THE PRINCIPLE OF WHOLISTIC REFERENCE*

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Abstract: In its strongest, unqualified form the principle of wholistic reference is that each and every proposition refers to the whole universe of discourse as such, regardless how limited the referents of its non-logical or content terms. Even though Boole changed from a monistic fixed-universe framework in his earlier works of 1847 and 1848 to a pluralistic multiple-universe framework in his mature treatise of 1854, he never wavered in his frank avowal of the principle of wholistic reference, possibly in a slightly weaker form. Indeed, he took it as an essential accompani-ment to his theory of concept formation and proposition formation. Similar views are found in later logicians, and some of the most recent formulations of standard, one-sorted first-order logic seem to be in accord with a form of it, if they do not actually imply the principle itself.

Key-words: Chateaubriand. Sentence. Proposition. Label. Pointer. Logically perfect language. Universe of discourse.

O PRINCÍPIO DA REFERÊNCIA UNIVERSALISTA

Resumo: Em sua forma mais forte e geral, o princípio da referência universalista afirma que toda proposição refere-se ao universo completo do discurso propriamente dito, independentemente de quão limitados sejam os referentes de seus termos não-lógicos ou com conteúdo. Embora Boole mudasse de um modelo monístico de universo-fixo nos seus trabalhos iniciais de 1847 e 1848 para um modelo pluralista de universo-múltiplo no seu tratado maduro de 1854, ele nunca hesitou em sua aceitação franca do princípio da referência universalista, possivelmente em uma forma ligeiramente mais fraca. De fato, ele considerou este princípio como um acompanhamento essencial para a sua teoria de formação de conceitos e de proposições. Visões semelhantes são encontradas em lógicos

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posteriores, e algumas das mais recentes formulações da lógica clássica de primeira ordem parecem estar de acordo com uma forma deste, se é que elas não implicam, de fato, o próprio princípio.

Palavras-chave: Chateaubriand. Sentença. Proposição. Rótulo. Indicador. Linguagem logicamente perfeita. Universo de discurso.

Boole's use of the symbol '1' in *Laws of Thought* (1854, pp. 42, 47) marks a milestone in logic. He had already used it in his earlier work (Boole 1847, p. 15), where '1' expressed "the universe". The universe was taken to be the most comprehensive class, the uniquely determined class of which every other class is a subclass[1]. This corresponds to using the words 'thing', 'entity', 'object' and their synonyms in their broadest, invariable sense. Thus in the earlier 1847 work, '1' is a constant, or label-word, like 'one', 'class', 'symbol', 'the universe' or 'Boole', an expression whose referent is considered to remain essentially fixed regardless of the context in which it is used. Today, using the terminology of Tarski (1986), the symbol '1' is a logical constant in Boole (1847). This 1847 usage was already a momentous innovation that had eluded Aristotle and the other earlier logicians; but the usage introduced in 1854 was part of an even more revolutionary sea change in logic[2].

Label-words remain, so to speak, attached to what they label. Label-words contrast with indexicals or pointer-words such as 'you', 'me', 'here', 'there', 'this', 'now', 'yesterday' and the like, as well as richer words such as 'father', 'mother', 'school' and 'home' whose referents are not

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[1] Kanamori (2003, p. 274) seems to imply that Boole discovered the null set denoted by '0' in Boole 1847, p. 21. As amazing as this may seem I know of nothing to the contrary. It seems certain that Boole was the first to use '0' and '1' in class-theoretic senses.

[2] Boole keeps to this usage throughout the 1847 treatment of "class logic". But, when he turns to "hypothetical logic" on page 48 of the 82 page booklet, he is forced by his redeployment of the same formalism to assign '1' another denotation, which he calls "the hypothetical Universe" (1847, p. 49), thus foreshadowing or setting the stage for the dramatic 1854 innovation.

fixed until they are used in a particular context at a particular time and place by a particular person for a particular audience. What a pointer points to depends on where it is, on what its orientation is and on what happens to be "in its way".

For example, the sentence 'Yesterday you told me that Mother was home' can be used to express many different propositions. On a given day it may be used to express a large number of propositions, each distinct from any proposition it could be used to express on any other given day. One given person could use the sentence to express to a second given person a proposition that no third person could express to a fourth person using the same sentence. Label-words are also called tags; pointer-words are also called deictics, egocentrics, egocentric particulars, emphatic particulars, demonstratives, token-reflexives or even here-nows (Lewis & Langford 1932, p. 312; Copi & Gould 1967, p. 128; Audi 1999, pp. 254, 423, 923; Anderson 1998, p. 132). Of course, 'mother' and 'home' are both ambiguous and in some senses they are used as label-words, e.g., when preceded by an article as in 'Every mother should have a home'.

In sharp contrast to the usage in Boole's 1847 booklet, in Boole's 1854 book '1' becomes an indexical and by translation so does 'entity', 'being', 'thing', 'individual', etc. In the 1854 *Laws of Thought*, '1' indicates not "the universe" but the limited subject matter of the particular discourse in which it is used, what Boole calls 'the universe of [the] discourse', something that varies from one discourse-context to another (Boole 1854, p. 42). Sometime between the time when Boole wrote the 1848 paper "The calculus of logic", which has no occurrences of the expression 'universe of discourse', and the writing of the 1854 book, he learned, probably from De Morgan, that a discussion need not concern "the universe of conceivable objects" but instead may have as its own special "ultimate subject" a more limited domain, in his own words:

> In every discourse, whether of the mind conversing with its own thoughts or of the individual in his intercourse with others, there is an

> assumed or expressed limit within which the subjects of its operation are confined. The most unfettered discourse is that in which the words we use are understood in the widest possible application, and for them the limits of discourse are co-extensive with those of the universe itself. But more usually we confine ourselves to a less spacious field. Sometimes, in discoursing of men we imply (without expressing the limitation) that it is of men only under certain circumstances and conditions that we speak, as of civilized men, or of men in the vigour of life, or of men under some other condition or relation. Now, whatever may be the extent of the field within which all the objects of our discourse are found, that field may properly be termed the universe of discourse. ... Furthermore, this universe of discourse is in the strictest sense the ultimate *subject* of the discourse. (Boole 1854, p. 42)

This is the first time in the history of the English language that the expression 'universe of discourse' was ever used[3]. In this way Boole opens up room for one and the same language to be used in many different interpretations in a way that had never been done before. In arithmetic discourses the universe of discourse is often the class of natural numbers, but it is sometimes broader or narrower; in geometric discourses the universes of discourse are various classes of geometrical figures, just points, just plane figures, just solids, etc.; and in set-theoretic discourses the universes of discourse include a class of sets but sometimes include other objects as well[4]. It is important to notice that Boole seems to regard the context sensitivity or context relativity of the

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[3] De Morgan (1847) wrote of "the universe of a proposition" and "the universe of thought", but it was Boole who coined 'universe of discourse'. See Audi 1999, p. 941.

[4] For more on universe of discourse, see any good philosophical dictionary. For example, see "Universe of discourse" in the 1999 *Cambridge Dictionary of Philosophy* (Audi 1999, p. 941) or the article of the same name in the 1996 *Oxford Dictionary of Philosophy* (Blackburn 1996, p. 387). The principle of universe relativity – that one and the same sentence can be used to express different propositions according to which universe the discourse of its use has – was resisted by certain schools of logic for many years. As late as 1903 a major logician, Bertrand Russell, could write a 500-page book without mentioning the concept of universe of discourse or even using the expression (1903).

class-name '1' to pertain only to its extension (reference or denotation) and not to its intension (meaning, sense or connotation). Thus, as a class name, '1' has a fixed meaning something like "the universe of discourse", whereas the extension of "the universe of discourse" is different in different contexts. This is remarkably similar to the functioning of personal egocentric words such as, e.g., 'I', which has the same sense whenever it is used but its reference is context-sensitive. When I say it, the reference is me; but when you say it, the reference is not me but you.

The idea of multiple universes of discourse is one of the key ideas of modern logic (Sagüillo 1999; Boolos *et al.* 2002, p. 103). Modern logic is almost inconceivable without the concept of universe of discourse. In each modern mathematical theory the special universe of discourse contains all of the entities that are subjects of the propositions of the theory. For example, in number theory there are propositions about zero, propositions about one, propositions about zero and one, and so on. As mentioned above, Boole made this point already in 1854, 'this universe of discourse is in the strictest sense the ultimate subject of the discourse' (1854, p. 42). But his intention went far beyond the view just attributed to modern mathematical logic. For Boole, except perhaps in a degenerate case such as "0 = 0", *not only was each proposition about some entity or entities in the universe of discourse, but also it was about the universe of discourse itself.*

Moreover, except perhaps in degenerate cases, both of the terms of an equation contained the concept of the universe of discourse and, indeed, each expression of a term of an equation contains at least one occurrence of the character '1' referring to the universe of discourse (1847, p. 15; 1854, p. 44). For example, Aristotle's "Every square is a rectangle", considered as a proposition of a general theory of geometry, would be treated by Boole as "Being an entity that is square is being an entity that is rectangular that is square" where the word 'entity' names the universe of geometrical discourse. One Boolean equation for this would be '$(s \cdot 1) = (s \cdot (r \cdot 1))$'.

To be explicit, Boole's *principle of wholistic reference*[5] is his view that (possibly excluding degenerate cases) *each and every equational proposition refers to the universe of discourse as such*[6]. As far as I know this aspect of Boole's semantics or theory of propositions does not seem to have been explicitly noted by Boole scholars. It is not mentioned in any of the

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[5] Here 'wholistic' is taken in the standard narrow sense of "pertaining to the whole as opposed to a part". This choice of orthography is intended to discourage any suggestion of sympathy with so-called 'holistic' philosophies that affirm such theses as that no particular experience could refute any particular belief, that no particular therapy could cure any particular illness or that no one word has any meaning except in the context of a whole sentence. The idea that the part depends for its "meaning" on the whole has nothing to do with this paper. However, as pointed out by Peter Hare, the noted historian of American philosophy, (per. comm.) Boole's view strongly suggests the proposition that it is impossible to refer to a proper part of the universe of discourse without referring to the whole, a proposition with strong holistic resonance.

[6] Apparently, instead of 'wholistic' the alternate spelling 'holistic' is much more common even though its sense pertains to a whole and not to a hole, or to holiness or anything else commonly spelled with initial 'hol'. The spelling 'wholistic' suggests, as would be appropriate, that the word is derived from the English word 'whole', which is never spelled without the initial double-u. The spelling 'holistic' suggests that the word is derived from the Greek word *holos* (whole). Interestingly, although *istic* is a standard Greek suffix (as in *eristic, logistic, sophistic,* etc.), there is no entry for *holistikos* in the Liddell-Scott-Jones *Greek-English Lexicon.*

The English word 'holistic' was coined in 1926 by Jan Smuts; it is not a borrowing from Greek. It seems hardly relevant to find out what the coiner had in mind, not that people have never been known to allege impressively learned reasons after the fact for decisions made spontaneously and without reflection on more humble grounds. For people like me without much Greek, the spelling 'wholistic' seems mnemonically superior for the intended meaning, and for the people well-versed in Greek it is hard to see what difference it would make. Nevertheless, when deciding which spelling to use it would be foolhardy to ignore the fact mentioned above that the overwhelming majority preference is for 'holistic' in the "richer" sense, but not in the present sense, as pointed out independently by David Hitchcock and Michael Scanlan (per. comm.) – a recent *Google* search turned up about thirty 'holistic' hits for each 'wholistic' hit.

articles selected for Gasser's *A Boole Anthology: Recent and Classical Studies in the Logic of George Boole* (Gasser 2000) nor is it in the article that Susan Wood and I wrote in the late 1970s (Corcoran & Wood 1980). The first place it is mentioned seems to be in the Introduction to the 2003 edition of *Laws of Thought* (Boole 1854/2003).

This strange and fascinating view that, in his words (1854, p. 42), "the universe of discourse is the ultimate subject" of every proposition is foreshadowed in Boole's earlier work (1847, pp. 15-16), where he explicitly says that when an "elective symbol" such as '*x*' occurs by itself it is to be taken as elliptical for the class-name '*x*(1)'. Thus in that work, where the universe is *the* universe of discourse, '1' occurs in every term expression except perhaps some 'involving '0', and maybe' '0' itself: if '(*x* · 0)' would be elliptical for '(*x*(1) · 0)'. In (1848, p. 184) he makes this a matter of principle by writing: "The universe of conceivable objects is represented by 1 or unity. This I assume as the primary and subject conception. All subordinate conceptions of class are understood to be formed from it by limitation ..." If he takes the null class as a "subordinate conception", then '0' might be taken as an elective symbol, so that in an equation '0' is elliptical for '0(1)', corresponding to the epistemic act of choosing nothing from the universe. If my memory serves me, Susan Wood (1976) discusses this point: but I do not have access to her work now.

But in *Laws of Thought* this view is in a way more explicit and in a way less explicit (1854, pp. 42-43). Boole's view is based on his insightful epistemic principle that the mental process of formulating a propositional thought begins with the act of conceiving of the universe of discourse[7].

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[7] In today's terminology this would correspond to getting clear about the range of the individual variables, excellent advice for writers as well as for readers, even those who may not accept Boole's principle of wholistic reference without qualification. It would be sad for an obsession with a supposed counterexample or limitation to mask sound and useful aspects of Boole's principle.

Any subsequent specialization of the subject of the proposition is construed as a concept based on the concept of the universe of discourse in addition to whatever else it involves. To be clear, Boole's view is that the meaning of, e.g., the common noun 'human' in a particular discourse – the common noun concept expressed by 'human' – has a logical form corresponding to 'entity that is a human' or 'entity that is human' (1854, p. 42). Boole (1854, p. 27) writes: it is the same thing to say "Water is a fluid thing" as to say "Water is a fluid". This means that for Boole the sentences not using the word 'entity' are elliptical for corresponding sentences that do use the word 'entity', or a synonym such as 'thing' or 'being'. For example, 'Being a square is being both a square and a polygon' is elliptical for something like 'Being an entity that is square is being both an entity that is square and an entity that is polygonal'. This in turn means that the equations not using the digit '1' are elliptical for equations that do use '1' (1847, p. 15). For example, the equation '$s = (s \cdot p)$' is elliptical for something like '$(s \cdot 1) = (s \cdot (p \cdot 1))$'. Likewise, 'Being a nonentity is being a square that is a circle' is elliptical for something like 'Being a nonentity is being an entity that is square that is a circle'. Thus, the equation '$0 = (s \cdot c)$' is elliptical for something using '1' like '$0 = ((s \cdot 1) \cdot c)$'. In Boole's language no fully expressed equation is composed entirely of a connector, letters and operation symbols without a digit. Every such equation contains at least two occurrences of digits, on each side there is at least one occurrence of '1' or at least one of '0'.

Even though Boole changed from a monistic fixed-universe framework "as in his 1847 and 1848" to a pluralistic multiple-universe framework in 1854, he never wavered in his frank avowal of the principle of wholistic reference. Indeed, he took it as an essential accompaniment to his theory of concept formation and proposition formation, as mentioned above.

Boole is not the only important logician to accept something similar to the principle of wholistic reference. Chateaubriand (2001, p. 53) says '... Frege held that statements ... could not refer to isolated

aspects of reality ... that their connection to reality must be ... [total]'. So it looks like Frege accepted something like the principle of wholistic reference even though he differed with Boole on many other points. Chateaubriand's own view is nearly diametrically opposed to the principle of wholistic reference (2001, Ch. 2). But he does not go so far as the principle of mentioned reference, i.e., that in a logically perfect language not every sentence mentions the universe of discourse and each sentence refers only to the referents of the non-logical expressions occurring in the sentence.

In *Principles of Mathematics* (1903, p. 79), Russell says that Peano held, in earlier editions of his *Formulaire*, a view that is in the neighborhood of the principle of wholistic reference, if we can extrapolate using classical logic. The explicit point attributed to Peano is that 'Every man is mortal' expresses the proposition that every entity is mortal, if it is a man, i.e., a proposition that is about all entities without exception, and not just about men, as Russell says is his view. To be clear about Russell's view it is useful to quote in full the sentence summarizing the discussion that he started with the reference to Peano: "But when we say 'all men are mortals', it seems plain that we are speaking only of men, and not of all other imaginable terms". Nevertheless, the last sentence at the end of the long paragraph containing the above points seems to say that it is Peano's view, not Russell's own, that is the one "most relevant to Mathematics", despite the fact that earlier in the book (1903, p. 34) he makes statements that seem to presuppose something like what he is here criticizing in Peano, viz. the principle of wholistic reference. For example, on page 34 he writes:

> In inferences from "Socrates is a man", it is customary ... to regard Socrates (*sic*) merely as a symbol, capable of being replaced by any other man; and only a vulgar prejudice in favor of true propositions stands in the way of replacing Socrates by a number, a table, or a plum-pudding.

The exact opposite of what seemed plain to Russell in 1903 (p. 79) seemed plain to many mainstream logicians a few years later when they

found themselves called on to defend the so-called "Boolean interpretation" according to which the universal affirmative sentence is supposed not to imply the corresponding existential. In their haste to explain why "Every square is a rectangle" does not imply "Some square is a rectangle", which was contrary to their readers' expectations, they found themselves alleging that the former proposition was the same as the one expressed by 'Every object is such that, if it is square then it is rectangular'. Of course there was no discussion of how it could be that such disparate sentences (one with no adjectives but two nouns neither being 'object', the other having two adjectives but neither of the formers' nouns) could express one and the same proposition or how it could be determined that a sentence apparently expressing a proposition about geometrical figures was actually expressing a proposition about everything without exception (in Russell's words, about all imaginable terms). Naturally the question did not arise as to whether "Every object is such that, if it is square then it is rectangular" implies "Some object is such that, if it is square then it is rectangular". Nevertheless, without realizing it, many of the standard logic texts presupposed something close to the Boolean principle of wholistic reference and in an unnuanced form not qualified by the Boole-De Morgan doctrine of multiple universes of discourse, in fact, in a form close to that of Boole's 1847 booklet. Cohen and Nagel's popular and respected *Introduction to Logic (*1934/1962/1993*)* is typical, see pp. 41-44.

Some readers might discern a form of the principle of wholistic reference in standard first-order logic that finds the same information in "Two is even" as in "Every number which is two is even" or in Quine's *Philosophy of Logic* (1970/1986, pp. 25, 26) where 'Plato is mortal' is paraphrased as 'Some object that platonizes is mortal'.

help of several logicians including Professor Oswaldo Chateaubriand of Rio de Janeiro, Brazil. This paper is part of a conversation that I have been having with Chateaubriand for many years. The two of us met in a philosophy of mathematics seminar at the University of California at Berkeley in the spring of 1965 when Berkeley was the unrivaled world center of Logic and the Methodology of Deductive Sciences, as they preferred to call it. We soon became close friends, and we have visited many times since. Chateaubriand and I are remarkably similar in philosophic temperament, standards of evidence, and intellectual interests. We both tend toward the platonistic, the analytical and the mathematical in philosophic orientation. We both value objectivity, clarity, rigor, and explicitness. We both value the history of ideas, not just the history of logic, or the history of philosophy, or the history of mathematics to mention three areas that we both love for themselves and for the richness that they lend to philosophic deliberation. We are thoroughly unmystical, unreligious and unsuperstitious. Perhaps it can be said that we are platonistic naturalists (or naturalistic platonists) who are thoroughly unsupernaturalist in our attempt to do justice to the human experience of mathematics, logic, and language. The conversation took on an added intensity with the publication of his 2001 book *Logical Forms*, which reexamines many of the themes that we have discussed over the years, including the theme of this essay.

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