Rule-following and the Objectivity of Proof

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Ideas on meaning, rules and mathematical proofs abound in Wittgenstein's writings. The undeniable fact that they are present together, sometimes intertwined in the same passage of *Philosophical Investigations* or *Remarks on the Foundations of Mathematics*, does not show, however, that the connection between these ideas is necessary or inextricable. The possibility remains, and ought to be checked, that they can be plausibly and consistently separated. I am going to examine two views detectable in Wittgenstein's works: one about proofs, the other about meaning and rules. The first is the denial of the objectivity of proof. The second is a conception of meaning stemming from the rule-following considerations. I shall argue that, though Wittgenstein seems to conjoin the two views, they can be, and should be, separated[1].

In *Wittgenstein's Philosophy of Mathematics*, published in 1959, Michael Dummett wrote: «Wittgenstein's main reason for denying the objectivity of mathematical truth is his denial of the objectivity of *proof* in mathematics»[2]. The view that Dummett called '*the denial of the objectivity of proof*' can be summarized as follows:

   i)   symbolic configurations are proofs if, and only if, they are treated as proofs by the community.

In Wittgenstein's words:

   This is a demonstration for whoever acknowledges it as a demonstration. If anyone *doesn't* acknowledge it, doesn't go by it as a demonstration, then he has parted company with us even before anything is said[3].

On the other hand, we can call '*the objectivity of proof*' this principle:

   ii)   *a*) symbolic configurations can be treated as proofs by the community without being proofs and *b*) symbolic configurations can be proofs without being treated as proofs by the community.

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[1] Crispin Wright has already remarked: «the ideas on rules can motivate much of what Wittgenstein says about Platonism in the philosophy of mathematics, and about mathematical objectivity, and logical compulsion – and in general can explain his opposition to ideas about mathematics that overlook what we might call the 'anthropological contribution'. What they cannot explain are his distinctive remarks about proof and the status, in point of certainty, of the conclusions of proof», Wright (1991), in Wright (2001) p. 421.

[2] Dummett (1959), now in Dummett (1978) p. 184.

[3] Wittgenstein (1956) I § 61.

The objectivity of proof in this sense is more moderate than the view defended by Dag Prawitz that a proof exists «in a tenseless or abstract sense of exists»[4] even if it is never actually constructed by any cognitive subject. Clearly Prawitz's view implies that symbolic configurations can be proofs (if they express an abstractly existing proof) without being treated as proofs by the community. However, the converse does not hold: admitting the possibility of proofs which are not treated as proofs does not necessarily lead to Prawitz's view. According to the principle which we call 'the objectivity of proof', a symbolic configuration can be a proof without being treated as such, but this principle can refer to symbolic configurations which *are* actually constructed by cognitive subjects, so that there may be no need for us to say that a proof exists in an abstract and tenseless sense.

The objectivity of proof is supported by an obvious pre-theoretical intuition that the community can be wrong. The community can be wrong because it can fail to understand a new method of proof, with the result that a proof is not treated as a proof; and the community can be wrong because it can erroneously believe that a symbolic configuration is a proof: in the history of mathematics it is often the case that something is accepted as a proof, but later a mistake is discovered. When this happens it is usually concluded that, though previously considered a proof, the argument in question is not (and never was) really a proof.

Since the objectivity of proof is supported by pre-theoretical intuitions, many would agree with Dummett's protest that the denial of the objectivity of proof is «extremely hard to swallow»[5]. The problem is that a denial of the objectivity of proof seems to be a consequence of Wittgenstein's rule-following considerations. Dummett's reaction is to claim that «the celebrated 'rule-following considerations' embody a huge mistake»[6]. The mistake, according to Dummett, is «a general internalist premiss, that there is nothing to truth beyond our acknowledgement of truth»[7]. I am not convinced that the latter thesis must be a premiss of the rule-following considerations, if they are suitably interpreted. Thus I should like to find a different way of dealing with the problem without rejecting the rule-following considerations.

Can we accept the rule-following considerations and their consequences without abandoning the objectivity of proof? This question leads to another question: what is the content of the rule-following considerations? They may be given several interpretations. In particular, interpretations differ in detecting different *targets* of Wittgenstein's objections, i.e. in drawing different conclusions about the view which should be dismissed if the import of the rule-following considerations were properly appreciated. I shall briefly survey the readings of three influential authors: John McDowell, Saul Kripke and Crispin Wright. They would agree that the rule-

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[4] Prawitz (1987) p.153, cf. Prawitz (1998a) p. 287.
[5] Dummett (1978) p. 173.
[6] Dummett (1990), now in Dummett (1993) p. 460.
[7] *Ibidem.*

following considerations concern sentences which we might call 'meaning-sentences':

    a) Fabio understands the symbol '+' ;
    b) The symbol '+' means addition;
    c) Fabio means addition by '+';
    d) Fabio uses '+' according to the addition rule;
    e) '5+7=12' means that 5+7=12.

The rule-following considerations are about linguistic meaning and understanding. Up to this point there is agreement. But opinions are different about the conception of understanding which Wittgenstein attacks. According to John McDowell the target of Wittgenstein's rule-following considerations is the view that «understanding is always interpretation»[8]. If this is right, it seems clear that the rule-following considerations do not affect the objectivity of proof. The objectivity of proof can be denied. But it can be preserved as well, because to grant that understanding is not (always) interpretation does not prevent one from consistently maintaining that the meaning of a mathematical statement is fully determined through some other meaning-conferring fact which is not an interpretation. A fully determined meaning can fix in advance what counts as a proof of a statement independently of what speakers do or believe about proof-candidates, so that the objectivity of proof is preserved. On the other hand, it seems equally clear that, if McDowell's reading is right, the rule-following considerations are not very important. We don't need the rule-following considerations to show that the assumption that «understanding is always interpretation» is wrong. Actually, we don't need any elaborate argument: understanding S^1 can consist in finding an interpretation S^2 only if one understands S^2, but then (if understanding is always interpretation) one needs a further interpretation S^3, and so on. Thus one is lead into an infinite regress or into a vicious circle, which both would make understanding impossible. Since understanding *is* possible, the assumption from which we started, which equated understanding with interpretation, is wrong. If the foregoing reasoning holds, however, the rule-following considerations are superfluous. (Or do the rule-following considerations boil down to this elementary reasoning?)

    The rule-following considerations of Saul Kripke's Wittgenstein, according to a widespread picture[9], are aimed at showing the untenability of the view that meaning-sentences express *facts*: their conclusion is that there is no fact of the matter about how Fabio understands the symbol '+', what Fabio means, what agrees with the rule of addition. Though it has gained general acceptance, I believe this is not a right picture of Kripke's Wittgenstein: another picture harmonizes better with Kripke's text. To see why, we should remember that sentences like

    a*)   It is a fact that Fabio understands the symbol '+' ,
    b*)   It is a fact that the symbol '+' means addition,

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[8] McDowell (1984) pp. 325-363; now in McDowell (1998) pp. 221-262, cf. in particular pp. 229, 236, 242.
[9] Cf. Miller (1998) chap. 5 and Hale (1997) §§ 2-3.

c*)   It is a fact that Fabio means addition by '+',
d*)   It is a fact that Fabio uses '+' according to the addition rule,
e*)   It is a fact that '5+7=12' means that 5+7=12,

are perfectly meaningful for Kripke's Wittgenstein. They are meaningful because we master their use and, in particular, know their assertability conditions, which are immediately determined by the assertability conditions of the corresponding sentences (a)-(e)[10]. If the assertion "Fabio understands the symbol '+'" is justified, we shall be entitled to affirm "It is a fact that Fabio understands the symbol '+'".

Here the reader should be warned not to think that Kripke's Wittgenstein is employing the same notion of assertability condition advocated by Dummett as a central notion of a verificationist theory of meaning[11]. According to Dummett the assertability condition for a given mathematical sentence fixes what can count as its proof once for all, before we encounter such a proof and decide that it is a proof[12]. A grasp of an assertability condition of Dummett's kind would constitute what Kripke's Wittgenstein calls a *superlative fact*: a fact that *determines in advance the correctness of future uses*. This notion is just the notion we should reject according to the rule-following considerations. The real target of Kripke's Wittgenstein, as I understand him, is not the idea that a meaning-sentence expresses a fact, but precisely the idea that it expresses a 'superlative fact' [eine übermässige Tatsache] in which all possible correct uses of an expression are already contained[13].

> "It is as if we could grasp the whole use of the word in a flash." Like *what* e.g.? – Can't the use – in a certain sense – be grasped in a flash? And in *what* sense can it not? – The point is, that it is as if we could 'grasp in a flash' in yet another and much more direct sense than that. – But have you a model for this? No. It is just that this expression suggests itself to us. As the result of the crossing of different pictures.[14]

> You have no model of this superlative fact, but you are seduced into using a super-expression. (It might be called a philosophical superlative.)[15]

If we try to explain what a fact of this kind might consist in, our attempts end in failure. Nothing in behaviour or in mental life could constitute such a fact. This idea, therefore, ought to be discarded. Repudiating superlative meaning-facts prevents us from endorsing Dummett's conception of assertability conditions, but it is compatible with a more modest Wittgensteinian conception, according to which assertability conditions do not determine use in advance, but simply coincide with an aspect of actual use[16].

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[10] Cf. Kripke (1982) p. 86. See also Byrne (1996).
[11] Cf. Dummett (1976), now in Dummett (1993) pp. 70-80.
[12] Cf. Dummett (1978) pp. 176, 185.
[13] Cf. Kripke (1982) p. 69.
[14] Wittgenstein (1953) I §191.
[15] Wittgenstein (1953) I §192.
[16] Cf. Kripke (1982) pp. 90-92.

If the latter picture of Kripke's Wittgenstein is correct, then the target of Kripke's Wittgenstein is not very different from the target of Crispin Wright's Wittgenstein, which Wright himself describes through an analogy:

> The target – what I shall call *objectivity of meaning* – is the conception that the meaning of an expression stands to the unfolding tapestry of the way it is used in our linguistic practices as a person's character, according to a certain misconception of it, stands to his or her unfolding behaviour. The misconception would have it that character is, as it were, a finished design for a person's life which they usually act out, but which their behaviour may, at any particular stage, somehow betray[17].

Leaving the analogy, the idea is that, after a finite number of uses of an expression, its meaning is completely determined, once for all, in such a way that a corresponding meaning-sentence expresses a fact which is *fully accomplished.* Such an accomplished meaning-fact determines in what cases an expression is correctly used *before* those cases are actually encountered: the meaning is fixed and future uses will be either faithful or unfaithful to that meaning; if they are faithful, they are correct, if they are unfaithful, they are incorrect. To avoid misunderstanding, I prefer to adopt a terminology which is different from the one chosen by Wright in the quoted passage: I am going to call the view which is the target of Wright's Wittgenstein '*the complete determination of meaning*':

iii) after a finite number of uses of an expression *X*, the meaning of *X* is *completely* fixed, in such a way that the correctness (or incorrectness) of future uses of *X* in new unconsidered circumstances is determined *in advance*[18].

It seems to me that a criticism of the complete determination idea is central in Wittgenstein's texts. Such a criticism is clearly present, for example, in the following passages of the *Philosophical Investigations*:

> "All the steps are already taken" means: I no longer have any choice. The rule, once stamped with a particular meaning, traces the lines along which it is to be followed through the whole of space. – But, if something of this sort really were the case, how would it help? No; my description only made sense if it was to be understood symbolically. – I should have said: *This is how it strikes me*. When I obey a rule, I do not choose. I obey the rule *blindly*.[19]

Wittgenstein adds that the description according to which a meaning-rule «traces the lines along which it is to be followed through the whole of space» is «mythological»:

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[17] Wright (1986) p. 273, now Wright (2001) p. 55.
[18] Cf. Wright (1980) pp. 21, 216.
[19] Wittgenstein (1953) I §219.

> My symbolical expression was really a mythological description of the use of a rule.[20]

Thus the corresponding conception of meaning is only a myth[21].

Though I believe that the criticism of complete determination is a crucial theme of Wittgenstein's thought, my concern in the present paper is not to establish whether one of the three above mentioned interpretations is better supported by textual evidence than the others. They were all suggested by Wittgenstein's writings and, in any case, one can hardly disagree with Wright's remark:

> Whether Wittgenstein actually ever had exactly these, or similar, considerations in mind is a question of much less interest than what force attaches to them[22].

The "mythological" idea of a complete determination of meaning is a view which many consider right, and sometimes even obvious. It is connected with the objectivity of proof. One can infer the objectivity of proof from the complete determination of the meaning of a mathematical sentence. This is Prawitz's train of thought in the following passage:

> Once we have laid down what counts as canonical proofs, it is a factual matter whether an alleged proof amounts to such a canonical proof. If it is not a canonical proof, then it is again a factual matter whether the alleged proof yields a method for finding a canonical proof. Hence it should be clear that it is not our treating it as a proof that makes it a proof. [...] the question of whether something is a proof is fixed when the meanings are given, that is, when it is given what counts as a canonical proof[23].

Here Prawitz assumes that meanings can be *completely* given at the beginning, after some training in the use of mathematical sentences. In other words, he assumes the complete determination of meaning. A criticism of complete determination is, therefore, very interesting, and the criticism stemming from Wittgenstein's rule-following considerations has a remarkable force, in my opinion. That is why it deserves a careful study. I am not going to develop such a study. My aim is only to probe the connection between a rejection of complete determination and the denial of the objectivity of proof. But I shall briefly recall the two problematic features of complete determination which the criticism highlights.

The first, as we have seen, is a difficulty to understand the *nature* of a completely determined meaning-fact: what *is* a meaning-fact which enjoys the mysterious non-causal power to fix the correctness (or incorrectness) of all future uses in advance? Kripke's Wittgenstein shows that we are far from having any clear conception of what such a fact could be. To take for granted that this idea is acceptable, without really

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[20] Wittgenstein (1953) I §221.
[21] Cf. Wittgenstein (1953) I §§ 188-189.
[22] Wright (1986) p. 272, Wright (2001) p. 54.
[23] Prawitz (1998b) pp. 49-50.

explaining it, is – I believe – dogmatic. McDowell's Wittgenstein seems to be guilty of dogmatism when he suggests that there is nothing problematic with the idea of grasping the whole use of a word in a flash and that the mistake lies precisely in considering the notion a problem[24].

The second problematic feature is expounded by Wright starting from the analogy between the meaning of an expression and the character of a person in the above quoted essay *Rule following, meaning and constructivism*. Of the complete determination of meaning he gives a *reductio ad absurdum* which I should roughly summarize as follows: complete determination implies that, after having associated a certain meaning with an expression *X*, it is possible that the whole community *always* uses *X* wrongly, and this is absurd[25].

These two problems may convince us that the complete determination of meaning is wrong. But rejecting complete determination does not involve a rejection of the notion of meaning. We may adopt a different conception of meaning, which I propose to call '*the plasticity of meaning*':

iv)   the meaning of *X* is never completely determined or fixed: it is continuously moulded and shaped by our common use (i.e. use accepted by the community).

The plasticity of meaning agrees with Wittgenstein's idea that concepts are «pliable»[26]. Wright seems to favour this view when he writes that a correct conception of meaning «has to be compatible with the capacity of ongoing use to determine meaning»[27]. If the plasticity of meaning is taken to be the outcome of the rule-following considerations, a reformulation of our original problem suggests itself. The original problem was: can we accept the rule-following considerations and their consequences without abandoning the objectivity of proof? The reformulated problem is: can we consistently endorse both the plasticity of meaning and the objectivity of proof?

It may seem that the answer must be in the negative. A philosopher, let's call him Fred, can offer *an argument from the plasticity of meaning to the denial of the objectivity of proof*.

1) The meaning of *X* is never completely determined or fixed, it is continuously moulded and shaped by our common use.
2) Every common use of *X* contributes to the meaning of *X*.
3) Every common use of *X* is constitutive of the meaning of *X*.
4) Every common use of *X* is correct.
5) If the community treats (or does not treat) a proof-candidate for *X* as a genuine proof, the community is right.

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[24] Cf. McDowell (1984) p. 355, McDowell (1998) p. 258.
[25] Cf. Wright (1986).
[26] Wittgenstein (1956) IV §4.
[27] Wright (1986) p. 274, cf. 289, 293.

6) Symbolic configurations are proofs if, and only if, they are treated as proofs by the community

Fred's argument, however, has two weak points. One is the inference from 3 to 4. From the fact that a certain use is constitutive of the meaning of an expression, it does not follow that the use in question is in any sense correct (true or valid). For example, the assertion "one can always collect together into a set all the things satisfying a given description" was constitutive of the meaning of "set" in Cantor's set theory, but it led to Russell's paradox, and thus it was not true. A supporter of the doctrine that meaning-constitutive principles are analytically true or valid, might try to resist this objection. Nevertheless, even if Fred obstinately insists on the inference from 3 to 4, there is another weak point: the inference from 1 to 2 is also wrong, because it surreptitiously transforms the plasticity of meaning into a more specific claim, which I name '*the holistic plasticity of meaning*':

v) the meaning of X is never completely determined or fixed: it is continuously moulded and shaped by *all* our common use.

Plasticity and holistic plasticity are not equivalent. The reason is that we can distinguish between primitive uses and non-primitive uses. To some extent, the distinction may be viewed as a free development of Wittgenstein's distinction between rigid and fluid propositions in *On Certainty*[28].

vi) A use U of X in a language L is a **primitive use**, *if, and only if,* speakers of L
a) treat U in such a way that they neither acknowledge the possibility, nor the need of giving any justification of U ;
b) treat any deviation with respect to U as indicating a lack of understanding of X.

Primitive uses may be assertions or inferences. They are those uses about which we can say: «I have exhausted the justifications, I have reached the bedrock, and my spade is turned»[29]. Here are some examples of uses which count for us as primitive:

PRIMITIVE USES

α) $1+1 = 2$

β) $$\frac{A \qquad \text{if } A, \text{ then } B}{B}$$

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[28] Cf. Wittgenstein (1969b) §§ 96-97.
[29] Wittgenstein (1953) I §217.

γ)
$$\frac{\text{the successor of } u = \text{the successor of } t}{u = t}$$

δ)
$$\frac{x \text{ is prime,} \qquad z \neq x, \qquad z \neq 1}{x \text{ is not a multiple of } z}$$

Examples of non-primitive uses are the following:

NON-PRIMITIVE USES

ε) Every composite number can be expressed as a product of primes in a unique way (apart from the order).

ζ)
$$\frac{x \text{ is prime,} \qquad x \text{ divides a product } a^1 \times a^2 \times ... \times a^n}{x \text{ divides at least one factor } a^i \quad (n \geq i \geq 1)}$$

η)
$$\frac{x, y, z, n \text{ are positive integers} \qquad x^n + y^n = z^n}{n \leq 2}$$

If someone rejected δ, we would say that he (or she) does not understand "prime". But if ε or ζ is rejected, we shall simply conclude that our interlocutor does not know a certain theorem. The reader can easily apply corresponding considerations to the other examples. The distinction between primitive and non-primitive uses allows us to advance a more moderate thesis of plasticity, which does not entail holistic plasticity. Let's call it '*the primitive plasticity of meaning*':

vii) the meaning of X is never completely determined or fixed: it is continuously moulded and shaped by our common **primitive** uses of X (and possibly of some other related words). Non-primitive uses are *not* constitutive of meaning.

The main thesis I want to defend in this paper is that *primitive plasticity of meaning is compatible with the objectivity of proof*. To substantiate my claim I describe two possible epistemic courses.

The *first epistemic course* consists of five stages. First stage: the community accepts a proof P^1. Second stage: a dissident D is sceptical about P^1 and calls on those who accept P^1 to further elaborate and articulate it. Third stage: attempts at elaborating and articulating P^1 show that some steps in P^1 conflict with primitive uses. Fourth stage: the community admits that D's doubts were right and rejects P^1. Fifth stage: the community admits that P^1 is not and never was a proof. Thus, it seems right to say that at stage 1 of this epistemic course P^1 <u>is treated as a proof, but it isn't a proof</u>.

Also *the second epistemic course* consists of five stages. First stage: a single mathematician M offers a proof P^2. Second stage: the community, after examination of P^2, does not accept P^2. Third stage: M explains the new proof: he (or she) emphasizes certain steps of P^2, the structure of P^2, articulates the crucial steps and shows that they can be linked to primitive uses. Fourth stage: the community gradually comes to agree that M is right. Fifth stage: the community accepts P^2 as a proof and admits that it was a proof also at stage 2, though its being a proof was not understood yet. Thus it seems right to say that at stage 2 of this epistemic course <u>P^2 is a proof even if P^2 is not treated as a proof by the community</u>.

Such epistemic courses are quite common in mathematical practice. They show that the community implicitly acknowledges a principle that might be called '*pre-theoretical principle of self-corrigibility*':

viii)　if, under pressure of critical discussion, we articulated our judgments about some uses, we could discover that these judgments are wrong and then we ought to correct them.

Endorsement of self-corrigibility doesn't contradict the primitive plasticity of meaning. It does not, even if one assumes that from the mere fact that a certain use is constitutive of meaning it follows that the use in question is correct, true or valid. (As the reader will remember, this was the assumption underlying the step from 3 to 4 in Fred's argument above.) Even on this assumption, it is still consistent to maintain that common *primitive* uses mould and shape our meanings and, at the same time, self-corrigibility holds as far as *non-primitive* uses are concerned. If acceptance or rejection of P is not a primitive use, the reaction of the community to the deviations from the common attitude exhibited by D and M is not to behave as if such deviations simply indicated lack of understanding. Both parties (the community and the deviant mathematician) acknowledge that the disagreement can be settled by further arguments. Hence, in particular, the community acknowledges the possibility of being mistaken about P. If we frame self-corrigibility into a theory of meaning which distinguishes between primitive and non-primitive uses, we obtain a *principle of self-corrigibility for non-primitive uses*:

ix)　the community acknowledges that if, under pressure of critical discussion, judgments about non-primitive uses were articulated, they could turn out to be wrong and then self-correction would be required.

We can already conclude that primitive plasticity is compatible with the objectivity of proof. A description of the first epistemic course illustrates that for some P^1, P^1 is treated as a proof, but it isn't a proof. A description of the second epistemic course illustrates that for some P^2, P^2 is a proof, even if P^2 is not treated as a proof by the community. In a variant of the second epistemic course M disappears at the second stage and later another mathematician M^* reproposes P^2 in such a way that the community finally accepts it. At the second stage of the variant course P^2 is a concatenation of signs to which the community associates a meaning, but, since M is not there any more and M^* is not there yet, nobody treats P^2 as a proof: it is a proof, but it is not treated as a proof by anyone.

The assumption that being meaning-constitutive implies being valid or true does not impair my thesis that primitive plasticity is compatible with the objectivity of proof. However, I think we should reject that assumption: though a primitive use is *treated* by the community as immediately valid or *a priori* true, this doesn't guarantee that it *is* true or valid. It may be wrong: for example, it may be paradoxical[30]. This runs counter to some of Wittgenstein's statements. For example, in *Remarks on the Foundations of Mathematics*:

> We can conceive the rules of inference – I want to say – as giving the signs their meaning, because they are rules for the use of these signs. So that the rules of inference are involved in the determination of the meaning of the signs. In this sense rules of inference cannot be right or wrong[31].

Here Wittgenstein says that certain linguistic uses are instances of meaning-constitutive rules; therefore, he apparently says, these uses are neither right nor wrong and are beyond criticism. In other passages he maintains that some logical principles, like the excluded middle[32], or the classical law of double negation elimination[33], cannot be criticized and thus are immediately valid. This Wittgensteinian view, however, does not follow from the plasticity of meaning, nor seems to be a consequence of the rule-following considerations. The fact that a linguistic use constitutes the meaning of an expression does not make that use absolutely immune from criticism. It only makes the relevant criticism a different kind of criticism: a criticism of meaning.

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[30] Other ways in which a meaning-constitutive use can be wrong are considered in Cozzo (2002) §§ 22-24.

[31] Wittgenstein (1956) VII.§ 30, p. 398.

[32] Cf. Wittgenstein (1969a) II § 23.

[33] Already in 1931, in a conversation with Waismann, Wittgenstein defended the view that double negation elimination is constitutive of the meaning of negation and is thus beyond criticism, cf. Waismann (1967) VI p. 184. With different nuances the idea can be detected in *Philosophische Grammatik*, cf. Wittgenstein (1969a) I §14, in *Philosophische Untersuchungen*, cf. Wittgenstein (1953) p. 148, footnote b), and in *Remarks on the Foundations of Mathematics,* cf. Wittgenstein (1956) I §11.

Primitive uses can be criticized. But the corresponding criticism is different from a criticism of non-primitive uses. Criticism of primitive uses is criticism of language itself (if you prefer, criticism of concepts, or meanings, themselves). In the first epistemic course the dissident *D* might call on interlocutors to further elaborate and articulate also a use that is primitive for the community at that stage of enquiry. This would amount to throwing a meaning-constitutive principle into question, i.e. throwing a fragment of language into question. We know from the history of science that this is sometimes a decisive move. As far as mathematics is concerned, Lakatos has called "concept-stretching"[34] such a criticism and the modifications and improvements of the language that can arise from it. In particular cases it can be difficult to distinguish the latter kind of criticism from the more usual criticism which only demands a justification of non-primitive uses on the basis of primitive uses which implicitly constitute the common meanings of words. Both aspects are present in our concrete practice of criticizing arguments[35]. But the distinction is in principle clear and very important. By exploiting Wittgenstein's metaphor in *On Certainty* we might say that both the waters (non-primitive assertions and arguments within the language) and the river-bed (the language with its meaning-constitutive primitive uses) may move (may be improved by criticism). In particular cases, it can be difficult to make the distinction between the movement of the waters and the shift of the river-bed, but the difficulty does not eliminate the distinction: the waters couldn't move if there were no relatively firm and solid river-bed functioning as a channel for their flux[36]. Leaving the metaphor: no agreement about the acceptability of new arguments and assertions would be possible without a relatively stable shared epistemic background. Therefore arguments must be constructed on the basis of a background of commonly accepted primitive practices. But primitive practices too may be revised, in special circumstances.

I am aware that the notion of a criticism of language would require a more elaborate analysis[37]. But if one agrees that primitive uses can be rationally criticized, one will also agree that primitive uses cannot constitute analytic necessities "grounded in meanings". The main thesis of this paper, according to which the plasticity of meaning can consistently cohabit with the objectivity of proof, does not depend on acceptance of the further tenet that primitive uses are exposed to rational criticism, but this tenet strengthens the main thesis. In any case, the question was: "can we accept the rule-following considerations and their consequences without abandoning the objectivity of proof?". The answer is in the affirmative.

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[34] Cf. Lakatos (1976) pp. 83-99.

[35] This is the reason why Lakatos speaks of «intrinsic unity between the 'logic of discovery' and the 'logic of justification'», Lakatos (1976) p. 37.

[36] Cf. Wittgenstein (1969b) §§ 95-99.

[37] On this topic see Cozzo (2002) § 22.

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