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NUMBERS AND PROPOSITIONS: REPLY TO MELIA

By TIM CRANE

IS THE WAY we use propositions to individuate beliefs and other intentional states analogous to the way we use numbers to measure weights and other physical magnitudes? In an earlier paper [2], I argued that there is an important disanalogy. One and the same weight can be 'related to' different numbers under different units of measurement. Moreover, the choice of a unit of measurement is arbitrary, in the sense that which way we choose doesn't affect the weight attributed to the object. But it makes little sense to say that one and the same belief can be related to different propositions: different proposition means different belief. So there is no analogous arbitrary choice.

Joseph Melia disputes this [5]. He claims that on two theories of propositions, the same belief can be related to different propositions, so that 'which object a thinker is related to by having a particular belief *is* relative to an arbitrary choice' ([5], p. 46). One such theory is the neo-Russellian theory of propositions as ordered n + 1-tuples. Fred's belief that *a bears R to b* can be considered as a relation between Fred and the ordered triple <a,R,b>, or as a relation between Fred and the ordered triple <R,a,b>. These are different abstract objects, but since the choice between them doesn't affect the attribution of the belief that *a* *bears R to b* to Fred, it is arbitrary which we choose as 'the object' to which Fred is related.

The other theory is that propositions are sets of possible worlds. Fred's belief that *a bears R to b* can be considered as a relation between Fred and the set of worlds where *a* bears *R* to *b*, or as a relation between Fred and the set's characteristic function. Again, these are different abstract objects, but again it is arbitrary which we choose as 'the object' to which Fred is related.

I agree with Melia that in both these cases, there is no interesting question about which abstract object Fred is 'really' related to in having this belief. But what is the significance of this?

Melia thinks it undermines my claim that 'which abstract object a thinker is related to by having a particular belief is not relative to an arbitrary choice' ([5], p. 46). But I did not make this general claim. In fact, in my original paper I gave a counterexample to it: the same belief can be 'indexed' by different abstract *sentence*-types ([2], p. 228). Fred's belief that *snow is white* can be considered as a relation to the Italian sentence 'La neve è bianca' or the German 'Schnee ist weiss'. As with Melia's examples, the choice between these objects is arbitrary, in the sense that whichever way we choose, we attribute to Fred the belief that *snow is white*.

Melia's examples, and this one, all illustrate that the same belief can be considered as a relation to some different *abstract objects*, relative to an arbitrary choice. I have no quarrel with this claim, since in effect I made it in my original paper.

What I did deny was that the same belief could be related to two different *propositions*. But if this is right, and I accept Melia's examples, then doesn't it follow that ordered n + 1-tuples and sets of possible worlds cannot really be propositions? And doesn't this mean that I cannot be neutral on conceptions of propositions, as I claimed to be ([2], p. 224)?

In a sense, of course, the question 'are ordered n + 1-tuples *really* propositions?' is a silly one. 'Proposition' is a technical term, and we can define it as we wish. Our definitions come to have significance only because of the roles propositions play in our theories of mind and language. As Melia correctly says, once we have said what these theoretical roles are, we then use the term 'the proposition that *p*' to refer to whatever object plays this role ([5], p. 47).

What are these roles? One role is that identified by Frege: a proposition is 'something for which the question of truth arises' ([3], p. 20). Propositions are supposed to be the bearers of truth-values — so differences in proposition must correspond to differences in truth-value bearer. Another role is that of being the objects of propositional attitudes. Propositions are ways the thinker takes (or wants etc.) the world to be — so differences in proposition must correspond to differences in the way the world is taken (or wanted etc.) to be. (For more on these roles, and tensions between them, see [6], pp. 66–70; [4], pp. 54–5, 57–9.)

How does this apply to the difference between <a,R,b> and <R,a,b> as the content of Fred's belief that *a bears R to b*? This difference cannot correspond to a difference in truth-value bearer. Nor can it correspond to a way the world is represented as being. The belief that *a bears R to b* represents the world as being such that *a* bears *R* to *b* – and on Melia's neo-Russellian theory this is expressed just as much by <a,R,b> as by <R,a,b>.

So if propositions are supposed to play the roles mentioned above, the difference between <a,R,b> and <R,a,b> is not a difference in proposition, in any theoretically interesting sense of that term. From the perspective of the theory of propositions, whatever <a,R,b> can do, <R,a,b> can do too, and vice versa. And likewise with a set of possible worlds and its characteristic function.

Perhaps we can minimize dispute here by talking of 'proposition-roles' instead of propositions. Proposition-roles are different when there is a difference in the theoretical roles they play in giving an account of some semantic or psychological fact. Given this distinction, I can then agree with the neo-Russellian that <a,R,b> and <R,a,b> are different propositions. But I can say that they play the same proposition-role. And likewise with a set of possible worlds and its characteristic function.

Melia and I should therefore agree that (a) the same belief can be indexed to certain distinct objects (e.g. sentences, ordered n+1-tuples etc.); and that (b) the same belief cannot be indexed to distinct proposition-roles. I think (a) was explicit in my original paper. (b) should have been, and I am grateful to Melia for alerting me to the need to make this point clear.

So once we distinguish between the abstract objects some philosophers call 'propositions' and the theoretical roles they play, it is apparent that there is no real dispute between Melia and me. However, it is worth noting that a loosely analogous distinction can be made in the case of numbers (see [1]). On the theory that numbers are sets, the number 3 can be thought of as the set $\{\emptyset,\{\emptyset\},\{\emptyset,\{\emptyset\}\}\}$ or as the set $\{\{\{\emptyset\}\}\}$. The choice between these two is arbitrary from the point of view of number theory, in the sense that what we can do with the number 3 is not affected by that choice. Yet they are different abstract objects.

We might conclude from this, a Benacerraf does, that whatever else they are, numbers 'could not be sets at all' ([1], p. 285). Or we might maintain that numbers are sets, but distinguish between numbers and number-roles, as we have distinguished between propositions and proposition-roles. We might then say (to echo Melia) that when we talk of the number 3, we are not talking about an object but about 'a certain role an object may play'. We would then *call* whatever object plays the number-3-role 'the number 3'. The set $\{\emptyset,\{\emptyset\},\{\emptyset,\{\emptyset\}\}\}$ or the set $\{\{\{\emptyset\}\}\}$ could play this role but the choice between them is arbitrary.

It doesn't matter here whether this is the right thing to say about numbers. I only want to point out that the choice between {Ø,{Ø},{Ø,{Ø}}} and {{{Ø}}} as occupants of the number-3-role is analogous to the choice between <a,R,b> and <R,a,b> as occupants of the proposition-that-*a bears R to b*-role. This provides a useful way of illustrating my original point, as follows.

Suppose we settle, by arbitrary stipulation, the choice between <a,R,b> and <R,a,b>, and the analogous choice between {Ø,{Ø},{Ø,{Ø}}} and {{{Ø}}}. Then in order to measure weight and other magnitudes, there is a further arbitrary choice to make: the choice of a unit of measurement. But there is no such further choice to make in the case of belief and other attitudes. Or to put the matter in terms of 'roles': the weight of a bag of sugar is preserved across variations in the 'number-roles' used to index it. But the belief that *a bears R to b* cannot be preserved across variations in 'proposition-role' — as Melia in effect allows ([5], p. 47). This is the disanalogy I argued for in my original paper. The mere fact that the same belief can be construed as a relation to many different abstract objects is irrelevant.

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