Propositions as (Flexible) Types of Possibilities

Nate Charlow

1

A proposition, for purposes of this chapter, is an object of thought.[1] When you think that Toronto is northeast of Detroit, the thing that you think to be, or evaluate as, true—*that Toronto is northeast of Detroit*—is what philosophers call a proposition.

You are no doubt familiar with the thesis that a proposition (in this sense) is a *function from possible worlds to truth values*. (The thesis is equivalent, in a classical setting, to the thesis that a proposition p is a set of possible worlds, namely, the set of worlds w such that $p(w)$ is true.) The thesis is common, even omnipresent, in certain corners of philosophy—in particular, areas that focus on "model-theoretic" analysis or modeling. More or less throughout the region of theoretical linguistics known as formal or compositional semantics, the analysis of propositions as sets of possibilities tends to be assumed, at the very least as the best working hypothesis in some model-building enterprise. Many philosophers—especially those whose work is formal semantics-adjacent, but also a great many whose work is in formal areas of decision theory and epistemology—follow suit.

This analysis remains—notwithstanding its widespread adoption—polarizing. Many critics have noted serious difficulties with analyzing so-called propositional attitude verbs like 'thinks' and 'believes' as expressing *universal quantification* over sets of possibilities—difficulties that are liable to appear intractable, in view of basic facts about the logic of the universal quantifier.[2] These critics tend to regard this analysis as involving a serious error, and unsuitable for use even as a working hypothesis. Many of course have suggested alternative analyses of propositions (Structured accounts, Fregean/Sense-based accounts, Act-based accounts, Relationalist accounts, and more) to supplant it.[3]

This chapter will endeavor to see (some) truth in both points of view (though from the perspective of the model-building enterprise of formal or compositional semantics). I will agree with the critics that the standard analysis of a proposition as a set of logically or metaphysically possible worlds is untenable (for reasons that will be covered in the first part of this chapter). But, rather than giving up on the analysis of the semantic argument of a propositional attitude verb as a type of possibility (and relinquishing the substantial progress made within this paradigm), I will here focus on presenting various strategies for refining it. One such strategy, due to Jaakko Hintikka, seeks to distinguish the kind of "worlds" over which propositional attitude verbs quantify from the kind of worlds over which modalities expressing logical or metaphysical necessity quantify. Another strategy, associated with David Lewis, draws a

---

1 Other authors (e.g. Kaplan 1989; Stanley 1997) center discourse-related notions (what a speaker says with a sentence) in elucidating the notion of a proposition. It is by now a philosophical cliché that a theory of propositions, qua objects of thought, may fail to satisfy as a theory of propositions, qua contents of assertion, and vice versa.

2 In particular, the fact that the universal quantifier is upward monotone: when ϕ entails ψ, $\ulcorner \forall x \phi \urcorner$ entails $\ulcorner \forall x \psi \urcorner$.

3 In this volume, see King (2022) on the Structured theory, Textor (2022) on Frege's theory, Hanks (2022); Soames (2022) on the Act-based theory, and Yoon (2022) on the Relationalist theory.

distinction *in semantic type* between the possibilities over which propositional attitude verbs quantify and the possibilities over which modalities expressing logical or metaphysical necessity quantify: a "doxastic" or "epistemic" possibility is a centered possible world (e.g., a pair $\langle w, x \rangle$, with w a possible world and x a "coordinate" within that world).

After reviewing these strategies (as a kind of proof of concept), I will turn my attention to a line of a theorizing that is an outgrowth of the recent literature on epistemic modality, according to which contentful thought is broadly "informational" in its nature and import. The general idea behind this line of theorizing is that an object of thought is not a way *the world* could be, but rather a way *one's perspective* could be (with respect to a relevant representational question). I will spend the middle part of this chapter motivating and developing a version of this strategy that is, I'll argue, well-suited to explaining clear phenomena concerning the attribution of perspectival attitudes—in particular, attitudes towards loosely information-sensitive propositions—with which other extant approaches struggle. My goal here will be to motivate a distinctive version of the "informational" approach (the "Flexible Types" approach), which is based on the theory proposed in Charlow (2020). According to the Flexible Types approach, propositional attitude verbs are quantifiers over sets of possibilities, but a possibility is a type-flexible notion—sometimes a possible world, sometimes a perspective, sometimes a set of possible worlds, sometimes a set of perspectives.

After introducing the Flexible Types approach, this chapter circles back to more traditional concerns for the analysis of propositions as types of possibilities. Here too the Flexible Types approach bears fruit. Although there are certainly significant differences—I note some in the concluding section—the gist of this chapter is Hinitkkan or Lewisian in spirit (if not quite in letter). We can make progress on addressing the challenges for the analysis of propositional content in terms of types of possibilities, through empirically driven refinement of our notion of what kind of thing a "doxastic possibility" is.

2

This chapter's topic is what we could call the *semantic* (or *logical*) *type* of propositions. The larger theoretical context is that of compositional semantic theorizing for natural language. In a compositional semantics, the semantic value of a complex constituent (e.g., a sentence) will be determined by (i) the semantic values of its sub-constituents and (ii) the compositional rules available in that language. The semantic value or extension of 'thinks' relative to a world of evaluation w, $[\![thinks]\!]^w$, will be treated as two-place function, mapping a propositional content p and an individual x to true (hereafter abbreviated $\top$) iff x thinks that p in w.

$$[\![thinks]\!]^w = \lambda p.\lambda x.x \text{ thinks that } p \text{ in } w$$

The ontology of a compositional semantic theory will generally include particular individuals (type: e), truth values (type: t), possible worlds (type: s), and any higher type that can be constructed stepwise from these basic types: if τ and τ' are types, then $\langle \tau, \tau' \rangle$ (a function mapping something of type τ to something of type τ') is a type.[4]

Where do propositions fit into this picture? What is the *semantic type of a proposition*? A well-known and still widely endorsed answer, due in essence to Hintikka (1962, 1969) and

---

4 For an entré to intensional compositional semantics, see von Fintel & Heim (2011).

associated closely with Stalnaker (1984), is that propositions are of semantic type $\langle s, t \rangle$: the type of a function that maps a possible world (type: s) into a truth value (type: t). According to this story, a proposition is (a function that characterizes) a set of worlds, namely, the set of worlds relative to which the proposition has the truth value $\top$. Relative to a world w, an individual x stands in the relation $[\![\text{thinks}]\!]^w$ to a set of possible worlds p if and only if the possible worlds that are compatible with what x thinks at w, here notated as B_x^w, are all p-worlds.

$$[\![\textit{thinks}]\!]^w = \lambda p_{\langle s,t \rangle}.\lambda x_{\langle e \rangle}.\forall v \in B_x^w : p(v) = \top$$
$$[\![x \textit{ thinks that } p]\!]^w = \top \text{ iff } \forall v \in B_x^w : [\![p]\!]^v = \top$$

The basic idea here is that $\ulcorner x$ thinks$\urcorner$ is a modal necessity operator; $\ulcorner x$ thinks $p \urcorner$ is true at w if and only if it must be that p, given what x thinks in w. Propositions are *domains* of possibilities, while attitude verbs are construed as universal (intensional) *quantifiers* over those domains.

3

What is the motivation for understanding propositions (and the ascription of propositional attitudes) along these lines? The core "conceptual" motivation[5] concerns what we might call the *essence of thought* (and its role in the broader cognitive activity of inquiry). Hintikka writes (cf. Stalnaker 1984: Ch. 1):

> What I take to be the distinctive feature of all use of propositional attitudes is the fact that in using them we are considering more than one possibility concerning the world. (This consideration of different possibilities is precisely what makes propositional attitudes propositional, it seems to me.)... My basic assumption... is that an attribution of any propositional attitude... involves a division of all the possible worlds... into two classes: into those possible worlds which are in accordance with the attitude in question and into those which are incompatible with it. (1969: 24-5)

Hintikka here articulates a common line of thought, on which an essential feature of "propositional" or "intentional" thought is the representation of *alternative possibilities* (implicitly, in the context of an *inquiry into which of these alternative possibilities represents actuality*). To think that p requires that you in some sense represent a situation in which p is the case and a situation in which p is not the case—you must distinguish possibilities in which p is true from possibilities in which p is false. When the alternative possibilities that are compatible with what one is thinking (in w) are all possibilities in which p is true, it seems you have ruled it out that p is false (in w). And if you've ruled it out—eliminated the possibility—that p is false (in w), it seems plausibly to follow that, according to what you think (in w), p is the case.

It is important right away to note an ambiguity in the theoretical notion of 'possibility': by 'possibility' we might mean either a *possible world* or a *point of semantic evaluation* (i.e., a point against which the extension of a syntactic constituent is determined, or an "index"). In the "single-indexed" semantic theory sketched above, no such distinction is drawn: points of evaluation *are* just possible worlds. But in a less idealized semantic theory, which relativizes extensions, not just to possible worlds, but additionally to times, individuals, etc., the distinction

---

will be important. Lewis (1979, 1986), for example, takes the type of propositional content to be the type of sets of indices (which he in turn takes to be the type of sets of world-time-individual triples), not the type of sets of possible worlds.[6] Others—myself included—propose more radical emendations of the Hintikkan framework. I will return to this below.[7]

4

My focus in this chapter is presenting, evaluating, and ultimately refining the thesis that propositional content should be typed, in the context of a compositional semantic theory for ascriptions of thought in ordinary language, as a set of possibilities. (According to one particular version of that thesis, the type of propositional content is the type of sets of possible worlds.) Over the decades, philosophers have raised a host of challenges to the general idea that propositional content should be typed as a set of possibilities, as well as to the more specific idea that it should be typed as a set of possible worlds. In this section I will review the two primary worries for this idea (and review a common technique for addressing them).

One prominent challenge is the **Problem of Logical Omniscience**. The problem is simple. How can a theory of (the semantics of) the attitudes accommodate the plain fact that we are not logically omniscient, i.e., that there are logical consequences of our beliefs which we nevertheless fail to believe? More precisely, it is manifestly the case that, for some x, p, q: x thinks that p; q is a logical consequence of p; and it is *not* the case that x thinks that q. The Hintikkan's challenge is to explain how this is possible, in light of the following.

   i. Suppose that $p \vDash q$. Then for any logically possible w, $[\![p \supset q]\!]^w = \top$.
  ii. Suppose further that x thinks that p (at w).
 iii. Then in any world v compatible with what x thinks (at w), $[\![p]\!]^v = \top$.
  iv. Since q is a logical consequence of p and $[\![p]\!]^v = \top$, $[\![q]\!]^v = \top$.
   v. Since v is an arbitrary world compatible with what x thinks (at w), and $[\![q]\!]^v = \top$, x thinks that q (at w). This contradicts the supposition that x is not logically omniscient.

As Hintikka (1975) stressed, however, this presentation of the Problem of Logical Omniscience relies on a substantive assumption: that the 'doxastic possibilities' over which attitude verbs quantify are ipso facto *logically* or *metaphysically* possible worlds. If we relinquish that assumption, then the move from (iii) to (iv) is simply invalid. The fact that q is a logical consequence of p tells us something about what holds at an arbitrary logically possible world. But it may not tell us anything about what holds at an arbitrary world compatible with x's beliefs, if belief-worlds are not assumed to be logically possible.[8]

Another prominent challenge arises out of the literature on what is known as **Frege's Puzzle**

---

6 I am here using 'index' to refer to a point that fixes the extension of a context-independent intension (compare the understanding of 'index' in Lewis 1980). I am therefore ignoring the phenomenon of hyper-intensional or context-dependent semantic values (though the phenomenon will make an appearance toward the end of this chapter). For relevant discussion, see Murray (2022).

7 We should also note an ambiguity in the theoretical notion of a possible world; 'possible' here might be taken as logical/metaphysical possibility or doxastic/epistemic possibility. More on this below.

8 For recent work on impossible worlds, see e.g. Restall (1997); Nolan (1997, 2013); Berto & Jago (2019).

([Frege 1948](#)).[9] Here is a simple statement. 'Hesperus' is a name for the Evening Star, while 'Phosphorus' is a name for the Morning Star. In fact, though unknown until the advent of Classical Greek civilization, Hesperus and Phosphorus are the same celestial body—the planet Venus. Although Homer would have believed that Phosphorus is visible in the morning, he would not have believed that Hesperus is visible in the morning. But if Hesperus is identical to Phosphorus, does it not follow (by, roughly, the substitutivity of identicals) that the proposition that Hesperus is visible in the morning is identical to the proposition that Phosphorus is visible in the morning (and therefore that Homer believed Phosphorus was visible in the morning exactly if he believed that Hesperus was visible in the morning)?

Apparently it does *not* follow. The puzzle again is to explain how that is possible. Interestingly, [Hintikka](#) ([1969](#)) thought his theory, understood aright, was suited to exactly that.

> [W]e can at once see why the familiar principle of the substitutivity of identity is bound to fail in the presence of propositional attitudes when applied to arbitrary singular terms. Two such terms, say *a* and *b*, may refer to one and the same individual in the actual world... thus making the identity '*a* = *b*' true, and yet fail to refer to the same individual in some other (alternative) possible world. (30)

Hintikka here claims that even when in reality $a = b$, the set of worlds where '$a = b$' is true (i.e., the set of w such that the referent of 'a' in w = the referent of 'b' in w) is distinct from the set of worlds where '$a = a$' is true. Now of course this appears to fly in the face of Kripkean orthodoxy, according to which singular terms (e.g., proper names) are rigid designators—the referent of a proper name is the same individual with respect to any possible world of evaluation ([Kripke 1980](#)). Thus, according to Kripkean orthodoxy, if 'a' and 'b' have the same referent, they have the same referent with respect to any possible world of evaluation, and it is impossible that the set of worlds where '$a = b$' is true differs from the set of worlds where '$a = a$' is true.

However this line of thought rests on the same assumption as the last: that belief-worlds are actually (metaphysically) possible. If we relinquish this assumption, then Kripkean Rigid Designation (the thesis that, given actually co-referring 'a' and 'b', 'a' and 'b' have the same referent in each metaphysically possible world) is compatible with Hintikka's remarks as quoted above (according to which, even when 'a' and 'b' actually co-refer, 'a' and 'b' may fail to co-refer in some doxastic possibility).[10]

This is a tentative pass at the issues raised by Logical Omniscience and Frege's Puzzle. We

---

9 Frege's Puzzle is a particular version of a more general problem: the problem of necessarily equivalent (i.e., co-extensional in every metaphysically possible world) propositions.

10 [Stalnaker](#) ([1984](#)) differs from Hintikka on these points—a fact that is due to his (according to a common view, which is shared by the author, misplaced) reductive ambitions with respect to intentionality. (Another philosopher with famously reductive ambitions is of course [Lewis](#) ([1986](#)).) According to Stalnaker, the content of belief (and of assertion) is typed as a set of metaphysically possible worlds. Stalnaker thus develops a different treatment of logical omniscience and Frege's Puzzle, the former relying heavily on the idea that thought is fragmented (i.e., one has/instantiates more than one belief state), the latter relying heavily on Diagonalization (here see also the "Two-Dimensionalist" strategy of e.g. [Jackson](#) ([1998](#)); [Chalmers](#) ([2002](#), [2004](#), [2011](#)), which I won't discuss in any detail in this chapter). Stalnaker's reliance on fragmentation as a general strategy for handling the problem of logical omniscience is problematic. It can explain why thinking that p and thinking that p implies q does not suffice for thinking that q (if the thought that p and the thought that p implies q are "housed in" different belief states). But it cannot explain why thinking that p does not suffice for thinking q, when q is a *logical consequence* of p. For discussion of this point, see [Yalcin](#) ([2018](#)).

will revisit them (in the context of a more contemporary take on the semantics of attitude verbs and their complements) below.

5

[Lewis](1979), building off of [Perry](1977, 1979) presents a different challenge to the idea that the content of a state of belief is to be typed as a set of possible worlds. Consider this thought experiment:

> [Two gods] inhabit a certain possible world, and they know exactly which world it is... Still I can imagine them to suffer ignorance: neither one knows which of the two he is. They are not exactly alike. One lives on top of the tallest mountain and throws down manna; the other lives on top of the coldest mountain and throws down thunderbolts. Neither one knows whether he lives on the tallest mountain or on the coldest mountain; nor whether he throws manna or thunderbolts. (520-1)

Let the world they inhabit be w. There is, by assumption, exactly one world compatible with what each god thinks, and it is w. But neither god knows which god he is. Both are uncertain about something. But their doxastic possibilities cannot be refined any further.

Lewis's analysis of this case (and similar cases of so-called "self-locating" or "*de se*" thought) is sensible (and has been enormously influential among philosophers and semanticists): doxastic possibilities are finer in "grain" than possible worlds.[11] For the god on the tallest mountain to think that he is the god on the tallest mountain, he must rule out the possibility that he is the god on the coldest mountain. Given Lewis's stipulations about this case, the possibility cannot be represented as a possible world; it instead needs to be represented as a kind of individual coordinate *within* a possible world. For each god, there are two possibilities (letting G_1 be the god on the tallest mountain, G_2 the god on the coldest mountain):

- $\langle w, G_1 \rangle$: the possibility that they are identical to G_1 in w.
- $\langle w, G_2 \rangle$: the possibility that they are identical to G_2 in w.

Lewis dubs possibilities of this type *centered worlds*. He proposes to modify the Hintikkan theory described above in a corresponding fashion, modeling attitude verbs as quantifiers over sets of centered worlds (see also [Lewis 1986](): §1.4). Letting $[\![\epsilon]\!]^{w,a}$ be the extension of ϵ relative to the centered world $\langle w, a \rangle$ (with a an individual), the core formal ideas are these:

$$[\![\text{thinks}]\!]^{w,a} = \lambda p_{\langle se,t \rangle}.\lambda x_{\langle e \rangle}.\forall \langle v, y \rangle \in B_x^w : p(\langle v, y \rangle) = \top$$
$$[\![x \text{ thinks that } p]\!]^{w,a} = \top \text{ iff } \forall \langle v, y \rangle \in B_x^w : [\![p]\!]^{v,y} = \top$$

In Lewis's framework, the semantic type of a proposition, $\langle se, t \rangle$, is the type of a set of centered worlds, which is to say: a function from sets of world-individual pairs into truth-values. The

---

11 [Cappelen & Dever](2013) argue that this case presents just another instance of Frege's Puzzle. They may be right about that, but note that this isn't by itself an objection to elucidating the attitudes the gods have in this case with sets of centered worlds. Lewis is (among other things) advancing an empirical or modeling hypothesis in semantics. If treating attitude verbs as quantifiers over centered worlds best fits the data in Lewis's Two Gods case (and this case is merely another instance of Frege's Puzzle), this suggests we can make progress on Frege's Puzzle by treating attitude verbs as quantifiers over centered worlds. More on this general theme below.

semantic type of a "doxastic state" B_x^w is the type of a set of centered worlds $\langle v, y \rangle$, where $\langle v, y \rangle \in B_x^w$ exactly if it is compatible with what x thinks in w that they are identical to y in v.

Although this represents an emendation of Hintikka's theory, the resulting theory is certainly Hintikkan in spirit. The essential feature of intentional thought remains the representation of alternative possibilities; for x to think that p is for p to be true at each doxastic possibility for x. The difference is that for Lewis a set of doxastic possibilities represents a property of an *individual*—a way for an individual to be (where for Hintikka a set of doxastic possibilities represents a property of the "objective" situation—a way that the objective circumstances, or the "world", could be). To think that p, for Hintikka, is to ascribe a property to the "objective" situation. To think that p, for Lewis, is to *self-ascribe* the property of being an individual y in an objective situation v such that p is true of y in v.[12]

The strongest case for understanding propositional content in terms of sets of centered worlds ultimately comes, not from recherché cases like Lewis's Two Gods, but from attention to *everyday attributions of perspectival thought*: thought taking as its object a "perspectival" or "subjective" proposition.[13]

(1) Biff thinks that okra is gross.

Claims like 'okra is gross' are commonly thought to resist understanding or analysis in "single-indexed" semantic theories, by which I mean semantics theories in which extensions (e.g., truth values) are relativized only to objective circumstances, or worlds. But whether okra is gross seems to depend, not on the "objective" state of the world—i.e., on okra's perspective-independent features—but on something "subjective", namely, the perspective (in particular, the tastes) of the individual who evaluates or judges the claim (in a case like (1), that is the subject of the attitude ascription). From Biff's point of view, the alternatives he rules out, when he thinks that okra is gross, are possibilities concerning the way okra tastes *to him* (not alternative possibilities for the "objective" state of the world). (1), on the intended reading, is true if and only if for each w and x (such that, given what Biff thinks, he could be x in w), x finds or evaluates okra as gross in w.

6

We have just seen how thought-ascriptions with perspectival contents might motivate an emendation of the Hintikkan account. In the next part of the chapter I'll come at the same point from a different direction (and in considerably greater detail), by considering thought-ascriptions with a different kind of perspectival (specifically, *information-sensitive*) content:

(2) Biff thinks that it's likely to rain.

(3) Biff thinks that it might pour.

(4) Biff thinks that if it rains, it will pour.

---

12 See Pearson (2020: §5) for a clear discussion of some compositional intricacies.

13 The literature here is vast, but some important references are Lasersohn (2005); Stephenson (2007a,b); Egan (2010); Pearson (2013); MacFarlane (2014); Coppock (2018). It is worth noting that Judge/*De Se* Relativists (Egan, Lasersohn, MacFarlane, Stephenson) generally treat cases like (1) and (2-4) below in similar ways.

As Yalcin (2007) observes, cases like these resist easy explanation in the Hintikkan theory. The rough reason is that attitude ascriptions whose complements are information-sensitive are naturally read in such a way that the following are *not* attitudes that Biff can have.

(5)　　??Biff thinks that it's likely to rain and that it won't rain.

(6)　　??Biff thinks that it might pour and that it won't pour.

(7)　　??Biff thinks that if it rains, it will pour; that it might rain; and that it won't pour.

The Hintikkan theory will struggle to explain why thoughts like the ones we attempt to attribute to Biff in (5-7) are not, on the intended (i.e., epistemic) readings of their complements, thoughts that Biff can have.[14] Let us see how the problem works for (6) (the problem generalizes pretty straightforwardly to examples (5) and (7)). Note first that the truth of 'it might pour' at a world w cannot imply the truth of 'it will pour' at w; to say otherwise would grossly distort the logic of a claim like 'it might pour'. So, there must be a non-empty class of possibilities w such that 'it might pour and it won't pour' holds at w. But, if that is correct, then the set of such w should be something Biff can think true, and a claim like (6) should have an acceptable reading.[15] But it does not. More generally, in the intuitive sense of 'possibility' off of which theorists like Hintikka, Stalnaker, and Lewis base their analyses of propositions, the complements of (5-7) don't appear to designate possibilities at all. The challenge for the Hintikkan theory is stark: to state a theory which delivers on judgments (5-7)—in particular, the judgment that their complements *don't designate something fit for being the object of any propositional attitude*[16]—without making a hash of the logic of information-sensitive claims, like the complements of (2-4).

7

The neo-Hintikkan/neo-Stalnakerian theory of Yalcin (2007) is designed to thread this needle.[17] According to Yalcin, the reason that Biff can't have the thought we attempt to attribute to him in (6) is simply that, for Biff to have the thought we attribute to him in (3), there must be *some* possibility w such that w is compatible with what Biff thinks and such that it pours in w. But

---

14 I allow that there are ways of reading a case like (6) on which it attributes a coherent thought to Biff. If, for example, we read 'it might pour' as saying that the *objective chance* of it pouring is non-zero, then (6) is coherent. That is *not* the intended (epistemic) reading of (6). (The intended reading is, very roughly, the reading according to which, for Biff to think that it might pour, there is a doxastic possibility for Biff in which it pours. Much more on this below. On the intended reading, I hope it is evident why (6) can't attribute a coherent doxastic state to Biff.)

15 To be clear: this argument is not airtight. With a bit of ingenuity, the Hintikkan provide an analysis of (2-7) that is equivalent to Yalcin's analysis (see fn17). I am here only attempting to provide some prima facie motivation for Yalcin's theory of cases like (2-7) against the Hintikkan's. Note that I will ultimately be arguing *against* Yalcin's theory of these cases in this chapter (though I will be agreeing with him that the complements of (5-7) don't designate possibilities that are fit for belief or acceptance).

16 As Yalcin (2007) emphasizes, this represents a crucial distinction between what he calls "epistemic contradictions" (like 'it won't pour, but it might pour') and Moore Paradoxical claims (like 'it won't pour, but I don't know that it won't pour'). Moore Paradoxical claims are fit for being the object of certain propositional attitudes—one can, e.g., suppose or imagine that it won't pour but that you don't know that. One cannot, however, suppose or imagine that it won't pour, but that it might pour. For extensive discussion, see Yalcin (2007: Sect. 2).

17 Ninan (2018); Beddor (2020) provide more orthodoxly Hintikkan theories that replicate the predictions of Yalcin (2007). But, as we will see in a bit, replicating these predictions is not a desirable feature for a theory to have.

for Biff to have the thought that it won't pour, there can be *no* possibility w such that w is compatible with what Biff thinks, and such that it pours in w. So Biff can't have the thought we attempt to attribute to him in (6).

That is the rough idea behind Yalcin's account. The remainder of this section will more carefully lay it out. For (6) these are the key clauses. (From here forward I will provide syncategorematic, rather than compositional, truth conditions for ascriptions of thought.)

$$[\![\textit{might } \phi]\!]^{w,i} = \top \text{ iff } \exists v \in i : [\![\phi]\!]^{v,i} = \top$$

$$[\![x \textit{ thinks that } \phi]\!]^{w,i} = \top \text{ iff } \forall v \in B_x^w : [\![\phi]\!]^{v,B_x^w} = \top \quad (B_w^x \subseteq W)$$

A distinctive feature of Yalcin's analysis is that the extension of $\ulcorner \textit{might } \phi \urcorner$ is relativized, not only to a possible world w, but additionally to an informational perspective i (typed, in the present theory, as a set of worlds). According to this theory, whether it might pour (in the "epistemic", not "objective chancy", sense), like whether okra is gross, depends, not on the objective state of the world, but on a perspective. The only real difference between Yalcin's and the Hintikkan proposal for 'thinks' is that, in Yalcin's theory, in evaluating $\ulcorner x \textit{ thinks } \phi \urcorner$, we shift the informational perspective against which ϕ is semantically evaluated, to x's perspective.

From these clauses it follows that:

$$[\![x \textit{ thinks that might } p]\!]^{w,i} = \top \text{ iff } \forall v \in B_x^w : [\![\textit{might } p]\!]^{v,B_x^w} = \top$$
$$= \top \text{ iff } \forall v \in B_x^w : \exists u \in B_x^w : [\![p]\!]^{u,B_x^w} = \top$$
$$= \top \text{ iff } \exists u \in B_x^w : p(u) = \top$$

According to this theory, x thinks that it might be that p (at w) exactly if, simply, the possibilities compatible with what x thinks at w, B_x^w, include possibilities at which p. This explains (6). For Biff to think that it won't pour (in w), it must be the case that, for each v compatible with B_{Biff}^w, it does not pour in v. So, if Biff thinks it might pour (in w), Bill cannot also think that it won't pour (in w). Just as desired.

Yalcin (2010, 2012a,b) generalizes this "informational" logic and semantics for epistemic modalities to probability operators and indicative conditionals. In the generalized theory, probability operators are evaluated against a special kind of doxastic or informational perspective—not a set of possibilities, but something that associates a set of possible worlds with a likelihood, i.e., a credal or probability measure.[18] Let $I = \langle W_I, S_I, \text{Pr}_I \rangle$ be an **informational perspective**, with W_I a set of possible worlds, S_I a set of subsets of W_I,[19] and Pr_I a probability measure mapping elements of S_I into $[0, 1]$ and which is normalized to S_I (i.e., $\text{Pr}_I(\cup S_I) = 1$). Then:[20]

$$[\![\textit{might } \phi]\!]^{w,I} = \top \text{ iff } \exists s \in S_I : s \cap \{v : [\![\phi]\!]^{v,I} = \top\} = s$$
$$[\![\textit{likely } \phi]\!]^{w,I} = \top \text{ iff } \text{Pr}_I(\{v \in \cup S_I : [\![\phi]\!]^{v,I} = \top\}) > .5$$
$$[\![\phi \Rightarrow \psi]\!]^{w,I} = \top \text{ iff } \text{Pr}_I(\{v \in \cup S_I : [\![\psi]\!]^{v,I} = \top\} \mid \{v \in \cup S_I : [\![\phi]\!]^{v,I} = \top\}) = 1$$

---

18 Similar or closely related ideas can be found in e.g. Swanson (2006); Lassiter (2011); Égré & Cozic (2011); Rothschild (2012); Moss (2013, 2015, 2018).

19 It is natural to require, and Yalcin does require (Yalcin 2011, 2018), that S_I partition W_I (though the latter reference also expresses reservations about this requirement). In general, S partitions T exactly if $\cup S = T$ and for any $s, s' \in S : s \cap s' = \emptyset$.

20 Here I depart slightly, but harmlessly, from Yalcin's treatment of the indicative conditional (which presupposes the existence of an update function $[\cdot]$, such that $[\phi]$ is a function from informational perspectives into updated informational perspectives).

It would at this juncture seem natural to state the rest of the theory exactly as before, with one small change: we type x's "state of belief" at w as a (probabilistic) informational perspective, here notated as I_x^w. Then we have the following prediction:

$$[\![x \text{ thinks } p \text{ is likely}]\!]^{w,I} = \top \text{ iff } \forall v \in \cup S_{I_x^w} : [\![likely\ p]\!]^{v,I_x^w} = \top$$
$$\text{iff } \forall v \in \cup S_{I_x^w} : \Pr_{I_x^w}(\{u \in \cup S_{I_x^w} : u(p) = \top\}) > .5$$
$$\text{iff } \Pr_{I_x^w}(\{u \in \cup S_{I_x^w} : u(p) = \top\}) > .5$$

According to this theory, x thinks that it is likely that p (at w) exactly if, simply, x regards p as *more likely than not*.

As desired, the theory generates the prediction that there is something amiss with Biff's thought in (5). The rough idea is simple: if all Biff's belief-worlds are worlds where it won't rain, then the degree of likelihood Biff assigns the proposition that it will rain must equal 0. But then it cannot be the case that Biff regards this proposition as more likely than not. (I leave working out the theory's explanation of (7) as an exercise for the reader.)

The theory seems to work like a charm. Another feather in its cap: a natural (if, I'll explain, partial) treatment of Logical Omniscience. Suppose that q is a logical consequence of p and that x thinks that p. Does it follow that x thinks that q? On the present version of the theory, yes—but on a very lightly modified version, no (see esp. Yalcin 2011, 2018). The modification Yalcin provides is simple: to treat attitude verbs (and epistemic modals) as semantically presupposing the "visibility" of their propositional complements in the relevant informational perspective. I won't spend any time on the formalities here; for our purposes, it suffices to note that, even when q is a logical consequence of p, p's visibility in I does *not* imply q's visibility in I. The basic idea here is simple and fairly natural: when q is a logical consequence of p and x thinks that p, x thinks that q if and only if x "sees" the possibility that q. But although there's a lot to like about this take on Logical Omniscience, its incompleteness is apparent: when q is a logical consequence of p, x thinking that p and x seeing the possibility that q does not appear to be sufficient for x to think that q, when x *doesn't think* (e.g., because x is ignorant of the fact) that q is a consequence of p (as noted by Hintikka 1962: 30-1). I'll return to this below.

8

I turn now to a different worry for theories of this general type.[21] Although Yalcin's theory is, we might say, "embedded" in a larger Hintikkan or Stalnakerian understanding of content, it actually appears to take a decidedly non-Hintikkan/non-Stalnakerian view about the metaphysics of the objects of thought. On the face of the theory, there does not appear to be *a propositional content* that a subject x accepts, when x thinks that it might be the case that p, or that it is likely that p. According to Yalcin's account, for x to think that it might be the case that p (or that it is likely that p) is not for x to have any attitude toward the proposition that it might (or that is likely to) be the case that p. It is rather for x's p-involving thoughts or credences *to be a certain way* (cf. Yalcin 2011).

---

21 Although this worry doesn't present a serious threat to the broad sort of theory of content Yalcin wants to defend, discussing it will provide the essential context for a different worry, that I do take to present such a threat.

A noted difficulty for this type of (*adverbial*, rather than *objectual*) construal of contentful thought is that it makes a hash of disjunctive belief.[22] Rothschild (2012) discusses this example:

(8) Ingrid thinks that [either it's likely Estonia will become a totalitarian state or it's likely Lithuania will become a totalitarian state].

Clearly, Rothschild observes, (8) does not imply either (9) or (10).[23] (More generally, it is possible to think that a disjunction, whose disjuncts are logically independent, is the case, without thinking, of either disjunct, that it is the case.)

(9) Ingrid thinks that it's likely Estonia will become a totalitarian state.

(10) Ingrid thinks that it's likely Lithuania will become a totalitarian state.

But it is (I should say *appears*) difficult to see how this implication could fail in the sort of theory offered above. In fact, assuming the natural treatment of disjunction...

$$[\![\phi \vee \psi]\!]^{w,I} = \top \text{ iff } [\![\phi]\!]^{w,I} = \top \text{ or } [\![\psi]\!]^{w,I} = \top$$

...the result that (8) implies either (9) or (10) follows directly.[24]

The solution to this problem that is offered in Rothschild (2012) (see also Yalcin 2012b) is simple and intuitive, and it involves—crucially, given this chapter's topic—a kind of theoretical *regrouping/retrenchment around the core Hintikkan thesis*: that the content of a doxastic state is to be represented with a set of possibilities. Per Rothschild's suggestion, however, possibilities are not to be construed as possible worlds—ways for the objective circumstances to be. Possibilities are instead to be construed as states of information—*ways of assigning probabilities*, or degrees of belief, to sets of (lower-type, or "alethic") possibilities. The general idea is nicely summarized in the following programmatic slogans. Propositional content[25] is essentially probabilistic in character; to accept a propositional content ϕ is to rule out possibilities that are incompatible with the probabilistic information contained in or expressed by ϕ (see also Swanson 2006; Yalcin 2012a; Moss 2013, 2015, 2018).[26]

---

22 The problem here is noted in Swanson (2006) and discussed at length in Schroeder (2015) and Rothschild (2012) (note that Schroeder's paper preceded Rothschild's in the literature). As Schroeder observes (and Rothschild and Yalcin (2012b) echo), there is a closely related problem concerning the treatment of negation in a theory like Yalcin's. Rothschild and Yalcin (2012b) propose to treat these problems in essentially identical fashion.

23 In Charlow (2015), I resisted the idea that predicting an implication similar to this one was problematic. This line involves us in theoretical and technical complexities that I'd now just as soon avoid.

24 *Proof.* Let '$\triangle$' abbreviate 'it is likely that', and **E** (or **L**) the set of E- (or L)-worlds in $\cup S_{I_x^w}$.

$$[\![x \text{ thinks } \triangle E \vee \triangle L]\!]^{w,I} = \top \text{ iff } \forall v \in \cup S_{I_x^w} : [\![\triangle E \vee \triangle L]\!]^{v,I_x^w} = \top$$
$$\text{iff } \forall v \in \cup S_{I_x^w} : [\![\triangle E]\!]^{v,I_x^w} = \top \text{ or } [\![\triangle L]\!]^{v,I_x^w} = \top$$
$$\text{iff } \forall v \in \cup S_{I_x^w} : \Pr_{I_x^w}(\mathbf{E}) > .5 \text{ or } \forall v \in \cup S_{I_x^w} : \Pr_{I_x^w}(\mathbf{L}) > .5$$
$$\text{iff } [\![x \text{ thinks } \triangle E]\!]^{w,I} = \top \text{ or } [\![x \text{ thinks } \triangle L]\!]^{w,I} = \top$$

25 Many philosophers, including myself (Charlow 2014, 2015), have used the phrase "propositional content" to pick out a particular kind of content—i.e., "factual" content, or content with an alethic truth condition. Schroeder (2011) advocates for a more general usage for this phrase, according to which a propositional content is nothing more than a relatum of an attitude like belief. I am following Schroeder's usage here.

26 Willer (2013) offers a similar theory, though only for epistemic 'might' and 'must', and within an Update/Dynamic Semantic setting.

Here is a theory of thought that is appropriate to this theoretical orientation. Let $\mathcal{I}_x^w$ designate the *set of states of information I*, such that I is compatible with what x thinks in w.[27]

$$[\![x \text{ thinks } \phi]\!]^{w,I} = \top \text{ iff } \forall I \in \mathcal{I}_x^w : \forall v \in \cup S_I : [\![\phi]\!]^{v,I} = \top$$

According to this theory, x's doxastic state at w, $\mathcal{I}_x^w$, isn't well-represented with a single way of assigning probabilities to sets of worlds. It is instead represented with a *set of* ways of assigning probabilities to sets of worlds. The general idea—which should be a familiar one for the Hintikkan—is that 'thinks' is a universal quantifier over states of information that are compatible with the subject's beliefs (and "derivatively", we might say, over the worlds that are compatible with those states of information). All told this represents a fairly minor modification of the theory described in the last section. (Note that the rest of that theory's clauses receive no modification at all.)

The resulting theory—I will call it the Imprecise Bayesian (or just Imprecise) theory—still explains why (5-7) are defective. For Biff to think that it's likely to rain and that it won't rain (at w), it will have to be the case that:

$$\forall I \in \mathcal{I}_{\text{Biff}}^w : \forall v \in \cup S_I : [\![\triangle R]\!]^{v,I} = [\![\neg R]\!]^{v,I} = \top$$

Let I be any possibility in $\mathcal{I}_{\text{Biff}}^w$. Note that if for any world v in $\cup S_I$, $[\![\neg R]\!]^{v,I} = \top$, then the probability I assigns R (at w) equals zero. But then it cannot be the case that $[\![\triangle R]\!]^{v,I} = \top$.

The theory achieves this prediction while avoiding the prediction that (8) implies either (9) or (10). Per the theory, Ingrid thinks $\triangle E \vee \triangle L$ (at w) exactly if, at each information state I compatible with that Ingrid thinks (at w), according to I it is likely that E or it is likely that L.

$$\forall I \in \mathcal{I}_{\text{Ingrid}}^w : \forall v \in \cup S_I : [\![\triangle E \vee \triangle L]\!]^{v,I} = \top$$
$$\text{iff } \forall I \in \mathcal{I}_{\text{Ingrid}}^w : \forall v \in \cup S_I : [\![\triangle E]\!]^{v,I} = \top \text{ or } [\![\triangle L]\!]^{v,I} = \top$$

It is evident that $\mathcal{I}_{\text{Ingrid}}^w$ can satisfy this condition, without it being the case that either of the following conditions hold:

$$\forall I \in \mathcal{I}_{\text{Ingrid}}^w : \forall v \in \cup S_I : [\![\triangle E]\!]^{v,I} = \top$$
$$\forall I \in \mathcal{I}_{\text{Ingrid}}^w : \forall v \in \cup S_I : [\![\triangle L]\!]^{v,I} = \top$$

The Imprecise theory is the state-of-the-art, when it comes to theorizing the content of a doxastic state (in the broad Hintikkan tradition as filtered through Stalnaker, Lewis, Heim). It aims to accommodate a truism about states of thought directed at claims with "epistemic", "informational", or "probabilistic" content—a truism with which the Precise version of the theory struggled, namely: agents are often not in a "fully" opinionated doxastic state with respect to such claims.[28] It accommodates this truism, while basically duplicating the Precise theory's success in accounting for data like (5-7).

---

27 This is in essence the theory proposed in Rothschild (2012). Again I take some minor liberties with the theory of acceptance (and acceptance-like states) stated in Yalcin (2012b).

28 In the epistemological literature it has been controversial whether this involves a defect of rationality. Joyce (2005, 2010) argues quite convincingly that it does not. The fact that this theory appears to dovetail perfectly with the picture of content that is implicit in the epistemological literature on Imprecise Credence is one of its more theoretically striking features.

9

Alas the Imprecise theory still makes a hash of "higher-order" states of uncertainty (of which thoughts with disjunctive contents are but a relatively peripheral sub-type). To see this, let us introduce numerical probabilistic operators with the following semantics.

$$[\![[\alpha]\phi]\!]^{w,I} = \top \text{ iff } \mathrm{Pr}_I(\{v \in \cup S_I : [\![\phi]\!]^{v,I} = \top\}) = \alpha$$

(C1)-(C3) below are all fairly direct predictions of the theory; the reader should work through the proofs independently (see discussion in Moss 2015; Charlow 2020). For any w and I (such that all relevant "visibility" presuppositions are satisfied by I):

(C1) $[\![[1]\Diamond\phi \vee [0]\Diamond\phi]\!]^{w,I} = \top$

(C2) $[\![[1]\triangle\phi \vee [0]\triangle\phi]\!]^{w,I} = \top$

(C3) $[\![[1](\phi \Rightarrow \psi) \vee [0](\phi \Rightarrow \psi)]\!]^{w,I} = \top$

Though it may be possible to defend (C1),[29] there is no sense in defending (C2) or (C3). These are false predictions: neither $\ulcorner[1]\triangle\phi \vee [0]\triangle\phi\urcorner$ nor $\ulcorner[1](\phi \Rightarrow \psi) \vee [0](\phi \Rightarrow \psi)\urcorner$ is in the neighborhood of logical truth (again see Moss 2015; Charlow 2020). Here of course my main concern is how these predictions affect the attendant semantic theory of propositional content. But, as you might imagine, the effect is not good.

The discussion of the prior section may have left you with the impression that the Imprecise theory *is* able to accommodate states of higher-order uncertainty. In the Imprecise theory, when Ingrid's information (at w) is compatible with both $\triangle E$ and $\neg\triangle E$, it appears straightforward to represent this feature of Ingrid's information by requiring that $\mathcal{I}^w_{\text{Ingrid}}$ contain an I such that according to I, E is likely, *and* an I' such that according to I', E is not likely.

That impression is however false. The reason is simple: it is possible for Ingrid to have a non-extremal degree of confidence in $\triangle E$—e.g., for Ingrid to have a high degree of confidence (falling short of certainty) in $\triangle E$—without Ingrid being inclined to think that either $[1]\triangle E$ or $[0]\triangle E$ is the case. According to the Imprecise theory, it isn't. Given (C2), according to every possibility compatible with $\mathcal{I}^w_{\text{Ingrid}}$, $[1]\triangle E$ is the case, or else $[0]\triangle E$ is (Charlow 2020: 774). So $\mathcal{I}^w_{\text{Ingrid}}$ cannot represent the sort of doxastic state we are attempting to attribute to Ingrid, when we imagine Ingrid as having high but non-extremal degree of confidence in $\triangle E$. The objection, in brief, is that when Ingrid's confidence in $\triangle E$ is of a non-extremal degree, Ingrid is not inclined to accept that $[1]\triangle E \vee [0]\triangle E$. But, according to the Imprecise theory, she is.

The reader may be skeptical about the idea of non-extremal degrees of confidence targeting explicitly probabilistic claims.[30] If that describes you, note that the point may also be made for indicative conditionals—a category of claims for which the idea of non-extremal confidence should be familiar and anodyne (note that on the present theory indicative conditionals are treated as a sub-type of probabilistic claim; for extensive discussion, see Charlow 2016, 2021). Given (C3), the dialectic sketched above can be recreated, mutatis mutanda, for indicative conditionals.

---

[29] In response to a critique by Sorensen (2009), Yalcin (2009) offers a defense of a principle that is very similar to (C1). See Charlow (2020: 770) for a rebuttal to Yalcin.

[30] Skepticism isn't warranted. In addition to Moss (2013, 2018), see Gaifman (1988) and the ensuing literature. According to Gaifman's theory, probabilistic claims are understood as claims about the "True" Chances (for a critique see Charlow 2020: 775). Note that an analysis like Gaifman's will struggle to make sense of data like (5).

10

This chapter reads like a catalogue of headaches for the Hintikkan. Even for areas where a Hintikkan view seems to promise some kind of theoretical advance, as in the case of perspectival thought, that promise has turned out to be false.

In the remainder, however, I want to lay out some reasons for optimism for the Hintikkan theory—more specifically, what I'll call a *Flexible Types* version of the ("Informational", rather than "alethic") Hintikkan theory. This section outlines the Flexible Types theory, and revisits the analysis of perspectival thought in light of it. The subsequent section revisits some of the central problems for the Hintikkan theory, in light of the Flexible Types version of it. To preview: the strategy there will be to analyze trouble cases, like Frege's Puzzle, as cases of perspectival thought.

A core idea of the Flexible Hintikkan theory is that the possibilities over which attitude verbs quantify come in a variety of types. A possibility can, depending on the "representational context", be a world or an informational perspective (or, more generally, a way of representing with respect to a relevant representational question). A possibility can in certain contexts even be a *set of* lower type possibilities (i.e., an object that is understood as a proposition in c may be understood as a possibility in a different context c'). The maximally "general" notion of a possibility is just the notion of a *point on which the extension of a clause depends*.

In the Flexible Hintikkan theory, a proposition is a set of possibilities, of any *one* of these types; so, the notion of a proposition, like the notion of a possibility, is a type-flexible notion. There is an attendant impact on the theory of propositional attitudes. According to the Flexible theory, in the course of evaluating an ascription of thought $\ulcorner x$ thinks that $\phi\urcorner$, context is required to specify a type of possibility among which x's doxastic state discriminates (i.e., between possibilities that are compatible with x's doxastic state in c, and possibilities that are ruled out by x's doxastic state in c).[31] For x to think that ϕ in c (relative to w), x's doxastic state with respect to the c-relevant representational question, $\Pi_x^{c,w}$, must entail ϕ's propositional content with respect to c.

$$\llbracket \mathit{thinks} \rrbracket^{c,w} = \lambda\omega_{\langle\pi,t\rangle} . \lambda x_{\langle e\rangle} . \Pi_x^{c,w} \subseteq \omega$$
$$\llbracket x \ \mathit{thinks} \ \phi \rrbracket^{c,w} = \top \text{ iff } \Pi_x^{c,w} \subseteq \llbracket\phi\rrbracket^c$$

For present purposes I'll formulate the remainder of the Flexible Theory for a language containing only epistemic/probability operators. (The reader can skim these clauses—I will talk through their important aspects as appropriate.) Then I will show the theory in action, by revisiting the issues that have occupied our attention for the last few pages. The Flexible theory's assignment of truth conditions is fairly standard (although it requires relativizing truth

---

31 So, per the Flexible theory, there is strictly no such thing as x's doxastic state "full stop" or "sans phrase".

to informational perspectives, in addition to contexts and worlds of evaluation).

$$\llbracket p \rrbracket^{c,w,I} = \top \text{ iff } p(w) = \top$$

$$\llbracket \neg\phi \rrbracket^{c,w,I} = \top \text{ iff } \llbracket \phi \rrbracket^{c,w,I} = \bot$$

$$\llbracket \phi \wedge \psi \rrbracket^{c,w,I} = \top \text{ iff } \llbracket \phi \rrbracket^{c,w,I} = \llbracket \psi \rrbracket^{c,w,I} = \top$$

$$\llbracket \Diamond\phi \rrbracket^{c,w,I} = \top \text{ iff } \exists s \in \mathcal{S}_I : s \cap \llbracket \phi \rrbracket^c = s$$

$$\llbracket \triangle\phi \rrbracket^{c,w,I} = \top \text{ iff } \mathrm{Pr}_I(\llbracket \phi \rrbracket^c) > .5$$

$$\llbracket \phi \Rightarrow \psi \rrbracket^{c,w,I} = \top \text{ iff } \mathrm{Pr}_I(\llbracket \psi \rrbracket^c | \llbracket \phi \rrbracket^c) = 1$$

The content mapping $\llbracket \cdot \rrbracket^c$ is defined recursively (basically as in Charlow 2020).

$$\llbracket p \rrbracket^c = \{w : p(w) = \top\}$$

$$\llbracket \neg\phi \rrbracket^c = \overline{\llbracket \phi \rrbracket^c}$$

$$\llbracket \phi \wedge \psi \rrbracket^c = \llbracket \phi \rrbracket^c \cap \llbracket \psi \rrbracket^c$$

$$\llbracket \Diamond\phi \rrbracket^c = \{I : \exists s \in \mathcal{S}_I : s \cap \llbracket \phi \rrbracket^c = s\}$$

$$\llbracket \triangle\phi \rrbracket^c = \{I : \mathrm{Pr}_I(\llbracket \phi \rrbracket^c) > .5\}$$

$$\llbracket \phi \Rightarrow \psi \rrbracket^c = \{I : \mathrm{Pr}_I(\llbracket \psi \rrbracket^c | \llbracket \phi \rrbracket^c) = 1\}$$

The main thing to note in this formal treatment is that contents come in two general types: sets of worlds, and sets of informational perspectives. (Note that an informational perspective is itself a type-flexible notion. More on this below.)

The Flexible theory agrees, broadly, with the Imprecise theory about the analysis of cases like (8). According to both theories, to say that Ingrid thinks that $\triangle E \vee \triangle L$, is to say that Ingrid('s doxastic state) rules out possibilities—for both theories, these are ways of assigning credence to sets of worlds—according to which it is neither the case that $\triangle E$ nor $\triangle L$.

$$\llbracket \textit{Ingrid thinks } \triangle E \vee \triangle L \rrbracket^{c,w,I} = \top \text{ iff } \Pi_{\mathrm{Ingrid}}^{c,w} \subseteq \llbracket \triangle E \vee \triangle L \rrbracket^c$$

$$\text{iff } \Pi_{\mathrm{Ingrid}}^{c,w} \subseteq \{I : \mathrm{Pr}_I(\llbracket E \rrbracket^c) > .5 \text{ or } \mathrm{Pr}_I(\llbracket L \rrbracket^c) > .5\}$$

However, none of (C1), (C2), or (C3) are predictions of the Flexible theory, assuming that we treat probability operators $\ulcorner[\alpha]\urcorner$, like the operator $\triangle$, as taking arguments of flexible types.

$$\llbracket [\alpha]\phi \rrbracket^{c,w,I} = \top \text{ iff } \mathrm{Pr}_I(\llbracket \phi \rrbracket^c) = \alpha$$

$$\text{So: } \llbracket [\alpha]\triangle E \rrbracket^{c,w,I} = \top \text{ iff } \mathrm{Pr}_I(\llbracket \triangle E \rrbracket^c) = \alpha$$

$$\text{iff } \mathrm{Pr}_I(\{I' : \mathrm{Pr}_{I'}(\llbracket E \rrbracket^c) > .5\}) = \alpha$$

Unlike the Imprecise theory, the Flexible theory contains no stipulations that rule out the possibility that for some I, $\mathrm{Pr}_I(\llbracket \triangle E \rrbracket^c)$ is a non-extremal credence (e.g. .7). And so it is possible, given the Flexible theory, that Ingrid thinks that $\triangle E$ is likely to such a degree.

$$\llbracket \textit{Ingrid thinks } [.7]\triangle E \rrbracket^{c,w,I} = \top \text{ iff } \Pi_{\mathrm{Ingrid}}^{c,w} \subseteq \llbracket [.7]\triangle E \rrbracket^c$$

$$\text{iff } \Pi_{\mathrm{Ingrid}}^{c,w} \subseteq \{I' : \mathrm{Pr}_{I'}(\llbracket \triangle E \rrbracket^c) = .7\}$$

$$\text{iff } \Pi_{\mathrm{Ingrid}}^{c,w} \subseteq \{I' : \mathrm{Pr}_{I'}(\{I'' : \mathrm{Pr}_{I''}(E) > .5\}) = .7\}$$

By allowing an agent's confidence to be distributed *over ways of assigning likelihood* to ordinary "factual" claims, the Flexible theory renders intelligible states of higher-order uncertainty directed at claims with "non-factual" or "perspectival" (in this case, probabilistic) content.

The perspectival aspect of the Flexible theory also allows it to accommodate data with which the original Hintikkan theory struggled—data like (5-7). I will consider (5) as a proof of concept. According to the Flexible theory:

$$[\![\textit{Biff thinks } \triangle R \wedge \neg R]\!]^{c,w,I} = \top \text{ iff } \Pi^{c,w}_{\text{Biff}} \subseteq [\![\triangle R \wedge \neg R]\!]^c$$

But, according to the Flexible theory, $[\![\triangle R]\!]^c$ and $[\![\neg R]\!]^c$ are of distinct semantic types; the first is a set of informational perspectives, the latter is a set of worlds. In a compositional semantic setting, $[\![\triangle R \wedge \neg R]\!]^c$ will be undefined, unless the semantic types of $[\![\triangle R]\!]^c$ and $[\![\neg R]\!]^c$ are *coordinated*. Supposing that $[\![\triangle R]\!]^c$ is the type of a function mapping informational perspectives into truth values (equivalently, a set of informational perspectives), semantic coordination is achieved by "lifting" the type of $\neg R$, so that, if $[\![\triangle R \wedge \neg R]\!]^c$ is defined, $[\![\neg R]\!]^c$ is of the same type as $[\![\triangle R]\!]^c$.[32] In particular, $[\![\neg R]\!]^c$ will be a property of an informational perspective (specifically, the property such a perspective has when, according to it, there is no chance that R).[33] But there is no I such that, according to I, the chance of R is greater than .5 and there is no chance that R. Therefore, given these assumptions, $[\![\triangle R \wedge \neg R]\!]^c = \emptyset$. As desired, then, the complement of (5) does not designate a content that is fit for being the object of a propositional attitude.

The Flexible theory is in essence a Hintikkan theory, with a twist in the theory of content. The theory of content treats the nature of content as a context-sensitive affair. Once a context is fixed—thereby fixing a type of possibility among which representational attitudes in that context must discriminate—the theory works in a fairly standard fashion. In particular, propositional contents are sets of possibilities, and propositional attitudes are analyzed in terms of the possibilities with which they are (in)compatible.

11

The Flexible theory offers a pleasingly general perspective on the semantic type of a proposition. In this section I will illustrate the point by arguing that the theory can be generalized to handle

---

32 For the classic discussion of type-lifting as a route to semantic coordination, see Partee & Rooth (1983).

33 Although type-lifting plays a very different role in their theories, both Moss (2015) and Charlow (2020) require that, if $[\![\triangle R \wedge \neg R]\!]^c$ is defined, the content of $\neg R$ in c be treated as the property an informational perspective has when, according to it, there is no chance that R. In either theory, the application of type-lifting mechanisms will be constrained in basically the way envisioned in Partee & Rooth (1983). The general idea will be to "use the lowest types possible" that are required for composing meanings via Function-Argument Application.

An important difference between Moss' theory and the Flexible theory is that, in Moss' theory (as well as the theories of Yalcin and Rothschild) content is univocally typed (approximately, as a set of probability measures). So, in Moss' theory, a type-shifting operator must be applied to the "classical" content of a sentence like 'it is raining' to yield something fit for belief/assertion. Apart from the worry that this is an over-generalization of Bayesian ideas/modeling techniques, this idea seems to involve a distortion in the theory of content; there are reasons to distinguish the content of p from the content of $[1]p$. Note, e.g., the fact that these contents embed differently under negation; the meaning of 'that's not the case' changes depending on whether 'that' refers to the claim that p or the claim that $[1]p$.

16

Frege's Puzzle. More precisely, I'll show that a promising approach to Frege's Puzzle can be smoothly integrated into the Flexible theory (and that there is an empirical benefit to doing so).

According to a rich vein in recent semantic theorizing (see in particular Cumming 2008; Santorio 2012; Schoubye 2020):

- Proper names are semantically variables, which is to say that they, like any variable over individuals, have their (type: e) extensions relative to assignment functions.
- Certain intensional operators in natural language—specifically, epistemic modals and attitude verbs—are quantifiers over assignment functions in addition to possible worlds.

The Variabilist approach to proper names aims to account for a range of data that other approaches have, in the main, neglected. In particular:

- The fact that proper names—like pronouns, which are the assignment-sensitive expression *par excellence*—have bound readings. (A well-known example from Geurts (1997): "If a child is christened 'Bambi', Disney will sue Bambi's parents.")
- The fact that proper names have "shifted" interpretations under epistemic modals. (An example from Schoubye (2020): supposing our knowledge doesn't rule out the possibility that Del Naja is responsible for the work of Banksy, there is a true reading of "Del Naja might be Banksy".)

Here I will be content to motivate the Variabilist approach by outlining its application to the version of Frege's Puzzle I described above. Letting g be a variable assignment—a function from variable expressions (according to the Variabilist theory, these include names like 'a', 'b' and 'c') into individuals—the key ideas (here following Cumming 2008) are these:

$$\llbracket a \rrbracket^{w,g} = g(a)$$
$$\llbracket a = b \rrbracket^{w,g} = \top \text{ iff } \llbracket a \rrbracket^{w,g} = \llbracket b \rrbracket^{w,g}$$
$$\llbracket c \text{ thinks that } \phi \rrbracket^{w,g} = \top \text{ iff } \forall \langle w', g' \rangle \in B_{\llbracket c \rrbracket^{w,g}} : \llbracket \phi \rrbracket^{w',g'} = \top$$

The application to Frege's Puzzle is relatively direct.

$$\llbracket Homer \text{ thinks } H=P \rrbracket^{w,g} = \top \text{ iff } \forall \langle w', g' \rangle \in B_{\llbracket Homer \rrbracket^{w,g}} : g'(H) = g'(P)$$

The notion that belief-states are sets of world-assignment pairs no doubt requires unpacking. Cumming suggests the following gloss: $\langle w', g' \rangle \in B_z$ exactly if it is compatible with what z thinks that the actual world is w' and "the reference function'—the function that semantically associates referring terms with individuals—is g' (2008: 545). The basic idea here—certainly not an unfamiliar one in the literature on Frege's Puzzle—is that agents have what we could gloss as metalinguistic/semantic beliefs, in addition to beliefs about matters of "worldly fact."[34] Although in actuality 'Hesperus' and 'Phosphorus' co-refer—i.e., $g(H) = g(P)$—this doesn't

---

34 The view here should be distinguished from Metalinguistic Descriptivism, according to which the semantic value of 'Del Naja' is, in any (bare-singular, argument-position) syntactic context, identical to the semantic value of 'the individual called Del Naja' (for a recent version of this view, see Fara 2015). A subtler but still important distinction: the Variabilist theory does not strictly say that someone who thinks that Del Naja is Banksy has a *de dicto* belief that 'Del Naja' and 'Banksy' refer to the same person. What the theory says is that they have beliefs whose truth-value depends, relative to the attributor's context, on a way of mapping referring terms to individuals.

imply that, according to what Homer thinks—that according to all (or any) of the assignments compatible with Homer's beliefs—'Hesperus' and 'Phosphorus' co-refer. Which is as desired.

The "Variabilist" theory will however founder on broadly the same considerations as the Imprecise theory, unless it is able to render intelligible states of uncertainty directed at claims whose content is, according to the theory, metalinguistic/semantic in nature.

(11)   Pythagoras thinks it's likely that Hesperus is Phosphorus.

A natural solution here is to marry the Variabilist theory to the Flexible theory—to permit estimates of likelihood to be distributed over ways of associating referring terms with individuals.[35] Here is an implementation requiring only a light emendation to the Flexible theory:

$$\llbracket a = b \rrbracket^{c,w,I} = \top \text{ iff } g_I(a) = g_I(b) \qquad \llbracket a = b \rrbracket^c = \{I : g_I(a) = g_I(b)\}$$

The type of perspective that is relevant to semantic evaluation of $\ulcorner x = y \urcorner$ is, according to this theory, a perspective I that associates individuals with referring terms (i.e., variables), so that the assignment g_I is defined. On this theory:

$$\llbracket Pythagoras \text{ } thinks \text{ } \triangle H = P \rrbracket^{c,w,I} = \top \text{ iff } \Pi^{c,w}_{\llbracket Pythagoras \rrbracket^{c,w,I}} \subseteq \llbracket \triangle H = P \rrbracket^c$$
$$\text{iff } \Pi^{c,w}_{Pythagoras} \subseteq \{I' : \Pr_{I'}(\llbracket H = P \rrbracket^c) > .5\}$$
$$\text{iff } \Pi^{c,w}_{Pythagoras} \subseteq \{I' : \Pr_{I'}(\{I'' : g_{I''}(H) = g_{I''}(P)\}) > .5\}$$

Roughly, Pythagoras thinks it's likely that Hesperus is Phosphorus if *he has high confidence in a perspective according to which 'Hesperus' and 'Phosphorus' co-refer.*

The reader will note that this doesn't yet account for the independence of (12) and (13).

(12)   Homer thinks Phosphorus is visible in the morning.

(13)   Homer thinks Hesperus is visible in the morning.

It is important to note, however, that (12) and (13) are independent only on a specific reading— the reading commonly known as *de dicto*. In the Flexible theory, it is natural to treat the content of 'Phosphorus is visible in the morning' as context-sensitive, in the sense that the *de dicto* reading, which is salient in certain contexts, characterizes a different kind of possibility than the *de re*, which can also be salient in certain contexts. More precisely, in some contexts, the sentence is used to characterize a perspective according to which (the object called) Phosphorus is visible in the morning, in other contexts it is used to characterize a set of "actually possible" worlds in which Phosphorus (and hence Hesperus) is visible in the morning. That is to say,

---

35 A slightly different way of effecting the marriage—distributing credence over sets of world-assignment or world-perspective pairs—is suggested by the treatment of "modal credence" in Goldstein (forthcoming). A different treatment still is suggested in Goldstein & Santorio (forthcoming). The theory offered here requires less of a departure from extant ways of theorizing about propositional attitudes (although the interested reader should certainly familiarize himself with these theories and form their own judgments). Our theory's use of type-shifting involves us in some complexities (see critical discussion in Mandelkern (2019) and the partial reply in Charlow (2021)). Apart from its use of type-shifting, the theory is otherwise quite standard in nature.

there exist c and c' such that:

$$\llbracket P \text{ is visible...} \rrbracket^c = \{I : \forall w \in \cup S_I : g_I(P) \text{ is visible in the morning in } w\}$$
$$\neq \{I : \forall w \in \cup S_I : g_I(H) \text{ is visible in the morning in } w\}$$
$$\llbracket P \text{ is visible...} \rrbracket^{c'} = \{w : g_{c'}(P) \text{ is visible in the morning in } w\}$$
$$= \{w : g_{c'}(H) \text{ is visible in the morning in } w\}$$

Though I don't here venture a theory of what, in general, distinguishes c and c' of this type, the rough idea I hope will be fairly intuitive: when we are talking about what Homer thinks, sometimes what matters is how Homer thinks of Phosphorus. Other times, what matters is what actual possibilities are (in)compatible with Homer's thoughts.

12

Before concluding let's revisit some loose ends, concerning the role of *impossibilities* in a theory of propositional content. The impossible worlds theorist has a distinctive way of talking about the sorts of possibilities that I've advocated representing as "higher-type" possibilities. We've understood a doxastic possibility for Homer (when Homer thinks Hesperus isn't Phosphorus) as a perspective I such that "according to" I Hesperus isn't Phosphorus. The impossible worlds theorist understands a doxastic possibility for Homer as a world *in which* Hesperus isn't Phosphorus. I am inclined to regard these as different theoretical idioms or *façons de parler*. That said we have some reason to prefer a theory that accounts explicitly for the empirical (e.g., binding) data for which Variabilist analyses of names generally excel in accounting.[36]

Something similar will go for Logical Omniscience. In lieu of a worked-out theory, here are some programmatic remarks. Suppose that x thinks that p, that q is a logical consequence of p, that x is aware of the possibility that q, but that x doesn't think that q. According to the impossible worlds theorist, this is explained by the fact that x thinks that p exactly if the doxastic possibilities compatible with what what x thinks are all worlds in which p, and some of these worlds are worlds in which it isn't the case that q. So, according to the impossible worlds theorist, it must be that some of x's doxastically possible worlds are logically impossible. According to my preferred treatment, possibilities—in particular, the higher-type possibilities that represent perspectives—needn't, as a matter of metaphysical or even psychological necessity, be *coherent* (though it is typically a useful idealization to assume that they are). So for example x's state of mind might be well represented with a set of I such that $Pr_I(p) = 1$ and $Pr_I(q)$ is less than 1—even though, for any *probabilistically coherent* measure Pr, $Pr(p)$ must be less than or equal to $Pr(q)$. In such a case, there is a sense in which, according

---

36 The impossible worlds theorist can also talk about data like (5-7) in their own idiom (none to my knowledge do). I have theorized the complement of (5) as characterizing an empty set of (higher-type) doxastic possibilities. For the impossible worlds theorist, $\ulcorner \triangle \phi \wedge \neg \phi \urcorner$ can characterize a set of *doxastically impossible* worlds—a way that things *metaphysically could* be, but *doxastically can't* be. The idea here might be something like this: w is doxastically possible only if w is conceivable. Since the complement of (5) isn't conceivable (cf. Yalcin 2007)...

   ??Imagine/suppose that it's likely to rain and that it won't rain.

...it characterizes no doxastic possibilities. As with Frege's Puzzle, I'm inclined to think that the data here is more perspicuously handled by treating higher-type entities as doxastic possibilities.

to *I*, something logically impossible (at least within the logic of probability) is the case, namely $[1]p \wedge \neg[1]q$, and so there is a sense in which such an *I* represents an impossible situation—a way *the chances could not in fact be*. In this sense, then, we might be inclined to say that *I* represents an impossible world. Does anything of genuine substance rest on such matters of theoretical interpretation? For what it is worth, my inclination here is: probably not.

13

I have here focused on a simple semantic question: what is the semantic type of the content argument of a propositional attitude verb? I have therefore prescinded from many "philosophical" questions about propositions—in particular, metaphysical questions concerning the nature of propositions. That isn't because I think these questions are illegitimate. It's because addressing these questions doesn't seem to get us much traction on the compositional or modeling questions that I am interested in. To conclude, though, I would like to revisit one or two.

Set-theoretic approaches to propositional content are generally regarded as being in competition with Structured, Fregean, and Act-Based approaches to the metaphysics of propositions. Do I regard the Flexible Types theory as being in this sort of competition? Not particularly, no. In the Flexible Types theory, a set of possibilities is used to model a *way of representing* (with respect to some contextually relevant representational question). If you would like to read me as taking a stand on the metaphysical question, there it is: *a proposition is a way of representing, with respect to a relevant representational question* (cf. Charlow 2020). More precisely, a proposition is the kind of thing that rules in/out ways of representing, adoption of which would resolve a relevant representational question. Although, with respect to any context, a proposition is modeled with a set in that context, the things that the proposition rules in/out in a context may nevertheless be very fine-grained (or coarse-grained) things indeed.

For this reason I am not driven to locate a more univocal, less context-dependent, answer to the question: what is a proposition? It depends. When, for example, we attribute to someone the thought that Samuel Clemens wrote *Huckleberry Finn*, do we need to distinguish the representational state of someone who is ignorant of the fact that Samuel Clemens was Mark Twain from the representational state of someone is not? Sometimes, yes—in which case we will avail ourselves of a class of possibilities of higher granularity, to allow us to differentiate a perspective according to which 'Samuel Clemens' and 'Mark Twain' co-refer from a perspective according to which they do not.[37] But, sometimes, no—in which case we are free to avail ourselves of a class of possibilities of lower granularity.

Because perspectives inhere in individuals, there is a natural temptation to read the Flexible Types theory of content as an extension of Lewis's *de se* theory of content. Although there are certainly affinities between Lewis's theory and the Flexible Types theory, my remarks here should indicate why this reading is inapt. Being in a state of mind wherein one rules in/out a way of representing with respect to a relevant question does not imply that one is in a *de se* state of mind. According to the Flexible Types theory, someone who is aware that Samuel

---

37 For philosophical reasons, we might wish to understand a perspective according to which 'Samuel Clemens' and 'Mark Twain' co-refer as a perspective involving the performance of a type of cognitive act whereby 'Samuel Clemens' and 'Mark Twain' are used to refer to the same individual (see e.g. Hanks 2011, 2022; Soames 2014, 2022). Though this can be regarded as an optional philosophical gloss on the theory of Frege's Puzzle sketched above, it should not be regarded as being in competition with it.

Clemens is Mark Twain is someone who rules out—whose state of mind is such that they have eliminated—any perspective I such that $g_I(SC) \neq g_I(MT)$. Ruling out any perspective I such that $g_I(SC) \neq g_I(MT)$ does not imply that one self-ascribes the property of having a perspective I such that $g_I(SC) = g_I(MT)$. It implies merely that we cannot correctly attribute to them a perspective I according to which 'Samuel Clemens' and 'Mark Twain' fail to co-refer. In general the state of having a perspective with a certain feature is distinct from the state of being aware that one has a perspective with that feature.

References

Beddor, Bob. 2020. Relativism and expressivism. In Martin Kusch (ed.) *The Routledge Handbook of Philosophy of Relativism*, 528–539. Routledge. https://philpapers.org/rec/BEDRAE.

Berto, Francesco & Mark Jago. 2019. *Impossible Worlds*. Oxford University Press.

Cappelen, Herman & Josh Dever. 2013. *The Inessential Indexical*. Oxford: Oxford University Press.

Chalmers, David J. 2002. On sense and intension. *Philosophical Perspectives* 16: 135–182. http://www.jstor.org/stable/3840912.

Chalmers, David J. 2004. Epistemic Two-Dimensional semantics. *Philosophical Studies* 118: 153–226. http://www.jstor.org/stable/4321464.

Chalmers, David J. 2011. Frege's puzzle and the objects of credence. *Mind* 120: 587–635. doi:10.1093/mind/fzr046.

Charlow, Nate. 2014. The problem with the Frege-Geach Problem. *Philosophical Studies* 167: 635–665. doi:10.1007/s11098-013-0119-5.

Charlow, Nate. 2015. Prospects for an expressivist theory of meaning. *Philosophers' Imprint* 15: 1–43.

Charlow, Nate. 2016. Triviality for restrictor conditionals. *Noûs* 50: 533–564. doi:10.1111/nous.12111.

Charlow, Nate. 2020. Grading modal judgement. *Mind* 129: 769–807. doi:10.1093/mind/fzz028.

Charlow, Nate. 2021. Triviality and the logic of restricted quantification .

Coppock, Elizabeth. 2018. Outlook-based semantics. *Linguistics and philosophy* 41: 125–164. doi:10.1007/s10988-017-9222-y.

Cumming, Samuel. 2008. Variabilism. *The Philosophical Review* 117: 525–554.

Egan, Andy. 2010. Disputing about taste. In Ted Warfield and Richard Feldman (ed.) *Disagreement*, 247–286. Oxford University Press.

Égré, Paul & Mikaël Cozic. 2011. If-clauses and probability operators. *Topoi* 30: 17–29. doi:10.1007/s11245-010-9087-y.

Fara, Delia Graff. 2015. Names are predicates. *The Philosophical Review* 124: 59–117. doi:10.1215/00318108-2812660.

von Fintel, Kai & Irene Heim. 2011. *Intensional Semantics*. MIT. http://web.mit.edu/fintel/fintel-heim-intensional.pdf.

Frege, Gottlob. 1948. Sense and reference. *The Philosophical Review* 57: 209–230. doi:10.2307/2181485.

Gaifman, Haim. 1988. A theory of higher order probabilities. In Brian Skyrms & William L Harper (eds.) *Causation, Chance and Credence: Proceedings of the Irvine Conference on Probability and Causation Volume 1*, 191–219. Dordrecht: Springer Netherlands.

Geurts, Bart. 1997. Good news about the description theory of names. *Journal of Semantics* 14: 319–348. doi:10.1093/jos/14.4.319.

Goldstein, Simon. forthcoming. Epistemic modal credence. *Philosophers' Imprint* https://philpapers.org/rec/GOLEMC.

Goldstein, Simon & Paolo Santorio. forthcoming. Probability for epistemic modalities. *Philosophers' Imprint* https://philpapers.org/rec/GOLPFE-2.

Hanks, Peter W. 2011. Structured propositions as types. *Mind* 120: 11–52. doi:10.1093/mind/fzr011.

Hanks, Peter W. 2022. The view of propositions as types of actions. In Chris Tillman & Adam Russell Murray (eds.) *The Routledge Handbook of Propositions*. Routledge.

Heim, Irene. 1992. Presupposition projection and the semantics of attitude verbs. *Journal of Semantics* 9: 183–221. doi:10.1093/jos/9.3.183.

Hintikka, Jaakko. 1962. *Knowledge and Belief: An Introduction to the Logic of Two Notions*. Ithaca: Cornell University Press.

Hintikka, Jaakko. 1969. Semantics for propositional attitudes. In J W Davis, D J Hockney & W K Wilson (eds.) *Philosophical Logic*, 21–45. Dordrecht: D. Reidel.

Hintikka, Jaakko. 1975. Impossible possible worlds vindicated. *Journal of Philosophical Logic* 4: 475–484. http://www.jstor.org/stable/30226996.

Jackson, Frank. 1998. *From Metaphysics to Ethics: A Defence of Conceptual Analysis*. Oxford: Clarendon Press.

Joyce, James. 2005. How probabilities reflect evidence. *Philosophical Perspectives* 19: 153–178. doi:10.1111/j.1520-8583.2005.00058.x.

Joyce, James. 2010. A defense of imprecise credences in inference and decision making. *Philosophical Perspectives* 24: 281–323. doi:10.1111/j.1520-8583.2010.00194.x.

Kaplan, David. 1989. Demonstratives. In J. Almog, J. Perry & H. Wettstein (eds.) *Themes from Kaplan*, 481–563. Oxford: Oxford University Press. https://philpapers.org/rec/KAPD.

King, Jeffrey C. 2022. Syntactically structured propositions. In Chris Tillman & Adam Russell Murray (eds.) *The Routledge Handbook of Propositions*. Routledge.

Kripke, Saul A. 1980. *Naming and Necessity*. Blackwell: Harvard University Press.

Lasersohn, Peter. 2005. Context dependence, disagreement, and predicates of personal taste. *Linguistics and Philosophy* 28: 643–686. doi:10.1007/s10988005-0596-x.

Lassiter, Daniel. 2011. Measurement and modality. Ph.D. Dissertation, NYU.

Lewis, David. 1979. Attitudes *de dicto* and *de se*. *The Philosophical Review* 88: 513–543.

Lewis, David. 1980. Index, context, and content. In S. Kanger & S. Ohman (eds.) *Philosophy and Grammar*, 79–100. Holland: D. Reidel. https://andrewmbailey.com/dkl/Index_Context_Content.pdf.

Lewis, David. 1986. *On the Plurality of Worlds*. Oxford: Blackwell.

MacFarlane, John. 2014. *Assessment Sensitivity: Relative Truth and Its Applications*. Oxford: Oxford University Press.

Mandelkern, Matthew. 2019. Bounded modality. *Philosophical Review* 128(1): 1–61. doi:10.1215/00318108-7213001.

Moss, Sarah. 2013. Epistemology formalized. *The Philosophical Review* 122: 1–43. doi:10.1215/00318108-1728705.

Moss, Sarah. 2015. On the semantics and pragmatics of epistemic vocabulary. *Semantics and Pragmatics* 8: 1–81. doi:10.3765/sp.8.5.

Moss, Sarah. 2018. *Probabilistic Knowledge*. Oxford: Oxford University Press.

Murray, Adam Russell. 2022. Propositional dependence and perspectival shift. In Chris Tillman & Adam Russell Murray (eds.) *The Routledge Handbook of Propositions*. Routledge.

Ninan, Dilip. 2018. Relational semantics and domain semantics for epistemic modals. *Journal of Philosophical Logic* 47: 1–16. doi:10.1007/s10992-016-9414-x.

Nolan, Daniel. 1997. Impossible worlds: A modest approach. *Notre Dame Journal of Formal Logic* 38: 535–572. doi:10.1305/ndjfl/1039540769.

Nolan, Daniel P. 2013. Impossible worlds. *Philosophy compass* 8: 360–372. doi:10.1111/phc3.12027.

Partee, Barbara H. & Mats Rooth. 1983. Generalized conjunction and type ambiguity. In R. Bäuerle, C. Schwarze & A. von Stechow (eds.) *Meaning, Use, and Interpretation of Language*, 361–383. Berlin: Walter de Gruyter.

Pearson, Hazel. 2013. A judge-free semantics for predicates of personal taste. *Journal of Semantics* 30: 103–154. doi:10.1093/jos/ffs001.

Pearson, Hazel. 2020. Attitude verbs. In D. Gutzmann, L. Matthewson, C. Meier, H. Rullmann & T. Zimmerman (eds.) *The Wiley Blackwell Companion to Semantics*. https://semanticsarchive.net/Archive/mIwMTc1Z/Attitude%20verbs%20Pearson%20to%20appear.pdf.

Perry, John. 1977. Frege on demonstratives. *The Philosophical Review* 86: 474–497. doi:10.2307/2184564.

Perry, John. 1979. The problem of the essential indexical. *Noûs* 13: 3–21. doi:10.2307/2214792.

Restall, Greg. 1997. Ways things can't be. *Notre Dame Journal of Formal Logic* 38(4): 583–596. doi:10.1305/ndjfl/1039540771.

Rothschild, Daniel. 2012. Expressing credences. *Proceedings of the Aristotelian Society* CXII, Part 1: 99–114. doi:10.1111/j.1467-9264.2012.00327.x.

Santorio, Paolo. 2012. Reference and monstrosity. *The Philosophical review* 121: 359–406. doi:10.1215/00318108-1574427.

Schoubye, Anders J. 2020. Names are variables. *The Philosophical Review* 129: 53–94. doi:10.1215/00318108-7890468.

Schroeder, Mark. 2011. Two roles for propositions: Cause for divorce? *Noûs* 47: 409–430. doi:10.1111/j.1468-0068.2011.00833.x.

Schroeder, Mark. 2015. Attitudes and epistemics. In *Expressing Our Attitudes*, 225–262. Oxford: Oxford University Press. doi:10.1093/acprof:oso/9780198714149.003.0010.

Soames, Scott. 2014. Cognitive propositions. In *New Thinking about Propositions*. Oxford: Oxford University Press. doi:10.1093/acprof:oso/9780199693764.003.0006.

Soames, Scott. 2022. Cognitive propositions. In Chris Tillman & Adam Russell Murray (eds.) *The Routledge Handbook of Propositions*. Routledge.

Sorensen, Roy. 2009. Meta-agnosticism: Higher order epistemic possibility. *Mind* 118: 777–784. doi:10.1093/mind/fzp105.

Stalnaker, Robert. 1984. *Inquiry*. Cambridge: MIT Press.

Stanley, Jason. 1997. Rigidity and content. In Richard G Heck (ed.) *Language, Truth, and Logic*. Oxford University Press. https://philpapers.org/rec/STARAC.

Stephenson, Tamina. 2007a. Judge dependence, epistemic modals, and predicates of personal taste. *Linguistics and Philosophy* 30: 487–525. doi:10.1007/s10988-008-9023-4.

Stephenson, Tamina C. 2007b. *Towards a Theory of Subjective Meaning*. Ph.D. thesis, MIT.

Swanson, Eric. 2006. Interactions with context. http://www-personal.umich.edu/~ericsw/Swanson,%20Interactions%20with%20Context.pdf. Ph.D. Dissertation, MIT.

Textor, Mark. 2022. Frege on thoughts. In Chris Tillman & Adam Russell Murray (eds.) *The Routledge Handbook of Propositions*. Routledge.

Willer, Malte. 2013. Dynamics of epistemic modality. *The Philosophical Review* 122: 45–92. doi:10.1215/00318108-1728714.

Yalcin, Seth. 2007. Epistemic modals. *Mind* 116: 983–1026. doi:10.1093/mind/fzm983.

Yalcin, Seth. 2009. More on epistemic modals. *Mind* 118: 785–793. doi:10.1093/mind/fzpl06.

Yalcin, Seth. 2010. Probability operators. *Philosophy Compass* 5: 916–937. doi:10.1111/j.1747-9991.2010.00360.x.

Yalcin, Seth. 2011. Nonfactualism about epistemic modality. In A. Egan & B. Weatherson (eds.) *Epistemic Modality*, 295–332. Oxford: Oxford University Press.

Yalcin, Seth. 2012a. Bayesian expressivism. *Proceedings of the Aristotelian Society* CXII, Part 2: 123–160. doi:10.1111/j.1467-9264.2012.00329.x.

Yalcin, Seth. 2012b. Context probabilism. In M. Aloni, V. Kimmelman, F. Roelofsen, G. Sassoon, K. Schulz & M. Westera (eds.) *Proceedings of the 18th Amsterdam Colloquium*, 12–21. ILLC. doi:10.1007/978-3-642-31482-7_2.

Yalcin, Seth. 2018. Belief as question-sensitive. *Philosophy and Phenomenological Research* 97: 23–47. doi:10.1111/phpr.12330.

Yoon, Chulmin. 2022. Semantic relationalism. In Chris Tillman & Adam Russell Murray (eds.) *The Routledge Handbook of Propositions*. Routledge.