The Synthetic Concept of Truth and Its Descendants

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Dedicated to Jozo and Rajka Ištuk, victims of war and victims of lies on the Internet

Abstract. The concept of truth has many aims but only one source. The article describes the primary concept of truth, here called the synthetic concept of truth, according to which truth does not belong exclusively to us nor does it belong exclusively to nature – it is the objective result of the synthesis of us and nature in the process of rational cognition. It is shown how various aspects of the concept of truth – logical, scientific, and mathematical aspect – arise from the synthetic concept of truth. Related to these aspects, i) the regression of truth is analysed and it is shown how the distinction between assertion and valuation resolves the regression, (ii) Tarski's definition of truth is analysed and its role in the concept of truth is identified, and (iii) the truth predicate is analysed and it is shown why paradoxes of truth arise.

keywords: truth; truth in logic; truth in science; truth in mathematics; the regression of truth; Tarski's definition of truth; the truth predicate; paradoxes of truth

"The ideal subject of totalitarian rule is not the convinced Nazi or the convinced Communist, but people for whom the distinction between fact and fiction (i.e., the reality of experience) and the distinction between true and false (i.e., the standards of thought) no longer exist."

Hannah Arendt [Arendt, 1973, p. 474]

1 Introduction

Many of the ambiguities associated with the concept of truth stem from the fact that the concept has various aspects that are not sufficiently differentiated. Tarski's T-scheme [Tarski, 1933] is a classic example of this. T-scheme is a set of T-sentences, the sentences of the form:

$$T(\lceil \varphi \rceil) \leftrightarrow \varphi^*$$

where "T" is the symbol of the truth predicate, φ any sentence of a language L (usually the language we are considering), $\lceil \varphi \rceil$ is the name of that sentence in a language ML (usually the metalanguage in which we consider L), while φ^* is a translation of that sentence into ML. To get a concrete example of a T-sentence, I will take the English sentence "Svrco is afraid of thunder" (the language L will be part of the English language), and my native language as the language ML:

$T("Svrco is afraid of thunder") \leftrightarrow \check{S}vr\acute{c}o se boji grmljavine$

where Svrco is afraid of thunder *= Švrco se boji grmljavine is a translation of the English sentence into my native language. Here the concept of truth appears in five places: as the truth value of the left and right sides of the biconditional, as the truth value of the whole biconditional, as the meaning of the truth predicate symbol "T", and as the truth value of the sentence "Syrco is afraid of thunder". Only the last sentence belongs to the language L, while the other sentences and the symbol "T" belong to the language ML. However, all of them have a semantic source in the sentence "Svrco is afraid of thunder" of the language L. The left side of the biconditional through the symbol T allows to speak in ML about the truth value of the sentence "Svrco is afraid of thunder" of the language L, the right side of the biconditional is related to the truth value of the translation of that sentence into ML, while the truth value of the whole biconditional is related to the success of the translation. Thus, the key aspect of the concept of truth is related to the truth value of the sentence "Syrco is afraid of thunder" of the language L, while other aspects are connected to this primary aspect for various reasons. In what follows, I will focus on this primary concept of truth – the truth values of the atomic sentences of the language L, leaving aside the truth values of the metalanguage in which I will carry the considerations. After analysing the primary concept of truth, I will consider other aspects of the concept of truth. Related to these aspects, in the last part of the article i) the regression of truth is analysed and it is shown how the distinction between assertion and valuation resolves regression, (ii) Tarski's definition of truth is analysed and its role in the concept of truth is identified, and (iii) the predicate of truth is analysed and it is shown why paradoxes of truth arise.

There is a vast philosophical literature on the concept of truth. Although various aspects of the concept of truth have been addressed [Glanzberg, 2021], I have not come across a differentiation of the concept of truth as done in this article. An analysis of the concept of truth will be conducted on sentences. A convincing argument for such a choice was given, for example, by Quine [Quine, 1986, Chapter 1].

2 The synthetic concept of truth

The whole scientific language can be understood as an extension and improvement of everyday language. So, I will begin the analysis with the sentence from everyday language: "Svrco is afraid of thunder". To determine the truth value of the sentence "Svrco is afraid of thunder" we must know the meaning of its parts. Knowledge of English grammar tells us which parts they are and what their linguistic meaning is: "Svrco" is the name of an object, and "is afraid of thunder" is a predicate expression. However, in order to determine the truth value of the above sentence, we must know exactly which object the word "Svrco" names and what the meaning of the predicate expression "is afraid of thunder" is. Svrco is my only pet, and every connoisseur of English knows the meaning of the word "is afraid of thunder", despite the fact that we do not know clearly enough what the "meaning of a predicate expression" means. Knowledge of these meanings is necessary but not sufficient to determine the truth value of the sentence "Svrco is afraid of thunder". We still have to do an appropriate experiment, let nature give its contribution, to determine that it is a true sentence.

This example illustrates the basic cognitive situation of putting an object a in a binary investigative framework ("experimental apparatus"): we generate an investigation (an "experiment") in which nature chooses one of the two offered values, yes or no, as the result of the investigation (of the "experiment"). I will term such a binary investigative framework a predicate P. We apply the predicate P to an object a and describe the situation with the declarative atomic sentence "P(a)". Two possible results of the application are the so-called truth values termed True and False. We take the result chosen by nature as the truth value of the language form "P(a)". True and False are designed by us as a part of the binary investigative design (the design of the binary "experiment") and selected by nature in the realization of the investigation (in the "experiment"). These binary investigations ("experiments") are the essence of our rational cognition. We make the question and offer two possible answers, and nature selects an answer. The selected truth value does not belong exclusively to us nor does it belong exclusively to nature. It is the objective result of the synthesis of us and nature in the process of rational cognition: it discriminates what is and what is not. That is why I have termed this primary concept of truth the synthetic concept of truth.

The cognitive situation illustrated and described above, simple as it might seem, has a number of underlying characteristics and assumptions that are essential for the process of rational cognition and that need to be clarified. First of all, it reflects our innate approach to the world which we divide into objects (elements upon which something is done) and into predicates (which determine what is done). This division is not absolute – something that is a predicate in one context can become an object to which other predicates are applied in another context. This object - predicate dualism is a fundamental characteristic of the cognitive framework described here. It is reflected in language through the structure of the atomic sentence "P(a)". Symbols "a" and "P" have different roles in the sentence. We use symbol "a" to name (mention) an object a. We use symbol "P" to say something about the

¹This is the language form of Einstein's claim that "The whole of science is nothing more than a refinement of everyday thinking" [Einstein, 1936, p. 349]

object a. Because of these different roles, I say that symbol "P" symbolizes a predicate \underline{P} rather than that it names the predicate. To emphasize this difference I will name " \underline{P} " the predicate symbolized by "P". This enables us to maintain a very important use and mention distinction between using a predicate in a sentence to say something about an object (when we use symbol "P") and mentioning a predicate in a sentence to say something about the predicate itself (when we use symbol " \underline{P} ").

To my knowledge, Whorf is the first one to recognise that the object-predicate dualism is a prominent feature of Indo-European languages: "Our language thus gives us a bipolar division of nature. But nature herself is not thus polarized." [Whorf, 1940, p. 247]. He also recognizes that the dualism and the way we analyse nature is not inherent to nature but to our approach to nature: "We dissect nature along lines laid down by our native language. The categories and types that we isolate from the world of phenomena we do not find there because they stare every observer in the face; on the contrary, the world is presented in a kaleidoscopic flux of impressions which has to be organized by our minds – and this means largely by linguistic systems in our minds. We cut nature up, organize it into concepts and ascribe it significance as we do, . . . " [Whorf, 1940, p. 231].

Frege [Frege, 1892] considers predicates (concepts) and objects to be fundamentally different entities. To him this division is absolute, not relative, dependent on context, as it is considered in this article. However, the essential difference between Frege's approach and the approach developed here is that Frege considers predicates to be metaphysical entities in the Platonic sense of the word [Frege, 1918], while they are considered here to be binary investigative frameworks that belong to our real activities.

Furthermore, the language form "P(a)" is not a passive description of the corresponding binary investigation – it is a part of the investigation. Although names for objects and symbols for predicates can be arbitrary, their presence in our rational cognitive processes is essential. Through names, we control our connection with objects and through predicate symbols, we control our connection with predicates. Moreover, objects and predicates do not exist by themselves – they exist as parts of our rational syntheses with nature. Since names and predicate symbols are a means of extracting objects and predicates in rational cognition, each name is part of the object it names and each predicate symbol is part of the predicate it symbolizes. Thereby, a particular syntactic form is not important. What is important is the very presence of the form together with the condition that different objects and predicates have different corresponding language forms.

To my knowledge, von Humboldt is the first to recognize the importance of the previously described connection between language forms and the formation of concepts, and who finds in this relation the key to understanding why language is essential for thinking: "Language is the formative organ of thought. Intellectual activity, entirely mental, entirely internal, and to some extent passing without trace, becomes, through sound, externalized in speech and perceptible to the senses. Thought and language are therefore one and inseparable from each other. But the former is also intrinsically bound to the necessity of entering into a union with the verbal sound; thought cannot otherwise achieve clarity, nor the idea become a concept. The inseparable bonding of thought, vocal apparatus and hearing to language is unalterably rooted in the original constitution of human nature, which cannot be further explained ... without this transformation, occurring constantly with the help of language

even in silence, into an objectivity that returns to the subject, the act of concept formation, and with it all true thinking, is impossible." [Humboldt, 1836, p. 50]. Umberto Eco says this poetically in the last sentence of the 1980 novel *The Name of the Rose*: "Stat rosa pristina nomine, nomina nuda tenemus."²

A fundamental semantic assumption of the use of the atomic sentence "P(a)" in rational cognition is that "a" names an object. This rests on the assumption that it is possible to extract from the world something to be named. I will term the named object the semantic value of the name, and our ability to identify the named with the help of nature I will term the full meaning of the name³. How we make the extraction and how we keep the connection between the name and the named in the flow of time is a very complex subject. For the purpose of analysing the concept of truth, my goal in this and the next paragraph is to establish two assertions. In doing so, I will not deal with defined names, but only with primitive names of a language, because the definition of a name ultimately reduces its full meaning to the full meanings of primitive names and primitive predicate symbols.⁴ When looking at my dog, I realize the connection between the word "Švrćo" and my pet almost with a pure perception. However, in the moments when I can't see him, I keep the connection on the basis of some definite knowledge and the theory that my pet exists somewhere as a distinct object. In everyday life, we keep the connection between the name and the named across time in such a way that, using some commonly established knowledge, we trace the named object and any changes made upon it until the moment when we decide that it is no longer the same object (because it is destroyed or it is transformed into something else). When this connection terminates depends on an accepted world view. For example, when Svrćo dies, whether the name "Svrćo" denote his bones or his spirit, or neither, depends on a world view. I like to call this "the problem of Trigger's broom". Trigger is a likeable street sweeper in a British TV Series "Only Fools and Horses". He has got a medal from local authorities because of his thriftiness - he has been using the same broom for the last twenty years. However, we soon learn that in those twenty years he has replaced the broom head 17 times and the broom handle 14 times. Is it the same broom despite the changes? In everyday situations the decision is a matter of an (established) convention, more or less. However, other obscurities emerge when we analyse the connection between names and objects we cannot perceive directly. Here, the connection is more complex and more dependent on a theory. When we investigate in an experiment if a particle x was an electron, how do we know (i): that there is a distinguished object we can investigate, (ii): that the connection between name "x" and the object is preserved during the experiment, and (iii) that another object didn't appear or the named object of the investigation hasn't changed? Even if we ignore changes over time, the connection between name and the named is a complex mechanism of our interaction with nature. To begin with, I would use the game of recognizing figures in the clouds. Not only does the recognition of a figure in the clouds depend on the place of observation, but two people in the same place will see different figures. In ordinary situations, we all recognize and name the same beings and objects, so it seems to us that we are only giving names to existing objects. But as soon as we move away from the usual situations,

²"Yesterday's rose stands only in name, we hold only empty names."

³Every name has the same general meaning – to name something.

⁴The predicate symbols are analysed below.

extracting from the situation what will be our object (the named) becomes more and more dependent on our approach. For example, in fluid dynamics, we distinguish between two approaches to the study of fluids, depending on what we have extracted for study – whether our object is a fluid that occupies a certain space and is constantly changing in time (Euler's approach) or always the same piece of fluid that is constantly changing space in to which it is located (Lagrange's approach). A step further in the analysis would require us to "dive" into the fluid and turn into, for example, a jellyfish, while retaining the same linguistic abilities. I believe that, due to different needs and perception, the world would look completely different to us: the naming abilities would be completely different and we would extract completely different parts of reality for the named objects.⁵ I believe these considerations are compelling enough to accept the first assertion about names: that, like the truth value of the associated primitive sentence, the process of naming is also a kind of synthesis of us and nature.

When I use the name "Švrćo", I exactly know what is named: my dog Švrćo. However, even in everyday situations, we use names for which we don't know the exact object they name, for example, the name of a person we don't know. Even worse, it is possible that such a person does not exist, as it the case today with fake profiles on the internet. In the same unwarranted way, we extend the language used in everyday situations to other situations, when we are involved in science and mathematics, or when we talk fairy tales to children. However, we think "with names" in the same way, whether we know what they name or not and whether they name anything at all. For example, when we are involved in the fairy tale Snow White and the Seven Dwarfs we think, discuss and make conclusions as if all the characters in the story exist, because we are "tuned" to think in this way in semantically clear everyday situations. Only, when we step out of the language of the story (and use another language) we acknowledge that there are no such objects. That's how we get to the second assertion about names: concerning names, the moral is that when we use language we assume that every name names an object, no matter how this connection is achieved and whether it is achieved at all. Expressed in terms of meaning: when we use language we use general meaning of names (that a name names something), but not necessarily the full meanings of names.

I consider that naming, together with the fundamental assumption of its use, is a key primitive element of language. I think it is wrong to minimize the importance of naming as in Russell's theory of descriptions [Russell, 1905], in Quine's reduction to values of variables [Quine, 1948] or more radically in Quine's reduction to "ideal nodes at the foci of interesting observation sentences" in his naturalized epistemology [Quine, 1990].

The next fundamental semantic assumption of the use of the sentence P(a) in rational cognition is that the predicate symbol "P" symbolizes a predicate \underline{P} . The predicate \underline{P} is a binary investigative framework that applied to an object a determines, through the intervention of nature, the result of the generated investigation – the truth value of the corresponding atomic sentence P(a). Thus, each predicate determines, through the intervention of nature, a mathematical function (in the mathematical extensional sense) from objects to truth values. I will call this function the semantic value of the predicate (and of the corresponding

⁵In [Atiyah, 1995], the famous mathematician Michael Atiyah made a thought experiment with an intelligent jellyfish, in which he showed that its mathematics would be significantly different from ours, thus arguing that mathematics is human invention, not discovery.

predicate symbol). However, we must not equate the predicate and its semantic value. Otherwise, we would destroy the whole language mechanism of rational cognition. The predicate is part of the process of rational cognition while its semantic value on a given object is the final result of this process, in which nature is substantially involved. Thus, the predicate is an intermediary, together with nature, between the predicate symbol and its semantic value. The predicate is our ability to identify the semantic value of the predicate symbol with the help of nature. Thus, we can talk of the predicate \underline{P} as the full meaning of the predicate symbol P.6 This connection between the language form and reality is even more complex than naming. However, as with names, for the purpose of analysing the concept of truth, my goal in this and the next paragraph is to establish two assertions. In doing so, I will not deal with defined predicate symbols, but only with primitive predicate symbols of a language, because the definition of a predicate symbol ultimately reduces its full meaning to the full meanings of primitive names and primitive predicate symbols. With the predicate expression "is a dog" we associate the whole mainly empirical mechanism, which, if applied to Syrćo gives, between two possible answers, the answer True. We can describe the meaning of the predicate expression "is a dog" by the meanings of other predicate expressions, for example "is an animal", "barks", etc. However, in this way we only connect the meanings that lie behind various predicate symbols, but we certainly don't use this as the primary way to learn the meaning of the expression "is a dog". From the moment of birth we form that meaning, I would say almost by perception, as a part of our ability to differentiate beings. The semantic mechanism of the predicate expression "is a dog" is deeply rooted in our sensory world, and only later do we complete it (make it more precise) with determinations which vary from everyday experience (for example that a dog does not necessarily have fur) to advanced theoretical knowledge (for example about its genetic code). Whatever that mechanism is, it is certainly not just an imprint of reality into our conscience, as we are an active party in the process. Concerning the predicate expression "is an electron", the semantic mechanism behind it is more obscure than the mechanism for the predicate symbol "is a dog", because the predicate is applied to objects out of our direct experience. We must develop adequate experimental tools, built on some theory (world view), to have an indirect experience of such objects. Does one of these experiments determine the mechanism behind the predicate expression "is an electron"? Or, is the essence of "is an electron" something else which only coincides with the concrete mechanism in the context of the experiment? We would like that "is an electron" have a deeper meaning than it manifests in particular experimental settings. However, is such a "transcendental" predicate expression independent of various experimental settings or is it just their "common denominator"? In other words, does the predicate attached to the expression "is an electron" exist independently of us or does it exist only through our cognitive interaction with nature? A simple picture is that all such predicates exist independently of us, and that we only discover them through our interaction with nature. However, we have no rational ground for this claim. We can believe in metaphysical nature of predicates (not me) and this belief can lead us in investigations, but it must not be a part of scientific approach – scientific approach must be based on reason. On the other hand, if we were to bound ourselves to predicates that correspond to experimental

⁶Every predicate symbol has the same general meaning – that its semantic value is a function from objects to truth values.

settings we would lose any power of deeper cognition of nature. However, regardless of the semantic mechanism, in order for "theoretical" predicates, like the predicate attached to the expression "is an electron", to have any cognitive value, they necessarily exist as part of our cognitive interaction with nature although in a more subtle way than "empirical" predicates. Moreover, there are no boundaries – every predicate is theoretical as well as experimental in some degree (I consider perception as the most basic kind of experiment). A dominantly experimental predicate attached to the expression "is a dog" has some elements of a theory (if not in implicitly assumed conceptual framework, then at least in some extreme situations when it is not so obvious that an object is a dog). A dominantly theoretical predicate attached to the expression "is an electron" has some elements of an experiment, because without it the expression would lose any meaning. Furthermore, even in common situations, different people use different predicates. Predicates are the basic means by which we abstract what is important to us from a given situation. Let's imagine a group of hikers who have decided to have lunch. They have found a stone with a flat upper surface which is adequate to put out food and consume it. For them the stone is a table. It is the same stone on which a ranger stood yesterday because he had a good view from it. For hikers the stone is a table, for the ranger it is an observation post. Each of them extracted what they needed from the stone using the appropriate predicate. Even when I described that object as a stone I have abstracted something from it by the predicate expression "is a stone". Even when I described it as an object I have abstracted something from it by the predicate expression "is an object". Further relativization would lead us to thought experiments in which we would analyse what kind of predicates other organisms (elephants or microbes) would develop in the same situation if they had our linguistic abilities. By means of their predicate expressions, they would surely create different abstractions and structure the situation differently. I believe these considerations are compelling enough to accept the first assertion about predicates: that, like the truth value of the associated primitive sentence and like the naming, predicates are also a kind of synthesis of us and nature.

With predicate symbols, as with names, we encounter uncertainty, too. When I use, for example, the predicate expression "to be a dog", in standard situations I know immediately how to perform the corresponding experiment. However, it could happen that in some exceptional situations I don't know how to determine if an object is a dog. This uncertainty happens especially in science. Let's take for example the predicate expression "is an electron", the meaning of which has changed over the centuries. We use various predicate expressions for which sometimes we are uncertain about how to apply them to various objects. In other words, we have predicate symbols which symbolize incomplete predicates: the predicate symbols don't have the full meaning. This situation will be analysed in more detail in Subsection 4. Here I just want to stress that in the same unwarranted way as with names, we extend the use of predicate symbols from everyday situations to other situations. However, it does not prevent us to think with predicate symbols as if they always have the full meaning – that they always symbolize complete predicates, predicates which, with the help of nature, unambiguously determine the semantic values of the associated predicate symbols. That's how we get to the second assertion about predicate symbols: concerning predicate symbols, the moral is that when we use language we assume that every predicate symbol symbolizes a complete predicate, without considering how this connection of language and reality is achieved and whether it is achieved at all. Expressed in terms of meaning: when we use language we use general meaning of predicate symbols (that their semantic values are functions from objects to truth values), but not necessarily the full meanings.

Due to the further analysis of different aspects of the concept of truth, it should also be pointed out that there are situations where we do not use predicates as an investigative tool to address questions to nature. Commonly, these are situations which we create and over which we have control, for example, in designing a game, a story or a mathematical world (as I will explain later in Section 5). Then, for some predicates, we directly decide on which objects they give True and on which objects they give False. For example, we can decide which character in a fairy tale will be good or which natural numbers less than 100 will have some (unimportant) property U (we will just enumerate such numbers). This is another use of predicates in which we directly reduce them to their semantic values. The role of these predicates in our rational activities is quite different than the original role of predicates as investigative frameworks in rational cognition.

As I have analysed one-place predicate symbols, I can also analyse multi-place predicate symbols. The analysis of function symbols is similar to the analysis of predicate symbols. Every function symbol symbolises a function, a procedure that, when applied to objects, determines an object, with the help of nature. A nice example of these functions are measurement functions, such as mass or temperature, which associate numbers with parts of nature through an appropriate measurement process.⁷ The semantic value of the function (and of the function symbol) is the corresponding mathematical (extensional) function between objects.

To conclude, the essence of the synthetic concept of truth is the following one. By disjoining the world into objects and predicates, which we control through names and predicate symbols, we put binary questions to nature. By selecting one of two offered answers, nature brings its contribution to the framework, besides its contribution to the processes of naming and of predicating. In a binary experiment of applying predicate \underline{P} to object a, when nature selects an answer, True or False, it "says" something about itself. With this valuation of the language form "P(a)", the form which describes and controls the binary investigation, we gain knowledge about nature. This is the starting point for the overall role of the concept of truth in our rational cognition.

Clearly, this concept of truth is not any kind of a deflationary conception of truth that diminishes the importance of the concept of truth.⁸ The synthetic conception of truth is of crucial importance for rational cognition. Also, the synthetic concept of truth is not a kind of a correspondence theory of truth where the truth value of the sentence is determined only by whether the sentence corresponds with reality or not. Thereby, reality is considered something independent of us and language: language only serves to describe reality.⁹ In the synthetic conception of truth, atomic sentences themselves, with their interpreted parts – names and predicate symbols – and with their truth values, where nature is involved, form

⁷These functions are analysed in [Čulina, 2022]

⁸Various formulations of the deflationary conception of truth can be found, for example, in [Stoljar and Damnjanovic, 2014].

⁹Various formulations of the correspondence conception of truth can be found, for example, in [David, 2020].

reality: reality is the result of the synthesis of us and nature through the creation and use of language. I consider that the synthetic concept of truth is the solution to the philosophical problem of truth – is there any connection between truth and reality and, if so, what is the connection. The synthetic concept of truth shows that there is a connection and precisely shows what the connection is.

Davidson points out the key problem of "unity of proposition" that the theory of truth and predication must solve.: "...if we do not understand predication, we do not understand how any sentence works, nor can we account for the structure of the simplest thought that is expressible in language. At one time there was much discussion of what was called the "unity of proposition"; it is just this unity that a theory of predication must explain. The philosophy of language lacks its most important chapter without such a theory, the philosophy of mind is missing its crucial first step if it cannot describe the nature of judgement; and it is woeful if metaphysics cannot say how a substance is related to its attributes." ([Davidson, 2005, p. 77]). I consider that the synthetic concept of truth provides a solution. The truth value of an atomic sentence, as the result of our synthesis with nature in the process of rational cognition, gives unity to the atomic sentence: it makes the atomic sentence to be something more than just the concatenation of its parts, the predicate symbol and the name involved in the sentence. This solution can be considered a solution that is obtained when we subtract metaphysics from Frege's solution [Frege, 1891, Frege, 1918, Frege, 1897].

The role of language in rational cognition is analysed in detail in [Culina, 2021a].

3 The logical aspect of truth

We can build various language structures over atomic sentences. The object-predicate dualism naturally leads to first order languages, which not only have a simpler and clearer semantics than other languages, but also prove to be the most important type of logical language. In what follows, I will assume this type of language.

The basic building blocks of a first-order language are atomic sentences which are analysed above. Consequently, all the assumptions of the use of atomic sentences are now the assumptions of the use of an interpreted first-order language.

Each complex sentence of an interpreted first order language describes a particular investigation which is a combination of investigations described by atomic sentences. Eg. the sentence $P(a) \wedge Q(b)$ describes a binary investigation composed of the binary investigations described by the sentences P(a) and Q(b). This investigation applied to a and b yields True when both atomic investigations yield True, otherwise it yields False. Likewise, $\forall x \ P(x)$ can be considered a new investigation that gives the value True when for each valuation of the variable x the investigation P(x) gives the value True, while otherwise it gives the value False. Why do we need these combinations at all, given that there is nothing new in them concerning rational cognition which is not present in atomic sentences? There are several reasons but by far the most important reason to combine binary experiments is to recognize and determine a regularity that is repeated in certain types of combinations. For example, every time when we assert that an object is a dog, we or somebody else, sooner or later, will also assert that the object is mortal. We combine the investigations "x is a

dog" and "x is mortal" into the investigation "if x is a dog then x is mortal". We capture in a simple way the observed regularity by claiming that the sentence "For all x, if x is a dog then x is mortal" is true. However, quantification poses the so-called problem of induction [Hume, 1740]. We can investigate the truth value of "if x is a dog then x is mortal" for every value of x (in principle) but we cannot do it for all (potentially infinite) values: this is a situation in which we can possibly get the answer "no" but never the answer "yes". We could conclude that this is not a binary investigation at all, and we could exclude this type of sentences from language. However, then we could not express regularities which we observe and which are the main sources of knowledge, as the history of science confirms.¹⁰ As with naming and predicating, we extend the use of language in ordinary situations and assume that every sentence of an interpreted first-order language is true or false, regardless of the way we find its truth value, and even regardless of whether we can find it at all. We accept such universal and existential sentences (and corresponding investigations) despite all uncertainty they bring. This assumption is of great importance for the scientific concept of truth, which will be described in the next section, but also for the logical concept of truth to which this section is dedicated. This assumption and all the assumptions of the use of atomic sentences I will term the external assumptions of an interpreted first-order language. Their fulfilment is crucial for the application of the language but not for the logic of the language. The only important thing for the logic of the language is that these assumptions are part of the specification of the language, not whether they are fulfilled. By the logic of a language, I mean the internal organization of the language – the connection of semantic values of language forms, which is independent of the reality that the language speaks about - together with the external assumptions of the language use.

For a first order language, a mathematical (extensional) function is connected with each language construction of a sentence from simpler sentences. The function determines the truth value of the constructed sentence on the basis of the truth values of the sentences from which it is constructed. For example, the construction of the conjunction $\varphi \wedge \psi$ is connected with two-place boolean function that outputs True only when both inputs are True. The important property of of any such function is that it is an internal semantic function, a function that connects semantic values independently of the reality the language speaks of. So, it belongs to the logic of the language. I will term such a function the semantic function of the construction. These semantic functions gives recursive conditions for truth values which, together with the truth values of atomic sentences, determine the unique mathematical function that assigns, in a given evaluation of variables, a truth value to each sentence. This means that in an interpreted first order language, under the external assumptions of its use, the truth value of each sentence is entirely determined by the truth values of atomic sentences. According to the synthetic concept of truth, the truth values of atomic sentences are primitive semantic elements of language determined by the process of rational cognition. In this way, the truth value of each sentence is connected with reality in a completely determined way.

Because the semantic functions of the sentence constructions in a first order language belong to the logic of the language, they determine the logical connection of truth values

¹⁰As C. D. Broad said: "induction is the glory of science and the scandal of philosophy" [Broad, 1952, p. 143]

of the sentences. This aspect of truth, the internal interconnectedness of the truth values of sentences of a language, I will term the logical aspect of the concept of truth. Important concepts of logical truth and logical consequence belong to this aspect. Logical truth is the sentence whose truth is determined by the internal semantic structure of the language regardless of its particular connection with reality. Eg. $\neg A \lor A$ is a logical truth, because its truth is determined by the internal semantics of the connectives \neg and \lor . Also, that from a set of sentences $\{A_1, A_2, \ldots\}$ logically follows a sentence B, means that starting from the truth of the sentences A_1, A_2, \ldots the internal semantic structure of the language, not the reality the language speaks of, determines the truth of B. Thus, for example, the internal semantics of the conjunction \land determines that B logically follows from $A \land B$. The relationship of logical consequence between sentences is one of the crucial language mechanisms in the development of rational cognition.

The logical elements of first order languages are analysed in detail in [Culina, 2021b].

4 The scientific aspect of truth

As analyzed above, the first order language built upon interpreted atomic sentences has the external assumptions of its use. These are: (i) the fundamental assumption of the language use of names – every name names an object, (ii) the fundamental assumption of the language use of function symbols – every function symbol symbolizes a function which applied to objects gives an object, (iii) the fundamental assumption of the language use of predicate symbols – every predicate symbol symbolizes a predicate which applied to objects gives one of the two possible results, "True" or "False", and (iv) the fundamental assumption of the language use of sentences – every sentence is true or false. In a real process of rational cognition, already in everyday situations and especially in science, we use names for which we do not know completely what they name, predicate and function symbols for which we do not know completely what they symbolise, and quantified sentences for which we do not know if they are true or not. However, it is important to emphasize that regardless of whether the exterior assumptions are fulfilled or not, the logic of the language demands that when we use the language we assume that they are fulfilled. In thinking itself there is no difference whether we think of objects that really exist or we think of objects that do not really exist and whether the predicate symbols we use can be applied to such objects at all or not. That difference can be registered only in a "meeting" with reality. Furthermore, although semantic values of the complex language forms are determined by semantic values of the simpler forms from which they are built, in the process of rational cognition we invert this original priority. An assertion about a particular object is more confident and more determined rational cognition then an assertion about all objects. However, we cannot apply all primitive (undefined) predicates to all objects, because there are too many objects, potentially infinitely many. Furthermore, some objects disappear, some come into existence. So, we cannot know the truth values of all atomic sentences. We rely more and more on the regularities which we notice. These regularities are formed by universal and existential sentences (laws). These sentences gradually become the main basis for rational cognition, although we cannot perform completely the complex binary investigations they determine.

Moreover, these sentences speak often about idealized situations and idealized objects using idealized predicates. For example, in classical mechanics, we analyse a motion of the so-called material particles which at each moment of time occupy exactly one point in space. Hence, we assert something about objects which even do not exist in the strict sense of this word. We make assertions about such objects without any corresponding atomic sentence we could verify experimentally. Despite this, such assertions are the result of a deeper analysis of real situations and, through a kind of synthesis, give us powerful knowledge of real situations. All this means that our real knowledge, regardless of the degree of its accuracy, is almost always only a fragment of some assumed semantically complete language. The whole dynamics of a scientific theory can be understood as the dynamics of completing and changing an appropriate language. In the process of rational cognition, we decrease unspecified parts of the language, even change the semantic values that had been already formed. However, this process is not chaotic, but it is, looking over longer periods, a constant advance in rational cognition of nature. 11 That is because it has powerful regulatory mechanisms which control and drive it - the exterior interaction with nature through experiments and the logic of language. Namely, for a theory to be a scientific one, at least some names and some function and predicate symbols must have an exterior interpretation, an interpretation in the exterior world, not necessarily a complete one. This partial external interpretation enables us to perform at least part of the binary experiments described by atomic sentences. This allows nature to put its answers into our framework, so that we can test our conceptions experimentally. Without this part the theory is unusable. On the other hand, the language disciplines us in a way that we shape our cognition and understanding into a set of sentences which we consider to be true. In an ideal case, we choose a not too big set of sentences we are pretty sure to be true, the axioms of the theory. Then, we are obligated, by the logic of the language, to consider true all sentences which logically follow from the axioms. So, another rationalized part of our conceptions consists of a set of sentences we consider to be true and to which we try to give an axiomatic organization. Therefore, a scientific theory about nature is a junction of a set of sentences (the sentence part of the theory) and partial external interpretation of the language (the interpreted part of the theory). From the axioms of the theory, we logically deduce the truth values of sentences. Particularly, we deduce the truth values of atomic sentence which belong to the external interpretation and which are, therefore, experimentally verifiable. If the truth values do not coincide with the truth values which nature gives, then the theory is wrong. If they are identical, it makes the theory trustworthy but, as we know, it is not proof that it is right. As Popper emphasizes, theories must be experimentally verifiable so that they can be falsifiable. In this interaction of the sentence part and the externally interpreted part of a theory, the real dynamics of the theory takes place: the axioms, as well as the interpreted parts, evolve, even change, and the same happens with the whole language framework. I will term this aspect of the concept of truth the scientific aspect of the concept of truth. At the core of this scientific dynamics is the synthetic concept of truth. It gives legitimacy and perspective to scientific research described above as a development of truth valuations of sentences and external interpretation of a language.

¹¹Even Kuhn's scientific revolutions [Kuhn, 1962] can be interpreted as radical changes of established language frameworks.

5 The mathematical aspect of truth

The concept of truth in mathematics essentially depends on the accepted philosophy of mathematics [Horsten, 2019]. Thus, the mathematical concept of truth presented here also depends on a certain philosophy of mathematics, which is elaborated in [Čulina, 2020b].

I consider mathematics primarily the internal organization of rational cognition, a thoughtful modelling of that part of the process of rational cognition that belongs to us. Building a logical language is one such modelling. So, I consider that logic is part of mathematics. A first order language is a mathematical model constructed for the use in rational cognition just like natural numbers are constructed for counting. It is the result of thoughtful modelling of intuition about our natural language. Thoughtful modelling of other intuitions about our internal world of activities, for example, intuitions about quantity, symmetry, flatness, nearness, etc., lead to other mathematical models. By "our internal world of activities" I mean the world that consists of activities over which we have strong control and which organize and design by our human measure (e.g., movements in space, grouping and arranging small objects, writing on paper, painting, playing music, ...). It is from these concrete activities that the idea of an idealized mathematical world emerges, the world that expands and supplements the internal world of activities. Let's take real numbers, for example. Although we can approximate irrational numbers by rational numbers with arbitrary precision (if we had enough space, time and materials - again idealization), their existence is outside our means of construction – we have just imagined irrational numbers. ¹² By choosing names, function symbols and predicate symbols, we shape the initial intuition into one structured conception. However, here the role of functional and predicate symbols, as well as the truth values of sentences, is different than in rational cognition. Predicates are not investigative tool to address questions to nature, there is no intervention of nature, and thus no synthesizing role of truth values. Truths are truths "by fiat". Because we create a mathematical world we have a complete control in its design. We determine on which objects the predicate will give truth, in the same way as we decide which character in a fairy tale will be good. It's the same with functions. We cannot experimentally verify that ||+||=|||| (2 + 2 = 4) because it is not the truth about nature – it is the way we add tallies. However, since the conception usually goes beyond our constructive capabilities, the constructed language has only partial interpretation in our internal world of activities. Since the interpretation is only partial, and because the imagined domain of interpretation is usually infinite, we cannot determine the truth values of all sentences of the language. Therefore, we must further specify the conception by appropriate choice of axioms. When we describe a mathematical world by some set of axioms, inferring logical consequences from the axioms, we establish what is true in that world. This can be very creative and exciting work and it seems that we discover truths about some existing exotic world, but we only unfold the specification. The inferred sentences are not true because the world they describe is such, but that world is so conceived that those sentences are true in it. They are the conditions that the world must satisfy. I will term this aspect of the concept of truth, as a specification of an imagined mathematical world that emerged from our internal activities, the mathematical aspect of

 $^{^{12}}$ In his book [Mac Lane, 1986], Sounders Mac Lane describes this process of idealization on a multitude of examples.

the concept of truth. Since I consider logic to be part of mathematics, the logical aspect of the concept of truth is also part of the mathematical aspect of the concept of truth. I would note that we have already encountered this mathematical aspect in logic on the example of a linguistic construction using the conjunction \wedge . This conjunction is directly associated with its semantic value, the corresponding Boolean function, without an intensional intermediate step.

6 The assertion-valuation distinction and the regression of truth

All previous considerations have been done in the appropriate metalanguage whose sentences also have their truth values. Using sentences of the language ML I discussed the truth values of sentences of a first order language L. The reader will reflect on the correctness of my considerations, that is, on the truth values of my assertions. The insights she will thus gain are composed of sentences which also have truth values, which may be the subject of other sentences. And so on indefinitely. However, since the pattern is repeated in this infinite regression, it is sufficient to look at one step, the transition from L to ML, that is, to analyse the connection of the sentences $T(\lceil \varphi \rceil)$ and φ (where "T" is the truth predicate symbol). Without loss of generality, we can concentrate on the connection between the sentences "Syrco is afraid of thunder" and " "Syrco is afraid of thunder" is a true sentence". The main difference in the use of these sentences is that when I say "Svrco is afraid of thunder", the subject of my expression and thought is my dog Svrco, and when I say "Svrco is afraid of thunder" is a true sentence", the subject of my expression and thought is the sentence "Syrco is afraid of thunder". This is a typical use-mention distinction. In the first case I use the sentence "Syrco is afraid of thunder" to say something about Syrco and in the second I mention the sentence to say something about it. What is specific here is that one sentence speaks about the truth of another sentence, where each of the sentences has its own truth value. If, for example, we were talking about the number of letters in the sentence "Syrco is afraid of thunder", nothing would be disputable. However, as far as the truth is concerned, there is also a difference between the above sentences. I will term it assertion-valuation distinction. Namely, the very way we use a (declarative) sentence conveys the information that we consider it true. So, when I assert "Svrco is afraid of thunder", in addition to the information about Syrco, I convey the information that it is a true sentence. So, there is no need to assert it in a roundabout way with the sentence "Svrco is afraid of thunder" is a true sentence" (by which I again convey the information that this sentence is true). However, if someone considers the truth of the sentence "Svrco is afraid of thunder" he will not use it but will mention it and evaluate its truth. If he concludes that it is true, he will end his analysis with the assertion "Svrco is afraid of thunder" is a true sentence". This assertion-valuation distinction is a mechanism for stopping or prolonging truth regression. The assertion aspect stops the regression, and the valuation aspect continues the regression. So if we agree on something, that's where the regression ends. Usually the regression stops in the metalanguage because, if disputes do occur, they are disputes about the truth of the sentences of the language L and not about the truth of the sentences of the metalanguage, so they are resolved by the assertions of the metalanguage. If someone disputes what I have said about the truths of sentences of the language L, he disputes the truth of the corresponding ML metalanguage sentence. But the subject of his analysis will again be the language L and the conclusion he draws will be the assertion of the metalanguage ML and not its metalanguage MML.

As far as I know, the importance of the linguistic mechanism of assertion was first pointed out by Frege [Frege, 1897]. How subtle and important the concept of assertion is in Frege can be read in [Pedriali, 2017].

7 Tarski's definition of truth

As analyzed in the introductory part of the article, Tarski's T-scheme is a classic example in which various aspects of the concept of truth are mixed. This extends to Tarski's definition of truth, too – some see the definition as an argument for the correspondence theory of truth, others for the deflationary theory of truth. A comprehensive analysis of Tarski's work and various critiques of the work can be found in [Patterson, 2012]. In this section, Tarski's T-scheme and Tarski's definition of truth are analysed in relation to the aspects of truth differentiated in this article, especially in relation to the synthetic concept of truth.

Regarding the analysis of the concept of truth, the assertion-valuation distinction shows that truth value occurs in two ways, implicitly as part of an assertion or explicitly through the truth predicate symbol, i.e. through mentioning the truth value of a sentence. So to assert the sentence $T(\lceil \varphi \rceil)$ which explicitly says that the sentence φ is true is to assert the sentence φ , and vice versa. If we ignore the translation problems and consider that the metalanguage ML is an extension of the language L, this means that all T-sentences are true, that is, to avoid regression, we can assert that for every sentence φ of the language L:

$$T(\lceil \varphi \rceil) \leftrightarrow \varphi$$

The nature of the truth of these T-sentences can be viewed in various ways, depending on how we view the truth predicate symbol through which the truths of the left and right sides of the biconditional are equated. However, regardless of these differences, the truth of T-sentences belongs to the mathematical aspect of the concept of truth because their truth belongs to the internal organisation of rational cognition. If we were to use the more general T-scheme $T(\lceil \varphi \rceil) \leftrightarrow \varphi^*$ related to a metalanguage that is not an extension of the language L, due to the question of correctness of translation, the scientific aspect of the concept of truth could be present, too.

It is common to consider T-sentences $T(\lceil \varphi \rceil) \leftrightarrow \varphi^*$ as partial definitions of the truth predicate. In this case, T-sentences are analytical truths of the metalanguage ML. So, this is a logical aspect of the concept of truth. This view is directly related to Tarski's analysis of the concept of truth. Tarski's definition of the truth predicate for the language L in the language L [Tarski, 1933] is a formally correct definition because it enables the elimination of the defined predicate symbol T in every sentence of the language L. The definition is also a materially adequate definition in the sense that all T-sentences logically follow from

it. However, Tarski's definition of truth has the role of a content-wise definition only when we want to set the truth of the sentences of one as yet uninterpreted language L by using the truth of the sentences of another language ML. This definition transfers the meaning, and thus the truth value of the sentence φ^* of ML, to the truth of the sentence φ of L via the appropriate T-sentence. That is why Tarski's definition is so important in mathematical logic. However, for the interpreted language, it has no content-wise sense because it defines something that has already been determined. In such a context, this definition simply gives a translation from the language L to the language ML via the T-scheme: each sentence φ of the language L is translated into the sentence φ^* of the language ML. If the translation is correct, it preserves the meanings and thus the truth values of the sentences. In this situation, Tarski's definition is simply a mathematical construction of the translation function. It makes possible to connect the truths of sentences of two different languages. But whether Tarski's definition is a substantive definition or just a mechanism of translation from one language to another, it only transfers the problem of the truth of a sentence of one language to the same problem of the truth of the corresponding sentence of another language. Instead of examining the truth of the statement "Svrco is afraid of thunder", we can now examine the truth of the statement "Švrćo se boji grmljavine". If the translation is correct, it is one and the same problem. This is best seen when the metalanguage ML is an extension of the language L, i.e. when we have a T-scheme $T(\lceil \varphi \rceil) \leftrightarrow \varphi$. Then Tarski's definition translates the problem of the truth of the sentence "Svrco is afraid of thunder" to the problem of the truth of the sentence "Syrco is afraid of thunder" ($\varphi^* = \varphi$).

The problem with Tarski's definition of the concept of truth and the interpretation of his contribution to the analysis of the concept of truth is as follows. Tarski says: "We should like our definition to do justice to the intuitions which adhere to the classical Aristotelian conception of truth – intuitions which find their expression in the well-known words of Aristotle's metaphysics: 'To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true'." [Tarski, 1944, p. 342]. However, Frege shows [Frege, 1897] that it is not possible to give an absolute definition of truth, because the application of such a definition depends on the truth of definiens, so it is a circular definition. As a special case, he shows that a correspondence theory of truth is impossible because it reduces the problem "is a sentence true" to the problem "is it true that the sentence corresponds with reality", which again leads to circularity. Tarski's definition of the truth of a sentence is not an absolute definition of truth neither does it refine an intuition about truth as correspondence with reality. It is a relative definition of the truth of sentences in one language (object language) by the truth of sentences in another language (usually metalanguage). The definition enables a translation of the truth for sentences in one language into truth of sentences in another language, as Tarski explicitly states in his T-convention [Tarski, 1933]. Hence, in Tarski, the intuition about a correspondence theory of truth is realized as a correspondence of truth between two languages and not between language and reality. Tarski's recursive definition of truth reduces the truth values of compound sentences to atomic sentences. Tarski's and the synthetic conception of truth differ in the way they treat atomic sentences. Tarski finishes his definition by giving a translation of atomic sentences to metalanguage, and by this transferring the concept of truth from language to metalanguage. Contrary to this, in the synthetic conception of truth, the truth values of atomic sentences are undefined primitive elements determined by the process of rational cognition. In this way, the truth value of every sentence is connected with reality in a completely determined way. Thus, Tarski's definition of the concept of truth correctly formulates recursive conditions that connect the truth of a constructed sentence with the truth of the sentences from which it is constructed, while by translating the truth of atomic sentences of language L into the truth of sentences of metalanguage it ceases to be a content-wise theory of truth.

8 The truth predicate and the paradoxes of truth

The basic purpose of the truth predicate symbol T is that we can use it, in the corresponding metalanguage ML, to describe the truth values of the sentences of the language L. According to the meaning of the truth predicate symbol T, the sentence $T(\lceil \varphi \rceil)$ is a true (false) sentence of ML when φ is a true (false) sentence of L. When the language L is not part of the language ML, the role of this predicate symbol is the same as, for example, the predicate expression "is a diesel engine". Just as in the language of mechanical engineering we speak about engines using the predicate expression "is a diesel engine", so in ML we speak about the truth values of sentences L using the predicate symbol T. T is a non-logical symbol of the language ML, just as "is a diesel engine" is a non-logical expression of the language of mechanical engineering. As "is a diesel engine" connects engine types with the truth values of the corresponding sentences of the language of mechanical engineering, so the truth predicate symbol T connects the truth values of the sentences of the language L with the truth values of the corresponding sentences of the language ML. However, when L is part of the language ML, then the truth predicate symbol T connects the truth values of sentences of the same language. Truth conditions on the truth predicate symbol T, that $T(\lceil \varphi \rceil)$ is a true (false) when φ is a true (false), where φ belongs to L, now belong to the internal semantics of the language in the same way as, for example, truth conditions on connectives. In this case, the truth predicate symbol T is a logical symbol of the language ML, like connectives and quantifiers. The only difference in relation to connectives and quantifiers is in universality. Only a language that has its own sentences in the domain of its interpretation (possibly through coding) can have a logical symbol of its own truth predicate. However, this situation, when ML is an extension of L, and so the truth predicate symbol is a logical symbol of ML, opens up the possibility of the paradoxes of truth. In a standard situation in science, atomic sentences of the language L do not contain the truth predicate symbol T. and they have a certain truth value as the result of rational cognition. Such a situation does not lead to paradoxes. Namely, according to the previously described truth condition on the logical symbol T, in order to examine whether the atomic sentence $T(\lceil \varphi \rceil)$ of the language ML is true, we need to examine whether the sentence φ of the language L is true, and its truth is completely determined by the truth of the atomic sentences of the language L. Thus the truth value of the sentence $T(\lceil \varphi \rceil)$ is unambiguously determined. However, in a natural language the truth predicate symbol is applicable to all its sentences (L = ML): L contains T. Now, too, by the truth condition on the logical truth predicate symbol, the examination of the truth of the atomic sentence $T(\lceil \varphi \rceil)$ is reduced to the examination of the truth of the sentence φ , and the examination of its truth is reduced to the examination the truth of atomic sentences. But now some of these atomic sentences can again be of the form $T(\lceil \psi \rceil)$, so that the process does not stop but continues again. While for the standard language L which speaks of some natural phenomenon and does not contain its own truth predicate symbol, this procedure gives a unique answer, now we have no guarantee that the reduction procedure will stop at some step or that we will get unique truth values of sentences covered by such procedure. Let us consider the two simplest examples where the truth determination procedure is not successful:

the sentence L: $\neg T(\overline{L})$ (The Liar) the sentence I: $T(\overline{I})$ (The Truth-teller)

For the sentence L we have the following chain of reduction:

$$L \mapsto \neg T(\overline{L}) \mapsto T(\overline{L}) \mapsto L \mapsto \dots$$

It is easy to see that no evaluation along this chain satisfies the truth conditions: the assumption that L is true gives that L is false, and the assumption that L is false gives that L is true. Thus we cannot assign any truth value to the sentence L. On the other hand, for the sentence I we get the following chain of reduction:

$$I \mapsto T(\overline{I}) \mapsto I \mapsto \dots$$

Now both evaluations, the evaluation according to which I is true and the evaluation according to which I is a false sentence, satisfy the truth conditions along the chain. So, this sentence can be both true and false in an equally (un)convincing way.

The paradoxes of truth stem precisely from the fact that the classical procedure of determining truth values does not always have to give a classically assumed (and expected) unique answer. Such an assumption is an unjustified generalization from common situations to all situations. We can preserve the classical procedure but we must reject universality of the assumption of its success. The awareness of that transforms paradoxes of truth to normal situations inherent to the classical procedure. Let's note that the paradoxes of truth arise from the internal organization of language, so they belong to the logical aspect of the concept of truth and do not concern the synthetic concept of truth. Thus the solution should be sought in the internal organization of the language.¹³

9 Epilogue

In 1991, Milošević and Tuđman, presidents of Serbia and Croatia, met in Karađorđevo, in the former Yugoslavia. They talked behind closed doors, with no witnesses, and no record was left of the conversation. Did they then make an agreement on the partitioning of Bosnia and Herzegovina along so-called ethnic lines, and thus destroy so many human lives and cause so much human suffering? The synthetic concept of truth gives us the legitimacy to ask that question, and all of the above aspects of the concept of truth can help us get the answer one day.

 $^{^{13}\}mathrm{A}$ good overview of various solutions to the paradox of truth can be found in [Beall et al., 2020]. The author's solution can be found in [Čulina, 2001, Čulina, 2020a].

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