The Simplicity of Physical Laws

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Abstract

Physical laws are strikingly simple, although there is no *a priori* reason they must be so. I propose that nomic realists of all types (Humeans and non-Humeans) should accept that simplicity is a fundamental epistemic guide for discovering and evaluating candidate physical laws. This principle of simplicity addresses several problems of nomic realism and simplicity. A consequence is that the oft-cited epistemic advantage of Humeanism over non-Humeanism disappears, undercutting an influential epistemological argument for Humeanism. Moreover, simplicity is shown to be more tightly connected to lawhood than to mere truth.

Keywords: empirical equivalence, induction, underdetermination, determinism, strong determinism, primitivism, Humeanism, non-Humeanism, laws of nature, Bayesianism, comparative probability, expert principle, theoretical virtues, IBE, nested theories, vagueness

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1 Introduction

Physical laws are strikingly simple, although there is no *a priori* reason they must be so. My goal in this paper is to articulate and clarify the view that simplicity is a fundamental epistemic guide for discovering and evaluating the fundamental laws of physics.

Many physicists and philosophers are realists about physical laws. Call realism about physical laws *nomic realism*. It contains two parts. First, physical laws are objective and mind-independent. Second, we have epistemic access to physical laws. Nomic realism gives rise to an apparent epistemic gap: if physical laws are really objective and mind-independent, it may be puzzling how we can have epistemic access to them, since laws are not consequences of our observations. The epistemic gap can be seen as an instance of a more general one regarding theoretical statements on scientific realism (Chakravartty 2017).

In response to the gap, nomic realists invoke super-empirical theoretical virtues, a familiar example of which is simplicity. Simplicity can be used to eliminate a large class of empirically equivalent theories with complicated laws. Together with other theoretical virtues, it may even yield a unique theory at the end of inquiry given the totality of evidence. However, there are several difficulties with simplicity (Baker 2022, Fitzpatrick 2022):

1. The problem of coherence: naive applications of the principle of simplicity lead to probabilistic incoherence (sometimes called the problem of nested theories, or the problem of conjunctive explanations).

2. The problem of justification: there is no plausible epistemic justification for the principle of simplicity.

3. The problem of precision: there is no precise standard of simplicity.

These problems are by no means unique to simplicity. One can ask similar questions about other super-empirical virtues such as unification and strength. We face similar problems whenever we use super-empirical virtues for guiding our theory choices. What one says about simplicity may well apply to other epistemic guides that nomic realism employs.

I develop a framework for thinking about simplicity as a fundamental epistemic guide to physical laws. When we analyze the content of nomic realism, we discover a straightforward solution to the problem of coherence. When we reflect on the applications of simplicity, we find good reasons for taking it as a fundamental principle that requires no further epistemic justification. When we think about the variety of cases we need to apply it to, we see it is necessary to maintain a vague principle that only takes on precise forms in specific domains.

Is this framework redundant for Humeans? No. It is sometimes believed that, on the best-system account of laws (BSA), we can get the simplicity of physical laws for free, because laws are defined to be simple and informative summaries of the mosaic. That is a mistake, because we are not given the mosaic. The principle of simplicity must be added to BSA as an epistemic norm that guides our expectations about the best system even when we do not have direct epistemic access to the mosaic. As we shall see, since both Humeanism and non-Humeanism need an independent epistemic principle concerning the simplicity of physical laws, they are on a par regarding the empirical discovery of laws. If there is no problem for Humeans to adopt the epistemic guide, there is no problem for non-Humeans either. As a consequence, the oft-cited epistemic advantage of the former over the latter disappears. (See (Hildebrand 2022, §8) for a similar perspective; see also (Chen and Goldstein 2022, §4.1).) This undercuts an influential epistemic argument for Humeanism (Earman and Roberts 2005). Nevertheless, the real targets of the simplicity postulate can be different: on non-Humeanism it is an epistemic guide about which laws we should entertain, while on Humeanism it is ultimately an epistemic guide about which type of mosaics we should take seriously.

Recent works in the foundations of physics and the metaphysics of laws have provided new case studies that call for a more systematic look on the methodological principles required for upholding nomic realism. I offer this framework as a lens to think about various commitments of nomic realism in a more unified way. This might not be the only lens possible, but it has a number of features that would be attractive to nomic realists. For one thing, simplicity is recognized as an important theoretical virtue in scientific practice (Schindler 2022), and it is one principle that nomic realists may already endorse. Its theoretical benefits, as I hope to show, justify the cost of the posit. Although this will not convince nomic anti-realists, they may still find this framework useful for understanding a position they ultimately reject.

Here is the plan. First, I clarify nomic realism and its metaphysical and epistemological commitments. I illustrate the epistemic gap by working through three algorithms for generating empirical equivalents. I introduce the principle of nomic simplicity as a tie-breaker and show how it solves the problem of coherence. Next, I examine five additional applications of the principle of simplicity: induction, symmetries, dynamics, determinism, and explanation. The upshot is that simplicity may be more fundamental than many methodological principles we already accept. The applications suggest that the principle is best taken as a fundamental yet vague epistemic principle. The analysis clarifies why Humeanism has no epistemic advantage over non-Humeanism with respect to our epistemic access to physical laws.

2 Nomic Realism

Nomic realism contains an epistemic gap. Let us start with a more precise formulation of nomic realism. Let nomic realism denote the conjunction of the following two theses:

Metaphysical Realism: Physical laws are objective and mind-independent; more precisely, which propositions express physical laws are objective and mind-independent facts in the world.[1]

Epistemic Realism: We have epistemic access to physical laws; more precisely, we can be epistemically justified in believing which propositions express the physical laws, given the evidence that we will in fact obtain.[2]

Nomic realists would like to hold on to both theses, and the puzzle is how. Ultimately, the puzzle is an instance of a more general puzzle about how we can be justified in believing anything beyond the logical closure of empirical evidence. One can already see that it is closely related to issues about the rationality of induction and scientific explanation, which will be discussed in §4. But first, we need to understand what the gap looks like in specific cases. For concreteness, let us look at a Humean account and a non-Humean account, both of which aspire to satisfy nomic realism. The character of the epistemic gap in these two accounts is representative of the situation across a large class of realist accounts of physical laws.

2.1 Two Accounts

First, consider the Humean best-system account of Lewis (1973, 1983, 1986), with some modifications:

Best System Account (BSA) Fundamental laws of nature are the axioms of the best system that summarizes the mosaic and optimally balances simplicity, informativeness, fit, and degree of naturalness of the properties referred to. The mosaic (spacetime and its material contents) contains only local matters of particular fact, and the mosaic is the complete collection of fundamental facts. The best system supervenes on the mosaic.[3]

BSA satisfies metaphysical realism, even though its laws are not metaphysically fundamental. Given a particular mosaic (spacetime manifold with material contents), there is a unique best system that is objectively best.[4]

[1] A weaker version of metaphysical realism maintains that laws are not entirely mind-dependent. That would accommodate more pragmatic versions of the Humean best-system accounts (e.g. Jaag and Loew, Hicks, Dorst, Callender CITE). The arguments below, with suitable adaptations, still apply.

[2] The terminology is due to Earman and Roberts (2005). Here I've added the clause "given the evidence that we will in fact obtain." My version of epistemic realism is logically stronger than theirs.

[3] A key difference between this version and Lewis's (Lewis 1973, 1983, 1986) is that the latter but not the former requires fundamental laws to be regularities. The other difference is the replacement of perfect naturalness with degree of naturalness. See (Chen 2022b, sect.2.3) for more in-depth comparisons. On Humeanism, the mosaic is often required to be about local matters of particular fact.

[4] For the sake of the argument, I set aside the worry of "ratbag idealism" and grant Lewis's assumption that nature is kind to us (Lewis 1994, p.479). Even if BSA satisfies the weaker version of metaphysical realism where laws are not entirely mind-dependent, it does not automatically secure epistemic realism, as we shall see. Hence, this discussion also applies to recent versions of pragmatic Humeanism.

Next, consider a recent non-Humean account according to which laws govern and exist over and above the material contents (Chen and Goldstein 2022):

Minimal Primitivism (MinP) Fundamental laws of nature are certain primitive facts about the world. There is no restriction on the form of the fundamental laws. They govern the behavior of material objects by constraining the physical possibilities.

MinP satisfies metaphysical realism, because the primitive facts about the world are taken to be objective and mind-independent. It is minimal in the sense that it places no restrictions on the form of fundamental physical laws. MinP is compatible with fundamental laws taking on the form of boundary conditions, least action principles, and global spacetime constraints.[5] (Chen and Goldstein 2022) also posit an epistemic principle called "Epistemic Guides" that we will return to in §5.3.

2.2 Epistemic Gaps

Now, do BSA and MinP vindicate epistemic realism? Their metaphysical posits, by themselves, do not guarantee epistemic realism. This should be clear on MinP. Since there is no metaphysical restriction on the form of the fundamental laws, if they are entirely mind-independent primitive facts about the world, how do we know which propositions are the laws? However, an analogous problem exists on BSA. This claim may surprise some philosophers, as it is often thought that BSA has an epistemic advantage over non-Humean accounts like MinP, precisely because BSA brings laws closer to us. BSA defines laws in terms of the mosaic, and the mosaic is all we can empirically access (Earman and Roberts 2005).

The problem is that *we are not given the mosaic*. Just like physical laws, the mosaic entertained in modern physics is a theoretical entity that is not entailed by our observations. Our beliefs about its precise nature, such as the global structure of spacetime, its microscopic constituents, and the exact matter distribution, are as theoretical and inferential as our beliefs about physical laws. They are all parts of a theory about the physical world. Just as defenders of MinP require an extra epistemic principle to infer what the laws are, defenders of BSA require a similar principle to infer what the mosaic is like. The latter, on BSA, turns out to be equivalent to a strong epistemic principle concerning what we should expect about the best system given *our limited evidence*, which because of its limitation pins down neither the mosaic nor the best system.

After all, on BSA laws are not summaries of our observations only, but of the entire spacetime mosaic constituted by the totality of microphysical facts, a small minority of which show up in our observations. The ultimate judge of which system of propositions is the optimal true summary depends on the entire mosaic, a theoretical entity. (For this reason, BSA should not be confused as a version of strict empiricism.) And in current physics, our best guide to the mosaic is our best guess about the physical laws. At the end of the day, MinP and BSA turn out to require the same super-empirical epistemic principle concerning physical laws. On neither account does the epistemic principle follow from the metaphysical posits about what laws are. (I return to this point in §5.3.)

[5] See Adlam (2022) and Meacham (2023) for related views, and Hildebrand (2020) for an overview of non-Humeanism.

To sharpen the discussion, let us suppose, granting Lewis's assumption of the kindness of nature (Lewis 1994, p.479), that given the mosaic ξ there is a unique best system whose axioms express the fundamental law L:

$$L = BS(\xi) \tag{1}$$

with $BS(\cdot)$ the function that maps a mosaic to its best-system law.[6] Let us stipulate that for both BSA and MinP, physical reality is described by a pair (L, ξ). For both, we must have that $\xi \in \Omega^L$, with Ω^L the set of mosaics compatible with L. This means that L is true in ξ. On BSA, we also have that $L = BS(\xi)$. So in a sense, all we need in BSA is ξ; L is not ontologically extra. But it does not follow that BSA and MinP are relevantly different when it comes to epistemic realism.

Let E denote our empirical evidence consisting of our observational data about physical reality. Let us be generous and allow E to include not just our current data but also all past and future data about the universe that we in fact gather. There are two salient features of E:

- E does not pin down a unique ξ. There are different candidates of ξ that yield the same E. (After all, E is a spatiotemporally partial and macroscopically coarse-grained description of ξ.)
- E does not pin down a unique L. There are different candidates of L that yield the same E. (On BSA, this is an instance of the previous point; on MinP, this is easier to see since L can vary independently of ξ, up to a point.)

Hence, on BSA, just as on MinP, E does not pin down (L, ξ). There is a gap between what our evidence entails and what the laws are. Ultimately, the gap can be bridged by adopting simplicity (among other super-empirical virtues) as an epistemic guide. Nevertheless, it helps to see how big the gap is so that we can appreciate how much work needs to be done by simplicity and other epistemic guides.[7]

2.3 Empirical Equivalence

The epistemic gaps can be illustrated by considering cases of empirical equivalence. If different laws yield the same evidence, it is puzzling how we can be epistemically justified in choosing one over its empirically equivalent rivals, unless we rule them out by positing substantive assumptions that go beyond the metaphysical posits of nomic realism. In the literature (see for example Kukla (1998)), there are suggestions about how to algorithmically generate empirically equivalent rivals, but some of them are akin to radical Cartesian skepticism. Here I offer three new algorithms, modeled after

[6] We might understand pragmatic Humeanism as recommending that we use another best-system function $BS'(\cdot)$ that is "best for us."

[7] It is worth contrasting the current setup with the influential framework suggested by Hall (2009, 2015). In one way of fleshing out the core idea of BSA, Hall imagines a Limited Oracular Perfect Physicist (LOPP) who has as her evidence all of ξ and nothing else. Her evidence E_{LOPP} contains vastly more information than E. On BSA, E_{LOPP} pins down (L, ξ). Notice that E_{LOPP} is as theoretical for Humeans as for non-Humeans. The Humean's best guess about what is in E_{LOPP} depends on her expectation about what L looks like given E. We return to this point in §5.3.

concrete proposals considered in recent discussions in philosophy of physics. They have much more limited scopes, but they should be less controversial.

I shall use a fairly weak notion of empirical equivalence, according to which L_1 and L_2 are empirically equivalent with respect to actual evidence E just in case E is compatible with L_1 and L_2. This criterion, with the emphasis on actual data E, is weaker than the notion of empirical equivalence according to which two laws should agree not just on the actual data but all possible data (data that can in principle be measured in the actual world as well as those in any nomologically possible world). I will drop the explicit reference to E in what follows. There are two reasons for focusing on the weaker notion. First, it is sufficient for illustrating the epistemic gaps. Second, what is in-principle measurable in the actual world and in nomologically possible worlds depends on what the laws are. Using actual data instead of all possible data provides a more level playing field when discussing different hypotheses about laws.

Algorithm A: Moving parts of ontology (what there is in the mosaic) into the nomology (the package of laws).

General strategy. This strategy works on both BSA and MinP. Given a theory of physical reality $T_1 = (L, \xi)$, if ξ can be decomposed into two parts $\xi_1 \& \xi_2$, we can construct an empirically equivalent rival $T_2 = (L \& \xi_1, \xi_2)$, where ξ_1 is moved from ontology to nomology.

Example. Consider the standard theory of Maxwellian electrodynamics, T_{M1}:

- Nomology: Maxwell's equations and Lorentz force law.
- Ontology: a Minkowski spacetime occupied by charged particles with trajectories $Q(t)$ and an electromagnetic field $F(x, t)$.

Here is an empirically equivalent rival, T_{M2}:

- Nomology: Maxwell's equations, Lorentz force law, and an enormously complicated law specifying the exact functional form of $F(x, t)$ that appears in the dynamical equations.
- Ontology: a Minkowski spacetime occupied by charged particles with trajectories $Q(t)$.

Our evidence E is compatible with both T_{M1} and T_{M2}. The outcome of every experiment in the actual world will be consistent with T_{M2}, as long as the outcome is registered as certain macroscopic configuration of particles (Bell 2004). We can think of the new law in T_{M2} as akin to the Hamiltonian function in classical mechanics, which is interpreted as encoding all the classical force laws, except that specifying $F(x, t)$ is much more complicated than specifying a typical Hamiltonian. Both $F(x, t)$ and the Hamiltonian are components of respective laws of nature that tell particles how to move.[8]

[8]Note that we can decompose the standard ontology in many other dimensions, corresponding to more ways to generate empirically equivalent laws for a Maxwellian world. This move is discussed at length by Albert (2022). Similar strategies have been considered in the "quantum Humeanism" literature. See Miller (2014), Esfeld (2014), Callender (2015), Bhogal and Perry (2017), and Chen (2022a).

Algorithm B: Changing the nomology directly.

General strategy. This strategy is designed for MinP. We can generate empirical equivalence by directly changing the nomology. Suppose the actual mosaic ξ is governed by the law L_1. Consider L_2, where $\Omega^{L_1} \neq \Omega^{L_2}$ and $\xi \in \Omega^{L_2}$. L_1 and L_2 are distinct laws because they have distinct sets of models. Since E (which can be regarded as a coarse-grained and partial description of ξ) can arise from both, the two laws are empirically equivalent. There are infinitely many such candidates for Ω^{L_2}. For example, Ω^{L_2} can be obtained by replacing one mosaic in Ω^{L_1} with something different and not already a member of Ω^{L_1}, by adding some mosaics to Ω^{L_1}, or by removing some mosaics in Ω^{L_1}. L_2 is empirically equivalent with L_1 since E is compatible with both.[9]

Example. Let L_1 be the Einstein equation of general relativity, with $\Omega^{L_1} = \Omega^{GR}$, the set of general relativistic spacetimes. Assume that the actual spacetime is governed by L_1, so that $\xi \in \Omega^{L_1}$. Consider L_2, a law that permits only the actual spacetime and completely specifies its microscopic detail, with $\Omega^{L_2} = \{\xi\}$. Since our evidence E arises from ξ, it is compatible with both L_1 and L_2. Since it needs to encode the exact detail of ξ, in general L_2 is much more complicated than L_1.[10]

Algorithm C: Changing the nomology by changing the ontology.

General strategy. This strategy is designed for BSA. On BSA, we can change the nomology by making suitable changes in the ontology (mosaic), which will in general change what the best system is. Suppose the actual mosaic ξ is optimally described by the actual best system $L_1 = BS(\xi)$. We can consider a slightly different mosaic ξ', such that it differs from ξ in some spatiotemporal region that is never observed and yet E is compatible with both ξ and ξ'. There are infinitely many such candidates for ξ' whose best system $L_2 = BS(\xi')$ differs from L_1. Alternatively, we can expand ξ to $\xi' \neq \xi$ such that ξ is a proper part of ξ'. There are many such candidates for ξ' whose best system $L_2 = BS(\xi')$ differs from L_1, even though E is compatible with all of them.

Example. Let L_1 be the Einstein equation of general relativity, with $\Omega^{L_1} = \Omega^{GR}$, the set of general relativistic spacetimes. Assume that the actual spacetime is globally hyperbolic and optimally described by L_1, so that $L_1 = BS(\xi)$. Consider ξ', which differs from ξ in only the number of particles in a small spacetime region R in a far away galaxy that no direct observation is ever made. Since the number of particles is an invariant property of general relativity, it is left unchanged after a "hole transformation" (Norton 2019). We can use determinism to deduce that ξ' is incompatible with general relativity, so that $L_1 \neq BS(\xi')$. Let L_2 denote $BS(\xi')$. $L_1 \neq L_2$ and yet they are compatible with the same evidence we obtain in ξ. Since ξ' violates the conservation of number of particles and perhaps smoothness conditions, L_2 should be more complicated than L_1.

To be sure, we can also combine these algorithms to produce more sophisticated examples of empirical equivalence.[11] Here we have three algorithms with different strategies and scopes that can establish the existence of empirically equivalent rival

[9] See Manchak (2009, 2020) for more examples.

[10] L_2 is a case of strong determinism. See Adlam (2021) and Chen (2022c) for recent discussions.

[11] For example, in certain settings, we can change both the ontology and the nomology to achieve empirical equivalence. For every wave-function realist theory, there is an empirically equivalent density-matrix realist theory Chen (2019). Their ontology and nomology are different, but no experiment can determine which is correct.

laws, for a world like ours. They are inspired by recent discussions in philosophy of physics. None of them requires Cartesian skepticism. If such algorithms are allowed, how can we maintain epistemic realism? We may summarize the puzzle about nomic realism:

Puzzle about Nomic Realism: In such cases of empirical equivalence, what justifies the acceptance of one candidate law over the other?

3 The Principle of Nomic Simplicity

It has been recognized, correctly on my view, that nomic realists need to invoke theoretical virtues as a way to choose among empirically equivalent laws underdetermined by evidence.

3.1 The Problem of Coherence

An important example is the *principle of simplicity* (PS). The basic idea is that simplicity is a guide to truth. It has an intuitive appeal, as the paradigm examples of physical laws are strikingly simple and simpler than other candidates that yield the same data. Moreover, in the examples of empirical equivalence discussed before, the simpler law does seem like the better candidate.

What does it mean for simplicity to be a guide? A guide is not a guarantee. Inferences in the context of uncertainty, even when epistemically justified, are fallible. We can make mistakes when relying on the principle of simplicity. Perhaps the actual physical laws appear less simple than the ones we regard as laws, based on the principle of simplicity. I think a realist should admit this possibility. Indeed it is a hallmark of realism that we can be wrong, even when we follow scientific methodology. This uncertainty can be formulated probabilistically:

Principle of Simplicity (PS) Other things being equal, simpler propositions are more likely to be true. More precisely, other things being equal, for two propositions L_1 and L_2, if $L_1 >_S L_2$, then $L_1 >_P L_2$, where $>_S$ represents the comparative simplicity relation, $>_P$ represents the comparative probability relation.[12]

PS regards simplicity as a guide to *truth*. A proposition being simpler raises its probability of being true relative to a more complicated proposition. Although this is close to the usual gloss of of the idea that simplicity as an epistemic guide, it is the wrong principle for nomic realists. PS faces an immediate problem—the problem of nested theories, or sometimes called the problem of logical constraints.[13]

[12]It may be too demanding to require a total order that induces a *normalizable* probability distribution over the space of all possible laws. It is less demanding to formulate PS in terms of comparative probability.

[13]The problem has been much discussed in the philosophy of science literature but not much in foundations of physics. It was first raised by Popper (2005) against the Bayesian proposal of Wrinch and Jeffreys (1921). For recent discussions, see Sober (2015), Schupbach and Glass (2017) and Henderson (2022).

Problem of Coherence PS leads to probabilistic incoherence.

Whenever two theories have nested sets of models, say $\Omega^{L_1} \subset \Omega^{L_2}$, the probability that L_1 is true cannot be higher than the probability that L_2 is true. For concreteness, consider an example from spacetime physics. Let Ω^{GR} denote the set of models compatible with the fundamental law in general relativity—the Einstein equation, and let Ω^{GR^+} denote the union of Ω^{GR} and a few random spacetime models that do not satisfy the Einstein equation. Suppose there is no simple law that generates Ω^{GR^+}. While the law of GR (the Einstein equation) is presumably simpler than that of GR^+, the former cannot be more likely to be true than the latter, since every model of GR is a model of GR^+, and not every model of GR^+ is a model of GR. This is an instance of the problem of nested theories, as Ω^{GR} is a subclass of and nested within Ω^{GR^+}.

3.2 The Correct Principle

I propose that *simplicity is a fundamental epistemic guide to lawhood*. Roughly speaking, simpler candidates are more likely to be laws, all else being equal. This principle solves the problem of coherence in a straightforward way. It also secures epistemic realism in cases of empirical equivalence where simplicity is the deciding factor. In particular, we should accept this principle:

Principle of Nomic Simplicity (PNS) Other things being equal, simpler propositions are more likely to be laws. More precisely, other things being equal, for two propositions L_1 and L_2, if $L_1 >_S L_2$, then $L[L_1] >_P L[L_2]$, where $>_S$ represents the comparative simplicity relation, $>_P$ represents the comparative probability relation, and $L[\cdot]$ denotes *is a law*, which is an operator that maps a proposition to one about lawhood.[14]

From the perspective of nomic realism, one can consistently endorse PNS without endorsing PS. Some facts are laws, but not all facts are laws. Laws correspond to a special set of facts. On BSA, they are the best-system axioms. On MinP, they are the constraints on what is physically possible.

We are ready to see how PNS solves the problem of nested theories. Recall the earlier example of GR and GR^+. Even though we think that the Einstein equation is more likely to be a law, it is less likely to be true than the law of GR^+. I suggest that what simplicity selects here is not truth in general, but truth about lawhood, i.e. whether a certain proposition has the property of being a fundamental law.

Let us assume that fundamental lawhood is factive, which is granted on both BSA and MinP. Hence, lawhood implies truth: $L[p] \Rightarrow p$. However, truth does not imply lawhood: $p \nRightarrow L[p]$. This shows that $L[p]$ is logically inequivalent to p. This is the key to solve the problem of coherence.

On PS, in the case of nested theories, we have probabilistic incoherence. If L_1 is simpler than L_2, applying the principle that simpler laws are more likely to be true,

[14]For example, $L[F = ma]$ expresses the proposition that *F=ma is a law*. The proposition *F=ma* is what Lange (2009) calls a "sub-nomic proposition."

we have $L_1 >_P L_2$. However, if L_1 and L_2 are nested with $\Omega^{L_1} \subset \Omega^{L_2}$, the axioms of probability entail that $L_1 \leq_P L_2$. Contradiction!

On PNS, the contradiction is removed, because *more likely to be a law* does not entail *more likely to be true*. If L_1 and L_2 are nested, where L_1 is simpler than L_2 but $\Omega^{L_1} \subset \Omega^{L_2}$, then $L_1 \leq_P L_2$. It is compatible with the fact that $L[L_1] >_P L[L_2]$. What we have is an inequality chain:

$$L[L_2] <_P L[L_1] \leq_P L_1 \leq_P L_2 \tag{2}$$

This is also a new and simple solution to the problem of nested theories / problem of coherence. It is compatible with but less demanding and perhaps more general than the recent proposal of Henderson (2022). Unlike Henderson's approach, my proposal works even when candidate laws are not structured in a hierarchy.

3.3 Simplicity as a Tie Breaker

PNS is useful for resolving cases of empirical equivalence constructed along Algorithms A-C in §2.

For Algorithm A, T_2 will in general employ much more complicated laws than T_1. For example, the laws of T_{M2} specify $F(x,t)$ in its exact detail. Given that $F(x,t)$ is not a simple function of space and time coordinates, the laws of T_{M2} are not simple. In contrast, the laws of T_{M1} need not specify something so complicated. PNS suggests that other things being equal, we should choose T_{M1} over T_{M2}. In a Maxwellian world, we should postulate the existence of fields in the ontology and not in the nomology.[15]

For Algorithm B, $L2$ will in general be more complicated than $L1$, if Ω^{L_2} is obtained from Ω^{L_1} by adding or subtracting a few models. For example, a strongly deterministic theory of some sufficiently complex general relativistic spacetime, as described in the example, needs to specify the exact detail of that spacetime, will employ laws much more complicated than the Einstein equation. PNS suggests that other things being equal, we should choose the Einstein equation over such strongly deterministic laws.[16]

For Algorithm C, even though the mosaics of $L1$ and $L2$ are not that different, if $L1$ is a simple system, then in general $L2$ will not be. In fact, given enough changes from the actual mosaic, there may not be any optimal system that simplifies the altered mosaic to produce a good system.

PNS is to be contrasted from the simplicity criterion in the Humean best-system account of lawhood. They are different kinds of principles: the latter is a metaphysical definition of what laws are, while the former is an epistemic principle concerning ampliative inferences based on our total evidence. Even if a Humean expects that the best system is no more complex than the mosaic, it does not follow that she should expect that the best system is relatively simple, since there is no metaphysical guarantee that the mosaic is "cooperative." Both Humeans and non-Humeans can be uncertain about the laws, and both need a new principle to justify epistemic realism. If Humeans

[15]In my view, earlier versions of quantum Humeanism with a universal wave function are like T_{M2}, and choosing them violates PNS. In contrast, the version of quantum Humeanism suggested in Chen (2022a) solves this problem, as the initial density matrix is as simple as the Past Hypothesis.

[16]Not all strongly deterministic theories are overly complex. See Chen (2022c) for a simple candidate theory that satisfies strong determinism.

are epistemically warranted in making such a posit, non-Humeans are too.

3.4 More General Principles

Saying that simplicity is a fundamental epistemic guide to lawhood does not mean it is the only such guide. Recall that PNS contains the proviso "other things being equal." But sometimes other factors are not held equal, and we need to consider overall comparisons of theoretical virtues (epistemic guides) and their balance. Other theoretical virtues can also serve as epistemic guides for lawhood. For example, informativeness and naturalness are two such virtues. A simple equation that does not describe much or describe things in too gruesome manners is less likely to be a law.

We can formulate a more general principle:

Principle of Nomic Virtues (PNV) For two propositions L_1 and L_2, if $L_1 >_O L_2$, then $L[L_1] >_P L[L_2]$, where $>_O$ represents the relation of overall comparison that takes into account all the theoretical virtues and their tradeoffs, of which of which $>_S$ is a contributing factor, $>_P$ represents the comparative probability relation, and $L[\cdot]$ denotes *is a law*, which is an operator that maps a proposition to one about lawhood.

Since $>_O$ need not induce a total order of all possible candidate laws, the corresponding $>_P$ need not induce a total order either.[17] What is overall better is a holistic matter, and it can involve trade-offs among the theoretical virtues such as simplicity, informativeness, and naturalness. PNV should be thought of as the more general epistemic principle than PNS. I shall mainly focus on PNS, but what I say below should carry over to PNV.

Perhaps the most general principle is this:

Principle of Explanatory Virtues (PEV) For two propositions L_1 and L_2, if $L_1 >_O L_2$, then $Exp[L_1] >_P Exp[L_2]$, where $>_O$ represents the relation of overall comparison that takes into account all the theoretical virtues and their tradeoffs, of which of which $>_S$ is a contributing factor, $>_P$ represents the comparative probability relation, and $Exp[\cdot]$ denotes *is an explanation*, which is an operator that maps a proposition to one about explanations.

If we endorse PEV, we may view epistemic guides for lawhood as similar to the criteria for inferences to the best explanation (IBE). Choosing a law over any other candidate on the basis of nomic virtues is similar to choosing an explanation over any other based on IBE. My solution to the problem of coherence can be adapted to solve a similar problem on IBE.

3.5 Further Questions

There are further questions we can ask about PNS.

First, what is the measure of simplicity invoked here? It is unrealistic to insist that there is a single measure of simplicity regarding physical laws. There are many

[17]Nevertheless, in the cases of empirical equivalence discussed in §2, there are clear winners in terms of overall comparison.

aspects of simplicity, as shown by recent works in computational complexity, statistical testing, and philosophy of science. Among them are: number of adjustable parameters, lengths of axioms, algorithmic simplicity, and conceptual simplicity.[18] Certain laws may employ more unified concepts, better achieving one dimension of simplicity, but require longer statements and hence do less well in other dimensions of simplicity. There need not be any precise way of trading off one over the other. Moreover, not all laws must take the form of differential equations; there can be boundary-condition laws and conservation laws. It is unreasonable to expect a single measure applied to all different forms of laws. I suggest that we take simplicity to be measured in a holistic (albeit vague) way, taking into account these different aspects of simplicity.[19]

The vagueness of simplicity might seem like as a problem for nomic realists. However, what matters to a realist who believes in simplicity is that there is enough consensus around the paradigm cases. There are hard cases of simplicity comparisons, but there are also clearcut cases, such as T_{M1} and its empirical equivalents generated by Algorithm A, or general relativity and its empirical equivalents generated by Algorithms B and C. This is similar to Lewis's assumption that Nature is kind to us and the borderline cases do not show up in realistic comparisons. The vagueness of simplicity here is no worse than the problem in the BSA account of lawhood.

The vagueness of simplicity does not imply that there are no facts about simplicity comparisons. Let us think about an analogy with moral philosophy. Judgments about moral values are also holistic and vague. While there are moral disagreements about hard cases, there can still be facts about whether helping a neighbor in need is morally better than poisoning their cat. Moral realists can maintain that we have robust moral intuition in paradigm cases, which are not threatened by the existence of borderline cases. Sometimes different moral considerations conflict. In such cases, we may need to trade-off one factor against another.

Given the vagueness of the simplicity postulate, we should not expect there be a total order that ranks theories from the most to the least simple. Instead, there may be multiple chains of partial orders, where each is connected by a "simpler than" relation $>_S$.[20] We return to this point in §5.2.

Second, why should we regard the Guide-to-Lawhood principle as an epistemic principle? Like many substantive (not merely structural) epistemic principles, it does not follow from self-evident premises. However, we may consider an argument from reflective equilibrium. There are many cases of empirical equivalence where the salient difference between the empirical equivalents is their relative complexity. For example, if we are epistemically justified in accepting T_{M1} over T_{M2} because the former has simpler laws, or in accepting GR over GR^+ because the former has simpler laws, simplicity has to be an epistemic guide.[21]

Reflecting on our judgments over those cases, we may conclude that simplicity as

[18]For an overview of these different measures, see Baker (2022) and Fitzpatrick (2022).

[19]Alternatively, we may take simplicity as a family of concepts, and the principle of simplicity as a family of principles.

[20]Hence, there may be pairs of theories that cannot be compared with respect to simplicity. If A and B are such a pair, then none of "$>_S$," "$<_S$," and "$=_S$" applies to them. We should allow cases of incommensurability or incomparability regarding simplicity. This relation can be denoted by $A \circledast B$.

[21]For a similar argument, see Lycan (2002).

a guide to lawhood is one posit we should make to justify epistemic realism about laws. It is what we presuppose when we set aside (or give less credence to) those empirical equivalents as epistemically irrelevant. For our preferences in the cases of empirical equivalence to be epistemically justified, the principle of simplicity should be an epistemic guide. As such, it is not merely a pragmatic principle, although it may have pragmatic benefits. Simpler laws may be easier to conceive, manipulate, falsify, and the like. But if it is an epistemic guide, it is ultimately aiming at certain truths about lawhood and providing epistemic justifications for our believing in such truths. There is, to be sure, the option of retreating from epistemic realism. But it is not open to nomic realists.

Finally, Why should we regard Guide-to-Lawhood as a *fundamental* epistemic principle? The reason is that in the cases of empirical equivalence discussed above, the principle cannot be reduced to other epistemic principles that are more closely connected to lawhood or truth. I suggest that it is a rock-bottom principle that need not be justified further; it is an assumption we ought to adopt prior to empirical investigation. What about the view that PNV or PEV is more fundamental than PNS? I do not think that is right, even though PNV and PEV are more general. Generality does not entail fundamentality. We may regard PNS as a member of a family of fundamental principles that together determine PNV and PEV. Nevertheless, it is useful to recognize how PNS is connected to the more general PEV and PNV.

4 Theoretical Benefits

To further illustrate the theoretical benefits of PNS, I discuss five issues that are important to nomic realists: induction, symmetries, dynamics, determinism, and explanation. The metaphysical posits of nomic realism leave it open what we should say about them. Accepting PNS allows us to say the right things in a systematic way.

4.1 Induction

Hume's problem of induction is closely related to the problem of underdetermination. We want to know the physical reality (L, ξ). Given our limited evidence about some part of ξ and some aspect of L, what justifies our inference to other parts of ξ or other aspect of L that will be revealed in upcoming observations or in observations that could have been performed? It does not follow logically. Without some *a priori* rational guide to what (L, ξ) is like or probably like, we have no rational justification for favoring (L, ξ) over any alternative compatible with our limited evidence. On a given L we know what kind of ξ to expect. But we are given neither L or ξ. Without further inferential principles, it is hard to make sense of the viability of induction.

What about the oft-cited principle of the uniformity of nature (PU)?

Principle of Uniformity (PU) Nature is uniform.

It is not clear what the principle says and how it relates to induction. When we consider three possible interpretations, we realize PU is not what we want or need for induction.

First, suppose PU means that we should expect that evidence E to be uniform, in the sense that given the same experimental setups, the outcomes are always the same. But this is not always useful for induction. Experimental setups are never exactly the same, and neither are the outcomes of experiments. They are similar in some respects but not others. Moreover, our evidence for physical theory typically consists in a variety of different types of experiments and observations, which allow us to cross-check the same theory from different angles. For example, our evidence for general relativity comes from a variety of sources (Misner et al. 1973, Thorne and Blandford 2017), and it is better that they are not all of the same type.

Second, suppose PU means that we should expect that the mosaic ξ to be uniform. This is often what people have in mind, but it is unclear what is meant. The mosaic we inhabit is manifestly not uniform; there are many different kinds of objects, properties, and structures. We do not live in an empty universe that is completely homogeneous and isotropic. The spacetime region we inhabit is quite different from regions with violent collisions of stars and merging of black holes. What happens on earth is quite different even from a nearby patch—the core of the sun, where nuclear fusion converts hydrogen into helium. The variety and complexity in the matter distribution does not diminish our confidence in the viability and the success of induction. In fact, non-uniformity of a certain kind is arguably necessary for the observed temporal asymmetries in our universe, which may be a precondition for induction.[22]

Finally, suppose PU means that we should expect that the law L to be uniform. This interpretation shifts our attention from the mosaic to the law. However, it is still the wrong principle. Some people might understand the uniformity of L to mean that it is of the form "for all x, if Fx then Gx," which is a regularity, i.e. a universally quantified statement about the mosaic, holding for everything, everywhere, and everywhen. The problem is that the principle is vacuous, as any statement can be translated into a universally quantified sentence. That I have five coins in my pocket is equivalent to the statement that, for everything and everywhere and everywhen, I have five coins in my pocket. Suppose we understand the uniformity of L to mean that it does not have refer to any particular individual, location, or time. That version is too restrictive. There are candidate laws that do refer to particular facts, such as the Past Hypothesis of statistical mechanics, quantum equilibrium distribution in Bohmian mechanics, the Weyl curvature hypothesis in general relativity, and the No-Boundary Wave Function proposal in quantum cosmology. These laws can be accepted on scientific and inductive grounds, and may be required to ultimately vindicate our inductive practice. (I say more about this in §4.3.) Suppose we understand PU to mean that the same law applies to everywhere in spacetime. That version is again vacuous, as even an intuitively non-uniform law can be described by a uniform law with a temporal variation, such as

$$F = ma \text{ for } (-\infty, t] \text{ and } F = (8m^9 - \frac{1}{7}m^5 + \pi m^3 + km^2 + m)a \text{ for } (t, \infty) \tag{3}$$

with F given by Newtonian gravitation. The same law can be applied to everywhere in spacetime.

[22]See Wallace (2010) and Rovelli (2019) for the importance of the hydrogen-helium imbalance in the early universe to the existence of the relevant time asymmetries.

Under various precisifications, PU is not what we want or need for induction to work. In contrast, PNS is the better choice. We can rationally prefer $F = ma$ to (3) when available evidence underdetermines them. What induction ultimately requires is the reasonable simplicity of the physical laws, and a simple law may well give rise to a complicated mosaic with an intricate matter distribution. This does not forbid laws about boundary conditions or particular individuals. Some simple laws may even have temporal variation, such as a time-dependent function $F = \frac{1}{t+k}ma$ for some constant k. As long as the temporal variation and spatial variation are not too extreme as to require complicated laws, we can still inductively learn about physical reality based on available evidence, even in a non-uniform spacetime with dramatically different events in different regions. The law of quantum mechanics can be checked in double-slit experiments, interferometers, and the stability of macroscopic objects such as tables and planets. PNS delivers the right results that we have rational expectation that induction can work, and it is not a vacuous or overly stringent requirement like PU. One way of thinking about Hume's problem of induction is that we need to adopt an epistemic, non-structural principle. PNS does the job better than the oft-cited PU. What ultimately backs induction is (our rational belief in) the simplicity of physical laws.

4.2 Symmetries

Symmetry principles play important roles in theory construction and discovery. Physicists routinely use symmetries to justify or guide their physical postulates. However, whether symmetries hold is an empirical fact, not guaranteed by the world *a priori*. So why should we regard symmetry principles to be useful, and what are they targeting? I suggest that certain applications of symmetry principles are defeasible guides for finding simple laws. In such cases, their epistemic usefulness may come from the fact that certain symmetries are defeasible indicators for simplicity.[23]

Consider again the toy example in (3). This law violates time-translation invariance and time-reversal invariance. In this case, we have a much better law that is time-translation and time-reversal invariant:

$$F = ma \text{ for all times} \tag{4}$$

The presence of the two symmetries in (4) and the lack of them in (3), indicate that all else being equal we should prefer (4) to (3). We can explain this preference by appealing to their relative complexity. (4) is much simpler than (3), and the existence of the symmetries are good indicators of the relative simplicity. However, in this comparison, we are assuming that both equations are valid for the relevant evidence (evidence obtained so far or total evidence that will ever be obtained). The preference is compatible with the fact that *if* empirical data is better captured by (3), we should prefer (3) to (4).

In the relevant situations where symmetry principles are guides to simplicity, they are only defeasible guides. Symmetry principles are not an end in itself for theory choice. I shall provide two more examples to show that familiar symmetry principles

[23]For a related perspective, see North (2021).

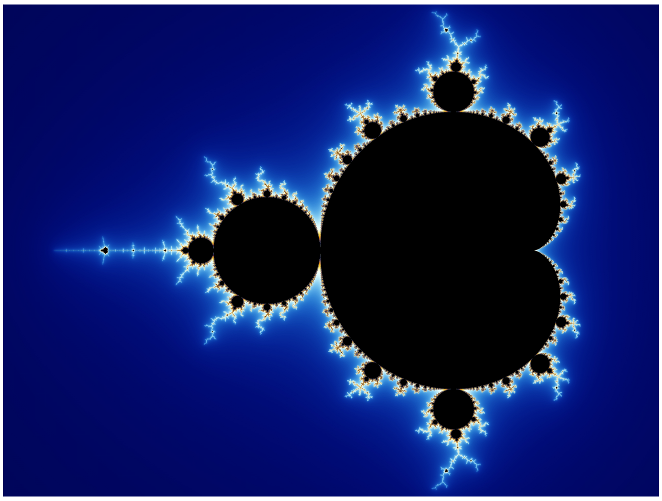

Figure 1: The Mandelbrot set with continuously colored environment. Picture created by Wolfgang Beyer with the program Ultra Fractal 3, CC BY-SA 3.0, https://creativecommons.org/licenses/by-sa/3.0, via Wikimedia Commons

are not sacred, but rather defeasible indicators of simplicity, that can be ultimately sacrificed if we already have a reasonably simple theory that is better than the alternatives.

The first is the toy example of the Mandelbrot world (Figure 1). Consider the Mandelbrot set in the complex plane, produced by the simple rule that a complex number c is in the set just in case the function

$$f_c(z) = z^2 + c \tag{5}$$

does not diverge when iterated starting from $z = 0$. (For example, $c = -1$ is in this set but $c = 1$ is not, since the sequence $(0, -1, 0, -1, 0, -1, ...)$ is bounded but $(0, 1, 2, 5, 26, 677, 458330, ...)$ is not. For a nice description and visualization, see (Penrose 1989, ch.3-4).) The pattern on the complex plane is surprisingly intricate and rich. It is a striking example of what is called the fractal structure. When we zoom in, we see sub-structures that resemble the parent structure. When we zoom in again, we see sub-sub-structures that resemble the sub-structures and the parent structure. And so on. Interestingly, they closely resemble, but they are not exactly the same. As we zoom in further, there will always be surprises waiting for us. Each scale of magnification will reveal something new.

Now, let us endow the Mandelbrot set with physical significance. We regard the Mandelbrot set on the complex plane as corresponding to the distribution of matter over a two-dimensional spacetime, which we call the *Mandelbrot world*, ξ_M. We stipulate that the fundamental law of the Mandelbrot world is the rule just described, which we denote by L_M. The fundamental law is compatible with exactly one world.[24]

The physical reality consisting of (L_M, ξ_M) is friendly to scientific discovery. If we were inhabitants in that world, we can learn the structure of the whole ξ_M from the

[24] It is worth noting that the patterns of the Mandelbrot world are not fine-tuned, as they are stable under certain changes to the law. For example, as (Penrose 1989, p.94) points out, other iterated mappings such as $f_c(z) = z^3 + iz^2 + c$ can produce similar patterns.

structure of its parts, by learning what L_M is. However, L_M is not a law with any recognizable spatial or temporal symmetries.[25] In fact, the usual notions of symmetries do not even apply to L_M, because it is not expressed as a differential equation. It is compatible with exactly one mosaic, namely ξ_M. Moreover, the physical reality described by (L_M, ξ_M) is a perfect example of an ultimate theory (though not of the actual world). It is an elegant and powerful explanation for the patterns in the Mandelbrot world. What could be a better explanation? I suggest that none would be, even if it had more symmetries. In this case, we do not need symmetry principles to choose the right law, because we already have a simple and good candidate law. The lack of symmetries is not a regrettable feature of the world, but a consequence of its simple law.

The second and more realistic example is the Bohmian Wentaculus (Chen 2018, 2022a, 2023). If we adopt the nomic interpretation of the quantum state, which is made plausible by the simplicity of the initial density matrix, we can understand the mosaic ξ_B as consisting of only particle trajectories in spacetime, with the fundamental dynamical law L_B as given by this differential equation:

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\text{Im}\frac{\nabla_{q_i} W_{IPH}(q,q',t)}{W_{IPH}(q,q',t)}(Q) = \frac{\hbar}{m_i}\text{Im}\frac{\nabla_{q_i} \langle q| e^{-i\hat{H}t/\hbar}\hat{W}_{IPH}(t_0)e^{i\hat{H}t/\hbar}|q'\rangle}{\langle q| e^{-i\hat{H}t/\hbar}\hat{W}_{IPH}(t_0)e^{i\hat{H}t/\hbar}|q'\rangle}(q = q' = Q) \quad (6)$$

Since the quantum state is nomic, as specified by a law, the right hand side should be the canonical formulation of the fundamental dynamical law for this world. Notice that the right hand side of the equation is not time-translation invariant, as at different times the expression

$$\text{Im}\frac{\nabla_{q_i} \langle q| e^{-i\hat{H}t/\hbar}\hat{W}_{IPH}(t_0)e^{i\hat{H}t/\hbar}|q'\rangle}{\langle q| e^{-i\hat{H}t/\hbar}\hat{W}_{IPH}(t_0)e^{i\hat{H}t/\hbar}|q'\rangle}$$

will in general take on different forms. However, the physical reality described by the Bohmian Wentaculus may be our world, and the equation can be discovered scientifically. The law is a version of the Bohmian guidance equation that directly incorporates a version of the Past Hypothesis. Hence, (L_B, ξ_B) describes a physical reality that is friendly to scientific discovery and yet does not validate time-translation invariance.

In the Bohmian Wentaculus world, symmetry principles can be applied, but the fundamental dynamical law explicitly violates time-translation invariance. In such cases, the lack of symmetries is not a problem, because we already have found the simple candidate that has the desirable features. Again, the time-translation non-invariance is a consequence of its simple law. PNS takes precedence over symmetry principles and are the deeper justification for theory choice.

4.3 Dynamics

We have good reasons to allow fundamental laws of boundary conditions. However, many boundary conditions are not suitable candidates for fundamental lawhood. Epistemic guides such as simplicity allow us to be selective in postulating boundary

[25]There is, however, the reflection symmetry about the real axis. But it does not play any useful role here, and we can just focus on the upper half of the Mandelbrot world if needed.

condition laws, and to give more weight to proposals that include dynamical laws.

To start, let us review some reasons to posit fundamental laws of boundary conditions. First, cosmologists have suggested that fundamental physical laws should include a law about the initial condition of the universe. This is implicit in Hartle and Hawking's papers about the No-Boundary Wave Function, a theory according to which (roughly speaking) "the boundary conditions of the universe are that it has no boundary" (Hartle and Hawking 1983, p.2975). Hartle (1996) summarizes his view as follows:

> "A present view, therefore, is that the most general laws of physics involve two elements:
>
> - The law of dynamics prescribing the evolution of matter and fields and consisting of a unified theory of the strong, electromagnetic, weak and gravitational forces.
> - A law specifying the initial boundary condition of the universe.
>
> There are no predictions of any kind which do not depend on these two laws, even if only very weakly, or even when expressed through phenomenological approximations to these laws (like classical physics) appropriate in particular and limited circumstances with forms that may be only distantly related to those of the basic theory."

Second, probabilistic boundary conditions are indispensable to the predictive power of certain physical theories (Ismael 2009). If they play the role of underwriting objective probabilities in physics, then probabilistic boundary conditions can earn the status of fundamental lawhood. The Past Hypothesis and Statistical Postulate of the Mentaculus theory (Albert 2000, Loewer 2007) are in this category. We can also consider another case that is independent of the Past Hypothesis, namely the quantum equilibrium distribution in Bohmian mechanics. It says that the initial particle configuration is distributed according to $\rho(q, t_0) = |\Psi(q, t_0)|^2$. This distribution postulate is made plausible but not entailed by the Bohmian dynamics. This probabilistic boundary condition arguably plays the role of a physical law in Bohmian mechanics (Barrett (1995), Loewer (2004), Callender (2007)).

The above examples of boundary condition laws have a common feature: they are simple to specify. Many boundary conditions contain a great deal of correlations, but only a select few are good candidates for fundamental laws, namely those that are also sufficiently simple. One may wonder why we choose the Past Hypothesis, a macroscopic description, over a precise microscopic initial condition of the universe. The answer is that the former is much simpler than the latter and is still sufficiently powerful to explain a variety of temporally asymmetric regularities. The simplicity of the boundary condition laws make it almost inevitable that we will have dynamical laws in addition to boundary condition laws. The scientific explanations of natural phenomena come from the combination of simple boundary conditions and dynamical laws. As such, dynamical laws have to carry a lot of information by themselves.

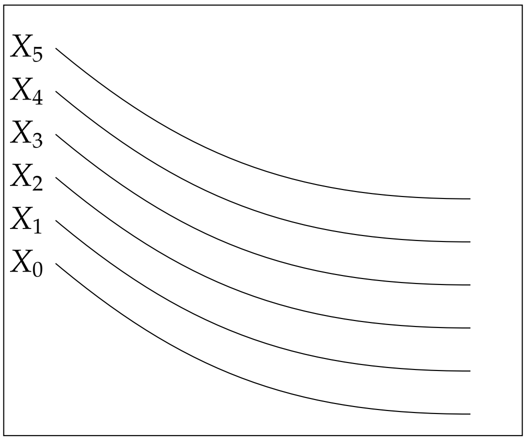

Figure 2: Schematic illustration of a deterministic theory T. Ω^T contains six nomologically possible worlds that do not cross in state space.

4.4 Determinism

Nomic realism is often accompanied with other reasonable expectations about physical laws. Here I discuss some issues related to determinism and superdeterminism.

Borrowing ideas from (Montague 1974, pp.319-321), (Lewis 1983, p.360), and (Earman 1986, pp.12-13), I define determinism as follows (also see Figure 2):

Determinism$_T$ Theory T is *deterministic* just in case, for any two $w, w' \in \Omega^T$, if w and w agree at any time, they agree at all times.

Intuitively, T is deterministic if and only if no two mosaics compatible with T cross in state space.

By using Ω_α as the set of mosaics compatible with the actual law, we can say that:

Determinism$_\alpha$ The actual world α is *deterministic* just in case, for any two $w, w' \in \Omega_\alpha$, if w and w' agree at any time, they agree at all times.

Determinism is true just in case the actual world α is deterministic in this sense.

On MinP, given any mosaic ξ, there are many possible choices of L such that $\xi \in \Omega^L$ and mosaics do not cross in Ω^L. Here is an algorithm to generate some of them: construct a two-member set $\Omega^L = \{\alpha, \beta\}$ such that α and β agree at no time (or any Cauchy surface). Any law with such a domain meets the definition of determinism. As long as α is not a world where every logically possible state of the universe happens some time in the universe, it is plausible to think that there are many different choices of β that can ensure determinism. Without a further principle about what we should expect of L, determinism is too easy and almost trivial on MinP.

On BSA, the problem is the opposite. It becomes too difficult and almost impossible for a world to be deterministic. Given the evidence E we have about the mosaic, even though E may be optimally summarized by a deterministic law L, it does not guarantee (or make likely, without further assumptions) that the entire mosaic is optimally summarized by a deterministic law L. Small "perturbations" somewhere in the mosaic can easily make its best system fail determinism.[26]

[26]See Builes (2022) for a related argument.

Hence, there is a question of what nomic realists should say that constitutes a principled reason to think that determinism is not completely trivial (on MinP) and not epistemically inaccessible (on BSA).

With PNS, determinism is no longer trivial on MinP. Given any mosaic ξ, even though there are many deterministic candidates compatible with and true at ξ. Not every mosaic will be compatible with a relatively simple law that is deterministic. The non-triviality of determinism on MinP is the fact that it is non-trivial to find a law that is simple and deterministic, as that is not guaranteed for every metaphysically possible mosaic.

With PNS, determinism is no longer epistemically inaccessible on BSA. This follows from the more general principle that PNS gives us epistemic justification to hold beliefs about parts of the mosaic that we have not observed and will never observe. We are justified in believing that the best system of the actual mosaic is relatively simple, even though the actual evidence does not entail that. If the actual evidence can be optimally summarized by a deterministic law restricted to the actual evidence, we have epistemic justification to make inferences about regions that will not be observed – the entire mosaic, ξ, can be summarized by a simple law that happens to be deterministic.

Related to determinism is the concept of superdeterminism, which has been much debated in quantum foundations. A superdeterministic theory is a deterministic one that has to violate statistical independence (Hossenfelder and Palmer 2020). Roughly speaking, a theory violates statistical independence just in case the probability distribution of the fundamental physical variables is not independent of the detector settings. The motivation is to evade Bell's theorem of non-locality. (See Chen (2021) for a critical overview.)

It is not metaphysically impossible, on BSA or on MinP, that the actual laws are superdeterministic. However, superdeterminism faces many objections. An oft-cited criticism focuses on the fact that endorsing violations of statistical independence would be bad for science in one way or another. After all, the assumption of statistical independence is integral to ordinary statistical inferences. Maudlin (2019) (and similarly Shimony et al. (1976)) suggests that rejecting it would make it impossible to do science:

> If we fail to make this sort of statistical independence assumption, empirical science can no longer be done at all. For example, the observed strong robust correlation between mice being exposed to cigarette smoke and developing cancer in controlled experiments means nothing if the mice who are already predisposed to get cancer somehow always end up in the experimental rather than control group. But we would regard that hypothesis as crazy.

This is compelling, but not decisive. The viability of scientific methodology is not a consequence of the metaphysical aspects of BSA or MinP. What about the charge that superdeterministic theories require initial conditions that are extremely fine-tuned? The worry for this response is that fine-tuning of initial conditions can occur even in acceptable theories, such as theories that posit the Past Hypothesis as a fundamental law. The Past Hypothesis plays a crucial role constraining what can and cannot happen in scientific experiments. (Moreover, together with the Statistical Postulate, it may partly explain why statistical independence holds in our world.) So, what is a principled ground for doubting superdeterminism in a world like ours, for nomic realists who endorse either BSA or MinP, but does not rule out laws about boundary conditions such as the Past Hypothesis?

PNS offers a principled reason for doubting superdeterminism. The constraints on empirical frequencies are so severe that it is hard to see how it can be written down in any simple formula. In order for the local theory to be compatible with the predictions of quantum mechanics, it would have to radically constrain the state space of the local theory so that only a very small class of histories will be allowed. (Such a constraint can be a joint effect of some lawlike initial conditions and the dynamical laws.) Not all arrangements of the local parameters will be permitted—otherwise one cannot guarantee perfect agreement with quantum predictions. What kind of constraints? They will have to encode as much information as the setup and non-local correlations. For example, they would need to entail that an experiment done today using randomization method based on the digits of π will somehow still result in statistically dissimilar sub-collections in such a way that produce the desired outcomes of experiments done at arbitrarily far away locations. Similarly it will be the case for randomization based on the letters of Act V of *Hamlet*, the Chinese characters in the *Analects*, or the hexagrams of *I Ching*. No matter what randomization method we choose, the superdeterministic mechanism must ensure that the chosen sub-collection is somehow just the right one for a particular experimental setup. Since the randomization methods seem to have nothing in common, it is hard to see how the constraints on initial conditions and dynamics can be simple at all. These give us reasons to think that they will be quite complicated. PNS suggests we give very little credence to theories with such complicated laws. Unlike the Past Hypothesis, which can be given a reasonably simple description of the matter distribution or spacetime structure of the initial condition, we have no reason to think that the superdeterministic laws can be simple at all. Given simpler alternatives such as Bohmian mechanics and objective collapse theories, we can be epistemically justified in assigning low credences in superdeterministic theories.

4.5 Explanation

There is a strong connection between nomic realism and scientific explanation. The point of postulating physical laws, on BSA and on MinP, is to provide scientific explanations. However, not all candidate laws provide the same quality of explanation or same kind of explanation. Hence, on both versions of nomic realism, we might wonder if there is a principled reason to think that we will have a successful scientific explanation for all phenomena.

On MinP, explanations must relate to us (Chen and Goldstein 2022, p.45). Constraints, in and of themselves, do not always provide satisfying explanations. Many constraints are complicated and thus insufficient for understanding nature. For example, the constraint given by just $\Omega^L = \{\xi_M\}$, which requires a complete specification of the mosaic, is insufficient for understanding the Mandelbrot world. Knowing why there is a pattern requires more than knowing the exact distribution of matter.

On MinP, many candidate laws can constrain the mosaic. But not all have the level of simplicity to provide illumination about the mosaic. With PNS, we expect the actual constraint to be relatively simple. The constraint given by the Mandelbrot law should be preferred to that given by $\Omega^L = \{\xi_M\}$. The simple law provides a successful explanation while the more complicated one does not.[27]

On BSA, it is built in the notion of laws that they systematize the mosaic. However, whether there is a systematization that is simpler than the mosaic is a contingent matter, depending on the detailed, microscopic, and global structures of the mosaic. Not every mosaic supports a systematization that provides illumination in the sense of unifying the diverse phenomena in the mosaic. BSA only entails that the best system is no more complex than the exact specification of the mosaic. For example, some mosaics may support no better optimal summary than the exact specification of the mosaic itself. Hence, on BSA, having successful explanations is not automatic. It requires the mosaic to be favorable.

On BSA, some mosaics are favorable: they support optimal summaries that are simpler than themselves and provide "Humean explanations" about the mosaic. In fact, in a sense, most mosaics are not favorable (Lazarovici 2020). There exist mosaics underdetermined by actual evidence that do not support any good summaries. Given the actual evidence, with PNS, we are epistemically justified in inferring that the actual best system is relatively simple such that it can provide a "Humean explanation" about the actual mosaic. In effect, we are expecting that the actual Humean mosaic is a favorable one that completely cooperates with our scientific methodology and is such that it can be unified in a reasonably simple best system.

On both MinP and BSA, the viability of scientific explanation can be justified by PNS.

5 Epistemic Fundamentality

I have argued that PNS yields substantive theoretical benefits. For that reason, it is reasonable to take it as a fundamental principle. In this section, I discuss three issues: the problem of justification, the problem of precision, and the epistemology of laws on Humeanism and non-Humeanism.

5.1 The Problem of Justification

One might think that it is odd to have a fundamental epistemic principle that is not a structural criterion. It is not analytic and not something that is empirically discoverable. (Roberts 2008, p.158) suggests that something like PNS would be a "synthetic *a priori*" principle concerning metaphysically contingent truths that is much stronger than what even Kant would affirm.

We might wonder what can possibly justify such a principle. It is natural to worry:

Problem of Justification There is no plausible epistemic justification for the principle of (nomic) simplicity.

In §3.5, I have argued that, by reflective equilibrium, PNS should be regarded as a plausible epistemic principle. The applications of PNS to a wider range of cases in §4

[27]PNS and other epistemic guides may be regarded as questions raised by Hildebrand (2013).

give further support to that idea.

Why think it is a *fundamental* epistemic guide that is not justified further? I think a compelling argument can be made by its vindication of induction ($4.1). The viability of induction is indispensable to scientific practice and nomic realism. We can make a transcendental argument that, since without believing in induction we would not be able to do science, we have to believe in induction. By Hume's argument we know that it is impossible to provide a non-circular justification of induction. Deductive and probabilistic justifications of induction would require premises that can be learnt only through induction. Therefore, whatever justification we offer for induction would not completely satisfy the skeptic. We have to start somewhere by making some fundamental posits that clarify how and why induction works.

I suggest that PNS is a good candidate for such a fundamental posit. If we are rational to believe that physical laws are reasonably simple, we can assume that they are completely uniform in space and time or else provide a simple rule that specifies how they change over space or time. We can discover, using standard scientific methodology, physical laws and the natural phenomena that follow from them. PNS is at the correct level of generality and makes the right connections to symmetries, time-dependence, laws, and explanation. It is hard to see what else can justify induction that strikes a better balance. (Similar arguments can be made based on considerations of scientific methodology (§4.4) and explanation (§4.5).)

For epistemological foundationalists, PNS may seem too strong. But it is worth remembering that we already need many similar principles, regarding the veridicality of perception, and absence of an evil demon, etc. PNS is just another foundational principle that we need to succeed in our epistemic lives.

How about approaches that aim to reduce simplicity to structural epistemic principles (such as the likelihood principle)? As far as I can see, the reductive approaches to simplicity do not apply in the case of empirical equivalence. For example, the AIC model-selection criterion advocated by Forster and Sober (1994) is designed for predictively inequivalent theories. In the absence of any successful reduction of simplicity to resolve cases of empirical equivalence, it is warranted to regard it as a fundamental epistemic guide. If someone presents a proof that simplicity can be reduced to something else, we should be open to the idea and regard the principle of simplicity as derivative of some deeper principle. However, the existence of reasonable algorithms that generate empirical equivalents should cast doubt on the existence of such proofs.

5.2 The Problem of Precision

One might think that it is also odd to have a fundamental yet vague principle. It is natural to worry:

Problem of Precision There is no precise standard of simplicity.

The assumption behind the worry is that vagueness is always an indicator for non-fundamentality; whatever fundamental epistemological principles there are, they must be exact.

It is unclear why the assumption is true. I am not aware of any non-structural epistemic principle that is exact. In the case of PNS, we have principled reasons to expect that it is vague, and its vagueness is appropriate. Consider its range of applications. It grounds our commitments about induction, symmetries, dynamics, determinism, and explanation. The measures we use in different cases may not always agree. Moreover, if we allow vague fundamental laws (Chen 2022b), it is natural to expect that the measure of simplicity will be vague.

There is another reason to tolerate some vagueness in simplicity. Consider algorithmic randomness, an active field of research in mathematics and computer science. Mathematicians and computer scientists start with a vague pre-theoretical concept of randomness. They propose a range of mathematically precise definitions, some of which turn out to be theoretically fruitful. These include the Kolmogorov notion of incompressibility, the Martin-Löf notion of typicality, and the game-theoretic notion of fair gambling (Dasgupta 2011). Surprisingly, in idealized circumstances they are provably equivalent. The equivalence shows that the vague concept is latching onto something in mathematical reality. But such notions do not completely eliminate vagueness. For example, when we apply any notion of algorithmic randomness to finite models, we see the reappearance of vagueness (Li and Vitányi 2019, p.56). Instead of having a completely exact boundary between random and non-random sequences, we need to adopt a vague boundary: a sequence is random just in case it cannot be represented by a significantly shorter algorithm, where what counts as "significantly shorter" is of course vague. Insofar as it is legitimate to apply randomness to finite strings, we can regard the vagueness here as entirely appropriate.

In cases where we can legitimately apply the concept of algorithmic randomness, it is also a measure for complexity. A non-random sequence may be considered simple. A non-random mosaic can always be captured by a suitably simple law. Hence, we may plausibly treat the two concepts as duals of each other.

Duality Simplicity and algorithmic randomness are duals of each other.

Since algorithmic randomness is appropriately vague, simplicity is too.

5.3 Humeanism vs. non-Humeanism

Does Guide-to-Lawhood follow from the metaphysical postulates of BSA? The answer is no. To see this, let us recall the comparison between T_{M1} and T_{M2}. Following Guide-to-Lawhood, a Humean scientist living in a world with Maxwellian data would (and should) prefer T_{M1} to T_{M2} because the laws of T_{M1} are simpler. However, on BSA, it is metaphysically possible that the actual ontology does not include fields. If that is the actual mosaic, the best system may in fact correspond to the enormously complicated laws of T_{M2}. It follows that what counts as the actual best system on the BSA may differ from what we should accept as the best system according to Guide-to-Lawhood.

There is no inconsistency, because *what the laws are* can differ from *what we should believe about what the laws are*. Hence, defenders of BSA are in a similar epistemic situation as defenders of MinP. Even if the nomology of T_{M2} represents the actual governing laws, a defender of MinP would and should regard T_{M1} as more likely than

T_{M2}. Humeans and non-Humeans can be mistaken about physical reality, even when they are completely rational. That is a feature and not a bug, because nomic realists should be fallible.

This has ramifications for the debate between Humeanism and non-Humeanism. According to an influential argument,[28] Humeanism has an epistemic advantage over non-Humeanism, because the former offers better epistemic access to the laws. The argument is that the Humean mosaic is all that we can empirically access, on which laws are supervenient, but non-Humeans postulate facts about laws that are empirically undecidable. But if the analysis in this paper is correct, such arguments are epistemically irrelevant. We never, in fact, occupy a position to observe everything in the mosaic. Our total evidence E will never exhaust the entire mosaic ξ. But if both Humeans and non-Humeans need to accept an independent substantive epistemic posit in order to ensure epistemic access to physical laws, there is no real advantage on Humeanism. The reason we have epistemic access to laws is by appeal to PNS, which does not follow from the metaphysical posits of either Humeanism or non-Humeanism. Humeanism and non-Humeanism are epistemically on a par, with respect to the discovery and the evaluation of laws.

Taking a step back, we can see that the relation between Humeanism and PNS is somewhat indirect. PNS is an epistemic principle regarding what system we should believe given the total evidence. BSA is a metaphysical principle regarding what the best system is given the total mosaic. Since $L = BS(\xi)$, with the full mosaic the Humeans can in principle solve for L. However, the Humeans do not have access to the full mosaic, because they are macroscopically and spatiotemporally limited. Humeans are essentially solving an inverse problem. Given evidence E, what is the simplest law (that also balances a host of other epistemic guides) compatible with E?[29] Suppose the epistemic guides recommend a unique candidate law given evidence E:

$$L_{\text{epistemic}} = EG(E) \tag{7}$$

with $EG(\cdot)$ the function that maps a set of evidence to a law recommended by the epistemic guides. Taking epistemic guides seriously is to have high confidence that

$$L = L_{\text{epistemic}} \tag{8}$$

Because epistemic guides do not guarantee the right answer, it is possible that

$$L \neq L_{\text{epistemic}} \tag{9}$$

In probabilistic terms, a Humean who believes in PNS and the other epistemic guides should have high prior credence in (8) and low prior credence in (9). Given the high probability of (8), the Humean would solve an inverse problem to determine the actual mosaic, up to a point:

Humean Inverse Problem What is the actual mosaic like, given we have epistemic

[28] For example, see Earman and Roberts (2005) and Roberts (2008).

[29] Recall the discussions of Hall (2009, 2015) about the Limited Oracular Perfect Physicist (LOPP). She has no inverse problem to solve, as her evidence E_{LOPP} completely pins down (L, ξ). That is entirely different from the situation of actual Humeans.

reasons to infer that it is optimally described by $L_{\text{epistemic}}$?

This can be answered by finding the following:

$$\xi_\alpha \in \Omega_{\text{BSA}}^{L_{\text{epistemic}}}, \text{with } \Omega_{\text{BSA}}^{L_{\text{epistemic}}} = \{\xi : BS(\xi) = L_{\text{epistemic}}\}^{30} \tag{10}$$

As a last step of finding out fundamental reality, Humeans then infer that the actual mosaic is a member of $\Omega_{\text{BSA}}^{L_{\text{epistemic}}}$. But this rational reconstruction makes it explicit that Humeans rely on the epistemic guides. To know what the fundamental reality (the Humean mosaic) is, they need to (1) collect empirical data, (2) make ampliative inferences using epistemic guides such as PNS, and (3) determine what the actual mosaic is like based on the best guess about physical laws.

Let us contrast that with the rational construction on the non-Humean view of MinP. Even though there is no metaphysical restriction on the form of fundamental laws, it is rational to expect them to have certain nice features, such as simplicity and informativeness. On BSA, those features are metaphysically constitutive of laws, but on MinP they are merely epistemic guides for discovering and evaluating candidate laws. At the end of the day, they are defeasible guides, and we can be wrong about the fundamental laws even if we are fully rational in scientific investigations. The second part of Chen and Goldstein (2022)'s MinP is an epistemic thesis:

Epistemic Guides Even though theoretical virtues such as simplicity, informativeness, fit, and degree of naturalness are not metaphysically constitutive of fundamental laws, they are good epistemic guides for discovering and evaluating them.

Just as on BSA, accepting Epistemic Guides is to have high confidence in (8), which is compatible with $L \neq L_{\text{epistemic}}$. The epistemic gap on BSA is the same as that on MinP; there is no relevant epistemic advantage of Humeanism over non-Humeanism.

Epistemic Parity Thesis Humeanism does not have an epistemic advantage over non-Humeanism regarding our epistemic access to physical laws.[31]

Finally, our discussion is relevant to the problem(s) of ratbag idealism. (Hall 2009, §5.6) suggests that, facing the problem that the simplicity criterion in the BSA is too subjective, Lewis and other Humeans can "perform a nifty judo move." If non-Humeans

[30]In general, $\Omega_{\text{BSA}}^{L} \neq \Omega^{L}$ as there are members of the latter that may not be members of the former (e.g. undermining histories).

[31]Still, some Humeans might argue that non-Humeanism opens up extra epistemic risks (Earman and Roberts 2005, p.280), because it is possible that we have the entire mosaic and yet not know what the laws are. However, it is too idealized to be an epistemic situation relevant to actual scientists. Perhaps such Humean would then reply that non-Humeanism opens up extra epistemic risks in a stronger sense, because given any set of evidence (such as the spatiotemporally partial and macroscopic E) non-Humeanism allows "more" distinct laws than Humeanism does. Now, the number of physical laws compatible with any finite and limited body of evidence is infinite. Talking about "more" in the infinite context requires a measure. Suppose this can be done rigorously. It does not follow that the set of extra laws has positive measure. One may rationally assign probabilities such that set of the extra laws has measure zero, so that:

$$P_{\text{BSA}}(L|E) = P_{\text{MinP}}(L|E) \tag{11}$$

where L is some particularly good candidate and E is our available evidence. If we characterize epistemic risks probabilistically, the Epistemic Parity Thesis can still hold.

regard simplicity as an epistemic guide to laws, "central facts of normative epistemology are *also* up to us." Hall argues that this is more objectionable than the ratbag idealism of BSA. A defender of BSA may reasonably embrace ratbag idealism and take laws to be pragmatic tools to structure our investigation of the world. With that viewpoint, we can expect that what laws are is somewhat up to us. However, there is no reason on non-Humeanism why epistemological and normative facts should be up to us. So the non-Humeans face a worse problem of ratbag idealism. My analysis suggests that both Humeans and non-Humeans need to adopt fairly strong epistemological principles such as PNS. Unless Humeans retreat to anti-realism about the mosaic (including its microscopic structure and unexplored regions of spacetime), they cannot avoid the problem that "central facts of normative epistemology" may be up to us.

6 Conclusion

Nomic realism is epistemically risky. There is an epistemic gap between metaphysical realism and epistemic realism. However, the gap is no smaller on Humeanism than on non-Humeanism. On both accounts, we need to decide what the physical laws are, in the vast space of possible candidates, based on our finite and limited evidence about the universe. The principle of nomic simplicity, as a fundamental epistemic guide to lawhood, encourages us to look in the direction of simpler laws. We need to add it to both Humeanism and non-Humeanism. It vindicates epistemic realism when there is empirical equivalence (at least in those cases discussed in the paper), avoids probabilistic incoherence when there are nested theories, and supports realist commitments regarding induction, symmetries, dynamics, determinism, and explanation. With many theoretical benefits for only a small price, it is a great bargain.

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