Pragmatic Nonsense

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Abstract

Inspired by the early Wittgenstein's concept of nonsense (meaning that which lies beyond the limits of language), we define two different, yet complementary, types of nonsense: formal nonsense and pragmatic nonsense. The simpler notion of formal nonsense is initially defined within Tarski's semantic theory of truth; the notion of pragmatic nonsense, by its turn, is formulated within the context of the theory of pragmatic truth, also known as quasi-truth, as formalized by da Costa and his collaborators. While an expression will be considered formally nonsensical if the formal criteria required for the assignment of any truth-value (whether true, false, pragmatically true, or pragmatically false) to such sentence are not met, a (well-formed) formula will be considered pragmatically nonsensical if the pragmatic criteria (inscribed within the context of scientific practice) required for the assignment of any truth-value to such sentence are not met. Thus, in the context of the theory of pragmatic truth, any (well-formed) formula of a formal language interpreted on a simple pragmatic structure will be considered pragmatically nonsensical if the set of primary sentences of such structure is not well-built, that is, if it does not include the relevant observational data and/or theoretical results, or if it does include sentences that are inconsistent with such data.

Keywords: Semantics, Truth, Pragmatic Truth, Quasi-truth, Nonsense

1 Introduction

The distinction between sense and nonsense should be trivial in the context of formal languages, as will be clear in our definition of formal nonsense, initially formulated within the context of the semantic theory of truth of Alfred Tarski. However, as a formal language is equipped with formal tools to better represent the reality of a certain linguistic practice, such distinction may become more blurred. This occurs in the context of the theory of pragmatic truth of Newton da Costa and his collaborators, which aims at providing a formal account for the conception of truth assumed in the scientific practice. In order to clarify the distinction between sense and nonsense in that context, we define the notion of pragmatic nonsense. Notice that the pragmatic type of nonsense extends the formal type, rather than replacing it.

The early Wittgenstein's concept of nonsense appears in the *Tractatus Logico-Philosophicus*, originally published in German in 1922. We will follow its second English translation, by David Pears and Brian McGuinness [1].

The notion of formal nonsense will be initially defined within the context of Tarski's semantic theory of truth, introduced in a 1933 paper originally published in Polish, the English translation with the title *The concept of truth in formalized languages* published as a chapter of *Logics, semantics, metamathematics* [2], and clarified in *The semantic conception of truth and the foundations of semantics* [3]. However, the definitions and nomenclature we will use here follow the *Introduction to mathematical logic* [4] and the *Introduction to set theory* [5]. For the purposes of clarification, we will eventually contrast the terminology we use from the terminology found in other authors.

The notion of pragmatic nonsense, by its turn, is inscribed within the theory of pragmatic truth. The semantic-theoretical concept of pragmatic truth first appeared in *Pragmatic truth and approximation to truth* [6]. That same year, it appears in *Pragmatic Probability* [7] with the name of quasi-truth, with a formalization later developed on *The Logic of Pragmatic Truth* [8], which will be the one we will follow.

With our definition of pragmatic nonsense, we intend to clarify the pragmatic criteria necessary for the theory of pragmatic truth to achieve its goal, which is formalizing an intuitive notion of truth tacitly assumed in the scientific practice:

> [...] to accept a theory is to be committed, not to believing it to be true per se, but to holding it as if it were true, for the purposes of further elaboration, development and investigation. Thus acceptance involves belief that the theory is partially or pragmatically true only and this, we believe, corresponds to the fallibilistic attitude of scientists themselves [8, p. 617].

Therefore, the notion of formal nonsense will apply to all expressions that do not meet the formal criteria required for the assignment of any truth-value (whether true, false, pragmatically true, or pragmatically false) to them, mostly because they are not (well-formed) formulas; and the notion of pragmatic nonsense will apply to all (well-formed) formulas that, though they are not considered formally nonsensical, do not meet the pragmatic criteria for the assignment of any truth-value to them, pragmatic criteria which are inscribed within the context of scientific practice.

Thus, in Section 2, *Truth and formal nonsense*, we will define the notion of formal nonsense for formal languages interpreted on total structures; then, in Section 3, *Pragmatic truth and formal nonsense*, we will define the notion of formal nonsense for formal languages interpreted on simple pragmatic structures; and finally, in Section 4, *Pragmatic nonsense*, we will the define the notion of pragmatic nonsense for formal languages interpreted on simple pragmatic structures.

2 Truth and formal nonsense

In order to define nonsense in the context of formal languages, we must first assume a definition of formal language or, more specifically, of a first-order language (see [4, p. 48-49]), a definition of sentence of a first-order language (see [4, p. 50]), and a definition of interpretation of a first-order language (see [4, p. 49]). Since Mendelson [4] defines an interpretation as consisting of a domain and assignments of relations to predicate letters, operations to function letters, and elements to individual constants, we must assume definitions of relation and operation or, in this case, function (see [5, p. 19-29]).

Assuming a definition of structure as a domain and a family of relations, we may say that a formal language is interpreted on a structure. Notice that Tarski and Vaught [9] refer to the concept of structure with the term "relational system" or just "system"; in Hodges [10], by its turn, what we simply call structure is called "relational structure". Notice also that what Mendelson [4] calls "interpretation" has the name of "structure" in Enderton [11], in which the term "interpretation" has an entirely different meaning.

Then, we assume a definition of satisfaction (see [4, p. 51-52]) and a definition of truth of a formula in a first-order language (see [4, p. 52]).

None of the definitions listed above will be exposed here for they can be easily found, not only in the literature we refer to, but on many other basic sources.

Once a formal language is properly built and interpreted on a structure, it is possible to determine which sequences of symbols of such language, or expressions, have a determinate meaning (the closed formulas, or sentences, which are necessarily true or false), which expressions have an indeterminate meaning (the open formulas, which may come to be true or false), and which expressions have no meaning at all (for they are not considered formulas). A meaningless expression, therefore excluded from the well-formed formulas of a formal language, may be called nonsensical, if we use the term in the general sense that the early Wittgenstein gave it on the *Tractatus Logico-Philosophicus*, as that which lies beyond the limits of language and has no possible truth-value [1]; though we will not use the concept with the exact meaning it has in the *Tractatus* (otherwise, this whole paper would be considered nonsense).

An example of a nonsensical expression would be "Socrates is identical", or, in a first-order formal language with equality, '$a =$'. "The reason why 'Socrates is identical' means nothing is that there is no property called 'identical'" [1, §5.473]. Of course, Wittgenstein adds, "The proposition is nonsensical because we have failed to make an arbitrary determination, and not because the symbol, in itself, would be illegitimate" [1, §5.473], that is, when building our formal language, we defined the symbol '=' as a substitute for the symbol 'P_1^2', meaning thus a two-place predicate of the form 'P_1^2 (a, a)' or '$a = a$', not a one-place predicate, or property.

Following Wittgenstein's conception of nonsense, in the context of Tarski's semantic theory of truth, the notion of formal nonsense would be simply defined as an expression which fails to be a (well-formed) formula.

Definition 2.1 (**Formal nonsense for formal languages interpreted on total structures**).

Let L be a formal language. An expression 'α' of L is *formally nonsensical* if, and only if, 'α' is not a (well-formed) formula. That is, given an interpretation I on a structure E with a domain D according to which L is interpreted, 'α' cannot be interpreted.

This definition is rather trivial, and will remain quite simple when we extend it in the next section, after defining pragmatic truth (see Definition 3.6). However, when we enter the domain of the theory of pragmatic truth, it will become evident that a stronger notion of nonsense, which we will call pragmatic nonsense, will be necessary in order to prevent a weakening of pragmatic truth.

3 Pragmatic truth and formal nonsense

As is the case with the Tarskian definition of true sentence, the definition of pragmatically true sentence only applies to sentences of formalized languages. Thus, the definition of formal language assumed on the previous section will be maintained, as well as every other previously assumed definition, with the exception of the definitions of relation and structure, which must be replaced with the definitions of partial relation and partial structure, with no loss, since a total relation is a particular case of partial relation and a total structure is a particular case of partial structure. Those definitions will be exposed here for clarity (a different definition of partial structure may be found in [6]).

Definition 3.1 (**Partial relation**).

Let D be a non-empty set. An *n-ary partial relation* R^n on D is an ordered triple $\langle R_1,\, R_2,\, R_3\rangle$, in which $R_i \cap R_j = \emptyset$, for $i \neq j$, i, $j \in \{1, 2, 3\}$, and $R_1 \cup R_2 \cup R_3 = D_n$, such that:

i) R_1 is the set of the ordered n-tuples of elements of D that we know that belong to R^n;

ii) R_2 is the set of the ordered n-tuples of elements of D that we know that do not belong to R^n;

iii) R_3 is the set of the ordered n-tuples of elements of D that we do not know if they belong to R^n or not.

Definition 3.2 (**Partial structure**).

A *partial structure* E is an ordered pair $\langle D,\, R_i^j\rangle_{i\in I,\, j\in J}$, such that:

i) D is a non-empty set, the domain of the structure;

ii) $(R_i^j)_{i\in I,\, j\in J}$, such that $J = \{1,\, 2,\, ...,\, n\}$ and $i \in I$ is a family of j-ary R_i^j partial relations on D.

Remark 3.1 (**Total relation and total structure**).

If, for a given partial relation R^n, $R_3 = \emptyset$, then R^n is a total n-ary relation, or just a relation in the usual sense, which may be identified with R_1.

If, for every partial relation of a given partial structure E, $R_3 = \emptyset$, then E is a total structure, or just a structure in the usual sense.

Having defined the notions of partial relation and partial structure, in order to define pragmatic truth we must then define the notions of simple pragmatic structure and $\mathcal{A}$-normal structure. Such definitions, which can be found in [8], will be exposed here since the reader may not be familiar with the theory of pragmatic truth (also known as quasi-truth).

Definition 3.3 (**Simple pragmatic structure**).

A *simple pragmatic structure* $\mathcal{A}$ is an ordered triple $\langle D,\, R_i^j,\, \mathcal{P}\rangle_{i\in I,\, j\in J}$, such that:

i) D is a non-empty set, the domain of the structure;

ii) $(R_i^j)_{i\in I,\, j\in J}$, such that $J = \{1, 2, ..., n\}$ and $i \in I$ is a family of j-ary R_i^j partial relations on D;

iii) $\mathcal{P}$ is a possibly empty set of sentences of a formal language L interpreted by the interpretation I on $\mathcal{A}$, the set of primary sentences of $\mathcal{A}$.

Definition 3.4 (**$\mathcal{A}$-normal structure**).

Let $\mathcal{A}$ be a simple pragmatic structure such that $\mathcal{A} = \langle D,\, R_i^j,\, \mathcal{P}\rangle_{i\in I,\, j\in J}$, $\mathcal{B}$ a total structure such that $\mathcal{B}$ $\langle D,\, R_k^j,\, \mathcal{P}\rangle_{k\in K,\, j\in J}$, and L a formal language interpreted by the interpretation I on $\mathcal{A}$ and on $\mathcal{B}$. $\mathcal{B}$ is an $\mathcal{A}$-*normal structure* if, and only if:

i) The domains D of $\mathcal{A}$ and D of $\mathcal{B}$ are the same;

ii) If 'a_i' is an individual constant of L, then 'a_i' is interpreted according to the interpretation I by the same element e_i from the domain D of $\mathcal{A}$ and from the domain D of $\mathcal{B}$;

iii) For each partial relation $R_i^j = \langle R_1,\, R_2,\, R_3\rangle$ of $\mathcal{A}$, there is a corresponding total relation $R_k^j = \langle R_1\text{'},\, R_2\text{'},\, R_3\text{'}\rangle$ of $\mathcal{B}$ that is an expansion of R_i^j (that is, in which the ordered n-tuples that in R_i^j belonged to R_3 in R_k^j either belong to R_1' or R_2', and thus R_3' is empty);

iv) If 'α' is a sentence of L that belongs to $\mathcal{P}$, then 'α' is true according to the interpretation I of L in $\mathcal{B}$.

Remark 3.2 (**The existence of $\mathcal{A}$-normal structures**).

The necessary and sufficient conditions for the existence of at least one $\mathcal{A}$-normal structure, given a simple pragmatic structure, can be found in [6].

With the above definitions, one is now able to set the conditions in which a sentence becomes pragmatically true and those in which a sentence becomes pragmatically false.

Definition 3.5 (**Pragmatic truth and pragmatic falsehood of formulas**).

Let $\mathcal{A}$ be a simple pragmatic structure, $\mathcal{B}$ an $\mathcal{A}$-normal structure, L a formal language interpreted by the interpretation I on $\mathcal{A}$ and on $\mathcal{B}$, and 'α' a formula of L.

A formula 'α' is *pragmatically true* (or *quasi-true*) according to the interpretation I in a simple pragmatic structure $\mathcal{A}$ if, and only if, 'α' is true according to the interpretation I in an $\mathcal{A}$-normal structure $\mathcal{B}_i$ (that is, if, and only if, there exists at least one $\mathcal{A}$-normal structure $\mathcal{B}_i$ in which 'α' is true).

A formula 'α' is *pragmatically false* (or *quasi-false*) according to the interpretation I in a simple pragmatic structure $\mathcal{A}$ if, and only if, 'α' is not pragmatically true according to the interpretation I in $\mathcal{A}$ (that is, if, and only if, there does not exist even one $\mathcal{A}$-normal structure $\mathcal{B}_i$ in which 'α' is true).

Let us see an example, in which a family of four (a father, a mother, and two sons) and some of their relationships are described as a partial structure, and then as a simple pragmatic structure and an $\mathcal{A}$-normal structure, which will allow us to state pragmatic truths about that family.

Example 3.1 (**Partial structure of a family F**).

The partial structure F is an ordered pair $\langle D_F,\, R_i^2\rangle_{\ i=\{1,2\}}$, with $R_i^2 = \{R_1^2,\, R_2^2\}$, such that:

i) $D_F = \{$Joseph, Mary, John, Peter$\}$;

ii) $R_1^2 = \langle R_1,\, R_2,\, R_3\rangle$, such that: $R_1 = \langle$Joseph, Mary$\rangle$, $\langle$Mary, Joseph$\rangle$; $R_2 = \langle$Joseph, Joseph$\rangle$, $\langle$Joseph, John$\rangle$, $\langle$Joseph, Peter$\rangle$, $\langle$Mary, Mary$\rangle$, $\langle$Mary, John$\rangle$, $\langle$Mary, Peter$\rangle$, $\langle$John, Joseph$\rangle$, $\langle$John, Mary$\rangle$, $\langle$John, John$\rangle$, $\langle$John, Peter$\rangle$, $\langle$Peter, Joseph$\rangle$, $\langle$Peter, Mary$\rangle$, $\langle$Peter, John$\rangle$, $\langle$Peter, Peter$\rangle$; and $R_3 = \emptyset$;

iii) $R_2^2 = \langle R_1,\, R_2,\, R_3\rangle$, such that: $R_1 = \langle$Joseph, John$\rangle$; $R_2 = \langle$Joseph, Joseph$\rangle$, $\langle$Joseph, Mary$\rangle$, $\langle$Mary, Joseph$\rangle$, $\langle$Mary, Mary$\rangle$, $\langle$Mary, John$\rangle$, $\langle$Mary, Peter$\rangle$, $\langle$John, Joseph$\rangle$, $\langle$John, Mary$\rangle$, $\langle$John, John$\rangle$, $\langle$John, Peter$\rangle$, $\langle$Peter, Joseph$\rangle$, $\langle$Peter, Mary$\rangle$, $\langle$Peter, John$\rangle$, $\langle$Peter, Peter$\rangle$; and $R_3 = \langle$Joseph, Peter$\rangle$.

So, we know that Joseph is married to Mary, and that is represented by the relation R_1^2 (which, since R_3 is empty, is a total relation); and we also know that Joseph is the biological father of John, but we do not know if Joseph is the biological father of Peter or not, and that is represented by the relation R_2^2 (which is, thus, a partial relation).

Consider, in the following example, built upon Example 3.1, that a DNA paternity test was made in order to answer that question, and that both the result of the test and the implication of such result were added to the set of primary sentences of a simple pragmatic structure.

Example 3.2 (**Simple pragmatic structure $\mathcal{G}$**).

The simple pragmatic structure $\mathcal{G}$ is an ordered triple $\langle D_{\mathcal{G}},\, R_i^j,\, \mathcal{P}\rangle_{i\in I,\, j\in J}$, such that:

i) $D_{\mathcal{G}}$ is a non-empty set, the domain of the structure (which extends the domain D_F of the partial structure F, as seen in the previous example, in order to include the domain of a formalization of genetics);

ii) $(R_i^j)_{i\in I,\, j\in J}$, such that $J = \{1, 2, ..., n\}$ and $i \in I$ is a family of j-ary R_i^j partial relations on $D_{\mathcal{G}}$ (which extends the family of relations R_i^2 of the partial structure

F, as seen in the previous example, in order to include the family of relations of a formalization of genetics);

iii) $\mathcal{P}_\mathcal{G}$ is a set of sentences of a formal language $L_\mathcal{G}$ interpreted by the interpretation $I_\mathcal{G}$ on $\mathcal{G}$, such that $\mathcal{P}_\mathcal{G} = \{$'$\alpha$', '$\alpha \to \beta$'$\}$ ('α' may be translated, in the metalanguage $M_\mathcal{G}$ of $L_\mathcal{G}$, as 'Joseph has at least 99,99% of chance of being the biological father of Peter'; 'β' may be translated, in the metalanguage $M_\mathcal{G}$ of $L_\mathcal{G}$, as 'Joseph is the biological father of Peter'; and, therefore, '$\alpha \to \beta$' may be translated, in the metalanguage $M_\mathcal{G}$ of $L_\mathcal{G}$, as 'If Joseph has at least 99,99% of chance of being the biological father of Peter, then Joseph is the biological father of Peter').

Now, consider that the simple pragmatic structure in question was extended to a total structure, forming an $\mathcal{A}$-normal structure.

Example 3.3 ($\mathcal{A}$**-normal structure** $\mathcal{H}$)**.**

The is $\mathcal{A}$-normal structure $\mathcal{H}$ is a total structure such that:

i) The domains $D_\mathcal{G}$ of $\mathcal{G}$ and $D_\mathcal{G}$ of $\mathcal{H}$ are the same;

ii) If 'a_i' is an individual constant of $L_\mathcal{G}$, then 'a_i' is interpreted according to the interpretation $I_\mathcal{G}$ by the same element e_i from the domain $D_\mathcal{G}$ of $\mathcal{G}$ and from the domain $D_\mathcal{G}$ of $\mathcal{H}$;

iii) For each partial relation R_i^j of $\mathcal{G}$, there is a corresponding total relation R_k^j of $\mathcal{H}$ that is an expansion of R_i^j; in this case, the total relation R_2^2' $= \langle R_1$', R_2', R_3'$\rangle$ of $\mathcal{H}$ is an expansion of the partial relation $R_2^2 = \langle R_1,\ R_2,\ R_3\rangle$ of $\mathcal{G}$, such that R_1' $=$ $\langle$Joseph, John$\rangle$, $\langle$Joseph, Peter$\rangle$;

iv) The sentences 'α' and '$\alpha \to \beta$' of $L_\mathcal{G}$ belonging to $\mathcal{P}_\mathcal{G}$ are true according to the interpretation $I_\mathcal{G}$ of $L_\mathcal{G}$ in $\mathcal{H}$.

Equipped with the partial structure F, the simple pragmatic structure $\mathcal{G}$, and the $\mathcal{A}$-normal structure $\mathcal{H}$ (and, of course, with a formal language $L_\mathcal{G}$ interpreted by an interpretation $I_\mathcal{G}$ on F, $\mathcal{G}$ and $\mathcal{H}$), one can assert, for instance, that the sentence 'P_2^2 ($Joseph$, $Peter$)' or, in a metalanguage $M_\mathcal{G}$, 'Joseph is the biological father of Peter', is pragmatically true in the simple pragmatic structure $\mathcal{G}$, since it is true in the $\mathcal{A}$-normal structure $\mathcal{H}$. That is possible even though one does not know if that sentence is true or false in the partial structure F, since the sentence is consistent with the primary sentences of $\mathcal{P}_\mathcal{G}$. In this example, it is not only consistent with but also a logical consequence of the sentences of $\mathcal{P}_\mathcal{G}$, but that is due to the simplicity of the example and must not always be the case; also, in this example, only one $\mathcal{A}$-normal structure is possible out of the two possible expansions of the simple pragmatic structure, but that is also due to the simplicity of the example.

Before we proceed to the definition of pragmatic nonsense, we must first elucidate the role of the set of primary sentences $\mathcal{P}$ of a simple pragmatic structure. The sentences of $\mathcal{P}$ constitute an empirical and/or theoretical framework which provides a foundation for pragmatic truth. Such primary sentences "[...] are based on observation and experience [...] or were instituted by previous investigations [...]" [6, p. 204]) and "[...] in general include both [...] true decidable sentences, such as observation statements, and certain general propositions encompassing laws already assumed to be true" [12, p. 162]. They may also constitute abductive hypotheses, and may be either true in the Tarskian sense, pragmatically true themselves, or "accepted" as true.

The sentences of $\mathcal{P}$ have, at the same time, the function of limiting the possible $\mathcal{A}$-normal structures a simple pragmatic structure may be expanded to, for those expansions that are inconsistent with the sentences of $\mathcal{P}$ are not considered $\mathcal{A}$-normal and thus disregarded.

For example, suppose the traditional set $\mathcal{S}$ of axioms of Peano Arithmetics (PA) is consistent, $L_{\mathcal{G}}$ is the first-order language of PA, and $\mathcal{G}$ is a partial structure with the same signature [10] of the standard model of PA plus an additional dyadic relational predicate $Pf(x,y)$ in which x is the Gödel number of a proof of the sentence y. The first component R_1 of the partial relation $Pf = \langle R_1, R_2, R_3\rangle$ is composed of all, and only, atomic sentences true in the standard model; and its third component R_3 is composed of all the true atomic sentences that are true only in non-standard models of PA. Thus, notice that, except for the relational symbol Pf, $\mathcal{G}$ is isomorphic to the standard model of PA. If one adds the sentence 'CON($\mathcal{S}$)' to the set $\mathcal{P}$, where CON($\mathcal{S}$) is the sentence that states the consistency of $\mathcal{S}$, then there is no total structure expanding $\mathcal{G}$ in which an atomic sentence from R_3 is added to R_1.

Therefore, even though we may not know exactly everything that is and is not the case in a given partial structure, we may know what may or may not be the case given certain data previously obtained, certain theoretical results previously accepted, or simply some investigation hypotheses.

However, from the formal point of view, $\mathcal{P}$ may be empty; in principle, there are also no restrictions or impositions for the inclusion of sentences in $\mathcal{P}$, except that they must be sentences of a formal language L interpreted by an interpretation I on the referred simple pragmatic structure. With the introduction of the notion of pragmatic nonsense, the present article aims at introducing pragmatic criteria capable of clarifying the inclusion of sentences in $\mathcal{P}$.

Before we move on to the notion of pragmatic nonsense, though, we must update our definition of formal nonsense one last time. For even with the introduction of the tools of the theory of pragmatic truth, there remains a case in which a (well-formed) formula will be considered formally nonsensical: when the existence of at least one totalizing expansion of the simple pragmatic structure in question, that is, of an $\mathcal{A}$-normal structure, is not decidable.

Definition 3.6 (**Formal nonsense for formal languages interpreted on simple pragmatic structures**).

Let L be a formal language interpreted according to an interpretation I on a partial structure E with a domain D, and on a simple pragmatic structure $\mathcal{A}$ built upon E. An expression 'α' of L is *formally nonsensical* if, and only if: either 'α' is not a (well-formed) formula; or, in case 'α' is a (well-formed) formula, one cannot determine the existence of at least one $\mathcal{A}$-normal structure $\mathcal{B}$ built upon $\mathcal{A}$.

4 Pragmatic Nonsense

Consider, for instance, the simple pragmatic structure $\mathcal{G}$ of our Example 3.2, but with a different set of primary sentences $\mathcal{P}_{\mathcal{G}} = \{$'$\alpha \rightarrow \beta$'$\}$, '$\alpha \rightarrow \beta$' meaning in the metalanguage 'If Joseph has at least 99,99% of chance of being the biological father of Peter, then Joseph is the biological father of Peter'. Let us call such simple pragmatic structure $\mathcal{G}$'. $\mathcal{G}$' may be expanded to two possible total structures: $\mathcal{H}$', in which the partial relation R_2^2 is expanded to the total relation R_2^2' so that the ordered pair $\langle$Joseph, Peter$\rangle$ is in the relation; and $\mathcal{H}$", in which the relation is expanded so that the ordered pair $\langle$Joseph, Peter$\rangle$ is not in the relation. Both total structures are consistent with $\mathcal{P}_{\mathcal{G}}$ and may, therefore, be considered $\mathcal{A}$-normal structures.

Consider now that the DNA paternity test was made and its result was positive, that is, that the sentence 'α' or 'Joseph has at least 99,99% of chance of being the biological father of Peter' could have been included in $\mathcal{P}_{\mathcal{G}}$, but it was not included. Thus, one may say that the sentence 'P_2^2 $(Joseph, Peter)$' or 'Joseph is the biological father of Peter' is pragmatically true in the simple pragmatic structure $\mathcal{G}$', since it is true in the $\mathcal{A}$-normal structure $\mathcal{H}$', and that the sentence '$\neg P_2^2$ $(Joseph, Peter)$' or 'Joseph is not the biological father of Peter' is also pragmatically true in the simple pragmatic structure $\mathcal{G}$', since it is true in the $\mathcal{A}$-normal structure $\mathcal{H}$". That is, once disregarded the observational datum constituted by the result of the DNA test, which is available, relevant and assumedly true, both hypotheses are formally admissible; nonetheless, from the pragmatic point of view, only one hypothesis is admissible: the one which takes into account the result of the DNA test.

Consider then the simple pragmatic structure $\mathcal{G}$ of our Example 3.2, but with a different set of primary sentences $\mathcal{P}_{\mathcal{G}} = \{$'$\neg\alpha$'$\}$, '$\neg\alpha$' meaning in the metalanguage 'Joseph is not the biological father of Peter', which is the verdict of a fortune teller. In this scenario, the paternity test was not made. Let us call such simple pragmatic structure $\mathcal{G}$". As is the case with $\mathcal{G}$, only one of the possible total expansions of $\mathcal{G}$" may be considered an $\mathcal{A}$-normal structure: $\mathcal{H}$"', in which the partial relation R_2^2 is expanded to the total relation R_2^2' so that the ordered pair $\langle$Joseph, Peter$\rangle$ is not in the relation. Thus, one may say that the sentence '$\neg P_2^2$ $(Joseph, Peter)$' or 'Joseph is not the biological father of Peter' is pragmatically true in the simple pragmatic structure $\mathcal{G}$", since it is true in the $\mathcal{A}$-normal structure $\mathcal{H}$"'. That is, once regarded such "testimonial" datum, which should, pragmatically, be considered false, but was assumed to be true, only one hypothesis is formally admissible; but, from a pragmatic point of view, the admissibility of the contradictory hypothesis, 'P_2^2 $(Joseph, Peter)$', should not be excluded, for the verdict of a fortune teller does not constitute scientific evidence.

Therefore, it seems necessary that, considering the aforementioned role of the set of primary sentences $\mathcal{P}$, depending on the domain of investigation represented by a partial structure, certain restrictions or impositions are made to the inclusion of sentences in $\mathcal{P}$. That is, $\mathcal{P}$ should include the observational data and/or the theoretical results, whether true or pragmatically true, that are considered relevant and are available; and $\mathcal{P}$ should not include sentences arbitrarily assumed as true if those are inconsistent with the observational data or the theoretical results, whether true or quasi-true, that are considered relevant and are available.

Of course, a central notion here is that of relevance: what is considered relevant or not depends not only on the subject of the investigation, but also on its context and objectives. Using the terminology introduced by Kuhn in *The Structure of Scientific Revolutions* [13], the set of primary sentences $\mathcal{P}$ is not the same for an investigation carried out in a context of normal science and for a research that aims at investigating a completely new hypothesis that, if confirmed, could question the current paradigm.

For instance, a research aimed at making observations that "[...] can be compared directly with predictions from the paradigm theory" [13, p. 26] would consider relevant the theoretical results in question. One example would be the first direct observation of gravitational waves (see [14]), which proved predictions made by Einstein one hundred years before the observation, within the context of the theory of general relativity.

On the other hand, a researcher may consider relevant the observational data of an anomalous phenomenon, like when Röntgen "[. . .] interrupted a normal investigation of cathode rays because he had noticed that a barium platino-cyanide screen at some distance from his shielded apparatus glowed when the discharge was in process" [13, p. 57]; that observation later led, serendipitously, to the discovery of X-rays.

If the set of primary sentences $\mathcal{P}$ added to a partial structure is built according to those general rules, the sentences about such simple pragmatic structure may be considered pragmatically true (or pragmatically false); otherwise, it seems to us that those sentences should not be considered neither pragmatically true, nor pragmatically false, but nonsensical, since the tools used to determine their pragmatic truth or falsehood, in view of the partial character of the structure and its relations, would not constitute a reliable foundation for pragmatic truth.

Such restrictions would not allow the construction of the simple pragmatic structures $\mathcal{G}$' and $\mathcal{G}$" of our examples (and, consequently, the construction of the $\mathcal{A}$-normal structures $\mathcal{H}$', $\mathcal{H}$" e $\mathcal{H}$"'), or, once they were built, would not allow one to consider the sentences about $\mathcal{G}$' and $\mathcal{G}$" pragmatically true.

Thus, before defining the notion of pragmatic nonsense, we must first define when a sentence must be included in set of primary sentences $\mathcal{P}$ (and consequently when a sentence must be excluded from $\mathcal{P}$), and when the set of primary sentences $\mathcal{P}$ is well-built (and consequently when $\mathcal{P}$ is not well-built).

Definition 4.1 (**Necessary inclusion and necessary exclusion of sentences to a set of primary sentences** $\mathcal{P}$)**.**

Let L be a formal language interpreted according to an interpretation I on a partial structure E with a domain D, and on a simple pragmatic structure $\mathcal{A}$ built upon E. Let $\mathcal{P}$ be the set of primary sentences of $\mathcal{A}$.

A sentence 'α' of L must be *necessarily included* in $\mathcal{P}$ if, and only if, it is a true, pragmatically true, or assumedly true sentence relevant to $\mathcal{A}$.

A sentence 'α' must be *necessarily excluded* from $\mathcal{P}$ if, and only if: either 'α' is inconsistent (that is, contradictory) with a necessarily included sentence of $\mathcal{P}$; or 'α' is formally nonsensical (according to Definition 3.6).

Definition 4.2 (**Well-built and not well-built sets of primary sentences** $\mathcal{P}$)**.**

Let L be a formal language interpreted according to an interpretation I on a partial structure E with a domain D, and on a simple pragmatic structure $\mathcal{A}$ built upon E. Let $\mathcal{P}$ be the set of primary sentences of $\mathcal{A}$.

A set of primary sentences $\mathcal{P}$ is *well-built* if, and only if, it includes every necessarily included sentence and does not include any necessarily excluded sentence.

A set of primary sentences $\mathcal{P}$ is *not well-built* if, and only if, it does not include any necessarily included sentence or it includes any necessarily excluded sentence.

Hence, we define the notion of pragmatic nonsense as follows.

Definition 4.3 (**Pragmatic nonsense for formal languages interpreted on simple pragmatic structures**)**.**

Let L be a formal language interpreted according to an interpretation I on a partial structure E with a domain D, and on a simple pragmatic structure $\mathcal{A}$ built upon E. Let $\mathcal{P}$ be the set of primary sentences of $\mathcal{A}$.

A sentence 'α' of L may be pragmatically true or pragmatically false according to the interpretation I on $\mathcal{A}$ if, and only if, $\mathcal{P}$ is well-built.

A sentence 'α' of L is *pragmatically nonsensical* according to the interpretation I on $\mathcal{A}$ if, and only if: either $\mathcal{P}$ is not well-built; or, in case $\mathcal{P}$ is well-built, by adding 'α' to $\mathcal{P}$, $\mathcal{P}$ becomes not well-built (that is, 'α' must be necessarily excluded from $\mathcal{P}$).

That is, both the sentence that, when added to a well-built set of primary sentences $\mathcal{P}$, makes it not well-built, and every sentence of a formal language interpreted on a simple pragmatic structure whose set of primary sentences $\mathcal{P}$ is not well-built, are considered pragmatically nonsensical.

5 Conclusion

When constructing a formal language, the logician usually does not care about the case in which the rules are not followed. For the work of the logician, the sequences of symbols that are not formulas are not relevant. Thus, defining nonsense as those expressions that are devoid of meaning may seem unnecessary from a formal point of view, though certainly not from a philosophical one.

It seems that, in that same spirit, the theory of pragmatic truth, as developed by da Costa and his collaborators, takes for granted the best scientific practices, as it expects the relevant empirical and theoretical data to be added to a simple pragmatic structure's set of primary sentences $\mathcal{P}$, disregarding the misuse of the tools it provides as irrelevant. However, we believe that, even though there can be no formal criteria for the inclusion or exclusion of sentences to $\mathcal{P}$, there may be clearer pragmatic criteria, which we have stated in our definition of pragmatic nonsense.

Our definition of pragmatic nonsense for formal languages interpreted on simple pragmatic structures (that is, Definition 4.3), thus, is an extension of our definition of formal nonsense for formal languages interpreted on simple pragmatic structures (that is, Definition 3.6); which, by its turn, is an extension of our definition of formal nonsense for formal languages interpreted on total structures (that is, Definition 2.1). Therefore, the notion of pragmatic nonsense extends the notion of formal nonsense, which we could call a Tarskian notion of nonsense, the same way as the theory of pragmatic truth extends the Tarskian theory of truth. It also allows us to strengthen the theory of pragmatic truth, by making it clear that it is not the case that any sentence about any partial structure can be considered pragmatically true simply with the addition of an empty or completely arbitrary or irrelevant set of primary sentences to such structure.

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