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Abstract: Roughly, iffication is the speech-act in which - by appending a suitable if-clause - the speaker qualifies a previous statement. The clause following if is called the qualiffication. In many cases, the intention is to retract part of the previous statement - called the preiffication. The modified statement - called the iffication - is never stronger than the preiffication. I can retract part of "I will buy three" by appending "if I have money". A degenerate iffication is one logically equivalent to its preiffication. There are two limiting cases of degenerate iffications. In one the qualiffication is tautological, as "I will buy three if three is three". In the other the negation of the qualiffication implies the preiffication, as "I will buy three even if I will not buy three". Reiffication is the iffication of an iffication. For example, the previously mentioned iffication is ifficated by appending "if there are three left". Deiffication is the speech-act in which - by appending a suitable and-clause - the effect of an iffication is cancelled so that the result implies the preiffication. The first iffication mentioned above can be deifficated by appending "and I have money". All examples in the body of the paper come from standard (one-sorted, tenseless, non-modal) first-order arithmetic. All theorems are limited to propositions expressible in a language of first-order arithmetic. An easy theorem, hinted above, is that an iffication is degenerate if and only if the negation of the qualiffication implies the preiffication. The iffication of a conjunction using one of the conjuncts as qualiffication need not imply the other conjunct: "Two is an even square if two is square" does not imply "Two is even".

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## Pedagogical Comments

## 1. Is Stronger Than

This is logical jargon for "superimplies" or "is a superimplicant of".
Q1. Given any two propositions, in order for the first to (superimply * be stronger than) the second it is necessary and sufficient for the first to contain all of the information in the second
( *and more).
Q2. Given any two propositions, in order for the first to (superimply * be stronger than) the second it is necessary and sufficient for the first to imply ( *and not be implied by) the second. Q3. "Two is a prime number that is even" (implies * superimplies* is logically equivalent to) "Two is an even number ( *that is prime)".
2. Is Weaker Than

This is logic jargon for "is superimplied by" or "is a superimplication of"
Q4. "Two is a prime number ( *that is even)" is (implied by * superimplied by * is logically equivalent to) "Two is an even number that is prime".
There are exercises in Cohen-Nagel on this.
2. Important Facts about Conditionals.

Q1. In every case, the (negation of * ) the (antecedent * consequent) implies the conditional. Q2. In every case, the (negation of * ) the conditional implies the (negation of * ) the (antecedent * consequent).

## Total Iffications

A total iffication, in which the entire preiffication is retracted, is one that is tautological. Total iffication theorem: In order for an iffication to be total it is necessary and sufficient for the qualiffication to imply the preiffication.
This is a "form" of the principle of corresponding conditional, also known - quite improperly as the deduction theorem.
One-premise principle of corresponding conditional: In order for a one-premise argument to be valid it is necessary and sufficient for the conditional of the premise with the conclusion to be tautological.
Principle of tautological conditionals: In order for a conditional to be tautological it is necessary and sufficient for the antecedent to imply the consequent.

Degenerate Iffications

| $(\mathrm{Q} \rightarrow \mathrm{P})$ | $\sim \mathrm{Q}$ | $\sim \mathrm{P}$ |
| :--- | :---: | :---: |
| ? P | ? P | ? Q |

If any proposition is put for P in each of the above three schemes and any proposition is put for Q in all three, then the three arguments are all valid or all invalid. Let us look at the limiting cases.

| $(0=0 \rightarrow 1<2)$ | $\sim 0=0$ | $\sim 1<2$ |
| :--- | :--- | :--- |
| $? 1<2$ | $? ~$ |  |

Remind me to show you nice deductions for these.
$(\sim 1<2 \rightarrow 1<2)$
$\sim \sim 1<2$
$\sim 1<2$
? $1<2$
? $1<2$
? $\sim 1<2$

The above are limiting cases. One qualiffication is tautological and the other is the negation of the preiffication. In the intermediate cases the qualiffications are informative propositions implied by the negation of the preiffication.
$((2<3 \mathrm{~V} \sim 1<2) \rightarrow 1<2)$
$\sim(2<3 \mathrm{~V} \sim 1<2) \quad \sim 1<2$
? $1<2$
? $1<2$
? $(2<3 \mathrm{~V} \sim 1<2)$

Degenerate Iffication Theorems

| $(\mathrm{Q} \rightarrow \mathrm{P})$ | $\sim \mathrm{Q}$ | $\sim \mathrm{P}$ |
| :--- | :---: | :---: |
| ? P | ? P | ? Q |

Degenerate Iffication Theorem: In order for a conditional to imply its consequent it is necessary and sufficient for the negation of the antecedent to imply the consequent.
Proof: We do necessity first and then sufficiency.
Assume that the conditional implies its consequent. Notice that the negation of the antecedent implies the conditional. Since implication is transitive, the negation of the antecedent implies the consequent.
Now assume that $\sim \mathrm{Q}$ the negation of the antecedent implies P the consequent. Thus, $(\sim \mathrm{Q} \rightarrow \mathrm{P})$ is tautological. Thus, $(\mathrm{Q} \rightarrow \mathrm{P})$ contains the information in $(\mathrm{Q} \rightarrow \mathrm{P})$ and $(\sim \mathrm{Q} \rightarrow \mathrm{P})$ together. But, these together imply $P$, the consequent. Since implication is transitive, $(Q \rightarrow P)$ implies $P$ the consequent. QED.

The Ifficational Spectrum
Every iffication of a given preiffication lies somewhere in the spectrum whose limiting cases are total iffications, at one end, and degenerate iffications on the other.

Conjunctive Iffications
The iffication of a conjunction using one of the conjuncts as qualiffication need not imply the other conjunct: "Two is an even square if two is square" does not imply "Two is even".

Argument
Two is an even square if two is square ? Two is even

Counterargument
Two is an odd square if two is square ? Two is odd

Conjunctive Iffication Theorem: In order for an iffication of a conjunction using one of the conjuncts as qualiffication to imply the other conjunct it is necessary and sufficient for one conjunct to be implied by the negation of the other.

## Reiffications

We can have ham and eggs if we have eggs if we have ham. This of course is not a total ification because having ham and having eggs is not sufficient for having ham and eggs.

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