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ARTICLE

ARISTOTLE ON MATHEMATICAL TRUTH

Phil Corkum

Both literalism, the view that mathematical objects simply exist in the empirical world, and fictionalism, the view that mathematical objects do not exist but are rather harmless fictions, have been both ascribed to Aristotle. The ascription of literalism to Aristotle, however, commits Aristotle to the unattractive view that mathematics studies but a small fragment of the physical world; and there is evidence that Aristotle would deny the literalist position that mathematical objects are perceivable. The ascription of fictionalism also faces a difficult challenge: there is evidence that Aristotle would deny the fictionalist position that mathematics is false. I argue that, in Aristotle's view, the fiction of mathematics is not to treat what does not exist as if existing but to treat mathematical objects with an ontological status they lack. This form of fictionalism is consistent with holding that mathematics is true.

KEYWORDS: Aristotle; mathematics; truth; fictionalism; literalism

Do mathematical objects exist in some realm inaccessible to our senses? It may be tempting to deny this. For how could we come to know mathematical truths, if such knowledge must arise from causal interaction with non-empirical objects? However, denying that mathematical objects exist *altogether* has unsettling consequences. If you deny the existence of mathematical objects, then you must reject all claims that commit you to such objects, which would seem to mean rejecting as false much of mathematics. For, as David Papineau (1990) vividly puts it, it is doublethink to deny that mathematical objects exist but to continue to believe, for example, that there are two prime numbers between ten and fifteen. Two current responses to this problem are literalism and fictionalism. Both literalists and fictionalists deny the existence of a world of mathematical objects distinct from the empirical world. But they differ markedly in this denial. Literalists argue that mathematical objects simply exist in the empirical world; on this account, mathematical assertions assert true beliefs about perceivable objects. Fictionalists, on the other hand, hold that, strictly

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speaking, mathematical objects do not exist at all, and so exist in neither the empirical world nor in some realm distinct from the empirical world. They argue that mathematical objects are not actual objects but rather harmless fictions; on this account, mathematical assertions do not assert true beliefs about the world but merely fictional attitudes.

Although these two positions are apparently quite opposed to one another, they nonetheless have been both ascribed to Aristotle. Indeed, Aristotle's philosophy of mathematics seems to exhibit some of the features characteristic of literalism and some of the features characteristic of fictionalism. However, Aristotle's position also exhibits features interestingly different from both positions. I will begin by quickly surveying the variety of descriptions which Aristotle uses to characterize the relation between mathematical objects and the perceivable world. This will help to explain how these apparently opposed positions have been ascribed to Aristotle. There are three classes of descriptions.

(1) The use of *qua* (ᾗ) as the adverbial modification of a verb of consideration.[1] One such use is in a negative or, less commonly, positive description with the object of consideration *sensible* (or mobile or physical) things. Thus Meta. M.3 (1077b20ff.):

> clearly it is possible that there should also be both propositions and demonstrations about sensible magnitudes, not however *qua* sensible but *qua* possessed of certain definite qualities.... and in the case of mobiles there will be propositions and sciences, which treat them however not *qua* mobile but only *qua* bodies, or again only *qua* planes, or only *qua* lines, or *qua* divisibles, or *qua* indivisibles having position, or only *qua* indivisibles.[2]

The other use of ᾗ is in a positive description with the object of consideration *mathematical*. These uses of ᾗ regularly modify the mathematical as separate; I will consider this use in more detail in conjunction with the class of separation descriptions, below. For example, see Meta. E.1 (1026a9–10): 'it is clear that some branches of mathematics are considered as immovable and separate (χωριστά).'[3]

[1]θεωρεῖν: Meta. E.1 (1026a10), K.3 (1061a35ff.), M.3 (1078a24ff.). Cf. νοεῖν: De An. III.7 (431b15); σκοπεῖν: Phys. II.2 (194a10); πραγματεύεσθαι: Phys. II.2 (193b31); ζητεῖν: Meta. E.1 (1025b1); ἐνδέχεσθαι περί: Meta. M.3 (1977b21). With the use of ᾧν, not ᾗ, De An. III.7 (431b12).

[2]Translations based on those collected in McKeon, R., *The Basic Works of Aristotle* (New York: Random House, 1941). Cf. Phys. II.2 (194a9–12): 'Geometry investigates physical lines but not qua physical.' ἡ μὲν γὰρ γεωμετρία περὶ γραμμῆν φνσικῆν σκοπεῖ, ἀλλ' οὐχ ᾗ φνσική. Also Meta. K.3. (1061a34).

[3]Cf. De An. III.7 (431b15–16); APo. I.18 (81b4–5). An exception, with ᾗ in a negative description with the object of consideration mathematical, occurs at APo. I.13 (79a7–10); I will consider this passage in detail later.

(2) The sensible is abstracted from the mathematical. Here, two compounds of αἱρεῖν are used.[4] First, compounds with περι are used to describe the 'stripping off' of the nonmathematical properties of sensible objects to leave only the mathematical. Thus Meta. K.3 (1061a28–33):

> the mathematician investigates abstractions for before beginning his investigation he strips off (περιελών) all the sensible qualities, e. g. weight and lightness, hardness and its contrary, and also heat and cold and the other sensible contrarieties, and leaves only the quantitative and continuous.[5]

The second compounds are with ἀπο. This is a rare use in verb form.[6] More common are substantive phrases such as τὰ ἐξ ἀφαιρέσεων λεγόμενα.[7]

(3) The mathematical is separated from the sensible. The verb of separation here is most commonly χωρίζειν thus Phys. 2.2 (193b31–34): 'the mathematician, though he too treats of these things [the properties of the earth and the world], . . .separates (χωρίζει) them; for in thought they are separable (χωριστά) from motion.'[8] Note that in separation descriptions the verb of separation is regularly qualified in some way. Thus in the Phys. passage quoted above, χωριστά is qualified with the dative, τῇ νοήσει.[9] More common is the use of ᾗ as a qualification, as in the Meta. E.1 passage classified under the consideration class of descriptions. A special case is the description at Meta. M.3 (1078a21–22): '[the mathematician] studies what has not

[4]Other compounds – for example, ἀνα–αἱρεῖν ('extract': Phys. I.4) and δια–αἱρέσιν ('division': Phys. III.6) – are used in senses unrelated to mathematical abstraction.

[5]Cf. Meta. Z.3 (1029a11).

[6]APo. I.5 (74a37-b1). Also Meta. Z.3 (1029a16), a passage whose credibility for the ascription of any view to Aristotle I will draw into question later. There are other uses of ἀφαίρεσιν unrelated to mathematical abstraction: for example, Meta. Δ.22 (1022b31), with ἀφαίρεσιν associated with privation. There are also uses of ἀφαιρεῖν in context of the definition of non-substance categories; these uses are not in explicit relation to mathematical abstraction.

[7]in explicit apposition with τὰ μαθηματικά, De Caelo (299a14–18); perhaps not referring to strictly mathematical abstract objects, APo. I.18 (81b3); as τὰ ἐν ἀφαιρέσει λεγόμενα, De An. III.7 (431b12–13), III.8 (432b5); as τὰ ἐν ἀφαιρέσει, ὄντα De An. III.4 (429b21); as τὰ δι' ἀφαιρέσεών ἐστιν in apposition with τὰ μαθηματικά and contrasted with ἐξ ἐμπειρίαν, NE (1042a18). The τὰ . . .λεγόμενα construction is ambiguous. Ross (*Aristotle's Metaphysics* (2 vols., Oxford: Oxford University Press, 1924, 2nd ed. 1956)) translates as 'the so-called . . .'; an alternative reading is 'the things said as a result of . . .'.

[8]χωρίζειν from ὕλη, De An. III.4 (429b21); without an indirect object, Meta. M.3 (quoted below); cf. ἀποτεμόμεναι, Meta. G.1 (1003a24–5); περιγραψόμεναι, Meta. E.1.

[9]Hardie and Gaye's translation of this dative phrase as 'in thought,' suggests a locative sense; this would support my subsequent argument against extension as a mathematical matter. However, this is a rare use of the dative (see Sonneschein §434); the sense may be instead instrumental.

been separated by separating (τὸ μὴ κεχωρισμένον θείη χωρίσαν);' here the participle of separation is qualified by a negative phrase.

These descriptions present an ambiguous picture of Aristotle's view of mathematical existence. Are mathematical objects properties or entities? Consider APo. 1.5 (74a35ff.):

> The angles of a brazen isosceles triangle are equal to two right angles: but eliminate brazen and isosceles and the attribute remains.

Here Aristotle speaks of taking away both properties (isosceles) and matter (bronze). If abstraction is the elimination of *properties*, mathematical objects would seem to be physical objects considered as if they did not have certain properties. Consideration descriptions with the object of consideration sensible substances tend to support this view: recall, in the Meta. M.3 (1077b20ff.) passage, quoted above, the mathematician is represented as considering sensible things (but not as sensible) and mobile things (but not as mobile). Those who take a literalist interpretation of Aristotle's philosophy of mathematics, and especially those who hold that mathematics studies in part a distinctly mathematical matter contained in the physical world, tend to emphasize these descriptions, as we'll see. On the other hand, if abstraction is the elimination of the matter of a sensible substance or the isolation of its mathematical features, mathematical objects would seem to be certain *properties* of sensible things – properties such as triangularity. Those who take a fictionalist interpretation of Aristotle's philosophy of mathematics tend to emphasize these descriptions, as we'll see. The ambiguity between the two pictures apparently presented by these descriptions is noted by Ian Mueller (1970, 162ff.) and Julia Annas (1976, 30), and is one reason why both literalism and fictionalism has been ascribed to Aristotle.

Here is a sketch of the argument of the paper. I will begin by discussing literalism in contemporary philosophy of mathematics and the ascription of literalism to Aristotle. The view faces challenges as an interpretation of Aristotle: the ascription commits Aristotle to the unattractive view that mathematics studies but a small fragment of the physical world; and there is evidence that Aristotle would deny the literalist position that mathematical objects are perceptible (§1). I will also consider in detail the best developed literalist interpretation of Aristotle. The interpretation rests on the claim that Aristotle has a doctrine of a uniquely mathematical matter; I will argue that this claim is false. Although the considerations of these two sections of the paper fall short of refuting the ascription of literalism to Aristotle, they do shift the burden of proof on those who would persist in the ascription (§2). I will then discuss fictionalism in contemporary philosophy of mathematics and the ascription of fictionalism to Aristotle. This ascription also faces a difficult challenge: there is evidence that Aristotle would deny the fictionalist position that mathematics is false (§3). I will then argue that, in Aristotle's view, the fiction of mathematics is not to treat what does not exist as if existing but to treat mathematical objects with an ontological status they lack. This form of fictionalism is consistent with holding that mathematics is true (§4).

1

I will begin by briefly discussing literalism in recent philosophy of mathematics. I have noted that mathematical truths, as standardly interpreted, commit us to the existence of mathematical objects. In contemporary philosophy of mathematics the most pressing difficulty with such commitment is epistemological. As Paul Benacerraf (1973) framed the issue, if there is a transcendent world of mathematical entities, it is unclear how such a world could cause our knowledge of it. The difficulty, then, is to reconcile our best current theories of truth with a causal theory of knowledge.

Mathematical literalists[10] accept that mathematics commits us to the existence of mathematical objects and attempt to avoid the epistemological difficulties resulting from this commitment by arguing that we indeed do have perceptual knowledge of these mathematical objects. On this account then, we simply are in causal interaction with mathematical objects. Penelope Maddy, for example, argues that we simply perceive sets.[11] She calls such set-theoretical realism Aristotelian, not Platonic in part 'since sets, on the view [she is] concerned with, are taken to be individuals or particulars, not universals.'[12] Donald Gillies has endorsed some of Maddy's views and also the designation of these views as Aristotelian.[13] Gillies writes that it seems 'highly plausible to claim that sets exist in the material world. Examples of naturally occurring sets would be: the stars of a galaxy, the planets of the solar system.... If sets exist in the material world, then it seems reasonable to suppose that we might on occasion perceive a set with our senses.'[14]

The ascription of literalism to Aristotle has textual support. Recall that, in passages such as 193b23–25, quoted above, Aristotle asserts that mathematical objects are part of the physical world. But the ascription

[10]The term is Chihara's (*Constructibility and Mathematical Existence* (Oxford: Oxford University Press, 1990), 3ff.).

[11]Maddy, P., 'Perception and Mathematical Intuition', *Philosophical Review*, 89 (1980): 163–96, 178ff.; also Maddy, P., *Realism in Mathematics* (Oxford: Oxford University Press, 1990).

[12]Maddy (1980, 163).

[13] Gillies, D., 'Do we need Mathematical Objects?', *British Journal for the Philosophy of Science*, 43 (1992): 263–78., 266ff.

[14]Gillies, D., An empiricist philosophy of mathematics and its implications for the history of mathematics', in *The Growth of Mathematical Knowledge*, edited by E. Grosholz and H. Breger (Synthese Library/Volume 289, Kluwer, 2000) 9. Gillies believes that sets are perceptible since observation is theory-laden. Set-theoretic literalism has received some critical attention; see, for example, Chihara (1990, 194–215).

faces two challenges. First, can Aristotle mean that all mathematical objects whatsoever are physical? The set-theoretical literalism of Maddy and Gillies may be a plausible position, but extending literalism to other branches of mathematics is problematic. For physical objects lack the exactitude characteristic of many kinds of mathematical objects. It seems, for example, that we do not encounter perfectly straight lines in the physical world. The ascription of literalism to Aristotle, then, appears to saddle him with an implausible view.

The best developed literalist interpretation of Aristotle's philosophy of mathematics is to be found in Mueller (1970). Mueller resolves the problem that most mathematical properties of sensible substances lack the exactitude characteristic of the subject matter of mathematics by arguing that Aristotle's claim that the physical world contains mathematical objects is merely the claim that the physical world contains a matter of pure extension – whose only features are length, width and depth – and that this is also the basis of geometric objects. So, on Mueller's view, Aristotle does not claim that all mathematical objects are contained in the physical world. Rather, he holds that the physical and mathematical realms overlap. Although physical lines and triangles lack the exactitude characteristic of geometric objects, the physical world shares with mathematics the precise extensional features of length, width and depth.

The Phys. 2.2 passage quoted above might be read so as to lend support to Mueller's interpretation. However, an unattractive result of this view is that Aristotle's claim that the physical world contains mathematical objects is severely restricted. The physical world only contains only a small part of geometry. In response, Jonathan Lear (1982) argues that it is not so implausible to ascribe to Aristotle the view that there are in fact exact mathematical objects such as triangles in the world. This allows much more of mathematics to be contained in the physical world than Mueller's view allows. However, there are unattractive results of this view as well. For although the physical world contains much of geometry, in Lear's view, standard geometry is a part of a mere sliver of the physical world. Moreover, Lear concedes that not all geometric objects are found in the physical world. So, although the overlap between the physical world and mathematics is less restricted than it is on Mueller's view, it is also restricted.

A second problem for ascribing literalism to Aristotle is that he would reject the literalist view that we can perceive mathematical objects. Aristotle's account of mathematical concept acquisition concerns a progression from common sensibles to objects for thought. The mathematical *genera* are among the common sensibles (*koina*) described at De An. 3.1 (425a14–19):

> that which we perceive incidentally through this or that special sense, e. g. movement, rest, figure, magnitude, number, unity; for all these we perceive by

> movement, e. g. magnitude by movement, and therefore also figure (for figure is a species of magnitude), what is at rest by the absence of movement, and number by the negation of continuity.

Aristotle seems to hold that we perceive geometric and arithmetic properties only incidentally through such special sense faculties as sight. To continuous *quanta* correspond the common sensible of magnitude; to discrete *quanta* correspond the common sensible of number. Since these are not the objects of any one sense faculty, we perceive mathematical properties through the *sensus communis*. However, it is not these common sensibles *per se* which the mathematician studies. See De Mem. 1(449b31ff.):

> Without a presentation intellectual activity is impossible. For there is in such activity an incidental affection identical with one also incidental in geometrical demonstrations. For in the latter case, though we do not for the purpose of the proof make any use of the fact that the quantity in the triangle [for example, which we have drawn] is determinate, we nevertheless draw it determinate in quantity... [for] one envisages it as if it had determinate quantity, though subsequently, in thinking it, he abstracts from its determinateness.... [This] presentation is an affection of the *sensus communis*.

The use of diagrammatic representation in geometric proofs is used as an example to illustrate Aristotle's claim that representations are a condition for any thinking. Just as a particular proof employs but does not concern just a particular diagram, so too an episode of thought employs but does not concern a particular representation. Notice that thought does not use a visual representation but a common sensible.[15] Since mathematics is a field of thought, mathematicians do not study the common sensibles which are employed in mathematical proof and which are the perceived mathematical properties of sensible substances. So mathematical objects are not themselves perceivable.

The ascription of literalism to Aristotle faces these two challenges. Before turning to the ascription of fictionalism to Aristotle, I will discuss the question whether there is a distinctly mathematical matter.

2

I've noted that the best developed literalist interpretation of Aristotle's philosophy of mathematics ascribes to him a doctrine of mathematical matter as pure extension. Aristotle would of course recognize that both physical and geometric objects have extensional features of length, breadth

[15]Although the power to represent is the faculty of the imagination: see Caston, V., 'Why Aristotle needs imagination', *Phronesis*, 41 (1996): 20–55.

and depth. But would he describe extension as a kind of mathematical matter? In this section, I turn to the evidence on which a doctrine of mathematical matter may be ascribed to Aristotle. Although some of the ablest commentators – ancient and recent – have advocated this interpretation, the textual support is tenuous; furthermore, what passages there are, are open to alternative readings.

The ascription to Aristotle of a doctrine of mathematical matter rests on two points. First, Aristotle is represented as holding a doctrine of either noetic matter or prime matter. Second, this kind of matter is identified with extension. Combining these two points, it is argued that for Aristotle mathematics concerns a kind of intellectual matter, extension, which is either itself prime matter or the first layer of form placed on prime matter. This interpretation stems from the ancient commentators and has been advocated in recent scholarship by Mueller (1990, 464–65), who writes:

> Mathematical objects are embodied in pure extension underlying physical objects; the geometer's abstraction of non-geometric properties enables him to apprehend these things which satisfy the mathematician's definitions.

Against this interpretation, I will first argue that the passages cited in support of the ascription to Aristotle of a doctrine of mathematical matter are inconclusive. Recent advocates of this interpretation candidly acknowledge this. Mueller (1990, 465), for example, acknowledges the poverty of textual support: 'This interpretation, which I [Mueller] have espoused, has the disadvantage of assigning to Aristotle a theory about which one might expect him to have been more explicit if he held it.' On the basis of this lack of evidence, I am sceptical that the ascription is correct. Second, I will address further difficulties for the position. On the basis of these difficulties, I believe that a mathematical matter could not explain mathematical existence even if the doctrine *could* be ascribed to Aristotle.

I turn first to passages which lend support to the ascription to Aristotle of a doctrine of noetic matter. Although Aristotle explicitly describes mathematical objects as separated both from motion (and so from what we may call kinetic matter) and from sensible matter,[16] there is evidence elsewhere for another kind of Aristotelian matter, noetic matter. I will argue that this evidence supports the view that mathematics must have for Aristotle a material *explanation*; but that the evidence is insufficient to ascribe to Aristotle a doctrine of mathematical *matter*.

There are only two sources for the ascription to Aristotle of a doctrine of noetic matter. The first is Meta. Z.10 (1036a9–12):

> And some matter is perceptible and some intelligible, perceptible matter being for instance bronze and wood and all matter that is changeable, and intelligible

[16]Phys. II.2 (193b34): Meta. E.1 (1025b34).

> matter being that which is present in perceptible things not qua perceptible, i. e. the objects of mathematics.

Here, mathematical objects and noetic matter are explicitly identified. Unfortunately, this passage is a *postea addita*, and reflects not Aristotle but the very tradition of Aristotelian interpretation with which I disagree.[17] The second source for a doctrine of noetic matter is Meta. H.6 (1045a33–5):

> Of matter some is intelligible, some perceptible, and in a formula there is always an element of matter as well as one of actuality; e. g. the circle is 'a plane figure'.

Here Aristotle refers to noetic matter. However, if we consider the context of this passage, I think that it will be clear that the further identification of this noetic matter with mathematical objects is implausible. The context of this passage is Aristotle's resolution of the problem of the unity of a binomial definition. The genus stands in relation to the species as something potential to something actual. This allows the composite to be a unity. A mathematical example is used to illustrate this principle. The definition of a circle, a plane figure with every point equidistant from the centre, has a generic element, 'a plane figure'. It is such an element of a definition which may be called noetic matter. The noetic matter referred to here is therefore not exclusively mathematical but generic. In so far as mathematical definitions require *genera*, mathematics has a material explanation; however, this in itself does not entail that there exists a uniquely mathematical matter.

I turn next to the second set of passages, cited in support of the identification of either noetic matter or prime matter with extension. I will argue that these passages do not support this identification. Here too there are only two relevant passages. The first passage is Meta. Z.3 (1029a7–19):

> We have now outlined the nature of substance, showing that it is that which is not predicated of a stratum, but of which all else is predicated. But ... on this view matter becomes substance. For if this is not substance, it baffles us to say what else is. When all else is stripped off evidently nothing but matter remains. For while the rest are affections, products, and potencies of bodies, length, breadth, and depth are quantities and not substances (for a quantity is not a substance), but the substance is rather that to which these belong primarily. But when length and breadth and depth are taken away we see nothing left unless there is something that is bounded by these; so that to those who consider the question thus matter alone must seem to be substance.

Much has been made of this passage by those who ascribe to Aristotle a doctrine of prime matter. I do not find this ascription convincing; but I will

[17] Jaeger, OCT.

not pursue this debate further here.[18] For my purposes it is sufficient to consider the use of this passage to ascribe to Aristotle a doctrine of mathematical matter of pure extension or dimension. The passage lends support to this interpretation only if we can take the phrase, 'something bounded by these [dimensions]' to positively assert the existence of a kind of matter which is extension and which is the subject of dimensions, and so to reject the preceding phrase, 'there is nothing left.' Sorabji, for example, argues that the phrase is such an assertion.[19] However, this reading of the passage is not entirely convincing when we take into consideration the context of the passage. Aristotle is entertaining the claim that subject (*hupokeimenon*) and substance (*ousia*) are identical. This claim is rejected; the passage, as I read it, supports this rejection by reducing the claim to an absurdity. On the account that subject and substance are identical, it seems that matter is substance. This would violate any interpretation of Aristotelian substance: although controversial, substance is taken to be either the individual composite of form and matter or the species-form.

Our second passage is Phys. 4.2 (209b5–11):

> If, then, we look at the question in this [preceding] way the place of a thing is its form. But, if we regard the place as the extension of the magnitude, it is the matter. For this is different from the magnitude: it is what is contained and defined by the form, as by a bounding plane. Matter or the indeterminate is of this nature; when the boundary and attributes of a sphere are taken away, nothing but the matter is left.

The argument here is that place seems to be matter in so far as place seems to be extension. Aristotle later dismisses both the view that place is matter and the view that place is extension.[20] But this in itself would not refute the identification of matter and extension. However, this passage does not identify matter and extension but rather draws a simile between the two: matter is like (*toiouton*) extension.[21] As such, little support for the identification of matter and extension can be drawn from this passage.

I have argued that the passages cited do not provide conclusive support to the ascription to Aristotle of a doctrine of mathematical matter. It might be

[18]Scholars who argue against the orthodox ascription to Aristotle of a doctrine of prime matter include King, H. R., 'Aristotle without *Prima Materia*', *Journal for the History of Ideas,* 17 (1956): 370–89 and Charlton, W., *Aristotle's Physics Books I and II* (Oxford: Clarendon, 1992). Scholars defending this orthodoxy include Solmsen (1958) and Robinson, H. M., 'Prime Matter in Aristotle', *Phronesis,* 19 (1974): 168–88. For a recent bibliography, see Bostock, D. *Aristotle's Metaphysics Books* [Zeta-Theta] (Oxford: Oxford University Press, 1996).

[19]Sorabji, R. R. K., *Matter, Space and Motion* (London: Duckworth, 1983, 6). I agree with Sorabji that the 'unless . . .' phrase is neither a gloss (contra Schofield, , M. '*Metaph.* Z 3: some suggestions', *Phronesis,* 17 (1972): 97–101, 97) nor a reference 'to a more familiar kind of subject, such as bronze' (contra Robinson (1974, 187)).

[20]At 211b29–212a2 and 211b14–29 respectively.

[21]Sorabji (1983) notes this difference.

said that there is a distinctively mathematical matter, but only insofar as extension is the genus of geometric species. For Aristotle views the genus-species relation as analogous to the matter-form relationship. As such, no conclusions can be drawn as to the point of contact between the sensible or material world and the realm of the mathematical. Although the considerations of the last two sections of the paper fall short of refuting the ascription of literalism to Aristotle, they do shift the burden of proof onto those who would persist in the ascription.

3

I turn to fictionalism in contemporary philosophy of mathematics, the ascription of fictionalism to Aristotle and the points of agreement and disagreement between Aristotle and fictionalists. As I've noted, if you deny the existence of mathematical objects, then it seems that you must reject all claims that commit you to such objects, which means rejecting most of mathematics as standardly understood. Contemporary mathematical fictionalists such as Hartry Field (1980) accept this consequence. According to mathematical fictionalism, mathematicians make the fictitious assumption that mathematical objects exist: such an assumption, they admit, is false; but the fiction, they assure us, is harmless and useful. Fictionalists disarm the apparent commitment to mathematical objects in mathematical statements by showing how in principle these statements could be rewritten into synonymous statements which do not have problematic ontological commitments. One strategy takes the form of a reduction to quantificational statements. Although the nature of these quantifiers is controversial, I will present an example using existential quantifiers: this is the simplest case. Consider the equation

$$\text{(A) } 2 + 3 = 5$$

A reductionist reading of this equation, with the numerical quantifier $(\exists n)$ an abbreviation for a sequence of n distinct existential quantifiers, would translate (A) as follows:

$$\text{(B) } (\forall V)(\forall W)[(\exists 2x)(Vx)\&(\exists 3x)(Wx)\& \sim (\exists x)(Vx\&Wx) \supset (\exists 5x)(Vx \vee Wx)].$$

That is, the equation $2+3=5$ can be read as saying merely that if there are two Vs and three different Ws, then there will be five things which are V or W. Where the arithmetical equation mentions abstract objects, the quantificational statement is free of such reference. Such quantificational statements are elephantine: this is partly why mathematics is a *useful* fiction.

Lear (1982) and others have ascribed mathematical fictionalism to Aristotle. The ascription has some initial plausibility. As we've seen, Aristotle sometimes describes the relation between mathematical objects and the sensible world in ways which suggest fictionalism. For example, Aristotle claims that mathematicians separate mathematical properties in thought. And this sounds rather like the claim that mathematical objects don't exist but mathematicians make the fictitious assumption that they do. Is this the right picture for Aristotle's philosophy of mathematics?

One difficulty with ascribing fictionalism to Aristotle is that the fictionalist holds that mathematics is, strictly speaking, false; but Aristotle explicitly holds that mathematics is true.[22] Those who would persist with the ascription are hard pressed to explain this apparent inconsistency. Lear (1982, 191), for example, writes:

> For Aristotle, mathematics is true, not in virtue of the existence of separated mathematical objects to which its terms refer, but because it accurately describes the structural properties and relations which actual physical objects do have. Talk of nonphysical mathematical objects is a fiction, one that may be convenient and should be harmless if one correctly understands mathematical practice.

However, that mathematics 'accurately describes the structural properties and relations which actual physical objects do have' explains why the assumption that nonphysical mathematical objects exists is a *useful* fiction; it doesn't show that, despite this, mathematics is nonetheless *true*. For example, Papineau (1990, 173) writes:

> Lear ... does seem to want it both ways. He shows how the possibility of sticking to beliefs which do not involve abstract objects makes it both harmless but useful to work with propositions that do. But then he claims that this yields a sense in which the latter propositions are true.

To give another example, Hussey puts forward the interesting and sophisticated interpretation that mathematics for Aristotle concerns representational objects.[23] A representational object is a nonexistent object which can stand for a variety of existing objects which approximate it. However, Hussey (1991, 127) offers the explanation that no falsity results from the false assumption of the existence of fictitious objects because the

[22]Phys. 193b35; Meta. 1078a18–19.

[23]For the original exposition on representational objects, see Kit Fine, *Reasoning with Arbitrary Objects* (Oxford: Blackwell, 1985). This form of fictionalism entails the rejection of the principle of bivalence; Aristotle would resist such a result (except possibly for future contingent statements).

assumption is 'eventually discharged.' This is less than satisfying. The difficulty from our perspective is that a mathematical claim is true in virtue of the object picked out by its referring expressions being correctly characterized by the predicate expression. So, to take again our example, 'This triangle has interior angles of 180 degrees' is true in virtue of there being a referent for 'this triangle' which has the feature ascribed to it by the predicate. On this view, it is difficult to see how *discharging* the fictional assumption of the existence of the referent for 'this triangle' would vindicate the truth of 'This triangle has interior angles of 180 degrees'.

So, like the fictionalist, Aristotle holds that mathematics ascribes to mathematical properties of sensible substances an ontological status they in fact lack. However, unlike a contemporary fictionalist, Aristotle does not believe that this requires that we deny that mathematical objects exist or that mathematics is false. To resolve this tension, I will next argue that the ascription to Aristotle of fictionalism is typically made within an anachronistic and misleading framework. For I am in broad agreement with those, such as Lear and Hussey, who ascribe fictionalism to Aristotle. But by placing the ascription in an appropriate context, I hope to contribute to this line of interpretation by providing a satisfying fictionalist account of Aristotle's views on mathematical truth.

4

I began this paper with Benacerraf's problem for the philosophy of mathematics: on the best current theories of knowledge and truth, our knowledge of mathematical truths requires causal interaction with mathematical objects; yet mathematical objects do not seem to exist among perceptible objects. Aristotle is not explicitly concerned with this problem. Rather, his concern is to explain in what sense mathematical objects exist, given that they can be neither separate from, nor present in, sensible substances. Aristotle raises this aporia at 997b12–34. I will discuss Aristotle's reasons for claiming that mathematical objects are inseparable from sensible substances in a moment. Geometric objects cannot be *present in* sensible substances for then there will be co-located solids, co-located lines, and so on. Notice that mathematical objects cannot be said of sensible substances, for items and what are said of them are synonymous in Aristotle's idiosyncratic sense of synonymy, articulated at 1a6–8: they share both a name and a definition. But the definition of a circle, for example, does not define a circular sensible substance. So it seems that mathematical objects fall outside the classification of beings in the *Categories*: they are not non-substances present in a sensible subject; they are not universals said of a subject; and they are not themselves independent subjects existing separately from sensible substances.

Aristotle returns to this aporia at 1077b12–33:

> It has been sufficiently stated that mathematical objects are neither substances apart from bodies, nor prior to perceptible things in being but only in definition, nor capable of existing somewhere separate. But since it was not possible for them to exist in perceptible things either, it is plain that they either do not exist at all or exist in a certain manner (*tropon tina esti*) and, because of this, do not exist without qualification (*ouch haplôs estin*). For being is said in many ways.... Thus since it is true to say without qualification that not only things which are separable but also things which are inseparable exist (for example, that moveable things exist), it is true also to say without qualification that the objects of mathematics exist.

Aristotle dismisses without comment the position that mathematical objects do not exist at all. (I will discuss this dismissal in a moment.) Rather, we can truly say that mathematical objects exist, for they exist in a qualified way. Aristotle sees the possibility that they have qualified existence as a consequence of his position that being is said in many ways. Aristotle expresses this ontological thesis in several passages. For example, at 1003a33-b10 he writes:

> being is said in many ways, but in relation to one certain nature and not merely homonymously. Just as everything which is healthy is related to health, one by preserving it, another by producing it, and another by being a symptom of health ... so too being is said in many ways but all in relation to one principle. For some are called beings because they are substances, others because they are affections of a substance, others because they are paths towards substance, or destructions or privations or qualities of substance, or productive or generative of substance, or of things which are relative to substance, or negations of one of these things or of substance itself.

I will call the claim that something 'is said in many ways' multivocity. The multivocity of being is at least the view that 'exists' is predicated variously.[24] I next will explain how these views – the position that mathematical objects are inseparable from sensible substances, the claim that being is a multivocal and the distinction between qualified and unqualified being – shed light on Aristotle's philosophy of mathematics.

I will begin with Aristotle's claim that mathematical objects are inseparable from sensible substances. Aristotle regularly uses separation terminology to indicate ontological independence. Fine (1984) argues per-

[24]An associated view is that being is a connected homonym. Irwin, T., 'Homonymy', *Review of Metaphysics*, 34 (1981): 523–44. and Shields, C., *Order in Multiplicity: Homonymy in the Philosophy of Aristotle* (Oxford: Oxford University Press, 1999, §1.3) argue that multivocity and homonymy are co-extensive.

suasively that separation in Aristotle is an asymmetric relation. Fine also holds that separation indicates a capacity for independent existence. But in my (2008) I've argued at length that one item can be separate from a second for Aristotle, even when it is impossible for the one to exist apart from the other. Rather, one thing is separate from another just in case an account of the former's ontological status as an existent can be made without reference to the latter. I can not rehearse fully this argument here. But let me sketch one consideration in favour of this interpretation of separation terminology. Consider *propria*, necessary but inessential properties. A classic example of a *proprium* for humans is risibility. An individual human can not exist apart from her *proprium* and so can not lack risibility. But by claiming that an individual substance is separate from its properties, Aristotle appears to be committed to holding that an individual human is separate from her property of risibility.[25]

Aristotle holds that there are a variety of different kinds of entities: individual substances such as Callias, universal substances such as humanity, and individuals and universals among such other categories as qualities and quantities are all among things that have an ontological status. Of all these, only individual substances have their ontological status independently of standing in a relation to some other kind of entity. All other entities have their ontological status in virtue of standing in a relation to some individual substance or other. Mathematical objects have their ontological status in virtue of standing in a relation to sensible substances. I believe that this is what Aristotle means by claiming at 1078a21–22 that mathematical objects are not separate from sensible substances. He does not mean that mathematical objects can not exist apart from sensible substances, but that they possess their ontological status in virtue of standing in some relation to sensible substances.[26] I will not discuss here the nature of this dependence. To do so would require a lengthy discussion of Aristotle's views on abstraction. However the precise nature of a mathematical object's dependence on

[25]Aristotle holds that substances, alone of the categories, are separate: see, for example, 185a31–2, 1029a27–8. I discuss the evidence for taking this to mean that substances are ontologically independent from all other entities, in my (2008).

[26]This interpretation of separation terminology helps to explain the inseparability of mathematical objects from sensible substances. Aristotle argues at 1077b12–39 that mathematical objects cannot be separate from sensible substances on pain of regress. If there is a geometric figure such as a circle separate from circular sensible substances, then there will be another circle in addition to the first two, and so on. The argument here is obscure. Aristotle may be offering an argument similar to his objection to Platonic Forms at Peri Ideon 84.23–4 and elsewhere. Cohen, S. M. ('The Logic of the Third Man', *Philosophical Review*, 80 (1971): 448–75) and others have argued that such arguments are explanatory regresses. On this interpretation of 1077b12–39, a geometric figure such as the circle is posited so to explain the circularity of physical circles. But Aristotle canvasses the worry that the geometric figure is itself circular and furthermore cannot itself be referenced in an explanation of that circularity. Hence the need for a third circle. It would be unclear why an explanatory independence would follow from a capacity for separate existence, but explanatory independence is plausibly a consequence of ontological independence.

sensible substances need not concern us. It suffices for our present purposes to show that mathematical objects are existents which are in some way or other ontologically dependent on sensible substances.[27]

For Aristotle, ontological dependence is closely connected to predicability. The predicability of an expression suggests that the referent of that expression is ontologically dependent on another entity; the impredicability of an expression, on the other hand, suggests that the referent of that expression is ontologically independent. Expressions referring to individual substances are the only expressions which cannot be predicated of another entity. Expressions referring to other kinds of entities are predicable of individual substances. This is Aristotle's methodology in the *Categories*: the predicability or impredicability of an expression provides a rationale for a preliminary classification of the referent of that expression as ontologically dependent or independent.

However, within mathematical discourse, certain mathematical objects play the role of impredicable subjects. Entities which are, strictly speaking, dependent on sensible substances are, in mathematics, the subjects of predications. Consider a mathematical claim such as 'This triangle has interior angles equal to 180 degrees'. Here a mathematical property is predicated of a subject which cannot be predicated of another mathematical entity. The impredicability of the subject, within mathematical discourse, suggests that the referent of the expression is ontologically independent with respect to other mathematical entities. I propose that this is, according to Aristotle, the conceit of mathematics—a conceit which resembles mathematical fictionalism insofar as the mathematician treats mathematical objects with an ontological status they in fact lack. However, where contemporary mathematical fictionalists hold that mathematics treats what does not in fact exist as if it does exist, Aristotle holds that mathematics treats what exists qualifiedly as if it exists unqualifiedly. I believe that this is what Aristotle means when he says, in the 1078a21–22 passage mentioned above, that the mathematician separates what, strictly speaking, is not separate from sensible substances, and when he claims at 193b31–34 that the mathematician separates mathematical objects in thought.

Aristotle is then an ontological pluralist. Among existents are both entities which are ontologically independent and are the referents of subject terms within a canonical discourse, and entities which are ontologically dependent. With this discussion of Aristotle's metaontological views in the background, I return now to the ascription of fictionalism to Aristotle. Fictionalism is a position in the contemporary philosophy of mathematics which arises within a framework that distances us from Aristotle. In

[27]On mathematical abstraction see Cleary, J. 'On the Terminology of "Abstraction" in Aristotle', *Phronesis,* 30 (1985): 13–45 and on Aristotle's use of the qua operator see Lear, J., 'Aristotle's Philosophy of Mathematics', *The Philosophical Review,* 91 (1982): 161–92.

particular, there are in this framework anachronistic assumptions of ontological monism, and a corresponding unitary account of ontological commitment.

Recall, the difficulty with this ascription is that Aristotle holds that mathematics is true but contemporary fictionalists hold that mathematics is false. We have seen that on contemporary theories of truth a sentence expresses a truth just in case the referent of its subject exists and is correctly characterized by the predicate of the sentence. The only demand on truth for Aristotle, however, is to say of what is that it is and of what is not that it is not.[28] And Aristotle is catholic in his acceptance of what is. Qualities, quantities, mathematical objects and so on are all things that are.[29] The interesting philosophical question is, for Aristotle, *in what way* things which are – that is to say, whether or not they have claim to their ontological status independently of standing in a relation to something else and so *simpliciter*.[30] The question of mathematical existence is not one of existence *per se* but of the dependence on, or independence from, those items which have uncontroversially independent status as beings, sensible individual substances.

On Quine's account of ontological commitment, we are committed to the values of the variables and range of the quantifiers in those true statements which are indispensible for science. Others have proposed alternative accounts of ontological commitment. Azzouni (2007), for example, holds that we are committed to the extension of the existence predicate. Cameron (2007) holds that we are committed only to the truth makers: on this view, mathematics may be true but, if the truth makers of mathematical truths are not mathematical objects, then the truth of mathematics fails to commit us to the existence of mathematical objects. Aristotle seems to view truths as committed to the referents of the terms employed in canonical statements expressing those truths. Being the referent of either a term which is an impredicable subject or a term predicable of an impredicable subject suffices for inclusion in the ontology of the *Categories*.

However, although Aristotle is catholic in his acceptance of what exists, he is not indiscriminate. There are filters for this ontology. First, we are committed to the referents of terms in *reputable* opinions.[31] And second, there are exceptions, and Aristotle can deny an existence claim even when they are consequences of reputable opinions, such as alleged truths with

[28]'To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true' (Metaphysics 1011b25).

[29]Categories 1.

[30]See 1077B33–5: 'We truly say without qualification not only that the separable things exist but also that the inseparable things exist.' I've argued in my (2008) that the sense of some separation terminology in Aristotle is that separate things have claim to the status of a being independently of standing in a relation to something else, and so have being simpliciter.

[31]For this interpretation of endoxa in Aristotle, see for example, Irwin (1989)

fictional or non-denoting terms and philosophical theses such as those purporting the existence of Love or Strife. Schaffer (2009, 352) characterizes Aristotle as taking a 'permissive disinterest' in such existence questions as whether there are numbers. In support of this characterization, Schaffer cites my (2008) observation, an interpretation of 1076a36–37, that 'the philosophical question is not *whether* such things exist but *how* they do.' Schaffer goes on to advocate what he calls the Aristotelian view that the task of metaphysics is to say not what exists but what grounds what, and its method is to deploy diagnostics for what is fundamental, together with diagnostics for grounding derivative entities on fundamental entities. There is much that I find attractive in Schaffer's characterization of metaphysics. But, although I ascribe to Aristotle the view that the philosophical work of metaphysics predominantly lies in articulating how things exist, I now hesitate to characterize Aristotle as disinterested in existence questions altogether. Aristotle's philosophical method is typically to begin by surveying reputable opinions, to tease out aporetic difficulties, and eventually to resolve the aporia by introducing a new distinction. Mathematical discourse, of course, is constituted by the expert opinions of mathematicians and so makes at least a *prima facie* commitment to the existence of mathematical objects. Recall that Aristotle dismisses without comment in 1077b12–33, quoted above, the option that mathematical objects do not exist at all. I believe that it is in this spirit that Aristotle does not consider the non-existence of mathematical objects: the reputable opinions of mathematicians give us a defeasible but *prima facie* reason to hold that they do exist. Aristotle offers the distinction between qualified and unqualified being so to resolve the difficulties of this commitment.

Because the contemporary fictionalist denies that mathematical terms refer to existing objects, she must say that mathematics is, strictly speaking, false. Aristotle appears to endorse some kind of fictionalism yet he holds that mathematics is true. This problem of ascribing fictionalism to Aristotle arises because the contemporary framework presupposes a unitary account of ontological commitment; and Aristotle, I suggest, would deny this presupposition. To flesh out this point, we might distinguish between weak and strong ontological commitment. *Weak ontological commitment* is to entities which exist but not necessarily to entities which unqualifiedly exist or are ontologically independent entities. S*trong ontological commitment*, by contrast, is to unqualifiedly existing objects. A term in a true statement carries for Aristotle *weak* ontological commitment to entities which exist. But a term in a true statement need not carry *strong* ontological commitment to unqualifiedly existing objects. So a sentence is true for Aristotle only if its terms are at least weakly ontologically committing. Strong ontological commitment is sufficient but unnecessary for truth. Correspondingly there are two kinds of fictionalism. Some fictionalists hold that the fiction of mathematics is to treat what is non-existent as if it is existing. Aristotle's fiction is that mathematicians treat what exists merely dependently as if it exists independently.

I will bring the paper to a conclusion. The distinction between weak and strong ontological commitment resolves our interpretative difficulty. There is good reason to ascribe fictionalism to Aristotle. And this ascription is consistent with Aristotle's view that mathematical assertions are true. This resolution does not in itself answer the questions, in what precise way do mathematical objects exist and how do mathematical objects depend on sensible substances for their ontological status. But I will leave discussion of this topic for another occasion.[32]

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