Is Logic Empirical?

Logical 'Conventionalism' from an Empirical Standpoint

Abstract

The laws of classical logic are taken to be logical truths, and logical truths are taken to objectively hold. However, we might question our faith in these truths: why are they true? One often avoided approach is logical conventionalism, because it makes the logical truths dependent on somewhat intersubjective linguistic conventions. Another approach, proposed by Putnam (1975) and more recently Dickson (2001) or Maddy (2007), is to adopt empiricism about logic. On this view, logical truths are true because they are true of the world alone – this gives logical truths an air of objectivity unlike logical conventionalism. Putnam and Dickson both take logical truths to be true in virtue of the world's structure, and the structure of the world is to be understood to be given by our best empirical theory, quantum mechanics. As it turns out, the structure of quantum mechanics apparently makes true the laws of quantum logic, and falsifies (one half of) the distributive law, something which was taken to be a logical truth under classical logic. Empiricists take this to indicate that the distributive law was not a logical truth to begin with. However, this argument assumes that there is a single determinate structure of the world prescribed by quantum mechanics. In this essay, I argue that this assumption is false, and that the structure of the world is underdetermined in quantum mechanics. Likewise, the choice of 'true' logic, as given by the world's structure, is also underdetermined. This leads to what I call empirical conventionalism: the world alone fails to determine our logical truths. We need something broadly intersubjective, and thus less than objective, to fix our choice of logic even under empiricism. An attempt to avoid one form of conventionalism has thus led us back to another.

Keywords: philosophy of physics, philosophy of logic, underdetermination, conventionalism, quantum logic.

1. Introduction

Consider the *distributive law over conjunctions* for all sentences *p, q,* and *r*:

(**CON**): *p* and (*q* or *r*) ↔ (*p* and *q*) or (*p* and *r*)

Alongside other 'laws' of classical logic, **CON** is usually taken as a *logical truth – regardless of the contents of p, q* or *r*, **CON** *objectively* holds.

We might ask: why are logical truths true? One approach takes logical truths to follow from meanings of subsentential operators. This seems to lead to logical conventionalism, (roughly) the thesis that logical truths, e.g. **CON**, are true 'in virtue of meaning' or 'true by convention'.[1] However, logical conventionalism is intuitively unsatisfactory for explaining **CON**'s *objective* truth since it makes **CON**'s truth dependent on (at best) *intersubjective* conventions.[2]

One attractive alternative is empiricism, which claims that the facts determining choice of logic are *not* conventional because they are given by the world alone, independent of human conventions. Empiricism, taken as the thesis that the world *alone* determines our logic, *prima facie* avoids the problem of intersubjectivity: a logic is objectively true because it is *validated* solely by a mind-independent world. Logical truths hold independent of us because there are empirical facts of the matter deciding the 'true' logic.

How is a logic validated by the world? Maddy (2007) proposes that 'logic is true of the world because of its *underlying structural features*'.[3] For example, I might say that **CON** is validated by the world's **CON**-structure: whenever I have a red ball *and* either a blue *or* green ball, I have *either* a red *and* blue ball, *or* a red *and* green ball. Conversely, to say that **CON** is not validated is to say that the world does not have a **CON**-structure.

The difficulty, then, is determining the world's structure: the empiricist strategy is to 'read off' logic from *our best (most empirically successful) sciences*[4], which I take to be quantum mechanics (**QM**).[5] Putnam thinks this approach is superior to logical conventionalism:

> Anyone who really regards the choice of a logic as a 'matter of convention', will have to say that whether 'hidden variables exist', or whether, perhaps, a mysterious' disturbance by the measurement exists', […] is likewise a matter of convention.[6]

If relevant empirical facts about **QM** determining the 'true' logic appear determinate and objective, empiricism has an edge over logical conventionalism in explaining **CON**'s objective truth. However, **CON** appears false in the logic of quantum mechanics, quantum logic (**QL**). Empiricists like Putnam (1975) and Dickson (2001) interpret this to mean that **CON** *is* false, and **QL** is instead the 'true' logic.[7] The objectivity of empiricism thus comes at a cost: **CON** is, after all, a *law* of logic, which we had hoped to establish as objectively true. The empiricist might bite the bullet

---

[1] Warren (2016), p. 2.
[2] Quine (1936) remains the starting point against *explicit* conventionalism. However, see Warren (2016) who argues for *implicit* conventionalism.
[3] Maddy (2009), p. 226.
[4] Putnam (1975), p. 179.
[5] I employ *non-relativistic* **QM** here, and assume that the conceptual problems afflicting various interpretations presented here in relativistic **QM** are resolvable – if so, the issues discussed later remain.
[6] Putnam (1975), pp. 191-192.
[7] Dickson (2001), p. 2

here and find forsaking **CON** a worthwhile price for reclaiming objectivity for logic.

Here, I re-examine this strategy, specifically its presupposition of a determinate world-structure prescribed by **QM**. In this essay, I show that the choice of world-structure in **QM** is *empirically conventional:* nothing within **QM**'s formalism, from which all empirical results are derived, can determine the choice of world-structure, or 'true' logic. The world alone fails to decide our logic. Putnam's challenge to the conventionalist thus fails: those relevant empirical facts which determine the 'true' logic *are* still conventional, leaving us with *yet* another form of conventionalism.

In **§2**, I introduce basic **QM** formalism – the backbone of **QM**'s empirical success. In **§3** I define **QL** on **QM**'s structure, and show why **CON** *prima facie* fails in **QL**. In **§4** I present two well-known interpretations of **QM,** each with a different interpretation of **QM** formalism and thereby different conclusions about **QL**'s status and **CON**. In **§5**, I argue that the empirical results of quantum mechanics *underdetermines* interpretation, and leads to empirical conventionalism about **QM's** interpretation. *A fortiori*, the 'true' logic is *underdetermined*. This leads to a conventionalism about logic, from *within empiricism*.

2. Basic Quantum Mechanical Formalism

Before I describe **QL**, I first present **QM**'s formalism underpinning it: "a set of equations and […] calculational rules for making predictions that can be compared with experiment".[8] The formalism alone is enough to explain all empirical results, and its empirical success is undisputed. As Cushing notes, most physicists, in experimental contexts, adheres *only* to the formalism and 'getting the numbers right'.[9]

Systems: a quantum system (the quantum analogue of classical physical systems) is represented by some *Hilbert space* $\mathfrak{K}$ (i.e. a *complex complete inner-product* vector space).

Observables: each *observable* (measurable property of the system), e.g. spin or momentum, is represented by a Hermitian operator[10] with an associated family of projection operators, each projecting onto (normalized) mutually orthogonal one-dimensional subspaces of some $\mathfrak{K}$. The set of these subspaces form an *orthonormal basis* of some $\mathfrak{K}$ (i.e. they generate the span[11] of that $\mathfrak{K}$).

States*:* Every one-dimensional subspace of an orthonormal basis is an *eigenstate* of the observable, and represents a possible *state* of the system (e.g. spin-up, spin-down). However, since $\mathfrak{K}$ is constructed from the span of such subspaces, all of their linear combinations are also inside $\mathfrak{K}$, and likewise possible states of the system: if ψ and ϕ are distinct eigenstates of a system, then the *superposition* of the two eigenstates, a vector $\boldsymbol{a}\psi + \boldsymbol{b}\phi$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are complex numbers such that $|\boldsymbol{a}^2| + |\boldsymbol{b}^2| = 1$, is *itself* a possible state of the system.

Dynamics*:* A wave equation (e.g. Schrödinger's equation) governs the dynamics of states in $\mathfrak{K}$ over time. A solution to this equation is a *wave-function* $\boldsymbol{\Psi}$ describing how a system *deterministically* evolves over time.

***Composite Systems*:** the tensor product $\otimes$ of multiple systems describes these systems. Given two systems **1** and **2** with the bases:

[8] Cushing (1993), p. 265.
[9] *Ibid.*
[10] An operator $\mathbf{A}$ on $\mathfrak{K}$ is *Hermitian* if, for all vectors $\mathbf{u}$ and $\mathbf{v}$, $\langle \mathbf{u}|\mathbf{A}\mathbf{v}\rangle = \langle \mathbf{A}\mathbf{u}|\mathbf{v}\rangle$. For more details, see Hughes (1993).
[11] The *span of vectors* is the set of *all their possible linear combinations.*

$$\{|+_x^1\rangle, |-_x^1\rangle\} \text{ and } \{|+_x^2\rangle, |-_x^2\rangle\}$$

a new basis for the composite system, $\mathfrak{K}_c$, is constructed with the following possible states:

$$\{|+_x^1\rangle \otimes |+_x^2\rangle, |+_x^1\rangle \otimes |-_x^2\rangle, |-_x^1\rangle \otimes |+_x^2\rangle, |-_x^1\rangle \otimes |-_x^2\rangle\}$$

Notably, these states are *irreducibly* composite: For example, $|-_x^1\rangle \otimes |+_x^2\rangle$ *cannot* be broken down into independent sub-states $|-_x^1\rangle$ or $|+_x^2\rangle$; these states are *entangled* and must be described *together*. This is the source of Einstein-Podolsky-Rosen correlations[12] and quantum non-locality.

***Measurements*:** Lastly, given a measurement on a system in state ψ, the *projection postulate* states that

$$\psi = \sum_k a_k \psi_k \rightarrow \psi_j$$

Upon measurement, ψ is 'collapsed' onto some one-dimensional subspace representing an eigenstate. If ψ is some superposed state $\psi = \boldsymbol{a}\psi_1 + \boldsymbol{b}\psi_2$, the postulate states that ψ 'collapses' into one of two eigenstates ψ_1 or ψ_2, with the *Born rule* prescribing probabilities for the states occurring as $|\boldsymbol{a}^2|$ and $|\boldsymbol{b}^2|$ respectively. Thus, in considering whether a system is in state ψ_1 *or* ψ_2, we must calculate it *via* calculating the probabilities of $|\boldsymbol{a}^2|$ and $|\boldsymbol{b}^2|$ from $\boldsymbol{a}\psi_1 + \boldsymbol{b}\psi_2$.

3. Basic Quantum Logic

3.1. 𝕶's structure, and QL

The set of all possible subspaces of $\mathfrak{K}$, $\boldsymbol{S}(\mathfrak{K})$. $\boldsymbol{S}(\mathfrak{K})$ has a structure: it is a *partially ordered lattice* $\boldsymbol{L}(\mathfrak{K})$, with $\boldsymbol{P} \leqslant \boldsymbol{Q}$ defined as $\boldsymbol{P}$ being a subspace of $\boldsymbol{Q}$ in $\mathfrak{K}$. For any two subspaces in $\boldsymbol{L}(\mathfrak{K})$, there is a greatest subspace common to both (the *infimum*), and a smallest subspace containing them both (the *supremum*). Following Hughes[13], I define meet (∧) and join (∨) on subspaces in $\boldsymbol{L}(\mathfrak{K})$:

$$\textbf{(Meet) } \boldsymbol{P} \wedge \boldsymbol{Q} = \boldsymbol{P} \cap \boldsymbol{Q}$$
$$\textbf{(Join) } \boldsymbol{P} \vee \boldsymbol{Q} = \bigcap\{\boldsymbol{N}: \boldsymbol{N} \in \boldsymbol{S}(\mathfrak{K}) \text{ and } \mathbf{P} \leqslant \mathbf{N}, \mathbf{Q} \leqslant \mathbf{N}\}$$

While the *meet/infimum* of two subspaces is equivalent to their intersection, the *join/supremum* of two subspaces is *not* their union in the classical sense. Rather, it is their *span*, *viz.* the *plane* containing the two subspaces *and* all their possible linear combinations. Indeed, a union of two subspaces is in general *not* a subspace in 𝕶.[14] This reflects **QM**'s principle that, if any two states are possible states of a system, then, *at the same time*, so is their linear combination.

In $\boldsymbol{L}(\mathfrak{K})$ every subspace is a subspace of $\mathfrak{K}$, and the subspace of every member of $\boldsymbol{L}(\mathfrak{K})$ is the origin vector $\mathbf{0}$. Hence, $\mathfrak{K}$ is the *maximum element*, and $\mathbf{0}$ the *minimum element*, of $\boldsymbol{L}(\mathfrak{K})$.

The *orthocomplement* $\boldsymbol{P}^\perp$ of any subspace $\boldsymbol{P}$ is such that $\boldsymbol{P} \vee \boldsymbol{P}^\perp = \mathfrak{K}$, $\boldsymbol{P} \wedge \boldsymbol{P}^\perp = \mathbf{0}$, $(\boldsymbol{P}^\perp)^\perp =$

[12] See Fine (2014) for an excellent historical summary on Einstein-Podolsky-Rosen correlations.
[13] See Hughes (1994) for a full formal account of **QL**
[14] Specifically, $P \cup Q$ is a subspace iff one of them is contained in the other.

$\boldsymbol{P}$, and $\mathbf{P} \leqslant \mathbf{Q}$ implies $\boldsymbol{Q}^{\perp} \leqslant \boldsymbol{P}^{\perp}$. Two subspaces $\boldsymbol{P}$ and $\boldsymbol{Q}$ are *orthogonal*, $\boldsymbol{P} \perp \boldsymbol{Q}$, iff $\mathbf{P} \leqslant \boldsymbol{Q}^{\perp}$.

We can quite naturally define **QL** as a formal logic on $\boldsymbol{L}(\mathfrak{K})$. First we start with a set of *logical vocabulary* {'$\vee_{QL}$', '&', '~'}, and take the *propositions* to be handled by **QL** to be all *experimental propositions* $\boldsymbol{x_p}$which may be asked of a system, of the form 'will the system pass a test for some possible state $\boldsymbol{P}$ with probability 1?'[15] A function $\boldsymbol{f}: \boldsymbol{x_i} \rightarrow \boldsymbol{L}(\mathfrak{K})$ then puts the set of these propositions $\boldsymbol{x_i}$ into bijective correspondence with $\boldsymbol{L}(\mathfrak{K})$. For each proposition $\boldsymbol{x_p}, \boldsymbol{x_q} \in \boldsymbol{x_i}$, and subspaces $\boldsymbol{P}, \boldsymbol{Q} \in \boldsymbol{L}(\mathfrak{K})$,

$$f(\boldsymbol{x_p} \,\&\, \boldsymbol{x_q}) \text{ iff } f(\boldsymbol{x_p}) \wedge f(\boldsymbol{x_q}) = \boldsymbol{P} \wedge \boldsymbol{Q}$$
$$f(\boldsymbol{x_p} \vee_{QL} \boldsymbol{x_q}) \text{ iff } f(\boldsymbol{x_p}) \vee f(\boldsymbol{x_q}) = \boldsymbol{P} \vee \boldsymbol{Q}$$
$$f({\sim}\boldsymbol{x_p}) \text{ iff } [f(\boldsymbol{x_p})]^{\perp} = \boldsymbol{P}^{\perp}$$

Clearly, '&', '$\vee_{QL}$' and '~' parallel the *meet* ($\wedge$), *join* ($\vee$) and orthocomplement ($^{\perp}$) operations on $\boldsymbol{L}(\mathfrak{K})$.

Lastly,[16] I define *logical consequence*:

$$\boldsymbol{x_p} \vDash_{QL} \boldsymbol{x_q} \text{ iff } f(\boldsymbol{x_p}) \leqslant f(\boldsymbol{x_q}) = \boldsymbol{P} \leqslant \boldsymbol{Q}$$

3.2. The Status of CON

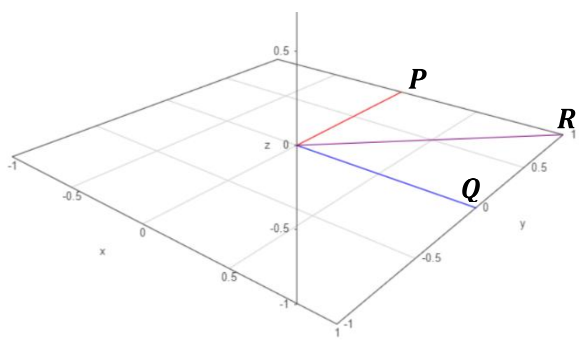
With **QL** set up, I return to the issue raised in **§1**. Recall the empiricist claim: **QL**, based off the structure of **QM**, apparently shows that **CON** is false. Consider **QL**'s equivalent of **CON**:

$$\textbf{(CON*)}: \boldsymbol{x_R} \,\&\, (\boldsymbol{x_P} \vee_{QL} \boldsymbol{x_Q}) \leftrightarrow (\boldsymbol{x_R} \,\&\, \boldsymbol{x_P}) \vee_{QL} (\boldsymbol{x_R} \,\&\, \boldsymbol{x_Q})$$

This holds iff:

$$\textbf{(CON}^{\dagger}\textbf{)}: \quad \boldsymbol{R} \wedge (\boldsymbol{P} \vee \boldsymbol{Q}) = (\boldsymbol{R} \wedge \boldsymbol{P}) \vee (\boldsymbol{R} \wedge \boldsymbol{Q})$$

Suppose that $\boldsymbol{P}$ and $\boldsymbol{Q}$ are orthogonal, and $\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q}$:



Clearly, $(\boldsymbol{R} \wedge \boldsymbol{P}) = \mathbf{0}$ and $(\boldsymbol{R} \wedge \boldsymbol{Q}) = \mathbf{0}$, i.e. they only intersect at $\mathbf{0}$. Therefore, $\mathbf{0} \vee \mathbf{0} = \mathbf{0}$ on the right-hand side of **CON**†. However, consider the left-hand side: the intersection of the plane containing $\boldsymbol{P}$ and $\boldsymbol{Q}$, and the subspace $\boldsymbol{R}$ is clearly $\boldsymbol{R}$ itself, since the entirety of $\boldsymbol{R}$ is on the plane.

[15] Bacciagaluppi (2009), p. 9.

[16] I ignore ultrafilters – **QL**'s analogue for truth-assignment – and logical truth due to space constraints. Nothing I discuss turns on them.

Since $\mathbf{0} \neq \boldsymbol{R}$, **CON**† is false. *A fortiori*, **CON*** is false.

Objection: *nothing so far* shows that the *classical* **CON**, employing 'and' and 'or', has broken down. Rather, I merely demonstrated the falsehood of **CON***, using '&' and '$\vee_{QL}$', on a restricted class of *experimental propositions*. Thus Maudlin complains that "quantum 'logic' isn't *logic,* i.e. isn't an account of conjunction and disjunction".[17] To show that **CON***'s failure entails the **CON**'s failure from an empirical perspective, proponents of **QL** must show that **QL** *is* classical logic – we just got the logical behavior of 'or' *wrong*.

I think this requires us to first claim that, in the context of experimental propositions, there is **(a)** no connective 'or' that is *meaningfully definable*, and **(b)** the *best replacement* for 'or' is '$\vee_{QL}$'. Furthermore, **(c)** we must show that the experimental propositions of **QL** *exhaust* the propositions about the world. In other words, the structure of **QM** *completely describes* the world. Without **(c)**, then the proponent of classical logic can still claim that the world is *really* classical, and the non-classical nature of **QL** only arises in the context of measurements: the world *alone* still gives us classical logic. Given **(a)** – **(c)**, though, the proponent of **QL** can assert that there is no other way to 'read off' disjunction from the structure of the world without using '$\vee_{QL}$'. This, together with the empiricist assumption that logic is given by the world alone, justifies the claim that 'or' was *really* '$\vee_{QL}$' all along: **CON*** is *really* **CON**, and since **CON*** is false, so is **CON**.

Within the context of experimental propositions, there is justification for **(a)**: there is no clear way to introduce 'or' within **QM**'s structure since there is, in general, no experimental proposition or subspace in $\mathfrak{K}$ corresponding to the classical disjunction of $\boldsymbol{P}$ and $\boldsymbol{Q}$.[18] Furthermore, the one special case where $\boldsymbol{P} \cup \boldsymbol{Q}$ *is* a subspace, viz. when one of the subspaces is contained in the other, can be interpreted in terms of the *span* of $\boldsymbol{P}$ and $\boldsymbol{Q}$ as well. Lastly, it is clear that the span of **P** and **Q** is widely used experimentally, in e.g. considering superposed states of **P** and **Q**. Thus, either there is no experimental proposition corresponding to $\boldsymbol{P} \cup \boldsymbol{Q}$, or $\boldsymbol{P} \cup \boldsymbol{Q}$ can be understood as the span of **P** and **Q** in the special case**,** which in turn applies generally in **QM**. This gives us reason to claim that we cannot even speak of the classical 'or' meaningfully in terms of experimental propositions.

Dickson (2001) further argues for **(b)**, claiming that '$\vee_{QL}$' is the only other plausible candidate[19] for replacing our classical 'or', since '$\vee_{QL}$' satisfies most of our constraints on 'or'. It is worth looking at the logical behavior of '$\vee_{QL}$' to see its similarity to 'or'. For example:

$$\boldsymbol{P} \leqslant \boldsymbol{P} \vee \boldsymbol{Q}$$
$$\boldsymbol{Q} \leqslant \boldsymbol{P} \vee \boldsymbol{Q}$$

Consequently:

$$\boldsymbol{x_P} \vDash_{QL} \boldsymbol{x_P} \vee_{QL} \boldsymbol{x_Q}$$
$$\boldsymbol{x_Q} \vDash_{QL} \boldsymbol{x_P} \vee_{QL} \boldsymbol{x_Q}$$

Notably, this is reminiscent of the introduction rules for 'or'. Furthermore:

$$\text{If } \boldsymbol{x_P} \vDash_{QL} \boldsymbol{x_R} \text{ and } \boldsymbol{x_Q} \vDash_{QL} \boldsymbol{x_R},$$
$$\text{then } \boldsymbol{x_P} \vee_{QL} \boldsymbol{x_Q} \vDash_{QL} \boldsymbol{x_R}$$

[17] Maudlin (2003), p. 491.
[18] Bacciagaluppi (2009), p. 19.
[19] Dickson (2001), p. 4.

This is also similar to the elimination rules for 'or'. '$\vee_{QL}$' thereby appears to behave like the classical 'or'. Of course, '$\vee_{QL}$' behaves differently in other contexts, notably **CON**.[20] However, given **(a)**, '$\vee_{QL}$' seems the closest substitute for our classical intuitions about disjunctions *in the context of experimental propositions.*

4. Two Interpretations: Quantum Logic as Global Logic?

(a) and **(b)** concludes that classical logic cannot be 'read off' the structure of *experimental propositions* in **QM**. However, what about **(c)** – do experimental propositions exhaust all propositions about the world? I argue that there are at least two ways[21] to understand the world-structure **QM** prescribes and **QL**'s experimental propositions: this suggests that philosophical claims based on **QM** are "highly dependent on the interpretational approach one adopts towards the theory".[22]

4.1. Bohmian 'Pilot-Wave' Interpretation

Bohm's 'pilot-wave' interpretation (**BM**) takes every particle to have *determinate* positions and trajectories. However, particles are guided by a 'pilot-wave' obeying the wave-function **Ψ**, causing Bohmian particles to evolve in a uniquely quantum fashion. This wave-function also generates a statistical distribution of the particles' positions, $\mathbf{P} = |\mathbf{\Psi}^2|$. This set-up allows **BM** to uncontroversially satisfy the constraints of **QM** formalism as introduced in **§2**, e.g. the Born rule, and recovers *all* empirical results of **QM**.

However, on **BM**'s view, **QM** formalism is merely *epistemic* in nature. As Bohm notes: "The use of statistics is […] not inherent in the conceptual structure, but merely a consequence of our ignorance of the precise initial conditions of the particle."[23] **QM** formalism is simply an effective tool for *us* to calculate the properties of particles, *given* that the *determinate* but *hidden*, level of phenomena – particles with determinate positions/trajectories – postulated by **BM** lies beyond the reach of measurement. However, in **BM**, particles are *really* ontologically *classical.*[24]

Particularly, in the case of superpositions and '$\vee_{QL}$': under **BM**, if a system is in a superposed state, then **Ψ** 'pilots' particles to two states with a frequency distribution obeying the Born rule. However, importantly, the *particles* themselves are *either* in 'support' of one state *or* another in the classical sense: "the [position/trajectory] configuration of the system is located only in one of these different components, and this is already a matter of classical logic".[25]

On this deeper level, particles are in *some* determinate position at any one time, and all other properties are further derived from position variables on **BM**'s view. The world is *fundamentally classical*, and **CON** remains true. The use of spans – and '$\vee_{QL}$' – instead of classical union in **QL** reflects not the world, but our inability to access the level of hidden variables: the 'non-classical' nature of **QL** arises from our epistemic limitations.

It is thus inadmissible to claim that **QL**'s experimental propositions exhaust all propositions

[20] For an in-depth analysis of the logical behavior of '$\vee_{QL}$', see Humberstone (2011), pp.913-917.
[21] I leave out the 'Copenhagen' interpretation here due to space-constraints – accepting it would not harm our case anyway.
[22] Bacciagaluppi (2009), pp. 36-37.
[23] Bohm (1952), p. 171
[24] Bacciagaluppi (2009), pp. 30-31.
[25] *Ibid*, p. 31.

about the world. Experimental propositions reflect not the totality of the world, but the limits of our epistemic access to the world. **QL** is, under **BM**, merely a logic of measurements and *cannot* be taken to conclusively replace classical logic (and thereby falsify **CON**).

4.2. Many-Worlds Interpretation

Contrariwise, the *many-worlds interpretation* (**MWI**) claims that **QM**'s formalism, and the experimental propositions of **QL**, completely describes the universe. However, on this view, *our* 'world' is but one 'branch' of the universe.

Consider a measurement device $\boldsymbol{\phi}$ which points *up* $|\uparrow_{\boldsymbol{\phi}}\rangle$ when an electron is spin-up, points *down* $|\downarrow_{\boldsymbol{\phi}}\rangle$ when an electron is spin-down, and points *nowhere* $|\varnothing_{\boldsymbol{\phi}}\rangle$ when there is no electron. Furthermore, an observer, $\boldsymbol{O}$, can likewise be considered a system: Suppose $\boldsymbol{O}$ observes $\boldsymbol{\phi}$ pointing a certain direction when $\boldsymbol{\phi}$ in fact points in that direction. Let $|\uparrow_{\boldsymbol{O}}\rangle, |\downarrow_{\boldsymbol{O}}\rangle$, and $|\varnothing_{\boldsymbol{O}}\rangle$ represent these $\boldsymbol{O}$-states. Given a system $\boldsymbol{E}$ of a spin$_x$-1/2 electron prepared in a superposed state $a|+_{\boldsymbol{E}}\rangle + b|-_{\boldsymbol{E}}\rangle$, we can construct a composite system $\boldsymbol{E} \otimes \boldsymbol{\phi} \otimes \boldsymbol{O}$. Thus, when $\boldsymbol{O}$ observes $\boldsymbol{\phi}$ measuring $\boldsymbol{E}$:

$$(\boldsymbol{a}|+_{\boldsymbol{E}}\rangle + \boldsymbol{b}|-_{\boldsymbol{E}}\rangle) \otimes |\varnothing_{\boldsymbol{\phi}}\rangle \otimes |\varnothing_{\boldsymbol{O}}\rangle \rightarrow \boldsymbol{a}|+_{\boldsymbol{E}}\rangle \otimes |\uparrow_{\boldsymbol{\phi}}\rangle \otimes |\uparrow_{\boldsymbol{O}}\rangle + \boldsymbol{b}|-_{\boldsymbol{E}}\rangle \otimes |\downarrow_{\boldsymbol{\phi}}\rangle \otimes |\downarrow_{\boldsymbol{O}}\rangle$$

However, instead of saying $\boldsymbol{a}|+_{\boldsymbol{E}}\rangle \otimes |\uparrow_{\boldsymbol{\phi}}\rangle \otimes |\uparrow_{\boldsymbol{O}}\rangle + \boldsymbol{b}|-_{\boldsymbol{E}}\rangle \otimes |\downarrow_{\boldsymbol{\phi}}\rangle \otimes |\downarrow_{\boldsymbol{O}}\rangle$ 'collapses' into $\boldsymbol{a}|+_{\boldsymbol{E}}\rangle \otimes |\uparrow_{\boldsymbol{\phi}}\rangle \otimes |\uparrow_{\boldsymbol{O}}\rangle$ or $\boldsymbol{b}|-_{\boldsymbol{E}}\rangle \otimes |\downarrow_{\boldsymbol{\phi}}\rangle \otimes |\downarrow_{\boldsymbol{O}}\rangle$ upon interaction with $\boldsymbol{\phi}$ (per the projection postulate), **MWI** claims that the universe is in *both* states simultaneously – the universe remains *superposed*. This seems absurd since our measurements show *one* definite result. However, the phenomenology of measurement, and the projection postulate, is explained away in **MWI** by saying that *we*, as $\boldsymbol{O}$, are entangled with one particular measurement outcome or another – from (one of) *our* perspective(s), only one outcome obtains. Furthermore, $\boldsymbol{a}|+_{\boldsymbol{E}}\rangle \otimes |\uparrow_{\boldsymbol{\phi}}\rangle \otimes |\uparrow_{\boldsymbol{O}}\rangle$ and $\boldsymbol{b}|-_{\boldsymbol{E}}\rangle \otimes |\downarrow_{\boldsymbol{\phi}}\rangle \otimes |\downarrow_{\boldsymbol{O}}\rangle$ rapidly *decohere* [26] following measurement, due to environmental interferences, and become *dynamically independent* of one another. For all practical purposes, then, there is only one definite state relevant to *us* ('one of' us). The other 'world' is effectively inaccessible.

Thus, **MWI** accounts for *exactly the same empirical results* as **BM**, and, as before, **QL** works as a logic of measurements. However, according to **MWI**, and unlike **BM**, the formalism of **QM** and the possible experimental propositions – corresponding to possible subspaces of $\mathfrak{K}$ – are *not* just propositions about measurements or reflections of our epistemic limits: rather, they are *genuine* propositions about the universe.

Prime example: **MWI** takes superposed states as *actual states of single particles*, *not* stochastic distributions of classical particles into possible states per **BM**. On **MWI**, systems which are in superposed states *stay so* after measurement unlike **BM**, where particles are in *either* some determinate state *or* another. On **MWI**, each component state of the superposition *actually* obtains, albeit in dynamically independent 'worlds'. In considering the classical union of two states of a system, then, we *must* consider the span of the two states where we find their linear combination, i.e. the *actual* state of the universe. Hence, we see that propositions about the universe behave like the experimental propositions of **QL** in that they map onto the lattice-structure of $\boldsymbol{L}(\mathfrak{K})$: **QL** under **MWI** *replaces* classical logic as the 'true' logic of the world.

[26] For more on decoherence in **MWI**, see e.g. Wallace (2010, 2012).

Furthermore, **MWI** explains why classical logic has been so successful from our perspective: **CON** *is* validated by *our* 'world', which is decohered from other worlds (in everyday macroscopic scenarios) – only one of the quantum disjuncts obtain *from our perspective*. However, we were *mistaken* to think that our 'world' is all there is to the universe. Both quantum disjuncts really *do* obtain in the universe, and the universe is described completely by **QM** and the experimental propositions in **QL**. Thus **(c)** obtains, and we might claim that **CON** is *really* not validated by the world. Turning **BM** on its head, on **MWI** it is *classical logic* that arises from our epistemic limitations.

5. Empirical (Under)-Determination of Interpretation and Logic

I began by asking why logical truths are true. Hoping to avoid the conventionalist path, I turned to empiricism. However, **§4** shows that the world alone, given by our best sciences, cannot give a decisive answer to whether experimental propositions exhaust all propositions about the world. Though **BM** and **MWI** are *empirically* equivalent, each reproducing **QM**'s empirical results, each interpretation includes postulations about the world (hidden variables or dynamically independent worlds) beyond empirical reach. These interpretations also take different logics to be *the* universally true logic. This leads us to conclude that empirical evidence alone fails to determine the logic of the world: The 'true' logic, under empiricism, remains underdetermined.

I propose this situation leads back to *conventionalism*. Here, conventionalism is not the thesis that logic is 'true by meaning' or 'convention'. It makes no substantive claims about the truth-status of logical truths. Rather, this conventionalism is analogous to the situation for our universe's 'ultimate' space-time structure. It is generally agreed that *that* is conventional, in the sense that general relativity "allows for a wide variety of cosmological models but that, due to structure internal to the theory itself, does not allow us to determine which of these models best represents our universe.[27]

This applies for **QM** and its various interpretations: "no amount of evidence will ever compel us to embrace a particular scientific claim"[28] about **QM**'s interpretation. This is what I call *empirical conventionalism*, which is described by Sklar as such: "insofar as the two theories have the same predictive content with regard to the directly observable facts, they ought to be viewed as merely conventional alternatives to one another and not as genuinely alternative theories about the nature of the world."[29] In other words, **QM** formalism and its empirical results is *in principle* indifferent between interpretations.

This is old news in physics – many have been willing to 'shut up and calculate', ignoring interpretative questions precisely because of empirical conventionalism. What is new to us is the result that the choice of 'true logic' is also *empirically conventional.* No empirical evidence can determine whether **QL** or classical logic is true; this choice is arbitrary *from an empirical perspective.* Thus Belousek concludes: "the 'book of nature' proves too ambiguous to be uniquely interpretable".[30]

Cushing's solution is to go beyond empirical facts of the matter "to include factors such as fertility, beauty, coherence, naturalness and the like."[31] However, it is unclear what evidence can

---

[27] Manchak (2009), p. 53.
[28] Ibid.
[29] Sklar (2004), p. 958.
[30] Belousek (2005), p. 673.
[31] Cushing (1993), p. 272.

empirically settle the debate here, since all empirical results (ever) available to the two interpretations are *equivalent.* In any case, to rely on such extra-empirical factors is to give up on empiricism. Rather, logic choice is determined partly by human factors, which are at best *intersubjective.* To go down this route is to lose the objectivity of logic even on the empiricist view, yet it seems that, at least within **QM**, we must go down this route.

This distinction between unempirical and empirical facts can be further clarified with Putnam's distinction between two types of facts constraining what he calls total science:[32]

(ICC) Internal Coherence Constraint:	Science must cohere with *simplicity*, and agreement with *intuition*, and so on.
(ECC) External Coherence Constraint:	Science must agree with *experimental checks*, i.e. empirical facts.

Here, an interpretation is chosen not only because it coheres with all possible empirical facts, viz. **ECC**, but also because of simplicity, intuitiveness, fecundity, etc., viz. **ICC**. Putnam suggests that **ICC** provides a *further fact of the matter* which decides between seemingly empirically conventional choices.

Two points: first, I think the acceptance of **ICC** simply makes the *un*empirical elements involved in interpretational choice more obvious. While something *can* be a determinate fact of the matter given such constraints, *these* constraints of simplicity, intuitiveness, etc., are exactly what appears to be *intersubjective*. Even if there could be a decisive fact of the matter given some choice of **ICC**, I am not sure we could ever find objective grounds for **ICC**s themselves. The choice of a determinate interpretation with **ICC** thus comes at a loss of objectivity.

Secondly, it is unclear whether there even *is* a fact of the matter under **ICC** whether **BM** or **MWI** is better. To me, at least, it is not apparent whether **BM** or **MWI**, presented above, is *simpler*, or more *intuitive.* Given the complicated nature of **QM**, and the technical and conceptual apparatus required for both **BM** (hidden level of phenomena, distinct pilot-waves guiding quantum particles, non-locality) and **MWI** (a world of infinite 'worlds', decoherence as a rough-grained process), neither **BM** nor **MWI** obviously satisfies any given **ICC** (e.g. simplicity, etc.) better than the other. One is ultimately left to one's metaphysical predilections.

In any case, the empiricist would have lost much in adopting **ICC**. Recall that empiricism aims to place logical truths on firmer grounds than logical conventionalism. Empiricism does so by appealing to the world because the relevant empirical facts determining a 'true' logic are intuitively *objective* in a way our linguistic conventions are not. However, even within empiricism, there is no determinate interpretational choice for **QM**. **ECC** does not suffice for any decision on the true world-structure and the 'true' logic; we must appeal to **ICC**, be it simplicity, intuition, or what-not. Regardless of the outcome of *that* debate, the resulting situation is certainly not objective as the choice seems to amount to something about *us*, as rational beings, as scientists, and so on. Consequently, the world *alone* has failed to give us the 'true' logic. Empiricism thus fails to obtain objectivity for logic, leading instead to empirical conventionalism. It is no longer clear whether this is more attractive than logical conventionalism.

[32] Putnam (1974), p. 33.

6. Conclusion

Empiricists who want to recover the objectivity of logic by appealing to the world alone must recognize that our best theory of the world, **QM**, is *underdetermined* when it comes to the world-structure it prescribes. This entails that the 'true' logic is likewise *empirically conventional* – we have no empirical reason to think that **CON** (and classical logic) is true of the world or otherwise, because the true structure of the quantum world is unknown (indeed, *unknowable*). Adopting **ICC** to determine our 'true' logic only ameliorates this situation by basing our choice of logic on *intersubjective* – not quite objective – facts. Thus, empiricism, with its associated empirical conventionalism about logic, appears no better off than logical conventionalism in accounting for the objectivity of logical truths: something broadly conventional lurks.

Cited Works

Bacciagaluppi, Guido. "Is Logic Empirical?" (2009) Retrieved from http://philsci-archive.pitt.edu/3380/ (preprint).

Belousek, Darrin. W. "Underdetermination, Realism, and Theory Appraisal: An Epistemological Reflection on Quantum Mechanics." *Foundations of Physics* 35, no. 4 (2005): 669-95.

Bohm, David. **"**A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables I." *Physical Review* 85, no. 2 (1952): 166-79.

Dickson, Michael. "Quantum Logic Is Alive ∧ (It Is True ∨ It Is False)." *Philosophy of Science* 68, no. S3 (2001). Retrieved from http://mdickson.net/pubs/Quantum_Logic.pdf, 1-17.

Cushing, James. "Underdetermination, Conventionalism and Realism: The Copenhagen vs. the Bohm Interpretation of Quantum Mechanics," in *Correspondence, invariance and heuristics: essays in honour of Heinz Post*, edited by Steven French and Harmke Kamminga**.** (Dordrecht: Springer, 1993), 261-278.

Fine, Arthur. "The Einstein-Podolsky-Rosen Argument in Quantum Theory", *The Stanford Encyclopedia of Philosophy* (Fall 2016 Edition), edited by Edward N. Zalta. URL = <https://plato.stanford.edu/archives/fall2016/entries/qt-epr/>.

Hughes, R. I. G. *The structure and interpretation of quantum mechanics.* (1994) Cambridge, Mass: Harvard Univ. Press.

Humberstone, Lloyd. *The Connectives*. (2011) Cambridge, Mass: MIT Press.

Maddy, Penelope. *Second Philosophy: A Naturalistic Method.* (2007) Oxford: Oxford University Press.

Manchak, John. B. "Can we know the global structure of spacetime?" *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 40, no. 1 (2009): 53-56.

Putnam, Hilary. "The Refutation of Conventionalism." *Noûs* 8, no. 1 (1974): 25-40.

—— "The Logic of Quantum Mechanics", in *Mathematics, Matter and Method Vol. 1 (*1975), Cambridge: Cambridge University Press, 174-197.

Quine, W. V. O. "Truth by Convention." *Philosophy of mathematics* (1936), 329-54.

Sklar, Lawrence. "Spacetime and Conventionalism." *Philosophy of Science* 71, no. 5 (2004**):** 950-59.

Wallace, David. "Decoherence and its role in the modern measurement problem." *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 370, no. 1975 (2012): 4576-593.

——"Decoherence and Ontology (or: How I learned to stop worrying and love FAPP)" in *Many Worlds?: Everett, Quantum Theory & Reality*, edited by Simon Saunders, Jon Barrett, Adrian Kent and David Wallace (Oxford: Oxford University Press, 2010), 53-72.

Warren, Jared. Revisiting Quine on Truth by Convention. *Journal of Philosophical Logic* (2016).