MEANINGS OF FORM

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Abstract: The expressions 'form', 'structure', 'schema', 'shape', 'pattern', 'figure', 'mold', and related locutions are used in logic both as technical terms and in metaphors. This paper juxtaposes, distinguishes, and analyses uses of these expressions by logicians. No such project has been attempted previously. After establishing general terminology, we present a variant of traditional usage of the expression 'logical form' followed by a discussion of the usage found in the two-volume Chateaubriand book *Logical Forms* (2001 and 2005)—the most comprehensive work on the subject ever written and in many ways the focus of this paper.

Keywords: Morphology. Character. Sense. Sentence. Proposition. Form. Structure. Schema. Alternative constituent. Type. Token.

SIGNIFICADOS DE FORMA

Resumo: As expressões 'forma', 'estrutura', 'esquema', 'formato', 'modelo', 'figura', 'molde', e termos relacionados são usados em lógica tanto como termos técnicos quanto como em metáforas. O presente artigo justapõe, diferencia e analisa o uso destas expressões por lógicos. Nunca se tentou levar a cabo um tal projeto antes. Após estabelecer uma terminologia geral, apresentamos uma variante do uso tradicional da expressão 'forma lógica', seguida de uma discussão do uso encontrado nos dois volumes do livro *Logical Forms* (2001 e 2005) de Chateaubriand - o trabalho mais abrangente já escrito sobre o assunto e, em larga medida, o foco deste artigo.

Palavras chave: Morfologia. Caráter. Sentido. Sentença. Proposição. Forma. Estrutura. Esquema. Constituinte alternativo. Tipo. *Token*.

INTRODUCTION

In several senses, form has been a central concern of logic for a long time. In 1931 Eaton wrote:

> But Aristotle *was* dealing with validating forms for inference, and he exhibited these forms in abstraction from the particular subject-matter to which they accidentally applied. The abstract or formal character of the Aristotelian logic, so admirably preserved in the Middle Ages, above all else makes it continuous with mathematical logic.

In *Logical Forms*, Chateaubriand discusses Aristotle's treatment of logical form (2001, 27, 60; 2005, 109) and he shows keen awareness of the contribution of the scholastics in the Middle Ages. He quotes an interesting but difficult passage from Albert of Saxony (1316-1390) (2005, 197):

> Of consequences, one kind is formal, another material. That is said to be a formal consequence to which every proposition which, if it were to be formed, would be a valid consequence, if similar in form, e.g., '*b* is *a*, therefore some *a* is *b*'. But a material consequence is one such that not every proposition similar in form to it is a valid consequence, or, as is commonly said, which does not hold in all terms when the form is kept the same; e.g. 'a man runs, therefore an animal runs'. But in these (other) terms the consequence is not valid: 'a man runs, therefore a log runs'.

On the same page, he quotes another interesting and problematic passage. But from the slightly earlier William of Ockham (1285-1349), who is known to have influenced Albert:

> Of consequences, one kind is formal, another material. Formal consequence is twofold, since one holds by an extrinsic medium concerning the form of the proposition, such as these rules: 'from an exclusive to a universal (proposition) with the terms interchanged is a correct consequence', 'from a necessary major and an assertoric

> minor (premiss) there follows a necessary (conclusion)', etc. The other kind holds directly through an intrinsic medium and indirectly through an extrinsic one concerning the general conditions of the proposition, not its truth, falsity, necessity or impossibility. Of this kind is the following: 'Socrates does not run, therefore some man does not run'. The consequence is called 'material' since it holds precisely in virtue of the terms, not in virtue of some extrinsic medium concerning the general conditions of the proposition. Such are the following: 'If a man runs, God exists', 'man is an ass, therefore God does not exist' etc.

Chateaubriand's own terminology sometimes resonates with that of Saxony and Ockham: "...logical implications ... are implications that hold in virtue of logical form alone" (2005, 274).

Many writers, notably, Cohen-Nagel (1934, 10f), Lewis-Langford (1932/1959, 512), and Tarski (1956, 414), use the unqualified word 'form' similarly. Church (1956, 2, 10) and others use 'logical form' in contexts similar to those in which Eaton used 'form'. It sometimes happens that a writer establishes the compound 'logical form' and then later uses the unqualified 'form' elliptically with 'logical' to be understood. On the other hand, a kind of reverse usage occurs: a writer establishes the unqualified 'form' and then later introduces the compound 'logical form' as if the unqualified uses had been elliptical with 'logical' to be understood. Some authors sharply distinguish the two expressions (Church 1956, 2, 10).

As noted above, the expressions 'form', 'structure', 'schema', 'shape', 'pattern', 'figure', 'mold', and related locutions are used in logic both as technical terms and in metaphors. All are ambiguous (multisense, or polysemic) in that each is used with multiple normal meanings (senses, or definitions). Sometimes one writer uses two or more as stylistic variants, but just as often two or more are used to mark philosophically or mathematically important distinctions. Moreover, it also happens that one writer uses one of these expressions in two or more senses. Explicit stipulative definitions are

rare. Several of their meanings are vague (imprecise, or indefinite) in that they admit of borderline (marginal, or fringe) cases. It is often difficult to determine what a given author intends by them.

One ontological question is: "what is a logical form?" Does the question presuppose that it is clear what a form is? Is it asking merely for the property in virtue of which a form is logical? Or does it regard the expression 'logical form' as a unitary common noun and not as a qualified common noun phrase? The question has a long history.

Some authors such as Quine (1970/1986, 48) do not answer the ontological question. Instead of saying what they take a logical form *to be*, they say only how something's logical form can be *represented*. Oswaldo Chateaubriand makes interesting proposals about representing forms and about saying what they are in his two-volume *Logical Forms* (2001 and 2005), probably the most thorough and comprehensive work on the subject, which can be called *logical morphology* taking *morphology* to be the general study of form in every sense. His book deserves to be regarded as a defining event in the history of logical morphology.

He pointed to a way of representing toward the end of volume II (2005, 438): "Set theory ... gives us a more concrete representation of the logical forms". He also accepts more traditional ways of representing logical forms. In volume I (2001, 29), he wrote that "the expression '*Fa*' represents the logical form" of the sentence 'Theaetetus is sitting'. He summarized his understanding of the current situation (2005, 109):

> The usual way of characterizing logical form is by specifying a grammar where certain notions are categorized as logical notions. One sense in which the formal languages of logic are formal is that their terms, formulas, and sentences are alleged to be logical forms. What is the status of these logical forms? What are they forms of, if anything? The standard (linguistic) view is that the logical forms are forms of sentences, and of other linguistic entities. Sentences, say,

> have a certain logical structure, and various logics, old and new, provide analyses of this logical structure. This structure is a syntactic feature of sentences; i.e., logical forms are syntactic structures.

This passage is no more puzzling than many other remarks on the subject. How can sentences be logical forms when logical forms are forms of sentences? Are some sentences logical forms of themselves or is every sentence a logical form of other sentences? Who characterized logical forms "in the usual way"? How did this alleged trend become established? And what satisfaction did the trend give its subscribers?

One thing that seems clear in *this* passage is that Chateaubriand is relating to the tradition in logic of using 'logical form' as a unitary expression and not as a qualification of 'form' admitting other qualifications including 'non-logical'. In other places, the opposite is the more likely interpretation.

Before we can understand any answer we must understand the question, and that requires knowing what the questioner means by 'logical form', a common noun phrase which might seem to refer to objects called logical forms. However, it might well be that an author uses the expression not as a sortal or substantive but as a part of a relational expression as in 'X is a logical form of Y' or as part of a functional expression as in 'X is the logical form of Y', where 'the logical form of' names a mapping defined on the class of things that have logical forms. Moreover, an author might reject all of the above suggestions and explain that 'logical form' is part of an expression for an equivalence relation as in 'X is in the same logical form as Y', 'X has the same logical form as Y', or 'X fits the same logical form as Y' (Cf. Chateaubriand 2001, 25). In the early 1970s, I subscribed to the last usage and, somewhat regrettably, I ridiculed those who asserted or postulated entities called logical forms (e.g., Corcoran 1974, 101-102)—a group to which I now belong.

As will be explained below, Chateaubriand (2001, 29; 2005, passim) complements his discussion of representation of logical forms by forthrightly identifying entities called logical forms with certain higher-order properties which he calls variously *logical properties*, *logical relations*, and *logical notions*. His repeated use of the special expression 'logical notion' invites comparison with Tarski's usage in the famous 1986 posthumous paper "What are logical notions?". Chateaubriand's extensive bibliographies (2001, 431-442; 2005, 479-503) do not list the Tarski paper. This is the case despite the fact that in 2004 he saw the relevance, usefulness, and importance of juxtaposing Tarski's view with his own (Chateaubriand 2004, 106).

Although most authors distinguish [one-place] properties from [multi-place] relations, some use 'property' for the broader category containing both and some use 'relation' for the broader category. Chateaubriand (2001, 72 fn. 15) used 'property' for the broader category reserving 'relation' for the multi-place cases. However, in certain contexts he seemed to construe multi-place relations as special one-place properties, e. g., taking a two-place relation *to have* an ordered pair *as an element* rather than as *relating* one thing *to* another (2001, 61, 191, 202). This strikes me as ontologically remarkable. I take it to be obvious that the identity relation—which relates each individual to itself—is not ontologically the same entity as the property that belongs to each ordered pair whose first member is identical to its second member. The 2-place *relation* that a first-order property bears to an object having it is routinely treated as if it is a higher-order 1-place *property* that belongs to pairings—of first-order properties with objects to which they belong. Of course, Chateaubriand is not the first to do something like this (Mendelson 1964/1997, 6). However, contrary to Mendelson and others, Chateaubriand does not intend an ontological reduction (personal communication). Understanding and evaluating his theories requires taking this into account.

Quite apart from the above ambiguity of 'property' and 'relation', the expressions 'logical property' and 'logical relation' have a special ambiguity. In Chateaubriand's usage, which is Frege's, they are exemplified by self-identity and identity, respectively. In another usage, they are exemplified by tautologousness and consequence, respectively, as in the 1934 classic Cohen-Nagel book, which has a section called "The Possible Logical Relations between Propositions". My own writings use the latter exclusively. When I wrote the lecture "What are logical relations?" (Corcoran 2001), I had completely forgotten the Frege-Chateaubriand usage—and no one in the several audiences that heard it reminded me.

Before dealing with 'logical form', however, it seems prudent to study uses of the unqualified expression 'form'. And as Chateaubriand (2001, 94ff) implied, it is important to review the various ways the quotation marks can be used.

TERMINOLOGY

One usual convention is to use *single quotes* for making names of sentences and other expressions, for example names of words, phrases, symbols, characters, etc. Thus, 'One plus two is three' is a string of twenty-one character occurrences that is a five-word English sentence and 'One two three is plus' is a string of twenty-one character occurrences that is not an English sentence; 'square' is a string of six letter occurrences that is an English word and 'sqruae' is a string of six letter occurrences that is not an English word. Both the sentence and the word were used by Boole, but neither would have been recognized by Aristotle. The single *string* 'word' has six occurrences in this paragraph and it has twelve imprintings in two printings of the paragraph—by *string* is meant a string type, not a string occurrence and not a string token (Corcoran

2004 and 2006). Some authors refer to a string of characters as a 'syntactic string' (Chateaubriand 2001, 379, 397; 113).

Following Bertrand Russell (1903, 53ff. and 1905/1967, 99), Carnap (1934/1937, 14 and 1954/1958, 7), and others, *double quotes* are used in naming propositions and other meanings. Thus, "One plus two is three" is a true proposition known both to Boole and to Aristotle and "square" is a concept also well known to both. The following two lines are from Carnap (1934/1937, 14) and the next two are from his (1954/1958, 7).

'te(3) = 5' means: "the temperature at the position 3 is 5"

'sum(3,4)' has the meaning: "3 plus 4"

'[*Stud*(*a*)] V [*Fl*(*a*)]' means "*a* is either a student or a female, or both"

'[*Stud*(*a*)] . [*Fl*(*a*)]' means "*a* is a female student"

In familiar cases, under an interpretation, certain expressions express meanings or senses and they name entities or things. Thus, under an intended interpretation, the sentence 'One plus two is three' expresses the proposition "One plus two is three" and the number-word 'three' names the number three. Chateaubriand (2005, 202) quotes Kleene (1967, 4): "We deal with propositions through declarative sentences which express them in some language (the object language); the propositions are the meanings of the sentences". He treats propositions as senses (2001, 377-378 and 420) as does Church in the well-known 1956 paper "Propositions and Sentences".

Just as the sentence 'One plus two is three' contains occurrences of the five words 'one', 'plus', 'two', 'is', and 'three', the proposition "One plus two is three" may be considered to contain occurrences of the five concepts "one", "plus", "two", "is", and "three". Of course in general, the number of words or symbol

occurrences in a sentence is not a reliable measure of the number of concepts in a proposition it expresses.

It is usual to drop quotation marks, whether single or double, in displayed material. Thus, typographically there is no difference between referring to the five-word sentence

one plus two is three

and referring to the following true proposition it expresses under one of its interpretations.

one plus two is three

Ideally, the context should be clear. In the first case above, the displayed sentence is explicitly preceded by 'sentence'. In the second case, the displayed proposition is explicitly preceded by 'proposition'. With single or double quote names there is no need to be explicit once the convention is established.

It is also usual to drop quotation marks when quoting or discussing an author who uses expressions *autonymously*—to name themselves (Church 1956, 61). For example, in his terminology (Church 1956, 10), 0 is a value of the form $(n + n)$. Here '0' is used in the *ordinary* way, or *heteronymously*, to refer to the number, but '$(n + n)$' is being used autonymously to refer to a five-character string, viz. '$(n + n)$'. In Tarski's terminology (1941/1994, 7), the commutative law for addition may be given in the following form: $x + y = y + x$. Chateaubriand's book uses the Church terminology in Chapter 6 (2001, 189, 199) and Chapter 18 (2005, 267-273) but contains few other occurrences of strings used autonymously (e.g., 2005, 150).

Another notational issue is whether a sentence begins with a capital and ends with a period. It is clear that if a sentence is to be

part of its own negation, then the sentence cannot routinely begin with a capital. The negation of the above sentence is the following.

it is not the case that one plus two is three.

Likewise, it is clear that if a sentence is to be part of any conditional of which it is the antecedent, then the sentence cannot routinely end with a period. The sentence devoid of initial capitalization and final period has been called the *clausal form* of a sentence to distinguish it from its *assertoric form* (Corcoran 2006). Unfortunately, getting accustomed to this nicety takes some time. Accordingly, it is not uniformly observed in this essay.

In the literature, single and double quotes are used as above and in other ways as well. One notable usage, called *cautionary* quotes, puts quotes, single or double, around a word to indicate an unusual, non-literal, improper, or problematic sense as in Chateaubriand (2005, 266). There are proofs and then there are "proofs". There are "proofs" of false propositions. One of Frege's "laws" was found to be inconsistent. Some propositions "known" to be true for years are later found to be false.

In another notable usage, single quotes are used—not for purely *syntactic* or *string-theoretic* objects exhaustively described by specifying the characters and their order—but for what Church (1956(2), 6) calls *composite* entities. Thus, 'square', with single-quotes, would be used not for the syntactic object, a string of six letter occurrences, but for a composite object composed of the string taken under one of its interpretations or meanings. It is only with respect to the composite usage that the following is a complete sentence.

the number one is denoted by the word 'one'.

Were the quotes name of the three-letter word taken purely syntactically the above would have to be regarded as incoherently incomplete (like the infamous 'one plus two equals') or as elliptical for something like the following.

> The number one is denoted by the word 'one' under
> its arithmetic interpretation.

The string 'one' does not denote anything except under an interpretation. The single-quote notation is thus ambiguous: on one hand, it is used to denote strings of characters and, on the other, to denote meaningful expressions, or composite entities. Chateaubriand noted that Frege referred to such entities as signs. He wrote "… for Frege a sign is not a purely syntactic thing, but something that already contains a sense …" (Chateaubriand 2001, 379). Although neither Church nor Chateaubriand seem to notice, Tarski uses quotes to name composites in his truth-definition paper starting with the second sentence (1956/1983; 152, 155).

In order to eliminate one source of confusion, slashes (virgules) can be used to refer to composite entities. The composite /one/ is a two-part system composed of the string 'one' and the sense "one". The number one is denoted by the word /one/ and determined by the sense "one". The composite /one plus two is three/ is "a composite entity, sentence plus (abstract) proposition" (Church 1956(2), 6), that expresses or *contains* the proposition that one plus two is three. As Chateaubriand (2001, 379) points out, composites such as /one/ give rise to what has been called a *semiotic triangle* whose three vertices are respectively a string, a sense, and a referent and whose sides are respectively the relations of string-to-sense, string-to-referent, and sense-to-referent. He says that a composite expression "contains" its sense.

I used the word /form/ autonymously in the title of this article, /Meanings of Form/. Incidentally, the word /autonymous/ coined by Rudolf Carnap should not be confused with the phonetically similar and older /autonomous/ which is not etymologically related (Chateaubriand 2005, 150).The slash notation cannot be used unless it is clear in the context which of the various interpretations is intended as in:

> the number one is denoted by the word /one/.

The sentence below admits of several standard interpretations.

> every number is not positive

In the first place, the six-letter word 'number' has been taken to refer to the natural numbers beginning with zero, or to the natural numbers beginning with one, or to the integers, or to the rationals, or to the reals, or to the complex numbers—to mention the most common interpretations. Several senses have been attached to 'positive'. In all examples used in this paper, 'number' is taken in a sense coextensive with "natural number beginning with zero" and 'positive' is taken in the usual way. Thus, the *lexical* ambiguity has been treated. But even with these stipulations the sentence in question is still ambiguous—not in regard to its lexical items but in regard to structure. Under one interpretation, it expresses a false universal proposition logically equivalent to the following:

> every number x is such that x is not positive
>
> every number is non-positive
>
> no number is positive

Zero is the only number *not* a counterexample. Using standard terminology, under this interpretation 'not' has *narrow scope* (Audi 1995/1999, 822).

Under another interpretation, it expresses a true negative proposition logically equivalent to the following:

not every number is positive

it is not the case that every number is positive

some number is not positive

Using standard terminology, under this interpretation 'not' has *wide* or *broad scope*. The slash notation can be used where there is no ambiguity whether lexical or structural. The (composite) sentence /not every number is positive/ is true and /no number is positive/ is false.

To take a less technical and perhaps more familiar type of example, consider the (string) sentence 'Abe does not believe Ben died'. If this sentence is interpreted as expressing the proposition that Abe believes that it is not the case that Ben died, the scope of the occurrence of 'not' is 'Ben died'; interpreted as "It is not the case that Abe believes that Ben died", the scope is the rest of the sentence, i.e. 'Abe believes Ben died'. In the first case we have narrow scope, in the second wide scope. The slash notation cannot be used with the structurally ambiguous sentence 'Abe does not believe Ben died'. But, where suitable meanings have been stipulated for the two proper names, we can say that /Abe believes that it is not the case that Ben died/ neither implies nor is implied by /it is not the case that Abe believes that Ben died/.

The writings of logicians are not immune to structural ambiguity. The following is taken from the Eaton passage cited above.

Aristotle was dealing with validating forms for inference

We can read this as being about our activity of validating forms of inference or as being about forms that validate our inference.

To be precise, it is important to distinguish two similar figure/ground phenomena found in logic. *Structural ambiguity* applies only to uninterpreted sentences and belongs only to some of them. No composite sentence is structurally ambiguous; in fact, no composite sentence is ambiguous. The property of being *reconceptualizable* applies to propositions and belongs to every one without exception. A sentence is *structurally ambiguous* if it can be used to express two or more propositions having different logical forms. Roughly, a proposition is *reconceptualizable* if it is the conclusion or a premise in deductions of different forms. More detail on reconceptualizations is found below in the section 'Conceptualization'. The two properties—being structurally ambiguous and being reconceptualizable—have different and, in fact, disjoint ranges of applicability; thus, they have different and disjoint extensions (ranges of exemplification).

Dictionaries tend to name words by using the word in italics or in capitals. The ninth edition of a popular dictionary uses capitals, but its tenth edition uses italics. Of course, italics are often used for emphasis and to indicate non-literal usage. In the passage below, Whitehead (1911/1948, 8) used italics first for variables, then for emphasis and, in the second sentence, to indicate non-literal usage: "Since $x + 2 = 2 + x$ for any number x, it is true for *some* number x. Thus, as used here, *any* implies *some* and *some* does not exclude *any*." Some works such as Cohen and Nagel (1934/1962/1993) do not

have a uniform notational scheme. Even here in this article double quotes and italics are used in various ways, but single-quotes are uniformly used for naming strings.

ALTERNATIVE CONSTITUENT FORMAT

In order to display the data of English in a succinct way that permits ready grasp of the subtle contrasts, this paper introduces the *alternative constituent format*: the place of one grammatical constituent of a sentence is supplanted by a sequence of alternatives. Let us start with an ambiguous question.

Is two closer to three than four?

Consider an initial sentence given as an affirmative answer.

Two is closer to three than four.

We can compare this with a negative answer by replacing the constituent 'is' with the choice between 'is' and 'is not'.

Two (is * is not) closer to three than four.

We can also compare the initial ambiguous sentence with two other unambiguous sentences that give its different *readings* or *interpretations*. The following replaces the constituent 'four' with the choice among three alternatives: 'four', 'to four', and 'four is'. The asterisk is used to separate the choices.

Two is closer to three than (four * to four * four is).

Here choosing the first alternative yields the initial, elliptically ambiguous sentence; the second and third alternatives give two of its readings (the first true, the second false). In this example, three

sentences have been presented in one *alternative constituent string* or *ACS*. Of course, two constituents can vary independently.

 (Zero * One* Two) is closer to three than (four * to four * four is).

Here, three times three or nine sentences are presented in one ACS: the three choosing 'four' are ambiguous; the three with 'to four' are true; and the three with 'four is' are false. Any number of constituents can be varied.   An ACS that varies two or more constituents is called a *sequential* ACS because each sentence it yields is the result of the choice of a sequence of simultaneous choices.

        Moreover, there are cases where a constituent is "zeroed" as in the following.

              Two is (* not) closer to three than four.

              Two is closer to three than (* to * is) four.

The last ACS presents three sentences—two the same presented in the second example. In its first alternative sentence the alternative constituent is the null string—in the second it is an occurrence of a preposition and in the third a verb. Note that the second and third do not restore the entire ellipsis that gave rise to the ambiguity of the first. Using the standard bracket notation for reporting a reading restoring an ellipsis, we have:

              Two is closer to three than [it is to] four

              Two is closer to three than four [is to it].

The two fully restored, unelliptical sentences can be treated more fully with the following ACS.

       Two is closer to three than (* it is to) four (* is to it).

This ACS represents the following four strings. The first, a sentence, is obtained by choosing the two blanks—spaces, or null strings. The second, also a sentence, is obtained by choosing the first blank and the second non-null string. The third, also a sentence, is obtained by choosing the first non-null string and the second blank. The fourth, which is not a sentence, is obtained by choosing the two non-null strings.

<div align="center">

Two is closer to three than four.

Two is closer to three than four is to it.

Two is closer to three than it is to four.

Two is closer to three than it is to four is to it.

</div>

The fourth alternative string is similar to many other deviant strings encountered in actual writing: it contains both of two separately acceptable alternatives that cannot both be taken together. Consider the following.

<div align="center">

It is not known whether (* or not) Goldbach's Hypothesis
is true (* or not).

</div>

Some of the above examples illustrate the fact that when two constituents are simultaneously varied, a choice of an alternative for one of the constituents can limit the range of grammatically or coherently acceptable alternatives for the other choice. More generally, there are cases in which something can be done by either of two methods; but once one method has been begun it is impossible to switch to the other without making a hash. For example, the first edition of Quine (1970/1986) started defining truth using one notion of satisfaction and then inadvertently

switched to another notion thereby garbling his treatment (Corcoran 1972, 97-99). The mess is corrected in the second edition.

(Numerically * Alphabetically * Temporally), (1 * AA * lightning)
precedes (2 * AB * thunder)

Sequential alternative constituent strings are ideal for detecting and exhibiting what linguists call *selection restrictions* and what philosophers call *incoherencies* or *category mistakes*.
The initial sentence sometimes contains parentheses.

0 is a number in the form $(n + n)$

In such cases, it often proves convenient to use a different form of bracket for the alternative constituents as in the following two examples whose alternative sentences are directly relevant to issues concerning form and logical form.

0 is a number [* that] [* is * has][* of * in * an instance of]

the [form * structure * schema] $(n + n)$

This ACS yields 72 alternative strings that can be "generated" from the initial sentence '0 is a number in the form $(n + n)$'. Before confusion can arise, I want to say that when I use this sentence I take it as an idiomatic way of saying an ordinary, one-sorted first-order proposition about numbers , roughly, that $0 = (n + n)$ for some number n. I would not use it to say something about the form $(n + n)$. However, following Church (1956, 10), I would not hesitate to say that 0 is a number that is a value of the form $(n + n)$, for some value of the variable n.

A conjunction is a proposition [* that] [* is * has] [* of * in * an
instance of]

the [pattern* form * structure * schema] (*P* & *Q*)

Of course, this ACS yields 96 alternative strings that can be generated from a corresponding initial sentence 'A conjunction is a proposition in the pattern (*P* & *Q*)'.

However, in this case I use the sentence as a fairly literal way of saying something not expressible in an ordinary one-sorted first-order language—a two-sorted proposition about propositions and propositional patterns. / (*P* & *Q*)/is a name of the pattern that every conjunction has. (*P* & *Q*) is a propositional pattern. Although this does not follow Church (1956, 10), he could have consistently made this point (1956, 2-3). See Boger 2004, Corcoran 2007, and Cohen-Nagel (1934/1962/1993, xxxi-xxxvii).

The alternative constituent format can also be used to juxtapose philosophical views.

According to (Chateaubriand * Quine), the logical (forms * structures) are (forms * structures) of linguistic (types * tokens *acts).

According to (Chateaubriand * Quine), (sentences * propositions * statements * assertions)   (have * are instances of * are tokens of) (grammatical * logical) (forms * structures).

According to (Chateaubriand * Quine), the logical (forms * structures) are (properties * relations * attributes) of (sentences * propositions * statements * assertions).

CONCEPTUALIZATION

There are several ways of *conceptualizing* a proposition, or a composite sentence containing a proposition, depending on how we are using it. For example, consider the following simple proposition.

zero is even.

Loosely speaking, it can be conceptualized as being primarily about zero or as being primarily about the property of evenness depending on what question it is taken to answer, or what it is being deduced from, or what is being deduced from it. Conceptualizing it as being primarily about zero is relevant if we are deducing any of the following from it.

some number is even

zero is both even and square if zero is square

the first number is even if zero is the first number.

This is ordinary one-sorted logic taking the universe of discourse to be the natural numbers. Thus, /for some x, x is even/ contains "Some number is even", where 'number' is the *range-indicator* for every bound variable (Corcoran 1999 and 2008; Church 1956, 318 fn.). Moreover, as Quine (1970/1986, 25) notes "zero is even" is logically equivalent to "zero is a number that is even", or more awkwardly "there exists a number that is zero and is even". Conceptualizing "zero is even" as being primarily about evenness is relevant if we are deducing any of the following second-order propositions from it.

being even belongs to zero

zero shares a property with two if two is even

the first number having the property of being even is zero if zero is the first number.

Roughly speaking, the more complicated the proposition involved, the more evident the importance of conceptualization—and the more different conceptualizations are possible. Consider the following.

one plus two is three.

From this any of the following can be deduced.

> one is a number such that it plus two is three
>
> two is a number such that one plus it is three
>
> three is a number such that one plus two is it
>
> there is a number such that it plus two is three
>
> there is a number such that one plus it is three
>
> there is a number such that one plus two is it.

An even more revealing example of the need for conceptualizing, indeed reconceptualizing, concerns deducing the first-order conclusion "Every number is either zero or the successor of a number" from the single second-order premise "Every number has every property that belongs to zero and to the successor of every number it belongs to". There are of course several ways of proceeding; only one is treated here. The first step is to see the conclusion as saying of every number that it has a certain complex property. This prepares the way to noticing that it is logically equivalent to the proposition that every number has the property of being either zero or the successor of a number. My next step was to consider the instance "zero has the property of being either zero or the successor of a number" in relation to "zero is either zero or the successor of a number", conceptualized as a disjunction having as a disjunct the tautology "zero is zero". Thus the *basis* step in an application of mathematical induction, the premise, is tautological. By a series of similar reconceptualizations it can be seen that the *induction* step is also tautological. By the induction step is meant "Every number that is a successor of a number that has the property

of being either zero or the successor of a number has the property of being either zero or the successor of a number". My point is that my deduction required several reconceptualizations.

As might be almost evident, this deduction originated as a formalized natural deduction using the language of Gödel 1931. Since this language is logically perfect, none of its sentences is ambiguous, whether structurally or lexically. Structural ambiguity has no role in constructing deductions even if it might play an accidental role in devising ways to express deductions in English. Although conceptualization of a proposition and interpreting a structurally ambiguous sentence are similar figure/ground activities, they are fundamentally different and should not be confused. Despite the fact that I discussed both with my classes over the years, I never had occasion to compare them and I never realized how easy it was to confuse them.

Of course, it is not necessary to use the expression 'structurally ambiguous' in my sense. The expression is found only once in the two Chateaubriand volumes. On page 63 of 2001, it is used in connection with the following sentence:

(9) Sherlock Holmes is not a dentist.

He juxtaposed it with:

(10) It is not the case that Sherlock Holmes is a dentist.

Since the interpreted sentence /Sherlock Holmes is not a dentist/ contains the proposition it expresses, it is not ambiguous. Thus, he seemed to regard the first sentence as the uninterpreted 'Sherlock Holmes is not a dentist' and as admitting of two interpretations expressing propositions of different logical forms. In one the

negation has narrow scope, perhaps as in the following interpreted sentence.

(9c) Sherlock Holmes is a non-dentist.

And in the other the negation has broad scope, perhaps as in (10) above or (10c) below, both interpreted as negations of /Sherlock Holmes is a dentist/.

(10c) That Sherlock Holmes is a dentist is not the case.

These passages foreshadow Chateaubriand's discussion of the structural ambiguity of a certain sentence /–*Ga* / in an unusual interpreted formal language apparently constructed for the purpose (2001, 98-99). In this language, the character '–' *is* like the word 'one' in English: it is lexically ambiguous and its occurrences give rise to structurally ambiguous sentences. On one hand it is a predicate operator like the adjectival prefix 'non-' as in 'non-tautologous', and on the other it is a propositional operator like the sentential prefix 'it is not the case that-'. Thus, one and the same sentence is used to express two propositions having different logical forms: one a negative proposition with a positive predicate, the other a predicate-subject proposition with a negative predicate. With proper use of parentheses, this particular structural ambiguity can be avoided: /–(*Ga*)/ for the negative proposition, /(–*G*)*a*/ for the predicate-subject proposition. However, as Landini (2008) points out in his review, Chateaubriand's formulations reintroduce structural ambiguity in a different place. (Notice that a formal language need not be logically perfect.) Again in this case Chateaubriand uses the peculiar and unprecedented terminology: "...there are two interpretations of its logical structure". Others including me would say that two interpretations of the structurally ambiguous sentence '–*Ga*' are propositions having distinct logical structures.

Chateaubriand might have been discussing structural ambiguity of uninterpreted sentences and reconceptualizations of propositions expressed by interpreted sentences without recognizing the difference. I will discuss this further below.

BUFFALO USAGE

Since the 1980s, roughly speaking, by *logical form* I have referred to the form obtained from a proposition, a set of propositions, an argument, an argumentation, or even an entire deductive theory by abstracting from the subject-matter of its content terms or by regarding the content terms as mere place-holders or blanks in a form. My intention was to refine and broaden traditional terminology as represented in Cohen and Nagel (1934/1962/1993, 10ff, 47, 70) and as later discussed in the post-war philosophical literature. Two typical articles are Kalish's 1952 paper "Logical Form" and Whiteley's 1951 "The Idea of Logical Form". In a *logically perfect language* there is no ellipsis, no ambiguity, and no incoherence or category mistakes (cf. Chateaubriand 2001, 204). In short, the logical form of a proposition, a set of propositions, an argument, or an argumentation is determined by the grammatical form of the sentence, the set of sentences or the argument-text expressing it. In this connection we should recall what Church wrote (1956, 3): "To adopt a particular formalized language thus involves adopting a particular theory or system of logical analysis". Chateaubriand makes similar observations; for example, he wrote (2005, 109): "Sentences, say, have a certain logical structure, and various logics, old and new, provide analyses of this logical structure". He is even more explicit, but less succinct, a little later on page 115.

Two *texts*, let us say, are said to have *the same grammatical form*, in this sense, if a uniform one-one substitution of content words transforms the one exactly into the other. The sentence 'Abe

properly respects every agent who respects himself' may be regarded as having the same grammatical form as the sentence 'Ben generously assists every patient who assists himself'. Substitutions used to determine sameness of grammatical form cannot involve change of *form words* such as 'every', 'no', 'some', 'is', 'that', etc. and they must be *category-preserving*, i.e., they must put a proper name for a proper name, an adverb for an adverb, a transitive verb for a transitive verb, and so on. Two texts, e.g., sentences, having the same grammatical form have exactly the same form words distributed in exactly the same pattern and, although they of course need not have, and usually do not have, the same content words, they do have exactly the same number of different content words. The most distinctive feature of form words, which are also called *syncategorematic* terms or *logical* terms, is their topic neutrality; the form words in a sentence are entirely independent of and are in no way indicative of its content or topic. No two of the following sentences have the same grammatical form.

> Every number is odd.
>
> Every prime number is odd.
>
> Every number that is prime is odd.
>
> Every prime number exceeding two is odd.
>
> Every number that is prime is odd except two.
>
> Every number is odd or precedes three.
>
> Every prime number is odd or precedes three.

Modern formal languages used in formal axiomatization of mathematical sciences are often taken as examples of logically perfect languages. Pioneering work on logically perfect languages was done by George Boole (1815-1864), Gottlob Frege (1848-1925),

Giuseppe Peano (1858-1952), Bertrand Russell (1872-1970) and Alonzo Church (1903-1995). According to the *principle of logical form for arguments*, an argument is [logically or formally] valid or invalid "in virtue of logical form" (Church 1956, 2). More explicitly, every two arguments in the same form are both valid or both invalid. Thus, every argument in the same form as a valid argument is valid and every argument in the same form as an invalid argument is invalid.

The argument form that a given argument fits (or has) is not determined solely by the logical forms of its constituent propositions; the arrangement of those propositions is critical because the process of interchanging a premise with the conclusion of a valid argument can result in an invalid argument. In the following two arguments, the two premise sets have the same form and the two conclusions have the same form. But only one argument is valid.

Every rectangle is a polygon.   Every rectangle is a polygon.

Every square is a rectangle.   Every square is a rectangle.

Every square is a polygon.   Every cube is a polyhedron.

It is extremely arduous to express something equivalent to the principle of form without referring to a complex composed of premise set and conclusion as the entity whose form is relevant. This is true even in case one limits oneself to the single-premise case. Consider the following:

Every square is a rectangle.   Every square is a rectangle.

Every square is a rectangle.   Every cube is a polyhedron.

The following propositions are false: (1) if one proposition implies another, then every proposition in the same form as the first implies every proposition in the same form as the second; (2) propositions

imply each other in virtue of their forms. The formulations in many books admit of being misunderstood as stating or implying the falsehoods (Lewis-Langford 1932/1959, 340; Cohen-Nagel 1934, 52; Kneale-Kneale 1962/1988, 294; Tarski 1956/1983, 414; Quine 1970/1986, 48; Audi 1995, "logical form"). It is not *the forms* of the two propositions that matters; what matters is *the form* of an ordered pairing of the one with the other.

Of course, the principle of logical form, from which formal logic gets its name, does not apply to non-formal conceptions of validity such as material, enthymematic, or analytic validity. In this formal sense of 'valid', an argument is valid if and only if the conclusion is a formal consequence of the premise set. Hence, the above formulation of it cannot be accepted by those who use 'valid' in a broader sense (Chateaubriand 2005, 197; Lewis and Langford 1932/1959, 340). In this formal sense, the range of applicability of validity is the class of premise-conclusion arguments. Chateaubriand (2005, Chapters 13, 14, 15, and 19) uses the word 'valid' in connection with composite sentences, with propositions, and with rules of inference.

The principle of logical form is commonly used in establishing invalidity of arguments and consistency of sets of propositions. In order to show that a given argument is invalid it is sufficient to exhibit another argument as being in the same logical form and as having all true premises and false conclusion. The two arguments below are in the same logical form. The one on the right has all true premises and false conclusion and is thus invalid. The argument on the left is therefore also invalid.

Every rectangle is a polygon.	Every rectangle is a polygon.
<u>Every square is a rectangle.</u>	<u>Every square is a rectangle.</u>
Every cube is a polyhedron.	Every triangle is a circle.

In order to show that a given set of propositions is consistent it is sufficient to exhibit another set of propositions as being in the same logical form and as being composed exclusively of true propositions. The two sets below are in the same logical form. The one on the right has all true propositions and is thus consistent. The one on the left is therefore also consistent.

Every rectangle is a polygon.	Every rectangle is a polygon.
Every square is a rectangle.	Every square is a rectangle.
Every triangle is a circle.	Every cube is a polyhedron.

The history of these methods traces back through non-cantorian set theory, noneuclidian geometry, and medieval logic (especially Saint Anselm) to Aristotle. These methods must be used with extreme caution in languages such as English which fail to be logically perfect as a result of ellipsis, amphiboly, ambiguity, etc. For example, "This is a male dog" implies "This is a dog" but "This is a brass monkey" does not imply "This is a monkey", as would be required in a logically perfect language. Likewise, of two propositions commonly expressed by the ambiguous sentence 'Ann and Ben are married' one does and one does not imply the proposition that Ann is married to Ben.

Modern logic has seen applications of logical form unprecedented in traditional logic. There are many formal properties of sets of propositions: categoricity, countable satisfiability, independence, completeness, and consistency—to name a few. In each case, there are principles of logical form. For example, according to one *principle of logical form for proposition sets*, a proposition set is categorical or non-categorical "in virtue of logical form" (Corcoran 1980). More explicitly, every two proposition sets in the same form are both categorical or both noncategorical. Thus,

every set in the same form as a categorical set is categorical and every argument in the same form as a noncategorical set is noncategorical.

Quine and other logicians are careful to distinguish, in effect, the unique logical form of a proposition (or of a composite sentence) from its multiple schematic forms or patterns. What I call *the* logical form of a proposition corresponds to what Quine calls *the logical structure* of a composite sentence containing it. He said (Quine 1970/1986, 49): "I have explained ...what I mean by the logical structure of a sentence...." Quine is careful about the distinction between the definite and the indefinite articles. He also hints at another sense of 'logical structure' in which a sentence is not said to have *a unique logical structure* but to have *logical structure*—much like a person can be said to have *a personality* and, in a different sense, to have *personality*. He wrote (loc. cit.): "... it can happen that one sentence has the entire logical structure of another and yet not vice versa". Chateaubriand (2001, 2005) uses 'logical structure' in analogous dual senses; he uses 'logical form' interchangeably with 'logical structure'.

In my terminology, the proposition (A) "if Abe is Ben, then if Ben is wise Abe is wise" has exactly one logical form, which it shares with (B) "if Carl is Dan, then if Dan is kind Carl is kind" whereas it has all of the following schematic forms: (1) if P, then if Q then R; (2) if P then Q; (3) P. The principle of form for propositions is that every two propositions in the same logical form are both tautological (logically necessary) or both non-tautological, using the word 'tautological' in the broad sense used by many logicians including Gödel and Chateaubriand (2001, 100)—not in the narrow sense of "truth-functional tautology". Thus, although propositions A and B are tautological, there are non-tautological propositions that fit the three patterns or schematic forms just mentioned.

Failure to distinguish logical form from schematic form has led to fallacies. According to the principle of logical form quoted

above, every argument in the same logical form as an invalid argument is invalid, but it is not the case that every argument sharing a schematic form with an invalid argument is invalid. Contrary to what would be fallaciously thought, the conclusion "Abe is Ben" is logically implied by the following two propositions taken together, "if Abe is Ben then Ben is Abe" and "Ben is Abe"—since it is implied by "Ben is Abe" alone—even though the argument shares a schematic form with invalid arguments "committing" the fallacy of affirming the consequent.

The above is my version of the Buffalo usage which was developed in Buffalo, New York in the late 1970s and early 1980s through the efforts of several colleagues, students, and former students largely but not exclusively from the University of Buffalo. It continues to evolve, and there are variants in use by other members of the group.

LOGICAL FORM: CHATEAUBRIAND

Early on it becomes clear that Chateaubriand's usage is not like any discussed above. At first he seems to distinguish the logical structure(s) of a sentence from its logical form(s) (I. 63) : "as the logical structure of the sentences gets more complex there are more and more different interpretations (or alternative readings) of its logical form [sic]". As far as I know expressions such as 'different interpretations of a sentence's logical form' had not occurred previously in the literature, and Chateaubriand does not explain it. From the examples, we can infer that an interpretation or reading of a logical form of a sentence is another sentence. But sometimes it seems that 'logical form' and 'logical structure' are mere stylistic variants. Let us consider an example.

According to him (I, 63), "At a certain level of analysis there are nine different interpretations of the logical structure [sic] of (2) [the following meaningful or composite sentence]".

<div align="center">Frege taught Carnap</div>

He uses the standard /Rab/ for (2) and then he gives four of his nine interpretations as follows (the numbering is his):

<div align="center">

(2) *Rab*

(11) [*Rxy*] (*a*, *b*),

(12) [*Rxb*](*a*),

(13) [*Rax*](*b*),

(14) [*Zab*]([*Rxy*]).

</div>

Chateaubriand explains his formulas as follows (2001, 63): "The notation should be read as follows. In each case we have a predicate of a certain logical type—the part within square brackets on the left—followed by an appropriate number of arguments that are listed within parentheses on the right. Lower case letters such as '*x*' and '*y*' are variables over objects, and capital letters such as '*Z*' are second order variables over relations among objects and over sets of objects." Thus, we have translations into forced English, each preceded by an attempt by me to indicate closely related reconceptualizations.

<div align="center">(2e) Frege taught Carnap</div>

Frege bears the having-taught relation to Carnap

(11e)   Frege *taught* Carnap

The having-taught relation relates Frege to Carnap

(12e)   *Frege* taught-Carnap

Being someone who taught Carnap belongs to *Frege*

(13e)   Frege-taught *Carnap*

Being someone Frege taught belongs to *Carnap*

(14e)   Frege *taught* Carnap

Being a relation of Frege to Carnap belongs to the having-taught relation.

I am grateful to Chateaubriand for confirming that e-formulations express the distinctions that he was making.

Chateaubriand goes beyond treating "interpretations of logical structure" of composite sentences to identifying logical forms. In 2005 using capitalization conventions established on page 27 of 2001, he wrote on pages 131 and 132:

> One should look, therefore, for a notion of logical form that is independent of notational systems, but that can be correlated with notational forms. In my view, the logical forms are just the logical Forms; i.e., the logical properties. For example, Application of type $<<2,2>,<<1,1>,0>>$ is the logical form that we usually represent as

(13) *Fx*.

And similarly for the other Application properties of all types. Existence of type $<<2,1>,<1,1>>$ is the logical form that we represent as

(14) $\exists x Fx$,

and Subordination of type $<<2,2>,<<1,1>,<1,1>>>$ is the logical form that we represent as

(15) $\forall x\ (Fx \rightarrow Gx)$.

My suggestion, therefore, is that the logical forms are the logical properties, and that we can, and do, use various systems of notation to represent these logical forms. It does not follow that all logical forms can be represented in any one system of notation, nor that given a system of notation there is always a logical form corresponding to what we think is the representation of a logical form. This has to be argued for, or established in some way.

On page 133, he wrote: "Logic is largely a theory of these abstract properties. We can see this in Aristotle, we can see it in Frege, and we can see it in contemporary logic." All of this harks back to Volume I (2001, 29) where he wrote the following:

> Take the sentence 'Theaetetus is sitting'.... Its logical form is represented by, say, '*Fa*'. ... it is not the expression '*Fa*' that can be considered to be a logical form, but the logical property Application (or Instantiation) of a certain type; this property is expressed linguistically by the juxtaposition of a property letter '*F*' to an object letter '*a*'... the expression '*Fa*' represents the logical form, but this is not a purely syntactic feature of it. To analyze the sentence 'Theaetetus is sitting' as having the logical form '*Fa*' is to analyze it as expressing the instantiation of a property by an object, with the specific association of the property 'sitting' to '*F*' and the object Theaetetus to '*a*'—where '*F*' and '*a*' are thought of as non-logical constants. We may also say that we are analyzing the form of the sentence 'Theaetetus is sitting' as corresponding to a certain type of state of affairs consisting of the instantiation of a property in an object.

This raises the question of the *range of applicability* (or *significance*) of the properties Chateaubriand calls logical forms. The range of applicability of a property is the class of things it *applies* to—is *coherently* (or *meaningfully*) predicable of. The class of numbers is the range of applicability of the property of being prime, in at least one of the senses of 'prime'. The range of applicability of a property is to be distinguished from its *range of belonging* or *extension*, which is the class of things it *belongs* to—is *truly* predicable of. It is incoherent to say that truth is prime; it is coherent but false to say that zero is prime. Chateaubriand (2001, Chs. 1 and 10) uses 'applies to' precisely in the sense of "belongs to" reserving 'belongs to' for "is a member of". The expression 'property of' followed by a plural substantive is ambiguous: in one sense a property of numbers is one having the numbers as its extension and in another sense it is one having the numbers as its range of applicability. Consider the ambiguous sentence: 'Being true is a property of tautologies'. The relevant contrasts can be brought out with the following alternative constituent strings:

Being (true * tautological) is a property of (propositions * tautologies).

Being true is a property whose (extension * range of applicability) is the class of propositions.

Being tautological is a property whose (extension * range of applicability) is the class of tautologies.

Being true is a property whose (extension * range of applicability) is the class of (propositions * truths).

There is nothing radical about the view that logical forms are properties of propositions in the sense that the class of propositions

is their common range of applicability. We speak of a proposition as having a logical form and as having a property, so it is natural to take logical forms as properties—conventionally ignoring the difference between, on one hand, the property of being in a certain logical form and, on the other, the logical form itself. I do not recommend this, but we have all seen worse "abuses of language". However, this is far from Chateaubriand's viewpoint. Yes, he takes logical forms to be properties, but *he does not take them to be properties of propositions*. What does he take to be the range of applicability of the properties he calls logical forms? What are the logical forms properties of? What is the logical form represented *Fx* (or *Fa*) a property of? What is its range of applicability? What are some examples that have this property or that this property belongs to?

Let us take the proposition that Socrates is wise. According to Chateaubriand's thesis the logical form of this proposition applies to pairings of properties (like being wise) with things (like Socrates). The range of applicability of this form/property is the class of such pairings. It applies truly (and thus coherently) to the pairing of being wise with Socrates and to the pairing of being Greek with Euclid. It applies falsely (and thus coherently) to the pairing of being wise with Spitzer and to the pairing of being Greek with Oswaldo. It does not apply coherently to the pairing of Socrates with Euclid. It would be fair to say that according to Chateaubriand the logical form of the proposition that Socrates is wise is the higher-order relation R that relates a first-order property P to an individual i if and only if P belongs to i. What he would say is a little different: the logical form of the sentence /Socrates is wise/ is the higher-order property R that belongs to the pairing of a first-order property P with an individual i if and only if P belongs to i.

OTHER USAGES

Besides the usage in the previous section, there are other passages in which Chateaubriand uses 'form'. Already in Chapter 1, there are at least four. On page 50 in a discussion of Plato's theory of truth, he wrote of "the form Sitting" and "the form Flying". On page 52, we read: "Frege's idea that truth is a form of denotation, which is a form of identification, is a natural interpretation of the notion of correspondence if one thinks of statements as descriptions—as Frege does...". On page 56, he mentions "the structure (or logical *form*) of the statement". On page 59, he referred to a sentence of the *form* '*a* is *F*', thereby presupposing that the string is a form. On page 65, he discussed "treating quantification as a form of predication". On page 73, he says that his notation is a *form* of abstraction notation. In addition, the same Chapter contains occurrences of 'structure': on page 50 (*structure* of statements), 51 (*structure* of negative facts), 56 (*structure* or logical form of statements), 57 (*structure* of facts or states of affairs), 59 (the *structure* of reality), 63 (logical *structure* of sentences). After distinguishing two uses of the definite article 'the' on pages 105-106 of Chapter 3, he refers on page 395 of Chapter 11 to the definite article in the form 'is the'—without saying how a string could be a form that has in it an article. In his discussion of Brouwer on the last page of Chapter 12, he wrote: "What mind does later is to grasp the logical in the form of logical properties such as Diversity, Identity, etc." The total number of different senses of the expressions 'form', 'structure', 'schema', 'shape', 'pattern', 'figure', 'mold', and related locutions used as technical terms and in metaphors in *Logical Forms* is in the double digits. A morphological analysis of the book would catalogue, analyze, and interrelate them.

It has been common for years to say that a step-by-step *deduction* of a given conclusion from given premises about a given subject can serve as a *template* for indefinitely many other

deductions of different conclusions from different premises about different subjects. Every argumentation in the same form as a deduction is a deduction (Corcoran 1989, 1994). In some cases it has been said that the logical form of a deduction is a template for constructing deductions.

We often read of the *structure* of a *proof*, e.g., that an indirect proof of a given proposition has a different structure from a direct proof. Chateaubriand (2005, 304) mentions "a general theory of proof structures" and "the structure of actual proofs". Three pages later, he wrote: "In any case, proofs are usually represented as syntactic structures consisting of steps made up of sentences that are also thought of as idealized syntactic structures." Are the structures of the proofs the very things the proofs are usually represented as? It would seem that we have two different senses of 'structure'. One can wonder whether these proof structures are logical forms of proofs, but there is no doubt that Chateaubriand recognizes such entities. He wrote: "A formal proof, or deduction, is a representation of the logical form of certain proofs, or arguments" (2005, 292).

There is also discussion of the form or structure of an entire *deductive science*, which is sometimes construed as a multi-part system composed of a class of basic propositions called postulates or axioms, a class of deducible consequences, and a class of deductions of consequences. Feferman (1980, 631) is somewhat typical in saying that in the formal axiomatic method "the meaning of the undefined terms of a postulational theory has been varied while holding the deductive structure fixed". The *deductive structure of a postulational theory* can be considered to be a complex logical form.

In the framework of pre-model theoretic foundations of mathematics, it was not only propositions, deductions, postulate sets, and postulational theories that were thought to have forms: forms were also attributed to the systems of objects that were the contents to which the propositions referred. Two such systems were

said to have the same form, or structure, to be *isomorphic* (the same in form) if a one-one function between their universes of discourse carried the distinguished entities of one to corresponding distinguished entities of the other (Corcoran 1980). These ideas have been adapted for the model-theoretic framework where forms are attributed to set-theoretic objects called *models* (Mendelson 1964/1997, 111).

Tarski (1956/1983, 156, 174) wrote that an expression could be considered to be a physical body of a particular *shape* or *form*. Earlier (31, 47), he wrote that two inscriptions are *equiform* instead of writing that they are tokens of one and the same type. By 'the form of a sentence' Tarski usually meant "the type of a sentence token"—he rarely if ever used it to refer to its grammatical form or to the logical form of the proposition expressed, or to anything other than its syntactic type. On 176, he wrote that some definitions "have a more complicated logical structure" than others that are equivalent. Later two similar usages are found. On page 282 after the second presentation of his axioms for string theory, he wrote that expressions are not concrete inscriptions but classes of inscriptions "of like form", and in the same paragraph he wrote about "the pattern of this axiom system". In "Truth and Proof" (1969/1993, 102), he wrote of the *mold* of sentences that say of something that it is or that it is not. For 'gramatical structure' see Whithehead-Russell, 1910, 1-3.

The expression 'Of consequences, one kind is formal, another material', which is the first sentence of both of Chateaubriand's medieval logic quotations, is but one instance of the scheme:

of Xs, one kind is formal, another material

According to the medievalist scholar Jorge Gracia (per. com.), this and similar schemes were instantiated as stock phrases in many

contexts in medieval science, logic, philosophy, and theology. Even today we sometimes see the expressions *suppositio formalis* and *suppositio materialis* used in connection with the distinction between heteronymous and autonymous use of a name (Church 1956, 61; Tarski 1969/1993, 104).

In /one (is * denotes) a number/, the word /one/ has suppositio
(*formalis * materialis*)

Incidentally, it is easy to get these expressions reversed: words are often called linguistic *forms* and, while being non-material, they often denote *material* or at least refer to subject matter. In medieval Latin, *formalis* often connoted "correct", "proper", "normal", etc. as often does 'formal' in modern English. The contrast also survives in a different sense in distinctions such as Tarski's material adequacy and formal adequacy (1956/1983, 152, 188, 402, passim).

To go beyond logic for a moment, we read of several arch-conservative Roman Catholic groups who disapproved of Pope Paul VI (1897-1978) and debated whether the Holy See of Rome was technically "vacant" during his tenure, i.e., whether there was a legitimate Pope. Apparently, some say that he was both formally and materially Pope, some say that he was formally but not materially so, some say that he was not formally but only materially, and some say that he was neither formally nor materially Pope. Contemporary Catholic theology distinguishes formal and material virginity as well as formal and material sufficiency of Scripture for salvation. I do not know whether Ockham was the first to employ this formula in logic or whether the formula pre-dated his use of it.

JOHN CORCORAN

CONCLUSION

Chateaubriand has produced the most informed and technically competent study of logical form to date. His massive two-volume treatise is replete with original insights and provocative suggestions. His investigation of the relations between interpreted sentences and the higher-order properties he calls logical forms is unparalleled in the history of the subject and will have to be dealt with by future scholars. However, the plausibility of his key proposal that the objects previously called logical forms of propositions (or of interpreted sentences) are actually properties of complex objects has not been established. One of his most basic problems is a failure to determine what the traditional views were and what role logical forms actually served in metalogical discourse. But to my mind, the most disappointing deficiency is his failure to treat the most prominent types of logical forms: the logical forms of arguments, argumentations, proposition sets, and theories—complexes having propositions as components.

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