{ 17 }

Pluralism and Paradox

Aaron J. Cotnoir

1. Introduction

The semantic paradoxes are as much of a problem for pluralists about truth as they are for any other theory of truth. Alethic pluralists, however, have generally set discussion of the paradoxes aside.[1] In what follows, I argue that considerations involving the paradoxes have *direct* implications for alethic pluralism.

More specifically, alethic pluralism has bifurcated into two main types: *strong* and *weak*. Weak theories accept a truth predicate that applies to every true sentence (a *universal* truth predicate) in addition to the many other truth predicates, $T_1, \ldots, T_n$. Strong theories reject a universal truth predicate in favor of $T_1, \ldots, T_n$.[2] This chapter has two parts. The negative part (§2) shows that *both* types of theories suffer from paradox-generated inconsistency given certain plausible assumptions. The positive part (§3) outlines a new, consistent way to be a strong alethic pluralist. The trick to avoiding paradox is rejecting infinitary disjunction, something we already have pluralism-independent (but paradox-motivated) reasons to reject. In §4, I conclude by comparing this theory with a Tarskian hierarchical view and discuss some directions for future research.

---

[1] There are one or two exceptions. The only pluralist theories that handle paradoxes are those who have come to alethic pluralism *as a result o*f dealing with paradoxes. Hartry Field (2008) endorses a plurality of 'determinate' truth predicates in order to handle certain *revenge* charges. Jc Beall (2008b) discusses a strong falsity predicate to avoid a *reveng*e charge as well. See Beall (2013), for more details.

[2] Strictly speaking, there are more types if one considers the predicate/property distinction. Pedersen (2006) is quite careful about this. In this chapter, however, I focus merely on truth predicates rather than truth properties. This is for three reasons. First, regardless of one's theory of truth *properties*, one will need truth *predicate*s to express them. Second, paradoxes arise most straightforwardly for predicates; although there may be parallel (Russell-like) paradoxes for truth properties, whatever they may be. Finally, I am unclear what considerations would make a property a *truth* property; that is, I am somewhat sympathetic to deflationary theories of truth. In order not to prejudge any of this, I stick to predicates throughout.

2. Problem: universal truth and paradox

Pluralists endorse many truth predicates $T_1, \ldots, T_n$. Usually, each predicate is a truth predicate *for* a certain 'domain of discourse.'[3] Here, domains are not what first-order quantifiers range over. For our purposes, we may treat them simply as *fragments* of a language, where fragments of a language are disjoint proper subsets of the sentences of that language.[4]

What does it mean to be a truth predicate *for* a domain? Pluralists have endorsed certain minimal constraints.[5] One such minimal constraint is the T-scheme:

$$(\textit{TS}) \vdash T_i(\ulcorner\alpha\urcorner) \leftrightarrow \alpha \text{ for all sentences } \alpha \text{ in domain}_i.$$

Here 'T_i' is a truth predicate for domain$_i$. '$\ulcorner\alpha\urcorner$' just signifies the code for sentence α generated some adequate coding scheme; any arithmetization that yields a language rich enough to 'talk' about its own syntax will do. And '$\leftrightarrow$' is constructed in the normal way from any conditional that satisfies modus ponens ($\alpha, \alpha \rightarrow \beta \vdash \beta$), identity ($\vdash \alpha \rightarrow \alpha$), and transitivity ($\alpha \rightarrow \beta, \beta \rightarrow \gamma \vdash \alpha \rightarrow \gamma$).

Pluralists have endorsed many other constraints, but let me focus only on (*TS*). Weak alethic pluralists—those pluralists who endorse a universal truth predicate $\mathbb{T}$—must decide whether this universal predicate obeys the T-scheme. That is, does the weak pluralist accept (*FULL-TS*)?

$$(\textit{FULL-TS}) \qquad \vdash \mathbb{T}(\ulcorner\alpha\urcorner) \leftrightarrow \alpha \text{ for all sentences } \alpha.$$

If the answer is 'yes,' then it is straightforward to derive a paradox. We have assumed an adequate coding scheme; this is guaranteed if the language has the expressive resources of first-order arithmetic. So, standard diagonalization techniques guarantee that any expression with one free variable will have a Gödel sentence that is equivalent to that expression predicated of itself. In this case, $\neg\mathbb{T}(x)$ is such an expression; call its Gödel sentence 'λ.' But then λ is equivalent to $\neg\mathbb{T}(\ulcorner\alpha\urcorner)$, and so we can prove (*GS*).

---

[3] This is how both Wright (1992; 2001) and Lynch (2001; 2004; 2009) set up their theories. But see Horgan (2001), who thinks truth predicates are true of sentences relative to 'contexts.'

[4] Domains are difficult to pin down. Lynch (2004) writes, 'Intuitively, a propositional domain is simply an area of thought. . . . Propositional domains are individuated by the types of propositions of which they are composed. Propositions are in turn individuated by the concepts we employ in thinking about different subject matters' (399–400). But in order to type propositions in this way we must already have a clear taxonomy of types of concepts. Lynch himself believes that concepts often cannot be individuated in a determinate manner. He admits, 'Here, like everywhere else, types of concepts shade off into one another' (2001: 731). Thus, we have reason to think these propositional domains will be (in some cases) indeterminate. But this conflicts with Lynch's (2004: 400) assertions that every atomic proposition is a member one and only one domain (and essentially so). See Sher (2005) and C. D. Wright (2005) for more objections, and Lynch's essay in this volume for an attempt to address them.

[5] Wright and Lynch both endorse a *platitude*-approach to alethic pluralism. For Wright, the platitudes define the concept of truth; for Lynch, they define the functional role of truth.

$$(\textsc{gs}) \qquad \vdash \neg\mathbb{T}(\ulcorner\lambda\urcorner) \leftrightarrow \lambda.$$

And in the presence of (*FULL-TS*) we have it that $\vdash \mathbb{T}(\ulcorner\lambda\urcorner) \leftrightarrow \lambda$. But this, combined with (*GS*), gives us the paradox: $\vdash \neg\mathbb{T}(\ulcorner\lambda\urcorner) \leftrightarrow \mathbb{T}(\ulcorner\lambda\urcorner)$. Unless the logic is *extremely* nonclassical, these paradoxes will explode into triviality. It will turn out that *everything* is true, which is hardly desirable for a theory of *truth*.[6] None of this is anything novel or controversial. It is surprising, then, that alethic pluralists would endorse (*FULL-TS*). But nearly all weak alethic pluralists have, including Wright, Lynch, and Sher.[7]

So, I claim that the weak alethic pluralists, if they wish to avoid paradox, ought to reject (*FULL-TS*). If there *is* a universal truth predicate, it better not satisfy the T-scheme unrestrictedly. Should the weak pluralist endorse (*TS*) for each truth predicate $T_1, \ldots, T_n$? That is, can each domain-specific truth predicate satisfy the T-scheme restricted to its own domain? If weak pluralists accept this, this puts them in a sufficiently similar position as the strong alethic pluralist who endorses TS for the truth predicates $T_1, \ldots, T_n$. So let us turn to this option now.

2.1 STRONG PLURALISM AND THE LIAR

The strong alethic pluralist accepts many domain-specific truth predicates $T_1, \ldots, T_n$, yet rejects any universal truth predicate. Now, the strong pluralist must also decide whether each truth predicate satisfies the T-scheme (*TS*). If so, however, each T_i needs to satisfy (*TS*) only for all α in domain$_i$. This also would appear to run straight into semantic paradox.

Consider the liar-like sentence λ_1 constructed via diagonalization using the truth predicate T_1.

$$\lambda_1: \neg T_1(\ulcorner\lambda_1\urcorner)$$

Since λ_1 is the Gödel sentence of the open expression $\neg T_1(x)$, we can prove the following:

$$(\textsc{gs}\mathbf{1}): \vdash \neg T_1(\ulcorner\lambda_1\urcorner) \leftrightarrow \lambda_1.$$

If we endorse (*TS*), we are committed to $T_1(\ulcorner\alpha\urcorner) \leftrightarrow \alpha$ for all α in domain$_1$. But then we can show that $\vdash \neg T_1(\ulcorner\lambda_1\urcorner) \leftrightarrow T_1(\ulcorner\lambda_1\urcorner)$, on the assumption that λ_1 is in domain$_1$. And that's bad.

---

[6] I should note that I have some sympathy for nonclassical truth theories. See Caret & Cotnoir (2008) for a defense of one paracomplete option.

[7] See Wright (1992; 2001: 760); Lynch (2001: 730; 2009: ch. 4, §1). Sher (2004) is not explicit, but her discussion of the *unity* of truth raises serious suspicion that she endorses (*FULL-TS*) (see pp. 26–35). To be fair, none of these pluralists are undertaking any discussion of the paradoxes. But, in this chapter, I am claiming that they should.

Contrary to the above derivation, however, the strong alethic pluralist has available a novel response to these paradoxes. The derivation depends crucially on the assumption that the λ_1-liar is actually in domain$_1$. But the pluralist, of course, is free to reject that λ_1 is in domain$_1$. If λ_1 is *not* a sentence of domain$_1$, then we do not have to commit to $T_1(\ulcorner\lambda_1\urcorner) \leftrightarrow \lambda_1$. Thus, we do not arrive at the paradoxical consequence that $\vdash \neg T_1(\ulcorner\lambda_1\urcorner) \leftrightarrow T_1(\ulcorner\lambda_1\urcorner)$.

Here is another way of stating the point. As strong pluralists, we are free to claim that λ_1 is *not* true$_1$. Of course, λ_1 actually says of itself that it is not true$_1$. And so intuitively, it ought to be true! But if λ_1 is actually in domain$_2$, it may very well be true$_2$. We can endorse $T_2(\ulcorner\lambda_1\urcorner)$ without paradox.

Of course, we will be able to define a *new* liar, λ_2, by diagonalization using T_2.

$$\lambda_2 : \neg T_2(\ulcorner\lambda_2\urcorner)$$

But notice that λ_2 is *not* the same sentence as λ_1. Indeed, the two use different truth predicates. Here again, the pluralist is free to reject that λ_2 is in domain$_2$, but rather in, say, domain$_3$. This process can continue, and the result is that the pluralist can consistently endorse (*TS*) for T_i over domain$_i$ for every natural number i.

2.2 STRONG PLURALISM AND REVENGE

There is trouble lurking with the above proposal. And the trouble is tied up with the fact that is more difficult to avoid a universal truth predicate than one might initially think. Given the resources of disjunction, one can always define a universal truth predicate thus:[8]

$$(\textit{T-DEF}) \quad \mathbb{T}(\ulcorner\alpha\urcorner) := T_1(\ulcorner\alpha\urcorner) \vee T_2(\ulcorner\alpha\urcorner) \vee \ldots \vee T_n(\ulcorner\alpha\urcorner).$$

In the case where the number of domains is countably infinite, we simply require infinite disjunction to yield the definition.

$$(\textit{T-DEF*}) \quad \mathbb{T}(\ulcorner\alpha\urcorner) := \bigvee_{i\in\mathbb{N}} T_i(\ulcorner\alpha\urcorner).$$

Notice that (*T-DEF*) and (*T-DEF**) are genuinely universal truth predicates, in that $\mathbb{T}$ will be true of α regardless of its domain.[9] It turns out that it is difficult to be a *strong* alethic pluralist.

More troubling, however, is that if T_i satisfies (*TS*) for each $i \in \mathbb{N}$, then $\mathbb{T}$ will satisfy (*FULL-TS*). Suppose $\vdash T_i(\ulcorner\alpha\urcorner) \leftrightarrow \alpha$ for all α in domain$_i$, for each

[8] In (2009), I defined such a truth predicate to show that the proposal in Edwards (2008) did not avoid one. Nikolaj Pedersen (2010) used the same technique to formulate the 'linguistic instability challenge.'

[9] Here I assume that each truth predicate is true of only of sentences in its domain. Pluralists may wish to reject this assumption. If so, then (*T-DEF*) needs an additional constraint: sentence domains must be made explicit. So, $\mathbb{T}(\ulcorner\alpha\urcorner) := (T_1(\ulcorner\alpha\urcorner) \wedge \ulcorner\alpha\urcorner \in D_1) \vee (T_2(\ulcorner\alpha\urcorner) \wedge \ulcorner\alpha\urcorner \in D_2) \vee \ldots \vee (T_n(\ulcorner\alpha\urcorner) \wedge \ulcorner\alpha\urcorner \in D_n)$ will do the trick. See Pedersen & Wright (2013), this volume, for discussion of this issue.

$i \in \mathbb{N}$. Then $\vdash(T_1(\ulcorner\alpha\urcorner) \vee T_2(\ulcorner\alpha\urcorner) \vee \ldots T_n(\ulcorner\alpha\urcorner)) \leftrightarrow \alpha$ for all α irrespective of the domain; or, more generally,

$$\vdash \bigvee_{i\in\mathbb{N}} T_i(\ulcorner\alpha\urcorner) \leftrightarrow \alpha$$

will hold for any sentence α. But the lefthand side just is the definition of $\mathbb{T}(\ulcorner\alpha\urcorner)$, and so we have $\vdash \mathbb{T}(\ulcorner\alpha\urcorner) \leftrightarrow \alpha$ for any sentence α. We have used (infinitary) disjunction to construct a universal truth predicate satisfying (*FULL-TS*). So any strong pluralist who thinks $T_1, \ldots, T_n$ must satisfy (*TS*), is actually a *weak* pluralist that endorses (*FULL-TS*). The two positions actually collapse into the same view.

And now notice that $\neg\mathbb{T}(x)$ is an open expression of the required kind for diagonalization. So we will have its Gödel sentence, call it λ_ω, such that $\vdash\neg\mathbb{T}(\ulcorner\lambda_\omega\urcorner) \leftrightarrow \lambda_\omega$. But in the presence of (*FULL-TS*), we get the paradox: $\vdash\neg\mathbb{T}(\ulcorner\lambda_\omega\urcorner) \leftrightarrow \mathbb{T}(\ulcorner\lambda_\omega\urcorner)$. To put the point plainly, given infinitary disjunction, we can construct a sentence that says of itself: 'I'm not universally true.' That sentence is λ_ω, equivalent to $\neg\mathbb{T}(\ulcorner\lambda_\omega\urcorner)$.

That is just an abbreviation for

$$\neg\bigvee_{i\in\mathbb{N}} T_i(\ulcorner\lambda_\omega\urcorner).$$

But given that we have DeMorgan negation, that is equivalent to

$$\bigwedge_{i\in\mathbb{N}} \neg T_i(\ulcorner\lambda_\omega\urcorner).$$

So, intuitively λ_ω is a sentence that says of itself that it is not true$_1$, *and* not true$_2$, *and* so on.

The point has failed to be noticed. I, myself, failed to notice this result in (2009) where I argue that Edwards's (2008) solution to the problem of mixed conjunction has a universal truth predicate. Edwards's solution would require infinitary disjunction and hence necessitates $\mathbb{T}$. As a result of the above, Edwards's solution is outright inconsistent if it accepts (*TS*).

But others have failed to notice the point as well. Consider, for example, Pedersen (2010), who uses a construction similar to (*T-DEF*) to argue that strong alethic pluralism collapses into weak pluralism.[10] Regarding (*TS*), he says,

> According to pluralists [. . .] what makes a given predicate a truth predicate is that it satisfies a series of platitudes, or truisms, which delineate the truth

[10] More accurately, he argues that it does so given a principle of 'linguistic liberalism' regarding language expansion. He seems to assume that the predicate $\mathbb{T}$ must be *added* to the language, and that such additions need to obey certain principles. However, given that $\mathbb{T}$ is defined merely out of linguistic items we already have available, the language needs no expansion. We may wish to add the symbol '$\mathbb{T}$' to our syntax, but we are stuck with the universal truth predicate even if no such symbol is added.

> concept. A non-exhaustive list would include as platitudes that ‘p’ is true if and only if p (‘disquotational schema’) [. . .]. (2010: 99))

While Pedersen’s argument does not require the platitude approach, he fails to note that the strong pluralist simply cannot, on pain of paradox, introduce $\mathbb{T}$. He claims,

> Nothing prevents us from introducing $\mathbb{T}$. It is syntactically well-formed and disciplined, as any legitimate predicate should be[. . .]. $\mathbb{T}$ is a universal truth predicate because it applies to *exactly* those sentences to which one of $T_1, \ldots, T_n$ applies. (2010: 99])

And again, regarding $\mathbb{T}$ he writes,

> It is syntactically well formed, and comes with a condition of application [the T-scheme]. In the light of this, there is simply no further question whether $\mathbb{T}$ is a legitimate addition[. . .]. Hence, I see no way to resist the introduction of $\mathbb{T}$. (2010: 99)

There *is*, however, something that prevents us from introducing $\mathbb{T}$—doing so introduces paradox and inconsistency. There *is* a further question about whether $\mathbb{T}$ is a legitimate addition.[11]

The alethic pluralist has three options. First, one may endorse a nonclassical logic to avoid paradox. Any such theory will have to be significantly different from usual pluralist theories; indeed, it will represent a significant departure from classical logic.[12] The second option is to reject that (TS) holds for some truth predicate T_i. On pain of paradox, the pluralist must admit that there is at least one T_i that fails to satisfy the T-scheme. The last option is to reject the linguistic resources for introducing $\mathbb{T}$, to reject infinitary disjunction.

3. Solution: rejecting infinite disjunctions

Alethic pluralists—both strong and weak—may respond to this problem by rejecting that (TS) serves as a constraint on being a truth predicate for a domain. Or they may respond by adopting a nonclassical logic that can handle such paradoxes. These are just the usual, well-explored responses found in literature regarding monistic theories of truth. I argue, however, that both

[11] These considerations apply equally well to the disjunctivist theory endorsed by Pedersen (with Cory Wright) in chapter 5 of this volume.

[12] Contrary to some, the nonclassical option is not the ‘easy way out’ of the paradoxes. Two nonclassical pluralist theories, along with the difficulties surrounding them, are given in detail in Beall (2008b) and Field (2008).

of these options are unnecessary. Instead of rejecting (*TS*) as a constraint on truth, one only needs to reject infinitary disjunction. Such a rejection is already well motivated by Curry's paradox. Moreover, the considerations that motivate such a rejection will apply to almost any nonclassical option for handling the paradoxes. And obviously, the *strong* alethic pluralist—who thinks there is no universal truth predicate—will have a vested interest in rejecting any method for constructing one.

The liar is not the only semantic paradox that proves difficult for truth theories. Curry's paradox, formulated by Haskell Curry (1942), relies on the conditional rather than negation. Given the usual diagonalization techniques, we can arrive at a self-referential sentence κ that is equivalent to $T(\ulcorner\kappa\urcorner) \to \bot$, where $\bot$ is some falsehood like "0 = 1."

Here is the problem. Assume for conditional proof $T(\ulcorner\kappa\urcorner)$. By the left–right direction of (*TS*), we get κ, which is just equivalent to $T(\ulcorner\kappa\urcorner) \to \bot$. By modus ponens, we have $\bot$. So, we have proved $T(\ulcorner\kappa\urcorner) \to \bot$. But then we've really also proved κ because they are equivalent. The right–left direction of (*TS*) gives $T(\ulcorner\kappa\urcorner)$. And we use this, by modus ponens, to yield $\bot$. But $\bot$ can't be true!

Greg Restall (2008) has given a general argument, based on very minimal constraints, on the difficulties that Curry's paradox brings. Here are the requirements:

(*TRAN*) $\vdash$ is transitive.

(*CONJ*) $\alpha \vdash \beta$ and $\alpha \vdash \gamma$ if and only if $\alpha \vdash \beta \wedge \gamma$.[13]

(*DISJ*) Infinitary disjunction is available in the language.[14]

(*WEAK-TS*) $T(\ulcorner\alpha\urcorner) \wedge \tau \vdash \alpha$ and $\alpha \wedge \tau \vdash T(\ulcorner\alpha\urcorner)$ where τ is any true sentence.

The assumptions are quite plausible, even for the nonclassical theorist. Moreover, the version of the T-scheme here is extremely weak. In fact, (*WEAK-TS*) requires only that from $T(\ulcorner\alpha\urcorner)$ *and* some conjunction of true background constraints τ, we can infer α. This is even weaker than what is sometimes called the 'rule-form' T-scheme.

The derivation of $\bot$ is a bit involved, but a few important points should be highlighted.[15] The reason we need infinitary disjunction is that it can be used as a *residual* of conjunction. A connective $\odot$ is the residual of conjunction if it satisfies (*RES*).

(*RES*) $\alpha \wedge \beta \vdash \gamma$ if and only if $\alpha \vdash \beta \odot \gamma$.

[13] This amounts to the algebraic constraint that $\wedge$ must be a greatest lower bound with respect to $\vdash$.

[14] Finite conjunctions satisfying (*CONJ*) must also distribute over infinitary disjunction. Algebraically, then, this requires the logic to be a distributive lattice, which is nearly always the case.

[15] For the full derivation, see Restall (2008: 265).

Many conditional connectives satisfy (*RES*), which is why conditionals are often used to generate Curry paradoxes. So the nonclassical option for the alethic pluralist will have to include only non-(*RES*) conditionals. But in the presence of infinitary disjunction, we can define a residual thus:

$$(\vee\text{-}RES)\quad \beta \odot \gamma := \bigvee \{\alpha \mid \alpha \wedge \beta \vdash \gamma\}$$

($\vee$-*RES*) defines a connective satisfying (*RES*).[16] And so, any theorist accepting (*TRAN*), (*CONJ*), (*DISJ*), and (*WEAK-TS*) will fall prey to Curry's paradox. Of these options, I think the lesson of this version of Curry is that (*DISJ*) must go.

Recall, however, the lessons of the liar. There were three rival options for the alethic pluralist: (i) rejecting (*TS*) for some truth predicate T_i; (ii) moving to a nonclassical logic; and (iii) rejecting infinite disjunctions required for constructing $\mathbb{T}$. However, Restall's Curry shows more.

First, for option (i), it will not be enough simply to reject (*TS*) for some truth predicate T_i. They must reject (*WEAK-TS*) for some T_i. That is, not even an *enthymematic* version of the 'rule-form' T-scheme can count as a necessary condition on being a truth predicate for a domain. That is a fairly drastic limitation, especially given the alternative options.

Secondly, consider option (ii): nonclassical logic. The choice to reject (*WEAK-TS*) will completely undermine the reason for going nonclassical when faced with the liar. So, the nonclassical alethic pluralist, too, will have to reject either (*TRAN*), (*CONJ*), or (*DISJ*). The former two are arguably essential features of validity and conjunction.[17] It is intriguing to note, however, that nonclassical pluralists might have an advantage over nonclassical monists: pluralists might endorse nonclassical logics as restricted only to a 'paradoxical' domain. While this route is intriguing, I will not explore it here.[18]

These results should cause the alethic pluralist to seriously consider option (iii). The pluralist can retain (*TS*) for each T_i by rejecting infinite disjunction given by DISJ. She can retain her uniquely pluralist response to the liar. Rejecting DISJ also solves the problem of Curry paradoxes constructible using

[16] *Proof* (due to Restall): For the left–right direction of RES, assume $\hat{\alpha} \wedge \beta \vdash \gamma$. Since $\hat{\alpha} \in \{\alpha | \alpha \wedge \beta \vdash \gamma\}$, we have it that $\hat{\alpha} \vdash \vee\{\alpha | \alpha \wedge \beta \vdash \gamma\}$. For the other direction, assume $\hat{\alpha} \vdash \vee\{\alpha | \alpha \wedge \beta \vdash \gamma\}$. So, $\hat{\alpha} \wedge \beta \vdash \beta \wedge \vee\{\alpha | \alpha \wedge \beta \vdash \gamma\}$. Distributing, we have $\hat{\alpha} \wedge \beta \vdash \vee\{\alpha \wedge \beta | \alpha \wedge \beta \vdash \gamma\}$. But obviously, $\vee\{\alpha \wedge \beta | \alpha \wedge \beta \vdash \gamma\} \vdash \gamma$, and so by transitivity of $\vdash$ we have the result.

[17] Neil Tennant (1994) has endorsed non-transitive systems of logic. However, none of his systems will help with Curry paradoxes. It should be noted that Alan Weir (2005) has argued for restricting a generalized cut rule, related to transitivity in order to avoid Curry paradoxes. His system is also nonclassical in other ways; it is paracomplete, and adjunction fails—$\alpha, \beta \nvdash \alpha \wedge \beta$.

[18] While distinct from the logical pluralism of Beall and Restall (2006), Lynch (2008) provides philosophical motivations for this domain-relative logical pluralism. In (forthcoming), I give a formal semantics consistent with this approach. I fully expect, however, that there will be expressive difficulties for such a pluralist. It may be hard to isolate the paradoxical sentences from the normal ones, for similar reasons as given in Beall (2013). See also the essays in Beall (2008a).

infinitary disjunction. Curry paradoxes constructed from the classical material conditional will take a form similar to $T_i(\ulcorner\kappa\urcorner) \supset \bot$. This is classically equivalent to $\neg T_i(\ulcorner\kappa\urcorner) \vee \bot$. But these paradoxes can be handled identically to liars: while κ is not true$_i$, it may well be true$_j$ for some $j \neq i$. Barring future and unforeseen paradoxes, the alethic pluralist may adhere to a fully classical logic.

The above considerations suggest that the alethic pluralist would do well to avoid infinitary disjunction. As I showed in §2, the *strong* alethic pluralist must deny infinitary disjunction in order to avoid a universal truth predicate. Moreover, the *weak* alethic pluralist is faced with limited options if she decides not to reject it.

4. Conclusion: looking ahead

The response above has some similarities to a Tarskian hierarchical view of truth (Tarski, 1983;[19] 1944). So it is worth pausing briefly to compare and contrast the views. At the start, one obvious difference between the two views is that a Tarskian view relativizes truth to a *language*, whereas the pluralist relativizes truth to a *domain* (defined here as disjoint proper subsets of a language). The Tarskian theory is constrained by the fact that languages are arranged hierarchically; language $\mathcal{L}_n$ is a proper subset of the distinct language $\mathcal{L}_{n+1}$. Domains, however, share no sentences in common, since they are disjoint from each other. Moreover, on the Tarskian view, truth-in-$\mathcal{L}_n$ is only well-defined in $\mathcal{L}_{n+1}$; that is, no two languages may share the same truth predicate. By contrast, domains may share the same truth predicate. There might be multiple domains for which, say, correspondence is the correct truth property.

This feature of the Tarskian theory is tied to a second difference between it and the pluralist theory outlined above. According to the Tarskian theory, Liar sentences are ruled out on *syntactic* grounds. No language can contain a truth predicate that applies to sentences in that language. So if T_o is the truth predicate for language $\mathcal{L}_o$, then any sentence containing the predicate T_o cannot be a sentence of L_o. A fortiori, no liar sentence $\neg T_o(\ulcorner\lambda_o\urcorner)$ is well-formed in L_o. According to the pluralist view, however, liar-like sentences arise at the syntactic level. Indeed, a sentence like $\neg T_1(\ulcorner T_1(\ulcorner\lambda_1\urcorner)\urcorner)$ is syntactically well formed. The only constraints regard which sentences can belong to which domains.

Third, one must consider why liar sentences involving a truth predicate T_i must be in domain$_j$ where $i \neq j$. Remember that for the Tarskian theory, truth-attribution involves semantic ascent. A pluralist, however, need not claim that truth-attribution requires *ascent* to a 'higher' language. She is free

[19] After this chapter was in press, Shapiro (2011) briefly suggested a similar approach.

to claim that a truth-attribution of a sentence in some domain must always be in a distinct, but not 'higher,' domain. Since a pluralist (usually) individuates domains by the what a sentence is about, the pluralist can claim that while a sentence like 'Torture is wrong' is about moral concepts, the sentence "'Torture is wrong' is true" is about the semantic properties of a sentence. This general answer extends straightforwardly to all sentences of the language, including paradoxical ones.[20]

Finally, it is worth noting that some alethic pluralists like Horgan (2001) think truth is relative not to domains, but to contexts. Pluralists of this stripe have very close ties to contextualist approaches to the semantic paradoxes. Rejecting (*DISJ*) could be seen as a consequence of rejecting absolutely unrestricted quantification,[21] the main difference being that pluralists view different contexts as inducing distinct truth predicates; this is something contextualists explicitly deny.[22]

If an alethic pluralist takes the recommended route, by rejecting infinite disjunctions and a universal truth predicate, there is still work to be done. To be sure that the proposal is completely free of any unforeseen paradoxes, it would be desirable to have a full consistency proof.

Second, since the rejection of infinite disjunctions blocks the most obvious route to a universal truth predicate, it can serve as a response to Pedersen's (2010) 'instability challenge' for strong alethic pluralism. Precise details would have to be given, including an explanation as to why the strong pluralist rejects the infinitary disjunction to generalize over the truth predicates she accepts.

Thirdly, the instability challenge is not the only problem to be answered; the problems of mixed compounds and mixed inferences pose difficulties to alethic pluralists. Indeed, strong alethic pluralism appears to be underpopulated in part due to these problems.[23]

Responses to each problem would have to be formulated. Fortunately, there are already some options on the table. In the last section of (2009), I outlined a solution to the problem of mixed compounds that avoids a universal truth predicate. It is compatible with the strong theory proposed above. Jc Beall (2000) appeals to designated values in many-valued logic to solve the problem of mixed inferences. It should be noted that Christine Tappolet (2000)

[20] Michael Lynch has pointed out in conversation (also in his essay in this volume) that such a view will clash with deflationary theories of truth, who generally accept that α and $T(\ulcorner\alpha\urcorner)$ have the same semantic content.

[21] See Rayo and Uzquiano (2006), and in particular Glanzberg's (2006) for arguments that could be marshaled in favor of the above approach.

[22] See, for example, Glanzberg (2004).

[23] The problem of mixed inferences is originally due to C. Tappolet (1997). The problem of mixed compounds is probably due to Tim Williamson (1994). Michael Lynch (2001; 2004; 2009) has given weak pluralist responses. For another proposed solution, see Edwards (2008); but see my (2009) and Edwards's (2009).

responded to Beall's solution by arguing that the notion of 'designatedness' amounts to a universal truth predicate, which apparently undermines the proposal. In order for the notion of designatedness, however, to be expressible in the object language, one would need the resources of infinitary disjunction. And so Tappolet's objection will not be a problem for the current proposal.[24]

To the alethic pluralist who is not sympathetic to rejecting infinite disjunctions: it is my hope that this chapter will lead the way for pluralists to discuss the semantic paradoxes and the uniquely pluralist options available. At the very least, they should not continue to be ignored.[25]

References

Beall, Jc. (2000). On mixed inferences and pluralism about truth predicates. *Philosophical Quarterly* 50: 380–382.

Beall, Jc. (ed.). (2008a). *Revenge of the Liar*. Oxford: Oxford University Press.

Beall, Jc. (ed.) (2008b). *Spandrels of Truth*. Oxford: Oxford University Press.

Beall, Jc. (2013). Deflated truth pluralism. In N. J. L. L. Pedersen & C. D. Wright (eds.), *Truth and Pluralism: Current Debates* (323–338). New York: Oxford University Press.

Beall, Jc & Restall, G. (2006). *Logical Pluralism*. Oxford: Oxford University Press.

Caret, C., & Cotnoir, A. J. (2008). True, false, paranormal, and 'designated'? A reply to Jenkins. *Analysis* 68: 238–244.

Cotnoir, A. J. (2009). Generic truth and mixed conjunctions: Some alternatives. *Analysis* 69: 473–479.

Cotnoir, A. J. (Forthcoming). Validity for strong pluralists. To appear in *Philosophy and Phenomenological Research*.

Curry, H. (1942). The inconsistency of certain formal logics. *Journal of Symbolic Logic* 7: 115–117.

Edwards, D. (2008). How to solve the problem of mixed conjunctions. *Analysis* 68: 143–149.

Edwards, D. (2009). Truth-conditions and the nature of truth. *Analysis* 69: 684–688.

Field, H. (2008). *Saving Truth From Paradox*. Oxford: Oxford University Press.

Glanzberg, M. (2004). A contextual-hierarchical approach to truth and the liar paradox. *Journal of Philosophical Logic* 33: 27–88.

Glanzberg, M. (2006). Context and unrestricted quantification. In A. Rayo and G. Uzquiano (eds.), *Absolute Generality* (45–75). Oxford: Oxford University Press.

Horgan, T. (2001). Contextual semantics and metaphysical realism: Truth as indirect correspondence. In M. Lynch (ed.), *The Nature of Truth* (67–95). Cambridge, MA: MIT Press.

Lynch, M. (2001). A functionalist theory of truth. In M. Lynch (ed.), *The Nature of Truth* (723–750). Cambridge, MA: MIT Press.

---

[24] Moreover, I have argued elsewhere (Caret & Cotnoir, 2008) that for purposes of the semantic paradoxes, designatedness need not be expressible. I give the problem of mixed inferences full treatmeant in my (forthcoming). See also Pedersen (2006) for another strong pluralist option.

[25] Thanks to Jc Beall, Colin Caret, Doug Edwards, Michael Lynch, Patrick Greenough, Nikolaj Pedersen, and Crispin Wright for many helpful discussions.

Lynch, M. (2004). Truth and multiple realizability. *Australasian Journal of Philosophy* 82: 384–408.

Lynch, M. (2006). ReWrighting pluralism. *Monist* 89: 63–84.

Lynch, M. (2008). Alethic pluralism, logical consequence and the universality of reason. *Midwest Studies in Philosophy* 32: 122–140.

Lynch, M. (2009). *Truth as One and Many: A Pluralist Manifesto*. Oxford: Oxford University Press.

Pedersen, N. J. L. (2006). What can the problem of mixed inferences teach us about pluralism? *Monist* 89: 103–117.

Pedersen, N. J. (2010). Stabilizing alethic pluralism. *Philosophical Quarterly* 60: 92–108.

Pedersen, N. J. L. L., & Wright, C. D. (2013). Pluralism about truth as alethic disjunctivism. In N. J. L. L. Pedersen & C. D. Wright (eds.), *Truth and Pluralism: Current Debates* (87–112). New York: Oxford University Press.

Rayo, A., and Uzquiano, G. (eds.). (2006). *Absolute Generality*. Oxford: Oxford University Press.

Restall, G. (2008). Curry's revenge: The costs of non-classical solutions to the paradoxes of self-reference. In Jc Beall (ed.), *Revenge of the Liar* (262–271). Oxford: Oxford University Press.

Shapiro, S. (2011). Truth, function and paradox. *Analysis* 71: 38–44.

Sher, G. (2004). In search of a substantive theory of truth. *Journal of Philosophy* 101: 5–36.

Sher, G. (2005). Functional pluralism. *Philosophical Books* 46: 311–330.

Tappolet, C. (1997). Mixed inferences: A problem for pluralism about truth predicates. *Analysis* 57: 209–210.

Tappolet, C. (2000). Truth pluralism and many-valued logics: A reply to Beall. *Philosophical Quarterly* 50: 382–385.

Tarski, A. (1944). The semantic conception of truth and the foundations of semantics. *Philosophy and Phenomenological Research* 4: 341–375.

Tarski, A. (1983). The concept of truth in formalized languages. In J. Corcoran (ed.), *Logic, Semantics, Metamathematics* (152–278). 2nd ed. Indianapolis: Hackett.

Tennant, N. (1994). The transmission of truth and the transitivity of deduction. In D. Gabbay (ed.), *What is a Logical System?* (161–177). Oxford: Oxford University Press.

Weir, A. (2005). Naive truth and sophisticated logic. In B. Armour-Garb & Jc Beall (eds.), *Deflationism and Paradox* (218–249). Oxford: Oxford University Press.

Williamson, T. (1994). A critical study of truth and objectivity. *International Journal of Philosophical Studies* 30: 130–144.

Wright, C. (1992). *Truth and Objectivity*. Cambridge, MA: Harvard University Press.

Wright, C. (2001). Minimalism, deflationism, pragmatism, pluralism. In M. Lynch (ed.), *The Nature of Truth* (751–787). Cambridge, MA: MIT Press.

Wright, C. D. (2005). On the functionalization of pluralist approaches to truth. *Synthese* 145: 1–28.