Aristotle's argument from truth in *Metaphysics Γ* 4

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Some of Aristotle's statements about the indemonstrability of the Principle of Non-Contradiction (PNC) in Metaphysics Γ 4 merit more attention. The consensus seems to be that Aristotle provides two arguments against the demonstrability of the PNC, with one located in Γ 3 and the other found in the first paragraph of Γ 4. In this article, I argue that Aristotle also relies upon a third argument for the same conclusion: the argument from truth. Although Aristotle does not explicitly state this argument, it is the best argument that he could use to defend some of his statements in the second paragraph of Γ 4. Since the argument relies on only a few of Aristotle's core views about truth, I propose that it is faithful to his considered position throughout his corpus, and it may be the strongest argument he could offer for the indemonstrability of the PNC.

Aristotle's primary formulation of the Principle of Non-Contradiction in its most universal form (PNC) is found in *Metaphysics* Γ 3 at 1005b18-22.[1] As translated by Christopher Kirwan, it is as follows:

> *For the same thing to hold good and not to hold good simultaneously of the same thing and in the same respect is impossible* (given any further specifications which might be added against the dialectical difficulties). (1993: 7)

Much of the literature dedicated to Aristotle's views on the PNC in Γ is understandably focused on his attempts there to refute those who deny that the principle is true. In this context, a refutation is a method of argumentation that, when successful, shows that one's opponent must assume or rely upon what they claim to deny (either in denying it or in virtue of their other commitments). There is, however, a further question about Aristotle's motivation for devoting much of his attention to refutations. The answer is that he thinks that the PNC is indemonstrable. It is normally taken for granted that he is right about that. He may be, but the arguments he marshals for this conclusion prove to be interesting and instructive.

According to Aristotle, demonstrations are sound deductive arguments with necessary truths for premisses (see *APr*. 24a21-b12 and *APo*. 71b9-24, 73a21-24 and 74b5-17, as well as *Top*. 100a25-b20). Moreover, the premisses of demonstrations must be better known than, prior to and asymmetric explanations of their conclusions (see *APo*. 71b20ff.). Although the premisses of a demonstration necessitate its conclusion, it cannot be the case that if all of the members of any proper subset of its premisses are true, then its conclusion must be true (see *APr*. 24b18-22, as well as *Top*. 100a25-26 and *Soph. Ref.* 165a1-3). This last condition rules out irrelevant premisses that do not belong to the set of premisses that necessitate a given conclusion. It also rules out question-begging premisses, which are either identical to the conclusion or entail it,

[1] Throughout, I follow the standard convention of using Bekker numbering to cite Aristotle's works. Bekker numberings consist of a work abbreviation and a line number (or line range). For instance, '*Met.* 1005b18-22' cites lines 1005b18 through 1005b22 of the *Metaphysics*.

since those premisses are true only if the conclusion is true. There are many other features of demonstrations, but these are sufficient for our purposes.

It is commonly thought that Aristotle gives two arguments in Γ in favour of thinking that the PNC is indemonstrable. One recently prominent interpretation of the first is that it turns on the claim that the PNC is the firmest first principle that 'all those who demonstrate go back to' and that it is the 'principle of all the other axioms also' (*Met*. 1005b33-34). In his development of this interpretation, M. V. Wedin calls the claim that 'all those who demonstrate go back to' the PNC 'the ultimacy claim' (2004: 258-62). Since there is a clear sense in which indirect proofs, in moving from contradictions in their premisses to their conclusions, 'go back to' the PNC, the mystery for Wedin is how the ultimacy claim is true of direct proofs. In short, Wedin argues that to display the validity of direct proofs, one must make an appeal to consistency, which just is an appeal to the truth of the PNC. A direct proof is valid if and only if it is logically impossible for the conjunction of the premisses and the negation of the conclusion to be true. Since all demonstrations are direct or indirect proofs, it follows that all those who demonstrate 'go back to' the PNC, and thus a demonstration of the PNC would 'go back to' it as well.

Whether or not Wedin is right about what it means to 'go back to' the PNC, it is clear that Aristotle's first argument expresses his worry that any purported demonstration of the PNC would be problematically reliant on its own conclusion. At the very least, if any axioms that could be used to demonstrate the PNC would have the PNC as a principle of them, then there is nothing from which the PNC could be demonstrated. The premisses of such a demonstration would not be better known than, prior to or asymmetric explanations of the PNC because it bears these relations to them in virtue of being the 'principle of all the other axioms'—it is commonly thought that this just is what it is to be a principle of something else. This first argument against the demonstrability of the PNC is found in Γ 3 (namely *Met*. 1005b8-34), and it has long been the subject of dispute due to ambiguities in Aristotle's presentation.

Aristotle's second argument for the same conclusion turns on the claim that not everything could be demonstrated because if everything could be demonstrated, then there would be an infinite regress of demonstrations. Aristotle indicates that if there were an infinite regress of demonstrations, it would undermine the status of the demonstrations as demonstrations. To stop this vicious regress, Aristotle argues that we need indemonstrables and then he proposes that the PNC is the best candidate for being an indemonstrable. This argument is found in the first paragraph of Γ 4 (namely *Met*. 1006a8-11) and it too is well-known. Aristotle considers a similar regress throughout the *Posterior Analytics* (see 72b5-73a20, 84a30-84b2 and 90b25-91a11).

In addition to these, Aristotle in fact has a third argument for the indemonstrability of the PNC, or so I will argue. I call this third argument the *argument from truth*. It crucially relies on the claim that at least one premiss of any possible demonstration of the PNC would entail the PNC. But if this so, then a purported demonstration of the PNC would violate the requirement that no demonstration have question-begging premisses. Aptly, the argument from truth relies on two core features of Aristotle views on truth: his definitions of truth and falsehood, and the Principle of Bivalence (PB). Aristotle does not explicitly formulate this third argument, but I maintain that it is the best argument that he could give to defend some of his statements in the second paragraph of Γ 4. Furthermore, I believe that this argument provides confirming justification for Aristotle's pivot to refutations of the denier of the PNC.

The statements that I take to indicate the argument from truth are only mentioned in passing by Kirwan in his commentary (1993: 91-92), and, like other interpreters, he does not read them as indicating the presence of a third argument. On the contrary, Kirwan reads them as

simply part of Aristotle's introduction to his refutations and in particular to the refutation from signification, wherein Aristotle tells us that the denier of the PNC is implicated in affirming the PNC in signifying anything at all. I do not deny this. I agree that effecting a transition to refutations is Aristotle's ultimate goal here. Yet, from the way that the second paragraph of Γ 4 is structured, it becomes clear Aristotle has a third argument in mind that necessitates the transition. After all, neither of the other two arguments seems to be decisive, since the first suffers from ambiguities and the second is quick and amounts to no more than the proposal that the PNC is the best indemonstrable to stop a vicious regress of demonstrations. Here is the relevant part of the paragraph, as translated by Kirwan (1993: 8-9):

> By 'demonstrating in the manner of a refutation' I mean something different from demonstrating, because in demonstrating one might be thought to beg the original [question], but if someone else is cause of such a thing it must be refutation and not demonstration. In response to every case of that kind the original [step] is not to ask him to state something either to be or not to be (for that might well be believed to beg what was originally at issue), but at least to signify something both to himself and to someone else; for that is necessary if he is to say anything. (*Met*. 1006a15-22)

The two important segments of this quotation are Aristotle's statements that 'in demonstrating one might be thought to beg the [original] question' and that 'in response to every case of that kind the original [step] is not to ask him to state something either to be or not to be (for that might well be believed to beg what was originally at issue)'. The first statement is puzzling. Aristotle may seem to claim that a demonstration of the PNC should not be sought because demonstrations in general beg the question. But this cannot be what he has in mind, because, as mentioned before, in demonstrations the conclusion must be true in virtue of all of the premisses, while in question-begging arguments the conclusion only depends on a proper subset of them. Rather, he must be claiming that there is something special about purported demonstrations of the PNC that makes them question-begging, no matter how they are formulated. And I do not think that Aristotle is here referring to the alternative kind of question begging he discusses in *Prior Analytics* II 16. That kind occurs 'whenever a man tries to prove by means of itself what is not known by means of itself' (64b34-39; A. J. Jenkinson translation in Barnes 1984), but if anything is known by means of itself, the PNC is. Therefore, Aristotle's concern here is entirely different. Let me explain.

Although Aristotle does not explicitly say what it is about demonstrations of the PNC that makes them question-begging, the second segment of the quotation is a vital clue. In the second segment, Aristotle is concerned with refutations of the denier of the PNC rather than demonstrations, but he again mentions begging the question, thus linking the segment to the one before it. This is not an odd connection. After all, Aristotle says elsewhere that 'both the demonstrator and the dialectician argue deductively after assuming that something does or does not belong to something' (*APr*. 24a26-27)—language that is strikingly similar to that found in the second segment—and he is clear that all deductions can be question-begging in the way I have discussed (see *Top*. 100a25-30 and *Soph*. *Ref*. 165a1-4). If we interpret the first segment in light of the second because of this commonality, the natural reading of the first segment is that it is an expression of Aristotle's belief that the premisses of demonstrations are the problem. It is because the premisses of demonstrations are truths that must 'state something either to be or not to be' that purported demonstrations of the PNC beg the question. As we will see, since the

premisses of demonstrations are truths, each of them *must* 'state something either to be or not to be' (with the 'or' in this clause interpreted as exclusive)—that is, they *cannot* state something *both* to be and not to be. Yet, it cannot be the case that for any premiss of a demonstration of the PNC, if the premiss is true, then the PNC must be true. That would be to beg the question. So, if any purported demonstration of the PNC would have a premiss that ranges over all things, then any such demonstration would be question-begging, and not a demonstration after all. Since the PNC ranges over all things in its most universal form—the form which is Aristotle's concern here—the antecedent of this conditional is true.

The second segment concerns refutations, but Aristotle's concern there is parallel: Aristotle is saying that he cannot demand his interlocutor—the denier of the PNC—to do anything but signify something, a demand contrasted with the (outlawed) demand from the quotation to 'state something either to be or not to be'. If his interlocutor were to state a truth, then, as we will see, Aristotle could deduce that the PNC must be true of what the truth ranges over. And if she were to state a falsehood, then a true conclusion that entails the falsehood of the PNC could not be deduced, because Aristotle holds that no truth entails the falsehood of the PNC. Thus, there would be nothing Aristotle's interlocutor could say that has the falsity of the PNC as a consequence.

As plausible as this reading may seem, one should wonder: why should we think that truths cannot state something both to be and not to be? That is, must we read the 'or' from the second segment as exclusive? It could be that some thing x is φ *and* not φ. If this is possible, then one could truthfully state that x is φ, that x is not φ or *that* x *is both* φ *and not* φ. The PNC is false when instantiated to x and φ (i.e., φ both holds good and does not hold good of x), and a true statement of what x is entails the negation of the PNC, or so it seems.

The apparent possibility of truthfully stating of some being that it violates the PNC seems to be evidence against there being a third argument for the indemonstrability of the PNC. If one could make such a truthful statement, then it would undermine my claim that Aristotle thinks that the PNC must be true of what truths range over. The principle of charity demands that we not attribute a view to Aristotle that is open to such a counterexample, even if his statements seem to suggest that he holds it.

However, I think these appearances are deceiving. There is a relatively simple and plausible argument that can be attributed to Aristotle that both avoids this problem and makes sense of his statements at *Met*. 1006a15-22. Getting clear about how this argument most plausibly would work by formulating it precisely and by providing justifications for its premisses shows that there is no problem after all. Following Aristotle, I treat the PNC as a universal that says for all x, where x is a thing, and for all φ, where φ is a (non-linguistic) predicate, it is impossible for φ to hold good and to not hold good of x (simultaneously, in the same respect, and so on). What it means for the PNC to be true of a thing x is for there to be no φ such that φ holds good and does not hold good of x. Throughout, I treat statements of the form 'φ holds good of x' to be equivalent to statements of the form 'x is φ'. Aristotle uses the former kind of language in his formulation of the PNC and the latter in his formulation of his definitions of truth and falsehood. The argument runs as follows:

Show: A demonstration of the PNC is impossible.

(P1) A demonstration is a sound deductive argument where the premisses are necessary truths and where it is not the case that if all of the members of any proper subset of the premisses are true, then the conclusion must be true.

(P2) It cannot be the case that for any premiss of a demonstration of the PNC, if the premiss is true, then the PNC must be true.
(P3) The PNC must be true of the things that truths range over.
(P4) For any premiss of a demonstration of the PNC, the PNC must be true of the things that premiss ranges over.
(P5) The PNC ranges over all things, and at least one premiss of any demonstration of the PNC must range over all things.
(C) A demonstration of the PNC is impossible.

P1 is part of Aristotle's definition of demonstration and P2 is its application in the present case. P3 is the crucial premiss and it is what is challenged by the apparent possibility of truthfully stating of some being that it violates the PNC. P4 follows from P1 and P3. C follows from P2, P4 and P5. P5 will be discussed below, but first, an argument for P3 is needed. As background, consider Aristotle's definitions of truth and falsehood and perhaps his clearest statement on bivalence, which are found in Γ 7, as translated by Kirwan:

> This will be plain if we first define what truth and falsehood are: for to say that that which is is not or that which is not is, is a falsehood; and to say that that which is is and that which is not is not, is true; so that, also, he who says that a thing is or not will have the truth or be in error. (*Met*. 1011b25-29)

In this passage, Aristotle argues that if you deny φ of something that is φ, or affirm φ of something that is not φ, then you will be stating a falsehood. And if you affirm φ of something that is φ, or deny φ of something that is not φ, then you will be stating a truth. Aristotle's transition from these definitions to a statement on bivalence is commonly seen as inferential (see Kirwan 1993: 117-18), and it is clear why, since it follows from these definitions that whether you affirm φ of something or deny φ of something, you will be stating a truth or a falsehood. That is, there are no affirmations or denials that are neither true nor false.

There are two forms of the PB. What I will call 'the dialetheic PB' (PBD) is the claim there are only two truth values, true and false. It is dialetheic because it makes room for the possibility that there is something that is both true and false. The stronger form of PB is what I will call 'the monoletheic PB' (PBM), which states there are only two truth values, true and false, *and nothing has them both*. In the above passage, Aristotle states the PBD and the argument he presents in that passage does not require the PBM. The same goes for the other contexts in which he most explicitly mentions the PB (namely *Cat.* 2a5-10 and *Int.* 17a1-3). However, at no point does Aristotle consider the possibility that there is something that is both true and false (simultaneously, in the same respect, and so on), much less does he argue against the PBM.

Why does this matter? Well, if the PNC could be false of some thing x that a truth ranges over, then there could be an x and a ψ such that x is ψ and x is not ψ. Given Aristotle's definitions of truth and falsehood, it follows there could be a truth T that states that x is ψ, but it also follows there could there be a falsehood F that states the same thing, namely that x is ψ. So T is F! Thus there could be a statement that is both true and false (namely 'x is ψ'). But if Aristotle holds the PBM, then there could not be any statements that are true and false. And so it follows that the PNC could not be false of some thing x that a truth ranges over. That is, P3: the PNC must be true of the things that truths range over.

Given that demonstrations have necessary truths for premisses, Aristotle must rely on his definitions of truth and falsehood in his definition of demonstration. Yet, as mentioned, the innocence of the assumption of the PBM is not quite as clear-cut, and especially in this context, since the PNC is true if and only if the PBM is true (and so anyone who denies the PNC would also deny the PBM). Although it may be sufficient for my interpretative purposes to simply note that the PBM is taken for granted by Aristotle throughout his corpus and that it is required to make sense of his statements at *Met*. 1006a15-22, it would be ideal if further reason could be provided for interpreting Aristotle as being committed to the PBM.

Perhaps the best reason that can be offered on Aristotle's behalf has to do with his view of falsehoods as erroneous assertions that fail to correspond to the world as it is (a sentiment expressed at *Met*. 1011b25-29, as quoted previously; see also *Met*. 1051b2-8). Saying '*x* is *φ*' about some thing *x* which is *φ* and not *φ* is to utter a truth according to Aristotle's definition, but there is a sense in which it differs from saying '*y* is *φ*' about some thing *y* which is *φ but not* not *φ*. The former truth is something of a half-truth—since it is also false—while the latter is not. But there is no alternative, at least on Aristotle's definitions. To say the whole truth about *x*, namely '*x* is *φ* and not *φ*', is to say the whole falsehood, since it is to say '*x* is neither *φ* nor not *φ*'. If Aristotle endorsed the PBD, there would be pressure on him to alter his definition of falsehood such that nothing false can be said of *x* with respect to *φ*. Since all of the purported falsehoods that we could say about *x* with respect to *φ* are truths, they are not erroneous in any sense. No assertion predicating some *φ* of some thing can fail to correspond with the world while simultaneously succeeding in doing so. Thus, cases where the PNC is violated erode the distinction that Aristotle makes between truth and falsehood. In this way, Aristotle's theory of what truth and falsehood are presupposes the PBM, and his definitions would need revision were he to accept that the PNC is false in any instance. Since such a wholesale adjustment is not forthcoming, we have further reason to interpret Aristotle as not being open to the PBD.

Now, with the argument for P3 in hand, it should be clear why the PNC must be true of the things that truths range over. If this were not the case, then violations of the PBM would be possible, but there is reason to think Aristotle maintains (or should maintain) that they are not. Thus the apparent possibility of truthfully asserting of some being that it violates the PNC (and so is *φ* and not *φ*) is only an appearance, and there is no room to read 'something either to be or not to be' from the second segment of *Met*. 1006a15-22 with an inclusive 'or'.

What about P5? It states that at least one premiss of any demonstration of the PNC must range over all things (which is what the PNC ranges over). In *Prior Analytics* I 4-6, after analysing the different forms that (syllogistic) deductions can take, Aristotle draws several conclusions about general features of deductions, one of which is the claim that a deduction with a universal conclusion requires (at least two) universal premisses. Since, as noted, demonstrations are a species of deduction, this conclusion applies to them. The most universal form of the PNC is the form that the argument from truth concludes is indemonstrable. In its most universal form, the PNC says that for all *x*, where *x* is a thing, and for all *φ*, where *φ* is a (non-linguistic) predicate, it is impossible for *φ* to hold good and not to hold good of *x* (simultaneously, in the same respect, and so on), so it ranges over all things (for more on this feature of the PNC, see *Met*. 1005a22-23 and surrounding passages). It follows that the PNC is a universal and the premisses of a demonstration of it would themselves have to be universals. Because Aristotle also argues in *Prior Analytics* I that the subject of the universal conclusion of an argument with universal premisses must be the subject of at least one of its universal premisses, and the subject of the PNC is 'things', this means that one of the premisses of a

demonstration of the PNC would have to have 'things' as its subject. Given it has already been shown that this premiss must be a universal, it must range over all things. So, P5, and thus, with P2 and P4 already in hand, the conclusion C that a demonstration of the PNC is impossible.[2]

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