.

§5 Interpretations

¶1 · In order to reduce computing to physics, we need to interpret, for example, some electrical potentials as symbols, and some physical conditions as a computational *halt*. When the engineers are designing a computing device, the intended interpretation is documented in the technical manual of the computer, but if we allow any arbitrary interpretation, as argued by Putnam (1988), page 96 with a proof in the Appendix, then we can interpret any physical system as any computational system.

¶2 · In other words, he presents us a proof that every physical system realizes every computational system. If this were true, then it would explain why a scientific theory cannot be computational. And only thus could Putnam affirm both that the computer is the right model for the mind, and that this model does not explain anything. But, is he right? Could we save cognitive science?

¶3 · The main conclusions by Putnam (1988) in this book are negative: computing cannot be reduced to physics, and consequently intentionality cannot be explained scientifically. If these negative conclusions could not be overcome, then cognitive science would be doomed. However, these are the conclusions of some arguments, and those arguments are based on some assumptions. So our plan now will be to analyze the assumptions, and to assess their validity. Our final aim is to find a way to overcome the negative results by Putnam, or as he himself writes at the very end of the Introduction, page xv, to find "a different —and better— way of thinking about these philosophical issues."

§6 Two assumptions

¶1 · In the Appendix to Putnam's book, pages 121–125, there is a proof that "every ordinary open system is a realization of every abstract finite automaton", showing that "*everything has every functional organization*" (page xv). That is, the Appendix contains his proof that computing cannot be reduced to physics because every physical system implements every computational system.
¶2 · In our analysis, which is in the Appendix to this paper in §36, we conclude that Putnam does not prove the two-part theorem stated, but rather this slightly augmented three-part theorem: *using suitable interpreting functions, every ordinary open system is a realization of every abstract finite automaton.* As a consequence, Putnam's theorem holds if we are free to choose the suitable interpreting function for each pair of system and automaton, but it does not hold if there is an intended interpretation. So, in this case, an assumption is that there is not any intended interpretation. And even if there were not any intended interpretation, we would not be free to choose the suitable interpreting function for each pair of system and automaton should some interpretations were impossible, or too much difficult. Then another assumption is that any interpretation is as easy as any other, so every interpretation is as easy as the easiest one, and this only happens to ideal interpreters. Are these two assumptions right? Let us see some examples.

§7 A wall and a rock

¶1 · Before presenting my own one in the next section, we should mention two famous examples based on Putnam's proof: Searle's wall implementing a word processor program, and Chalmers' rock implementing a mind.
¶2 · Searle (1992), pages 208–209, writes:

> *Thus for example the wall behind my back is right now implementing the Wordstar program, because there is some pattern of molecule movements that is isomorphic with the formal structure of Wordstar. But if the wall is implementing Wordstar, then if it is a big enough wall it is implementing any program, including any program implemented in the brain.*

¶3 · Chalmers (1996), page 309, writes:

> *Specifically, he [Putnam] claims that every ordinary open system realizes every abstract finite automaton. He puts this forward as a theorem, and offers a detailed proof. If this is right, a simple system such as a rock implements any automaton one might imagine.*

Chalmers adds that "this would imply that a rock has a mind".

§8 A grenade

¶1 · Instead of using a wall as Searle or a rock as Chalmers, let me instead use an impact grenade. An impact grenade is a small explosive weapon typically thrown by hand, and its function is simple: if the safety lever is released and the impact sensor detects an impact (greater than, say, a 0.9 g shock), then the explosion mechanism is triggered and the grenade explodes; otherwise the grenade keeps its non-exploding state. To function this way, there was an engineer who took this functional description and implemented a physical device that conforms to it. This is the intended interpretation that relates both ways the functional description with the physical device. In the case of engineered devices, the intended interpretation is even written in a technical document, so its existence cannot be denied, I hope.

¶2 · Nevertheless, if you were a godlike subject, that is, an ideal and unintentional subject, then you would be able to interpret the grenade as a word processor to write this very paper, as Searle teaches us and Putnam allows us, because you could compute effortlessly any possible interpretation, and you would not be afraid of being killed by its explosion.

¶3 · However, if you, dear reader, are unfortunately a human subject, then you should better restrict yourself to the intended interpretation. Firstly because the computations that you would need to execute in order to interpret the grenade as a word processor are possibly non-feasible for you and surely absurdly complex, and secondly and most importantly because you could even die during the exercise.

¶4 · Therefore, in this case, the two assumptions apply in the case of godlike subjects, but not in the case of real and intentional subjects as ourselves.

¶5 · And although we would not die, we will waste too many computational resources interpreting a wall as a word processor, or a rock as a mind. The point again is that, while this would not affect a godlike subject computing effortlessly, in our case we could not use those wasted resources for calculating more relevant decisions, some possibly crucial to our life, and in any case better assigned to other tasks. For us, interpreting is expensive, or at least it is not free, and it has consequences, some perhaps deadly.

§9 Your dog

¶1 · After the wall, the rock, and the grenade, the next case will be a biological object as your dog, for example, which was engineered by evolution and lacks technical manuals. And let us name a specific neuron firing pattern happening in the brain of your dog pattern D. According to Putnam, we can interpret pattern D arbitrarily, and then, using Searle's example, pattern D could be interpreted as the function that deletes the second character of a string, let us call it function d_2. That is, if this function d_2 takes the string "good", then it returns the string "god".

¶2 · Now, suppose that we ask a cognitive scientist to investigate your dog's pattern D, that she set an experiment, and that after some thousand tests she finds that there is a perfect correlation between pattern D and your dog's heart increasing its beat rate. That is, she observes that it is only when pattern D is developing that she measures a heart beat rate increase, and it is only when she measures a heart beat increase that she observes pattern D. Her report also indicates correlations, though not perfect, between pattern D and your dog seeing your neighbor's cat, between pattern D and higher levels of adrenaline, and many others.

¶3 · Why do we understand that the intended interpretation of your dog's neuron firing pattern D cannot be function d_2? Why do we understand that the intended interpretation can instead be that the function of pattern D is to increase the beat rate of the heart of your dog? In the end, because we cannot imagine any way that calculating that function d_2 will bring more or better survival chances to a dog. And, on the other hand, increasing the beat rate of the heart will increase the oxygen supply to the muscles that is needed to run, and then we can easily imagine why computing the need of increasing the heart beat rate would have a value for survival.

§10 Life

¶1 · Every living being is intentional. The intention of every living being is to keep being alive. The previous sentence is nearly tautological because life is best defined as an intention. And it is a bit surprising that Putnam (1988), elaborating on intentions, does not make any reference to this fact.
¶2 · We are just formulating in abstract terms what is plainly known: that the computations executed by living beings are finally intended to keep that living being alive. And to live, sometimes the animal needs to run, for example, to flee from a predator. And to run, the muscles need more oxygen. And to provide more oxygen to the muscles, they have to receive more blood. And for the muscles to receive more blood, the heart has to pump more blood. And for pumping more blood, the heart has to beat more frequently. This chain of intentions links the increase of the bit rate of the heart with the final intention of every living being, which is to be alive, which is to not die.
¶3 · The intended interpretation of every living being is its final intention, to keep being alive. As this final intention is embodied in the living being, every one of its parts and organs and every one of its processes and computations get their intended interpretations. That living beings are intentional can be reformulated this way: every living being is an embodied intention.

§11 Intentions and meanings

¶1 · Furthermore, when the interpreting subject is a living being, everything is evaluated against its final intention: or it is needed to survive, or it only helps to survive, or it is indifferent for survival, or it is an inconvenience for survival, or it is a deathly danger. As result of this evaluation for survival, everything gets its intended interpretation. For example, according to Lettvin et al. (1959), a frog interprets that any dark point that moves rapidly in its field of vision is a fly which it will try to eat, so frogs interpret flies as needed to survive. Of course, this evaluation for survival does not need to be computed, but it has to be somehow embodied; green plants need sunlight to survive.
¶2 · The general idea is that *death* is the final meaning, because avoiding *death* is the final intention, and that this absolute meaning propagates. Those other meanings depend on the species and, for species that can learn, also on the individual. Although by the above explanations it could seem that everything has a meaning for every living being, this is not necessarily true. For example, the planet Saturn can be meaningless for frogs, because Saturn is inconsequential for frogs survival. If even more remote astronomical objects than Saturn have some meaning for us, it is because we accumulate meanings just in case, a strategy sometimes useful for survival, which we use.

§12 Human subject

¶1 · For any ideal and unintentional subject, any interpretation is as easy as the easiest one and there is not any intended interpretation. From the point of view of such a godlike subject, Putnam's theorem is true and "*everything has every functional organization*." However, for real and intentional subjects as ourselves, both assumptions are false: for a real subject not every interpretation is as easy as any other, and for an intentional subject there is an intended interpretation. In principle, this does not show that the theorem is false, it only means that the theorem does not apply to us.

¶2 · In any case, now that we know of two assumptions that prevent to explain the intentional realm scientifically, we can explore what would result if we negate those two assumptions. Therefore, we will assume instead that the subject is real and intentional, that is, we will assume that the subject is an embodied intention, a living subject. In particular, we will investigate the case of a human subject, because we know that Putnam's conclusions do not apply to living subjects as ourselves, and because we are the subjects doing science.

¶3 · In what follows, we will show constructively that a science of intentions is possible and that cognitive science is not doomed. As in any constructive proof, the construction presented is not necessarily unique, but only an instance.

§13 The requirement of subjectivity

¶1 · The first casualty of assuming a human subject is the requirement of objectivity, see §3. As we saw above, for living subjects everything has an intended interpretation, and for human subjects everything has a meaning. And, although the final intention of every living subject is to keep being alive, the intended interpretations and meanings that propagate from the final intention depend on the species, and for the species that can learn also on the individual. As we can learn, in our case the intended interpretations and meanings depend on the species and also on the individual. In any case, the conclusion is that, for a human subject, science has to be subjective.

¶2 · This conclusion is quite natural. If you reread *Representation and Reality* from this point of view, you will find that nearly every issue disappears. Intentions and meanings, which Putnam cannot imagine inside science, are naturally included inside science even before we start doing science, because intentions and meanings, finally rooted on life and death, are how we human subjects see everything through.

¶3 · At first, it could seem us that letting science to indulge into subjectivity is too high a price to pay. In ancient times, authority, who in medieval Europe was Aristotle tailored by the Church, was revered so much that science was stagnated. It was only when science theories were judged on how good they predicted measurements, instead of being sanctioned by the authority, that science was able to grow steadily. This is the method of science, on which Popper and Feyerabend can shed some light.

§14 The method of science

¶1 · It was Popper who gave the most precise definition of science: scientific theories have to be falsifiable by measurements. This definition requires objectivity, because if something cannot be measured, then it is out of science. And mental events cannot be measured, since they require an interpretation, so they are out of science.

¶2 · For Popper the method of science is based on trial and error. He explained it several times, but we will copy here the explanation that is included in page 3 of his 1994 book *All Life Is Problem Solving*:

> *The natural as well as the social sciences always start from* problems, *from the fact that something inspires* amazement *in us, as the Greek philosophers used to say. To solve these problems, the sciences use fundamentally the same method that common sense employs, the method of trial and error. To be more precise, it is the method of* trying out *solutions to our problem and then discarding the false ones as erroneous. This method assumes that we work with a large number of* experimental *solutions. One solution after another is put to the test and eliminated.*

¶3 · However, scientists do not always practice science following Popper's method, which is mechanical, objective, ideal. Being intentional, scientists try to explain how nature works by using whatever helps them. In *Against Method*, page 19, Feyerabend (1988) summarizes the situation: "*anything goes*". Being himself a polemicist, this is an overstatement. By exaggerating it, Feyerabend makes his point clearer: science is not objective, because science is practiced by real and intentional subjects.

¶4 · Should we then admit Feyerabend's anarchist conclusion that there is no scientific method? I do not think so. Although I will not present here a complete reformulation for the scientific method, I am sure that science has some limits. Let us take, for example, homeopathy, and let us suppose that homeopathic remedies by themselves neither cause harm nor benefit. Now take any naturally mortal disease that medicine cures. In this case, if medicine is used, then the patient will survive, and otherwise the patient will die. That is, the patient will die if homeopathy is used instead of medicine, or if neither homeopathy nor medicine is used. This would prove that medicine is science and homeopathy is not science, because medicine is effective against death while homeopathy is not.

¶5 · That we are still debating today about the scientific status of something as homeopathy that can cause death by omission shows that, in fact, Feyerabend is closer than Popper to define the method of science. Furthermore, let us make another supposition: it is found that some investigators who had published scientific papers suggesting some benefits of homeopathy are paid by a company producing homeopathic remedies. This would be considered a conflict of interest because scientists are real intentional subjects. Therefore, in the demarcation of science and its method we have to include the purpose of science, perhaps producing unbiased knowledge, and we should take into account that its agents are human, and therefore intentional.

§15 Relativism

¶1 · Subjectivism implies relativism only when the subject is ideal and unintentional. When the subject is alive, then subjectivism does not imply relativism.

¶2 · Usually subjectivism is neglected by arguing that, if we let that every subject be his own judge, then anything would be as good as anything: *anything goes*. This could be true for unintentional subjects who find anything as good as anything, but it is not true for living subjects, because for every living subject life is better than death, pleasure is better than pain, and eating is better than starving. Death is absolute for us.

¶3 · Furthermore, if a judge is in fact intentional, then it is better to be explicit on his intentions than to ignore them. It happens the same in politics, where it is not a good idea to assume that politicians will always work for their people needs, and it is much better for the society that its political system takes into account that politicians have their own personal needs, aspirations, and ambitions, which can conflict with those of their people. When there are intentional agents playing, to ignore the fact that they are intentional will distort everything. We should repeat it: the method of science has to consider that their actors are intentional.

¶4 · However, we human subjects have much more commonalities than differences, where the commonalities are given by our genome, and our differences by what each of us has learned, which depends on our culture and on our individual way. It is only because our human interests and intentions are basically the same that an extincted and long forgotten language can be reconstructed from its written record.

¶5 · Let us summarize the situation of science. There would be no science without some human beings purposely doing science. Then, the subject of science is human. So science, as any other human enterprise, is defined by its purpose, and its method is any that human beings could find to achieve that purpose. Although it has some merit, it is not completely true that *anything goes*, because we human subjects are intentional living beings sharing much more commonalities than differences.

§16 The requirement of biologism

¶1 · The second casualty of assuming a human subject is the requirement of physicalism, see §4. This, in fact, is just a consequence of the first one, because physicalism entails objectivism. Assuming a living subject requires biologism, that is, that science be reduced to biology. However, it is more than that. Today, under the physicalist requirement, biology is a branch of chemistry, which is a branch of physics. Therefore, biology itself should firstly be fully reoriented towards intentions; life is an intention, in fact it is the final intention. And only after that redefinition, should science be reduced to biology. The conclusion is that, for subjective science, science has to be based on subjective biology in order to be intentional.

¶2 · All this is easier to say than to implement. As science is currently based on the requirement of physicalism, this requirement permeates everything scientific. For example, the mathematical function is found nearly everywhere, because it models causality: as in causality, where the same cause always produces the same effect, in a function the same argument always produces the same value. Functions are used extensively in deterministic theories, as Einstein's relativity theory, and also in probabilistic theories, as quantum mechanics, where the arguments and values can be probabilities. In any case, functions lack intentionality, and then it is not possible to build a subjective science on them.

§17 Intuitionism

¶1 · Our task now will be to look for a mathematical concept that models intention. And, from the beginning, we have to avoid formalism, because mathematics is purely syntactic for formalism. That is, for formalism, mathematics is just the manipulation of strings of symbols, which are free of meaning, according to exact rules. What we are looking for is intuitionism, because for intuitionism mathematics is the result of human thought. Consequently, in intuitionism a mathematical statement can be true, or false, or unknown, which corresponds respectively to say that some mathematician has proved that the statement is true, or that someone has proved that the statement is false, or that no one has found a mathematical proof on the truth or falsity of the statement.

¶2 · Let us translate set theory, which is the foundation of mathematics, to intuitionism. There are two ways of defining a set:

- By extension (denotation): when we enumerate the elements of the set.
- By intension (connotation): when we express the precise condition that anything has to fulfill to be an element of the set.

Whenever we have a set defined by intension, a set defined by extension, and a proof that both are equal, we will say that both sets are the same set, but defined differently. For formalism and other objectivisms, a set defined by intension and a set defined by extension are either the same set or they are not the same set. However, for intuitionism and other subjectivisms, there is a third possibility: that the subject does not know whether they are the same or not. Only by allowing this third possibility can problems exist, because a problem is a state of ignorance.

¶3 · The most abstract way of seeing a problem is as the list of requirements that anything has to fulfill to be a solution to that problem. Therefore, we will say that a set defined by intension is a *problem*, and the same set but defined by extension is *its set of solutions*. And we will say that to *resolve* a problem is to calculate its set of solutions, though most times it will be enough to calculate one of its solutions to solve the problem. In any case, in subjectivism, set theory has three parts: A problem, which is a state of need, a solution, which is a state of satisfaction, and a resolution, which is a transition from uncertainty to certainty.

$$\text{Problem} \xrightarrow{\text{Resolution}} \text{Solution}$$

§18 The law of Turing

¶1 · Solutions are elements, as in set theory. Problems are predicates on the set of elements that are true for solutions and false for everything else. And resolutions, what are resolutions? Resolutions have to model the calculating capacity of the subject doing mathematics, and therefore we have to choose something that models our human calculating capacity. The answer to this seemingly impossible question was given by Turing (1936), who postulated that a problem is unresolvable if and only if there is not any Turing machine that can calculate its solution. That is, he posited that 'resolvable' is synonymous with 'calculable by a Turing machine', which is nowadays synonymous with 'computable'. Therefore, for Turing, computing is the model for the resolutions.

¶2 · In order to be formal and ceremonial, we will raise an aspect of this statement to the status of the first law of cognition under the name of his champion.

The law of Turing: computing is for solving problems.

¶3 · The law of Turing is twice intentional. The first time because it declares the intention of computing. Under objectivism, computing cannot have any intention, and then Putnam (1988) and Searle (1980 and 1992) are right when they argue that computing by itself is meaningless. However, under Turing's law, computing has an intention, which is to solve problems, and then computing is a means to an end, and consequently the meaning of any computation comes from the problem that the computation resolves, or tries to resolve.

¶4 · The law of Turing is twice intentional. The second time because the intention of computing is solving problems, and problem solving is our mathematical model for intentionality. A problem expresses a need, an intention, that is satisfied by a solution, and to find a solution to the problem we have to compute a solving resolution.

¶5 · Problem solving, being our mathematical model for intentionality, and life, being an intention, then life is the problem of survival and evolution is its resolver.

¶6 · Going back to the beginning of this section, mathematically under the law of Turing, solutions are elements, problems are predicates, and resolutions are computable functions, which are also known as (general) recursive functions.

§19 A kind of miracle

¶1 · As cited by Davis (1982), page 16, for Gödel, by a kind of miracle, computability is absolute, that is, independent of the language used to define it:

> *Tarski has stressed in his lecture (and I think justly) the great importance of the concept of general recursiveness (or Turing's computability). It seems to me that this importance is largely due to the fact that with this concept one has for the first time succeeded in giving an absolute definition of an interesting epistemological notion, i.e., one not depending on the formalism chosen. In all other cases treated previously, such as demonstrability or definability, one has been able to define them only relative to a given language, and for each individual language it is clear that the one thus obtained is not the one looked for. For the concept of computability however, although it is merely a special kind of demonstrability or decidability[,] the situation is different. By a kind of miracle it is not necessary to distinguish orders, and the diagonal procedure does not lead outside the defined notion.*

¶2 · Computing is independent of the language used to define it because, in fact, for any language to exist, a computing device has to implement the language. In other words, languages cannot exist by themselves, but every language is computed.

¶3 · That every language is computable provides us with another point of view from which to observe Putnam's difficulties with intentionality. That every language is computable implies that every language has to be implemented by a computing device. To ignore this fact, by assuming that there can be a language without a computing device implementing it, was one of Putnam's two assumptions that we are nullifying. And the other one was assuming no intentionality, also taken into account by Turing's law. Thus, the law of Turing goes straight against Putnam's assumptions preventing a cognitive science.

§20 Turing completeness

¶1 · Aside from computing absoluteness, and also by kind of miracle, computing shows another surprising property: *Turing completeness*, or completeness for short; you can find more details in Casares (TC). It happens that there are some computing devices that can imitate the calculations of any computing device, and we will say that those imitating computing devices are Turing complete. To do the imitation, part of the data entered to the Turing complete device is a description of the imitated computing device. We will call that description the *program*, which is written in a complete language. That is, a *complete language* is the language of a Turing complete device, and then we can describe any computing device in any complete language.

¶2 · The universal Turing (1936) machine is the mathematical model for Turing complete devices, and then Turing completeness is also known as *universal computing*.

¶3 · Using the software-hardware split, where hardware refers to the computing device and software refers to its data, Turing completeness can be defined as the capacity of some hardware devices to compute by software whatever hardware can compute. It will be important for what follows to note that, in the software-hardware split, reality and its objects are hardware, while language and its concepts are software.

§21 Problem theory

¶1 · Taking advantage of the law of Turing, it is easy to devise a problem theory by mixing intuitionist set theory and computing theory. The details are in Casares (PT), and now we will summarize some results that are interesting here.

¶2 · For each problem there is a corresponding set of solutions, its set of solutions. The set of problems is a Boolean algebra, and the set of sets of solutions is a Boolean algebra, too. For each problem there is a corresponding set of solutions, but there are some sets of solutions that are not the set of solutions of any problem. In other words, there are sets that can be defined by extension that cannot be defined by intension. This unexpected result, which is a reformulation of Gödel's (1930) incompleteness theorem as pointed out by Post (1944), precludes the isomorphism between the set of problems and the set of sets of solutions; they are quasi-isomorphic.

¶3 · Given a problem, to find a resolution that computes its solutions is another problem, which we call *its meta-problem*. Therefore, a solution of its meta-problem is a solving resolution of the original problem. For example, in a problem of arithmetical calculation, the solution is a number and the solving resolution is an algorithm, which is, for example, the algorithm for multiplication when 'by multiplying' is the solution to its meta-problem.

It takes 3 eggs to make a cake. How many eggs do you need to make 24 cakes?

¶4 · As the meta-problem of a problem is also a problem, the meta-problem of its meta-problem is the meta-meta-problem of the original problem, and so on. However, after mixing set theory with computing, every set is countable and "it is not necessary to distinguish orders". In particular, the set of problems, the set of sets of solutions and the set of resolutions are infinite enumerable, and meta-meta-problems, meta-meta-meta-problems and so on are all meta-problems.

¶5 · The problem theory defines three ways of resolving a problem: by routine, by trial, and by analogy.

- You resolve a problem by *routine* when you know a solution, so you only need to apply the solution to solve the problem. One element that is a solution is enough to resolve by routine.
- Resolving by *trial* is testing a set of possible solutions until finding one that solves the problem. A set of elements that can be solutions is needed to resolve by trial.
- Resolving by *analogy* is transforming a problem into another problem, the analogue problem. The analogue problem can also be transformed by analogy and so on, but when a solving resolution is known, transforming a problem into its meta-problem by analogy terminates the loop.

§22 The law of evolution

¶1 · When the three ways of resolving, which are routine, trial and analogy, are applied to problems and meta-problems we get a series of five resolvers of increasing problem solving power: mechanism, adapter, perceiver, learner, and subject.

¶2 · This series provides a framework for the evolution of cognition on the assumption that sometimes solving more problems is evolutionarily better. It is only sometimes, because design is always a trade-off between resources and achievements, independently of it is done by evolution or by a human engineer. In the case of this series, an evolution of resolvers —mechanism to adapter to perceiver to learner to subject— should follow provided that a bigger problem solving range means more survival opportunities, that in each step the increasing of complexity is compensated by its increased fitness, that software is cheaper than hardware, and that the adapter, the perceiver, the learner, and the subject conditions are satisfied in some environments. You can find more details about these conditions in Casares (PT). For us here, it is important to note that the *subject condition* is to be Turing complete,

¶3 · It is with those qualifications on the word 'sometimes', that we can express another law of cognition.

> *The law of evolution: sometimes, solving more problems is evolutionarily better.*

§23 The law of Einstein

¶1 · That software is cheaper than hardware can be eliminated from the list of provisions to the law of evolution, because it is another law of cognition.

> *The law of Einstein: software is much cheaper than hardware.*

¶2 · This law is named after Einstein (1905) because it is the computational version of the $E = mc^2$ equation of his theory of relativity. As hardware is matter seen from the point of view of computing, and software is energy seen from the same viewpoint, then software is c^2 times cheaper than hardware. You can find more details in Casares (TC).

§24 The evolution of cognition

¶1 · Using problem solving as the mathematical model for intentionality, and being life an intention, then life is the problem of survival and evolution is its resolver. Furthermore, under the provisions of the law of evolution, an evolution of resolvers should develop from mechanism to adapter, then to perceiver, next to learner, and finally to subject.

	Body	**Brain**	
one	Routine **Mechanism** $\Re_0$ $s \in \mathbb{S}$	Meta-routine **Perceiver** $\Re_2$ $f \in (\mathbb{S} \to \mathbb{S})$	element
some	Trial **Adapter** $\Re_1$ $S \in \mathbb{S} \to \mathbb{B}$	Meta-trial **Learner** $\Re_3$ $F \in (\mathbb{S} \to \mathbb{S}) \to \mathbb{B}$	set
internal	Analogy **Perceiver** $\Re_2$ $f \in \mathbb{S} \to \mathbb{S}$	Meta-analogy **Subject** $\Re_4$ $\mathfrak{f} \in (\mathbb{S} \to \mathbb{S}) \to (\mathbb{S} \to \mathbb{S})$	function
	Behavior $\mathbb{S}$	Model $\mathbb{S} \to \mathbb{S}$	

¶2 · A *mechanism* is an implementation in hardware of a single element, and then it only solves those problems for which that element is a solution. In that case, a mechanism implements a solution, and it represents the case when we know a solution to the problem, so we can apply it routinely. A living mechanism has a body implementing a single fixed behavior, so it can only survive in a specific, benign, and very stable environment, as it is the case of some extremophile archaea. Mechanisms resolve problems by routine.

¶3 · An *adapter* is an implementation in hardware of a predicate on elements, or equivalently of a set of elements. It represents a trial on a set of possible solutions. A living adapter has a body that can execute a set of behaviors, so that, in order to adapt itself to the current situation, it can choose one of them by using a trial and error procedure or a trigger condition, as a deciduous tree does. Adapters resolve problems by trial.

¶4 · A *perceiver* is an implementation in hardware of a single fixed function on elements to elements. It implements an analogy that transforms a problem into an analogue problem, which also needs a solution. When the implemented function is a solving resolution, the problem can be solved, representing the case when, though we do not know a solution, we know a way to find one. In this case, the analogue problem is the meta-problem, and knowing only a solving resolution of the problem is knowing a single and fixed solution for its meta-problem. A living perceiver has a body and a brain. The perceiver brain implements a single fixed function that translates sensations into perceptions, so the perceiver chooses its body behavior on its perceived reality, which is a fixed model of the exterior. For perceivers, reality is hardwired in their brains, but inside their brains, perceivers resolve in software, either by routine or by trial. From Lettvin et al. (1959) description, we conclude that a frog is a perceiver. Perceivers resolve problems by analogy or meta-problems by routine.

¶5 · A *learner* is an implementation in hardware of a predicate on functions from elements to elements, or equivalently of a set of functions on elements to elements. It represents a trial on a set of possible analogies. A living learner has a body that can execute a set of behaviors and a brain that implements a set of functions. Thus a learner can modify its perceived reality, which is therefore an updatable model of the exterior. When its current model fails, it selects a new one, thus learning something new about its external environment. Inside their brains, learners resolve in software by computing against current reality, which is also in software, at least in part. A dog is a living learner. Darwinian evolution is also a learning process, because it resolves and models in genetic material, which is software. The brain-body distinction in living learners corresponds to the genotype-phenotype distinction of evolution. Eventually, a theory of the evolution of evolution should explain how evolution reached its current state, and meanwhile we can already note that learning is not the first stage in problem solving. Learners resolve meta-problems by trial.

¶6 · A *subject* is an implementation in hardware of a function on functions to functions. It represents an analogy on analogies. When the subject condition is met, that function is the universal function, which is the function implemented by a universal Turing machine, and then the subject is Turing complete. A Turing complete subject can reason about any model and about any learning strategy, effectively translating to software the whole problem solving process, because it can compute in software whatever hardware can compute. A living subject has a body and a Turing complete brain. We humans are Turing complete subjects. Subjects resolve meta-problems by analogy.

§25 A joke

¶1 · This is not for your amusement, but because it will be important in the following pair of sections. We quote it from Barwise & Etchemendy (1999), pages 304–305:

> *The ambiguities become especially vexing with quantified noun phrases. Consider, for example, the following joke, taken from Saturday Night Live:*
>
> *Every minute a man is mugged in New York City.*
> *We are going to interview him tonight.*
>
> *What makes this joke possible is the ambiguity in the first sentence.*

¶2 · Ambiguity is resolved pragmatically, so for example if the replace the word ‘minute’ with the word ‘year’, then it would not be a joke, because then it would be possible though unlikely, and then news, that they were referring every time to one and the same man. We will note ‘â’ this use of the indefinite article; you can read ‘â’ as ‘a specific’. We will note ‘ã’ the other case, that is, when the reference does not point every time necessarily to one and the same referent, but each time to a possibly different element belonging to the set. So ‘â’ refers to a specific member of the set, and ‘ã’ refers to a generic member of the set, which can change each time.

§26 The law of Post

¶1 · That computing is the model for resolutions has another aspect that is not in the law of Turing: that the resolver and the subject doing mathematics is human. Turing (1936) argues that his computing model, the Turing machine, which is extremely simple and lacks any intelligence, can calculate anything that any human being can calculate. The thesis is that anything that we can calculate can also be calculated by ã Turing machine. An equivalent formulation says that anything that we can calculate can also be calculated by â universal Turing machine. In any case, we will call this thesis Turing's thesis.

¶2 · Church (1935) had previously proposed that anything that we can calculate can also be calculated by â λ-calculus interpreter, a thesis known as Church's thesis, but Gödel preferred computing, see Davis (1982), and in the end Turing (1937) proved that computing and λ-calculus are equivalent, and therefore Turing's thesis and Church's thesis are equivalent.

¶3 · This time we will be subjective, and we will raise Turing's and Church's theses to a cognitive law, the fundamental one, under the name of Post. Note that technically, as shown in Casares (CT), Church's thesis is a logical consequence of the law of Post as expressed here.

> *The law of Post: in calculating capacity, we are just Turing complete.*

¶4 · Why Post? It is easy to be sympathetic with Post, because in spite of his difficult life he anticipated Gödel and Turing, though being always second, he got no credit. Citing Stillwell (2004), page 3:

> *In the 1920s he discovered the incompleteness and unsolvability theorems that later made Gödel and Turing famous. Post missed out on the credit because he failed to publish his results soon enough, or in enough detail.*

This is nice, but again not enough. However, he indeed deserves to name the fundamental law of cognition, because Post (1936) was the first one to affirm that Church's thesis is a natural law stating the limitations of the mathematicizing power of our species *Homo sapiens*. In this he was against Church (1937) himself, for whom his thesis is just a definition, thus disqualifying Church to name this law.

¶5 · From other points of view, Turing completeness seems to be the final achievement absolutely possible in computing, but from a subjectivist point of view, completeness is just a consequence of our limited calculating power. We cannot imagine more calculating power because we have not more calculating power, so we conclude that our calculating power is the maximum calculating power. In any case, that we see completeness as the maximum calculating power is an argument in favor of the law of Post.

§27 Hardware and software

¶1 · Some people confuse Turing completeness with computing. I put part of the blame on using ‘unrestricted computing’ to refer to computing in general, as done by the influential Chomsky (1959), page 143: “A *type 0 grammar* (*language*) is one that is unrestricted.¶ Type 0 grammars are essentially Turing machines”. As Turing machines are the mathematical models for hardware devices, then ‘unrestricted computing’ is whatever hardware can compute, while ‘universal computing’ is a property of some hardware devices that can compute by software whatever hardware can compute. But the difference gets blurred if the distinction between hardware and software is ignored, and we Turing complete subjects are perhaps prone to ignore it because for us software is as capable as hardware.
¶2 · If we fail to notice the accents added over the indefinite article ‘a’ in the dual formulation of Turing’s thesis above, then ‘Turing machine’ and ‘universal Turing machine’ seem to be the same, thus confusing unrestricted computing with universal computing. So I put another part of the blame on this ambiguity of English.
¶3 · Let us see an example. Berwick & Chomsky (2016) write, in page 134:

> *Consider a Turing machine, which after all ought to be able to carry out any computation at all.*

This is ambiguous, misleading, and confusing. It is true that, for any specific computation, there is a specific Turing machine that carries it out. However, a specific Turing machine carries out one specific computation, and one is not any. In other words, the considered Turing machine is â hardware device that carries out just one specific computation. And a specific universal Turing machine can be programmed to carry out any computation at all, but not every Turing machine is a universal Turing machine, meaning that not every hardware device is universal. Part of the trick is to understand that the specific universal Turing machine is also carrying out *one* specific computation, that of emulating any specific Turing machine, which is given to the universal one as software.
¶4 · Only confusing unrestricted computing with universal computing can Berwick & Chomsky (2016) fall into the Turing machine trap, as they call it (pages 131–132):

> *It seems that insect navigation, like that of dead-reckoning ants returning with food to their nests, requires the ability to “read from” and “write to” simple tape-like memory cells. But if so, that’s all one needs for a Turing machine. So if all this is true, then ants have already climbed all the way up Nature’s ladder.*

They are implying that because ants already do computations, which could be qualified as *unrestricted* computations following Chomsky (1959), then ants are “able to carry out any computation at all”, without any restriction. The implication is wrong, because it confuses unrestricted computing with universal computing. Let us compare, for example, ants computing power with our universal computing power. By being Turing complete, we can be instructed to compute any algorithm whatever, for example the algorithm for multiplication, while ants can neither receive instructions nor be programmed in any way, but ants always compute the same fixed algorithm. Using the “ladder” presented in §24, ants are perceivers and we are Turing complete subjects.

§28 Philosophy

¶1 · After identifying life with the problem of survival, the law of Turing entails that resolving the survival problem is computing. A frog perceiving a fly is computing, a dead-reckoning ant is computing, and a human thinking and speaking is computing, too. Therefore, computability is a category, as conceived by Kant (1781, 1787), that is, a condition that any mental construct has to satisfy necessarily.

¶2 · While for Kant the categories were transcendental, for us the categories are just a consequence of the way our brain is and works, which is a consequence of its evolutionary design, and then intentional. In other words, the categories neither guide us to seek truth, nor towards ethical justice, nor to wise knowledge, but they are what they are just to keep us alive. Of course, Kant, having preceded Darwin (1859), was not in a position to say that the categories are evolutionary, but we are and we do: the categories are evolutionary. This is further elaborated in Casares (SP).

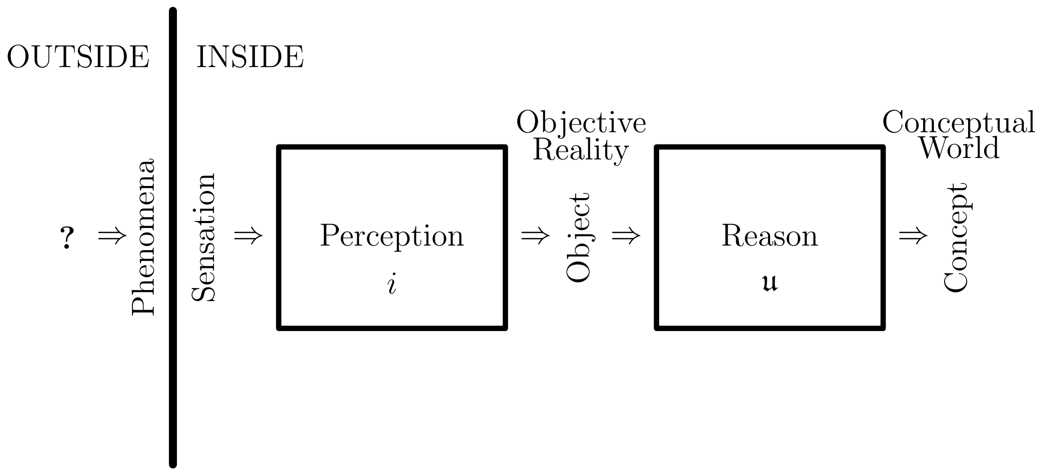
¶3 · Computability is the only category that extends all over brainy life, but there are other subcategories. For example, Turing completeness qualifies only our human conceptual machinery, which is responsible of our linguistic and reasoning abilities. According to the law of Post, we are just Turing complete. Being Turing complete means that we can think and say anything that is computable, and being just Turing complete means that anything that we can think or say has to be computable.

¶4 · Other subcategories are requirements imposed by perception, which is also computed by the brain. Here we will only mention that our percepts are objects, that is, substantive things with contingent properties. Although this should be the source of another law of cognition, perhaps the law of perception, we will not go into more details here.

¶5 · In summary, our Post-Kantian position differs from Kant in two points:

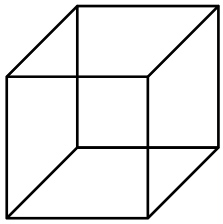
- what is the list of the categories, and
- that for us the categories are evolutionary, rather than transcendental.

However, Kant's Copernican revolution is as true now as it ever was.



§29 Reality is an illusion

¶1 · Exaggerating a bit, Kant's Copernican revolution is acknowledging that reality is an illusion. What we take for real, for example a gray stone, is not out there. According to physics, there are perhaps some strange particles and fields out there, but there is not for sure any stone out there, and therefore the gray stone we are seeing is an illusion fabricated by our brain.



¶2 · As the Necker cube illusion, the illusion of reality is bistable. Before experiencing Kant's Copernican revolution, objective reality is outside. And Kant's Copernican revolution is switching to realize that objective reality is inside. Although it was more than thirty years ago, I still remember vividly the first time I switched, the moment I underwent my Kant's Copernican revolution, the moment of my enlightenment.

¶3 · The illusion of reality is not easy to accept. Any philosopher for whom the problem of qualia refers to colors but not to things has not experienced Kant's Copernican revolution. According to the law of perception, both the thing and its properties are products of our perception, and then neither the thing (the stone) nor its properties (its color gray) are out there.

¶4 · I guess that Putnam in 1988 had not experienced Kant's Copernican revolution either, because starting at its very title, *Representation and Reality*, he ignores the subject who is representing reality.

§30 Splits

¶1 · *Representation and Reality* by Putnam (1988) is about the representation-reality split. And while we were investigating that fissure, we have found another four splits:

- The mind-body split by Descartes, where the mind is free and the body is unfree.
- The subject-object split of epistemology, where the subject is the agent observing and the object is the patient being observed.
- The software-hardware split of computing, where software is data and hardware is the physical device.
- The energy-matter split of physics, which was unified by Einstein.

¶2 · Though they are very different in origin, the partitions generated by these five splits are the same, as they divide the world in the same two parts: one which is perceptual and the other which is conceptual.

- Perceptual: reality, body, object, hardware, and matter.
  These are perceivable, that is, products of our perception.
- Conceptual: representation, mind, subject, software, and energy.
  These are not perceivable, but they are available through our language.

We will refer to these five splits without distinctions as the concept-percept split.

¶3 · Whenever it is considered that the gap caused by a split is unbridgeable, there is a dualism. The mind-body dualism caused by Descartes' split is the most famous dualism. However, for us, the concept-percept split is just a consequence of the way our brain is and works, that is, another Kantian category. And our brain is and works as it is and works because of our evolutionary history, which is reconstructed in §24:

- from mechanism to adapter: the body extends its inventory of behaviors.
- from adapter to perceiver: a simple brain models the exterior so adaptation is interiorized.

- from perceiver to learner: the brain extends its inventory of models.
- from learner to subject: the brain achieves Turing completeness allowing a fully linguistic modeling so learning is interiorized.

Therefore, the perceiver finishes the first interiorization and the Turing complete subject finishes the second. In us, the first interiorization produces percepts, the second one concepts, and both together the concept-percept split.

§31 Physics

¶1 · From our Post-Kantian point of view, the laws of nature are not ruling what happens outside, they are not out there waiting for us to be discovered. For us, the laws of nature are written by scientists, so they are linguistic concepts under the law of Post, that is, algorithms. Therefore, for physics and for any empirical science which relies on measurements, the laws of nature are the best algorithms that scientists have written till now for forecasting what we would measure should we set a scenario.

¶2 · That the laws of nature are written by scientists is what should be expected from any subjectivist science. And this is what the Copenhagen interpretation of quantum theory is saying. For example, as cited by Gregory (1988) in *Inventing Reality*, page 185, Bohr said: "It is wrong to think that the task of physics is to find out how nature is. Physics concerns only what we can *say* about nature."

¶3 · And Heisenberg (1958), chapter III in pages 18–32, who is the other founder of the Copenhagen interpretation, expounds on the subjectivism that quantum theory requires. In page 29, he writes: "This division [between the 'object' and the rest of the world] is arbitrary and historically a direct consequence of our scientific method; the use of classical concepts is finally a consequence of the general human way of thinking." He also explains how to reinterpret Kant's "a priori" under the light of modern science, pointing to biology (page 65), as we are doing here.

¶4 · In addition, the energy-matter split of physics was explained away by Einstein (1905) in his theory of relativity. While energy and matter are equivalent, they are different physical concepts because they seem completely different to us, the subjects of science.

§32 Self reference

¶1 · If for physics subjectivism is a requisite found only when reaching the subtleties of the matter, for cognitive science subjectivism is a prerequisite. It is not possible to start doing cognitive science without taking into account the subject who is doing cognitive science, because the object of cognitive science is its subject. Self reference and cognitive science are indissoluble.

¶2 · Self reference is tricky, and nearly paradoxical. The simplest paradox, which is the sentence "this sentence is false", is self referential, while its conjugate "this sentence is true" is as simple and as self referential, but it is not paradoxical; it is empty. In any case, self reference creates a loop, and whenever the loop is not finished there is a paradox. In computing, the only way to define a paradox is by expressing a loop that never *halt*s.

¶3 · Reference is one of the three intentional concepts investigated by Putnam (1988), because reference links a real object with its name, where the name is the representation of the real object in the language. So reference is the fundamental source of language meanings, and the primary link between representation and reality.

¶4 · For self reference to be possible, a linguistic concept, typically a sentence, has to refer to itself, that is, a linguistic concept has to refer to a linguistic concept. In objectivism, which keeps separated representation and reality, there is no room for self reference, because in objectivism a reference always points to a real object, and real objects are not linguistic concepts, thus preventing loops and consequently avoiding paradoxes, which are infinite loops.

¶5 · Contrariwise, in subjectivism under the law of Post, self reference is unavoidable, because every complete language is full referable, as explained in Casares (TC), which includes being self referable. Full reference means that in our complete language we can refer to any real object and to any linguistic concept. We can refer to your dog, to a specific neuron firing pattern happening in the brain of your dog, to this very sentence, to this paper, to the set of all papers that you have not read, to the set of all sets that are not members of themselves, and so on and on. And the conclusion is that, by allowing self reference, cognitive science is possible in subjectivism under the law of Post. Another not-so-nice conclusion is that paradoxes are unavoidable in subjectivism under the law of Post.

§33 Cognitive science

¶1 · With self reference making cognitive science possible, we close the circle. For Putnam (1988), intentionality cannot be scientific, thus dooming cognitive science, and our aim here was to find a way to save cognitive science. Analyzing Putnam's argument, we found that his idealization ignores the subject representing reality, by assuming a noninterfering subject who is unintentional and for whom any calculation is as easy as the easiest one. But we are the subjects doing science and we are not that way. So our plan to save cognitive science was to replace Putnam's ideal subject with a human subject.

¶2 · Although some parts of our proposal are more tentative, this subjectivist movement seems necessary: in cognitive science the object is the subject, so the subject cannot be obviated, and the subject has to be self referring. How to characterize the human subject is, of course, more tentative. In our case, in order to deal mathematically with intentions, we have devised a problem theory by incorporating computing to an intuitionist set theory.

¶3 · Computing kind-of-miraculous properties are crucial:

- Computing is absolute because, for any language to exist, a computing device has to implement the language.
- Computing can be universal, or Turing complete, because there are some computing devices that can be programmed to imitate any computing device.
- We are just Turing complete, so we can compute by software whatever hardware can compute, and our language is complete and therefore full and self referable.
- The software-hardware split of computing is readily open to inspection, and then it is easier to examine than the other splits.

¶4 · Then, we defined life as *the* intention; all other intentions derive from life. Therefore, using the concepts of the problem theory, life is the problem of survival and evolution is its resolver. Individual living beings are embodied intentions, and then also resolvers for the theory of problems. Under certain circumstances, an evolution of resolvers happens that undergoes two interiorizations, the first one producing percepts and the second one concepts, resulting in a Turing complete species, as ours.

¶5 · So, as illustrated by the figure in page 19, we are living beings who transform the external phenomena that interact with us, which are the raw sensations, into a reality of perceived objects, which are substantive things with contingent properties, and who on top of that fabricated reality can create linguistic concepts thus extending objective reality into a wider conceptual world. However, we are still living beings, embodied intentions, and therefore we are equipped with all this machinery for just one single purpose: to not die.
¶6 · It is only after this evolutionary history that has designed our human brain in the way that biology and cognitive science should explain, that some human beings begin to do science and physics, and that some other human beings begin to question the concept-percept split. This is why biology can explain physics, by explaining the subject doing physics, but physics cannot explain biology.
¶7 · Physics cannot explain biology because intention is out of physics, where nothing is good or bad. In physics, we cannot even express that death is bad. Contrariwise, each problem expresses an intention, which is to find at least one of its solutions. And then, after equating life to the problem of survival, life provides *the* intention. A consequence is that, under problem solving, a model is evaluated by itself and not by comparing it with any original: a model is good if and only if the modeling subject can use it to solve the problem that he is facing, ultimately a survival sub-problem.

§34 Putnam's solutions revisited

¶1 · Before concluding, we should revisit Putnam's (1988) solutions for the intentional concepts and check them against our subjectivist proposal. Intentional concepts, as those three that Putnam analyzes —truth, reference, and meaning—, are linguistic. Therefore, to explain them it is required to investigate language, which is a human peculiarity. And then, using our subjectivist approach, the key to explain those three concepts is to study the subject of language.
¶2 · The key insight about language was given by Tomasello (2008): a language is not useful for competition, it is only useful for collaboration. From our problematic point of view, collaboration happens when two or more individuals share a problem that they are resolving together, that is, when they share a common intention. It is only in these circumstances that communicating true information is valuable. In competition, confusing the enemy is most valuable. And basically, as Tomasello points out, the main difference between humans and apes is that we are collaborative and they are competitive.
¶3 · Under this light, Putnam's solution for truth, "truth is idealized rational acceptability" (page 115), sounds odd. We need the concept of truth because what is said can be false, for example when a seller is lying to get more money for something. So language can be used to collaborate or to compete, to help or to confuse, and then what is said in language can be useful to solve our problems or not, that is, it can be true or false. This is not the truth by Tarski, nor by anyone who ignores the subject of language, who is also the subject of logic.
¶4 · Putnam's reference is more focused: "it is a matter of *interpretation*" (page 118), which should call for an interpreter, but "interpretation is an essentially holistic matter", which is to say that reference cannot be analyzed. However, from our point of view, interpretation is a purely subjective matter. And, as we have already said above, in §15, subjective does not always mean relative.

¶5 · Putnam's solution for meaning is linked to his solution for reference (page 119):

> *[. . .]; knowing what the words in a language mean (and without knowing what they mean, one cannot say what they refer to) is a matter of grasping the way they are used. But use is holistic; [. . .].*

Again, from "meaning is use" and "use is holistic", he is saying that meaning cannot be analyzed. And again for us, meaning is a purely subjective matter, see Casares (BL). And then meaning can be analyzed by taking into account the subject of language.

¶6 · Putnam is insisting in the holistic nature of reference and meaning because, only if a concept can be analyzed, can this concept be part of a scientific theory. Therefore, if reference and meaning were holistic or non-analyzable concepts, as Putnam argues, then they could not be scientific, which is his point. And our point is that they can be analyzed, subjectively.

¶7 · Benefiting from Tomasello's key insight, we infer that language is useful only when two or more individuals are resolving a problem collaboratively together, resulting that truths, references and meanings have to be shared among the collaborating resolvers. Then, and only in this sense, can we agree with Putnam and affirm, see §2, that 'meanings are not in the head' but shared by the community of speakers. Nevertheless, this explanation is deceptive because, though shared and negotiated, the meanings are in the heads of the individuals who compose the community of speakers. However deceptive, at least we can understand Putnam's mantra in that sense.

§35 Conclusion

¶1 · For Putnam (1988), there cannot be any intentional science, thus dooming cognitive science. His argument is that intentional concepts are functional, and that functionalism cannot explain anything because "*everything has every functional organization*" (page xv), with a proof in the Appendix (pages 121–125).

¶2 · Trying to save cognitive science, we analyze Putnam's proof, finding that his proof uses an ideal interpreting subject who can compute effortlessly and who is not intentional. These two simplifying assumptions are rooted in the mind-body split by Descartes, which was the cause of the huge success of objective science, as described by Popper's method. However, this shows that Putnam's proof does not apply to us, because we are real and intentional subjects, as described by Feyerabend, so our plan to nullify Putnam's argument is to replace the ideal subject with a human subject. In summary, our plan is to update subjectivism by Kant.

¶3 · We assume that all intentions derive from life, which is the intention of not to die, or paraphrasing Shakespeare: 'to die, or not to die, that is *the* problem.' A consequence is that our reason is evolutionary as by Darwin, rather than transcendental as by Kant. In other words, our reason neither guide us to seek truth, nor towards ethical justice, nor to wise knowledge, but it is what it is just to keep us alive.

¶4 · To deal mathematically with intentions, we need intuitionism, because problems require ignorance. So, in order to model intentionality, we use a problem theory which is an intuitionist set theory where the resolving subject is a computing device, as suggested by Turing and Post. Computing kind-of-miraculous properties, thus qualified by Gödel, argue in favor of its leading rôle in cognitive science. Consequently, we propose the laws of Turing and Post as the first and the fundamental laws of cognitive science.

¶5 · We have also presented two other laws of cognition, which could explain the evolution of cognition in five main stages: mechanism, adapter, perceiver, learner, and subject. Five stages require four transitions, which are: body extension, body interiorization inside the brain, brain extension, and brain interiorization inside the brain. The first interiorization generates perceptions which are organized as an objective reality, and the second interiorization, fully enabled by Turing completeness, generates a linguistic conceptual world. This evolution would explain the concept-percept split, and its variants, as the mind-body dualism by Descartes.

¶6 · I hope that all these arguments can convince you that cognitive science is possible. Fortunately, for cognitive science to be possible, it is not necessary that every one of my proposals here is right because, though I am doing my best, sadly some will be wrong. Nevertheless, I would bet that, in the end, evolutionary subjectivism modeled as problem solving by computing will be the "different —and better— way of thinking about these philosophical issues."

¶7 · Lastly, I do not know what is the intention of intention. In the theory presented in this paper, intention is an undefined primitive concept on which other concepts are built. However, the final intention is to not die. It is certain that, for all us living beings, death is absolute. Therefore, I cannot say why there is intentionality instead of no intentionality, but I can say why there is something instead of nothing. There is something instead of nothing because we perceive things in order to survive, but I do not know why we want to survive. To me, the intention of intention is where every explanation finally breaks and fails.

§36 Appendix: Putnam's theorem

Our plan

¶1 · Putnam (1988), pages 121–125, proves the following theorem:

Every ordinary open system is a realization of every abstract finite automaton.

¶2 · An abstract finite automaton, also known as finite-state automaton, is a mathematical concept, so it does not need further specifications, see for example Mealy (1955), but an ordinary open system requires some additional physical stipulations in order to fit it into a mathematical theorem. So before the proof, Putnam explains the two Physical Principles that he needs: the Principle of Continuity and the Principle of Noncyclical Behavior. The Principle of Continuity is needed to adapt the ordinary system, which is modeled by physics as existing in an analog world, to the digital world of abstract finite automata. And Putnam assures us that the Principle of Noncyclical Behavior is true for any physical system affected by a clock, and that every open system is affected.

¶3 · We will proceed otherwise. Instead of assuming that every ordinary open system is affected by a clock, we will use a clock as system. This way we can completely ignore the physics of the theorem, including the two Physical Principles, and we can instead focus on its mathematical content. So we will restate Putnam's theorem in a simpler way by using a simple clock instead of the ordinary open system, we will prove it following his trail, and we will analyze the consequences.

Simpler theorem

¶4 · We will prove the simpler theorem:

Every simple clock is a realization of every abstract finite automaton.

Definition

¶5 · A *simple clock* is one that implements the successor function, so its starting state is 0, and the next state is the next natural number. So for any simple clock $s_0 = 0$, $s_1 = 1$, $s_2 = 2$, and in general $s_n = n$, where $n \in \mathbb{N}$.

Proof

¶6 · Any finite string of states, generated by any abstract finite automaton, can be realized by any simple clock using the following rule: assign the disjunction of the positions where it appears to each state in the string. For example, for string $ABABABA$, define $A = 0 \vee 2 \vee 4 \vee 6$ and $B = 1 \vee 3 \vee 5$. Using these definitions, the clock realizes the string. Q.E.D.?

Analysis

¶7 · As you can see, our proof has distilled the essence of Putnam's proof, avoiding any possible distraction. The analysis is now easier.

¶8 · The simple clock is perhaps the simplest clock, but the only requirement that any clock has to fulfill is that it cannot repeat any state. If, for example $s_j = s_n$, where $j \neq n$, that is, if state at time j were repeated at time n, not necessarily consecutive, then it could not realize every finite-state automaton, but only those that were at the same state at times j and n. The simple clock does not repeat any state, thus providing a unique index. In Putnam's proof, the Principle of Noncyclical Behavior grants a unique index.

¶9 · Using the simple clock, the rule used in our proof defines a partial function f that assigns a state of the finite-state automaton to the first natural numbers. Following the example, $f(0) = A$, $f(1) = B$, $f(2) = A$, $f(3) = B$, $f(4) = A$, $f(5) = B$, and $f(6) = A$. We will call this function the *interpreting function*.

¶10 · So the simple clock generates a series $0, 1, 2, 3, 4, 5, 6$, which is transformed by the interpreting function into the series $f(0), f(1), f(2), f(3), f(4), f(5), f(6)$ that realizes the string generated by the finite-state automaton $ABABABA$.

¶11 · By the simple clock definition above, we know that the next state of the simple clock will be $s_7 = 7$, but we cannot use the simple clock to foresee what will be the next state of the finite-state automaton. To make that prediction we will need to know the value of the interpreting function for number 7, that is, $f(7)$. And in general, to know the state of the finite-state automaton at any instant, we will need to know the total version of the interpreting function, that is, the value of $f(n)$ for any $n \in \mathbb{N}$.

¶12 · What indeed realizes the finite-state automaton is not the simple clock, which only provides a temporal index, but the interpreting function, which is not just a device of the proof, as it seemed to be. In other words, what the proof shows is this slightly augmented three-part theorem: *Using suitable interpreting functions, every simple clock is a realization of every abstract finite automaton.* Putnam's theorem requires the same amendment.

¶13 · Since Turing's (1937) proof, we know that realizing a function is equivalent to computing the function, where 'realizing' means 'effectively calculating' as in Church's (1935) thesis. As this is true for any function, it applies also to every interpreting function:

Realizing any interpreting function is computing it.

Comment

¶14 · In Casares (PR), you can find a discussion about a similar simplification by Chalmers (1996), and references to other three-part solutions to the problem of implementation by Blackmon (2013) and Giunti (2017).

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