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Abstract

The paper aims to add contextual dependence to the new directival theory of meaning, a functional role semantics based on Kazimierz Ajdukiewicz's directival theory of meaning. We show that the original formulation of the theory does not have a straight answer on how the meaning of indexicals and demonstratives is established. We illustrate it in the example of some problematic axiomatic and inferential directives containing indexicals. We show that the main reason why developing the new directival theory of meaning in this direction is difficult is that the theory focuses on the notion of a sentence (and not the notion of an utterance). To add the latter notion to the theory, we introduce the idea of admissible contextual distribution being an interpretation of the hybrid expression view on indexicals and demonstratives. We argue that this idea introduces a small but important modification to the concept of language matrix and gives way to define two distinct concepts of meaning: for an expression type and for a use of an expression type.

Keywords: Directival theory of meaning, Context, Indexicals, Demonstratives, Hybrid expressions.

1. New Directival Theory of Meaning

The new directival theory of meaning (henceforth the nDTM) is a functional role semantics based on a theory of meaning originally proposed by Kazimierz Ajdukiewicz in the 1930s. The easiest way of explaining the theory is to start with a slogan that it defines the meaning of words based on a combination of syntax and pragmatics. Let us now see how both aspects contribute to the theory. The nDTM begins with an observation about a specific type of disputes language users sometimes engage in. Since this part of the theory deals with users and their

¹ Cf. Ajdukiewicz 1934, 1978. A detailed description of all differences between the original theory and the nDTM is beyond the scope of this paper. A reader interested in learning about these differences as well as learning about the details of the new take on Ajdukiewicz's theory can find both in Grabarczyk 2017, 2019. The summary provided below focuses on those aspects of the theory that are necessary for the understanding of the problem described in section 2 as well as the consequences to the theory that follow from the solution proposed in section 4.

Argumenta (2022): 35-53 ISSN 2465-2334 © 2022 Tadeusz Ciecierski & Paweł Grabarczyk DOI 10.14275/2465-2334/202200.cie behavior, it can be classified as pragmatics. The nDTM points out that it is not uncommon for both sides of an argument to suspect that their dispute may be purely verbal—that it is only a result of using words with two different meanings. What the users do in these cases is shift their focus from the original dispute towards something the nDTM calls semantic trials. The trials confront the user with sentences that contain the disputed expression. The nDTM assumes that a competent language user knows that what they are expected to do during the trial is accept the invoked sentence, provided the circumstances are right. The titular 'directives' are rules of language that correlate specific sentences with correct circumstances and instruct the user to accept the sentence whenever it is uttered in these circumstances. The unusual nature of these sentences is that they are typically never used outside of the context of a semantic trial; they would have been seen as too trivial in a normal conversation. To make it clearer, imagine the following situation. Imagine two people who have a heated discussion about tables. At some point, one of them feels that the opinion of the other side is so outlandish, that they may be using the word "table" differently. To be sure, the person asks the following question: "Tables are a piece of furniture, right?". This is how a semantic trial can look like. It is aimed at checking if the interlocutors share the same directives. Directives function as cornerstones or boundaries of language because every competent user knows that once they fail to accept the sentence indicated by a directive, they will be treated by other language users as linguistically incompetent. It is important to point out that the directives function as a tacit linguistic mechanism; they do not have to be (and typically are not) known to the users. The users are trained to behave in a certain way and can formulate directives only as a post-factum observation of patterns of their behavior. In this respect, the directives function like the rules of syntax. The language user can gain full linguistic competence and not be able to formulate the rules of syntax.

Depending on the type of circumstances indicated by the directives, the nDTM differentiates between four types of directives: axiomatic, inferential, empirical, and promotive. The first category is the easiest to explain, as it is the one that does not specify any particular circumstances; the only thing the *axiomatic* directives ensure is that the user accepts a particular sentence, regardless of the circumstances they are in.² The table example we have just used falls under this category because the sentence "table is a piece of furniture" is expected to be accepted on all occasions. Building on this simple structure, it should now be easy to picture *inferential* directives as rules that instruct the user to accept a given sentence if they accepted some other, specified sentence—imagine the *modus ponens* rule as a model example of this. What it forces the user to do is that they must accept a given sentence if they already accepted an implication that has this sentence as a consequence and if they accepted the antecedent of this implication. In this case, the whole notion of circumstances correlated with the acceptance of a sentence boils down to the prior acceptance of other sentences.

Things get more interesting (and, admittedly, more complicated) once we proceed to the third and the fourth type of directive, which add extra-linguistic circumstances to the mix. *Empirical* directives instruct the language users to accept

² As we are going to see, the problems associated with contextual dependence challenge this simplified picture, as some unusual contexts combined with context-sensitive expressions may affect the reaction of the user.

a particular sentence if they happen to be in a certain perceptual state.³ To see it in an example, imagine the sentence "This is red" accepted by users while they look at a vivid red surface. The last type of directive (one that we might call *promotive*) states that in situations where the users accept a certain sentence, they should perform a certain bodily action: enter a certain motor state. To see this type of directive in an example, imagine the sentence "Stop!" accepted by the user. Once they do that, it is required of them to accompany their acceptance with a particular reaction; to put it bluntly, they must stop moving.

Following the original theory, the nDTM assumes that for every simple expression in language, there is a set of directives that contains this expression. Roughly speaking, the main idea of the nDTM is that the meaning of a given simple expression is its placement within 'its' directives—the directives it appears in. For a more detailed picture, imagine the set of all directives of a given language encoded in some way that retains their structure, that indicates the circumstances and the sentence that is to be accepted. Once we have it, we end up with something that the nDTM calls a language matrix. This construct enables us to define the meaning of an expression as its distribution in the language matrix. Even though the language matrix reflects the behavior of the users, the distribution of terms is given in a purely formal manner; it refers only to the structure of sentences and the placement of the expression within them. Because of this, as mentioned at the beginning, the theory can be said to be a combination of syntax and pragmatics. What must be specifically noted is that the nDTM does not make any use of the notion of reference; it is a purely non-referential theory of meaning. Needless to say, this does not mean that the nDTM claims that language is non-referential. The only thing the nDTM claims is that it is possible to explain the notion of linguistic meaning without invoking the notion of reference. The nDTM is best seen as a "prohibitive semantics"—a theory that construes linguistic meaning as something that regulates only the boundaries of language, something that helps us avoid misuse of language (as opposed to determining its use). The point of such a minimalist approach to semantics is that it removes a lot of pressure from the notion of linguistic meaning. We should not expect the notion of meaning to explain all the mechanisms that are present in the language. As argued in Grabarczyk (2019) since prohibitive semantics can give us notions such as "synonymy" or "translation", there is no need to postulate a richer theory of meaning. Still, if we wanted to explain linguistic phenomena that go beyond the minimalistic meaning defined in the nDTM, the theory does not prevent us from doing so. For example, there is nothing that prevents us from adding theory of reference as an auxiliary theory because the fact the nDTM does not make use of this notion does not mean it cannot be useful for explaining other linguistic facts. The solution we present in this article should be treated as one such addition to the original theory—an extension that helps it cover cases it was not originally designed to cover.

Since the language matrix is a theoretical construct of the linguist—a description of patterns of acceptance that can be observed in the linguistic community—it can be represented in a variety of ways. One simple way of representing it is a table that contains two main sections: the input and the output parts. Both parts can contain linguistic and extra-linguistic parts, and the four types of meaning directives we described above can now be distinguished depending on which

³ The way this state is understood is rather complex, but we do not have the space to discuss it in more detail. See Grabarczyk 2019 for a thorough discussion of its nature.

combination of input and output they contain. Axiomatic directives are the ones that result in linguistic output, regardless of the type of input. Inferential directives tie linguistic output with linguistic input; empirical directives present linguistic output when a specified extra-linguistic input is present; and, finally, promotive directives combine linguistic input and non-linguistic output.

To use a very crude simplification of the relation between a language matrix and the meaning of expressions, imagine that you observe a community of language users and discover sentences they always accept in certain circumstances. Let us now say that you fill in a spreadsheet table that indicates circumstances and the sentence to be accepted. The meaning of a given expression could then be given via a complete list of all the cells it appears in. Let us illustrate it on a very simple example that uses only two types of directives: axiomatic and inferential. Consider the following toy language that contains the following directives (assume that the symbol + signifies that a given sentence has been accepted by the users and the symbol ++ signifies that it should be accepted).

Axiomatic directives:

```
1.1 + + P(a).
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1.2 + +P(a) & Q(b).

Inferential directives:

$$2.1 + P(a)$$
; ++ $Q(b)$.

$$2.2 + P(a) & Q(b); ++ Q(c).$$

$$2.3 + O(b)$$
: ++ (Od).

According to the nDTM, the language matrix for this toy language is any systematic way that enables us to dismantle the language syntax and divide the directives into circumstances and a sentence that needs to be accepted. The method used by Ajdukiewicz, that I copy below is that for every expression we start with the expression followed by its main connective, its first argument, its first connective, etc. We decompose expressions as long as we get to the level of atomic expressions. An example of the result of this procedure for the above toy language could look like this:

		(Circumst	ances					Response								
1.1									P(a)	Р	a						
1.2									P(a)&Q(b)	&	P(a)	P	a	Q(b)	Q	b	
2.1																	
2.2	P(a)&Q(b)	&	P(a)	P	a	Q(b)	Q	b	Q(c)	Q	С						
2.3	Q(b)	Q	b						Q(d)	Q	d						

Now, if we wanted to define the meaning of a given simple expression, for example, Q, it would be identified with the distribution of this expression in the matrix. To make it easier to grasp we can now represent this distribution graphically. According to the nDTM, any expression that happens to have the same distribution as Q is synonymous with Q and any expression that has an identical distribution in some other language is a translation of Q.

		C	Circumst	ances					Response								
1.1									P(a)	P	a						
1.2									P(a)&Q(b)	&	P(a)	P	a	Q(b)	Q	b	
2.1																	
2.2	P(a)&Q(b)	&	P(a)	P	a	Q(b)	Q	b	Q(c)	Q	С						
2.3	Q(b)	Q	b						Q(d)	Q	d						

A definition of this type may disappoint people who expect the theory of meaning to provide something akin to a dictionary entry, but, as argued by Grabarczyk (2019), it can still deliver many other features of meaning, such as an explanation of synonymy, translation, or the difference between meaningful and nonsensical expressions.

Two additional explanations are in order. First, the language users are not expected to know the directives. As we already said, the directives are a theoretical construct of the linguist. What exists in the community is a certain regularity of reactions: the users recognize that they are challenged to the semantic trial and that if they fail to accept the sentence enclosed in the directive, they will not be treated as competent users of a given expression. Second, the axiomatic and inferential directives differ from the two other types of directives in that they contain schemas of sentences (and not particular substitutions of schemas). Once again, this will be most easily explained in the case of axiomatic directives. Imagine a simple case of identity. It is plausible to assume many languages to expect their users to accept sentences of the form A=A.⁴ What it means in practice is that the users know that any sentence of this form where the variable A has been replaced with an expression of a correct syntactical category should be accepted.

What is crucial for our purposes is that neither the nDTM nor the original Ajdukiewiczian theory engages with the notion of *context*, even though it touches upon it indirectly at least twice. Firstly, when the notion of the semantic trial is introduced. Recognizing semantic trials may be difficult, as they can be signaled in different ways in different linguistic communities. The most conspicuous way of signaling them is that the disputants engage in the semantic discourse and start to talk about 'meanings' or 'senses' of the word. These expressions function as markers of a contextual switch from a normal conversation to a semantic trial. One notorious sign of a semantic trial is that the users who fail to pass it—those who reject the sentence indicated by a directive are never treated seriously—will be seen as misunderstanding the sentence they reject or simply as using a different vocabulary. Semantic trials can thus be seen as a very specific context that competent language users must recognize.

The second aspect of the nDTM that reminds us of the notion of *context* is the idea of the circumstances C that the user must recognize as correct ones for a given directive. To provide an example of this, imagine a perception of a typical red patch that accompanies an empirical directive "this is red". Both aspects can be described as contextual, as they refer to extra-linguistic aspects of the utterance that must

⁴ The reason we are so cautious here is that whether a given directive functions in a given language demands the linguist to do the actual field work and test the users. It cannot be decided *a priori*.

accompany it. It is nonetheless important to remember that, apart from these two aspects, the nDTM does not take *contextual* dependence into account. Most importantly, it does not consider typical contextual parameters that are required to explain the meaning of expressions that are contextually dependent, such as indexicals. The main reason why this omission must be analyzed is that the lack of discussion of contextual dependence in the nDTM prevents us from asking if it is possible for some contextual parameters to influence semantic tests or directives, such as if it is possible for some of them to cancel the markers of a semantic test.

2. Indexicals and Demonstratives in the nDTM: The Problem

The ambition of the nDTM is to provide a theory of linguistic meaning. The realization of this task, however, faces a serious theoretical challenge when it comes to the analysis of indexicals and demonstratives (as well as context-sensitive expressions in general). Roughly speaking, the standard view on the semantics of such expressions assumes a difference between the linguistic meaning (analogous to the principle stating that in the case of 'I', 'I' refers to the speaker of the context; in Kaplanian semantics, such rules are represented as functions from contexts to contents), content expressed on a particular occasion (in Kaplanian semantics represented as functions from circumstances of evaluation—possible worlds in the simplest possible case—to extensions), and the extension of an expression with respect to particular circumstances of evaluation. One might expect, therefore, that the nDTM will provide at least a partial method of arriving at the rules that intuitively are linguistic meanings of indexicals and demonstratives. This is, however, highly problematic due to the following reason: apart from the highly specific case of a semantic trial mentioned earlier, the nDTM uses the notion of accepting a sentence in a situation that makes no reference to the context of use.

As we explained in section 1 among the types of meaning directives considered in the nDTM, the axiomatic and the inferential ones are usually mentioned. The former class contains the directives that specify that certain sentences must be accepted in every situation, while the latter specifies that certain sentences must be accepted if certain other sentences have been accepted. The additional constraint that axiomatic and inferential directives must follow the classic logic (and its standard extensions) does not have to be met by every possible language and set of directives. However, we assume that languages that meet such a constraint are much closer in spirit to natural language, at least if we consider some minimal set of logical truths and rules of inference. If this is the case, the nDTM must say something about the acceptance of sentences that contain indexical and demonstrative constituents and, at the very same time, are either exemplifications of logical truths or exemplifications of components of logically valid inferences. We know, however, that without the appropriate specification regarding the role of context, every such exemplification is prone to counterexamples.

Consider, for instance, the law of identity $A \Rightarrow A$ and its (*prima facie*) indexical exemplification:

(*) If you are a philosopher, then you are a philosopher.

(*) is false in cases in which the addressee of the first occurrence of 'you' in (*) is a philosopher and the addressee of the second occurrence of 'you' in (*) is not. Thus, at least on some occasions, (*) should be rejected. The matrix of the

language that contains (*) and respects the law of identity must therefore tell the difference between the occasions and the corresponding uses of (*).

To see another example, consider an inference that (again, *prima facie*) puts into question the validity of the conjunction elimination rule:

This is a conjunction, and Rome is situated on the Tiber.

Therefore: This is a conjunction.

The proponent of the nDTM should explain why this is not a real exemplification of the conjunction elimination rule at all, just like some uses of (*) are not exemplifications of the law of identity. One may produce examples of this sort concerning every tautology and rule of inference that any system of logic treats as valid.⁵

Note that some intuitive analyses are not available for the proponents of the nDTM. Saying, for instance, that tautologies and rules of inference represent propositions, not sentences, is not helpful at all because, firstly, our task is to construct a matrix of a language by appealing to the notion of an *acceptance of a sentence* and, secondly, define a linguistic meaning in terms of such a matrix. Propositional contents simply cannot do that job. The only way of solving the problem, it seems, is to introduce the context of utterance into the nDTM.

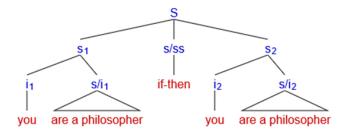
3. Indexicals and Demonstratives in the nDTM: A Partial (non-)Solution

In order to address the problem, we shall introduce the notion of *admissible contextual distribution* of an utterance. We borrow the idea in question from the theory of *contextual perdurantism* (Ciecierski 2019). Contextual perdurantism is one of several interpretations of the *hybrid name* view (other interpretations can be found in: Künne 1992, 2010; Textor 2007, 2015; Kripke 2011; see also Penco 2013 for a comparison between some of the theories) developed originally by Frege and according to which—in the case of indexicals and demonstratives—the circumstance becomes a part of the expression of the thought, that is they might be constituents of linguistic expressions themselves. This results in a special class of expressions being the hybrids of a purely linguistic component and external circumstances.

Contextual perdurantism treats each relevant aspect of context as an independent dimension of contextual space and treats utterances as aggregates of contextual parts across actual distributions of contextual parameters or, as one might put it, contextually perduring objects. Roughly speaking, admissible contextual distribution of an utterance is the sequence of potential values of contextual parameters. The parameters in question determine the semantic values of all indexicals and demonstratives⁶ that are terminal elements of the syntactic analysis of the uttered sentence. Consider (*) again and assume that it is used in the context c, which embraces two potential candidates for the role of addressee: Lauben and Ligens. Given the (categorial) syntax of (*), that is:

⁵ The problem in relation to Kaplanian semantics for indexicals and demonstratives has been studied extensively by many authors; some important works include: Kaplan 1989; Braun 1996; Predelli 2005; Radulescu 2015; Georgi 2020; McCullagh 2020.

⁶ For the sake of simplicity, we ignore here important differences between the representation of indexicals and demonstratives (we shall sometimes speak about deictic expressions in order to cover both classes). Let us assume that in the former case, the distributions concern demonstrations (considered as aspects of contexts) conceived as complex actions constituted by intentions and an elementary action of indication.



We associate the following set of admissible contextual distributions with it:7

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\{\langle i_1 - Lauben, i_2 - Lauben \rangle, \langle i_1 - Ligens, i_2 - Ligens \rangle, \langle i_1 - Lauben, i_2 - Ligens \rangle, \langle i_1 - Ligens, i_2 - Lauben \rangle\}.
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For terminal elements of the syntactic structure, we shall sometimes represent the fact that a particular terminal element is associated with a value of a particular contextual parameter in terms of an ordered pair: <expression, distribution>. We assume that distribution jointly with the meaning of an expression type determines the reference of an expression in several cases, the reference is just the object that is a value of a contextual parameter, so <expression, distribution> pairs might be indistinguishable from <expression, reference> pairs. 8

We imagine that the last paragraph could have confused some of the readers. Wasn't nDTM supposed to be a non-referential theory? As explained earlier, the solution we propose should be understood as one of the extensions of the nDTM that go beyond its minimalist nature. We believe that it is quite interesting that the addition of indexicals and demonstratives forces us to add reference to the theory.

We assume that, depending on the speaker's intentions, only one such distribution corresponds to what is communicated in the utterance. We shall call such distributions *relevant*. Relevant contextual distributions represent actual contexts of utterance (note that this concept is defined partially in terms of the speaker's intentions). We might, therefore, treat a pair consisting of a sentence and a contextual distribution as a representation of a particular utterance.

In the original nDTM, the concept of acceptance is defined for sentences and circumstances understood in terms of input data to which the language user is exposed. Introducing contexts into the nDTM requires substantial modifications of that concept. Firstly, the notion of acceptance should now apply not to sentences but to utterances (our sentence-contextual distribution pairs). Secondly, the circumstances of acceptance must be separated from the context of utterance. Although we assume that within the frame of a particular semantic trial both cooccur, they are distinct (but potentially overlapping) aspects of the circumstances in which the utterance occurs and in which it is assessed. The proper subject of acceptance is the sentence with its (fixed) contextual distribution.

⁷ Here the aspects of the context are identical with referents of a particular indexical, but in other cases it does not have to be like this: expressions like 'today' or 'yesterday' are linked to moments of utterance while referring to days that are appropriately related to the moments of utterance.

⁸ One might also suggest (cf. Ciecierski 2019) that particular aspects of the contexts (contextual parameters) are qua-objects rather than regular objects (cf. Poli 1999; Werner 2020). If this is the case, then Aristotle ≠ Aristotle-addressee, but <*you*, Aristotle-addressee> necessarily refers to Aristotle.

We might observe that among all the distributions, there are the homogeneous ones: they assign the same value of a contextual parameter to each occurrence of a particular indexical ($<i_1$ – Lauben, i_2 – Lauben> and $<i_1$ – Ligens, i_2 – Ligens> in our example). We might, therefore, in the first attempt of adding the context to the new directival theory of meaning, consider the modification of the idea behind the notions of axiomatic and inferential meaning directives and define both concepts relative to homogeneous contextual distributions and use the modified notion of acceptance. In the case of axiomatic directives, we might consider:

If cd is the relevant and homogeneous contextual distribution of the sentence S, the user accepts the utterance S, cd in every circumstance of acceptance c.

The extension of the idea to inferential directives looks as follows:

If the user accepts utterances $\langle s_l, cd_l \rangle ... \langle s_n, cd_n \rangle$, then she accepts the utterance $\langle s_k, cd_k \rangle$ (for k = 1...n+1).

Here, however, we lose any track of the (inferentially relevant) connections between contextual distributions. For instance, the idea given above does not distinguish between (the reader is kindly asked to keep in mind that the first argument of $\langle x, cd \rangle$ pair is an expression that is mentioned, not used):

<*This*, cd₁> is a conjunction, and Rome is situated on the Tiber.

Therefore: $\langle This, cd_1 \rangle$ is a conjunction.

And (for cd1 different from cd2):

< This, cd1> is a conjunction, and Rome is situated on the Tiber.

Therefore: $\langle This, cd_2 \rangle$ is a conjunction.

Additionally, it does not distinguish between:

If $\langle you, cd_1 \rangle$ are a philosopher, $\langle you, cd_1 \rangle$ are poorer than $\langle me, cd_1 \rangle$.

< You, cd1> are a philosopher.

Therefore: $\langle You, cd_1 \rangle$ are poorer than $\langle me, cd_1 \rangle$.

And (again: for cd_1 different from cd_2):

If $\langle you, cd_1 \rangle$ are a philosopher, $\langle you, cd_1 \rangle$ are poorer than $\langle me, cd_1 \rangle$.

< You, cd₂> are a philosopher.

Therefore: $\langle You, cd_1 \rangle$ are poorer than $\langle me, cd_1 \rangle$.

One might think that to fix this, we assume that among all homogeneous contextual distributions there is a maximal one, which is the one that embraces distributions of all premises and the conclusion of the argument. Since the maximal distribution enables tracking the relevant connections between the values of contextual parameters instead of considering potentially different distributions of premises and conclusion, we *might assign to all of them single maximal distribution*. If cd_m is such a maximal distribution, this results in the following characteristics:

If the user accepts utterances $\langle s_1, cd_m \rangle ... \langle s_n, cd_m \rangle$, then she accepts the utterance $\langle s_k, cd_m \rangle$ (for k = 1...n+1).

⁹ In the case of both axiomatic and inferential directives, the complete formulations should include information that the user accepts a sentence or will be treated by the language community as not participating in a semantic trial or not to use a sentence in the meaning it has in a particular language (cf. Grabarczyk 2019: 177). For the sake of simplicity, we use here a shorter and more intuitive formulation.

So far so good. A moment's reflection shows that the solution works for some cases but definitely cannot be generalized into others. Consider an instantiation of the law of conjunction elimination:

(**) If you₁ are a philosopher and you₂ are a philosopher, then you₃ are a philosopher.

The requirement that the homogeneous contextual distribution is required here is way too strong. A particular utterance of (**) is an instantiation of the law of simplification if and only if the third occurrence of 'you' gains reference relative to the same value of the addressee parameter as any of the two other occurrences of 'you'. So, the homogeneity of contextual distribution is not required to arrive at the (intuitively) logically valid instantiation of the law of simplification. We must, therefore, drop the idea of homogeneous distributions and look for the solution elsewhere.

4. Indexicals and Demonstratives in the nDTM: Taking the Hybrid Syntax Seriously

To see what went wrong, we shall use Geoff Georgi's notion of referential promiscuity (Georgi 2020: 129). The characterization of the notion is:

REFERENTIAL PROMISCUITY (RP)

An expression e of a language L is referentially promiscuous if and only if there are distinct free occurrences O_1 and O_2 of e in a sentence s, and some context c, such that the content of O_1 relative to c is distinct from the content of O_2 relative to c.

According to these characteristics, *s* ranges over all sentences, both compound and non-compound. So, it is in principle possible that an expression meets the characteristics only if its content differs across occurrences in *distinct* atomic sentences that are constituents of some compound *s*. But there are actually two ideas of RP that are combined into (RP), namely:

ATOMIC REFERENTIAL PROMISCUITY (ARP)

An expression e of a language L is atomically referentially promiscuous if and only if there are distinct free occurrences O_1 and O_2 of e in an *atomic* sentence s, and some context c, such that the content of O_1 relative to c is distinct from the content of O_2 relative to c.

COMPOUND REFERENTIAL PROMISCUITY (CRP)

An expression e of a language L is compoundly referentially promiscuous if and only if there are distinct free occurrences O_1 and O_2 of e in atomic sentences s_1 and s_2 , respectively, which are constituents of the compound sentence s, and some context c, such that the content of O_1 relative to c is distinct from the content of O_2 relative to c.

(ARP) entails (CRP): if an expression meets (ARP), then combinatorial syntax warrants that there will be cases of (RP) involving this expression in the domain of compound sentences. On the other hand, the reverse entailment depends on the syntactic status of e and the vocabulary of the language L: if, for instance, e is a singular expression and L contains relational predicates, (CPR) entails (APR).

The idea of (ARP) points out that the phenomenon that needs to be captured in the logic of indexicals and demonstratives concerns the occurrences of deictic expressions within atomic formulas. This suggests that our concept of contextual

distribution should apply to atomic formulas rather than to compound formulas. Our initial example (*) should, therefore, be associated with the following analyses that provide us with the list of all admissible contextual distributions (the '[' and ']' brackets mark the syntactic structure):

- (1) [if-then, [<[you, are a philosopher], <Ligens, Ligens>>], [<[you, are a philosopher], <Ligens, Ligens>>]]
- (2) [if-then, [<[you, are a philosopher], <Ligens, Lauben>>], [<[you, are a philosopher], <Ligens, Lauben>>]]
- (3) [if-then, [<[you, are a philosopher], <Lauben, Ligens>>], [<[you, are a philosopher], <Lauben, Ligens>>]]
- (4) [if-then, [<[you, are a philosopher], <Lauben, Lauben>>], [<[you, are a philosopher], <Lauben, Lauben>>]]

from which—if we note that the second argument of the contextual distribution is vacuous when paired with the antecedent of the conditional, while the first argument of the contextual distribution is vacuous when paired with the consequent of the conditional, that is on both occasions there are no indexical expressions with which the respective arguments combine to constitute the hybrid expression—we might easily get:

- (1*) [if-then, [<you, Ligens>, are a philosopher], [<you, Ligens>, are a philosopher]]
- (2*) [if-then, [<you, Ligens>, are a philosopher], [<you, Lauben>, are a philosopher]]
- (3*) [if-then, [<you, Lauben>, are a philosopher], [<you, Ligens>, are a philosopher]]
- (4*) [if-then, [<you, Lauben>, are a philosopher], [<you, Lauben>, are a philosopher]]

It might be useful to coin some terminology here: let us call the forms (1)-(4) (and analogous forms) *hybrid propositional forms* and the forms (1)*-(4)* (and analogous forms) *hybrid nominal forms*. The process of arriving at the list of hybrid sentences is partially bottom-up (atomic sentences are paired with contextual distributions; the occurrences of indexicals and demonstratives select the non-vacuous aspects of contexts) and partially top-down (a contextual distribution as a whole takes into account the entire compound sentence).

Pairing contextual distributions with atomic sentences enables us to track down the occurrences of indexicals and demonstratives as well as to pair them with different values of contextual parameters. The result is the propositional language of *utterances* (LU) with the following *hybrid syntax*.

The class of symbols of LU:

$$\langle P, CD, \{\neg, \land, \lor, \Rightarrow, \Leftrightarrow\}, \{(,)\} \rangle$$

where **P** is the set of propositional variables, and **CD** is the set of potential contextual distributions. We assume now that not all elements of $\mathbf{P} \times \mathbf{CD}$ are possible utterances of LU: in order for some $\langle p, cd \rangle \in \mathbf{P} \times \mathbf{CD}$ to count as a possible utterance, the contextual parameters of cd must provide values for all indexicals and demonstratives in p. For the purpose of the current presentation, we might assume that we have at our disposal a relation \mathbf{R} (we might treat it as primitive or attempt to define it) that selects all and only such pairs from $\mathbf{P} \times \mathbf{CD}$. Having this, we might define a potential non-compound utterance:

Definition (potential non-compound utterance) u is a potential non-compound utterance of LU iff $u \in \mathbf{P} \times \mathbf{CD} \mid \mathbf{R}$

(that is: *u* counts as a potential non-compound utterance if and only if it is a pair consisting of a propositional variable and a contextual distribution such that the contextual distribution provides all values for indexicals and demonstratives occurring in the propositional variable).

Next, we define the class of potential utterances of LU in the following manner.

- 1. All non-compound potential utterances of LU are potential utterances of LU.
- 2. If <*p*, *cd*> is a non-compound potential utterance of LU, then '¬<*p*, *cd*>' is a potential utterance of LU.
- 3. If $\langle p, cd \rangle$ and $\langle p, cd' \rangle$ are non-compound potential utterances of LU, then ' $\langle p, cd \rangle \land \langle p, cd' \rangle$ ', ' $\langle p, cd \rangle \lor \langle p, cd' \rangle$ ', ' $\langle p, cd \rangle \Rightarrow \langle p, cd' \rangle$ ', and ' $\langle p, cd \rangle \Leftrightarrow \langle p, cd' \rangle$ ' are potential utterances of LU.
- 4. If u is a potential utterance of LU, then ' $\neg u$ ' is a potential utterance of LU.
- 5. If u and u^* are potential utterances of LU, then ' $u \wedge u^*$ ', ' $u \vee u^*$ ', ' $u \Rightarrow u^*$ ', and ' $u \Leftrightarrow u^*$ ' are potential utterances of LU.
- 6. Nothing else is a potential utterance of LU.

Clauses 2-5 are necessary to allow formulas like ' $(\langle p, cd \rangle \lor \langle p, cd' \rangle) \Rightarrow \langle p, cd'' \rangle$ ', as potential utterances of LU while disallowing as potential utterances formulas such us ' $\langle p, cd \rangle$, $cd' \rangle$ ': we assume that contextual distributions apply to propositional variables only.

Thus conceived, the (propositional) language of LU is just a propositional language that replaces standard propositional variables with hybrid expressions of the form $\langle p, cd \rangle$. So, there cannot exist any logical differences between what counts as a logically valid formula or inference in the standard language of propositional logic and LU. For instance, the tableaux system for LU is just the standard system but allows instantiations of correct elimination rules, like:

```
[\neg \Rightarrow \text{Elimination}]
\neg(\langle p, cd \rangle \Rightarrow \langle p, cd' \rangle)
\langle p, cd \rangle, \neg \langle p, cd' \rangle
[\land \text{Elimination}]
\langle p, cd \rangle \land \langle p, cd' \rangle
\langle p, cd \rangle, \langle p, cd' \rangle
```

which enable, for example, the following derivations:

$[1] \neg [\langle p, cd \rangle \land \langle p, cd' \rangle) \Rightarrow \langle p, cd \rangle]$	[Assumption]
$[2] < p, cd > \land < p, cd' >$	$[\Rightarrow$ elimination, $[1]$
[3] ¬< <i>p</i> , <i>cd</i> >	$[\Rightarrow$ elimination, $[1]$
[4] <p, cd=""></p,>	$[\land elimination, [2]]$
[5] < <i>p</i> , <i>cd</i> '>	$[\land elimination, [2]]$
Contradiction: [3]-[4].	
$[1]^* \neg [< p, cd > \land < p, cd' >) \Rightarrow < p, cd'' >]$	[Assumption]
$[2] < p, cd > \land < p, cd' >$	$[\Rightarrow$ elimination, $[1]$
[3] ¬< <i>p</i> , <i>cd</i> ''>	$[\Rightarrow$ elimination, $[1]$

```
[4] < p, cd > [\land elimination, [2]] [5] < p, cd' > [\land elimination, [2]] No contradiction.
```

This shows that [1] is and [1]* is not an instantiation of the ' $(A \land B) \Rightarrow A$ ' law. This is exactly what we want to have when it comes to propositional tautologies and inferences. This, for instance, enables also distinguishing valid cases of:

This is a conjunction, and Rome is situated on the Tiber.

Therefore: This is a conjunction.

Namely:

<This is a conjunction, "This is a conjunction, and Rome is situated on the Tiber"> and <Rome is situated on the Tiber, cd>10

<This is a conjunction, "This is a conjunction, and Rome is situated on the Tiber">

and invalid ones, that is:

<This is a conjunction, "This is a conjunction, and Rome is situated on the Tiber"> and <Rome is situated on the Tiber, cd>

<This is a conjunction, "This is a conjunction">

(Note that the contextual distribution must contain as a demonstration or demonstratum the expression without the contextual distribution; otherwise, we arrive at an infinite regress.)

This, of course, leaves the problem we are facing partially open as we also need an account that would enable us to have adequate treatment of cases in which we are dealing with distinct but appropriately related contextual distributions, as potentially in:

This is bigger than this. This is bigger than this.

Therefore: This is bigger than this.

which might be an analytic statement also in cases different than:

< This is bigger than this, cd>, < This is bigger than this, cd>.

Therefore: < This is bigger than this, cd>

which is just a matter of propositional logic.

The solution naturally appeals to our distinction between hybrid propositional and *nominal propositional forms*. The solution that works for propositional entailments made use of the former by identifying the non-compound propositional formulas with formulas having hybrid propositional forms. *Per analogiam*: we might dwell into the structure of non-compound indexical formulas by identifying them with formulas having hybrid nominal forms. This would enable us to trace referentially the same occurrences of indexicals and treat them as occurrences of a single referential expression. In that manner, we might arrive at:

```
< This, a> is bigger than < this, b>, < This, b> is bigger than < this, c>.
```

Therefore: < This, a> is bigger than < this, c>

which becomes the case of:

x is bigger than y, y is bigger than z.

¹⁰ Since the second conjunction contains no indexicals, it might be paired with an arbitrary contextual distribution.

Therefore: x is bigger than z. which cannot be said of:

< This, a> is bigger than < this, b>, < This, c> is bigger than < this, d>.

Therefore: < This, a> is bigger than < this, d>

The solution we arrived at bears interesting relation to the view on the logical form developed by Iacona (2013; see also Georgi 2020: 138-39), which assumes that distinct symbols refer to distinct objects. On the one hand, nothing in our view prohibits co-referential hybrid expressions like <I, a> and <you, a>, which directly contradicts Iacona's assumption. At the same time, our view automatically secures the referential difference for any pair of expressions <x, a> and <x, b> by treating them—if a and b are parameters that point to different objects—as distinct hybrid symbols. What is important here is that enabling hybrid syntax enables us to keep the intrinsicality assumption regarding logical form, which is the claim that the logical form of an argument is intrinsic to the syntax of the argument.

Our considerations suggest that the properties such as validity and analyticity bear no special complications for indexical languages. From the viewpoint of the nDTM, we, therefore, have at our disposal the method of telling apart the uses of deictic sentences that are candidates for axiomatic and inferential rules from the uses of the very same sentences that are not such candidates. However, one might question this conclusion, if there are exemplifications of correct inferences or indexical tautologies that cannot be distinguished—using the method outlined above—from incorrect inferences or possibly false uses of sentences of the same form. In other words, the opponent of the view presented above must indicate cases of uses of indexical sentences or indexical reasonings that are sometimes valid and sometimes invalid and show that the difference between them cannot be captured by appealing to the idea of hybrid syntax.

Let us, therefore, consider two candidates for deictic reasonings:

- (5) Josh says "It is raining today". The next day, Josh says "It rained yesterday" (Radulescu 2015: 1844).
- (6) I utter, "David Kaplan is older than I". Then somebody addresses me with: "Therefore, you are not older than David Kaplan" (cf. Bar-Hillel 1963).

It is probably not obvious that the examples are cases of reasonings but let us suspend the skepticism regarding that issue and stick rather to the observation that there is a sense in which one could correctly claim that—in the appropriate contexts—one cannot consistently hold one of the utterances listed in (5) or (6) while denying the second. This, we think, suffices for treating (5) and (6) as cases of inferences. Our hybrid nominal forms of both arguments are respectively (with enthymematic assumptions made explicit):

It is raining < today, $d_0 >$.

It rained < yesterday, $d_1 > 11$

David Kaplan is older than $\langle I, a \rangle$.

For every pair of objects: if the first is older than the second, then the second is not older than the first.

< You, a> are not older than David Kaplan.

 $^{^{11}}$ d_0 and d_1 are the consecutive days at which today and yesterday are respectively uttered.

Here is the way in which one might attempt to analyze such cases. Since '<today, d_0 >' and '<yesterday, d_1 >', <I, a> and <You, a> are respectively co-referential, in order to secure the inferences, we need nothing more than the assumption that co-referential hybrid terms are interchangeable salva veritate. In fact, the idea in question is nothing more than the idea of direct reference, and indeed, we believe that indexicals and demonstratives are directly referential. One may worry, however, that something is missing here because we do not have an explanation of the fact that the appropriate change of indexicals (from to 'today' to 'yesterday' and from 'I' to 'you') is required for arguments to be valid. Moreover, it says nothing about reasonings such as:

(7) Josh says, "Today is Monday". Then he says, "Therefore, tomorrow is Tuesday"

which contain indexical expressions that are not co-referential. This, however, is not a problem, as the theory might claim that inferences of this sort might be simply described by appealing to perfectly general enthymematic assumptions of the arguments. Note that the complete hybrid nominal form of the argument looks as follows:

```
< Today, d_1 > is Monday.
```

< Today, d₁> precedes < Tomorrow, d₁>.

For any pair of days: if the first precedes the second and the first is Monday, then the second is Tuesday.

<*Tomorrow*, $d_1>$ is Tuesday.

All specifically indexical premises of the argument are empirical, and the validity of the entire argument hangs on the correctness of the perfectly general assumption three, which describes the simple arithmetic of days and which, without any doubt, should be included among the class of axiomatic directives for 'Monday' and 'Tuesday'.

Other authors working on the logic of indexicals prefer to analyze the cases in terms of constraints on relations between contexts for premises and the conclusion (Radulescu 2015) or the coordination of contexts of premises and the conclusion of an argument (Georgi 2020). This approach avoids this: it aims to achieve precisely the same results only by appealing to hybrid syntax, the interchangeability principle (and this is a referential principle!) (or principles, if other types of designators are at stake), and perfectly general meaning postulates.

It should be observed additionally that the existence of specific indexical tautologies¹³ (the thesis that such tautologies exist was the subject matter of intensive criticism in recent decades) is not the problem for the nDTM unless the sentences occurring in such tautologies have no corresponding false uses *and* unless the nDTM is unable to describe the difference correctly. We simply deny that there are such cases. The language matrix we shall arrive at will contain all the connections and regularities for indexicals and demonstratives and, by the same token,

¹² It also reminds of the version of Fregean semantics for indexicals that incorporates the idea of the primitive sense (cf. Tichy 1986) according to which (as originally for Frege), for instance, 'yesterday' and 'today' as considered in the appropriately related contexts have the same sense.

¹³ Among the candidates, one might mention:

I exist.

I am self-identical (Yagisawa 1993: 480-82).

will enable us to read all the regularities and systematic dependencies between uses of indexicals in various contexts. We see no reason for claiming that some of such regularities will not be reflected in the matrix.

The solution sketched above might be treated as a general account of indexical validity, but since our motivation was to apply it to the nDTM, we now should consider the issue of the general form of axiomatic and inferential directives in the nDTM. The approach sketched above entails that no special modification—in addition to introducing the hybrid syntax into the picture—is needed. In the case of axiomatic directives, we have:

If cd is the nominal contextual distribution of the sentence S, the user accepts the utterance $\langle S, cd \rangle$ in every circumstance of acceptance c.

while in the case of inferential directives, we simply have:

If the user accepts utterances $\langle s_1, cd_1 \rangle ... \langle s_n, cd_n \rangle$, then she accepts the utterance $\langle s_k, cd_k \rangle$ (for k = 1...n+1) for nominal contextual distributions: $cd_1...cd_k$.

However, this gives us a completely new idea of the language matrix and two distinct concepts of the meaning of an expression. Let us explain. From now on, language matrices apply to utterances (i.e., sentences in their hybrid nominal forms). The context of utterance becomes, therefore, a part of the syntactic structure. The circumstances of acceptance apply now to utterances. So, distribution within the language matrix is defined in terms of utterances in which an expression (hybrid or non-hybrid) occurs. It follows that notions such as meaning or synonymy are also defined relatively to utterances. Therefore, we have at our disposal the idea of the utterance's or use's meaning that replaces the original idea of an expression-type meaning. But one might still, if necessary, look at the distribution of expression types within the language matrix. In cases of indexicals and demonstratives, the distribution will simply involve an abstraction from the contextual parameter that is present in the hybrid nominal forms of a particular expression type. This is the second idea of the meaning of an expression that we might arrive at within the theory. The latter concept of meaning is the closest we might get to the intuitive idea of the linguistic meaning of an indexical.

5. Conclusion

As we saw, the original nDTM does not have an easy way of handling indexicals and demonstratives and remains prone to counterexamples, such as the one presented in (*). In this paper, we have sketched the way in which context can be added to the nDTM. We argue that the main reason why this cannot be done easily is that the theory operates only on the notion of an acceptance of the sentence (and not the notion of acceptance of an utterance). We presented an extension of the nDTM that modifies its key notion of a meaning directive, adding the parameter of contextual distribution to it. This, among other things, enables introducing into the nDTM the representations of particular utterances and distinguishing between the context of an utterance and circumstances of acceptance. The resulting view that wholly incorporates the idea of hybrid syntax avoids special complications regarding indexicals and demonstratives as occurring in the axiomatic and inferential directives.

The framework presented in this paper says nothing about various other functions the expressions might play in particular contexts, nor about the broadly conceived idea of context-dependence. It has been suggested by some (cf. Ciecierski 2021) that this would require a generalization of the notion of meaning that incorporates the idea that, on the one hand, the expressions might be used in several manners and that, on the other, its meaning is something that enables *all* such different uses and tells us when uses become abuses of an expression. This would require a theory of context sensitivity that presupposes a particular account of the functions of language. Such a theory should also describe how contexts affect the change of particular linguistic functions and how they affect the change of one use of an expression into another. Our modest concept of contextual distribution is not designed for this role. It is also not designed as a tool for dealing with the examples discussed within contemporary contextualism-minimalism debates. We do not consider this feature as a drawback, however, as narrowly conceived indexicality generates problems that are challenging themselves. Considering them in the context of the problem of alleged context-sensitivity of *red*, *know* or *tasty* might only obscure the nature of the problems with *I*, *here* or *now*.

The approach sketched above says also nothing about the functioning of indexical expressions in empirical and promotive directives. This issue—due to its complexity—requires a separate study. The main reasons for this are ambiguities related to the question of whether certain epistemological and motivational aspects that systematically accompany the use of indexical expressions should be represented at the level of their meaning. For example, certain direct speech acts seem to require particular personal pronouns. Requests, thanks, congratulations, and advice require either a second person pronoun, promises require the first-person pronoun, thanks require both types of pronouns, etc. Uses of these pronouns (given the appropriate attitudes of the participants of the situation) are systematically connected with the actions of the speaker and the hearer. Should the matrix of a particular language take these actions and behaviors into account when a list of promotive directives is considered? If so, should the category of the promotive directive include not only behaviors but also a change in the normative situation of the sender and receiver of an utterance, an aspect stressed by speech act theorists?

Considering the empirical directive category also leads to problems. For instance, in a semantic trial, we might show someone an object (e.g., a book) and ask: "Is this a book?" or "Is this a zebra?". We will consider a negative or affirmative answer to these questions as relevant for the meaning of nouns book and zebra, but rather not to the meaning of the demonstrative this. Someone may explain this asymmetry and say that the pronoun this does not occur in the indicated sentences in an essential manner. But what determines the essential or nonessential occurrence of this pronoun? Let us imagine, for example, that in a certain situation there are two objects in the environment—a book and a zebra, the first one closer and the second one far from the participants of the semantic trial. If the question is now asked, "Is this (we gesture to point to a book) a zebra, and that (we gesture to point to a zebra) a book?" and the user denies, we seem to be faced with a dilemma: either we recognize that she does not know the meaning of one of the respective nouns, or that she does know their meaning but somehow takes into account that aspect of the meaning of this and that which makes the former pronouns refer to closer and the latter to farther located objects. A directival theory of meaning that accounts for the phenomenon of indexicality and applies to languages with empirical and promotive directives must somehow resolve such issues.

Regardless of directions the further analysis should take, we would like to emphasize that the approach presented in this paper, which takes into account only axiomatic and deductive directives, can also be applied outside the new directival theory of meaning. This is because it enables us to solve the problem of the existence of specifically indexical tautologies and inferences in an elegant way. The cost we pay here is the adoption of the idea of hybrid syntax, the principle of interchangeability, and the inclusion of enthymematic premises in the description of indexical inferences. The solution presented in this paper is, for this reason, not only an attempt to supplement the directival theory of meaning but also a voice in the general discussion about the logic of indexicals and demonstratives. ¹⁴

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