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★**The method of analysis.**
Its geometrical origin and its general significance.
With an appendix by Árpád K. Szabó.
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Boston Studies in the Philosophy of Science, Vol. XXV.
D. Reidel Publishing Co., Dordrecht-Boston, Mass., 1974. xviii+144 *pp*. $14.90; $9.50 paperbound.

The "method of analysis" is a technique used by ancient Greek mathematicians (and perhaps by Descartes, Newton, and others) in connection with discovery of proofs of difficult theorems and in connection with discovery of constructions of elusive geometric figures. Although this method was originally applied in geometry, its later application to number played an important role in the early development of algebra [Jacob Klein, English translation, *Greek mathematical thought and the origin of algebra*, especially pp. 154–157, M.I.T. Press, Cambridge, Mass., 1968; Zbl **159**, 3]. The present authors sketch a radical interpretation (or reconstruction) of the method, and present elaborate arguments in favor of the various components of their own interpretation and against several competing views. These arguments are predicated on encyclopedic knowledge (of the ancient Greek language, of geometry, of history and philosophy of sciences, of modern logic and recursive function theory). Evaluation of the arguments requires familarity with vast areas of Western thought from the Platonic dialogues through the modern mathematico-philosophical work of Gentzen, Jaśkowski, Gödel, Church and others. This is a book written for experts.

It is universally agreed that the method of analysis begins by "assuming the thing sought after" (e.g., in geometry, the truth of the proposition to be proved or the existence of the geometric figure to be constructed). Aside from this, little else can be taken for granted. There is disagreement concerning the "direction of analysis", i.e. whether one is to seek implications of the assumption or whether one is to seek implicants of it. There is also disagreement concerning what is to be "anatomized" (analyzed), i.e., whether one analyzes mathematical objects (figures), mathematical propositions (the axioms, known theorems, and analytic assumption) or an imagined proof (of the analytic assumption from axioms and known theorems). The authors hold that the directional problem is a confusing pseudo-issue, and that both directions are more or less simultaneously pursued. They also hold that it is mathematical objects (not propositions or deductions) which are the primary focus of analysis both when the result being sought is a construction and when it is a theorem. In fact, the traditional distinction between "theoretical analysis" and "problematic analysis" is taken to be a distinction between logical forms of theorems rather than a distinction between propositions and figures.

The authors thus hold that analysis is not "running the syllogism backward" (or, in a more modern setting, that it is not an attempt to construct a "natural deduction" by simultaneously working forward from the premises and backward from the conclusion). They further hold that it is not an attempt to develop a chain of equivalences starting with the theorem and ending with the axioms, which is then convertible into a direct deduction. Rather, they emphasize that, within geometry, the main thrust of the method of analysis is a special kind of manipulation of geometric figures.

Despite the fact that the notion of a “geometric figure” plays a central role in the development of the main theses of the book, there is no attention paid to the problem of defining it. This lack is especially glaring when one notes that the authors seem to hold that figures are the focus of analysis even in the case of a negative analysis which results in a proof that the “required figure” does not exist.

Most previous writers have made a sharp distinction between heuristics (“logic” of discovery) and deduction (logic of justification), and they have attempted to understand the method of analysis as belonging totally to heuristics. The authors argue that the method of analysis bridges heuristics and deduction at least to the extent that the heuristic aspect of the method carries with it a distinctive style of deductive justification, the “analytical proof system” (page 27), which might prove to be a major breakthrough in our understanding of the method of analysis. It is certainly a new and promising idea which is likely to influence future research in this area.

The authors reassert the epistemic and ontological importance of geometric figures, something which had been denied in favor of exclusive concern with the formal structure of geometric propositions and deductions. In addition, they assume that the geometric propositions dealt with by the ancients correspond in some measure to formulas of modern symbolic logic and that the geometric proofs (deductions) sought by the ancients correspond in some measure to the highly structured formal deductions of modern “natural deduction systems”.

{Reviewer’s remarks: This book is a fresh and sophisticated examination of historically and philosophically important issues. In addition to the new ideas it sets forth, it also presents clear restatements of older ideas and new arguments for them. But the reviewer feels that the book is not nearly as successful as it could have been. There is a scarcity of examples and those that are presented are hard to follow, partly because of their intrinsic complexity, partly because they are incompletely stated, and partly because large steps of reasoning are omitted. Examples of negative analyses (leading to indirect proofs) are missing entirely and, likewise, there are no examples of cases where analysis fails. But the book falls short most basically because it lacks a clear and complete formulation of its results, because it is neither carefully written nor carefully proofread (page 53 has ten misprints), and because its 144 page length is not nearly long enough to cover the topic. Nevertheless, this book is a solid contribution and it is probably the most learned and most creative treatise ever written on the method of analysis. (The reviewer gratefully acknowledges the cooperation of R. Vesley, S. Shapiro and the first author in the preparation of the review.)}

Reviewed by *J. Corcoran*