SEMANTIC EPISTEMOLOGY REDUX: PROOF AND VALIDITY IN QUANTUM MECHANICS

Arnold CUSMARIU

ABSTRACT: Definitions I presented in a previous article as part of a semantic approach in epistemology assumed that the concept of derivability from standard logic held across all mathematical and scientific disciplines. The present article argues that this assumption is not true for quantum mechanics (QM) by showing that concepts of validity applicable to proofs in mathematics and in classical mechanics are inapplicable to proofs in QM. Because semantic epistemology must include this important theory, revision is necessary. The one I propose also extends semantic epistemology beyond the 'hard' sciences. The article ends by presenting and then refuting some responses QM theorists might make to my arguments.

KEYWORDS: Richard Feynman, J.M. Jauch, paradigm shift, proof, quantum mechanics, semantic epistemology, validity

1. Introduction

In an earlier article,[1] I presented and defended definitions of semantic evidence in science and in mathematics as part of a general semantic approach in epistemology. The first clauses of these definitions read as follows ('SL' and 'ML' abbreviate 'scientific language' and 'mathematical language,' respectively):

> (SES1) Where *z* is a wff of a scientific language *SL*, <u>*z*-is-evident-in-*SL* for S</u> =Df (i) There is a derivation-in-*SL* of *z* from true-in-*SL* instrumental-accuracy-law-sentences-of-*SL* and initial-condition-sentences-of-*SL*.
>
> (SEM1) Where *z* is a wff of a mathematical language *ML*, <u>*z* is evident-in-*ML* for a person S</u> =Df (i) There is a derivation-in-*ML* of *z*.

Three assumptions were made that seemed obvious at the time:

<u>Assumption 1</u>: SES(i) and SEM(i) are entitled to employ the same concept of derivability from standard logic. This assumption was made so that a semantic evidence predicate could be formulated along deductivist lines for both science and mathematics.[2]

---

[1] Arnold Cusmariu, "Toward a Semantic Approach in Epistemology," *Logos & Episteme. An International Journal of Epistemology* III, 4 (2012): 531-543.

[2] Cusmariu, "Toward a Semantic Approach," 533.

Assumption 2: The same concept of derivability from standard logic holds across all mathematical disciplines and their respective languages. This assumption was proved by Bertrand Russell and A.N. Whitehead in *Principia Mathematica*.
Assumption 3: The same concept of derivability from standard logic holds across all scientific disciplines and their respective languages. Physics has taken this assumption for granted ever since Newton's derivation of Kepler's Laws from his own.

I will show that Assumption 3 is not true for quantum mechanics (QM), likewise Assumption 1. Using as case study a proof by J.M. Jauch in his *Foundations of Quantum Mechanics*,[3] I show that standard concepts of validity applicable to proofs in mathematics and in classical mechanics are inapplicable to proofs in QM; therefore, SES1 must be revised to include this important theory. The one I propose also extends semantic epistemology beyond the 'hard' sciences. The article ends by showing the inadequacy of some responses QM theorists might make to my arguments and suggests that the unavailability of standard logic is a reason QM may represent a paradigm shift. Assessing what exactly that entails is beyond the scope of this article.

2. Case Study Preliminaries

Mathematical proofs work by establishing logical links to previous results. A *reductio ad absurdum* proof, which Pythagoras used to show that √2 is irrational, goes about it in a special way. Here a proposition *A* is proved by showing that its negation, ~*A*, leads to contradiction, *C*, from which *A* follows because contradictions are false. G.H. Hardy thought the *reductio* was "one of a mathematician's finest weapons."[4]

Though QM is an empirical theory – as is classical mechanics – there have also been efforts to prove results by purely logical means, including the *reductio* method. In his book, J.M. Jauch proved the following by *reductio*:

Proposition 1: Every dispersion-free state is pure.

Jauch proves Proposition 1 by deriving a contradiction from its negation,

Proposition 2: There is a dispersion-free and mixed state.

Here is the language of his proof:[5]

[3] Josef M. Jauch, *Foundations of Quantum Mechanics* (Reading, Massachusetts: Addison-Wesley, 1968). See also Constantin Piron, *Foundations of Quantum Mechanics* (London: Benjamin, 1976).
[4] G.H. Hardy, *A Mathematician's Apology* (Cambridge: Cambridge University Press, 1940), 94.
[5] Jauch, *Foundations of Quantum Mechanics*, 115.

Suppose the state p is a mixture. Then there exist two different states p_1 and p_2, as well as two positive numbers λ_1 and λ_2 such that $\lambda_1 + \lambda_2 = 1$ and $p = \lambda_1 p_1 + \lambda_2 p_2$. Since the two states p_1 and p_2 are different from one another, there exists a proposition a such that $p_1(a) \neq p_2(a)$ (cf. Property 5b of Section 6-3). Since the states are dispersion-free, there are two possibilities only: $p_1(a) = 1$, $p_2(a) = 0$ or $p_1(a) = 0$, $p_2(a) = 1$. In either case we have $\sigma(\mathrm{a}) = \lambda_1 + \lambda_2 \neq 0$. Thus the state is not dispersion-free, contrary to the assumption. This proves the proposition.

In a tradition going back to Euclid, Jauch presents only information he thinks is sufficient to make it apparent that the argument is logically correct (valid) – or, as Fermat famously put it, to 'compel belief' (*forcer á croire*).[6] Proving that the argument is valid, however, is another matter entirely.[7] A standard way of doing that in logic is by means of a formal proof of validity (FPV), which entails making argument steps explicit all the way to the conclusion and stating the rules of logic used to derive inferred steps.

3. A Formal Proof of Validity of Jauch's *Reductio*

Jauch's Proposition 1 is a universally quantified material conditional of first-order logic and may be symbolized as

(P1) $(x)(Fx \rightarrow \mathrm{G}x)$,

while its negation, Proposition 2, is an existentially quantified conjunction of first-order logic and may be symbolized as

(P2) $(\exists x)(Fx \,\&\, {\sim}Gx)$.

This construal of P1 and P2 is reasonable because Jauch uses the terms 'every' and 'there exist,' which denote universal and existential quantifiers, respectively, and has them apply to states characterized as dispersion-free, pure or mixed. This suggests that states are the objects of quantification in P1 and P2

[6] Paul Tannery and Charles Henry, eds., *Œvre de Fermat* (Paris: Blanchard, 1891-1912), Vol. II, 483.

[7] The idea that syntactic validity is proved by reference to rules of inference is due to Aristotle. Unfortunately, his list of valid syllogisms, which effectively function as such rules, turned out to be inadequate for general mathematical purposes – including the FPV of Jauch's *reductio* below – and there the matter rested until Frege's discovery of quantification. For more on these issues, see Arnold Cusmariu, "A Methodology for Teaching Logic-Based Skills to Mathematics Students," *Symposion: Theoretical and Applied Inquiries in Philosophy and Social Sciences* 3, 3 (2016): 259-292, esp. 259-261.

rather than the properties of being dispersion-free, pure and mixed. Quantification in P1 and P2 is first, not second order.[8]

However, strictly speaking, the negation of P1 is not P2 but rather

(P3) $\sim(x)(Fx \rightarrow Gx)$.

An FPV from P1 to P2 is needed before the full argument can get under way. Working with logical-form versions of Propositions 1 and 2 is sufficient for this purpose.

Steps	**Justification**
(1) $\sim(x)\Phi x \equiv (\exists x)\sim\Phi x$	*Quantifier Negation Law* (QN)
(2) $\therefore \sim(x)\Phi x \rightarrow (\exists x)\sim\Phi x$	From 1 by *Material Equivalence*, *Simplification* (Simp.)
(3) $\therefore \sim(x)(Fx \rightarrow Gx) \rightarrow (\exists x)\sim(Fx \rightarrow Gx)$	From 2 by *Substitution*
(4) $\therefore (\exists x)\sim(Fx \rightarrow Gx)$	From P3, 3 by *Modus Ponens* (MP)
(5) $\therefore (\exists x)\sim(Fx \rightarrow Gx) \rightarrow (\exists x)\sim(\sim Fx \vee Gx)$	From 4 by *Material Implication*
(6) $\therefore (\exists x)\sim(\sim Fx \vee Gx)$	From 4, 5 by MP
(7) $\therefore (\exists x)\sim(\sim Fx \vee Gx) \rightarrow (\exists x)(\sim\sim Fx \,\&\, \sim Gx)$	From 6 by *De Morgan's Theorem*
(8) $\therefore (\exists x)(\sim\sim Fx \,\&\, \sim Gx)$	From 6, 7 by MP
(9) $\therefore (\exists x)(\sim\sim Fx \,\&\, \sim Gx) \rightarrow (\exists x)(Fx \,\&\, \sim Gx)$	From 8 by *Double Negation*
(P2) $\therefore (\exists x)(Fx \,\&\, \sim Gx)$	From 8, 9 by MP

The FPV of Jauch's argument can now proceed.

Steps	**Justification**
$\therefore$ (1) p is a dispersion-free and mixed state.	From Proposition 2 by *Existential Instantiation*
$\therefore$ (2) p is a dispersion-free state.	From 1 by Simp.
(3) If p is a dispersion-free and mixed state, then p consists of dispersion-free states $p_1 \neq p_2$ such that $p = \lambda_1 p_1 + \lambda_2 p_2$ for positive numbers $\lambda_1 + \lambda_2 = 1$.	Assumption
$\therefore$ (4) p consists of dispersion-free states $p_1 \neq p_2$ such that $p = \lambda_1 p_1 + \lambda_2 p_2$ for positive numbers $\lambda_1 + \lambda_2 = 1$.	From 1, 3 by MP
$\therefore$ (5) If p consists of dispersion-free states $p_1 \neq p_2$ such	From Property 5b of Section 6-3,

[8] Jauch's identity conditions for states suggest a first-order interpretation: "Two states are identical if the relevant conditions in the preparation of the state are the same." (Jauch, *Foundations of Quantum Mechanics*, 92.) The ontological status of states does not affect the arguments presented below.

that $p = \lambda_1 p_1 + \lambda_2 p_2$ for positive numbers $\lambda_1 + \lambda_2 = 1$, then, $p_1(\text{a}) \neq p_2(\text{a})$ for some proposition *a*.	by *Universal Instantiation*[9]
∴ (6) $p_1(a) \neq p_2(a)$ for some proposition *a*.	From 4, 5 by MP
(7) If $p_1(a) \neq p_2(a)$ for some proposition *a*, then, either $p_1(\text{a}) = 1$ and $p_2(\text{a}) = 0$; or, $p_1(\text{a}) = 0$ and $p_2(\text{a}) = 1$.	Assumption
∴ (8) Either $p_1(\text{a}) = 1$ and $p_2(\text{a}) = 0$; or $p_1(\text{a}) = 0$ and $p_2(\text{a}) = 1$.	From 6, 7 by MP
(9) If Either $p_1(\text{a}) = 1$ and $p_2(\text{a}) = 0$; or $p_1(\text{a}) = 0$ and $p_2(\text{a}) = 1$, then, $\sigma(\text{a}) = \lambda_1 + \lambda_2 \neq 0$.	Assumption
∴ (10) $\sigma(\text{a}) = \lambda_1 + \lambda_2 \neq 0$.	From 8, 9 by MP
∴ (11) If $\sigma(\text{a}) = \lambda_1 + \lambda_2 \neq 0$, then *p* is not a dispersion-free state.	From the definition of 'dispersion-free' state.[10]
∴ (12) *p* is not a dispersion-free state.	From 10, 11 by MP
∴ (13) *p* is a dispersion-free state and *p* is not a dispersion-free state.	From 2, 12 by *Conjunction*
∴ (14) ~(*p* is a dispersion-free state and *p* is not a dispersion-free state.)	From (13) by the *Law of Non-Contradiction*
∴ (15) If ~(*p* is a dispersion-free state and *p* is not a dispersion-free state), then, it is not the case that *p* is a dispersion-free and mixed state.	From (1) and (14), shortcut[11]
∴ (16) It is not the case that *p* is a dispersion-free and mixed state.	From (14), (15) by MP
∴ (17) It is not the case that there is a dispersion-free and mixed state.	From (16) by *Existential Generalization*
∴ (18) Every dispersion-free state is pure. Q.E.D.	From (17) by QN[12]

[9] Jauch, *Foundations of Quantum Mechanics*, 94. This is one of several properties postulated.
[10] Jauch, *Foundations of Quantum Mechanics*, 114.
[11] We omit the laborious process of deriving (15) to avoid cluttering the text. The points made below do not require a full expansion of the argument for step (15).
[12] An FPV of (18), which has the form $(x)(Fx \rightarrow Gx)$, from (17), which has the form $\sim(\exists x)(Fx \,\&\, Gx)$, is analogous to the previous FPV and may be omitted.

4. Syntactic and Semantic Validity

The above is a proof of validity in the syntactic sense, according to which an argument is syntactically valid if and only if all inferred steps are derived according to rules of logic. A concept distinct from syntactic validity is semantic validity, according to which an argument is semantically valid if and only if its conclusion is true if the premises are true for any truth-functional interpretation of premises and conclusion. That is, an argument from premises $\{p_1, p_2, p_3 \ldots p_n\}$ to conclusion c is semantically valid just in case the corresponding material conditional $(p_1\&p_2\&p_3\& \ldots p_n) \rightarrow c$ is a tautology.

Let us provide a proof of semantic validity in the context of Jauch's argument. It will be sufficient to do so only for a portion of the argument because it is elementary how the proof can be generalized for the entire argument.

Thus, consider the inference to step (12) from premises (10) and (11):

(10) $\sigma(a) = \lambda_1 + \lambda_2 \neq 0$.

(11) If $\sigma(a) = \lambda_1 + \lambda_2 \neq 0$, then p is not a dispersion-free state.

∴ (12) p is not a dispersion-free state.

For ease of reference, let us first abbreviate (10) as r; (11) as $r \rightarrow \sim s$; and (12) as $\sim s$. The material conditional corresponding to the argument from (10) and (11) to (12) is $(r \& (r \rightarrow \sim s) \rightarrow \sim s)$. Next we enter this sentence and its components into a truth table configured according to standard semantics for logical connectives.

	1	2	3	4	5	6
1	r	s	$\sim s$	$r \rightarrow \sim s$	$r \& (r \rightarrow \sim s)$	$(r \& (r \rightarrow \sim s)) \rightarrow \sim s$
2	T	T	F	F	F	T
3	T	F	T	T	T	T
4	F	T	F	T	F	T
5	F	F	T	T	F	T

If the argument from (10) and (11) to (12) is semantically valid, then we should find that the material conditional corresponding to this argument, $(r \& (r \rightarrow \sim s) \rightarrow \sim s)$, is a tautology, meaning that column 6 should show only the truth value True, which it does. This completes the proof of semantic validity for the argument from (10) and (11) to (12) and, by implication, Jauch's entire argument.

It is not a coincidence that a syntactically valid argument has turned out to be semantically valid as well. Though there is no need here to address the general problem of equivalence between syntactic and semantic validity, the following points are relevant to the arguments that will emerge shortly.

First, we note that it is standard to define logical connectives by means of binary truth values as shown in the above truth table. Thus, column 3 shows the definition of negation; column 4 of material implication; and column 5 of conjunction. Though disjunction is not shown, its definition is set by binary truth values in similar fashion. Logical connectives are in general defined by standard truth-table semantics.

Second, standard semantics for logical connectives also define the concept of a tautology.

Third, the fact that standard semantics for logical connectives define the concept of a tautology means that the definition of semantic validity also assumes such semantics.

Fourth, the definition of syntactic validity also assumes standard semantics for logical connectives because: (a) logically compound sentences occur routinely in arguments, certainly mathematical arguments; and (b) rules of logic applied to derive inferred steps assume such semantics.

Point (b) may be obviated by noting that rules of logic such as MP, applied in the FPV above, assume standard semantics for material implication; and by noting that MP works because it is itself a semantically valid argument, meaning that a tautology corresponds to it. A truth table will show that the symbolic sentence corresponding to MP, $(p \,\&\, (p \rightarrow q)) \rightarrow q$, is indeed a tautology. It is apparent that standard semantics for material implication and conjunction must be assumed to show that this sentence is a truth-table tautology.

5. Logical Connectives and QM's Uncertainty Principle

According to truth tables defining standard semantics for logical connectives, the following sentences are tautologies:

(D1) $p \,\&\, (q \vee r) \rightarrow (p \,\&\, q) \vee (p \,\&\, r)$

(D3) $p \rightarrow ((p \,\&\, q) \vee (p \,\&\, {\sim}q))$

Thus, all eight rows of column 9 linking D1 components in columns 7 and 8 by means of material implication show the truth-value True.

	1	2	3	4	5	6	7	8	9
	p	q	r	$p \,\&\, q$	$p \,\&\, r$	$q \vee r$	1 & 6	4 v 5	7 → 8
1	T	T	T	T	T	T	T	T	T

2	T	T	F	T	F	T	T	T	T
3	T	F	T	F	T	T	T	T	T
4	T	F	F	F	F	F	F	F	T
5	F	T	T	F	F	T	F	F	T
6	F	T	F	F	F	T	F	F	T
7	F	F	T	F	F	T	F	F	T
8	F	F	F	F	F	F	F	F	T

Likewise, all four rows of column 7 linking D3 components in columns 1 and 6 by means of material implication show the truth-value True.

	1	2	3	4	5	6	7
	p	q	$\sim q$	p & q	p & $\sim q$	4 v 5	$1 \rightarrow 6$
1	T	T	F	T	F	T	T
2	T	F	T	F	T	T	T
3	F	T	F	F	F	F	T
4	F	F	T	F	F	F	T

We wish to show that the Uncertainty Principle (UP) of QM is not consistent with the tautological status of D1 and D3. Let us consider them in turn.

UP-D1 Inconsistency: The equivalence

(D) p & (q v r) ≡ (p & q) v (p & r),

is the rule of replacement *Distribution*, which is a conjunction of material conditionals:

(D1) p & (q v r) → (p & q) v (p & r)

(D2) (p & q) v (p & r) → p & (q v r)

UP is inconsistent with the tautological status of D1 because it implies that the antecedent of D1, p & (q v r), can be true but the consequent of D1, (p & q) v (p & r), is false because both disjuncts are false.

Thus, consider the following scenario:

- *p* is the proposition that the momentum of particle *x* is in the interval [0, +1/6],
- *q* is the proposition that the position of particle *x* is in the interval [−1, +1],
- *r* is the proposition that the position of particle *x* is in the interval [+1, +3][13]

Note first that the scenarios described by the three propositions *p*, *q* and *r*, taken singly, are consistent with UP. Second, scenario *p* & (*q* v *r*), the antecedent of D1, is also consistent with UP, and is in fact one of several truth-functional combinations of *p*, *q* and *r* that are consistent with UP.

However, scenario (*p* & *q*) v (*p* & *r*) is not consistent with UP because it asserts restrictions on simultaneous values of position and momentum of a particle that are not consistent with UP. If UP is true, the two components of the consequent of D1, (*p* & *q*) and (*p* & *r*), are both false, hence their disjunction is false.[14]

It is important not to misunderstand how this UP-based counterexample to the status of D1 as a tautology arises. It does not arise from the two scenarios taken singly because several truth-table rows for both scenarios show truth values consistent with UP.

Thus, column 7 of the truth table shows three rows where the scenario *p* & (*q* v *r*) has the truth value True:

	1	2	3	6	7
	p	*q*	*r*	*q* v *r*	1 & 6
1	T	T	T	T	T
2	T	T	F	T	T
3	T	F	T	T	T

Likewise, column 8 of the truth table below shows five rows where the scenario (*p* & *q*) v (*p* & *r*) has the truth value False:

[13]Scenario details are from a *Wikipedia* article, https://en.wikipedia.org/wiki/Quantum_logic, accessed July 28, 2015.

[14]Note that D2 is consistent with UP, because a conditional is trivially true if it has a false antecedent, which is the case if (*p* & *q*) v (*p* & *r*) is false according to UP because both disjuncts are false. Note also that 'rejecting D1' is shorthand for "rejecting that D1 is a tautology."

	1	2	3	4	5	8
	p	*q*	*r*	*p* & *q*	*p* & *r*	4 v 5
4	T	F	F	F	F	F
5	F	T	T	F	F	F
6	F	T	F	F	F	F
7	F	F	T	F	F	F
8	F	F	F	F	F	F

Rather, the problem is that there are no rows where <u>both</u> disjuncts of (*p* & *q*) v (*p* & *r*) are false (columns 4 and 5) <u>and</u> *p* & (*q* v *r*) is true (column 7).

	1	2	3	4	5	7
	p	*q*	*r*	*p* & *q*	*p* & *r*	1 & 6
2	T	T	F	T	F	T
3	T	F	T	F	T	T

Nor are there rows where <u>both</u> disjuncts of (*p* & *q*) v (*p* & *r*) are false (columns 4 and 5) <u>and</u> *p* & (*q* v *r*) is true (column 7).

	1	2	3	4	5	6	7
	p	*q*	*r*	*p* & *q*	*p* & *r*	*q* v *r*	1 & 6
4	T	F	F	F	F	F	F
5	F	T	T	F	F	T	F
6	F	T	F	F	F	T	F
7	F	F	T	F	F	T	F
8	F	F	F	F	F	F	F

<u>UP-D3 Inconsistency</u>: UP rejects the tautological status of D3

D3. $p \rightarrow (p \,\&\, q) \vee (p \,\&\, {\sim}q)$,[15]

as well because UP blocks the material implication of $(p \,\&\, q) \vee (p \,\&\, {\sim}q)$ from p under the following circumstances. Let B_1 and B_2 be Borel sets and p, q and ${\sim}q$ be the following propositions:

- p is the proposition that a measurement of the momentum of a particle will yield a value in B_1.
- q is the proposition that a (simultaneous) measurement of the position of a particle will yield a value in B_2.
- ${\sim}q$ is the proposition that a (simultaneous) measurement of the position of a particle will not yield a value in B_2.[16]

Even though p is consistent with UP, nevertheless according to UP it is neither the case that $p \,\&\, q$ nor that $p \,\&\, {\sim}q$. That is, according to UP, we should find at least one row in the truth table of D3

	1	2	3	4	5	6	7
	p	q	${\sim}q$	$p \,\&\, q$	$p \,\&\, {\sim}q$	4 v 5	$1 \rightarrow 6$
1	T	T	F	T	F	T	T
2	T	F	T	F	T	T	T
3	F	T	F	F	F	F	T
4	F	F	T	F	F	F	T

where p is true and $(p \,\&\, q)$ and $(p \,\&\, {\sim}q)$ are both false, i.e., where column 7 would show one F. However, inspection shows no such row in the truth table.In rows 3 and 4 where $(p \,\&\, q)$ and $(p \,\&\, {\sim}q)$ are both false, p is also false; in rows 1 and 2 where p is true, $(p \,\&\, q) \vee (p \,\&\, {\sim}q)$ is also true even though the two disjuncts alternate truth values.

Now, D1 and D3 are tautologies if and only if standard semantics for logical connectives are assumed. Therefore, QM, which must accept UP and must reject the tautological status of D1 and D3, must also reject standard semantics for logical connectives. What does this imply for Jauch's *reductio*?

---

[15] The equivalence class of D3 includes such tautologies as $p \rightarrow (p \,\&\, (q \vee {\sim}q))$ and $p \rightarrow (p \,\&\, (q \rightarrow q))$, whose status is also excluded by UP. Logical connectives being interdefinable, D1 also has an equivalence class with the same consequences.

[16] This example is adapted from David W. Cohen, *An Introduction to Hilbert Space and Quantum Logic* (New York: Springer-Verlag, 1989), 93.

6. A Problem for Jauch's *Reductio*

Jauch no doubt would have claimed that his argument for Proposition 1 valid in the intuitive sense that no errors of logic were committed, which is true as we have seen. If asked, he would have claimed further that his argument was valid in the sense of 'valid' that applies to all valid mathematical proofs. I wish to show that Jauch's acceptance of UP[17] and his implicit rejection of D1 and D3 as tautologies undermines the second claim.[18]

Because validity has a syntactic as well as a semantic meaning, proving this point requires two arguments. Let us take them in turn.

<u>Argument 1: Syntactic Validity</u>

> (a1) Jauch's argument is valid according to the standard definition of syntactic validity only if inferred steps are derived according to standard rules of logic.
>
> (b1) If inferred steps are derived according to standard rules of logic, then standard semantics define logical connectives in standard rules of logic applied.[19]
>
> (c1) If standard semantics define logical connectives in standard rules of logic applied, then standard semantics define logical connectives in all rules of logic.
>
> (d1) If standard semantics define logical connectives in all rules of logic, then they define logical connectives in D1.[20]
>
> (e1) If standard semantics define logical connectives in D1, then D1 is a tautology.
>
> (f1) If D1 is a tautology, there are no truth-value assignments under which both disjuncts of $(p \& q) \vee (p \& r)$ are false and $p \& (q \vee r)$ is true.
>
> (g1) If there are no truth-value assignments under which both disjuncts of $(p \& q) \vee (p \& r)$ are false and $p \& (q \vee r)$ is true, the Uncertainty Principle is false.
>
> (h1) The Uncertainty Principle is true.

Therefore,

> (i1) Jauch's argument is not valid according to the standard definition of syntactic validity.

---

[17] Jauch, *Foundations of Quantum Mechanics*, 162.

[18] Hans Reichenbach, a proponent of three-valued logic in QM, stated that in QM "[t]he two distributive rules hold in the same form as in two-valued logic." (Hans Reichenbach, *Philosophic Foundations of Quantum Mechanics* (Berkeley and Los Angeles: University of California Press, 1944), 156.)

[19] This premise would need to be restated slightly to run the argument with D3 because D3 is not a rule of logic. There is no need to do that for present purpose.

[20] It is sufficient for present purposes to focus only on the inconsistency between UP and D1.

Argument 2: Semantic Validity

(a2) Jauch's argument is valid according to the standard definition of semantic validity only if for any truth-functional interpretation of logically compound premises and conclusion, if the premises are true, then the conclusion is true.

(b2) For any truth-functional interpretation of logically compound premises and conclusion, if the premises are true, then the conclusion is true only if standard semantics define logical connectives in premises and conclusion.

(c2) If standard semantics define logical connectives in premises and conclusion, then standard semantics define conjunction, disjunction and material implication.

(d2) If standard semantics define conjunction, disjunction and material implication, then D1 is a tautology.

(e2) If D1 is a tautology, then there are no truth-value assignments under which both disjuncts of $(p \,\&\, q) \vee (p \,\&\, r)$ are false and $p \,\&\, (q \vee r)$ is true.

(f2) If there are no truth-value assignments under which both disjuncts of $(p \,\&\, q) \vee (p \,\&\, r)$ are false and $p \,\&\, (q \vee r)$ is true, then the Uncertainty Principle is false.

(g2) The Uncertainty Principle is true.

Therefore,

(h2) Jauch's argument is not valid according to the standard definition of semantic validity.

Therefore,

(G) Jauch's argument is not valid according to standard definitions of semantic and semantic validity.

Both arguments are logically correct. But are their premises true?

7. Defense of Argument 1

Premise (a1): This follows from the definition of syntactic validity.
Premise (b1): Rules of logic applied to derive steps in the FPV of Jauch's argument are logically compound, so that standard semantics for logical connectives are automatically assumed.
Premise (c1): It cannot be the case that some rules of logic assume standard semantics for logical connectives and some do not.
Premise (d1): This premise is true because D1 is part of a rule of logic.
Premise (e1): This follows from the truth table of D1.
Premise (f1): This also follows from the truth table of D1.
Premise (g1): To see that this premise is true, consider its contrapositive:

(g1*) If the Uncertainty Principle is true, there are truth-value assignments under which both disjuncts of (p & q) v (p & r) are false and p & (q v r) is true.

(g1*) is true based on the three propositions p, q and r specified above:

p is the proposition that the momentum of particle x is in the interval [0, +1/6],

q is the proposition that the position of particle x is in the interval [−1, +1],

r is the proposition that the position of particle x is in the interval [+1, +3]

If the Uncertainty Principle is true, then (g1*) is true given p, q and r as above. Since (g1*) and (g1) are equivalent, it follows that (g1) is also true.

Premise (h1): UP must be assumed to be true, certainly by QM theorists.

8. Defense of Argument 2

Premise (a2): This premise states a necessary condition of semantic validity.
Premise (b2): The discussion above of semantic validity justifies this premise.
Premise (c2): The truth table proving semantic validity of a portion of Jauch's argument,

	1	2	3	4	5	6
1	r	s	$\sim s$	$r \rightarrow \sim s$	r & $(r \rightarrow \sim s)$	$(r$ & $(r \rightarrow \sim s)) \rightarrow \sim s$
2	T	T	F	F	F	T
3	T	F	T	T	T	T
4	F	T	F	T	F	T
5	F	F	T	T	F	T

shows that if standard semantics define logical connectives in premises (columns 1 and 4) and conclusion (column 5), then standard semantics define conjunction, disjunction and material implication. Disjunction occurs in the column 5 sentence because material implication is definable in terms of it.
Premise (d2): This follows from the truth table of D1.
Premise (e2): This is the same as premise (f1) above.
Premise (f2): This is the same as premise (g1) above.
Premise (g2): This is the same as premise (h1) above.

9. Rescuing Semantic Epistemology

It would be quite an undertaking to devise a non-standard concept of derivability – call it 'derivability*' – and then complicate SES1 as follows:

> (SES2) Where *z* is a wff of a scientific language *SL*, <u>*z*-is-evident-in-*SL* for S</u> =Df (i) Either there is a derivation-in-*SL* or a derivation*-in-*SL* of *z* from true-in-*SL* instrumental-accuracy-law-sentences-of-*SL* and initial-condition-sentences-of-*SL*.

This is unnecessary. We can borrow the disjunctive form of SES2 and then rely on the fact that QM makes essential use of the concept of probability:

> (SES3) Where *z* is a wff of a scientific language *SL*, <u>*z*-is-evident-in-*SL* for S</u> =Df Either (i) there is a derivation-in-*SL* of *z* from true-in-*SL* instrumental-accuracy-law-sentences-of-*SL* and initial-condition-sentences-of-*SL*, or (ii) the probability of *z* is certainty or practically certainty relative to true-in-*SL* instrumental-accuracy-law-sentences-of-*SL* and initial-condition-sentences-of-*SL*; and either (iii) the derivation-in-*SL* of *z* is believed-in-*SL* by S, or (iv) or the relative probability of *z* as certainty or practically certainty is believed-in-*SL* by S.

A link weaker than deduction may enable us to widen the circle of semantic knowledge to include fields not considered 'hard' sciences such as psychology, anthropology and sociology. Thus, SES3 addresses a concern raised in the earlier article.[21] The semantic definition of scientific knowledge can remain unchanged.[22]

10. Some QM Responses Considered

It is far beyond the scope of this article to evaluate efforts to cope with the unavailability of standard logic in QM by replacing it with what has come to be called 'quantum logic,'[23] including how quantum logic might formulate an FPV of Jauch's *reductio* argument. Instead, let us consider two interesting strategies that QM proponents might suggest to counter Arguments 1 and 2.

---

[21] Cusmariu, "Toward a Semantic Approach," 542. I realize more needs to be said to make clear how SES3 would cover semantic knowledge in fields not considered 'hard' sciences. However, the matter is too complex to treat adequately in an article of this scope.

[22] Cusmariu, "Toward a Semantic Approach," 536.

[23] The *locus classicus* of what came to be called 'quantum logic' is Garrett Birkhoff and John von Neumann, "The logic of quantum mechanics," *Annals of Physics* 37 (1936), 823-43. An excellent review of the issues is Peter Gibbins, *Particles and Paradoxes* (Cambridge: Cambridge University Pres, 1987). Philosophical issues in QM are addressed by contributors to *The Wave Function*, eds. Alyssa Ney and David Z. Albert (Oxford: Oxford University Press, 2013). See also Gabriel Târziu, "Quantum vs Classical Logic: The Revisionist Approach," *Logos & Episteme. An International Journal of Epistemology* III, 4 (2012): 579-590; and Pierre Uzan, "Logique Quantique et Intrication," *Logos & Episteme. An International Journal of Epistemology* V, 3 (2014): 245-263.

STRATEGY 1: Redefine logical connectives using three-valued logic.

Comment: Hans Reichenbach has proposed redefining logical connectives using three-valued logic as a way of avoiding having to characterize statements about unobserved entities as meaningless.[24] Does adding a third truth-value, *Indeterminate*, and building new truth tables for the usual logical connectives resolve the problem?[25]

It does not, for D1 as well as D3. In the D1-associated scenario, the three propositions *p*, *q* and *r* are all true.

> *p* is the proposition that the particle has momentum in the interval [0, +1/6],
>
> *q* is the proposition that the particle is in the interval [−1, +1], and
>
> *r* is the proposition that the particle is in the interval [+1, +3].

The truth value of *p*, *q* and *r* is not *Indeterminate*; nor are *p*, *q* and *r* incompatible with the Uncertainty Principle taken singly. Given that the truth value of *p*, *q* and *r* taken singly is *True* and not *Indeterminate*, it follows from Reichenbach's own revised truth tables[26] that truth-functional combinations of *p*, *q* and *r* will also not be *Indeterminate*, including material implication in D1. Moreover, if *p*, *q* and *r* all have the truth value True, D1 would not turn out to have the truth value *Indeterminate* even if material implication in D1 is replaced by counterparts that Reichenbach calls 'alternative implication' and 'quasi implication.'[27] Thus, Strategy 2 does not enable QM to avoid having to deny that D1 is a tautology.

STRATEGY 2: QM needs only 'the mathematics of approximation.'

Comment 1: This strategy would appeal to a distinction the mathematician Felix Klein drew in a book originally published in 1908:[28]

---

[24] Reichenbach, *Philosophic Foundations*, 144-168.

[25] Peter Gibbins writes: "There are those that try to impose on quantum mechanics a logic that does not arise naturally from the formalism of the theory. Such is Reichenbach's interpretation which employs a 3-valued truth functional logic and which is generally admitted to be a nonstarter." Gibbins, *Particles and Paradoxes*, 124. Gibbins also makes a startling admission: "… what the [logical] connectives mean is a real problem in the philosophy of quantum mechanics. All attractive routes for defining them independently of the formalism of quantum mechanics seem to be blocked (I think they are blocked.) If this is so, the scope of quantum logic as a 'logic of the world' will be restricted (as I think it is)." Gibbins, *Particles and Paradoxes*, 140.

[26] Reichenbach, *Philosophic Foundations*, 151.

[27] Reichenbach, *Philosophic Foundations*, 151.

[28] Felix Klein, *Elementary Mathematics from an Advanced Standpoint: Arithmetic, Algebra, and Analysis* (New York: Cosimo Classics, 2009), 36.

> [I]t is natural to divide mathematics into two parts, which have been called *mathematics of approximation* and the *mathematics of precision*. If we desire to explain this difference by an interpretation of the equation $f(x) = 0$, we may note that, in the mathematics of approximation one is not concerned that $f(x)$ should be *exactly* zero, but merely that its absolute value $|f(x)|$ should *remain below the attainable threshold of exactness* ε. The symbol $f(x) = 0$ is merely an abbreviation for the inequality $|f(x)| < \varepsilon$, with which one is really concerned. It is only in the mathematics of precision that one insists that the equation $f(x) = 0$ be exactly satisfied.

Comment 2: Strategy 2 suggests an instrumentalist approach to mathematics, according to which mathematical resources are merely tools for computation, measurement, approximation, and the like. In QM, such a viewpoint is sometimes expressed by the admonition to "shut up and calculate." Thus, QM physicists have argued that the theory has been confirmed by experiments, has undeniable explanatory and predictive power, and that can be the end of it as far as physics is concerned.

Well and good but instrumentalism does not imply that any of the premises of Arguments 1 and 2 are false. Indeed, how could it? Those premises belong in the realm of logic alongside the 'mathematics of precision' – just as Plato thought. In any case, Klein was surely correct to note, at the end of the passage quoted above, that the mathematics of precision provides "valuable and indeed indispensible support for the development of mathematics of approximation."[29] The claim that QM needs only the mathematics of approximation is wishful thinking.

11. Concluding Remarks

Physics from Newton to Einstein is compatible with the logic that Russell and Whitehead placed at the foundations of mathematics in *Principia Mathematica*, including the concepts of proof and validity they worked hard to clarify. If the arguments presented here are sound, such compatibility does not hold for quantum mechanics – a disconnect that seems radical enough to warrant considering the theory a paradigm shift and may help explain why Richard Feynman famously quipped[30] "I think I can safely say that nobody understands quantum mechanics."[31]

---

[29] Klein, *Elementary Mathematics*, 36.

[30] Richard Feynman, *The Character of Physical Law: The Messenger Lectures* (New York: Modern Library, 1994), 129.

[31] Takis Hartonas and Gary Rosenkrantz provided helpful comments on earlier versions of this article.