Probability, Evidential Support, and the Logic of Conditionals

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Abstract

Once upon a time, some thought that indicative conditionals could be effectively analyzed as material conditionals. Later on, an alternative theoretical construct has prevailed and received wide acceptance, namely, the conditional probability of the consequent given the antecedent. Partly following critical remarks recently appeared in the literature, we suggest that evidential support—rather than conditional probability alone—is key to understand indicative conditionals. There have been motivated concerns that a theory of evidential conditionals (unlike their more traditional counterparts) cannot generate a sufficiently interesting logical system. Here, we will describe results dispelling these worries. Happily, and perhaps surprisingly, appropriate technical variations of Ernst Adams's classical approach allow for the construction of a logic of evidential conditionals with distinctive features, which is also well-behaved and reasonably strong.

Keywords: Conditionals, Probability, Evidential support, Suppositional, Transitivity.

1. Introduction

According to a very influential view, the assessment of a conditional statement amounts to the assessment of the conditional probability of its consequent given its antecedent. Put forward by Adams in the Sixties and Seventies (see Adams 1966, 1975), this idea has spread over the decades in many research areas in philosophical logic and the philosophy of language (see Bennett 2003 and Edgington 2020 for useful overviews), and ended up becoming a matter of substantial consensus in the psychology of reasoning (see Evans and Over 2004, and Oaksford and Chater 2010). In addition, it served as a building block for popular logical systems (e.g., Hawthorne 1996, Leitgeb 2004, Thorn and Schurz 2014).

The analysis in terms conditional probability may seem natural, but it is bound to miss a crucially important feature, namely the presence of a link of relevance between antecedent and consequent in a compelling conditional statement (see, e.g., Douven 2008, Krzyżanowska, Wenmackers, and Douven 2013, Skovgaard-Olsen, Singmann, and Klauer 2016, Spohn 2015). In this paper, we take this critical remark seriously and explore an alternative view. As we will see, the

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notion of evidential support can be represented in a probabilistic framework and exploited to characterize a logic of conditionals—*evidential* conditionals—conveying the idea that the antecedent provides evidence that the consequent holds.

Probabilistic approaches have been popular among authors (such as Edgington 1986, 2007) who firmly reject the idea of truth-conditions for non-material conditionals. Even though our work is consistent with this line of thought, we do not see our arguments here as incompatible with the view that conditionals have truth-conditions. Conversely, that view does not necessarily rob a probabilistic analysis of its legitimacy and theoretical potential, or so we believe (a similar approach is illustrated in different ways by Crupi and Iacona 2020, Égré, Rossi, and Sprenger 2021, and Douven 2016 himself). As a consequence, we take our treatment of conditionals to be essentially neutral with respect to the question whether conditionals have truth-conditions.

The paper is organized as follows. First, we outline a traditional approach to a probabilistic logic of conditionals (2.), following Adams's (1975, 1998). Second, we explain why evidential support should play a critical role in a satisfactory analysis of indicative conditionals and we reconstruct Douven's (2016) attempt to develop this project (3.). We then address some key principles of conditional logic (4.). Here, we argue that the logic of evidential conditionals should differ from both the traditional account and Douven's (2016) earlier attempt, and a logic with the desired features is presented accordingly (5.). We also analyze one specific case more closely (transitivity, 6.), and finally collect some brief concluding remarks (7.).

2. Conditional Probability and Suppositional Conditionals

Adams (1965) famously argued that the material conditional of classical propositional logic is not satisfactory as a model of the logical behavior of indicative conditionals.[1] Informally, Adams started from the idea that an indicative (simple) conditional can be assigned a degree of "assertability" (Adams 1966). The degree of assertability of "if α then β", in turn, is strictly related to the conditional probability $Pr(\beta | \alpha)$, namely, the probability of the consequent *on the supposition* that the antecedent holds. For this reason, we will call *suppositional* this kind of conditionals. For our purposes, a precise rendition of the ensuing probabilistic logic of conditionals can be given as follows.

Syntax. Let $\boldsymbol{P}$ be a propositional language with a (finite) set of sentence letters p, q, r … and the usual connectives, $\sim$, $\wedge$, $\vee$, $\supset$. Formulas in $\boldsymbol{P}$ are called *propositional formulas*, and $\vDash_{PL}$ will denote classical logical consequence in $\boldsymbol{P}$. We then define a language $\boldsymbol{L}_{\Rightarrow}$ including a further conditional symbol $\Rightarrow$:

– if $\alpha \in \boldsymbol{P}$, then $\alpha \in \boldsymbol{L}_{\Rightarrow}$;
– if $\alpha,\beta \in \boldsymbol{P}$, then $\alpha \Rightarrow \beta \in \boldsymbol{L}_{\Rightarrow}$;
– if $\alpha \in \boldsymbol{L}_{\Rightarrow}$, then $\sim\alpha \in \boldsymbol{L}_{\Rightarrow}$.

Note that $\boldsymbol{L}_{\Rightarrow}$ so defined leaves out embeddings and compounds of formulas with $\Rightarrow$, but allows for (iterated) negation of such formulas.

Semantics. For any standard probability function Pr over $\boldsymbol{P}$, we define a valuation function $V_{Pr} : \boldsymbol{L}_{\Rightarrow} \rightarrow [0,1]$ as follows:

[1] This negative conclusion is now popular. Williamson (2020) is a notable recent exception.

– for every $\alpha \in \boldsymbol{P}$, $V_{Pr}(\alpha) = Pr(\alpha)$;
– $V_{Pr}(\alpha \Rightarrow \beta) = Pr(\beta | \alpha)$, with $V_{Pr}(\alpha \Rightarrow \beta) = 1$ in case $Pr(\alpha) = 0$;
– $V_{Pr}(\sim\alpha) = 1 - V_{Pr}(\alpha)$.

V_P can be seen as representing the degree of assertability of sentences, including simple non-material suppositional conditionals of the form $\alpha \Rightarrow \beta$, and their (possibly iterated) negations.

Validity. For $\alpha_1, \ldots, \alpha_n, \beta \in \boldsymbol{L}_{\Rightarrow}$, $\alpha_1, \ldots, \alpha_n \vDash \beta$ if and only if, for any Pr, $\sum_1^n[1 - V_{Pr}(\alpha_i)] \geq 1 - V_{Pr}(\beta)$. In Adams's (1975, 1998) terminology, the *lack* of assertability of α, namely $1 - V_{Pr}(\alpha)$, is labelled *uncertainty*. So, according to the definition above, an argument is valid if and only if the uncertainty of the conclusion cannot exceed the total uncertainty of the premises. More informally, one can say that in a valid argument a high degree of assertability of the premises implies a high degree of assertability of the conclusion (at least when the premises are not too many). Importantly, as Adams (1998: 151) points out, if $\alpha_1, \ldots, \alpha_n, \beta \in \boldsymbol{P}$, then $\alpha_1, \ldots, \alpha_n \vDash_{PL} \beta$ if and only if $\alpha_1, \ldots, \alpha_n \vDash \beta$, so the probabilistic notion of validity recovers all classically valid inferences for the propositional fragment of the language (also see Suppes 1966). Moreover, substitution of (classically) logically equivalents also holds in this logic (Crupi and Iacona 2021).

The logic of the suppositional conditional $\Rightarrow$ is well understood (see, e.g., Leitgeb 2004, Ch. 3, and Crupi and Iacona 2021). In particular, $\Rightarrow$ is known to validate the following ten important principles, where "$>$" denotes a generic conditional, "T" stands for tautology, and the long arrow "$\Longrightarrow$" indicates valid inference:

1. *Superclassicality* (SC):	If $\alpha \vDash_{PL} \beta$, then $\alpha > \beta$ must hold
2. *Modus Ponens* (MP):	$\alpha > \beta, \alpha \Longrightarrow \beta$
3. *Conjunction of Consequents* (CC):	$\alpha > \beta, \alpha > \gamma \Longrightarrow \alpha > (\beta \wedge \gamma)$
4. *Disjunction of Antecedents* (DA):	$\alpha > \gamma, \beta > \gamma \Longrightarrow (\alpha \vee \beta) > \gamma$
5. *Cautious Monotonicity* (CM):	$\alpha > \beta, \alpha > \gamma \Longrightarrow (\alpha \wedge \gamma) > \beta$
6. *Right Weakening* (RW):	If $\beta \vDash_{PL} \gamma$, then $\alpha > \beta \Longrightarrow \alpha > \gamma$
7. *Limited Transitivity* (LT):	$\alpha > \beta, (\alpha \wedge \beta) > \gamma \Longrightarrow \alpha > \gamma$
8. *Rational Monotonicity* (RM):	$\alpha > \beta, \sim(\alpha > \sim\gamma) \Longrightarrow (\alpha \wedge \gamma) > \beta$
9. *Conjunction Sufficiency* (CS):	$\alpha \wedge \beta \Longrightarrow \alpha > \beta$
10. *Conditional Excluded Middle* (CEM):	$\sim(\alpha > \beta) \Longrightarrow \alpha > \sim\beta$

It is also well-known that the following principles are *not* valid for the suppositional conditional:

11. *Monotonicity* (M):	$\alpha > \gamma \Longrightarrow (\alpha \wedge \beta) > \gamma$
12. *Transitivity* (T):	$\alpha > \beta, \beta > \gamma \Longrightarrow \alpha > \gamma$
13. *Contraposition* (C):	$\alpha > \beta \Longrightarrow \sim\beta > \sim\alpha$

3. The Role of Evidential Support

In order to assess the logical profile of $\Rightarrow$ relative to principles (1)-(13), one of course has to keep in mind what $\alpha \Rightarrow \beta$ is meant to represent in the first place, that is, roughly, a statement that the consequent β is credible *on the supposition that* the circumstance described by the antecedent α obtains. Adams's extensive work has rather effectively supported the adequacy of his logic relative to its target explicandum (see Adams 1965, 1998: Ch. 6), and we can take for granted his view here. However, whether or not the assertability of indicative conditionals is fully captured by the theory of suppositional conditionals is a separate matter. According to McGee (1989), for instance, Adams's theory "describes what English speakers assert and accept with unfailing accuracy" (485), but Douven (2008) has put forward a forceful counterargument. One of Douven's key illustrations involved a long series of tosses of a fair coin, and a comparison between two statements:

(*) If there's a head in the first ten tosses, then there will be a head in the first 1.000.000 tosses.

(**) If Barcelona wins the Champions league, then there will be a head in the first 1.000.000 tosses.

The probabilities of the consequent given the antecedent in (*) and (**) are only minutely different (and one can make them converge at will, by just increasing the number of tosses). And yet, Douven (2008) points out, (**) appears vastly less compelling than (*), which then raises a crucial problem.[2]

Douven's proposed solution is that the plausibility of a (simple, indicative) conditional is accounted for by the conditional probability of the consequent given the antecedent, but only on the additional proviso that the antecedent gives *evidential support* to the consequent. Following the standard probabilistic construal of evidential support or incremental confirmation (e.g., Earman 1992, Fitelson 1999, Crupi 2015, 2020), this means that the conditional probability of the consequent given the antecedent must be *higher than* the unconditional probability of the consequent itself. For our purposes, Douven's own theory (developed in Douven 2016) can be specified as follows.

Syntax. Let $\boldsymbol{P}$ be a propositional language as above. We then define a language $\boldsymbol{L}_{\rightarrow}$ including a further conditional symbol $\rightarrow$:

– if $\alpha \in \boldsymbol{P}$, then $\alpha \in \boldsymbol{L}_{\rightarrow}$;
– if $\alpha,\beta \in \boldsymbol{P}$, then $\alpha \rightarrow \beta \in \boldsymbol{L}_{\rightarrow}$;
– if $\alpha,\beta \in \boldsymbol{L}_{\rightarrow}$, then $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \supset \beta \in \boldsymbol{L}_{\rightarrow}$;
– if $\alpha \in \boldsymbol{L}_{\rightarrow}$ then $\sim\alpha \in \boldsymbol{L}_{\rightarrow}$.

Language $\boldsymbol{L}_{\rightarrow}$ so defined allows for formulas with $\rightarrow$ to appear in the scope of all other connectives, but still leaves out embeddings, i.e., formulas with $\rightarrow$ occurring within the scope of $\rightarrow$ itself.

Semantics. For any standard probability function Pr over $\boldsymbol{P}$ and a threshold t ($1/2 \leq t \leq 1$) we define a valuation function $V_{(Pr,t)} : \boldsymbol{L}_{\rightarrow} \rightarrow \{0,1\}$ as follows.

– For every $\alpha \in \boldsymbol{P}$, $V_{Pr,t}(\alpha) = 1$ if and only if $Pr(\alpha) > t$; and $V_{Pr,t}(\alpha) = 0$ otherwise.

[2] One reaction could be to acknowledge that (*) is largely more assertible than (**) while insisting that they are both *true*, thus semantically on a par (Douven 2016: 105-07, discusses a Gricean variant of this reply, and finds it eventually defective). However, there also exist truth-conditional treatments of conditionals making (*) true and (**) false on very plausible assumptions. A major example is the strict conditional view (see, e.g., Lycan 2001, Gillies 2009, and Kratzer 2012), another one is the modal semantics for the evidential conditional presented in Crupi and Iacona (2020). So the relevance of Douven's example is not limited to a non-propositional view of conditionals, and reliance on the example does not presuppose such a view.

– $V_{Pr,t}(\alpha \rightarrow \beta) = 1$ if and only if: (i) $Pr(\beta | \alpha) > t$ and $Pr(\beta | \alpha) > Pr(\beta)$, or (ii) $Pr(\alpha) = 0$, or (iii) $Pr(\beta) = 1$; $V_{Pr,t}(\alpha \rightarrow \beta) = 0$ otherwise.
– $V_{Pr,t}(\alpha \wedge \beta) = 1$ if and only if $V_{Pr,t}(\alpha) = 1$ and $V_{Pr,t}(\beta) = 1$; $V_{Pr,t}(\alpha \wedge \beta) = 0$ otherwise.
– $V_{Pr,t}(\alpha \vee \beta) = 1$ if and only if $V_{Pr,t}(\alpha) = 1$ or $V_{Pr,t}(\beta) = 1$; $V_{Pr,t}(\alpha \vee \beta) = 0$ otherwise.
– $V_{Pr,t}(\sim\alpha) = 1$ if and only if $V_{Pr,t}(\alpha) = 0$; $V_{Pr,t}(\sim\alpha) = 0$ otherwise.

In Douven's (2016) favorite terminology, $V_{Pr,t}(\alpha) = 1$ and $V_{Pr,t}(\alpha) = 0$ are meant to represent that α is acceptable / not acceptable, respectively, relative to probability distribution Pr and threshold t. So this framework embeds a version of so-called Lockean thesis for qualitative rational acceptability.

Validity. For $\alpha_1, \ldots, \alpha_n, \beta \in \boldsymbol{L}_{\rightarrow}$, $\alpha_1, \ldots, \alpha_n \vDash \beta$ if and only if, for any Pr and any t, if $V_{Pr,t}(\alpha_1) = \ldots = V_{Pr,t}(\alpha_n) = 1$ then $V_{Pr,t}(\beta) = 1$, too. According to this definition, an argument is valid if and only if it preserves rational qualitative acceptability. Substitution of (classically) logically equivalents also holds in this logic. Importantly, however, there exist straightforward cases in which $\alpha_1, \ldots, \alpha_n \vDash_{PL} \beta$ for $\alpha_1, \ldots, \alpha_n, \beta \in \boldsymbol{P}$, and yet $\alpha_1, \ldots, \alpha_n \nvDash \beta$, so many classically valid inferences for the propositional fragment of the language are *not* recovered.

4. A Second Look at Logical Principles

Before presenting a discussion of $\Rightarrow$ and $\rightarrow$ and our own alternative proposal, it is important to emphasize our premises and current goals. First, here we will not challenge the idea that Adams's theory captures the logic of the suppositional conditional as a target explicandum. However, and second, we concur with Douven that a non-material indicative conditional conveying evidential support deserves being modeled to account for certain reliable and important intuitions. In fact, the idea of evidential support clashes with some of the principles that are valid for the suppositional conditional. The principles (6) to (10), in particular, turn out to be increasingly questionable in this perspective. Let us discuss them in turn.

Consider Right Weakening first, one of the most entrenched and technically powerful rules of conditional logic (see, e.g., Nute 1980: 52-3). At least since the debate between Hempel (1945) and Carnap (1962), it is clear that evidential support must fail so-called "special consequence condition": take contingent $\alpha,\beta \in \boldsymbol{P}$, probabilistically independent for some Pr, then surely α will support $\alpha \wedge \beta$ but not β—namely, $Pr(\alpha \wedge \beta \mid \alpha) > Pr(\alpha \wedge \beta)$, while $Pr(\beta | \alpha) = Pr(\beta)$—despite the fact that, obviously, $\alpha \wedge \beta \vDash_{PL} \beta$. Such failure of the special consequence condition arguably carries over to conditionals in the evidential sense. For an illustration, consider an uncertain election with five candidates, Anna, Barbara, Robert, Steven, and Ted, in strictly decreasing order of strength. The conditional "if Anna does not win the election (α), then a woman (Anna or Barbara) will win the election (γ)" is odd in terms of evidential support: in fact, finding out that α must make γ less probable than it was otherwise, and thus provides evidence *against* γ, if anything.[3] Given the assumed ranking of the candidates, however, one might

[3] Proof: given that $\sim\gamma \vDash_{PL} \alpha$ and $1 > Pr(\alpha)$, $Pr(\sim\gamma) > 0$, we have (by Bayes's theorem) that $Pr(\sim\gamma | \alpha) > Pr(\sim\gamma)$, and therefore $Pr(\gamma | \alpha) < Pr(\gamma)$.

well consider "if Anna does not win the election (α), then Barbara will win (β)" compelling, for the antecedent might increase the probability of the consequent significantly. So, intuitively, we have a case where "if α then β" is compelling while "if α then γ" is not, even if β entails γ—which is against RW.

Puzzling cases also arise concerning the related principle of Limited Transitivity (LT entails RW, given SC, see Crupi and Iacona 2020). Here is an illustration. Suppose you don't know who won Wimbledon in July 2018, and consider the following:

α = ATP #2 player in early July 2018 (Nadal) didn't win Wimbledon 2018
β = ATP #1 player in early July 2018 (Federer) did win Wimbledon 2018
γ = ATP #3 player in early July 2018 (Zverev) didn't win Wimbledon 2018

Take the instance of LT consisting in the inference from "if α then β" and "if α and β, then γ" to "if α then γ". The first premise, "if α then β", is plausible: for someone who does not know the outcome, getting to know that Nadal didn't win definitely is favorable evidence for Federer having been the winner.[4] The second premise, "if α and β, then γ", is straightforward; given the obvious background assumption that only one player wins Wimbledon, the consequent γ is implied by the antecedent, α and β. And yet the conclusion "if α then γ" is completely unsound in terms of evidential support: if Nadal didn't win, then the likelihood that a competitor such as Zverev *did* win must increase. As small as such increase may be, it definitely implies that α is *not* evidence in favor of γ.[5] In a case like this, assuming that conditionals convey evidential support, LT would license an inference from plausible premises to a very implausible conclusion.

Rational Monotonicity is also a rather popular principle in the literature. Still, we submit that an evidential conditional "if α then β" does not license the conclusion that "if α and γ, then β", even under the additional proviso that "not: if α then not-γ". Adapting a well-known example (Pearl 1988: Ch. 2), suppose a house alarm (α) can be triggered (normally and appropriately) by burglary (β), but also (rarely and accidentally) by an earthquake (γ). Then it makes sense to say that "if the alarm is activated (α), then burglary is happening (β)" in the evidential sense. But it would not be sensible to conclude that "if the alarm is activated (α) and an earthquake occurred (γ), then burglary is happening (β)". Yet the additional premise "not: if α then not-γ" would also be sound in this case, for the alarm is surely not evidence against the occurrence of an earthquake (in fact, it is an indication in favor of that, although a feeble one).

Conjunction Sufficiency, in turn, is a textbook case of a logical principle which should not be valid for a conditional implying evidential support. Two statements α and β can well jointly hold—say, "Mary went to the party last night" and "there was a full moon last night"—in absence of any connection of evidential support between them, so that "if α then β" (or "if β then α", for that matters) is pointless in the evidential sense. For similar reasons, CEM is clearly ruled out. By denying, for instance, the conditional "if Planet Nine exists, then the European Union will collapse within 5 years" for lack of a connection of evidential support between antecedent and consequent one is in no way logically committed

[4] As a matter of fact, the 2018 winner of Wimbledon was Novak Djokovic, then ranked #21.

[5] Here again, $Pr(\gamma | \alpha) < Pr(\gamma)$ because $\sim\gamma \vDash_{PL} \alpha$ (see footnote 1).

to accept "if Planet Nine exists, then the European Union will not collapse within 5 years". Here again, an underlying formal relationship should be noted, because CEM entails CS (given MP, see Crupi and Iacona 2020).

There is plenty of reasons, then, to think that the logic of a non-material conditional conveying the idea of evidential support should depart from the logic of $\Rightarrow$. Douven's theory accounts for the fact that principles (6)-(10) seem unsound as explained above, but at a very high cost: in fact, only one of principles (1)-(13) is validated by the conditional $\rightarrow$, namely SC (see Douven 2016, § 5.2). This is surely too much of a sacrifice. A failure of Modus Ponens, for instance, makes one doubt whether the very name 'conditional' is appropriate (see, e.g., van Fraassen 1976, p. 277). We conclude that, while Douven's project is important, his specific proposal suffers from significant limitations. The question naturally arises, then, if the idea of evidential support can be developed so that a more robust logic ensues.

5. The logic of Evidential Conditionals

Our proposal is to recover the role of evidential support by revamping a much older view, sometimes associated with the ancient Stoic logician Chrysippus (see Sanford 1989, p. 25, and Lenzen 2019, pp. 15-19, for discussion). According to this view, whether a (simple) conditional statement "if α then β" holds has to do with a relationship of incompatibility between the antecedent, α, and the negated consequent, $\sim\beta$. To flesh this out in probabilistic terms, we will need a probabilistic measure of "incompatibility". How should this be defined? First, it should be symmetric, so that the degree of incompatibility of α with $\sim\beta$ equals the degree of incompatibility of $\sim\beta$ with α for any given probability distribution Pr. Second, it should be maximal (that is, 1) in case $Pr(\alpha \wedge \sim\beta) = 0$. Third, it should be minimal (that is, 0) in case α and $\sim\beta$ are either probabilistically independent or positively correlated, namely, if $Pr(\alpha \wedge \sim\beta) \geq Pr(\alpha)Pr(\sim\beta)$. The simplest way to meet these constraints, it turns out, is to represent the degree of incompatibility between α and $\sim\beta$ as:

$$1 - \frac{Pr(\alpha \wedge \sim\beta)}{Pr(\alpha)Pr(\sim\beta)}$$

provided that $Pr(\alpha \wedge \sim\beta) \leq Pr(\alpha)Pr(\sim\beta)$, and 0 otherwise. For the limiting cases in which $Pr(\alpha) = 0$ or $Pr(\beta) = 1$, and thus $Pr(\alpha)Pr(\sim\beta) = 0$, the default option is to say that incompatibility is still maximal (i.e., 1) for then again $Pr(\alpha \wedge \sim\beta) = 0$. Our proposal is just to equate the assertability of an *evidential conditional* "if α then β" to the degree of incompatibility between α and $\sim\beta$ thus defined. Appropriate connections with evidential support are thereby promptly recovered. First, as long as $Pr(\beta) < 1$, "if α then β" turns out to be assertable to some degree at least only if assuming α increases the probability of β, that is, only if $Pr(\beta | \alpha) > Pr(\beta)$. Moreover, "if α then β" is maximally assertable in case α makes β certain, that is, when $Pr(\beta) < 1$ and $Pr(\beta | \alpha) = 1$. In fact, our measure of the incompatibility of α and $\sim\beta$ is identical to the measure of Bayesian confirmation as partial entailment of β by α investigated by Crupi and Tentori (2013, 2014). A probabilistic logic of evidential conditionals can now be spelled out accordingly, as follows.

Syntax. Let $\boldsymbol{P}$ be a propositional language as defined above. We then define a language $\boldsymbol{L}_{\rhd}$ including a further conditional symbol $\rhd$:

– if $\alpha \in \boldsymbol{P}$, then $\alpha \in \boldsymbol{L}_{\rhd}$;
– if $\alpha,\beta \in \boldsymbol{P}$, then $\alpha \rhd \beta \in \boldsymbol{L}_{\rhd}$;
– if $\alpha \in \boldsymbol{L}_{\rhd}$, then $\sim\alpha \in \boldsymbol{L}_{\rhd}$.

Language $\boldsymbol{L}_{\rhd}$ so defined leaves out embeddings and compounds of formulas with $\rhd$, but allows for (iterated) negation of such formulas.

Semantics. For any standard probability function Pr over $\boldsymbol{P}$, we define a valuation function $V_{Pr} : \boldsymbol{L}_{\rhd} \rightarrow [0,1]$ as follows.

- For every $\alpha \in \boldsymbol{P}$, $V_{Pr}(\alpha) = Pr(\alpha)$.
- $V_{Pr}(\alpha \rhd \beta) = 1 - \frac{Pr(\alpha \wedge \sim\beta)}{Pr(\alpha)Pr(\sim\beta)}$, if $Pr(\alpha \wedge \sim\beta) \leq Pr(\alpha)Pr(\sim\beta)$, with $V_{Pr}(\alpha \rhd \beta) =$ 1 in case $Pr(\alpha) = 0$ or $Pr(\beta) = 1$; otherwise $V_{Pr}(\alpha \rhd \beta) = 0$.
- $V_{Pr}(\sim\alpha) = 1 - V_{Pr}(\alpha)$.

V_P is meant to represent the degree of assertability of sentences, including simple non-material evidential conditionals of the form $\alpha \rhd \beta$ (and their possibly iterated negations).

Validity. For $\alpha_1, \ldots, \alpha_n, \beta \in \boldsymbol{L}_{\rhd}$, $\alpha_1, \ldots, \alpha_n \vDash \beta$ if and only if, for any Pr, $\sum_1^n[1 - V_{Pr}(\alpha_i)] \geq 1 - V_{Pr}(\beta)$. Here, just like in Adams's theory as specified above, an argument is valid if and only if the uncertainty (lack of assertability) of the conclusion cannot exceed the total uncertainty of the premises. So once again we can say that in a valid argument a high degree of assertability of the premises implies a high degree of assertability of the conclusion (at least when the premises are not too many). Just like in Adams's theory, moreover, we have both $\alpha_1, \ldots, \alpha_n \vDash_{PL} \beta$ if and only if $\alpha_1, \ldots, \alpha_n \vDash \beta$ (for $\alpha_1, \ldots, \alpha_n, \beta \in \boldsymbol{P}$) and the substitution of (classically) logically equivalents (Crupi and Iacona 2021).

	$\supset$	$\Rightarrow$	$\rightarrow$	$\rhd$
1. Superclassicality	✓	✓	✓	✓
2. Modus Ponens	✓	✓	✗	✓
3. Conjunction of Consequents	✓	✓	✗	✓
4. Disjunction of Antecedents	✓	✓	✗	✓
5. Cautious Monotonicity	✓	✓	✗	✓
6. Right Weakening	✓	✓	✗	✗
7. Limited Transitivity	✓	✓	✗	✗
8. Rational Monotonicity	✓	✓	✗	✗
9. Conjunctive Sufficiency	✓	✓	✗	✗
10. Conditional Excluded Middle	✓	✓	✗	✗
11. Monotonicity	✓	✗	✗	✗
12. Transitivity	✓	✗	✗	✗
13. Contraposition	✓	✗	✗	✓

Table 1: ✓ = valid; ✗ = invalid. The proofs for $\rhd$ are in Crupi and Iacona 2021.

A summary table of the logical profile of our evidential conditional is displayed above, along with the suppositional conditional $\Rightarrow$ and Douven's conditional $\rightarrow$. (The classical material conditional $\supset$ is also included for comparison.)

6. Shades of Transitivity

The pattern of validities / invalidities in Table 1 illustrates that our evidential conditional $\rhd$ overcomes the weakness of Douven's, and it also shows that its logic substantially differs from the logic of the suppositional conditional. A key feature in this respect has to do with Contraposition, which is validated by $\rhd$ but not by $\Rightarrow$. Originally, Adams argued that Contraposition should fail for indicative conditionals (e.g., Adams 1975, pp. 14-15, and Adams 1998, § 6.3). However, the clearest and strongest counterexamples against Contraposition imply that $\alpha > \beta$ can convey an "even if α, β" (or "if α, still β") construction in natural language (see Lycan 2001, p. 34, and Bennett 2003, pp. 32, and 143-144; also see Gomes 2019). This means that such counterexamples become innocent for a non-material conditional connective which clearly *rules out* "even if α, β" as a target explicandum. And that, we submit, is just the case for $\rhd$ (see Douven 2016, p. 119, for an important discussion along the same lines).

Other principles, as we know, are validated by the suppositional conditional but not by the evidential conditional. Our discussion of such cases is not complete yet. Above, we have provided intuitive motivations why principles (6)-(10) are naturally seen as invalid if $\alpha > \beta$ is meant to capture a relation of evidential support from α to β. One could, however, retain a general worry about the failure of Right Weakening and Limited Transitivity. Concerning the suppositional conditional, what such principles tell us is that, while full transitivity fails, there are weaker forms of transitivity that survive in the logic. In fact, the validity of both RW and LT turn out to follow from the validity of Transitivity under unproblematic assumptions. Concerning RW, posit $\beta \vDash_{PL} \gamma$ and assume $\alpha > \beta$. Given Superclassicality, we then have $\beta > \gamma$, and if Transitivity holds we immediately conclude that $\alpha > \gamma$. As for LT, we assume $\alpha > \beta$ and $(\alpha \wedge \beta) > \gamma$, since $\alpha > \alpha$ by SC, we derive $\alpha > (\alpha \wedge \beta)$ from the first assumption by Conjunction of Consequents, and again we conclude that $\alpha > \gamma$ by Transitivity. In the case of Monotonicity and Cautious Monotonicity, the attractive feature of preserving a weaker but discernable variant of a more traditional logical principle is shared by $\Rightarrow$ and $\rhd$. But given that $\rhd$ fails both RW and LT, a natural question is whether there exist *any* weak but non-trivial form of transitivity which is preserved in the logic of evidential conditionals.

Interestingly, at least two of them exist. The first one is appropriately seen as a weakening of Right Weakening itself:

Weak Right Weakening (WRW): If $\beta \vDash_{PL} \gamma$, then $\alpha > \beta$, $\gamma > \beta \vDash \alpha > \gamma$.

As a form of weakened transitivity, this inference rule demands something more from the second link of the transitive chain: not only has γ to follow logically from β (as in plain RW), one must also have the "reverse" evidential conditional $\gamma > \beta$ as a separate premise. As shown by the derivation below, WRW holds given CM, C, and Substitution of Logical Equivalents (SLE), all of which are valid principles for the evidential conditional (Crupi and Iacona 2020).

Assume $\beta \vDash_{PL} \gamma$.

1	$\alpha > \beta$	
2	$\gamma > \beta$	
3	$\sim\beta > \sim\alpha$	[1, C]
4	$\sim\beta > \sim\gamma$	[2, C]

5	$(\sim\beta \wedge \sim\gamma) > \sim\alpha$	[3,4, CM]
6	$\sim\sim\alpha > \sim(\sim\beta \wedge \sim\gamma)$	[5, C]
7	$\alpha > (\beta \vee \gamma)$	[6, SLE]
8	$\alpha > \gamma$	[7, SLE, because $\beta \vDash_{PL} \gamma$]

A symmetric maneuver generates another inference rule:[6]

Weak Left Strengthening (WLS): If $\alpha \vDash_{PL} \beta$, then $\beta > \alpha$, $\beta > \gamma \vDash \alpha > \gamma$.

In this case, the *first* link of the transitive chain is the target of a stringent demand: β has to follow logically from α, and the "reverse" evidential conditional $\beta > \alpha$ must be in place, too. As shown by the derivation below, rule WLS holds given CM and SLE.

Assume $\alpha \vDash_{PL} \beta$.

1	$\beta > \gamma$	
2	$\beta > \alpha$	
3	$(\beta \wedge \alpha) > \gamma$	[1,2, CM]
4	$\alpha > \gamma$	[3, SLE, because $\alpha \vDash_{PL} \beta$]

Interestingly, one can also easily show that WRW and WLS are interderivable given C. We illustrate below by the right-to-left derivation.

Assume $\beta \vDash_{PL} \gamma$

1	$\alpha > \beta$	
2	$\gamma > \beta$	
3	$\sim\beta > \sim\alpha$	[1, C]
4	$\sim\beta > \sim\gamma$	[2, C]
5	$\sim\gamma > \sim\alpha$	[3, 4, WLS, because $\sim\gamma \vDash_{PL} \sim\beta$]
6	$\alpha > \gamma$	[5, C]

So in the logic of the evidential conditional, weakened forms of transitivity require that the first (WLS) or second (WRW) link in the chain be strengthened in a similar way. This neat symmetry is broken, instead, in the logic of $\Rightarrow$: due to RW, strengthening the second link with propositional logical entailment suffice to preserve validity; due to the failure of Monotonicity, however, strengthening the first link with propositional logical entailment does *not* preserve validity.

7. Conclusion

A suppositional view understands a conditional as a statement that the consequent is credible given the antecedent. In many cases, however, a stronger connection between antecedent and consequent seems to be required, one that goes beyond a high conditional probability of the latter given the former. In fact, a number of suggestions broadly along these lines have flourished in recent times, spanning a variety of approaches (Andreas and Günther 2019, Berto and Özgün 2021, Crupi and Iacona 2020, 2021, Raidl 2021, van Rooij and Schulz 2019, Rott 2019). Here, we have suggested a new route to a logic of evidential conditionals, which hinges on the idea that the evidential support from α to β amounts to the

[6] Given SC, WLS can also be seen as a weakened form of another popular rule known as Conditional Equivalence: $\alpha > \beta$, $\beta > \alpha$, $\beta > \gamma \vDash \alpha > \gamma$ (see Adams 1998: 156, Krauss, Lehmann, and Magidor 1990: 179).

degree of incompatibility between α and $\sim\beta$, which also complies with earlier work in Bayesian confirmation theory (Crupi and Tentori 2013, 2014). We have then attached degrees of evidential support to non-material indicative sentences in a suitable formal language, and otherwise retained Adams's definition of validity unscathed. The features of the resulting logical theory are attractive enough, we submit, to motivate serious consideration.

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