Logical Refutation of the EPR Argument

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Abstract

On the grounds that the Einstein-Podolsky-Rosen argument is an example of reasoning by *reductio ad absurdum*, and that a counterexample is unacceptable, unless all its elements meet all the necessary conditions, its conclusions are invalidated. The arguments in this paper are strictly logical. Einstein, Podolsky and Rosen made a mathematical assumption that is incompatible with quantum mechanics.

Résumé

Parce que l'argument d'Einstein-Podolsky-Rosen est un exemple du raisonnement par *reductio ad absurdum*, et qu'un contre-exemple est inacceptable, à moins que tous ses elements rencontrent toutes les conditions nécessaires, ses conclusions sont infirmées. Les arguments en cet article par sont strictement logiques. Einstein, Podolsky et Rosen ont fait une prétention mathématique qui n'est pas compatible avec la mécanique quantique.

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In the second section of their celebrated paper [1], Einstein, Podolsky, and Rosen, consider a system made of two subsystems I and II. They attempt to prove that the entire system can be prepared in such state that when the two subsystems are not supposed to interact, "as a consequence of different measurements performed upon the first system, the second system can be left in states with different wave functions" that can be the eigen-functions of non-commuting operators.

They suppose that the subsystems I and II have been interacting from $t = 0$ to $t = T$ and no longer after that. To represent the state of the complete system at a time $t_f > T$, they consider an expansion of the form:

$$\Psi(x_1, x_2) = \sum_{n=0}^{\infty} \psi_n(x_2) u_n(x_1) \tag{1}$$

Where "$\psi_n(x_2)$ are to be regarded merely as the coefficients of expansion of Ψ into a series of orthogonal functions $u_n(x_1)$" that are the eigen-functions of an operator $\hat{A}$, with eigen-values $a_1, a_2, \cdots$. (We have introduced the symbol t_f instead of t, used by Einstein *et al*, because we will use t as a variable in Schrödinger's equation.)

A measurement of the physical quantity corresponding to $\hat{A}$ is performed at time t_f, giving a result of a_k. In consequence, subsystem II is left in a state described by the wave function $\psi_k(x_2)$ and the whole system is described by the wave function

$$\Psi(x_1, x_2) = \psi_k(x_2) u_k(x_1) \tag{2}$$

Therefore, the state of subsystem II at time t_f depends on the state of the physical quantity of subsystem I that is chosen to be measured at that time. (Because we can select any other physical property of subsystem I to measure at time t_f, getting similar results, but with a different set of eigen-functions.)

Later, Einstein *et al* introduce an assumption that is not compatible with quantum mechanics, as we will prove. They consider a particular case where the system is made of two particles and (1) can be written as:

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} e^{i/\hbar(x_1 - x_2 + x_0)p} dp \tag{3}$$

where x_0 is some constant.

They choose this wave function because they are interested in proving that, in particular, subsystem II, after measuring x or p for subsystem I, can be left in states where x or p, respectively, have definite values. This, according with their *criterion of reality*, would imply that those two physical magnitudes have simultaneous *reality*. However, they do no provide any proof that such a state could emerge as a result of temporal evolution, as specified by Schrödinger equation, which is explicitly required by their general description of the physical situation: "…We can then calculate with the help of Schrödinger's equation the state of the combined system I+II at any subsequent time; in particular, for any $t > T$. Let us designate the corresponding wave function by Ψ …"

Instead of providing that proof, as it is logically necessary, they limit themselves to write "Let us suppose that the two systems are two particles and that $\Psi(x_1, x_2) = \int e^{i/h(x_1 - x_2 + x_0)p} dp$ … " This cannot but cast doubts on the validity of assumption (3), unless we are willing to accept it because it has not been proved to be invalid.

The argument by Einstein *et al* is an example of reasoning by *reductio ad absurdum* [2]. This method of proof proceeds by stating a proposition and then **demonstrating** that it leads to a contradiction. As to the logical structure of this kind of argument, we remind the reader that [3 p. 57]:

> "We must be certain that, in showing that the negation of a statement we wish to prove leads to a false conclusion, *we use only premises known to be true as well as a logically valid argument.* For if the negation of our statement is not the only statement whose truth value is uncertain, or if the

argument used is not logically valid, then we could arrive to a false conclusion, even if the negation of our statement happens to be true.”

The wave function (3) is just one of the elements of an elaborated counterexample. An argument that is intended to prove that the statement that *quantum mechanics is a complete theory* is false, on the grounds that a quantum system can evolve by itself to a state where two complementary physical properties have *simultaneous reality*.

Below we prove that (3) cannot emerge as a result of temporal evolution, as described by Schrödinger equation.

Our first consideration is that, according to (3), the wave function at time t_f is equal to

$$\Psi(x_1, x_2) = 2\pi\hbar\,\delta(x_1 - x_2 + x_0) \tag{4}$$

Let us introduce the new coordinates:

$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \tag{5}$$

And

$$x = x_1 - x_2 \tag{6}$$

The hamiltonian between T and t_f is given by:

$$\hat{H} = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial X^2} - \frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2}\,. \tag{7}$$

Where m_1 is the mass of subsystem I, m_2 is the mass of subsystem II, $M = m_1 + m_2$, and $\mu = m_1 m_2 / \left(m_1 + m_2\right)$.

The first part of (7) corresponds to the kinetic energy of the center of mass, and the second to the kinetic energy of the internal motion.

Using (5) and (6) we have

$$\Psi\left(X, x\right) = 2\pi\hbar\,\delta\left(x + x_0\right) \tag{8}$$

Which depends only on x. Therefore, at time t_f, Ψ is an eigen-function of the operator

$$P_X = -i\hbar \frac{\partial}{\partial X} \tag{9}$$

with the eigen-value $P_X = 0$. And it must be so for any time between T and t_f, because $\left[\hat{H}, \hat{P}_X\right] = \hat{0}$. In consequence, the wave function has the form:

$$\Psi\left(X, x, t\right) = \psi\left(x, t\right) \tag{10}$$

for any time between T and t_f, where $\psi\left(x, t\right)$ is a solution of the Schrödinger's equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2\mu} \frac{\partial^2 \psi}{\partial x^2}. \tag{11}$$

This equation is formally identical to the equation of a single free particle, and quantum mechanics predicts that the corresponding wave functions spread in space as time goes on, instead of collapsing themselves into delta functions [4, pp. 104-106].

The solution of (12) with the final condition $\psi\left(x,t_f\right)=2\pi\hbar\delta\left(x+x_0\right)$ can be obtained from the non-relativistic propagator, using time reversal. It describes a temporal succession of non-normalizable states, where the square of the modulus of the wave function depends only on time. According to quantum mechanics, the square of the modulus of a physically acceptable wave function has to be integrable.

Notice that we had enough grounds in our last remark to reject (4), as soon as we wrote (8). This failure of (4) to serve its intended purpose in the argument by Einstein *et al*, cannot be fixed by replacing it by a *narrow superposition* of eigen-states of the operator $\hat{x}_1-\hat{x}_2$. Because a *narrow superposition* has not a definite value of x_1-x_2.

In addition, the function $\delta\left(x+x_0\right)$ is symmetric with respect to $-x_0$. Therefore, the expected value of the operator $\hat{v}=-i\hbar\partial/\partial x$, corresponding to the relative velocity, is equal to zero at time t_f, and at any other time between T and t, because $\left[\hat{H},\hat{v}\right]=\hat{0}$. Therefore, the wave function (10) does not correspond to the general idea of a system made of two particles that are flying apart after they have interacted. In consequence, unless we assume that the forces of nature can be turned off at will, the wave function (4) does not match the (incomplete) description of the physical situation, as given by Einstein *et al*.

Were our arguments not enough to have our claim accepted, we state that it is not sound to believe that assumption (3) is consistent with quantum mechanics on the grounds that it has not been proved that it is inconsistent [*ad ignorantium* fallacy].

The final conclusion of the EPR paper, that the wave function does not provide a complete description of physical reality, is strongly dependent on the validity of (3) at time t_f, as well as on some other philosophical considerations that we have decided not to address in our main argument. Therefore, that conclusion is invalidated.

Concluding Remarks

The invalidation of (3) casts doubts on the validity of the assumption that "…*If without in any way disturbing a system, we can predict with certainty (i. e. with probability equal to unity) the value of a physical quantity, then there is an element of physical reality corresponding to this physical quantity*…" introduced by Einstein, Podolsky, and Rosen in the same paper, with no more foundation than (3), on the basis that "we regard (it) as reasonable". The same doubts fall then over Bohm's conclusion [5 pp. 619-23] that the world cannot "correctly be analyzed into elements of reality, each of which is a mathematical quantity appearing in a complete theory" and, furthermore, on the current interpretation of Bohm's experiment.

Actually, according to quantum mechanics, if there is something we can predict without (in absolutely any way) perturbing a quantum system, it is the wave function, using Schrödinger's equation. Then, all we need is to accept that the square of the modulus the wave function is a physical quantity, which might be justified on the grounds *that it is reasonable* [6 p. 2], to jump to the conclusion that the square of the wave function, and therefore, the quantum potential, corresponds to an *element of reality*. Then we should accept that the de Broglie-Bohm theory is true, in the sense that it corresponds to reality. However, the de Broglie-Bohm theory was never accepted by Einstein, whose views about reality were addressed—by himself—in [7 p. 2]:

> "We are accustomed to regard as real those perceptions which are common to different individuals, and which, therefore, are in some sense impersonal. The natural sciences, and in particular, the most fundamental of them, physics, deal with such sense perceptions."

Here we have a (necessary and) sufficient condition of reality, stated in terms of what we perceive and measure, not in terms of what we predict. If we agree with the implicated meaning of the term *reality*, again, as a formal definition, we have to endorse Bohr's opinion that there is not a microscopic *quantum reality*: because we cannot perceive subatomic particles, neither measure them; all we can perceive and measure are the changes of state of macroscopic bodies, that we presume to be induced in them by their interactions with quantum corpuscles. However this conclusion is not as relevant as it might seem at first sight: Because it is grounded on the very definition of the term *reality*, and the definition of a term can always be revised.

As a final remark we have to say that from the fact that the EPR paper does not prove that quantum mechanics is incomplete we cannot conclude that quantum mechanics is complete. We consider that a theory is complete if: (a) the representation used by the theory is complete and the corresponding interpretation is unambiguous; (b) the axioms are logically consistent; (c) the theory does not leave any sensitive question without a sensitive answer; and (d) the theory is true. As D. Aerts has proved [2], there is evidence that quantum mechanics does not meet the conditions (a) and (c).

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