COSMIC SKEPTICISM AND THE BEGINNING OF PHYSICAL REALITY

by

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To my father, Paul Linford, who unfortunately passed away during the final year that this dissertation was being written, and to my grandmother, Maria Merels, who passed during my undergraduate degree. I owe much to both of them.

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In several ways, this dissertation represents the culmination of my previous life experiences. Though I spent many years of my life carving out an identity independent of my parents, in retrospect, I am the person they created. My father was a practically minded engineer. He taught me to be skeptical from an early age; he wisely channeled and redirected my childhood love for *The X-Files* and UFO documentaries into a scientific mindset that refused to take fantastical claims seriously without significant evidence. Although he was a practically minded engineer, he was also a photographer and artist who married the sciences and the humanities in his own way. Though he did not live to see the completion of this dissertation, I believe he would have been proud of me for writing it. My mother's religious identity is important to her and, although I do not share her religious commitments and do not interact with religion in the way that she does, I doubt that I would have developed my profound interest in religion and ultimate questions were it not for her influence.

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ABSTRACT

This dissertation is concerned with two of the largest questions that we can ask about the nature of physical reality: first, whether physical reality begin to exist and, second, what criteria would physical reality have to fulfill in order to have had a beginning? Philosophers of religion and theologians have previously addressed whether physical reality began to exist in the context of defending the Kalám Cosmological Argument (KCA) for theism, that is, (P1) everything that begins to exist has a cause for its beginning to exist, (P2) physical reality began to exist, and, therefore, (C) physical reality has a cause for its beginning to exist. While the KCA has traditionally been used to argue for God's existence, the KCA does not mention God, has been rejected by historically significant Christian theologians such as Thomas Aquinas, and raises perennial philosophical questions – about the nature and history of physical reality, the nature of time, the nature of causation, and so on – that should be of interest to all philosophers and, perhaps, all humans. While I am not a religious person, I am interested in the questions raised by the KCA. In this dissertation, I articulate three necessary conditions that physical reality would need to fulfill in order to have had a beginning and argue that, given the current state of philosophical and scientific inquiry, we cannot determine whether physical reality began to exist.

Friends of the KCA have sought to defend their view that physical reality began to exist in two distinct ways. As I discuss in chapter 2, the first way in which friends of the KCA have sought to defend their view that physical reality began to exist involves a family of a priori arguments meant to show that, as a matter of metaphysical necessity, the past must be finite. If the past is necessarily finite, then the past history of physical reality is necessarily finite. And if having a finite past suffices for having a beginning, then, since the past history of physical reality is necessarily finite, physical reality necessarily began to exist. I show that the arguments which have been offered thus far for the view that the past is necessarily finite do not succeed. Moreover, as I elaborate on in chapter 5, having a finite past does not suffice for having a beginning.

As I discuss in chapter 3, the second way in which friends of the KCA have sought to defend their view that physical reality began to exist involves a family of a posteriori arguments meant to show that we have empirical evidence that physical reality has a finite past history. For example, the big bang is sometimes claimed to have been the beginning of physical reality and, since we have excellent empirical evidence for the big bang, we have excellent empirical evidence for the beginning of physical reality. The big bang can be understood in two ways. On the one hand, the big bang can be understood as a theory about the history and development of the observable universe. Understood in that sense, then I agree that the big bang is supported by excellent empirical evidence and by a scientific consensus. On the other hand, some authors (particularly science popularizers, science journalists, and religious apologists) have wrongly interpreted big bang theory as a theory about the beginning of the whole of physical reality. As I argue, while a beginning of physical reality may be consistent with classical big bang theory, classical big bang theory does not provide good reason for thinking that physical reality began to exist.

In part II, I turn to discussing three necessary, but not necessarily sufficient, conditions for physical reality to have a beginning. Before discussing the three conditions, in chapter 4, I introduce three metaphysical accounts of the nature of time (A-theory, B-theory, and C-theory) as well as some formal machinery that will subsequently become useful in the dissertation. I introduce the first of the three conditions in chapter 5. According to the Modal Condition, physical reality began to exist only if, at the closest possible worlds without time, physical reality does not exist. I show that this condition helps us to make sense of various views in both theology and philosophy of physics. In chapter 6, I introduce the second of my three conditions, the Direction Condition, according to which, roughly, physical reality began to exist only if all space-time points agree about the direction of time, so that all space-time points can agree that physical reality's putative beginning took place in their objective past. In chapter 7, I discuss the third condition, the Boundary Condition, according to which physical reality began to exist only if there is a past temporal boundary such that physical reality did not exist before the boundary. I show that there are two senses in which physical reality could be said to have had a past temporal boundary. Lastly, in chapter 8, I show that there is a relationship between my three conditions and classical big bang theory, even though the relationship is not the one usually identified in the literature.

In part III, I present four arguments for the view that, at the present stage of philosophical and scientific inquiry, we cannot know whether physical reality satisfies the three necessary conditions to have had a beginning and, consequently, we cannot know whether physical reality had a beginning. As I will prove in chapter 9, no set of observations that we currently have, when conjoined with General Relativity, entails that physical reality satisfies the Direction or Boundary Conditions. As I show in chapter 10, considerations in the philosophical foundations of statistical mechanics entail either that the Cosmos violates the Modal Condition or else that there is a transcendental condition on the possibility of our knowledge of the past that prevents our access to data we would need to gather to determine whether physical reality satisfies the Boundary Condition. In chapter 11, I show that there are a number of live cosmological models according to which physical reality does not satisfy the Boundary Condition. As long as we don't know whether any of those cosmological models are correct, we do not know whether physical reality satisfies the Boundary Condition. Lastly, I turn to confirmation theory and show that, at our present stage of inquiry, ampliative inferences for the conclusion that physical reality satisfies the Modal, Direction, and Boundary Conditions are not successful.

1. INTRODUCTION

The Cosmos is all that is or ever was or ever will be. Our feeblest contemplations of the Cosmos stir us — there is a tingling in the spine, a catch in the voice, a faint sensation of a distant memory, as if we were falling from a great height. We know we are approaching the greatest of mysteries.

— Carl Sagan, *Cosmos*

Following Sagan, I will understand *Cosmos* to mean the totality of physical reality and to exclude any supernatural or abstract entities should they exist.[1] According to the Kalám Cosmological Argument (KCA),

1. Everything that begins to exist has a cause for its existence.

2. The Cosmos began to exist.[2]

3. Therefore, the Cosmos has a cause for its existence.

An investigation into the soundness of the KCA sets a research agenda.[3] In order to determine whether the KCA is sound, we need to answer two questions. First, whether we have reason to think that anything that begins to exist has a cause for its existence; to answer that question, we would need to interrogate the concept of causation and determine the contexts in which we ought to invoke causes. There are reasons to doubt the KCA's first premise that I have taken up elsewhere (Linford, 2020), but here I set the

[1]↑By 'Cosmos', I mean to non-rigidly designate the collection of whatever physical objects exist. The Cosmos exists just in case any physical entity exist.

[2]↑The KCA is often written in terms of the universe and not in terms of the Cosmos. However, the meaning of the term 'universe' widely varies. For example, physicists have developed so-called "multiverse theories" which are said to entail that there are universes other than our own. On the most popular versions of multiverse theories, for example, the inflationary multiverse, the other universes are proper parts of the same space-time manifold that our universe is part of. For that reason, the various universes are proper parts of one overall physical reality. To avoid confusion, I have stipulatively defined the term 'Cosmos' to mean the totality of physical reality.

[3]↑The KCA is a deductive argument. Deductive arguments are said to be *sound* just in case the argument satisfies two conditions. First, the premises of the argument are true. Second, the argument is *valid*, that is, there is no possible situation in which the argument's premises are true and the conclusion is false. Since the KCA is a valid argument, we have left to investigate whether the KCA is sound.

first premise aside. The second question concerns whether we have reason to think that the Cosmos began to exist. This dissertation investigates the empirical case for the view that the Cosmos began to exist.

The research agenda set by the KCA is open to everyone regardless of whether they endorse God's existence. I am not a theist, myself, but my research agenda has been set by investigating the KCA. Certainly, the foremost proponents of the KCA, either in the present or historically, have been Christian or Muslim apologists or theologians – e.g., John Philopponus, Al-Kindi, Al-Ghazali, Bonaventure, William Lane Craig, Robert Koons, J.P. Moreland – and they have argued that the cause of the Cosmos must be God. In fact, the Arabic word *kalám* originates in Islamic theology. Nonetheless, God appears nowhere in the KCA, itself; theists, atheists, and agnostics alike are free to affirm the KCA. Moreover, while the conclusion of the KCA is officially endorsed by all three Abrahamic religions (Judaism, Christianity, Islam), theists can reject the KCA either by rejecting one of the KCA's premises, e.g., perhaps God caused the Cosmos but the Cosmos did not begin, or by rejecting the conception of God as Creator. In fact, some historically important Christian theologians, such as Thomas Aquinas, did reject the KCA. Thus, theists, atheists, and agnostics alike can reject the KCA. The mystery as to the ultimate origins of the totality of physical reality motivates a family of perennial philosophical questions that should interest nearly all philosophers and possibly nearly all members of our species. Consequently, whether the KCA is sound is of broad philosophical interest and should not be relegated to a narrow discussion among philosophers of religion and theologians.

This dissertation includes thirteen chapters and is divided into three parts. Part I discusses the KCA. Chapters 2 and 3 cover the a priori and a posteriori defenses of the KCA respectively. The a priori defense has been (in my view) adequately and convincingly addressed elsewhere; the best responses to the a priori defense have been provided by Wes Morriston (2000, 2003, 2010, 2013, 2022), Alex Malpass, e.g., (2021, unpublished), and in co-authored work by the two of them together (2020). I discuss some of the best arguments against the a priori defense of the KCA in chapter one. My discussion of previous responses to the a priori defense is meant to motivate the rest of the dissertation, where I turn to the a posteriori defense. In chapter two, I turn to the a posteriori defense of the KCA, that is, attempts to utilize resources from physical cosmology – particularly from Big Bang cosmology – to support the premise that the Cosmos began to exist.

In part II, I turn to clarifying the concept that the Cosmos had a beginning. Note that since I've stipulatively defined the Cosmos as the totality of physical reality, the existence of any physical entities at all suffices for the existence of the Cosmos. Consequently, another way to understand my goal in part II is to articulate a set of general conditions for all physical entities to have had a beginning. Instead of developing a full set of necessary and sufficient conditions for the Cosmos to have had a beginning, I develop three conditions that are at least necessary for the Cosmos to have had a beginning; in order for those conditions to be adequate, the conditions should be useful in determining whether the Cosmos had a beginning and should help to elucidate the concept of a beginning. Moreover, while an explication of the notion that the Cosmos had a beginning should be of intrinsic philosophical interest, I am focused on a sense of 'beginning' that renders the conjunction of the KCA's two premises as plausible as possible. In developing the notion that the Cosmos had a beginning, we face a trade-off. On the one hand, to help the second premise – that the Cosmos began to exist – 'beginning' should be understood as broadly as possible. On the other hand, to help the first premise – that anything that begins to exist has a cause for its beginning – 'beginning' should be understood narrowly as possible so as to avoid making the first premise obviously false.

Part II includes chapters 4 through 8. Chapter 4 discusses conceptions of the beginning of the Cosmos that require a specific metaphysical theory of time. Since my aim is to develop a conception of the beginning of the Cosmos that does not require a specific metaphysical theory of the nature of time, I set aside those conceptions that do require a specific metaphysical theory of the nature of time. In chapter 5, I develop my first necessary criterion for the Cosmos to have had a beginning by turning to a problem that can be posed in both analytic theology and in philosophy of physics. As I discuss, some analytic theologians have argued that God is in time, time is finite to the past, and God did not begin to exist. If God is in time and time is finite to the past, then God's past is finite. And if God did not begin to exist, then some entities with a finite past are beginningless. This leaves us with a question: what criteria distinguishes entities with a finite past but that are beginninglesss from entities with a finite past that have a beginning? Likewise, according to a burgeoning literature in philosophy of physics, time might not be fundamental to physical reality. Timeless entities are beginningless. So, if physical reality is fundamentally timeless, then physical reality is fundamentally beginningless. Ergo, philosophers of physics, like philosophers of religion and theologians, have discussed entities whose past might be finite but that are beginningless. Using the Lewis-Stalnaker semantics for counterfactual conditionals, I develop a condition – that I call the Modal Condition – that distinguishes beginningless entities with a finite past from entities with both a finite past and a beginning. As I will argue, the Cosmos began to exist only if there is nothing that suffices for the Cosmos's existence and which would have existed if time had not existed. Moreover, the Modal Condition provides us with another reason for thinking that the a priori case for the beginning of the Cosmos fails, namely, that all of the a priori arguments have attempted to show only that, as a matter of metaphysical necessity, past time is finite. Even if the Cosmos's past is finite, the Cosmos could still fail to satisfy the Modal Condition and so could still be beginningless.

In chapter 6, I discuss a second necessary condition – the Direction Condition – for the Cosmos to have a beginning. If the Cosmos did have a beginning, then the beginning must be to the collective past of the rest of the Cosmos. As Geoffrey Matthews (1979) and Mario Castagnino, Olimpia Lombardi, and Luis Lara (2003) have shown, the Cosmos has a global direction of time – roughly, a "shared" direction of time throughout all of space-time – only if specific chronogeometric criteria are satisfied. The Direction Condition is the conjunction of their chronogeometric criteria for a global direction of time.

Chapter 7 discusses the final criterion for the Cosmos to have had a beginning, viz, that there is a boundary to the Cosmos's history. For example, one intuitive sense in which the Cosmos could have a beginning is just that there is a finite interval of time such that no physical entities exist before that interval. However, there are other ways in which the Cosmos could include a boundary; for example, the Cosmos's history might include a Cosmos-wide closed boundary infinitely far to our past. I summarize all of the relevant ways in which the Cosmos could have a past boundary through a disjunctive condition, that is, either the Cosmos includes a past closed boundary (the topological conception) or else the Cosmos includes a finite initial segment (the metrical conception).

In chapter 8, I turn to the relationship between classical Big Bang models and the Direction/Boundary Conditions. According to science popularizations, religious apologists, and some philosophers, Big Bang cosmology is a theory about the origins of the Cosmos. While I disagree, Big Bang cosmology is not altogether irrelevant to our notion that the Cosmos had a beginning. As I discuss, a variety of classical Big Bang models satisfy a technical condition for being singular, i.e., b-incompleteness. I prove a theorem that connects the Direction and Boundary Conditions to b-incompleteness, namely, that all classical space-times that satisfy the Direction and Boundary Conditions are b-incomplete. While space-time is likely not classical, and so the theorem does not necessarily have direct physical or metaphysical implications, the theorem clarifies why some authors have thought the Big Bang was the beginning of the Cosmos. Since the Modal Condition is one of the novel contributions made to the literature by this dissertation, past authors have only had access to the Direction and Boundary Conditions. If the Direction and Boundary Conditions were the only criteria needed for the Cosmos to have had a beginning and we assume (incorrectly) that General Relativity is a final theory of space-time, then the Cosmos having a beginning would turn out to entail b-incompleteness. If we added the additional assumptions that the Cosmos is spatially homogeneous and isotropic, then we would be able to *derive* classical, singular Big Bang models.

Part III turns to discussing Cosmic Skepticism, the view that the provinciality of our current knowledge of the physical facts with respect to scale, spatio-temporal location, or energy prevents us from having empirical access to whether the Cosmos satisfies the Modal, Direction, and Boundary Conditions. I develop four arguments that, collectively, mount a case for Cosmic Skepticism.

In chapter 9, I show that classical space-times satisfying the Direction and Boundary Conditions are observationally indistinguishable from classical space-times that do not satisfy the Direction and Boundary Conditions. Despite the fact that space-time indistinguishability has enjoyed a multi-decade long discussion in philosophy of physics and may threaten to undermine many of the results on which the a posteriori defense of the KCA depends, space-time indistinguishability has only twice, to my knowledge, been discussed in relationship with the KCA: first, in one of my own recent publications, i.e., Linford, 2021, and, second, in relation to Kant's first antinomy, i.e., Beisbart, 2022. Moreover, while some friends of the KCA have discussed specific cosmological models proposed by physicists at length, e.g., Craig and Sinclair, 2009, 2012, we should not think that any specific cosmological is probable. There are a large number of mutually incompatible cosmological models in the current literature that are compatible with all of the observational data gathered thus far. Many of those models were developed as toy models or to explore physical possibilities and so were not intended as probable descriptions of the Cosmos as a whole. Moreover, many of the best models appear to be equally well supported by the data. Assuming that the model with the greatest epistemic probability is not significantly more probable than at least one other model, since the probabilities of the models must add to 1, the epistemic probability of the most probable hypothesis is no greater than approximately 0.5.[4] Moreover, even if we do suppose that one cosmological model is significantly more probable than other cosmological models, which seems unlikely, there can be no more than one hypothesis with an epistemic probability greater than 0.5; thus, even if some live cosmological model is probable, the majority of live cosmological models are improbable. Since the majority, or perhaps all, of live cosmological models are improbable, we should not be surprised if friends of the KCA are able to show that a wide selection of cosmological models that lack a beginning are improbable.

Instead of investigating whether any particular cosmological model is probable, philosophers interested in how physical cosmology might be brought to bear on the KCA should instead discuss what we can say about the global structure of space-time given the observational data available to us or that might become available in the future. That question – what can we say about the global structure of space-time on the basis of our observations? – is the central question that has been investigated in the literature on observationally

[4]↑This result is trivial to prove. Suppose that there are three hypotheses A, B, and C, where A is the most probable hypothesis and B is the second most probable hypothesis. Also, assume that the difference in the probabilities of A and B are small, so that $Pr(A) \approx Pr(B)$. Since their epistemic probabilities must sum to 1, we have that $Pr(A) + Pr(B) + Pr(C) = 1$. Consequently, $Pr(A) + Pr(B) < 1$. Since $Pr(A) \approx Pr(B)$, we have that $2Pr(A) \lessapprox 1$, which entails that $Pr(A) \lessapprox 0.5$. Since the most probable hypothesis has a probability less than approximately 0.5, we know that the other hypotheses have a probability no greater then 0.5.

indistinguishable space-times. Therefore, one of my goals is to bring the literature on observationally indistinguishable space-times to the attention of philosophers interested in using physical cosmology to either support or reject the KCA.

In chapter 10, I discuss a set of conditions that constrain our knowledge of the Cosmos's past, including a transcendental condition on the possibility of our knowledge of the past. Chapter 11 shows that, despite claims made by two of the KCA's proponents, a variety of contemporary cosmological models do not satisfy the Boundary Condition. We don't know whether one of those models is correct and so we don't know whether the Cosmos does satisfy the Boundary Condition. Lastly, chapter 12 completes my case for Cosmic Skepticism by turning to confirmation theory. I discuss two kinds of inductive arguments that might be used to establish that the Cosmos has a beginning or features relevant for determining whether the Cosmos has a beginning. First, there are part-to-part inferences, which involve inferring from a portion of the Cosmos to which we have empirical access to a portion of the Cosmos to which we do not have empirical access and whose features might relevantly bear on whether the Cosmos satisfies the Modal, Direction, or Boundary Conditions. Second, there are part-to-whole inferences, which involve inferring from an empirically accessible portion of the Cosmos to the Cosmos as a whole. I argue that, at least at the present stage of philosophical and scientific inquiry, part-to-part inferences and part-to-whole inferences do not succeed. As a consequence of the fact that we cannot determine, at the present stage of physical inquiry, whether the Cosmos satisfies the Modal, Direction, or Boundary Conditions, a wholly empirical case for the KCA's second premise, i.e., that the Cosmos began to exist, does not succeed.

Part I

THE KALÁM COSMOLOGICAL ARGUMENT

2. THE A PRIORI DEFENSE OF THE KCA

In order to motivate the project for the rest of this dissertation, this chapter summarizes some of the reasons that I regard the *a priori* defense of the KCA's second premise as weak. There are three ways of building the *a priori* defense of the view that the Cosmos began to exist: first, one can argue that there are no actually infinite collections, second, one can argue that beginningless series are not metaphysically possible, and, third, one can argue that no actually infinite collection can be formed by successive addition. I turn to discussing all three in turn.

2.1 Actually infinite collections and beginningless series

In this section, I will consider two of the three families of arguments against an infinite past; first, the argument that the Cosmos's past must be finite because actually infinite collections are metaphysically impossible and, second, the argument that beginningless series are not metaphysically possible. I will discuss the two arguments together because there is a powerful objection – the unsatisfiable pairs diagnosis – that applies to both. Let's begin by considering arguments whose aim is to establish that the Cosmos had a beginning on the grounds that there are no actually infinite collections. A collection of objects is said to be *actually infinite* just in case the collection has more than any finite number of members and all of the members of the collection collectively exist together, whereas a collection is said to be *potentially infinite* just in case the collection has a finite number of members but grows without bound. In order to defend the claim that the Cosmos began to exist, proponents of the KCA have offered the following argument:

1. If past time is infinite, then there is an actually infinite collection.

2. There are no actually infinite collections.

3. Therefore, the past is not infinite (modus tollens from 1, 2).

4. If the past is not infinite, then the Cosmos has only existed for a finite period of time.

5. If the Cosmos has only existed for a finite period of time, then the Cosmos began to exist.

6. Therefore, the Cosmos began to exist (modus ponens from 3, 4, 5).

This is a valid argument and so we have left to determine whether the argument is sound. In subsequent chapters (particularly chapter 7), I will challenge the notion that if the Cosmos has only existed for a finite period of time, then the Cosmos began to exist. For now, let's consider how proponents of the KCA have defended the first subargument, that is, step 3. Since 3 deductively follows from premises 1 and 2, we should examine how KCA proponents defend premises 1 and 2. According to KCA proponents, premise 1 follows from the observation that if past time is infinite, then there is an actually infinite collection of past events, and so an actually infinite collection. One can challenge this premise on the grounds that on some metaphysical accounts of time, e.g., presentism, the past does not exist and so there is no collection of past events. Perhaps this objection can be overcome; at any rate, let's turn to premise 2. In support of premise 2, supporters of the KCA attempt to show that actually infinite collections are metaphysically impossible. There are at least two strategies for showing that actually infinite collections are impossible and I turn to each, in turn, below; I reply to the argument that beginningless series are metaphysically impossible in my discussion of the first strategy.

2.1.1 The first strategy for showing that actually infinite collections are impossible

On the first strategy, one constructs a scenario that would be metaphysically possible if actually infinite collections were metaphysically possible. One then shows the constructed scenario leads to an absurd consequence. If there are independent reasons for thinking that the absurd consequence is metaphysically impossible, then we have reason to think that the scenario, itself, is metaphysically impossible. And if the scenario, itself, is metaphysically impossible, then actually infinite collections are metaphysically impossible. Throughout this section, I will assume that having a finite past and beginning to exist are co-extensive; nonetheless, this conception will be revised in part II. For example, in chapter 5, I will argue that even if the Cosmos's past history were finite, the Cosmos might still be beginningless.

One popular choice for a thought experiment is Hilbert's Hotel (HH). HH is a hotel with an actually infinite number of rooms, e.g., a room for every positive integer. A variety of counterintuitive consequences follow from HH. For example, supposing that HH is full, one can accommodate any number – including an infinitude – of additional guests. To accommodate one more guest, have the guest in room 1 move to room 2, the guest in room 2 move to room 3, and so on, up the chain of rooms. To accommodate an infinitude of additional guests, have the guest in room n move to room $2n$. Since there is a one-to-one mapping from the positive integers to the even integers, all of the current guests can be moved into an even numbered room, and a countable infinity of new guests can be moved into the odd numbered rooms.

Proponents of the KCA claim that HH is absurd because HH violates intuitively plausible principles. According to a prima facie intuitively plausible principle, if a hotel is *full*, then the hotel cannot accommodate additional guests. HH can accommodate new guests even when full. One can object that this analysis relies on a systematic ambiguity in the concept of *fullness*. One way in which a hotel can be full is if no additional guests can be added to the hotel. Another way that a hotel can be full is if every room in the hotel is occupied. For hotels with a finite number of rooms, the two senses of 'fullness' are coextensive. But in the case of a hotel with an infinitude of rooms, the two senses are not coextensive. In contexts where the two senses of 'fullness' fail to be coextensive, there are two senses of the principle that a full hotel cannot accommodate new guests. In one sense, 'full' means that the hotel cannot accommodate additional guests. Clearly, HH is not full in that sense, so that the principle that a full hotel cannot accommodate new guests is inapplicable. Alternatively, 'full' can mean that all of the rooms are occupied. Taken in that sense, while HH is full, the principle that a full hotel cannot accommodate new guests turns out to be false.[1]

[1]↑There is a second systematic ambiguity in the presentation of the HH. Two consequences of the light cone structure of relativistic space-times are that (i) there is no absolute simultaneity and that (ii) there is an absolute maximum speed for the propagation of any signal. For those of us who are realists with respect to relativity, light cone structure is a good candidate for a metaphysically necessary feature of space-time since,

Let's set this ambiguity aside; I think that there is a deeper objection to arguments that utilize HH in attempting to show that actually infinite collections are impossible. At most, HH shows that actually infinite collections have counterintuitive consequences. While the counterintuitive features of the HH might provide us reason to be surprised to empirically discover an infinitely large hotel drifting somewhere out in space, the mere fact that a scenario is counterintuitive does not, in itself, provide us reason for thinking that the scenario is not metaphysically possible. While we might note that contemporary physics has provided us with reason to endorse a variety of counterintuitive scenarios as actual, the barrier to entry for metaphysical possibilia is quite low. For example, a galaxy-sized elephant that recites the Star Spangled Banner in perpetuity is presumably metaphysically possible, but is not a serious candidate as an empirical hypothesis.

Given the low barrier for inclusion as a metaphysically possible scenario, KCA proponents need to provide us a scenario whose consequences are more than counterintuitive. For example, KCA proponents might provide us with a scenario that results in a contradiction. Proponents of the argument that beginningless series are metaphysically impossible have offered scenarios, such as the Grim Reaper scenario (e.g., Koons, 2014, 2017),[2] that do result in a contradiction.

I offer two comments before describing a version of the Grim Reaper scenario. First, authors who utilize the Grim Reaper, and other related, scenarios, in defense of the KCA have a narrower aim than showing that *all* infinite collections are metaphysically impossible; their aim has been to show either that all temporal or all causal series have finite past extension. Nonetheless, their argument is qualitatively similar to the Hilbert Hotel

for example, light cone structure determines the objective ordering of events and the distinction between space and time. If light cone structure is a metaphysically necessary feature of space-time, then the guests couldn't switch rooms all *at once* for doing so requires the guests to switch rooms *simultaneously*. Moreover, since signals can propagate only at finite speed, the "wave" of guests transferring rooms must travel through the hotel at finite speed. To accommodate even one additional guest would require infinite time. In chapter 7, I will consider the possibility that the Cosmos includes two events between which an infinite amount of time elapses. Provided that such a scenario is not metaphysically possible, while each guest in the hotel will eventually move, there will never be a time, from any reference frame, from the perspective of which all of the guests will have moved. If accommodating one additional guest means that there will be a time when the process of accommodating the new guest has completed, then a full HH might not be able to accommodate a new guest after all.

[2]↑Koons, and others, were inspired by discussion in José Benardete's (1964) book.

argument that I previously considered in this section, namely, that, given the consequences of some specific scenario(s) that we construct, we are supposed to infer that infinite past series are metaphysically impossible.

Second, although the thought experiment, as presented by Koons and others, involves grim reapers and other fantastical details, I will present the thought experiment in terms of a mechanical device that I will refer to as *Pam*. Pam has existed for every time that there has been. If time began in the Big Bang, then Pam began to exist at the Big Bang. And if past time is infinite, then Pam's past history is infinite. Pam contains a clock, a digital camera, a computer, a stylus, and a piece of paper. At the end of every hour as recorded by Pam's clock, the camera takes a photo of the paper, the computer checks the photo to see if the stylus has previously written on the piece of paper, and, if the stylus has not written on the piece of paper, the stylus writes on the piece of paper. Otherwise, Pam does nothing. And now we ask – at the present day, has the stylus written on the piece of paper? Suppose time never began, so that Pam is presently infinitely old. The stylus must have written on the piece of paper, for if there ever was a time when the stylus had not written, then the stylus would write. When did that happen? Suppose the stylus wrote on the piece of paper at 1pm today. In that case, at noon, Pam checked to see if Pam had previously written on the piece of paper. Finding that Pam had not written on the piece of paper, Pam would have written on the piece of paper at noon. That's a contradiction; surely, if Pam had written on the piece of paper at noon, then Pam would not have written on the paper at 1pm. No contradiction results if our original assumption – that time never began – is false, for in that case, there would be a first hour.

The scenario involving Pam is not metaphysically possible because the scenario entails a contradiction. Proponents of the KCA say that there are scenarios involving beginningless series that entail contradictions for a more fundamental reason, namely, that beginningless series are metaphysically impossible. To the contrary, consider that scenarios in which actually infinite collections or beginningless series entail a contradiction are described by a conjunction of several propositions, e.g., $P \& Q \& R$. Supposing $P \& Q \& R$ entails a contradiction, we can conclude that $\neg \diamond (P \& Q \& R)$. If we push the negation past the $\diamond$-operator, we can infer $\Box(\neg P \vee \neg Q \vee \neg S)$, but we cannot infer that (for example) $\Box \neg P$.

Thus, the fact that various scenarios involving beginningless series or actually infinite collections are metaphysically impossible does not, in itself, show that beginningless series or actually infinite collections are metaphysically impossible;[3] we can construct other scenarios involving beginningless series or actually infinite collections that do not entail contradictions. Moreover, there is a simpler, unifying explanation as to why scenarios like the one I've constructed are not possible, viz, simply that such scenarios are contradictory. This is the basis for a convincing reply to scenarios meant to show that beginningless past series are metaphysically impossible called the *unsatisfiable pairs diagnosis* (UPD).

As Shackel (2005) and Malpass (unpublished) unpack the UPD, all of the scenarios meant to show that beginningless series are metaphysically impossible involve the following two principles:

P: The set S has no first member.

Q: For all x in S, E at x iff E nowhere before x.

P applies to beginningless series because beginningless series have no first member, e.g., beginningless temporal series have no first moment. Q applies to beginningless series because for all of the moments in the series, e.g., Pam writes at that moment only if Pam has not written at a previous moment. As Shackel and Malpass have proven, the two principles cannot be jointly satisfied, that is, $\neg\diamond(P\&Q)$. According to UPD, the fact that the two principles cannot be jointly satisfied explains why the scenario involving Pam, as well as a variety of similar scenarios, are not possible; but, given the UPD, we are left without a reason for thinking that beginningless series, themselves, are metaphysically impossible. While we can infer $\Box(\neg P \vee \neg Q)$, we cannot infer $\Box\neg P$. A similar diagnosis can be offered for the scenarios involving actually infinite collections that entail either absurdities or contradictions; supposing that some infinite collections (e.g., infinitely large hotels) are impossible, we do not thereby have a reason to think that other infinite collections, whose existence does not entail an absurdity or a contradiction, are impossible. On my view, the UPD comes close to a demonstration that there is little hope for the first strategy for

[3]↑Landon Hedrick (2022) has recently published a similar argument.

denying the existence of a beginningless series or of an actually infinite collection. By way of analogy, after learning Euler's proof that there is no circuit that traverses the seven bridges of Königsberg without doubling back, no one is surprised that there is no possible world that includes such a circuit. Nothing of metaphysical significance follows for the nature of bridges, paths traversing bridges, or the like. Likewise, why do we require any more explanation as to why the scenario involving Pam is metaphysically impossible than that the scenario fails to be self-consistent?

Nonetheless, I will offer one additional objection to the first strategy. Proponents of the first strategy, such as Robert Koons, utilize a modal recombination principle – of the sort famously defended by David Lewis – in order to construct the Grim Reaper scenario. According to the recombination principle Koons utilizes, given a space-time region and its contents s_1 from some possible world w_1 and a distinct space-time region s_2 with distinct contents from some other possible world w_2, another possible world w_3 can be constructed that includes duplicates of s_1, s_2, and their contents. The modal recombination principle can be used to construct a kind of inverse Grim Reaper scenario. Consider that for any negative integer, there is a possible world where an angel says that integer. (If one would prefer, one can instead consider a mechanical device that prints out a negative integer.) Using the modal recombination principle, we can string together events in which angels state distinct integers in order to construct a possible world $\mathbb{W}$ that includes a series of angels counting down through all of the negative integers; $\mathbb{W}$ includes a beginningless series since there is no first negative integer. If proponents of the first strategy are correct in endorsing the modal recombination principle to construct their thought experiments, there is nothing – so far as I can tell – that bars us from constructing $\mathbb{W}$. And if $\mathbb{W}$ is a legitimate metaphysical possibility – and $\mathbb{W}$ is a legitimate metaphysical possibility so long as the modal recombination principle is true – beginningless series are not metaphysically impossible.

2.1.2 The second strategy for showing that actually infinite collections are impossible

Whereas the first strategy utilizes the consequences of thought experiments, the second strategy utilizes general principles in an effort to provide us reason to think actually infinite collections or beginningless series are metaphysically impossible. Since the second strategy does not utilize thought experiments, the second strategy is not susceptible to the objections discussed in the previous section. Consider the following triple of jointly incompatible principles:

> *Hume's Principle*: Any two collections have the same size just in case their members can be put into 1-to-1 correspondence.
>
> *Euclid's Principle*: The whole of any collection is larger in size than any proper sub-collection.
>
> *Actually Infinite*: There is an actually infinite collection, that is, there is a collection C such that there is a 1-to-1 map between C and a proper sub-collection of C.

Since the three principles are mutually incompatible, we must deny at least one principle; since the three principles are not logically exhaustive, there is at least logical space to deny all three. Since all three principles are intuitively plausible but mutually incompatible, they jointly generate a paradox, sometimes called *Galileo's Paradox*, e.g., Parker, 2009.

Proponents of the second strategy claim that we should endorse both Hume's Principle and Euclid's Principle. If Hume's Principle and Euclid's Principle are each metaphysically necessary, then Actually Infinite is not metaphysically possible. Thus, advocates of the second strategy argue that we should resolve Galileo's Paradox by rejecting Actually Infinite. The trouble is that one could endorse other combinations of principles in order to solve Galileo's Paradox and there is, as far as I can tell, little reason – other than one's private intuitions – for favoring the choice made by friends of the second strategy over the available alternatives. For example, consider the following principle:

> *The Cardinality Principle*: The size of any collection is the cardinality of that collection.

If one denies the Cardinality Principle, then one should deny Hume's Principle. Moreover, consider that although the segment of the real line from 0 to 1 has the same cardinality as the segment of the real line from 0 to 2, the latter has twice the Lebesgue measure as the former. Since the Lebesgue measure of a point set captures one notion of the size of that point set, there is a well-defined sense of 'size' in which the size of a point set is not determined by the cardinality of that point set. And given that sense of 'size', we have reason to deny the Cardinality Principle and, consequently, Hume's Principle. In that case, we can consistently endorse Euclid's Principle and Actually Infinite.

Relatedly, Paul Draper (2008, p. 49) points out that there are at least two distinct senses of 'larger than'.[4] First, there is a sense of 'larger than' consistent with Hume's Principle, viz, collection A might be said to be larger than collection B just in case (i) there is no 1-to-1 correspondence between A and B and (ii) there is a 1-to-1 correspondence between B and a proper sub-collection of A. Second, there is a sense of 'larger than' that is inconsistent with Hume's Principle, namely, the "all-and-then-some" sense. For example, all of the integers are contained in the rationals, but the rationals include elements that are not included in the integers. In fact, a number of authors (e.g., Bellomo and Massas, 2021; Benci et al., 2006, 2007; Mancosu, 2009; Nasso and Forti, 2010; Trlifajová, 2018; VieriBenci and Nasso, 2003) have developed conceptions of set size that differ from the notion recommended by the Cardinality Principle in the case of infinite sets. I will refer to these alternatives as *Euclidean conceptions*. According to Euclidean conceptions, the set of rationals has a larger size than the set of integers, because the rationals include the integers as a subset, even though both sets have the same cardinality. Likewise, Euclidean conceptions entail an alternative conception of 'smaller than' that is not tied to cardinality in the case of infinitely large sets. Given that there are two analyses of 'larger than' and 'smaller than', there are two analyses of 'equal size'; only one analysis of 'equal size' is consistent with Hume's Principle. As Draper points out, the analysis of 'equal size' that is inconsistent with Hume's Principle is consistent with Euclid's Principle. Thus, if we endorse Euclid's Principle, affirm a Euclidean conception, and thereby deny that cardinality successfully

[4] ↑As Richard Sorabji (1983, pp. 217–218) has shown, the distinction between the two senses of 'larger than' has been known since at least the medieval period.

captures the notion of the size of a collection, then we ought to reject Hume's Principle. In that case, we are left without a reason to reject Actually Infinite.

We could instead deny Euclid's Principle and endorse Hume's Principle and Actually Infinite. As I've discussed, various authors have developed Euclidean conceptions of the size of infinite sets. For example, Matthew Parker (2013) has argued that there are a series of problems that plague Euclidean conceptions of the size of infinite sets. While Parker admits that the problems he identifies are not necessarily insurmountable, one person's modus ponens is another person's modus tollens. So, while one philosopher might take Hume's Principle and Euclid's Principle to jointly show that Actually Infinite is false, another philosopher might take Hume's Principle and Actually Infinite to jointly show that Euclid's Principle is false. In that case, one could consistently endorse Actually Infinite.

Lastly, consider the following alternative to Euclid's Principle:

> *The Modified Euclid's Principle*: The whole of any *finite* collection is larger in size than any proper sub-collection.

The Modified Euclid's Principle is consistent with both Euclid's Principle and its denial, so that accepting the Modified Euclid's Principle need not involve rejecting Euclid's Principle. However, Euclid's Principle is a logically stronger principle than the Modified Euclid's Principle in the sense that the former entails the latter but the latter does not entail the former. When we can make do with a logically weaker principle without a logically stronger principle, all else being equal, we should endorse the logically weaker principle without endorsing the logically stronger principle. Both principles are consistent with all of the same evidence, since all of the cases that confirm Euclid's Principle involve finite collections. Thus, accepting Euclid's Principle involves taking an additional and (apparently) unnecessary step; friends of the second strategy need to tell us why we should take that step. They have thus far failed to convincingly do so.

So far, we've seen that one can respond to Galileo's Paradox by denying either Hume's Principle or Euclid's Principle instead of denying Actually Infinite and that friends of the second strategy have yet to successfully defend their view that we should adopt both principles. While there is no clear reason why we should endorse both principles, there are at least two reasons in favor of denying at least one of the principles. First, consider that the physically possible worlds are those worlds which are consistent with the laws of physics and are typically understood to be a subset of the metaphysically possible worlds. Thus, if there is a model M that is both (i) self-consistent and (ii) consistent with the known laws of physics, we have defeasible reason for thinking that M represents a metaphysically possible state of affairs. There are models – such as de Sitter space-time – that are self-consistent, consistent with known physical laws, and include actually infinite collections.[5] Thus, since we have defeasible reason for thinking that actually infinite collections are physically possible, we have defeasible reason for thinking that actually infinite collections are metaphysically possible.

Second, as previously mentioned, friends of the second strategy endorse potentially infinite collections while denying that there are any actually infinite collections. But as Cantor argued, the potentially infinite depends on the actually infinite. For example, we can rigorously define the convergence of an infinite sequence without defining the value of the sequence at infinity. For example, in introductory calculus, we might express the convergence of some sequence $\{S_1, S_2, ...\}$ to some value S as $\lim_{n\to\infty} S_n = S$. In that context, we customarily tell students that S is the value that S_n has when $n = \infty$. Put that way, the convergence of a sequence seems to require that the sequence has a specific value at infinity, i.e., $S_\infty = S$. But, (in)famously, defining the value of a limit at infinity is a conceptual error. Mathematicians prefer to say that S_n approaches, but never reaches, S. When students return to convergent series in a subsequent Real Analysis class, they learn that the convergence of a sequence can be rigorously defined without defining the value of the sequence at infinity. We can say that any sequence $\{S_1, S_2, ...\}$ converges to a value S just in case, for any $\epsilon > 0$, there exists a positive integer N such that, for all $n > N$, $|S_n - S| < \epsilon$. The rigorous definition of the convergence of a sequence makes use of the potentially infinite because the definition describes a sequence that perpetually grows closer to a

[5]↑To see that de Sitter space-time does include an actually infinite collection in the relevant sense, consider any complete time-like geodesic in de Sitter space-time. From the perspective of an observer located at any point on that geodesic, there are an infinite number of hours (for example) to their past.

limit. To make the point clearer, for increasing values of n, the function $f(n) = |S_n - S|^{-1}$ grows without bound and therefore models potential infinity; that is, the value of $f(n)$ is potentially infinite. Nonetheless, as Cantor would remind us, the sequence's index ranges over an actually infinite set, i.e., n is defined as a positive integer. For that reason, the rigorous definition of the convergence of a sequence *presupposes* the existence of an actually infinite collection. This feature of the rigorous definition of the convergence of a sequence can be generalized: potential infinities presuppose actual infinities.

Cantor's notion that the potential infinite presupposes the actual infinite is echoed in the reply that Swinburne offers to Craig's arguments against actually infinite collections. As Swinburne (2004, p. 139) points out, Craig's arguments rely on the premise that if the past is beginningless then the collection of past events is an actually infinite collection. The collection of past events is an actually infinite collection only if there is some sense in which past events have reality. Swinburne goes on to point out that if the collection of past events has reality at least in some sense, then the collection of events within the past hour equally has reality in the same sense. There were an infinite series of periods of unequal length in the past hour, e.g., the past 1/2 hour, the past 1/4 hour, etc. Craig argues that the entire interval of the past hour is more fundamental than any subdivision of the past hour; we can subdivide the past hour only as a potential infinite. Nonetheless, Craig endorses the view that the past hour can be arbitrarily subdivided in whichever way one would like;[6] for that reason, Craig must presuppose that the past hour already includes an actual infinitude of subdivisions. (Cantor offered a similar argument involving the bisection of a line; see, e.g., Shapiro, 2011, p. 105.) Likewise, the "gunky" view of time endorsed by Craig entails that every subinterval of time includes proper subintervals;

[6]↑Despite Craig's presentism, Craig has long argued that instants do not exist. As I've described, Craig denies that any physical collection could be infinite while also denying the view that time is discrete. If time is continuous, one might have thought that any finitely long interval of time includes an infinitude of instants. In order to avoid the consequence that any interval of time includes an infinitude of instants, Craig adopts the Aristotelian position that intervals of time are fundamental and instants are a kind of mental fiction we arrive at as the boundary points of any given interval. Craig writes that "only intervals of time are real or present and that the present interval (of arbitrarily designated length) may be such that there is no such time as 'the present' *simpliciter*; it is always 'the present hour', 'the present second', etc. The process of division is potentially infinite and never arrives at instants" (Craig, 1993a, p. 260; also see Craig, 2000, pp. 179–180, Craig and Sinclair, 2009, pp. 112–113). For discussion, see Dumsday, 2016; Loke, 2016; Puryear, 2014, 2016; Zarepour, 2021.

every subinterval includes proper subintervals only if an actual infinitude of subintervals already exist in the original interval.[7] Unless friends of the second strategy are willing to deny the potentially infinite – which they are not usually willing to do – friends of the second strategy ought to accept the actually infinite.

2.2 Forming actually infinite collections by successive addition

I've discussed two strategies for showing that there are no actually infinite collections (or for showing that beginningless series are not metaphysically possible) and why those two strategies do not suffice for showing that the Cosmos has a finite past. In this section, I turn to a strategy which attempts to establish that the Cosmos has a finite past and that involves the thesis that an actually infinite collection cannot be formed by successive addition. A collection is formed by *successive addition* just in case one element is added to the collection at a time. On some metaphysical views about the nature of time – as discussed below – the past is formed by successive addition since the past forms by present moments passing away one at a time. And since no infinite collection can be formed by successive addition – or so the argument goes – the past, being a collection formed by successive addition, cannot be infinite. Call this the Successive Addition Argument.

There are two reasons as to why this argument is not convincing. First, as proponents of the Successive Addition Argument recognize, e.g., Craig, 2013, p. 13, the argument assumes a controversial view about the metaphysics of time, namely, the A-theory of time, according to which time objectively passes. If, instead of time passing, there is an eternal space-time block, then moments are not added to the past and so our past did not form by successive addition. I will have more to say about the A-theory in chapter 4; here,

[7] ↑Craig could reply that "cutting" the past hour into subintervals introduces divisions that were not already present. I don't see how a reply of that sort could work. The past hour isn't a spatial extension that we can subdivide; there isn't a physical process that can cut up intervals of time and produce something new, i.e., an instant, at the point at which the cut is made. Similarly, the notion that one could cut up a temporal interval in order to make an instant present is surely a category mistake. Instead, for each time, the totality of past events is a completed collection, and whatever subdivisions can be made of a past temporal interval, even if the subdivisions are not fundamental, must already have reality, at least in some sense, before the subdivisions are mentally made by the human intellect. For example, consider a particle that travels from point A, at time t_A, to point B, at time t_B, such that $t_A < t_B$. At any moment during the particle's journey, we can consider how much time has elapsed thus far; so long as the particle's motion is continuous, the particle's motion continuously marks out subdivisions of time.

I will note simply that the A-theory of time is at least controversial and does not enjoy popularity among (for example) philosophers of physics. Note the following four facts about the Successive Addition Argument: (i) the Successive Addition Argument requires a controversial premise, (ii) given that the premise is controversial, whether the premise enjoys a high probability is at least unclear, (iii) the conclusion of an argument, all else being equal, is as probable as the conjunction of the argument's premises, and (iv) the probability of a conjunction is no more probable than the least probable conjunct.[8] Given (i)-(iv), the Successive Addition Argument is not a persuasive reason to think that the past is finite.

Second, regardless of whether the A-theory of time is true, the Successive Addition Argument is question begging. While proponents of the Successive Addition Argument are on safe ground when they argue that an infinite collection cannot form from a finite collection by successive addition, they are on shakier ground when they argue that there cannot be an infinite collection each of whose members were added by successive addition. As I will argue in chapter 7, one way for the Cosmos to lack a beginning is that infinite time precedes every past moment. But, in that case, the Cosmos's past did not form, at least in the sense that a finite collection forms, since the Cosmos's past was always infinite.[9] Since the Cosmos's past did not obviously form in the relevant sense, the Cosmos's past did not obviously form by successive addition. And if the past did not form by successive addition, even if actually infinite collections could not form by successive additions, the Cosmos could still have an infinite past.

One could object that although the past was always infinite, each moment was added to the past by successive addition. Since the past is comprised by nothing other than moments, all parts of the past were added by successive addition. And if all parts of the past were added by successive addition, the past was formed by successive addition.

Two replies can be offered. First, consider an analogous inference: since each feather in a pile is light, the entire pile of feathers is light; we know this inference is not valid

[8]↑The fact that the probability of a conjunction is no more probable than the least probable conjunct follows from the conjunction rule, i.e., $Pr(A\&B) = Pr(A|B)Pr(B)$. The conjunction rule entails that $Pr(A\&B) \leq Pr(B)$. Note, also, that $Pr(A\&B) = Pr(B)$ only if $Pr(A|B) = 1$.
[9]↑Sorabji (1983, p. 220) makes a similar objection to the successive addition argument.

because some piles of light feathers are heavy. Similarly, the inference from each moment having been added by successive addition to the conclusion that the entire series formed by successive addition may be fallacious. Nonetheless, I'm not sure if the inference is fallacious because we know that some inferences from parts to wholes are not fallacious. For that reason, set this reply to one side. Second, consider an analogy commonly used by proponents of the Successive Addition Argument. The claim goes that one cannot count to infinity (Craig and Sinclair, 2012, p. 116). The reason that one cannot count to infinity is that no matter how many numbers one has counted, there is an infinitude of numbers left to count. Likewise, no matter how many individual elements one has added by successive addition, a finite collection cannot be made into an infinite collection. Nonetheless, if the collection that one is adding to is *already* infinite, then there is no need to turn a finite collection into an infinite collection. On one version of the previously mentioned hypothesis that the past was always infinite, for any past moment, there was only a finite span of time to the present.[10] Thus, unlike attempting to count from zero to infinity, there need be no problem in reaching the present from any past moment.[11]

Let's consider another analogy to bolster the intuition that if the past is infinite, then the past did not form, and so did not form by successive addition. On a metaphysical view about the nature of time called *growing block theory*, the past and present exist but not the future. The past is a block that "grows" by moments coming into being at the present and passing into the past. The Successive Addition Argument has an easy interpretation in terms of the growing block theory, namely, that the block of the past grows by successive addition. In the case of a finite past, there was a first moment to which successive moments were added via successive addition. No matter how many moments are added to the block of the past, the block of the past will never be transformed from being finite to being infinite. No progress can be made in gathering together an infinite collection when gathering together one element at a time, just as no progress is made in counting to infinity by counting one integer at a time. Proponents of the Successive Addition

[10]↑An alternative version of the hypothesis includes moments that are infinitely far to the past, as discussed in chapter 7.
[11]↑Wes Morriston (2022) and Paul Draper (2008, p. 47) have previously made similar arguments.

Argument want to generalize this conclusion; they would tell us that since no collection formed by successive addition can be actually infinite, the block of the past cannot be extended infinitely far to the past.

Contrast growing block theory with another metaphysical view about the nature of time called *shrinking block theory*, namely, the view that the present and future exist but not the past. There is a nearly complete symmetry between the growing block theory and the shrinking block theory; where the growing block theory says that the past grows by successive addition, the shrinking block theory says that the future shrinks by successive subtraction. If the future is finite, then, as each moment passes, the future shrinks; eventually, the last moment will pass. Nothing will follow. The situation is different with respect to an infinite future. In the case of an infinite future, the future block never truly *shrinks*, in the sense of decreasing in cardinality. Just as one cannot make progress in constructing an infinite series when gathering one element at a time, one cannot make progress in removing elements, one by one, from an infinite collection. No matter how many elements have been removed, an infinitude remains. Since the future block never truly shrinks in cardinality and no progress is made in unmaking the future block, the future is not *unmade* by successive subtraction. If, in the case of shrinking block theory, the future cannot be unmade by successive subtraction, then, in the case of growing block theory, the past was not *made* through successive addition. Adding moments to an infinite past no more grows the past, in the cardinal sense, than taking away moments reduces an infinite future, in the cardinal sense.

In correspondence, Alex Malpass considered Andrew Loke's (2014) thought experiment in which a Hilbert Hotel is constructed over an infinitude of past time, by, for example, constructing one room per year. Rooms are added to Loke's HH by successive addition since one room is added per year. Let's suppose that infinite past time precedes every past year and that there is finite time between any past year and the present. In that case, supposing that rooms stop being built this year, there would be a hotel with an infinitude of rooms, and so a completed HH. But suppose instead that rooms stopped being made five years ago. In that case, there would likewise be an infinitude of rooms and so a completed HH. In fact, if rooms stop being built in any past year whatsoever, the

HH would already have been completed. Thus, supposing that infinite past time precedes every past year and that there is finite time between any past year and the present, the HH is already complete prior to any year that there has ever been. The addition of any room to the HH during any past year fails to make any progress in expanding the size of the hotel. Since (i) the HH is already complete prior to any year that there has ever been and (ii) the addition of a room during any past year fails to expand the size of the hotel, the process of constructing rooms did not make the hotel; instead, the hotel always already existed.[12] Likewise, if the Cosmos's history includes an infinitude of past time prior to every moment, then the Cosmos has always already existed.

As I will discuss in chapter 5, all of the a priori arguments for a beginning of the Cosmos fail for a reason that I haven't discussed thus far. Proponents of the KCA typically endorse the views that God is in time, past time is finite, and so the view that God has existed only for finite time, while also endorsing the view that God is beginningless. Thus, KCA proponents typically endorse the view that some entities that have existed only for finite time are beginningless. The a priori arguments for the beginning of the Cosmos show, at most, that the Cosmos's past is finite; thus, the a priori arguments are incomplete because the a priori arguments do not address whether the Cosmos is a beginningless entity with a finite past. In chapter 5, I argue that there is a specific condition – the Modal Condition – that can be used to distinguish entities whose pasts are finite but are beginningless from entities whose pasts are finite and have a beginning. Thus, supposing my argument for the Modal Condition is successful, KCA proponents will need to conjoin their arguments for the finitude of the past with an argument that the Cosmos satisfies the Modal Condition. Since KCA proponents haven't even attempted to show that the Cosmos satisfies the Modal Condition, their a priori argument for the beginning of the Cosmos remains at best incomplete.

[12]↑Loke'e HH bears some resemblance to a conception of divine eternity put forward by Brian Leftow. As I will discuss in chapter 5, for Leftow (2005, p. 58), a proposition is already true at any given time *t* just in case the proposition is true at *t* and would have been true had time never reached *t*. On Leftow's view, God already exists at the first moment of time because God would have existed even if time had not. Similarly, there is a sense in which Loke's HH precedes time altogether because, for any given time *t*, Loke's HH already exists at *t*.

2.3 Summary

In this chapter, I summarized three families of a priori arguments for the beginning of the Cosmos, viz, that either actually infinite collections are metaphysically impossible, that beginningless series are metaphysically impossible, or that, as a matter of metaphysical necessity, no actually infinite series can be formed by successive addition. I went on to summarize a variety of reasons for rejecting all three a priori arguments. Having rejected all three families of a priori arguments, in the next chapter, I summarize the a posteriori defense of the KCA's second premise together with some standard reasons for thinking that classical Big Bang cosmology, though not altogether irrelevant for the KCA's second premise, does not adequately support the KCA's second premise.

3. THE A POSTERIORI DEFENSE OF THE KCA

While I maintain little hope for the a priori defense of the KCA, the KCA can be defended on a posteriori grounds. KCA proponents have appealed to various results from physical cosmology, which they claim succeed in showing that the Cosmos began to exist. Some proponents of the KCA, e.g., Andrew Loke (2017), argue that the KCA should primarily be defended on a priori grounds and have sought only to show that results from physical cosmology are consistent with the Cosmos having a beginning. Nonetheless, a number of the KCA's foremost proponents have argued that the KCA can be defended on a posteriori grounds; moreover, as someone who is skeptical of our ability to reach subsantive metaphysical conclusions without consulting the sciences, I view the a posteriori defense as more worthy of our time and reflection than the a priori defense.

Consider Craig's (2007a) comments with respect to Swinburne's rejection of the a priori defense of the KCA. As Craig summarizes, Swinburne makes the claim that the KCA's first premise, viz, that whatever begins to exist has a cause, enjoys only inductive support. Craig writes that he is "more than happy to accept the truth of [the first premise] on purely inductive grounds. While the kalam [sic] argument itself is a deductive argument, that does not imply that its premisses are not to be supported by inductive evidence". As Craig continues to explain, he has "made extensive appeal to the inductive evidence supplied by science as justification for both premisses of the kalam argument". Craig notes that he agrees with Swinburne in that the "present state of science" supports "the conclusion that the universe came into existence at some time in the finite past". Thus, as Craig interprets Swinburne, Swinburne endorses a wholly empirical defense of the KCA. Moreover, Craig signals that he would be happy with a wholly empirical defense of the KCA. Indeed, Swinburne has defended a cosmological argument on wholly empirical premises in two books; see Swinburne's (2004) and his (2010).

The Craig/Swinburne view that the beginning of the Cosmos can be provided a wholly a posteriori defense appears to many to be supported on excellent grounds. Science popularizations, no less than religious apologists, often report that twentieth century physical cosmology established the physical world – and so the Cosmos – had a beginning.

According to the story we are often told, Big Bang cosmology tells us that the universe – taken by the public to mean the totality of physical reality – began in a cataclysmic event fourteen billion years ago. Although science popularizers and religious apologists overstate their case, Big Bang cosmology is not irrelevant for thinking about the beginning of the Cosmos. In this chapter, in order to summarize the a posteriori defense of the KCA's second premise, I review Big Bang cosmology, introduce the notion of a cosmological singularity (which I make rigorous in chapter 8), and describe the relevance that both have for the a posteriori defense of the second premise of the KCA. Although Big Bang cosmology is not altogether irrelevant for the KCA's second premise, Big Bang cosmology does not adequately support the KCA's second premise. I also present a set of standard arguments as for why most physicists do not seriously endorse the Big Bang as the beginning of the Cosmos.

3.1 The Historical Narrative

Prior to the twentieth century, few authors expected that a case for the beginning of the Cosmos could be constructed utilizing wholly empirical premises. For example, arguments meant to establish that God created our world depended either on establishing that actually infinite collections were impossible (e.g., Al-Kindi, Al-Ghazali, Bonaventur, John Philoponus), that an infinite regress of essentially ordered elements was impossible (Thomas Aquinas), or that a sufficient reason is required for explaining the existence of the totality of contingent entities (e.g., Samuel Clarke and Gottfried Wilhelm Leibniz).[1] The relative absence of pre-twentieth century empirical arguments for the view that God created our world is easy to explain. On the one hand, if time were arbitrarily truncated – e.g., if we arbitrarily postulate that time began a few seconds ago with everything, including our memories and this dissertation, in their current state – then a radical skeptical catastrophe results in which we cannot trust the evidence we have for the past Earman, 1977, pp. 119–122. On the other hand, the general expectation had been that moments

[1]↑Helge Kragh (2008) describes a notable exception, i.e., a community of nineteenth century German theologians who utilized thermodynamics to construct a quasi-empirical case for the Cosmos's beginning.

of time, or perhaps the contents of moments of time, are sufficiently homogeneous that nothing empirically distinguishes some specific moment as the moment of Creation.

This situation changed dramatically in the early twentieth century with the advent of General Relativity. In General Relativity, space-time is a dynamical entity coupled to the matter-energy distribution. Historically, that the Cosmos began was taken to be more or less synonymous with the notion that the Cosmos had an initial finitely long period in its life. Whether the Cosmos had an initial finitely long period is a bit of unobservable chronogeometric structure. For scientific realists, we are justified in endorsing the unobservables entailed by a given physical theory provided we have sufficient independent evidence for that theory. Thus, for scientific realists, given the dynamical coupling between matter-energy and chronogeometry, unobservable chronogeometric structure can be inferred by examining the matter-energy distribution. Scientific realists might hope that empirical data, in conjunction with physical theory, may be able to tell us that the Cosmos began. We will see in subsequent chapters that this hope is dashed in various ways, even for the scientific realist. For now, set that aside and retain hope.

In the early twentieth century, Edwin Hubble discovered that galaxies are, on average, receding from one another. On the assumption – now confirmed to high precision for the observable universe – that galaxies are distributed homogeneously and isotropically throughout space, the Einstein Field Equations simplify to a pair of ordinary differential equations called the Friedmann-Lemaître-Robertson-Walker (FLRW) equations. The FLRW equations predict that unless the matter-energy density populating space-time has a specific critical value, space-time will either expand or contract. Given Hubble's observation of galactic recession, together with other data (e.g., elemental abundances, the Cosmic Microwave Background Radiation, etc), physicists reached the conclusion that the observable universe must have been in a radically different state in the distant past. The Einstein Field Equations, interpreted literally, suggest that space-time has an open boundary at a finite time in the past beyond which space-time cannot be extended. According to the Einstein Field Equations, to ask what was before that open boundary would be analogous to asking what is north of the north pole. This at least superficially seems like what many intuitively mean by the beginning of the universe, i.e., a first finitely long period in the history of the universe.

And this point is worth pausing over for three reasons. First, the popular misconception that science has told us the Cosmos had a beginning is based on efforts to explain the consequences of General Relativity to a mathematically unsophisticated general public. Second, General Relativity is the first mathematically sophisticated theory of chronogeometry to explicitly deal with the notion that space-time could have had a beginning. Even if General Relativity is not a final theory of space-time, General Relativity – by the theory's own lights – purports to be a fundamental theory of space-time. For that reason, one desideratum for an account of the beginning of the Cosmos is that the account should at least be consistent with General Relativity and should allow us to either make sense of or to refine the intuition that singular Big Bang models include a beginning; I will take up that project in chapter 8. Third, as I will also unpack below, proponents of the a posteriori defense of the KCA have understood Big Bang cosmology to provide evidence for the KCA's second premise.

3.1.1 The Big Bang and the KCA

Having summarized some of the relevant history of Big Bang cosmology, I turn to unpacking how Big Bang cosmology has featured into the a posteriori defense of the KCA. The reader should also note that this section helps to support one of the points I've already made, namely, that some of the foremost defenders of the KCA have held that the KCA's second premise can be supported on a posteriori grounds alone. Let's begin by considering how, in 1992, Craig described the role of Big Bang cosmology in supporting the KCA's second premise:

> The discovery during this century that the universe is in a state of isotropic expansion has led, via a time-reversed extrapolation of the expansion, to the startling conclusion that at a point in the finite past the entire universe was contracted down to a state of infinite density, prior to which it did not exist. The standard Big Bang model, which has become the controlling paradigm

> for contemporary cosmology, thus drops into the theologian's lap just that crucial premiss which, according to Aquinas, makes God's existence practically undeniable (Craig, 1992, pp. 238–9).

In this context, when Craig uses the term 'universe', he means what I have called the Cosmos. As Craig interprets Aquinas, Aquinas rejected the KCA in part because Aquinas could not foresee that empirical evidence for a beginning of the Cosmos would one day become available;[2] nonetheless, Craig claims that, *given Big Bang cosmology*, we ought to support the second premise of the KCA. The following year, Craig wrote, "What a literal application of the Big Bang model requires, therefore, is *creatio ex nihilo*. A literal interpretation of the Big Bang model in which the universe originates in an explosion from a state of infinite density, that is, from nothing, provides a simple, consistent, and empirically sound construction of how the universe began" Craig, 1993c, p. 44. That is, at least as Craig understood the matter in the early 1990s, classical Big Bang cosmology conclusively establishes the second premise of the KCA.

As I discuss throughout this dissertation, the reason that classical Big Bang models are thought to include a beginning – as opposed to merely depicting the observable universe's transition from some previous physical state – involves the fact that Big Bang models are singular, that is, that the models depict space-time as having an open boundary to the past beyond which, as a matter of physical and mathematical necessity, space-time cannot be extended. In this section, I offer a rough, intuitive conception of singularities as they

[2]↑Thomas Aquinas famously rejected the KCA on the grounds that there are no purely philosophical (or non-theological) arguments which establish that Creation is finitely old (Aquinas, n.d., IQ46A1; Aquinas, 1965). In Thomas's view, that Creation is finitely old is a doctrine which, like the doctrine of the Trinity, is available only through divine revelation (Aquinas, n.d., IQ46A2). Aquinas did offer cosmological arguments for God's existence, but Aquinas's cosmological arguments do not rely on the finitude of the past. For example, Aquinas's prime mover argument is based on the Aristotelian principle that any object which is not purely actual cannot move from potentiality to actuality by itself; instead, non-purely actual objects require a purely actual being (i.e., God) for their existence. While Aquinas thought that a purely actual prime mover is needed to explain the existence of the Cosmos, Aquinas denied that the coming into being of non-purely actual objects is an event that needs to have happened at some particular point in the finite past (Aquinas, n.d., IQ46A1). And while Aquinas argued that, based on the principle that there cannot be an infinite chain of efficient causes, there must be a first efficient cause, Aquinas did not argue that the first efficient cause must precede Creation either in time or in the order of explanation; in fact, Aquinas argued that the efficient cause of Creation can co-exist with Creation for each time that Creation exists (Aquinas, 1965); for this reason, Aquinas argued, Creation could be co-eternal with God in the sense that there was no first moment in time at which Creation began to exist.

appear in classical Big Bang models, with the parenthetical remark that many other kinds of singularities appear in mathematical physics. In chapter 8, I provide a more rigorous description of what makes a space-time singular after I've had a chance to introduce and define some technical machinery.

As I've said, General Relativity provided us with the first scientific theory in which space-time, itself, plays a dynamical role within the theory; moreover, the classical Big Bang models are General Relativistic models. General Relativity tells us that there is a class of space-times said to be *singular*. For the sake of intuition, consider the function $f(x) = 1/x$. Because there is no value of $f(x)$ at $x = 0$, there is a well-defined sense in which $x = 0$ represents an open boundary between the positive and negative real numbers. Likewise, a specific class of singular space-times – those containing so-called *curvature singularities* – include open boundaries where space-time comes to an end. There are non-singular solutions to the FLRW equations. However, for singular FLRW models, when we trace time backwards, we find that the energy density grows without bound and, in consequence, the Ricci scalar curvature grows without bound. FLRW space-times become ill-defined when the scalar curvature diverges. For that reason, one encounters an open boundary beyond which space-time cannot be extended. As I will discuss in chapter 7, a past boundary to the Cosmos is a necessary (but not sufficient) condition for the Cosmos to have had a beginning. Traditionally, authors have often focused on the boundary to the exclusion of all other criteria that might be thought necessary for the Cosmos to have had a beginning and so the fact that (some) classical Big Bang models include a past boundary has often been taken to indicate that the Cosmos likely had a beginning.

A decade after Craig's previously quoted remarks, Craig and his co-author James Sinclair discuss, but do not endorse, the following argument for the view that the universe began to exist:

P1. If space-time is singular, the universe began to exist.

P2. Space-time is singular.

C. Therefore, the universe began to exist (Craig and Sinclair, 2012, p. 98).

Here, Craig and Sinclair mean that if the Cosmos includes a space-time with a past singular boundary, then the Cosmos began to exist. Call this the Singularity Argument. Craig and Sinclair do not endorse the Singularity Argument because, as they acknowledge, there are non-singular cosmological models and singularities will likely be replaced by some other structure in a successor theory to General Relativity. Although Craig and Sinclair do not endorse the Singularity Argument, they do maintain that singular cosmological models provide strong evidence for a beginning of the Cosmos because they argue that there will be features in a future quantum gravity theory that correspond to the cosmological singularities in FLRW models. As Craig and Sinclair write, "There may be no such things as singularities per se in a future quantum gravity formalism, but the phenomena that [General Relativity] incompletely strives to describe must nonetheless be handled by the refined formalism, if that formalism has the ambition of describing our universe" (Craig and Sinclair, 2012, p. 106). That is, even if cosmological singularities are not real features of the universe, they are approximations of real physical features of the universe, and the way in which cosmological singularities approximate the universe suggests that our universe had a beginning. I don't find this argument convincing. While scientific realists would argue that presently well supported physical theories approximate their successor theories, no scientific realist claims that all of the entailments of current scientific theories approximate features that will appear in future scientific theories. What reason do we have for thinking that the structure which replaces singularities in a successor theory will have any relevant relationship to a beginning of the Cosmos? I cannot see any such reason and Craig and Sinclair have certainly not attempted to provide one. I will return to this issue in chapter 12.

Nonetheless, on Craig and Sinclair's interpretation, twentieth century cosmology was largely motivated by attempts to overcome or "evade" mathematical results concerning singularities in classical space-times, i.e., the singularity theorems, and perhaps motivated by a prejudice against theistic hypotheses. I do not endorse Craig and Sinclair's historical narrative. As we will see, there are good reasons for thinking that the appearance of singularities in a physical theory is an indication that the theory will be replaced by a successor theory; insofar as we have good reason for thinking that a current theory will be replaced, we have reason to look for the successor theory – importantly, reasons that have nothing at all to do with a prejudice against theistic hypotheses. Thus, cosmologists were unlikely to have been motivated by a prejudice against theistic hypotheses since there are other and better explanations for their actions.[3] In any case, Craig and Sinclair have offered various reasons why attempts to evade the singularity theorems have ended in failure. In the early 1990s, Craig argued that non-singular cosmologies are overly speculative and are implausible compared to singular cosmologies (Craig, 1993b). By 2009, Craig had conceded that non-singular cosmologies have been successfully produced. Nonetheless, Craig and Sinclair argue that such attempts have (typically) failed to produce cosmologies without beginnings or have otherwise been empirically ruled out (Craig and Sinclair, 2009, p. 180). Craig and Sinclair have gone on to provide the argument discussed above, namely, that while cosmological singularities might not survive future physical inquiry, the feature of the world picked out by the singularity theorems – apparently, that there is a boundary to the Cosmos's temporal existence – should survive into future physical inquiry (Craig and Sinclair, 2012, pp. 105–6).

An important point should be made here that is often lost in the literature on the KCA and that helps to explain one of the ways my dissertation contributes to the literature on the KCA. Supposing that Craig and Sinclair are right that all of the cosmological models developed thus far have been either singular, include a non-singular beginning, or have already been empirically ruled out, we cannot then infer that the Cosmos likely had a beginning. The collection of possible space-times is not exhausted by the collection of cosmological models thus far developed. Supposing that all of the cosmological models developed thus far on which the Cosmos is beginningless are implausible, cosmological models on which the Cosmos has a beginning may likewise be implausible.[4] The family of cosmological models thus far developed might not even be a representative subset of

[3]↑One can also check this conclusion against the history of the discipline. For example, as John Earman (1995, pp. 11–21) has described, Einstein's reasons for rejecting singularities had nothing to do with a prejudice against theism.

[4]↑By way of analogy, consider the hypothesis h_1:= 'my bedroom floor is covered with dust in the shape of Mickey Mouse' and h_2:= 'my bedroom floor is not covered in dust but is covered with mud in the shape of Mickey Mouse'. Obviously, h_1 and h_2 cannot both be true. Nonetheless, the two hypotheses are both improbable. Thus, we cannot infer that h_1 is probable by showing that h_2 is improbable.

the collection of all possible space-times, including all of the space-times consistent with our empirical data. Authors who have previously written on the KCA have neglected important mathematical results about the global properties of space-time that are independent of specific cosmological models; I consider some results about the global features of space-time in chapter 9.

3.2 Four Reasons Not to Take Curvature Singularities Seriously

As I've summarized, proponents of the KCA – such as Craig and Sinclair – now acknowledge that physical cosmology has moved on from an understanding of singularities as actually physically realized within nature. Nonetheless, friends of the a posteriori case for the KCA's second premise continue to argue that various results concerning singularities – particularly the singularity theorems developed by Roger Penrose, Stephen Hawking, Arvind Borde, Alan Guth, and Alexander Vilenkin – do have relevance for addressing whether the Cosmos began to exist.

In contrast, physicists usually interpret divergences, such as curvature singularities, in physical theories as an indication that the theory will be supplanted in future inquiry by a non-divergent theory. There are already excellent independent reasons for thinking that General Relativity will be supplanted by a non-singular theory. As Enrico Cinti and Vincenzo Fano (2021, p. 112) write, "most approaches to Quantum Gravity point in the direction of singularities, including that connected to the big bang, not being a genuine feature of spacetime at the quantum level". As Sean Carroll (2010, pp. 50–51) describes, since physicists do not yet know what physical theory will replace singularities, physicists do not yet know whether the universe includes a past boundary: "if someone asks you what really happened at the moment of the purported Big Bang, the only honest answer would be: 'I don't know.' Once we have a reliable theoretical framework in which we can ask questions about what happens in the extreme conditions characteristic of the early universe, we should be able to figure out the answer, but we don't yet have such a theory." As Carroll goes on to describe, "It might be that the universe didn't exist before the Big Bang, just as conventional general relativity seems to imply. Or it might very well be

[...] that space and time did exist before the Big Bang; what we call the Bang is a kind of transition from one phase to another." There are other possibilities; for example, spacetime might have somehow "emerged" from a primordial non-spatio-temporal state. In any case, the point is that physicists generally think that results concerning singularities are spurious and do not represent good reasons for thinking that the Cosmos includes a beginning of its existence.

Briefly, we can identify at least three reasons for denying that we should think the singularities that sometimes appear in FLRW models have physical significance. I will add a fourth reason that should appeal to authors who endorse the a priori case for the KCA's second premise.

The History of Physical Inquiry. Other theories that have appeared in the history of physical inquiry have included singularities. When those theories were supplanted by a successor theory, the singularities vanished. Thus, the history of physical inquiry provides us with some inductive support for the conclusion that the singularities which appear in General Relativity will vanish when General Relativity is supplanted by a successor theory.

For example, as Steinhart and Turok (2007, pp. 37–38) point out, there are singularities that appear in the equations describing fluid flow (the Navier Stokes equations) because the equations assume that fluids are continuous and do not adequately take into account their atomic composition. When the Navier Stokes equations are replaced by a more accurate description in terms of molecular dynamics, the singularities vanish. As Erik Curiel describes,

> This attitude [that singularities represent defects in physical theories and not genuine physical phenomena] is widely adopted with regard to many important cases, e.g., the divergence of the Newtonian gravitational potential for point particles, the singularities in the equations of motion of classical electromagnetism for point electrons, the singular caustics in geometrical optics, and so on. No one seriously believes that singular behavior in such models in those classical theories represents truly singular behavior in the physical world. We

should, the thought goes, treat singularities in general relativity in the same way (Curiel, 2021).

While we do not currently possess an accepted quantum gravity theory, a quantum gravity theory is generally expected to replace curvature singularities by some other structure, just as other divergent physical theories have been replaced by non-divergent theories. Indeed, the most popular candidates for a quantum gravity theory, such as string theory and loop quantum gravity, replace curvature singularities and allow for the development of cosmological models without a past boundary.

Mass-energy density considerations. For a second reason for denying that the singularities that sometimes appear in FLRW models have physical significance, consider that Quantum Field Theory and General Relativity are mutually incompatible theories. For that reason, physicists expect General Relativity to be supplanted by a successor theory in future physical inquiry. General Relativity is more likely to be supplanted because Quantum Field Theory deals with the "building blocks" of nature and has been confirmed in a wider domain.

Given that General Relativity is generally expected to be supplanted by another physical theory, we can ask in which domains General Relativity provides a good approximation. General Relativity is typically thought to provide a good approximation for small curvature and low energy. For example, dimensional analysis supports the notion that quantum gravity effects are important when the De Broglie wavelength approaches the Planck length. De Broglie wavelengths on the order of the Planck length are associated with an energy of approximately 10^{28} electronvolts. In the vicinity of the curvature singularities appearing in FLRW models, the curvature and mass-energy density become unboundedly large. Consequently, as one approaches a curvature singularity, one encounters energies arbitrarily larger than 10^{28} electronvolts (or, indeed, larger than any finite energy). Thus, in order to know what might have happened at earlier times, we would need to have in hand a description of whatever exotic physical theory should replace General Relativity at energies greater than 10^{28} electronvolts.

General Relativity should not be trusted at energies beyond 10^{28} electronvolts; in particular, the prediction that space-time has a past temporal boundary cannot be trusted. We do not yet possess a successful theory for energies beyond 10^{28} electronvolts, or at least there is no consensus as to what theory should supplant General Relativity in that domain, and so we do yet know what sort of exotic physics there might be for energies that exceed 10^{28} electronvolts.

Finite domain. There are another set of considerations closely related to the concerns about curvature and mass-energy density and that provide another reason for denying that the singularities that sometimes appear in FLRW models have physical significance. No matter how we think about the domain of validity of General Relativity, the domain of validity of physical theories is generally understood to be finite. No physicist should expect to be able to accurately extrapolate a physical theory over an actually infinitely large domain. General Relativity predicts that as we approach a curvature singularity, the mass-energy density and curvature become arbitrarily large. Thus, no matter where the boundaries on the domain of validity of General Relativity might be, General Relativity predicts that there is some location closer to the curvature singularity where the energy-density and curvature are larger. For that reason, we should not be realists with respect to the curvature singularities appearing in FLRW models and have no good reason for accepting the prediction that space-time has a past temporal boundary.

Counting down from infinity. The last reason for rejecting the physical significance of cosmological singularities is one that does not enjoy wide support – and certainly is not well-supported by physicists – but which should be taken seriously by friends of the a priori case for the KCA's second premise. Recall that, according to one of the a priori arguments, we cannot count up to infinity and cannot count down from infinity. However, curvature singularities – if they were real – would provide a physical realization of counting down from infinity. Let's recall again the function $f(x) = 1/x$ that I used to explain the notion of a curvature singularity. Imagine placing ourselves on the x axis at $x = 1$ and moving slowly towards $x = 0$. Pick any number you want larger than 1; call that number N. No matter what number you pick, as we move towards $x = 0$, we will eventually encounter a value of x such that $f(x)$ is larger than N. For example, if you choose $N = 5$, then, when x is equal to or smaller than $1/5$, $f(x)$ is equal to or greater than 5. One consequence is that all of the positive integers can be mapped to values of x between 0 and 1. In classical Big Bang models, the Ricci curvature, the matter-energy density, and the temperature grow without bound as we move backwards in time. Just as $f(x)$ maps all of the positive integers to values of x between 0 and 1, so, too, all of the positive integers appear as values of the Ricci curvature, the matter-energy density, and the temperature as we approach the Big Bang singularity. If counting down from infinity – as in counting down through all of the negative integers – cannot be physically realized, then there must be a finite maximum value for the Ricci curvature, matter-energy density, and temperature. But if there are finite maximum values for the Ricci curvature, matter-energy density, and temperature, then there is no Big Bang singularity. For example, our current universe might have emerged from a prior universe bouncing through a highly (but not infinitely) compressed state – as I discuss in chapter 11 – instead of having a singular boundary. Ergo, defenders of the a priori case for the KCA's second premise themselves have reason not to endorse the reality of curvature singularities.

3.2.1 Responses by Philosophers

No physicist that I have met is surprised by the three reasons I've offered for not endorsing a realistic interpretation of space-time singularities. For physicists, the three reasons that I have offered are obvious, well-known, and are not tremendously interesting or novel. However, I have encountered philosophers who, in casual conversation or correspondence, express surprise that physicists do not endorse a realistic interpretation of space-time singularities. Reactions expressed by philosophers, in correspondence or casual conversation, have tended to be of three sorts. First, some philosophers are simply unaware that physicists mean the term *singularity* in a technical sense and do not merely mean that the Big Bang was initiated by a special point. For example, some philosophers I have spoken to appear to think that the Big Bang singularity was a point, and so a part of space-time, having infinite matter-energy density instead of an open boundary to space-time. I'm not entirely sure why they would think that space-time including a point with infinite mass-energy density would have some sort of relationship to whether the Cosmos had a beginning. In any case, the mistaken view that the Big Bang singularity was some sort of special point is not an argument and so there is nothing to respond to, other than to say that, hopefully, this chapter provides enough of an introduction to the issues involved to dissuade philosophers from thinking that a singularity is merely a special point of some kind.

Second, some philosophers have heard that there are curvature singularities within black holes and they have read that we now possess excellent evidence that black holes exist, e.g., images captured by the Event Horizon telescope, gravitational wave data from LIGO, and the like. In reply, the three arguments I have provided for not taking curvature singularities seriously in FLRW models *do* apply equally to the curvature singularities General Relativity predicts for black holes. However, we should be careful when we say that physicists have excellent evidence for black holes. Physicists have evidence that there are astrophysical systems which obtain sufficiently high mass-energy densities that they develop horizons. For readers unfamiliar with the notion of a *horizon*, roughly, a horizon is a surface beyond which we cannot receive signals. If we were to approach a black hole, we would find that the velocity we would need to escape the black hole increases as we approach the black hole's center. At some point, we would find that the velocity we would need to escape the black hole is the speed of light; at that moment, we would be crossing the black hole's horizon. At points still closer to the black hole's center, the velocity needed to escape the black hole is greater than the speed of light. Thus, a signal that originates from within the horizon of a black hole would need to move faster than the speed of light to reach outside observers; since no signal can move faster than light, no signal can be transmitted from points within the horizon to points outside the horizon. If a black hole includes a curvature singularity, the singularity is clothed within the horizon. We do not have compelling observational evidence that any black hole includes a curvature singularity clothed within that black hole's horizon and, given that no signal can exceed the speed of light, we likely could not have compelling observational evidence that any given black hole includes a curvature singularity. Physicists generally think that while black holes exist – that is, while there exist compact objects with sufficient mass-energy density to have developed a horizon – physicists do not generally think that the interior of those objects is accurately described by General Relativity. While black holes exist, they likely do not contain singularities. Thus, the arguments that I've provided here do not provide reason to deny that there are astrophysical black holes, at least when that claim is correctly interpreted.

Let's move to a third response I've sometimes heard philosophers express. Science journalists will sometimes say that singularities are points to which physical laws do not apply. In casual conversation or correspondence, philosophers sometimes repeat that statement and are surprised to learn that, according to General Relativity, curvature singularities are open boundaries and not parts of space-time. Charitably, science journalists are expressing the notion that our *knowledge* of physical law is thought to run out as we approach the locations where General Relativity predicts the occurrence of a curvature singularity and not that fundamental physical law somehow stops applying. General Relativity does not have the power to literally predict points beyond the reach of physical law – how could a physical theory possibly do *that*? – but suppose that, somehow, General Relativity *did* predict that there are points where physical law no longer applies. One (perhaps defeasible) desideratum for a final physical theory is that the theory has unlimited scope. If General Relativity predicted the existence of points beyond the scope of physical law, wouldn't we understand such a prediction as a *defect* of General Relativity? And if we did understand such a prediction as a defect of General Relativity, wouldn't this provide us with another reason to deny that we should endorse a realistic interpretation of curvature singularities?

3.3 Some other philosophical responses

In this section, I briefly turn to some philosophical arguments which attempt to show that the Big Bang is relevant for whether the Cosmos began to exist but which do not draw upon results about singularities.[5] To begin, let's compare the situation in which present physical cosmologists find themselves with the situation that nineteenth century

[5] ↑The arguments in this section were suggested by Paul Draper in correspondence.

geologists found themselves in with respect to the debate between catastrophism and uniformitarianism. Suppose that Greg is a nineteenth century geologist who thinks that there is some minimal evidence for catastrophism and that Greg wants to know whether there is *some* evidence that the Earth began in the finite past. Greg might point out that if the Earth did begin to exist in the finite past, then there was a castrophe in the Earth's history before which we cannot trace the history of the Earth.

Throughout this dissertation, I will assume the *relevance theory of evidence*, according to which some datum e is evidence for hypothesis h relative to background knowledge K just in case the datum raises the probability of h, i.e., $Pr(h|e\&K) > Pr(h|K)$. Data that raises the probability that the Earth hasn't always existed in the Earth's present state raises the probability that the Earth began and so is evidence for the Earth's beginning. Likewise, I would concede that Big Bang cosmology provides us with some evidence that the universe has not always existed in the universe's present state and so some evidence for the conclusion that the universe began. Moreover, as I will discuss in chapter 7, one necessary condition for the Cosmos to have had a beginning is that the Cosmos includes a past boundary. While the universe is a proper part of the Cosmos, the Cosmos could not have a past boundary unless the universe has a past boundary. For that reason, that the universe includes a past boundary raises the probability that the Cosmos includes a past boundary. Thus, all else being equal, since Big Bang cosmology provides us with evidence that the universe has undergone change over time, we have some evidence that the Cosmos includes a past boundary and so some evidence that the Cosmos includes a beginning.

A few things can be said in reply. First, the relevance theory of evidence provides a very minimal threshold for data to count as evidence for a hypothesis. Consider LEPRECHAUN, that is, the hypothesis that the grass outside my apartment was made green by invisible leprechauns casting a green-making spell over the grass. In ordinary language, we might say that there is no evidence for LEPRECHAUN. But, according to the relevance theory of evidence, the fact that the grass outside my apartment is green *is* evidence for LEPRECHAUN. The trouble is just that the evidence is not strong enough to render LEPRECHAUN more epistemically probable than its negation. Likewise, while the fact that the universe has changed over time might provide some evidence that the Cosmos began, the evidence is not so strong as to render the Cosmos having a beginning more probable than the Cosmos not having a beginning. Second, whether we endorse a specific hypothesis ought to be decided by the *total* evidence and not merely in virtue of the fact that there is *some* data supporting the hypothesis.

I turn to considering a second philosophical argument. Gottfried Leibniz (1956, pp. 26–27) considers (and rejects) a view according to which the Cosmos was preceded by empty time. Since Leibniz endorses the Principle of Sufficient Reason, Leibniz asks for a sufficient reason for the Cosmos beginning at the specific time at which the Cosmos began and not at some other. Suppose that we trace the expanding universe backwards and suppose that there is some maximal matter-energy density to the universe. (For example, loop quantum gravity implies that there is a maximum physically possible matter-energy density.) Since the matter-energy density cannot be higher than the maximum and, as we trace the expansion backwards, the matter-energy density increases, there must be some time when the universe began to expand.

Consider two possibilities: first, that the universe existed in that maximally dense state for a past eternity or, second, that the maximally dense state corresponds to the first moment of time. If the maximally dense state existed for a past eternity, then we face Leibniz's problem; why did that maximally dense state begin expanding at some particular time instead of some other? But if the maximally dense state corresponds to the first moment of time, then we avoid Leibniz's problem because, in that case, the universe was not preceded by time. If there was a first moment of time, then the Cosmos must have a past boundary, and we have at least some evidence that the Cosmos began to exist.

There are at least three problems with this argument. First, the argument requires the adoption of a philosophically controversial premise, i.e., the Principle of Sufficient Reason. Second, provided that we accept the Principle of Sufficient Reason, as with the previous philosophical argument, the argument provides only weak evidence for the conclusion that the Cosmos began to exist. Third, the argument involves a false dichotomy. The maximally dense state could have been preceded by a contracting universe, as I consider in chapter 11.

3.4 Summary

In this chapter, I've summarized the a posteriori case for the KCA's second premise. At first glance, Big Bang cosmology offered to physicists what many authors from previous centuries considered infeasible, viz, an empirically well supported physical theory one of whose consequences is that there is a finite interval of time to our past prior to which the observable universe did not exist. Empirically oriented defenders of the KCA, such as Craig, Sinclair, and Swinburne, have argued that Big Bang cosmology lends strong empirical support to the KCA's second premise. However, the view that Big Bang cosmology lends strong empirical support to the KCA's second premise depends on various mathematical results in General Relativity. Physicists have a standard set of reasons for rejecting those mathematical results. General Relativity is likely to be supplanted in a future physical theory. Whether or not the universe, let alone the Cosmos, should be said to have a beginning according to whatever theory supplants General Relativity is not currently known.

I've rehearsed some standard reasons for doubting that curvature singularities are physically realized. However, as I have already discussed, friends of the KCA's second premise have argued that the KCA's second premise can be defended without appealing to curvature singularities. Thus, I have not yet shown that the KCA is without merit. In order to examine whether we can determine that the Cosmos had a beginning, we should articulate and clarify the concept of the Cosmos having a beginning. Unfortunately, most philosophers, physicists, and theologians who have previously discussed the beginning of the Cosmos have failed to articulate a conception of the beginning of the Cosmos and, so far as I know, no author has previously articulated a fully adequate conception. In the next section, I develop three necessary (though not necessarily sufficient) conditions for the Cosmos to have had a beginning. While I make no claim about the sufficiency of the three conditions, the three conditions do push the investigation of the beginning of the Cosmos substantially forward.

Part II

GETTING CLEAR ON THE BEGINNING OF THE COSMOS

4. THE BEGINNING OF THE COSMOS AND THE METAPHYSICS OF TIME

4.1 Introduction

In part II of this dissertation, I address how we should understand the notion that the Cosmos began to exist. One might have thought that the concept of the beginning of the Cosmos could be analyzed from the armchair. As a first pass, the Cosmos began to exist if there is a moment of time such that the Cosmos exists at that moment and the Cosmos does not exist at any prior moment. As we will see, this definition will not do and, unfortunately, a full suite of necessary and sufficient conditions for the Cosmos to have a beginning are surprisingly difficult to come by. Instead of developing a full suite of necessary and sufficient conditions, I will sketch and defend a set of conditions that meet the following three desiderata: (i) the conditions should be necessary for the Cosmos to have a beginning, (ii) the conditions should be useful in determining whether the Cosmos had a beginning, and (iii) the conditions should help to elucidate the concept of a beginning. The three conditions that I will sketch and defend are:

1. The Modal Condition: At all of the closest possible worlds where time does not exist, the Cosmos does not exist.

2. The Direction Condition: The Cosmos has a global direction of time.

3. The Boundary Condition: Either there is a closed boundary to the past of every non-initial space-time point (*the topological conception*) or there is an initial objectively finite portion of the Cosmos's history (*the metrical conception*).

I do not claim that these three conditions are logically independent. In this dissertation, I remain neutral on their logical interrelationships and my discussion of each the three conditions will be relatively self-enclosed.

Throughout, I will frequently discuss how these three conditions can be understood in the context of classical space-times, that is, relativistic and pre-relativistic space-times. As I've already explained, the majority of physicists and philosophers of physics agree that General Relativity will be supplanted in future physical inquiry by a quantum theory of gravity. We do not yet possess a universally agreed upon quantum theory of gravity. In some places, particularly in discussion of the Modal Condition, I will discuss some of the proposals for quantum gravity theories. However, in other places, I discuss only how my account applies to classical space-times.[1] For that reason, my comments in part II should be regarded as provisional and subject to revision in light of future physical (and philosophical) inquiry.

In this chapter, I begin a discussion of what 'beginning to exist' means by discussing whether beginning to exist requires a specific metaphysical theory about the nature of time to be true. In their sophisticated defenses of the KCA, William Lane Craig and James Sinclair have argued that beginning to exist is an irreducibly tensed notion, so that the Cosmos could have begun to exist only if the A-theory of time – that is, the view that there are objectively and irreducibly tensed facts – is true (Craig and Sinclair, 2009, pp. 183–184; Craig, 1990, pp. 337–338; Craig, 2007b); this conclusion is shared by many other philosophers, including William Godfrey-Smith (1977), Bradley Monton (2009, p. 94), David Oderberg (2003, p. 146), Ryan Mullins (2016, pp. 135–136, 143, 147; 2011, p. 43), and Felipe Leon (2019, p. 62). Other authors, e.g., Hans Reichenbach (1971, p. 11), have maintained that B-theory entails that nothing objectively begins or changes and so are implicitly committed to the view that if anything does objectively begin or change, A-theory is true.

Although some authors have claimed that beginning to exist is a kind of change and that change requires the truth of A-theory, B-theorists have developed an alternative account of change that does not require A-theory. The most popular B-theoretic account of change is the at-at theory, that is, the theory that a change occurs just in case (i) some state of affairs α obtains at time t_1, (ii) some state of affairs β mutually incompatible with α obtains at time t_2, and (iii) $t_1 \neq t_2$. In order to use the at-at theory in a conception of the beginning of the Cosmos, there would need to be a time before the Cosmos exists and a

[1] ↑Quantum gravity may provide further obstacles to a clear notion that the Cosmos began that I do not discuss in this chapter. For example, if the Cosmos can be in a superposition of having a beginning and not having a beginning, there may not be a determinate fact about whether the Cosmos began.

subsequent time when the Cosmos does exist. Some authors (e.g., Richard Swinburne) maintain that the Cosmos's beginning was preceded by empty time and so they can accommodate an at-at conception of the Cosmos's beginning. However, other authors maintain that time is a physical phenomenon; if they are right, time could not have preceded the Cosmos. In order to accommodate the intuitive notion that the Cosmos and time could have begun together, we need an alternative to the at-at conception. An alternative can be developed according to which, roughly speaking, a change occurs just in case either the three conditions from the at-at conception obtain *or* some state of affairs α obtains during some period of time and there is no prior period of time in which α obtains. In that case, the Cosmos could have begun if there was once a finite period of time before which the Cosmos did not exist. This notion will need to be made rigorous, and consistent with relativity, in subsequent chapters.

The present chapter sets the stage for the rest of part II. In order for my account of the notion that the Cosmos had a beginning to accommodate as many metaphysical views about the nature of time as possible, the present chapter leaves us with three desiderata: (i) the account should be consistent with both A- and B-theory, (ii) the account should be consistent with our best physical theories concerning the nature of time, including Special and General Relativity, and (iii) the account should be consistent with, but should not require, the view that there was time before the Cosmos's existence. While I will not assume in this dissertation that beginning to exist is an objectively tensed notion, some readers may find that view attractive. If they do, they can take solace in the fact that the necessary conditions for the Cosmos to have had a beginning that I defend throughout part II (the Modal, Direction, and Boundary Conditions) are not assumed to be sufficient conditions; the reader may, if they choose, add a Tensed Condition. Additionally, this chapter has the aim of introducing the various metaphysical accounts of time that I will make use of throughout the rest of this dissertation.

4.2 Metaphysical Accounts of Time

In this section, I will describe three families of metaphysical accounts of time – A-theories of time, B-theories of time, and C-theories of time – that will be useful both in this chapter and throughout the rest of this dissertation. According to the A-theories of time, time objectively "passes" or "flows" and grammatical tenses express objective and irreducible truths. Objective passage is usually understood to involve an absolute distinction between the past, the present, and the future; events are either absolutely past, absolutely present, or absolutely future. The passage of time involves future events becoming present and present events becoming past. The objective relations that *past*, *present*, and *future* bear to each other are termed the *A-relations* and, according to A-theorists, the tenses appearing in natural languages express the A-relations. The series of events arranged according to the A-relation is called the *A-series*.

B-theories of time conjoin two theses. First, B-theories of time deny that time passes or flows and that there are any irreducibly tensed truths. Second, B-theories of time endorse the view that there are absolute relations of *before* and *after*. B-theory is sometimes said to include the relation *simultaneous-with*. The orthodox Minkowskian interpretation of relativity precludes the possibility that any two spatio-temporally non-overlapping events are absolutely simultaneous. While the absolute simultaneity of spatio-temporally overlapping events is trivial, relativity is typically understood to preclude any absolute and non-trivial simultaneity relation. For that reason, I will leave absolute simultaneity out of the definition of B-theory.[2] Since B-theorists deny that time passes or flows and deny that there are any irreducibly tensed truths, B-theorists are typically understood to deny that there is an objective (or non-indexical) distinction between the past, present, or

[2]↑One may wonder how there could be absolute before/after relations if, according to relativity, there are no non-trivial absolute simultaneity relations. Pre-relativistic conceptions of time mandate that any two numerically distinct space-times points p_1 and p_2 are such that p_1 is either before, after, or simultaneous with p_2. Readers unfamiliar with relativity may therefore think that if p_1 and p_2 fail to be simultaneous, then p_1 must be either before, after, or else not absolutely temporally related to p_2. Relativistic space-times escape this intuition by introducing a distinction between space-like and time-like separated points absent from pre-relativistic space-times. In relativistic space-times, any two numerically distinct space-time points p_1 and p_2 are such that either p_1 and p_2 are time-like related – in which case either p_1 is either absolutely before or after p_2 – or p_1 and p_2 are space-like related, in which case p_1 is not absolutely simultaneous-with, before, or after p_2.

future. B-theorists can accommodate an indexical distinction between the past, present, and future similar to the spatial relations of *here* and *there*, but B-theorists cannot hold that there is an objective "passage" of time from future to present to past. As I've said, B-theorists do endorse objective relations of *before* and *after*; in turn, before and after are termed the *B-relations*. The series of events arranged according to the B-relation is called the *B-series*.

The claim is often made that relativity supports the B-theory of time. While there are non-traditional versions of B-theory that drop the notion that any two non-overlapping events can be absolutely simultaneous, the B-series, as originally formulated by John McTaggart (1908) and as often presented in introductory metaphysics textbooks, postulates a distinct formal structure for space-time than does the standard Minkowskian interpretation of relativity. Consider that Michael Loux (1998, p. 213), in his introductory metaphysics textbook, describes *B*-theory as entailing that "time is a dimension along with the three spatial dimensions; [time] is just another dimension in which things are spread out." Since the time dimension and the three spatial dimensions are independent, we might think of each instant of time – or each point along the temporal dimension – as corresponding to an arrangement of objects in space, so that any two events are simultaneous just in case they exist together in the same three-dimensional space. Philosophers of physics will recognize that the view described by Loux most closely matches Newtonian space-time, that is, the view that space-time consists of a series of three dimensional spaces located at successive times, and does not match the Minkowskian view in two important respects. First, on $B-$theory as described by Loux, time is an additional dimension to our familiar three spatial dimensions. As Minkowski (1952, p. 75) argued, in relativity, both space and time disappear as independent existences, so that we are left with a kind of union of the two that is neither spatial nor temporal. To be sure, space-time, as understood by Minkowski, is a four-dimensional manifold, but, since the division between space and time cannot be formulated without adopting a reference frame, the dimensions are themselves neutral between space and time. Second, to the extent that a time parameter appears in orthodox relativity, time is measured along trajectories (i.e., the so-called *proper time*) traversing space-time and not as an additional dimension to the three spatial dimensions.

Lastly, there are the C-theories. Like B-theories, C-theories deny that time passes or flows and that there are any irreducibly tensed truths. But C-theories additionally deny that that there are absolute or objective relations of before and after. That is, C-theories deny that time has any absolute or objective direction. C-theories endorse the view that there is an objective *betweeen-ness* relation called the *C-relation*. The series of events arranged according to the C-relation is called the *C-series*.

A-, B-, and C-theories can be distinguished by the arity of the objective relations postulated by each theory. A-theories postulate three monadic predicates (past, present, and future). For example, let β represent the time of my birth and let $\mathcal{P}$ be the predicate representing past-ness. In that case, A-theorists will agree that $\mathcal{P}(\beta)$ is now true, though $\mathcal{P}(\beta)$ was once false (or, on the view that there are no determinate truths about the future, that $\mathcal{P}(\beta)$ once had no truth value). The passage of time is reflected in the fact that there is some collection of non-indexical sentences, e.g., $\mathcal{P}(\beta)$, whose truth value changes. B-theory postulates two binary relations (before and after). For example, let $<$ represent the before relation and let Π represent the time at which I am writing this sentence. In that case, B-theorists will agree that $\beta < \Pi$. The fact that, according to B-theory, time does not objectively pass is reflected in the fact that there is no collection of non-indexical sentences whose truth value changes, e.g., $\beta < \Pi$ is timelessly true.[3] C-theory postulates one trinary relation (between-ness). Let Γ represent Lincoln's delivery of the Gettysburg address, Ω represent the 2024 American presidential election, and let $\mathcal{B}$ represent the between-ness relation. C-theorists will agree that $\mathcal{B}(\Gamma, \Pi, \Omega)$. But, due to the symmetry of $\mathcal{B}$, C-theorists will also agree that $\mathcal{B}(\Omega, \Pi, \Gamma)$. The fact that, for C-theorists, time does not objectively pass is reflected by the fact that, for the C-theorist, there is no set of non-indexical sentences whose truth value changes. And the fact that, for C-theorists, there is

[3]↑B-theorists allow that the truth values of *indexical* sentences can change. For example, the indexical sentence 'I graduated from the University of Rochester thirteen years ago' is true when stated in 2022 but false when stated in any other year. Nonetheless, according to B-theory, the truth of that sentence can be exhaustively explained by a set of non-indexical sentences whose truth value does not change. Compare: the sentence 'I am in Indiana' is true when uttered in Indiana, false when uttered outside of Indiana, and entirely explicable without positing some special metaphysical status to Indiana's border.

no direction to time is reflected by the symmetry of the between-ness relation, e.g., that $\mathcal{B}(\Gamma,\Pi,\Omega) \iff \mathcal{B}(\Omega,\Pi,\Gamma)$ is timelessly true.

4.3 Beginning of Existence and Metaphysical Accounts of Time

Having laid out the various metaphysical theories concerning the nature of time, in this section, I turn to considering whether the notion that the Cosmos began to exist requires a specific metaphysical account of the nature of time. As I will show, the notion that the Cosmos began to exist is incompatible with C-theory. And while some authors, whose views I will summarize, have thought that the beginning of the Cosmos requires the A-theory of time, I will argue that we should develop a notion of the beginning of time that is consistent with both A-theory and B-theory.

4.3.1 C-Theory and Beginning to Exist

Let's first turn to showing that the notion that the Cosmos had a beginning is incompatible with C-theory. Intuitively, beginning to exist is an asymmetric notion; if I began to exist at my birth and endure or perdure for some time, then I did not also cease to exist at my birth.[4] Consequently, if one of the C-theories is true, nothing begins to exist.

Whether we should say that C-theories of time are theories of *time* is at least somewhat controversial. When McTaggart first introduced and defended the view that our world is ordered according to a C-series and not the A- or B-series, he thought that he had abolished time altogether by showing that time is "unreal". In chapter 5, I will consider views according to which the Cosmos is fundamentally timeless. Most of the theories that I will consider are more radical than C-theory. Nonetheless, in chapter 10, I will

[4]↑Jeffrey Brower raised the objection that, intuitively, an object *O* which exists for a single instant (and satisfies the other conditions for beginning to exist that I discuss in chapters 5, 6, and 7) begins to exist. Fair enough, though, in that case, time asymmetry is still important for distinguishing the notion that *O* began from the notion that *O* ceased to exist. We can say that *O* began for several reasons, each tied to a temporal asymmetry, e.g., there is a time before *O*'s existence, there is a time *t* such that *O* did not exist before *t*, etc. Likewise, the destruction of *O* is tied to temporal asymmetry, e.g., there is a time after *O*'s existence, there is a time *t* such that *O* does not exist after *t*, etc. Consequently, if *O* exists only for one instant *T*, then *T* can be understood as both the "birth" and "death" of *O*, where birth and death are tied to distinct ways in which *T* bears an asymmetric relation to other (real or unreal) times.

assume that if C-theory is correct, then the Cosmos is fundamentally timeless, or at least not fundamentally temporal in any sense that is relevant for whether the Cosmos began. For present purposes, let's set aside the C-theories of time.

4.3.2 A-theoretic Accounts of Beginning

Some authors have argued that beginning to exist requires an A-theory of time. Consider that most A-theorists understand the passage of time to involve states of affairs coming into being. For example, growing block theorists maintain that only the present and the past exist. Future events do not yet exist, but come into being by becoming present. On B-theory, all moments of time exist *simpliciter* and so do not come into being by becoming present. Consequently, if 'my apartment began to exist' expresses the proposition that my apartment came into being by becoming present, then my apartment beginning to exist is inconsistent with B-theory. Nothing begins, in this sense, unless the A-theory is true. Likewise, Craig and Sinclair (2009, pp. 183–184) have argued that if there are no tensed facts, then there is no fact about the universe beginning to exist and the quest to find a cause of the universe is confused. Monton (2009, p. 94) puts the point in terms of four-dimensionalism. If space-time is a timeless four-dimensional block, in which time is another direction of space, then a boundary of time is not a beginning in the relevant sense. However, as I've discussed, relativistic four dimensionalism should not be understood as the view that time is another dimension of space. In any case, Monton's point can be made without geometrizing time. Monton's point can be re-phrased as follows: B-theory is sometimes interpreted to imply that space-time is an eternal and changeless four-dimensional block; if the four-dimensional block is eternal and changeless, then nothing ever changes. If nothing ever changes, then nothing begins to exist. Thus, on at least some conceptions of beginning to exist, beginning to exist requires A-theory. In the next section, I turn to examining an alternative B-theoretic account of beginning to exist.

4.3.3 B-theoretic Accounts of Beginning

The At-At Conception

If beginning to exist requires the A-theory of time, then there may be good reason to deny the view that the Cosmos began to exist. For example, there are a variety of arguments against the A-theory of time, including the fact that the A-theory of time is at least difficult to render compatible with relativity (Putnam, 1967; Rietdijk, 1966; Penrose, 1989, pp. 201, 303–304; Petkov, 2006; Romero and Pérez, 2014). Insofar as we have reason to be realists about relativity, and so to think that space-time has the formal structure postulated by relativity, we would have reason to deny that the Cosmos began to exist. Likewise, insofar as there are philosophical arguments against tensed theories of time, we would have reason to deny that the Cosmos began to exist. Nonetheless, despite the claims made by A-theorists, there are B-theoretic accounts of change that allow B-theorists to accommodate a different sense of beginning to exist. The fewer controversial assumptions that an account requires, the better off the account is. If an account of beginning to exist can be formulated that remains neutral between A- and B-theory, that account of beginning to exist would be superior to an account that requires either A- or B-theory.

For that reason, let's turn to considering one standard B-theoretic account of change. According to the *at-at theory*, for change to occur requires only that there (tenselessly) exists a time t_1 *at* which state of affairs s_1 obtains and a numerically distinct time t_2 *at* which state of affairs s_2 obtains such that s_1 and s_2 are incompatible states of affairs.[5] Perhaps we can say that x began to exist only if there exists a time t_1 at which x did not exist, a numerically distinct time t_2 at which x exists, and t_1 is absolutely before t_2. On this interpretation, for my apartment to begin to exist requires only that there is a time when my apartment does not exist that occurs absolutely before another numerically distinct time at which my apartment does exist. Call this account the *at-at account of beginning*.

[5]↑The at-at theory was originally developed as a theory of motion in reply to one of Zeno's Paradoxes (Russell, 1918, pp. 83–84; Arntzenius, 2000; Huggett, 2019; Salmon, 1980). However, motion is a kind of change (i.e., change in position over time) and, as pointed out in, e.g., Salmon, 1977, p. 222 the at-at theory is easily generalized into a theory of change.

The at-at account of beginning requires that there was a moment of time before the Cosmos existed. Swinburne has offered a substantivalist account of time in which time precedes what Swinburne calls the "Universe" – roughly equivalent to my "Cosmos" – and in which time has no beginning even though the Universe had a beginning. And Swinburne (1996) has argued that the Universe began just in case there was a time preceding the Universe. Alan Padgett (2000; 2001a, p. 109; 1989, 1991, 2010, 2013), Ryan Mullins (2014, 2016, 2020), Garrett DeWeese (2016), and other members of the so-called "Oxford School" have likewise defended views on which there was amorphous time prior to Creation. Members of the Oxford School can endorse the at-at account of the Cosmos's beginning because, on their view, there is a time that precedes the Cosmos.[6] While I think the Oxford School's view that time is non-physical is implausible, I do not argue against the Oxford School in this dissertation. For that reason, one desideratum for the necessary conditions for the Cosmos to have had a beginning that I develop in this dissertation is that the conception be consistent with the Oxford School's conception of time.

While some authors, such as the members of the Oxford School, maintain that time is non-physical and so could have preceded the Cosmos's existence, other authors maintain that time is a physical phenomenon that could not have preceded the Cosmos's existence. Contemporary physical theory appears to suggest that time is a physical phenomenon, so that time could not have preceded the Cosmos. For example, we can distinguish two ways of understanding the standard Minkowskian interpretation of relativity. *Substantival Minkowskians* understand space-time as a physical object (or substance). If Substantival Minkowskianism is true, then there is no time before the existence of physical objects and so no time before the Cosmos exists. As I've said, the at-at account of beginning requires two times, e.g., one time at which the Cosmos does not exist and a subsequent time at which the Cosmos does exist. Thus, if both Substantival Minkowskianism and the at-at account of beginning are correct, then the Cosmos did not begin to exist. On the other hand, *Relational Minkowskians* deny that space-time is a physical object. For the Relational

[6]↑To be clear, most, perhaps all, members of the Oxford School endorse the A-theory of time. Nonetheless, their view that time is not a physical phenomenon and precedes the Cosmos is compatible with both A-theory and B-theory.

Minkowskian, space-time should be understood in terms of the relations between material bodies or in terms of the properties (or attributes) of a single material body. Note that if space-time should be understood in terms of the relations between material bodies, then there is no time before the existence of material bodies. For that reason, if both Relational Minkowskianism and the at-at account of beginning are correct, then the Cosmos did not begin to exist. Therefore, if the at-at account of beginning to exist is correct and either version of Minkowskianism is correct, then the Cosmos did not begin to exist. Prima facie, this appears to rule out a beginning of the Cosmos, since one is hard pressed to imagine a better alternative interpretation of relativistic physics that avoids the view that time is a physical phenomenon.

The trouble is that even if time is a physical phenomenon, we should still be able to say that the Cosmos began. For example, supposing that time is a physical phenomenon, we should be able to say that the Cosmos and time began together. The at-at conception of the beginning of the Cosmos is inadequate because the at-at conception would inappropriately rule out such a possibility.

There is an additional reason to rule out the at-at conception of the beginning of the Cosmos. We can say that an entity *E* has a *beginning-in-time* just in case *E* has a beginning and, during all of the times in which *E* exists, *E* is a content of moments of time and so is distinct from time itself. In contrast, let's use *beginning-of-time* to refer to the beginning of time, itself. The at-at conception should be thought of as a conception of beginning-in-time as opposed to a conception of the beginning-of-time (Draper, 2008). Supposing that time is a physical phenomenon, so that time begins when the Cosmos begins, the beginning of the Cosmos is the beginning of time, itself. Thus, in order to be consistent with the view that time is a physical phenomenon, an analysis of the beginning of the Cosmos should accommodate the view that the beginning of the Cosmos was the beginning-of-time. For that reason, we need an alternative to the at-at conception for discussing the Cosmos's beginning.

Consider, again, the aforementioned desideratum that our conception of the Cosmos's beginning should be consistent with the Oxford School's conception of time. This desideratum reflects a more general thought about what sort of criteria a good conceptual analysis of the Cosmos's beginning should involve. In the introduction to this dissertation, I stated that I am interested in a conception of the beginning of the Cosmos that fulfills two desiderata. First, to help the second premise of the KCA -– that the Cosmos began to exist -– 'beginning' should be understood as broadly as possible. Second, to help the first premise — that anything that begins to exist has a cause for beginning – 'beginning' should be understood as narrowly as possible so as to avoid making the first premise obviously false. We can refine our intuitions, in light of sophisticated philosophical, scientific, and mathematical inquiry, about which epistemically possible worlds include a beginning of the Cosmos. For example, above, I showed that both Substantival Minkowskianism and Relational Minkowskianism, when conjoined with the at-at conception of the beginning of the Cosmos, led to the intuitively wrong conclusion about whether the Cosmos began to exist. While this dissertation does not take up whether Substantival Minkowskianism or Relational Minkowskianism are true, one of my goals for developing a conceptual analysis of the beginning of the Cosmos is that the concept be neutral with respect to as many metaphysical theories as possible.

Other B-theoretic Conceptions

The at-at account of the Cosmos's beginning leads to counterintuitive consequences about which possible worlds include a beginning of the Cosmos. Thankfully, there is an alternative analysis of the Cosmos's beginning that does not lead to the same counterintuitive consequences. The trouble with the at-at conception was the invocation of two times, including a time before the Cosmos's existence. In order to retain consistency with the Oxford School, we shouldn't rule out the possibility that there are times before the Cosmos. But to avoid the problems posed by the at-at conception, we need an account that does not make explicit reference to times before the Cosmos's beginning. We are thereby led to the following proposal:

> *If the Cosmos began to exist, then there was a time (or perhaps a finite interval of time) such that there were no prior times (or prior intervals of time) at which the Cosmos existed.*

This proposal requires only one time (or one finite interval of time) and so escapes the worries that were introduced by the at-at account. Nonetheless, this proposal remains consistent with the Oxford School by allowing for the possibility that there were prior times at which the Cosmos did not yet exist.

Nonetheless, this account is not fully satisfactory. Recall that, on the standard Minkowskian interpretation of relativistic space-times, we should not understand temporal series as a sequence of three dimensional spaces; instead, time is measured along trajectories that pass through space-time so that every temporal series is indexed to a specific trajectory through space-time. For that reason, in relativistic space-times, there is no such thing as a moment or interval of time simpliciter. Since the account we've developed thus far requires a moment, or interval, of time simpliciter, we will need to develop a more sophisticated conception in subsequent chapters. We will return to this issue in chapters 6 and 7.

4.4 Summary

The present chapter sets the stage for the rest of part II. I identified three desiderata that will need to be fulfilled in order for an analysis of the notion that the Cosmos had a beginning to be consistent with as many metaphysical views about the nature of time as possible: (i) the account should be consistent with both A- and B-theory, (ii) the account should be consistent with our best physical theories concerning the nature of time, including Special and General Relativity, and (iii) the account should be consistent with, but should not require, the view that there was time before the Cosmos's existence. In addition, this chapter allowed me to introduce the various metaphysical accounts of the nature of time that I will make use of throughout the rest of this dissertation.

As I've discussed, a variety of A-theorists have argued that nothing begins to exist if B-theory is true. However, B-theorists have developed an alternative account of change, the at-at theory, according to which change involves the existence of two mutually incompatible states of affairs at two distinct times. While some authors maintain that time is not a physical phenomenon and so may have preceded the Cosmos, contemporary physical theory suggests that time is a physical phenomenon and so could not have existed before the Cosmos. In order to accommodate the intuitive notion that the Cosmos and time could have begun together, B-theorists need an alternative to the at-at conception for articulating the notion that the Cosmos began to exist. I developed an alternative conception according to which the Cosmos could have begun if there was once a finite period of time before which the Cosmos did not exist. This notion will need to be made rigorous, and consistent with relativity, in subsequent chapters.

In the next chapter, I will further develop my analysis of the notion that the Cosmos began in a different way, i.e., by, first, turning to a debate in philosophy of religion concerning God's relationship to the beginning of time and, second, considering a similar issue in the literature on the philosophical foundations of relativity, quantum gravity, and quantum interpretations.

5. THE MODAL CONDITION

5.1 Introduction

At first glance, theologians and philosophers of physics are unlikely bedfellows. Nonetheless, both theologians and philosophers of physics are interested in understanding the claim that the whole of physical reality – the Cosmos – began to exist. For theologians, the claim that the Cosmos began to exist should be contrasted with the claim that God did not begin to exist. Some analytic theologians and philosophers of religion have defended the view that while there is a first finitely long period of time in God's life, God's life was beginningless (Craig, 2001b; Erasmus, 2021; Loke, 2017). This view is conceptually problematic because, prima facie, to begin to exist just means that one's life included a finitely long initial period of time. On the other hand, as discussed below, a variety of contemporary physical theories and research programs are committed to the claim that the Cosmos is not fundamentally spatiotemporal (Barbour, 1994, 1999; Bihan, 2017a, 2017b, 2019, 2020; Butterfield and Isham, 2006; S. Carroll, 2019, 2022; S. Carroll and Singh, 2019; Earman, 2002a; Healey, 2002, 2021; Huggett, 2022; Huggett and Wüthrich, 2013, 2018; Oriti, 2014, 2020, 2021; Rovelli, 2020; Wilson, 2021).[1] If the Cosmos is not fundamentally spatiotemporal, then, even if there were an initial finitely long period of

[1]↑Throughout this chapter, I make use of the notion of *fundamentality*. For example, I will examine theological theories according to which there is a fundamental aspect of God that is non-temporal and I will examine speculative physical theories according to which there is a fundamental aspect of physical reality that is non-spatio-temporal, or at least non-temporal. I do not provide an account of fundamentality here – in part because providing a conceptual analysis of fundamentality turns out to be non-trivial – but I will provide the reader with some intuition pumps for thinking about what I mean when I say that A is a fundamental aspect of some entity E. To say that A is a fundamental aspect of some entity E means that, at the level of metaphysical explanation, A is a non-derivative aspect of E; while there are other aspects of E whose explanation is in terms of A, A does not have a further and more basic explanation in terms of other aspects of E. We can identify a set of formal properties obeyed by the fundamentality relation. The fundamentality relation is transitive, i.e., if x is fundamental to y and y is fundamental to z, then x is fundamental to z. Fundamantality is irreflexive, i.e., nothing is fundamental to itself. And fundamentality is asymmetric, i.e., if x is fundamental to y, then y is not fundamental to x.

One way that A could be fundamental to E would be if A is the reductive base for E. For example, H_2O molecules are fundamental to water. However, fundamentality is more general than the relation of *being-a-reductive-base-for* since (for example) God is not reducible to God's fundamental aspect(s), but God's less fundamental aspects are explained in terms of God's more fundamental aspects. As another example, the relation of *being-functionally-realized-by* is another example of fundamentality, so that (for example) if mental states are functionally realized by, but not reducible to, neuronal states, then neuronal states are fundamental to mental states.

time in the life of the Cosmos, the Cosmos would be fundamentally beginningless. Thus, both theologians and philosophers of physics are interested in theories according to which there was an initial, finitely long period of time in the life of some x, even though x is beginningless.

Consequently, both theologians and philosophers of physics should be interested in developing necessary criteria for beginning to exist that distinguish beginningless entities whose lives include an initial finite period from entities that did begin to exist. In this chapter, I defend a necessary, but not sufficient, condition for beginning to exist that distinguishes the two classes of entities. According to the Modal Condition, the Cosmos had a beginning only if at all of the closest possible (or counterpossible) worlds where time does not exist, the Cosmos does not exist. To articulate the Modal Condition, I begin by discussing a theological debate concerning God's relationship to time and I develop the Modal Condition using the Lewis-Stalnaker semantics for counterfactual conditionals. Although I am not myself a theist, the theological reflections contained in this chapter were useful for thinking through a novel necessary condition for the beginning of existence; for that reason, I invite naturalists to read through the theological sections of this chapter with an open mind. After developing the Modal Condition in the theological context, I turn to a discussion of the Modal Condition in philosophy of physics. One upshot of this chapter is that, despite frequent claims to the contrary, establishing that physical reality has a finite past is not sufficient for establishing that physical reality had a beginning.

5.2 The Theological Problem

5.2.1 A survey of views on God's relationship to time

As I explained in the introduction, this chapter is concerned with two problems that have a common solution: one problem in philosophy of religion and another problem in philosophy of physics. In order to explicate the problem in philosophy of religion, I need to first explicate how, assuming that God exists, God might be thought to relate to time. There are three views about how God might be related to time (Deng, 2018; Ganssle, n.d. Leftow, 2005; Padgett, 2013). First, as defended by most classical theologians, God might be absolutely timeless, in the sense that God's life does not begin or end and God is not subject to temporal succession. Proponents of the absolutely timeless God sometimes say that God inhabits a timeless present that never passes into or out of either being or God's experience. This is contrasted with temporal entities, which experience successive presents. Second, God might be temporal but everlasting (or sempiternal), in which case God's life is subject to temporal succession but extends infinitely into the past and infinitely into the future.[2] Third, there is a family of hybrid views according to which God is in some sense timeless and in some sense temporal. I will refer to theories maintaining that God is in some sense timeless and in some sense temporal as *hybrid views*.

The family of hybrid views can be further subdivided in at least two ways. First, there there is the so-called *Oxford School* (DeWeese, 2016; Mullins, 2014, 2016, 2020; Padgett, 1989, 1991, 2000, 2001a, 2010, 2013; Swinburne, 1996). According to the Oxford School, time did not begin with the Cosmos. However, the Oxford School distinguishes between two distinct kinds of time: *physical time* and *metaphysical time*. Physical time is time as described by and measured within the physical sciences. Since physical time is time as described by and measured within the physical sciences, physical time could not exist without physical entities. According to the Oxford School, absent the laws of physics, there would be no fact about the ratio in duration between two non-overlapping intervals of time, so that, without the Cosmos, there would be no fact about the duration of any given temporal interval. That is, according to the Oxford School, without the physical universe, time is *amorphous*. Later in this chapter, I will discuss the views of one member of the Oxford School – Alan Padgett – at some length. Padgett refers to physical time as *Measured Time* and refers to metaphysical time as "eternity" (Padgett, 1989, 1991, 2000, 2001a, 2010); for Padgett, metaphysical time is time as experienced by God independent of physical reality.

The Oxford School can, itself, be subdivided into two groups: first, a group I will call the *Oxford Identificationists*, who maintain that time is numerically identical with an attribute of God, and a group I will call the *Oxford Creationists*, who maintain that time is

[2]↑A history of the first two views in ancient and medieval philosophy, and their relationship to contemporary philosophy, is provided in Kukkonen, 2015.

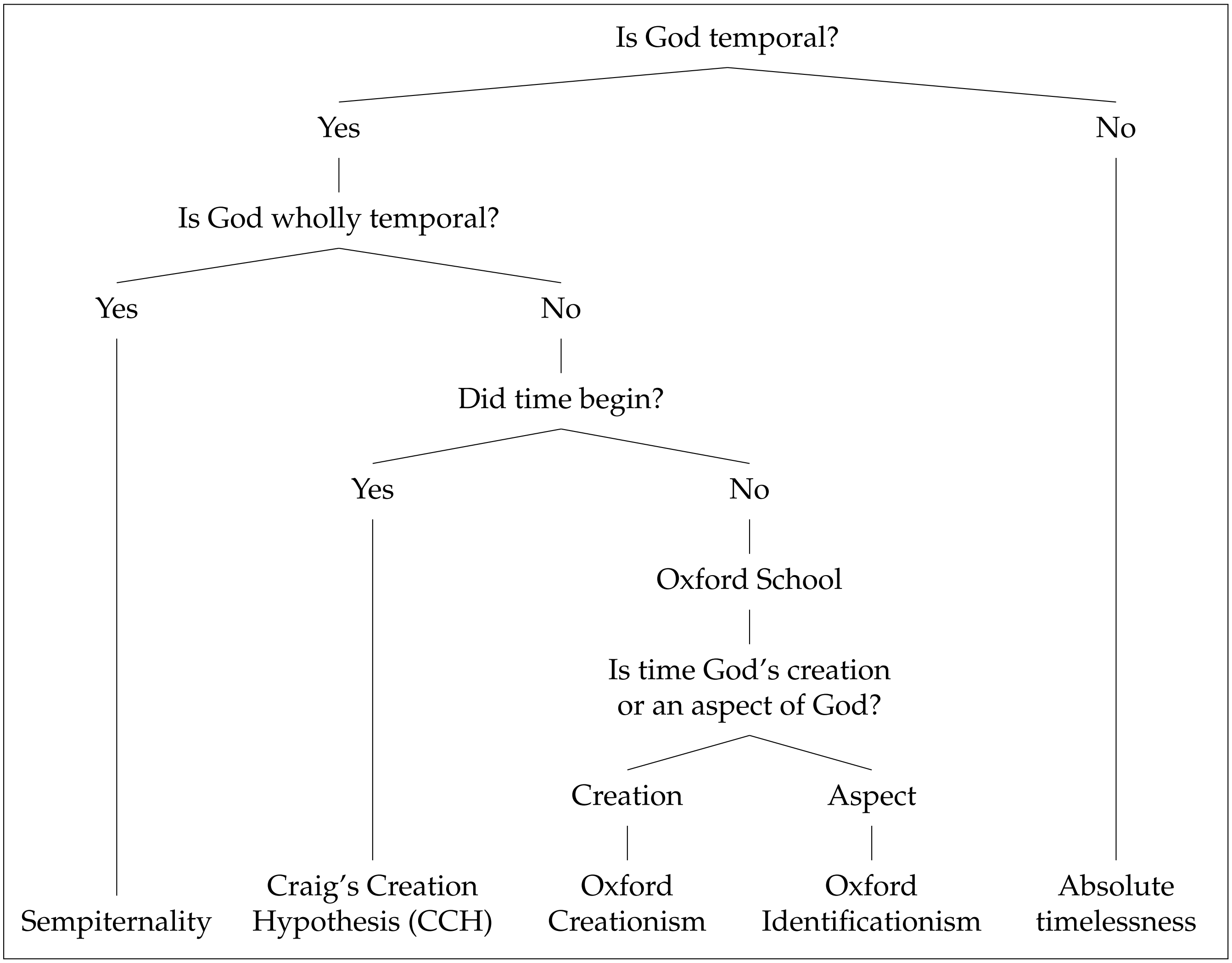

Figure 5.1. The most popular proposals in analytic theology concerning how God might be related to time.

not numerically identical with God but was created by God. Oxford Creationists argue that God transcends time because, on their view, God serves as the ground of time, God is unchanged by time, God has full control over the course of history, and God's aseity demands that God be understood as prior in the order of being to the existence of time. As Padgett describes the view, God is "relatively timeless", in that, while God is subject to change in God's non-essential characteristics, God's life is not measured by time and is not affected or contained by time (Padgett, 2000, p. 126).

Recall that I said there were two versions of the hybrid view. So far, we've discussed one version of the hybrid view – the Oxford School – as well as two subgroups within the Oxford School – i.e., the Oxford Identificationists and the Oxford Creationists. The second version of the hybrid view is a perspective championed by William Lane Craig according to which God is timeless sans Creation and temporal with Creation (see, for example, Craig, 2001b, pp. 270–275, Erasmus, 2021, and chapter 6 in Loke, 2017).[3] In this chapter, I adopt Jacobus Erasmus's name for that perspective, i.e., *Craig's Creation Hypothesis* or CCH (Erasmus, 2021, p. 197). Unlike the Oxford School, CCH involves the claim that time did begin with Creation. But, like the Oxford Creationists, CCH proponents affirm that God is prior in the order of being to time, that God transcends time, and that God is causally responsible for time. Importantly, according to CCH proponents, God somehow became temporal in virtue of having created time. As CCH proponents ordinarily explicate their view, the actual world includes a state of affairs in which God, alone, exists and, in that state of affairs, God is timeless. On the view of time endorsed by CCH proponents, change suffices for the existence of time. In the timeless state of affairs, God initiated the first change and, in doing so, brought time into being. The timeless state of affairs, qua timeless, cannot temporally precede the Cosmos; nonetheless, according to CCH

[3]↑Another hybrid view has sometimes been suggested that draws on the distinction Gregory Palamas drew between the divine essence (or nature) and the divine energies. A Palamite theologian might say that while the divine essence (or nature) is timeless, the divine energies are temporal. See, for example, Dumsday, 2021, p. 37. I will set this view aside for the purposes of this chapter, in part because the resulting hybrid view has not – as far as I have been able to find – been well developed in the analytic theology literature and in part because I am not sufficiently familiar with the view to competently comment on it. Readers who think that the Palamite view resolves the theological problems that I raise better than the views that I consider can interpret this chapter as articulating the destination of theological views alternative to their own.

proponents, the timeless state of affairs *causally* preceded both time and the Cosmos. Moreover, by initiating the first change, God initiated the beginning of time. One of my goals in this chapter is to offer a better articulation of CCH than has previously been offered; to do so, I will, in some places, make use of arguments presented by the Oxford School and particularly by Oxford Creationists.

I will ultimately argue that the version of CCH previously offered is incoherent. In particular, as I will argue, the view that the actual world contains a state of affairs in which God is timeless as well as a state of affairs in which God is temporal is problematic. However, my aims are not completely destructive; I want to offer CCH proponents an alternative version of CCH that I think is coherent. To that end, I will offer an alternative version of CCH that does not include the thesis that the actual world includes a state of affairs in which God is timeless. For my purposes, I will consider any view to be a version of CCH if, according to that view, (i) God is atemporal sans Creation and temporal with Creation and (ii) God is prior in the order of being to time, that God transcends time, and that God is causally responsible for time.

5.2.2 Theological accounts of the beginning of the Cosmos

Having surveyed the various ways that God has been proposed to relate to time, I turn next to how CCH proponents have thought about the notion that the Cosmos had a beginning. The Oxford School and CCH proponents differ in a variety of ways. For example, Oxford School proponents say that a duration of beginningless, amorphous time temporally preceded God's creation of the Cosmos whereas CCH proponents say that a state of affairs in which God, alone, exists and exists timelessly causally, but not temporally, precedes the Cosmos. Nonetheless, both the Oxford School and CCH proponents agree on three theses: (i) God is actually temporal, (ii) time is wholly explicable in terms of God, and (iii) while God did not begin to exist, the Cosmos did begin to exist. While the Oxford School and CCH proponents do disagree about why time is wholly explicable in terms of God,[4] let's put that difference to one side. I am interested in how the Oxford School

[4]↑The Oxford School is committed to the view that time is wholly explicable in terms of God either because time is an aspect of God (the Oxford Identificationists) or because God created time (the Oxford Creationists),

and CCH proponents might explicate the notion that while God did not begin to exist, the Cosmos did begin to exist. One is tempted to say that:

> Beginning-to-exist-1 $:=_{def}$ x began to exist just in case x is temporal and there was some finite period of time such that there were no previous finitely long periods of time during which x existed.

If so, then:

1. The Cosmos began to exist just in case the Cosmos is temporal and there was a finitely long period of time T such that the Cosmos did not exist before T and

2. If God is actually temporal, then, since God did not begin to exist, there is no initial finitely long period of time in God's life.

However, this account is incompatible with CCH. CCH proponents are committed to the claims that:

3.God is actually, but not necessarily, temporal,

4.There was a first finitely long period of time, and

5.God did not begin to exist.

If there is a first finitely long period of time and God is temporal, then, contrary to 2, there must have been a first finitely long period of time in God's life. Therefore, one may argue that 3-5 are collectively inconsistent with Beginning-to-exist-1.[5] The Oxford School avoids this problem because the Oxford School rejects 4; for the Oxford School, the Cosmos was

whereas, for CCH proponents, God initiated the first change and the existence of change suffices for the existence of time.

[5]↑When this chapter was submitted as an article for publication with *Erkenntnis*, an anonymous reviewer asked whether this problem can be resolved by compartmentalizing the first finitely long period of time in God's life to God's temporal life. Note that the problem under discussion concerns whether having a first finitely long period of time in the life of x suffices for showing that x began to exist; if the reviewer is correct that God did not begin to exist because we can compartmentalize the first finitely long period of time in God's life to God's temporal life, then that x has a first finitely long period of time in its life does not suffice for showing that x began to exist. That is, if the reviewer's suggestion is correct, then beginning-to-exist-1 is incorrect.

preceded by amorphous time, so that time lacks a first finitely long period. Thus, one tempting way to resolve this difficulty would be to simply affirm the Oxford School – or perhaps some other view of God – and reject CCH as incoherent. Let's forego the option of rejecting CCH in order to further investigate CCH.

To reiterate the incompatibility between CCH and Beginning-to-exist-1, suppose that Beginning-to-exist-1 is true. In that case, if God entered time in virtue of God's creation of time, as CCH proponents allege, then God's life includes a first finitely long period of time. If God's life did include a first finitely long period of time, then Beginning-to-exist-1 entails that God began to exist. CCH proponents want to avoid the conclusion that God began to exist; therefore, they need to identify a plausible alternative to Beginning-to-exist-1. Here is one alternative Craig has considered:

> Beginning-to-exist-2 $:=_{def}$ x begins to exist at t just in case "x exists at t; there is no time immediately *prior* to t at which x exists; and the actual world contains no state of affairs involving x's timeless existence" (as quoted in Morriston, 2000, p. 155).

Beginning-to-exist-2 does not seem to be adequate for Craig's purposes and Craig has since abandoned it (Craig, 2002).[6] Though Craig has abandoned Beginning-to-exist-2, Christopher Bobier's arguments against Beginning-to-exist-2 are instructive for articulating an adequate notion of beginning to exist.

Beginning-to-exist-2 consists of three conditions. Let's focus on the third condition, that is, that there is no actual state of affairs involving x's timeless existence. According to Bobier, the notion that there is no actual state of affairs involving x's timeless existence can be analyzed two ways. On the first analysis, the notion that there is no actual state of affairs involving x's timeless existence means that "[t]he actual world contains no *possible* state of affairs involving x's timeless existence" (emphasis is Bobier's; see his 2013, p. 597). Bobier argues that Craig cannot mean that x began to exist only if the actual world contains no possible state of affairs involving x's timeless existence. Bobier thinks that a timeless basketball is metaphysically possible. If a timeless basketball is metaphysically possible, then there is a possible state of affairs involving a basketball's timeless existence.

[6]↑Bobier (2013) argues persuasively that Craig's latest criteria will not work either.

So, the first option would entail that basketballs do not begin to exist and surely Craig does not think that basketballs are beginningless. I do not agree with Bobier that timeless basketballs are possible, though I grant Bobier's point; the *mere* possibility that *x* timelessly exists does not entail that *x* did not actually begin.

On the second analysis, the notion that there is no actual state of affairs involving *x*'s timeless existence means that a state of affairs involving *x*'s timeless existence does not obtain in the actual world Bobier, 2013, p. 597. This analysis will not fit Craig's purposes either. As I've discussed, on Craig's view, God did not begin to exist. Suppose that beginning-to-exist-2 did provide the correct analysis of beginning to exist. On Craig's view, God satisfies the first two conditions in Beginning-to-exist-2. That is, since Craig endorses a first moment (or interval) of time *t*, God exists at *t* and there is no time prior to *t* at which God exists. Thus, in order for God to be beginningless, God must violate the third condition, that is, there must obtain a state of affairs in the actual world in which God exists timelessly. Bobier argues that there cannot be such a state of affairs. As Bobier argues, no state of affairs obtains in which God exists timelessly prior to Creation because, according to Craig, time began with Creation and there are no states of affairs *temporally prior* to Creation. Moreover, no state of affairs obtains in which God exists timelessly after Creation because, on CCH, God is in time after Creation. Therefore, according to Bobier, the second option entails that there are no actual states of affairs involving God existing timelessly. If so, then, on the conception of beginning to exist we are considering, God began to exist.

One might object that Bobier has moved too quickly in concluding that no state of affairs obtains in which God exists timelessly. While Bobier has argued that no state of affairs obtains in which God exists timelessly before, simultaneous with, or after Creation, one might argue that if a state of affairs in which God exists timelessly did obtain, then, in virtue of being timeless, that state of affairs cannot be before, simultaneous with, or after Creation. Why couldn't a state of affairs obtain in the actual world that simply did not enter into before, after, or simultaneous-with relations? Thus, instead of showing that such a state of affairs does not obtain, perhaps Bobier has merely drawn out an implication of such a state of affairs. In the next section, I elaborate on why we should not commit ourselves to the view that the actual world includes both a state of affairs in which God is timeless sans Creation and a state of affairs in which God is temporal with Creation.

Does God's Life Have Two Portions?

I am addressing the notion that there obtain two states of affairs in the actual world: one state of affairs in which, sans Creation, God exists timelessly and another state of affairs in which, with Creation, God exists temporally. The question becomes in virtue of what the two states of affairs hang together in such a way that both states of affairs include numerically one deity. One could propose that the two states of affairs are two portions of God's life, that is, the portion of God's life in which God is timeless and the portion of God's life in which God is in time.[7] As I argue in this section, I have difficulty seeing how God's life could include both portions; without an adequate conception of how the two states of affairs could hang together, an alternative version of CCH – one that involves only the state of affairs in which God is in time – is preferable. Subsequently, I develop that alternative version of CCH and show the Modal Condition can be utilized in defense of that alternative.

Supposing that God's life includes both temporal and non-temporal portions, we should not say that the atemporal portion of God's life precedes the temporal portion since the atemporal portion cannot enter into temporal relations such as *before* or *after* (Craig, 2001b, pp. 267–268, Helm, 2001a, p. 49, Leftow, 2009, pp. 290–291). Friends of CCH, such as Craig, Erasmus, and Loke, have themselves argued that the atemporal portion of God's life is not before the temporal portion. On an A-theory of time, when one says that an event is past, one means just that the event has already passed. So, if the atemporal portion of God's life has passed away when God became temporal, then we

[7] ↑Some theologians will object that, given the doctrine of divine simplicity, God's life cannot be divided into portions. Craig, and other friends of CCH, reject the doctrine of divine simplicity. Moreover, since friends of CCH think that there is a state of affairs in which God is in time, and that God is subject to temporal succession, friends of CCH are already committed to the view that God's temporal life can be divided into successive moments. But to say that God's temporal life can be divided into successive moments is just to say that God's temporal life can be divided into portions. If God's temporal life can be divided into portions, then I have difficulty seeing why friends of CCH wouldn't simply say that the two states of affairs are portions of God's life simpliciter.

would have the logically impossible conclusion that the atemporal portion of God's life is past.[8] Thus, if there is an atemporal portion of God's life, then, however that portion may be related to the temporal portion of God's life, the atemporal portion, qua atemporal, cannot pass away. (Similar remarks were made in Kabay, 2009, p. 128 and Helm, 2001b, p. 163.) So, instead, the suggestion might be that the portion of God's life that is in time is present while the timeless portion is not present but, nonetheless, exists *simpliciter*.

This interpretation faces apparently insurmountable problems. For example, the identity conditions between the two portions of God's life are utterly mysterious. God cannot perdure or endure – let alone retain psychological continuity or maintain God's personal identity in some other way – between the two portions of God's life because one portion is not in time. One might instead suggest that there is a kind of continuity between the two portions of God's life because the atemporal portion timelessly causes the temporal portion. Setting aside difficult philosophical issues about whether an atemporal entity can cause a temporal entity, a mere causal relation does not suffice for establishing continuity between the two portions of a life. Without perduring or enduring, I have difficulty seeing how the two portions could be understood as two portions of the life of numerically one entity as opposed to the lives of two deities.

Craig and other friends of CCH are monotheists and so will want to avoid the conclusion that there is more than one deity. However, at the level of logical consistency, there is no tension that I can see between polytheism and CCH. Happily, there is a second difficulty for the view that God's life includes both temporal and non-temporal portions. To reiterate, we have been considering a view according to which God did not begin to

[8]↑According to one popular argument for the view that God is timeless, there is a tragedy in our own temporal existence because, for those of us in time, parts of life fall away from us and can never be recovered. We might look back on our loved ones who are no longer with us, but, so long as we are limited to the present life, we cannot experience, once more, the loved ones who are no longer with us. Proponents of the timeless God point out that God, as a perfect being, must not experience the tragedy of time passing and so no part of God's life falls away from God's experience. This implies that no part of God's life has passed away and that no part of God's life is before any other part, so that God's life is not subject to $A-$ or $B-$relations (or so the argument goes). If God is not subject to $A-$ or $B-$relations, then God is timeless. When Craig (2001a, pp. 132–136) replies to this argument on behalf of the view that God has a temporal portion of God's life, Craig does not object to the notion that, for a timeless God, no part of God's life passes away. This seems to be an implicit admission that timeless entities cannot pass away so that the timeless portion of God's life, qua timelessness, could not pass away. Elsewhere, Craig (2001a, p. 159) explicitly tells us that for the atemporal portion of God's life, there is no before or after and time does not pass.

exist because a state of affairs in which God timelessly exists obtains and a state of affairs in which God is temporal obtains as well as the possibility that the atemporal portion of God's life timelessly causes the temporal portion of God's life. The portion of God's life that is in time is in time essentially; the temporal portion of God's life, qua temporal, cannot exist in any possible world from which time is absent. Beginning-to-exist-2 entails that x began to exist only if there obtains no state of affairs in which x timelessly exists. Therefore, since there couldn't be a state of affairs in which the temporal portion of God's life timelessly exists, even if God can be said not to have a beginning, the portion of God's life that is in time would have a beginning. Craig is committed to the principle that anything that begins to exist requires a cause for its existence (S. Carroll and Craig, 2016; Craig, 1979; Craig and Sinclair, 2009, 2012; Craig and Smith, 1995). If anything that begins to exist does require a cause for its existence, then the portion of God's life that is in time requires a cause for its existence. The only plausible candidate for the cause of the temporal portion of God's life is the atemporal portion of God's life. Craig has argued that any cause of a temporal entity must itself be temporal and that God is temporally related to – in fact, simultaneous with – the Cosmos when God causes the Cosmos to begin Craig, 2001b, p. 276. Thus, the cause of the temporal portion of God's life must likewise be temporally related to – in fact, simultaneous with – the beginning of the temporal portion of God's life. Nonetheless, the timeless portion of God's life cannot be temporally related to, let alone simultaneous with, anything, so that the timeless portion of God's life cannot be the cause of the temporal portion of God's life.[9] Consequently, unless we give up CCH, Beginning-to-exist-2 fails and we need a different analysis for beginning to exist.

There is a third difficulty for proponents of CCH who maintain that God's life includes both a temporal and an atemporal phase. Consider one argument that both the Oxford School (e.g., Mullins, 2016; Padgett, 2000) and CCH proponents (e.g., Craig, 1998) have offered against the view that God is absolutely timeless. Some proponents of divine timelessness have argued that if the *A*-theory of time is true, then, even though God cannot undergo intrinsic change in virtue of being timeless, God does undergo changes in God's extrinsic relations (i.e., Cambridge changes) in virtue of God's relationship to

[9]↑Similar points were previously made in Mullins, 2020, p. 225 and Helm, 2011, p. 222.

a changing temporal reality. To the contrary, friends of the Oxford School and of CCH have argued that the A-theory of time is incompatible with the existence of a timeless entity that is either extrinsically or intrinsically related to temporal entities. For example, suppose that God exists, God was the Creator of some temporal entity E, and that the A-theory of time is true. In that case, even if God does not undergo any changes in God's intrinsic characteristics, as time passes and E ages, God undergoes Cambridge changes with respect to E. As Craig (1998, pp. 222–223; 2001a, pp. 140–141) puts the point, when God created the Cosmos, God was not timeless in virtue of the fact that God acquired a new characteristic. But, on Craig's view, any entity that acquires a new characteristic – even if that new characteristic solely involves entering into a new extrinsic relation – is temporal.[10] Therefore, even if God is immutable in God's intrinsic characteristics, Craig concludes that God is subject to temporal passage. Notice that a parallel argument can be provided for the atemporal portion of God's life. If the atemporal portion of God's life is either intrinsically or extrinsically related to the temporal portion of God's life – as is presumably required for the two phases to be portions of numerically one life – and the $A-$theory of time is true, then the timeless portion would acquire a new extrinsic relation when the temporal portion begins to exist. In that case, the timeless portion would not actually be timeless.

Erasmus (2021) and Craig (2001b, pp. 272–273) have each attempted to explain how the atemporal portion of God's life might be related to the temporal portion of God's life. Erasmus draws upon a distinction between an instant and an event. As Erasmus describes the distinction, an *instant* is an indivisible temporal point while an *event* is a change from one instant to another. On a discrete view of time, time can be understood as a series of instants, i.e., $t_1, t_2, t_3, ..., t_n$, and as a series of events, i.e., e$(1_1, t_2)$, e(t_2, t_3), ..., e(t_{n-1}, t_n), where e(t_i, t_{i+1}) is the event of changing from instant t_i to t_{i+1}. Erasmus then asks us to consider

[10]↑For example, suppose that God bears an extrinsic relation R to Adam-at-time-t_1 and bears extrinsic relation $\neg R$ to Adam-at-time-t_2. Let's also suppose that Craig's preferred version of $A-$theory, presentism, is true so that only the present moment exists. When t_1 is present, God bears extrinsic relation R to Adam-at-time-t_1 but, since t_2 does not yet exist when t_1 is present, God does not yet bear $\neg R$ to Adam-at-time-t_2. Subsequently, t_1 passes out of existence and t_2 passes into existence. Since God bears $\neg R$ to Adam-at-time-t_2, we know that God must take on the extrinsic relation $\neg R$ to Adam by t_2 and that God must no longer bear R to Adam. But that's just to say that there is succession in God's life and so that God is temporal.

God at t_1. On a relational conception of time, there is time only if there is change, that is, transition from one instant to another. Therefore, the state of affairs, involving God, at t_1 is the same state of affairs, involving God, as there is at the closest possible world without time. For Erasmus, the distinction between the closest possible world without time and the actual world consists just in the fact that God actualizes the change from t_1 to t_2, that is, $\mathrm{e}(t_1, t_2)$. Since the state of affairs, involving God, at t_1 is the same state of affairs as there is at the closest possible world without time, Erasmus understands the state of affairs, involving God, at t_1 as a timeless state of affairs.

Erasmus's response does not adequately address the objection that I have raised. On Erasmus's view, t_1 is *before* t_2 and *passes into* t_2. Therefore, t_1 is temporally related to t_2. If the state of affairs, including God, at t_1 were a timeless state of affairs, then that state of affairs, in virtue of being timeless, could not pass away or into t_2 and could not occur before t_2. Furthermore, I doubt that all friends of the CCH can take up Erasmus's response; for example, Craig has denied both that instants exist and that time is discrete.[11]

Although Erasmus intends for his discussion to be a loose summary of Craig's response, Craig's response is distinct from the response that Erasmus has described. In fact, while Craig agrees with Erasmus that, in the closest possible world without time, the state of affairs involving God at t_1 would have obtained, Craig denies that t_1 obtains in such a world (Craig, 2001b, p. 272). Craig's response draws upon two analogies with physical cosmology. In one analogy, Craig (2001b, p. 272, 2001a, p. 160) compares God's relationship to time to relativistic cosmological models featuring an initial singularity. As Craig rightly points out, according to General Relativity, the initial singularity is not a part of the space-time manifold but should instead be understood as an open boundary to the

[11] ↑The reader might be perplexed that Craig denies the existence of instants, given Craig's presentism, but Craig has long argued that instants do not exist. Craig denies that any physical collection could be infinite while also denying the view that time is discrete. If time is continuous, one might have thought that any finitely long interval of time includes an infinitude of instants. In order to avoid the consequence that any interval of time includes an infinitude of instants, Craig adopts the Aristotelian position that intervals of time are fundamental and instants are a kind of mental fiction we arrive at as the boundary points of any given interval. Craig writes that "only intervals of time are real or present and that the present interval (of arbitrarily designated length) may be such that there is no such time as 'the present' *simpliciter*; it is always 'the present hour', 'the present second', etc. The process of division is potentially infinite and never arrives at instants" (Craig, 1993a, p. 260; also see Craig, 2000, pp. 179–180; Craig and Sinclair, 2009, pp. 112–113). For discussion, see Dumsday, 2016; Loke, 2016; Puryear, 2014, 2016; Zarepour, 2021.

space-time manifold. Since the open boundary is not part of the space-time manifold, the boundary cannot be said to temporally precede any of the space-time points within the manifold. Craig claims that while the singularity is not temporally prior to space-time, the singularity is causally prior to space-time.

However, this cannot be a good analogy because the reason that the open boundary does not temporally precede any space-time point is that the open boundary does not exist, that is, the open boundary is an absence. Presumably, Craig does not want to commit himself to the view that God lacks being in any portion of God's life, regardless of whether that portion is temporal or atemporal. Moreover, it's at least not obvious to me that the singularity causally precedes space-time. While the nature of causation is philosophically controversial, a variety of theories of causation deny that absences can be causes; if an absence cannot be a cause, then, since an open boundary is an absence, an open boundary cannot be a cause either. Even if we should accept an analysis of causation on which absences can be causes, Craig and other friends of the CCH would be unlikely to accept the view that the Cosmos could have been caused by sheer nothingness; thus, while they might admit absences as causes, they would not admit an absence as the cause of the Cosmos.

In a second analogy, Craig (2001b, pp. 272–273) compares God's relationship to time to the Hartle-Hawking model (1983). As Quentin Smith (e.g., 1997) interprets that model, the initial singularity is replaced by a region featuring "imaginary time". Within that region, the space-time metric has Euclidean signature, with the consequence that there is no metrical distinction between space and time. On Smith's interpretation, that region features four dimensions of space instead of featuring one dimension of time and three dimensions of space. Smith argues that the timeless four-space region is topologically, but not temporally, connected to space-time. Craig (2001b, p. 273) speculates that perhaps the atemporal portion of God's life is (somehow) topologically, but not temporally, connected to the temporal portion of God's life. I'm not convinced that Smith correctly interpreted the Hartle-Hawking model,[12] but set that aside. If there is an atemporal portion of God's

[12]↑For example, Smith's interpretation involves the view that what distinguishes space from time is the distinction between Lorentzian and Euclidean signature. While the signature does provide *a* distinction

life that is (somehow) topologically but not temporally related to the temporal portion of God's life, then, once more, that atemporal portion can neither pass away nor into nor be placed before the temporal portion of God's life. Moreover, unless Craig can provide adequate reason to think that the topological joint between the two portions of God's life can support perdurance or endurance between the two portions of God's life, much less psychological continuity or other ways in which personal identity persists, I do not see how the topological joint suffices for showing the portions are the life of numerically one deity. Furthermore, the supposition that there is a topological joint between the two portions of God's life would not suffice for showing that the timeless portion could be related to the temporal portion without the timeless portion undergoing extrinsic change.

Loke (2017, p. 172) defends the coherency of the view that there is a causally prior timeless portion of God's life in a different way than either Erasmus or Craig. Recall that, according to the way in which CCH proponents have previously described their view, the actual world includes a state of affairs in which God exists alone, exists timelessly, and, in that timeless state of affairs, begins time by initiating the first change. CCH proponents often argue that only an entity with libertarian freedom could have the power to initiate the first change from a timeless state. According to the objection that Loke considers, an entity E, with libertarian freedom, cannot freely initiate change from a timeless state. According to Loke's imagined objector, for some entity E to change is just for E to have property p at some time t_1 and property $\neg p$ at some time t_2, such that $t_1 \neq t_2$. If E changes from a timeless state, then E did not change from one time to another. Loke replies that friends of the CCH can provide a disjunctive definition of change: for some entity E to change is just for E to have property p at some time t_1 and property $\neg p$ at some time t_2, such that $t_1 \neq t_2$, *or* for E to have property p in a timeless state and property $\neg p$ at some time t. Loke's reply does not appear to be adequate for defending the coherency of changing from a timeless state. If E is in a timeless state, then E cannot pass from that

between space and time, the signature is an implausible candidate for providing a complete explanation of the distinction between space from time for at least two reasons: (i) the signature cannot explain any sort of past/future asymmetry and so cannot explain *A*- or *B*-relations and (ii) we can construct (anachronistically) a model of Newtonian or Galilean space-time that include a space/time distinction while also featuring a metric with Euclidean signature.

timeless state and into a temporal state since a timeless state cannot, qua timeless, *pass away*. For that reason, Loke is incorrect when he writes, "there is nothing absurd about a personal timeless being deciding to leave His state of timelessness and enter into time" (2017, p. 175). Moreover, Loke has not provided a way for an entity to perdure, endure, or to persist in personal identity from a timeless state to a temporal state.[13]

The preceding problems evaporate if we suppose that God does not timelessly coexist with the temporal portion of God's life in possible worlds where God is temporal. On the condition for 'beginning to exist' that I propose in this section, in the actual world, God could be beginningless and yet only have a temporal portion of God's life. That is, on my proposal, an entity can have a finite past and yet, even though the actual world includes no atemporal portion of that entity's life, the entity may still be beginningless. Thus, even though God's life may include a first period of time, God could still be said not to have begun to exist. Like Craig, Padgett (2001a, p. 106) denies the view that if God is temporal, God could exist only if time exists. According to Padgett (2000, pp. 122–123), God could "live" in a timeless world and has freely and timelessly chosen to live in a temporal world. Since God timelessly chooses for our world to be one that includes time, there is no time at which God makes our world a temporal world and consequently no transition in God's life from an atemporal phase to a temporal phase. On Padgett's view, there is only one phase of God's life. Despite having only one phase in God's life, God includes atemporal aspects alongside temporal aspects, and the atemporal aspects of God are responsible for the existence of time.

One of the objections previously considered to the view that God's life includes both an atemporal portion and a temporal portion was that if *A*-theory is true, then, once the temporal portion begins, the atemporal portion acquires a new relation. This led to the

[13]↑Loke (2017, pp. 172–173) goes on to consider whether the First Cause of the Cosmos could be a physical state and argues that the First Cause must be able to prevent itself from "initially changing". According to Loke, only a timeless person with libertarian freedom, and not a timeless physical state, could prevent itself from initially changing and therefore could not be a physical state. Set aside the fact that a timeless entity should not be described in temporal terms, e.g., as *initially* anything. The real trouble seems to be opposite to the problem that Loke discusses. As a matter of logical consistency, a timeless entity cannot literally *become* anything else and therefore lacks the capacity to change *from* one state *into* some other. Consequently, a timeless physical state, qua timeless, would have no more difficulty "preventing" itself from coming to occupy some non-initial state than would a timeless person with libertarian freedom.

contradiction that the atemporal portion is both atemporal and temporal. The reader might worry that a similar objection can be provided for the view that God includes both atemporal and temporal aspects. If the atemporal aspect is related to the temporal aspect and we suppose that *A*-theory is true, why wouldn't the atemporal aspect acquire new relations as the temporal aspect changes?[14] In reply, the CCH proponent could say that God includes an atemporal aspect just in case there is an aspect of God that suffices for God's existence and that would have existed even if time did not. (As we will see, this is just to say that the CCH proponent could adopt the Modal Condition.) In that case, all aspects of God are undergoing relational changes throughout the entirety of God's life – the entirety of which is temporal – even though some of those aspects – importantly, aspects that suffice for God's existence – would have existed even if time had not existed.

For proponents of the CCH and unlike the Oxford School, past time is finite, so that the life of any temporal entity includes an initial finitely long period. In that case, there is an initial finitely long period of God's life. If God's life only includes the temporal phase, how could God's life be beginningless? Let's turn back to Bobier. Bobier comes close to suggesting the correct solution when he recognizes that what we require is a "modal fact". According to CCH, in the actual world, 'God is timeless sans Creation' is true. Bobier wonders what fact in our world could make 'God is timeless sans Creation' true. One candidate answer is a modal fact, that is, that had God not created the Cosmos, God would have existed timelessly (Bobier, 2013, p. 598).

Padgett similarly offers a modal analysis as part of his study of God's relationship to time. Consider how Padgett argues for his view that while God is in time, God is not necessarily in time. Padgett considers a possible world from which time is absent, but in which God is the Creator of all things other than Godself. As the Creator of all things other than Godself, all things other than God in the timeless world ontologically depend upon God. Padgett grants that such a world is logically possible and, since Padgett believes

[14]↑Padgett has made a similar criticism of Whitehead's "dipolar" conception of God, wherein God is conceived as having an absolutely timeless aspect (which Whitehead identifies as God's "primordial nature") and a temporal but everlasting aspect (which Whitehead identifies as God's "consequent nature"). Padgett (2000, p. 140) states, "It is hard to see how one 'actual entity' can exist in two antithetical modes of being, without destroying the unity of that entity. Since timelessness as Whitehead and most thinkers have understood it is the antithesis of time, no one being can be both timeless (in this sense) and temporal".

God can do any logically possible task, Padgett concludes that God could have actualized the timeless world but freely chose to actualize a temporal world instead (Padgett, 2001a, p. 106; Padgett, 2001b, pp. 106–107).

Padgett (2001a, p. 106) goes on to say that we have two possibilities for relating God to time, i.e., that either "God's time is a *necessary precondition* to God's Being" or "that God's Being is a necessary precondition to God's time (eternity)". Padgett (2001a, p. 107) rejects the possibility that time is a necessary precondition to God's Being. When Padgett proceeds to tells us that "God is not contained within time", Padgett clearly does not mean that God is atemporal. As I've discussed, Padgett is an Oxford Creationist and so agrees with Craig that God is temporal. Instead, Padgett means that God's being is prior in the order of ontological dependence to the existence of time, so that the existence of time should be understood in terms of God's existence and not vice versa. Craig (2001b, pp. 271–272; 2001a, p. 138) similarly offers a thought experiment that he uses to affirms that, had God not initiated time, our world, including God, would have been timeless. Craig and Padgett agree that God is prior in the order of being to the existence of time; on their view, that God is prior to time explains why, even if time began and God is temporal, God lacks a beginning. In light of Bobier's, Padgett's, and Craig's comments, I propose that the relation of ontological priority between God and time can be understood in terms of a modal fact. I turn to characterizing that modal fact in the next section.

5.2.3 Theology and the Modal Condition

What modal fact would be adequate for Padgett's or Craig's views? Let T = 'time exists'. Using the standard Lewis-Stalnaker semantics for counterfactual conditionals,[15] let $\Box\!\!\rightarrow$ represent the would-counterfactual conditional. That is, if, in all of the closest possible worlds where A is true, B is also true, then $A \Box\!\!\rightarrow B$. Moreover, let $\Diamond\!\!\rightarrow$ represent the might-counterfactual conditional. That is, if, in at least one of the closest possible worlds where A is true, B is also true, then $A \Diamond\!\!\rightarrow B$. On Craig's or Padgett's accounts, time only exists in virtue of God's contingent and freely-willed act of creation, that is, time

[15]↑Nothing crucial hangs on the Lewis-Stalnaker semantics. Therefore, the reader can, if they would like, substitute their favorite theory of counterfactual conditionals.

is asymmetrically explained by God. Assuming that God necessarily exists, as endorsed by most Christian philosophers and theologians, God exists at all of the nearest possible worlds without time.[16] Without time, God would have existed anyway. Consequently, we have that $\neg T \mathrel{\Box\!\!\to} \exists x . x = God$. Using the modal condition, we can articulate an argument for the CCH proponent's view that, even though God's life may have included an initial finitely long segment, God is nonetheless beginningless:

P1) If any entity is non-temporal, then that entity did not begin to exist.

P2) God is fundamentally non-temporal.

C1) So, God fundamentally did not begin to exist.

P3) Any entity that fundamentally did not begin to exist did not begin to exist simpliciter.

C2) Therefore, God did not begin to exist simpliciter.

(P1) is true because any entity that is timeless is beginning. (P2) is true because God is metaphysically prior to the existence of time and, for that reason, satisfies the Modal Condition. That is, there is an aspect of God that suffices for God's existence and which would have existed even if time had not. (C1) follows from (P1) and (P2) by modus ponens. (P3) is true because for any entity *E*, if there is an an aspect of *E* that suffices for the existence of *E* but which did not begin to exist, then *E* did not begin to exist. Lastly, (C2) follows from (C1) and (P3) by universal instantiation. Notice that this argument is independent of whether God's life includes an initial finitely long segment and so establishes the CCH proponent's view that God is beginningless even if God's life includes an initial finitely long segment.

Brian Leftow (2005, p. 58) comes close to articulating the Modal Condition in a discussion of Boethius's conception of divine eternity. According to Leftow, "For all *t*, a

[16]↑Padgett (2000, p. 123) agrees that God necessarily exists, but argues that God freely chose to create the Cosmos. According to Padgett (2000, p. 122), Duns Scotus showed that a timeless world is metaphysically possible and that God could have "lived" in such a world. For that reason, even though God necessarily exists, "the actual world could have been timeless". There was no time prior to God's free choice to create a temporal world and so God eternally and contingently wills that our world be temporal. For that reason, Padgett (2000, p. 123) writes, "God's choice [...] to live a certain kind of life – to be dynamic, active, changing – is the ground of the temporality of the universe".

proposition is already true at *t* just in case it is true at *t* and would have been true had time never reached *t*". As Leftow explains, a proposition can then be said to already be true at the first moment of time just in case that proposition would have been true had time not existed. For that reason, at the first moment of time, we can say that God already exists because God would have existed even if time had not. And since, at every time, we should say that God already exists, we should say that God did not begin to exist. Boethius (of course) differs from either proponents of the Oxford School or of the CCH in that, for Boethius, God is not temporal. Nonetheless, if God includes both temporal and atemporal aspects, then, supposing that God's atemporal aspects suffice for God's existence, the Modal Condition arrives at more or less the same analysis of the claim that God did not begin to exist as Leftow's Boethius.[17]

Recall that Erasmus's and Craig's proposals for relating the atemporal portion of God's life to the temporal portion of God's life involved the notion that the atemporal portion is (somehow) a boundary to the temporal portion. There is another important reason that the CCH proponent should not describe the atemporal portion as a boundary. According to CCH proponents, God created the Cosmos. If the life of the Cosmos included a finite initial period of time, then that finite initial period, itself, has a boundary. If the Cosmos has a past boundary, why shouldn't we conclude that the Cosmos, like the CCH proponent's God, has an atemporal portion of the Cosmos's life and was therefore beginningless? Consider, again, Erasmus's construction. We can imagine a sequence of instants t_1, t_2, ..., t_n comprising the history of the Cosmos. If the state of affairs involving the Cosmos at t_1 had never changed to the state of affairs involving the Cosmos at t_2, then, on a relational theory of time, the Cosmos would have been atemporal. Thus, through reasoning parallel to that which Erasmus provides in the case of God, we should conclude that the Cosmos's initial state of affairs was a timeless state of affairs. Consequently, if Erasmus's argument had been successful, we should say that the Cosmos is beginningless.

[17]↑Likewise, Gregory Ganssle (2001, p. 11) writes, "Now I have to admit that it is strange to say that God *was* timeless. It sounds as if I am claiming that there was a point in time at which he was timeless. What I mean to stress here is it is possible for God to exist without time. If past time is finite, and if God brought time into being, he is independent of time in this way".

Likewise, suppose Craig's analogy between God and singular relativistic space-times was successful. Craig has elsewhere taken Big Bang cosmology to show that the Cosmos had a beginning. But if the singular boundary is an atemporal portion of the Cosmos's life – as Craig's analogy seems to suggest – then the Cosmos was beginningless. (Similar points were previously made in Mullins, 2020, p. 226 and Kabay, 2009, p. 121.) Moreover, consider that having a temporal boundary is likely to itself be a necessary condition for beginning to exist. Therefore, the claim that either God's life or the Cosmos did not begin to exist *because* God's life or the Cosmos has a temporal boundary should strike us as intuitively absurd and implausible. I think there is a clear reason that CCH proponents say that God was beginningless and that the Cosmos had a beginning. Importantly, according to CCH proponents, while God is prior to time in the order of being, CCH proponents deny that the Cosmos is prior to time in the order of being. On their view, God necessarily exists, so that God would have existed even if time did not, whereas the Cosmos does not exist at the closest possible worlds without time.[18] In other words, CCH proponents appear to already implicitly endorse the Modal Condition.

Let's turn to three possible objections. First, note that friends of the CCH typically endorse the view that the span of past time is finite. If only the temporal phase of God's life is actual – so that God has only a temporal life and no atemporal phase – what explains the fact that time began a finite temporal interval to the past? Here, I think a variety of proposals can be offered. Suppose, as many friends of the CCH think, the series of past events grows by successive addition and successive addition cannot produce an actually infinite collection of past events. In that case, there is no need to postulate some state that God has prior to time; instead, we need only to postulate that God created an initial state while existing simultaneous to that initial state and then ensured the initial state was added to by successive addition. Since CCH proponents believe an infinitude of past time is metaphysically impossible, CCH proponents should say there is no special explanation required for the fact that, in worlds that include time, past time is finite. (This is not to deny

[18]↑Paul Kabay (2009) has argued that if God exists at all actual times (that is, God is omnitemporal) and time began, then God began to exist ex nihilo. However, Kabay assumes that God has no atemporal mode of being in the actual world. (See Kabay, 2009, pp. 122–123.) On the view under consideration in this chapter, God does have an atemporal aspect.

the CCH proponent's claim that the beginning of the Cosmos does require explanation.) Alternatively, if $B-$ or $C-$theory are true, the entire space-time block exists simpliciter and our place a finite distance from one boundary in the block is a purely indexical fact. No particular need for explanation of that indexical fact arises. Thus, whatever metaphysical view of time turns out to be correct, I don't see why a finite past would require God to occupy a timeless state prior to the beginning of time.

The second and third objection are resolved by one solution. For that reason, I will first discuss the two objections and then discuss their common solution. For the second objection, suppose that, perhaps for reasons beyond our ken, the world is better if time exists than if time does not exist. In that case, at any metaphysically possible world w, God knows w is better if time exists, and so God creates time. Time would necessarily exist, even though time would ontologically depend upon God. In other words, the Modal Condition would not be satisfied, even though God would be prior in the order of being to time.

For the third objection, consider that some members of the Oxford School, e.g., Swinburne, depart from the traditional view that God necessarily exists. In that case, we can either suppose that God does not create time in all possible worlds where God exists or that God does create time in all possible worlds where God exists. In the former case, the Modal Condition is satisfied. In the latter case, God would exist at all of the metaphysically possible worlds where time exists. Once more, the Modal Condition would not be satisfied, even though God would be prior in the order of being to time.

As I previously said, both the second and third objections can be handled by a common solution, namely, by generalizing the Modal Condition from including only counterfactual possibilities to including counterpossibilia. In the case that God necessarily exists and necessarily creates time, the closest world without time would be a counterpossible world where God exists but fails to create time. On the other hand, if God contingently exists but creates time in every world in which God exists, then the closest world without time would again be a counterpossible world where God exists but fails to create time. In any case, on the counterpossible version of the Modal Condition, we should still say that, without time, God would have existed anyway.

5.3 The Disappearance of Time in Physical Cosmology

The proper conception of the Cosmos's beginning is likewise an important question for philosophers of physics. Naturalists are unlikely to find theological arguments appealing, but, as I argue in this section, naturalists can take away an important lesson and thereby derive the Modal Condition for their own non-theological purposes. There are live physical theories, or at least interpretations of physical theories, according to which space-time is reducible to, functionally realized by, emergent from, or otherwise wholly explicable in terms of, some more fundamental non-spatiotemporal physical substructure. If so, whether a given proper part of the Cosmos is spatiotemporal will depend upon whether that part's substructure has the appropriate configuration, just as whether some body of water occupies a gaseous, liquid, or solid state depends upon the configuration of that body's molecular constituents (Oriti, 2021, p. 27). In that case, a spatio-temporal proper part of the Cosmos might include the Cosmos's first period of time. Since the Cosmos's existence would be prior in the order of being to the existence of time, there is a deeply intuitive sense in which the Cosmos would lack a beginning – just as a temporal God lacks a beginning if God is prior to time in the order of being – even if there is a first period of time in a non-fundamental proper part of the Cosmos. Thus, just as the theologian can offer an argument for the view that God is beginningless even if God's life includes an initial finitely long segment, so, too, the naturalist can say that the Cosmos is beginningless even if the Cosmos's history includes an initial, finitely long segment:

P1) If any entity is non-temporal, then that entity did not begin to exist.

P2*) The Cosmos is fundamentally non-temporal.

C1*) So, the Cosmos fundamentally did not begin to exist.

P3) Any entity that fundamentally did not begin to exist did not begin to exist simpliciter.

C2*) Therefore, the Cosmos did not begin to exist simpliciter.

As in the theological case, since this argument is independent of whether the Cosmos's history includes an initial, finitely long segment, this argument demonstrates that the

Cosmos would be beginningless so long as (P2*) is true, that is, so long as the Cosmos is fundamentally non-temporal.

Why think that (P3) is true? If there is a part (or aspect) p that suffices for the existence of some entity E and p is beginningless, then E is beginningless. For example, consider the Cosmos. If there is a part (or aspect) of the Cosmos that suffices for the Cosmos's existence, then the Cosmos is beginningless. Recall that I've stipulatively defined the term 'Cosmos' to mean the totality of physical reality so that the existence of anything physical at all suffices for the Cosmos's existence. For that reason, if any physical entity at all lacks a beginning, then the Cosmos lacks a beginning.

While the view that physical entities are essentially, and so fundamentally, spatio-temporal has been a long held dogma, there are several distinct ways in which the view has been put into doubt by developments in both philosophy of physics and theoretical physics. Space prohibits me from offering more than a brief survey. Moreover, I do not claim that a decisive case has been made for the view that space and time are non-fundamental.[19] Several of the arguments that I describe remain controversial and, at least in this chapter, I do not hope to settle live disputes concerning how to interpret the physical theories that I discuss. Nonetheless, an analysis of beginning to exist should at least be *consistent with* possible future directions of physical inquiry. As such, my aim in this section is to describe several possible avenues of future inquiry with which an analysis of beginning to exist should be consistent.

5.3.1 An Analogy for the Non-Fundamentality of Space-time

To ease our way into a discussion of the notion that space-time is not fundamental to the physical world, let's begin with an intuitive analogy. Suppose that something like the scenario depicted in *The Matrix* were actual, so that what we ordinarily take to be the external world is, in fact, a computer simulation. Let's call the people who are plugged into the Matrix *victims*. The set $\mathcal{S}$ of spatial relationships within the simulation are

[19]↑Neither quantum gravity nor quantum foundations are areas in which we have reached the end of inquiry. Moreover, given the provinciality of the energy scales that are available to us, we might not be able to probe quantum gravity in sufficient detail to know which quantum gravity theory is correct.

functionally realized by computers. The set S of spatial relationships between, and within, the physical components comprising the computers might have nothing at all to do with $\mathcal{S}$. Consider, too, the set of temporal relationships $\mathcal{T}$ between the events experienced by the victims plugged into the Matrix. Let's suppose that the computers control the length of the specious present experienced by the victims, so that the duration between two events within the Matrix might have little to do with the temporal durations between events as witnessed by those who have been liberated from the Matrix. In that case, the Matrix functionally realizes $\mathcal{T}$, even though there is a distinct set of temporal relations T outside the Matrix. In other words, by functionally realizing $\mathcal{S}$ and $\mathcal{T}$, the Matrix functionally realizes all of the spatio-temporal relations available to the victims. However, we have not yet envisioned a scenario in which physical reality is fundamentally non-spatio-temporal because the computers running the Matrix are themselves immersed in space-time.

Let's take this thought experiment one step further by considering George Berkeley's God. In Berkeley's metaphysics, all of the objects in our ordinary experience exist, but they are realized within God's mind. Presumably, Berkeley's God would have no difficulty realizing the code running on the computers in the aforementioned thought experiment. But, unlike the computers in the aforementioned thought experiment, God is not, herself, immersed in a spatio-temporal world. Instead of altering how the people within God's mind experience time by modifying their specious present, we can suppose that God is metaphysically responsible for time itself. In that case, God functionally realizes all of the spatio-temporal relations within God's mind and so functionally realizes space and time. For David Spurrett and David Papineau (1999) as well as Barbara Montero (2005), x is physical just in case x is not irreducibly mental; thus, if fundamental reality were not a person, did not instantiate folk psychological predicates, and did not otherwise instantiate irreducibly mental predicates, then fundamental reality would be purely physical. Therefore, to construct a view on which physical reality is not fundamentally spatio-temporal, we need take only one more step beyond Berkeley's God and suppose that, unlike Berkeley's God, fundamental reality is not a person, does not instantiate folk psychological predicates, and does not otherwise instantiate irreducibly mental predicates.

In the following subsection, I will survey how the view that the Cosmos is not fundamentally spatio-temporal arises in three contexts: first, in the interpretation of relativistic space-times; second, in the interpretation of quantum gravity theories; and, third, in the interpretation of quantum mechanics.

5.3.2 Non-Fundamental Space-time in Three Contexts

Relativistic Space-times

Relativistic space-times have been interpreted as not being fundamentally temporal. For example, contrary to how General Relativity is often presented today, Einstein offered an interpretation in which space-time is functionally realized by the gravitational field ("Space-time does not claim existence on its own, but only as a structural quality of the field", 1961, p. 176). Moreover, on the standard Minkowskian interpretation of relativity, space and time each disappear and we are left with a kind of union of the two (Minkowski, 1952, p. 75). The demand for general covariance in General Relativity is standardly interpreted to mean that the division of space-time into space and time depends upon the adoption of a specific reference frame, with an associated set of coordinates, with the consequence that the division of space-time into space and time lacks metaphysical significance (Oriti, 2021, p. 21). If the division of space-time into space and time lacks metaphysical significance, then we should not interpret space-time points as either spatial or temporal points; instead, we should interpret space-time points as belonging to a new category of entities *neutral* between space and time. And if space-time points are neutral with respect to either space or time, relativistic space-times are not fundamentally temporal.

On the view that space-time points are themselves neutral with respect to space or time, fundamental physical reality would satisfy the Modal Condition. In order to show that fundamental physical reality would satisfy the modal condition, one needs to show that in the closest possible worlds without time, the temporally neutral space-time points would still exist. Since the points are not fundamentally temporal, the points could have existed without exemplifying $A-$ or $B-$relations and so would have existed even if time had not.

Quantum Gravity

While the view that relativistic space-times are not fundamentally temporal is controversial, live proposals for quantum gravity theories provide still more reason to suspect that physical reality is not fundamentally temporal. For example, if one applies the canonical quantization procedure to the Hamiltonian formulation of General Relativity, one can write down an analogue of the Schrödinger Equation for the universe, called the Wheeler-DeWitt Equation, whose solution is the wavefunction (or the wavefunctional) of the universe. In the Wheeler-DeWitt equation, the Hamiltonian annihilates the universal wavefunction, in turn implying that the universal wavefunction has no time dependence (Barbour, 1994, 1999; Butterfield and Isham, 2006; Earman, 2002a; Healey, 2002). Consequently, according to the Wheeler-DeWitt equation, the universe occupies a timeless quantum state. The result is the so-called *Problem of Time* (e.g., Thébault, 2022), wherein physicists ask whether one can recover time in the appropriate limit from a timeless quantum state or if one should give up the approach leading to the Wheeler-DeWitt Equation altogether. While the Wheeler-DeWitt equation remains controversial, one accepted solution is to say that time should be replaced by a parameter internal to the Cosmos and that can play time's functional role (Barbour, 1994; Butterfield and Isham, 2006; Healey, 2002; Thébault, 2022; Oriti, 2021, p. 22). As Carlo Rovelli describes, "An accepted interpretation of [the disappearance of time] is that physical time has to be identified with one of the internal degrees of freedom of the theory itself (*internal time*)" (1991, p. 442). If time should be recovered as a parameter internal to the Cosmos, then the Cosmos is not fundamentally temporal.

A number of approaches to quantum gravity exacerbate the problem still further (Bihan, 2017a, 2017b, 2019, 2020; Butterfield and Isham, 2006; Healey, 2002, 2021; Huggett, 2022; Huggett and Wüthrich, 2013, 2018; Oriti, 2014, 2020, 2021; Rovelli, 2020; Wilson, 2021). For example, some approaches to quantum gravity replace the continua (space-time and fields) available in either classical General Relativity or in a quantized gravitational field with new fundamental degrees of freedom that are not spatio-temporal in any traditional sense (Oriti, 2021, pp. 23–27). As Oriti writes, "The main point should be clear: in quantum gravity, the fundamental degrees of freedom are not continuum fields and spacetime dissolves into pre-geometric, non-spatiotemporal entities, from which space, time, and geometry have to emerge in some approximation" (Oriti, 2021, p. 23).

As an example, consider Loop Quantum Gravity (LQG). LQG roughly tells us that space-time structure is underwritten by a discrete network of spins. An initial temptation is to think that LQG merely tells us that space-time has a discrete structure instead of the continuous structure postulated by General Relativity. If so, LQG does not deny that physical reality is fundamentally spatio-temporal. This initial temptation is at least not obviously correct for two reasons, to which I now turn.

First, I turn to disordered locality, as originally discussed in Markopoulou and Smolin, 2007. Suppose that the discrete structure found in LQG is a discrete space-time structure. In that case, the spatio-temporal relationships found in General Relativity might be expected to correspond to network structure in a straightforward way. For example, two objects that are contiguous in the General Relativistic description might be expected to sit at adjacent nodes in the underlying network structure or, at the very least, would be "closer" together in the network than objects that are spatio-temporally separated. However, LQG postulates no systematic correspondence between the spatio-temporal ordering of events and the adjacency relations in the underlying spin network. Some adjacent nodes correspond to space-time points separated by large spatio-temporal distances. For that reason, Le Bihan (2020, p. 12) has argued that LQG leads to a new form of eternalism ("atemporal eternalism"), on which the structure underlying space-time lacks the formal properties of the space-time block and, consequently, should not be understood as a space-time block.[20] This argument is not decisive; consider that, in the Matrix example I previously gave, the physical world outside the Matrix's structure might not straightforwardly correspondence to the spatio-temporal structure of the Matrix, even though the

[20]↑Nick Huggett (2022) has similarly argued that Group Field Theory postulates a structure underlying space-time with an altogether different formal structure from that of space-time.

external world might still be spatio-temporal. However, the argument is suggestive in that if the Cosmos lacked spatio-temporal structure, we would expect the fundamental formal structure of the world to substantially differ from that of the effective space-time available to ordinary empirical observations.

In addition to the fact that we might have expected disordered locality (or something close to it) if the Cosmos fundamentally lacked spatio-temporal structure, if disordered locality did turn out to be correct, then we would lose much of the justification we would otherwise have had for thinking that the Cosmos is irreducibly ordered according to either an *A*-series or a *B*-series and so much of the justification we would have otherwise had for thinking that the Cosmos is fundamentally temporal. Consider how *A*-theory is typically defended. *A*-theory is typically defended by appealing to our phenomenological experience of time. If loop quantum gravity is true, and so disordered locality is true, then the Cosmos is not fundamentally structured according to the *A*-series found in our phenomenological experience. While the possibility might remain that the Cosmos is fundamentally structured according to some other *A*-series, I have difficulty seeing how one could *justify* the view that the Cosmos is fundamentally structured according to an *A*-series. Likewise, consider how *B*-theory is typically understood, e.g., as a series of moments related one to another by *B*-relations. If what we ordinarily take to be moments ordered by *B*-relations turn out not to be reflected in the Cosmos's fundamental structure, as would turn out to be the case if disordered locality turns out to be correct, then we lose much of the justification we might have otherwise had for thinking that the Cosmos is fundamentally organized according to a *B*-series. We would be left with a view according to which the *B*-series we are familiar with is a derivative feature of our world and an open question as to whether fundamental reality is structured according to some other *B*-series.

I now turn to one last reason one might think loop quantum gravity is not fundamentally spatio-temporal. This last reason draws on the fact that loop quantum gravity is a quantum mechanical theory. In virtue of being a quantum mechanical theory, the spin network exists in a superposition state, so that, unlike classical space-time, the spin network does not have a definite or unique structure. Nonetheless, even though the *network* doesn't have a definite or unique structure in virtue of being in a superposition state, the wavefunction describing the superposition state *does* have a definite and unique structure. This suggests (again without definitively establishing) that the wavefunction is the fundamental object and not the spin network. Given that a variety of authors (as discussed below) have argued that we should understand the wavefunction as a non-spatio-temporal object, the object fundamental to loop quantum gravity might be understood as non-spatio-temporal. Whether this is the correct way to interpret the wavefunction remains a live dispute.

In quantum gravity theories where space-time is not fundamental, space-time can be recovered only by considering a sufficiently large collection of nodes, that is, by considering the network's hydrodynamic limit. Since space-time appears in the hydrodynamic limit only when the fundamental non-spatiotemporal degrees of freedom are arranged in an appropriate configuration, there may have been a physical process, termed *geometrogenesis* (Oriti, 2021, pp. 29–32; also see Oriti, 2014), whereby the early universe (or the Cosmos) "transformed" from a non-spatiotemporal phase into a spatiotemporal phase. Nonetheless, such a process is conceptually problematic because the non-spatiotemporal phase, qua non-spatiotemporal, cannot stand in the 'before' relation to the spatiotemporal phase. However, we may be able to replace our usual notion of time with a kind of "proto-time" and thereby allow "proto-temporal" evolution from the non-spatiotemporal phase into the spatiotemporal phase (Oriti, 2021, p. 31).

Consider the following toy model for geometrogenesis. Suppose that a cosmological model can be parametrized by some parameter T such that, for values of $T \geq T_0$, T can be interpreted as time, but, for values of $T < T_0$, T should not be thought of as time, since the sub-spatiotemporal degrees of freedom do not "coalesce" in the way required for space-time to emerge in the hydrodynamic limit. Candidates for such a parameter include the universe's volume or the scale factor (Oriti, 2021, p. 32). T should not be thought of as time because T cannot be globally interpreted as time. There is a domain, i.e., $T \geq T_0$, where T plays the functional role of time in our physical theories. Moreover, if one is committed to B-theory, one could postulate that, for $T \geq T_0$, event A is before event B just in case $T(A) < T(B)$. However, when we trace T "backwards" beyond T_0, we encounter a non-spatiotemporal domain where the ordering of the values of T should not be interpreted to correspond to *B*-relations, but can perhaps be interpreted as proto-*B*-relations, that is, as ordering relations that are (somehow) more fundamental than *B*-relations. In some sense, this is analogous to the theologian's thought that there is a kind of conceptual or explanatory priority in God's mind prior to Creation, even though there might not be time prior to Creation. In any case, even though T_0 would be the beginning of time, there is a clear intuition according to which T_0 would not be the beginning of the Cosmos.[21]

One might object at this point that I've previously rejected a similar model of God. I rejected the possibility that there is both an atemporal phase and a temporal phase of God's life on the basis that the continuity conditions between the two phases of God's life are utterly mysterious. God cannot perdure or endure from the atemporal phase to the temporal phase, the atemporal phase cannot pass away or into the temporal phase, and the atemporal phase cannot be before the temporal phase. Why shouldn't we reject the possibility that the Cosmos has two phases in its life for the same reasons? First, note that many (perhaps most or all) of the proponents of the CCH are committed to the *A*-theory of time. The view that space-time is not fundamental sits uncomfortably with *A*-theory so that proponents of the view that space-time is not fundamental are much more likely to be *B*- or *C*-theorists. On *B*- and *C*-theory, there is no temporal passage and so nothing passes away or into anything else. Thus, for *B*- and *C*-theorists, there is no problem for the view that the non-spatio-temporal phase does not pass away or into the spatio-temporal phase.[22] Moreover, while we might metaphorically speak about the life of the Cosmos, the Cosmos does not have a life in the sense that God would have a life. For that reason, the Cosmos's life does not need to be unified in the sense that God's life

[21]↑When this chapter was submitted as an article to *Erkenntnis*, an anonymous reviewer objected to my toy model of geometrogenesis. As the reviewer notes, one reason that one might think that T cannot be interpreted as a time parameter for $T < T_0$ is that the state of affairs such that $T < T_0$ does not satisfy the Einstein Field Equations. However, $T < T_0$ might still be interpretable as, for example, a $B-$series and so is interpretable as a time parameter after all. Supposing that the reviewer's objection suffices for showing that $T < T_0$ *can* be interpreted as a temporal series, the reviewer's objection does not suffice for showing that $T < T_0$ *should* be interpreted as a temporal series. For my purposes in this chapter, I need only to show that the emergence of time from metaphysically prior, but not temporally prior, non-temporal phenomenon is a live option that would be premature to rule out from the arm chair; again, I am not attempting to show which interpretation of loop quantum gravity is the correct interpretation. Instead, I am summarizing a live option that has been discussed in the literature.

[22]↑See the related set of remarks Craig makes in his (1998, pp. 246–248).

needs to be unified in order to be the life of numerically one deity. Furthermore, consider that, in Galilean and relativistic space-times, space-time points do not perdure or endure.[23] There seems to be a category mistake in supposing that space-time, itself, either endures or perdures. If there is a category mistake involved in the view that space-time, itself, either endures or perdures, then there is a category mistake involved in the view that the Cosmos endures or perdures. If there is a category mistake involved in the view that the Cosmos endures or perdures, there is no demand for the Cosmos to endure or perdure through geometrogenesis.

When this chapter was submitted as an article to *Erkenntnis*, an anonymous reviewer raised an objection to my use of the quantum gravity proposals that I considered in this section. According to the reviewer, the quantum gravity literature considers space-time non-fundamental because the fundamental entities postulated by quantum gravity theories (e.g., strings, causal sets, or whatever) do not satisfy the Einstein Field Equations. For example, when the claim is made that space-time is recovered only as part of a hydrodynamic limit, part of what is being claimed is that the Einstein Field Equations are recovered only as part of a hydrodynamic limit. However, in a discussion of the metaphysics of time, one might argue that we should allow that time has wider application than the Einstein Field Equations. For example, couldn't the $A-$ or $B-$theory of time be true even if the Einstein Field Equations do not apply? At least two replies can be offered to the reviewer's objection.

First, I do not claim that any specific quantum gravity theory is true or that any specific interpretation of any particular quantum gravity theory is the correct interpretation. There may be quantum gravity theories, e.g., causal set theory, that should be interpreted in A-theoretic terms. For my purposes in this chapter, I claim only that the non-fundamentality of time remains a live option that should not be ruled out from the arm chair. So long as philosophers of physics are seriously considering the possibility that physical reality is not fundamentally temporal, we need an analysis of the notion that the Cosmos had a

[23]↑If a space-time point did endure or perdure, then an object could be at absolute rest by occupying the same space-time point at successive times. Objects cannot be at absolute rest in Galilean or relativistic space-times. Therefore, space-time points do not endure or perdure in Galilean or relativistic space-times.

beginning consistent with the possibility that the Cosmos is not fundamentally temporal. Second, while the reviewer might be correct to say that the reason for thinking the entities fundamental to some specific quantum gravity theory are not spatio-temporal involves the failure of the Einstein Field Equations, there are quantum gravity theories whose fundamental entities should plausibly be thought of as non-temporal for other reasons. For example, the failure of the Einstein Field Equations does not appear among the reasons Baptiste Le Bihan (2020) surveyed for thinking that the entities fundamental to loop quantum gravity or string theory are non-temporal.

Quantum Interpretations

In addition to relativity and quantum gravity, quantum mechanics has sometimes been claimed to show that space and time are not fundamental. Some of the revolutionaries who first developed quantum mechanics, e.g., Pascual Jordan and Max Born, thought that quantum mechanics had revealed that microphysical entities are not spatiotemporal (Capellmann, 2021; Kragh, 1996, p. 47; Luminet, 2011, pp. 2915–2918). In turn, the notion that microphysical entities are not spatiotemporal inspired Georges Lemaître in the development of an early version of the big bang theory in which the universe originated in a timeless entity (the primordial "atom") (Lemaître, 1931; Kragh, 1996, p. 47; Luminet, 2011).

Several contemporary approaches to the foundations of quantum mechanics likewise suggest that space and time are not fundamental. For example, *wavefunction monism* is the view that all that ultimately exists is the universal wavefunction. (Some wavefunction monists are additionally committed to a "marvelous point" guided by the universal wavefunction or to the "space" inhabited by the wavefunction, though that space should not be thought of as space-time). We can distinguish at least three versions of the view. In one version of the view, defended by David Albert (1996, 2013, 2015, 2019a, 2019b), Barry Loewer (1996), Alyssa Ney (2012, 2013, 2020, 2021), and Jill North (2013), the universal wavefunction is a field either defined on configuration space or on some more exotic state space (Ney, 2020; also see chapter 4 in Ney, 2021). On this view, the wavefunction is typically thought of as fundamentally temporal and to occupy some kind of space, even if not the space of our ordinary experience. However, other versions of wavefunction monism entail that the universal wavefunction is not temporal. For David Bohm (1980, p. 211), the universal wavefunction is again a field defined on some high dimensional state space but time results as a consequence of projecting to a lower dimensional space. For Julian Barbour (1999), the universal wavefunction is a field defined on superspace, that is, the space of possible configurations of space-time, and with a distribution and amplitude defined by the Wheeler DeWitt Equation. For Sean Carroll (2019, 2022) and co-author Ashmeet Singh (2019), the universal wavefunction is a state vector in Hilbert Space. For Bohm, Barbour, Carroll, and Singh, the universal wavefunction is not a temporal object. If all that ultimately exists is the universal wavefunction, and the universal wavefunction is not temporal, then space-time is reducible to, functionally realized by, emergent from, or otherwise wholly and asymmetrically explained by the universal wavefunction.

Thus, there are a variety of live research programs according to which space-time is not fundamental to the Cosmos and is instead asymmetrically explicable in terms of some non-spatiotemporal structure. The non-spatiotemporal structure would be timeless, just as the molecules that comprise liquids lack the property of liquidity. Just as God is beginningless if God stands prior to time in the order of being, so, too, the Cosmos is beginningless if the Cosmos stands prior to time in the order of being.

5.3.3 Physical Cosmology and the Modal Condition

Recall the lesson that the naturalist can take from the theological discussion in section 5.2. Timeless entities are beginningless. So, fundamentally timeless entities are fundamentally beginningless. To reiterate, consider an entity A that is fundamentally timeless. In that case, there is an aspect of A – that is, the fundamental aspect – that is timeless. There could be another aspect of A – that is, a non-fundamental aspect – that is not timeless. Moreover, suppose that the existence of the fundamental aspect suffices for the existence of A but A could have existed without the non-fundamental aspect. Supposing that the non-fundamental aspect of A is in time in the actual world, A would still exist at one or more of the closest possible worlds lacking time. Because the fundamental aspect of A is beginningless, and the existence of the fundamental aspect suffices for A's existence, A is beginningless, even if the non-fundamental aspect of A existed for an initial finitely long period of time. Note that the non-fundamental aspect could have had a beginning, but a beginning of the non-fundamental aspect of A is not the beginning of A *simpliciter*.

Recall that for God to be beginningless required $\neg T \; \Box\!\!\rightarrow \; \exists x.x = God$. So, for A to lack a beginning even though A has an initial finitely long period of time requires that $\neg T \; \Diamond\!\!\rightarrow \; \exists x.x = A$, that is, had time not existed, A might have existed. Let C represent the statement that the Cosmos exists. Thus, the statement that had time not existed, the Cosmos might have existed anyway, is represented as $\neg T \; \Diamond\!\!\rightarrow \; C$. We want a necessary (but not sufficient) condition for the Cosmos to have a beginning. To derive such a condition, we should negate $\neg T \; \Diamond\!\!\rightarrow \; C$. The negation of $\neg T \; \Diamond\!\!\rightarrow \; C$ is equivalent to $\neg T \; \Box\!\!\rightarrow \; \neg C$. So, the Cosmos had a beginning only if

> At all of the closest possible worlds where time does not exist, the Cosmos does not exist.

Unfortunately, this criterion has not been given serious enough attention in philosophical arguments for the beginning of the Cosmos, such as those that I discussed in chapter 2, where authors swiftly move from the proposition that the Cosmos has a finite past to the conclusion that the Cosmos began to exist. Likewise, the arguments that I considered in chapter 3 swiftly moved from the view that the past history of the universe has a singular boundary and is therefore finite to the conclusion that the Cosmos began to exist. Or consider that, as Norman Kretzmann (1985), William E. Carroll (2007), and Jon McGinnis (2015) point out, Scholastic philosophers assumed a conception of beginning to exist that resembled beginning-to-exist-1. The Scholastic debate concerned whether God's creation of the Cosmos was consistent with the Aristotelian view that the Cosmos had an infinite (and so, on their view, beginningless) past. Scholastics assumed that either the Cosmos had a beginning – in which case they assumed the past must be finite – or else the Cosmos was beginningless – in which case they assumed the past must be infinite. A moment's reflection shows that both friends of the CCH and the Scholastics are incorrect. Supposing that one could show merely that the Cosmos had a finite past, one could not infer that the Cosmos had a beginning; one must also show (among other criteria) that the Cosmos is fundamentally temporal and therefore show that the Cosmos satisfies the aforementioned Modal Condition. We have then a general reason for rejecting all of the versions of the KCA that have thus far been offered in the literature. If proponents of the KCA want to establish that the Cosmos began to exist, they will have to do much more than they have done thus far.

5.4 Summary

In section II, we started with the tensed conception of beginning to exist. I rejected the tensed conception because it was desirable to identify a conception of beginning consistent with B-theory. We then turned to the at-at conception of beginning, which I rejected because the at-at conception requires a time before the Cosmos's existence. On the assumption that time is a physical phenomenon, time began with the Cosmos, whereas if, as the Oxford School supposes, time is non-physical, then there is at least a moment (or interval) before which the Cosmos did not exist. In this section, we embarked on developing a more sophisticated conception of the Cosmos's beginning. By examining a debate concerning God's relationship to time, I developed the intuition that the Cosmos had a beginning only if a specific modal condition were fulfilled. This intuition turns out to be useful in understanding a debate concerning the philosophical foundations of various physical theories that claim that time is (somehow) not fundamental to the Cosmos. Finally, the Modal Condition was articulated by using the standard Lewis-Stalnaker semantics for counterfactuals.

6. THE DIRECTION CONDITION

Having established the Modal Condition, i.e., that the Cosmos has a beginning only if, at all of the closest possible worlds where time does not exist, the Cosmos does not exist, I turn to the Direction Condition, i.e., that the Cosmos began to exist only if the Cosmos has a global direction of time. As a first, rough pass, the Cosmos has a global direction of time just in case the entire Cosmos "shares" a direction of time. In this chapter, I borrow the chronogeometric conditions for a global direction of time previously defended by Geoffrey Matthews (1979, p. 84) and Mario Castagnino, Olimpia Lombardi, and Luis Lara (2003). As they explain – and as I will unpack below – spacetime S has a global direction of time just in case (i) a unique temporal orientation – or, e.g., past-to-future direction – can be defined at each point of S, that is, S is *temporally orientable*, (ii) for any point p in S, there is a locally defined direction of time at p, and (iii) for all pairs of points p and q in S, the future (past) direction defined at p agrees with the future (past) direction defined at q.

Some readers may be perplexed by the idea that time could lack a global direction. For that reason, I will spend some time unpacking a few senses in which time could lack a global direction. If our Cosmos began, then the beginning of the Cosmos must be prior to all non-initial space-time points that the Cosmos includes. In order for the beginning to be prior to all non-initial space-time points, all non-initial space-time points must agree that the putative beginning is located to their past. And in order for all non-initial space-time points to agree that the putative beginning is located to their past, all non-initial space-time points must agree on the direction of time. There are two ways for space-time to lack a global direction of time. First, space-time could fail to have a global direction of time by failing to be temporally orientable. Second, given that space-time is temporally orientable, space-time could fail to have a global direction of time if the direction of time varied from one space-time region to another. I will consider each of these conditions in turn. However, in order to discuss either notion, I need to first put a bit of formal machinery on to the table.

There are three relations defined between any two points p and q in a relativistic space-time and that should be thought of in terms of the line, or the closest analogue to a line, connecting the two points.[1] If light can travel from *p* to *q* or vice versa, we say that *p* and *q* are light-like related. If an object, traveling slower than light, can travel from *p* to *q*, we say that *p* and *q* are time-like related. And if *p* and *q* are neither time-like nor light-like related, we say that *p* and *q* are space-like related. Points that are time-like or light-like related to *p* can be divided into two classes. As light falls in on a point, the light forms a sphere whose radius contracts with time. A cross section of a given sphere is a circle, so that the process of light falling in on a point forms a cone when represented using successive cross-sections. On the standard Minkowskian interpretation of relativity, the points that are *p*'s past are the points that can transmit a signal to *p*; thus, the points that in *p*'s past are said to be in *p*'s past light cone. Likewise, the light originating at *p* forms concentric circles and forms a cone when represented using successive cross-sections. Thus, since the points that are in *p*'s future are the points to which *p* can transmit a signal, the points that fall in *p*'s future are said to be in *p*'s future light cone. The points that are space-like related to *p* are said to be in *p*'s absolute elsewhere and, at least on the standard Minkowskian interpretation, are not absolutely to the future of, to the past of, or simultaneous with *p*. For that reason, we can say that there are no absolute temporal relations between *p* and any points that are space-like related to *p*, that is, there are no temporal relations between *p* and any of the points that do no fall into *p*'s past or future light cones. At *p*, we can define future pointing, past pointing, and space-like pointing vectors. For example, a future pointing vector at *p* points into the future light cone of *p*.

For readers who may not be familiar with relativistic space-times, I will stress that two points being space-like related is disanalogous with two points being spatially related to each other in space in a pre-relativistic conception of space-time. For example, at one point of time in my life, all of the moments in the entire life of some other person, all 95 years from birth to death, could be space-like related to me.[2] Moreover, at a given point of time in my life, in the reference frame that I occupy, the space-time points that are simultaneous relative to my reference frame – and so might be said to be co-exist with me

[1]↑In Euclidean geometry, we can sensibly define a line as the shortest path between two points. In a relativistic space-time, the closest analogue to a line – that is, a geodesic – turns out to be the longest path between the two points.

[2]↑This example previously appears in Gilmore et al., 2016, p. 108.

in space – are only a subset of the points that are space-like related to me. This is another reason for thinking that, as I discussed in chapter 4, relativistic four-dimensionalism is not the four-dimensionalism from metaphysics textbooks.

The future direction of time at a given space-time point p can be represented by a vector that points into p's future light cone. In order to compare the direction of time at one space-time point with the direction of time at another space-time point, we need a way to "move" a future-pointing vector (for example) from one space-time point to another. Mathematicians originally developed the notion that a vector could be assigned to the points of a space for flat, Euclidean spaces. In order to generalize the notion to spaces with arbitrary curvature, mathematicians imagine that we assign a flat space tangent to each point, called the *tangent space*. For example, ancient peoples thought that the Earth was flat. Upon discovering that the Earth is round, we can still construct a plane – the tangent plane – that approximates the Earth's surface at any given point on the Earth's surface.

The trouble is now that vectors at distinct points occupy distinct tangent spaces. In order to compare the vectors at one point with the vectors at another point, we need a mathematical operation that translates from one tangent space to another. The translation is easier to perform when the two tangent spaces correspond to points that are located closer together. Thus, to translate between the tangent spaces at two arbitrary points, mathematicians imagine a continuous series of translations, from one tangent space to the next, along a path. This operation – called *parallel transport* – can be thought of as moving a vector through a space, while keeping the vector's orientation fixed, so that the vector can be compared to a vector at some other point.

Having put some of the requisite technical machinery on to the table, I now proceed to a discussion of the two conditions for a space-time to have a global direction of time.

6.1 Temporally Orientable Space-times

As I've said, the first way that a space-time may lack a global direction of time would be if the space-time failed to be temporally orientable. Orientable surfaces have the feature that we can objectively distinguish the perpendicular direction from the surface. For example, the plane is an orientable surface. Given a piece of paper lying flat on a desk, we can objectively distinguish the direction from the paper's surface to the ceiling and the direction from the paper's surface to the floor. To see this, imagine parallel transporting a vector pointing towards the ceiling around the paper's surface. Without crossing the paper's edge – an operation that is not mathematically allowed – parallel transport cannot be used to turn a vector pointing towards the ceiling into a vector pointing towards the floor.

Suppose that we take a one foot long piece of ribbon and connect both ends of the ribbon without twisting the ribbon. The resulting surface – a cylinder without top or bottom – is another orientable surface. Construct a vector $\vec{v}$ pointing perpendicular from the ribbon's outer surface. Parallel transport $\vec{v}$ around the ribbon without crossing the ribbon's edge and we eventually return $\vec{v}$ to $\vec{v}$'s starting location. Upon returning, $\vec{v}$ will be restored to $\vec{v}$'s original orientation. Without crossing the ribbon's edge, there is no way to turn an outward pointing vector into an inward pointing vector. To put the point another way, a surface is orientable just in case parallel transport around a closed loop will never reverse the orientation of the vector.

Now consider the surface formed if, instead of gluing the two ends of the ribbon together in order to form a cylinder, we first rotate one end by π radians (180°) before connecting the two ends. The resulting geometrical object is a non-orientable surface mathematicians call a *Möbius Strip*. Notice that the π radians twist connected the ribbon's outside surface to the ribbon's inner surface. When we move an outward pointing vector around the ribbon, we will find that the vector eventually points inward, despite the fact that the vector was never made to move over an edge. In fact, if an outward pointing vector is made to "orbit" the surface of the ribbon in a fixed direction, we would find that the first time the vector returns to its starting location, the vector is inward pointing. By parallel transporting a vector around a closed loop, we were able to reverse the vector's orientation. For that reason, we cannot identify an objective orientation for the surface of a Möbius Strip, that is, Möbius Strips are non-orientable surfaces.[3]

We can ask an analogous question about the temporal orientability of relativistic space-times. Suppose that when we parallel transport a future directed vector around a closed loop in four-dimensions and return the vector to the point at which the vector started, the vector becomes past directed. In that case, we would not be able to objectively identify the past-to-future direction at a given point just as we cannot objectively distinguish the orientation of a Möbius Strip. Some solutions to the Einstein Field Equations are not temporally orientable. Although I am not sure how to metaphysically interpret non-temporally orientable space-times, I think that any space-time with a beginning must be temporally orientable.

That a space-time S is temporally orientable implies only that S is logically consistent with defining an absolute past-to-future direction at every point of the space-time. But logical consistency is not sufficient for showing that there *is* an objective past-to-future direction at every point. If we knew which light cone was the past light cone and which was the future light cone at any given space-time point p in S, then we would know the past-to-future direction at p. Given the past-to-future direction at p, we could then project the past-to-future direction to all other space-time points in S via parallel transport. However, the chronogeometry specified by General Relativity is symmetric with respect to the direction of time, so that the relativistic description does not suffice for telling us which light cone we should label as past and which light cone we should label as future.

[3]↑There are some additional clarifications that can be made to distinguish the examples that I've offered and relativistic space-times. We can distinguish between two distinct kinds of curvature, i.e., intrinsic and extrinsic curvature. Even though the ribbon is a two-dimensional object, we can "bend" the ribbon in three-dimensional space because we've embedded the ribbon into a three-dimensional space. This sort of curvature – which requires a higher dimensional embedding space – is extrinsic curvature. In the absence of intrinsic curvature, extrinsic curvature never *deforms* the contents of a surface. For example, if a pattern is printed on the ribbon but the ribbon is made of, e.g., cardboard, then bending the ribbon into a cylinder or a Möbius Strip leaves the pattern unaltered. On the other hand, intrinsic curvature *does* deform the contents of a surface. For example, if the ribbon were made of rubber, then we could deform a pattern printed on the ribbon's surface by stretching the ribbon. While extrinsic curvature requires a higher dimensional embedding space, intrinsic curvature can be defined without a higher dimensional embedding space. In General Relativity, the curvature responsible for gravitation is intrinsic curvature. Likewise, parallel transport can be defined without reference to a higher dimensional embedding space.

In order to describe an objective direction of time, we need to add additional structure to the relativistic description.

Suppose that we do add whatever additional structure suffices for specifying the past-to-future direction at p. And now suppose that q is a space-time point space-like related to p. If the past-to-future direction at p agrees with the past-to-future direction at q, then, to find the past-to-future direction at q, we need only to parallel transport a future directed time-like vector from p to q. However, as we will see in the next section, nothing I've said so far guarantees that the past-to-future direction at p agrees with the past-to-future direction at q. Although temporal orientability guarantees that parallel transport around a closed loop would never turn a future directed time-like vector into a past directed time-like vector, temporal orientability is not sufficient for the future direction at p to agree with the future direction at q. In order to guarantee that p and q – and all other points in the space-time – agree on the absolute direction of time, we need to fix the temporal direction over the entire space-time.

6.2 Fixed Temporal Direction

If the Cosmos began, then the Cosmos's beginning is prior to all non-initial events in our Cosmos's history. A number of authors have argued that fundamental physics does not provide the distinction between past and future directions found in the macrophysical world (Albert, 2000, 2017; S. Carroll, 2010; Farr, 2020; Farr and Reutlinger, 2013; Loewer, 2012a, 2012b, 2020; Price, 1997). If past and future directions are not fundamentally distinguished, then no event is fundamentally prior to all other events. Moreover, many physicists and philosophers of physics have argued that, even though there is no microphysical (or fundamental) direction of time, we can recover a macrophysical (or non-fundamental) direction of time. Macrophysical processes that happen only in one direction, e.g., the diffusion of gases into a room, involve an increase in entropy. Given two times at which the entropy differs, we can define an entropy gradient between the two times. Perhaps a local macrophysical direction of time should be understood in terms of or should share a reductive explanation with the local entropy gradient. But, if so, since the entropy gradient can change from one region of the Cosmos to another, perhaps there is no globally definable direction of time. Without a globally definable direction of time, no event can be macrophysically (or non-fundamentally) prior to all other events. In the nineteenth century, Ludwig Boltzmann imagined that the entropy has fluctuated up and down over time and fluctuates from one region of space to another:

> There must then be in the universe, which is in thermal equilibrium as a whole and therefore dead, here and there relatively small regions of the size of our galaxy (which we call worlds), which during the relatively short time of eons deviate significantly from thermal equilibrium. [...] For the universe as a whole the two directions of time are indistinguishable, just as in space there is no up or down. However, just as at a certain place on the earth's surface we can call "down" the direction toward the centre of the earth, so a living being that finds itself in such a world at a certain period of time can define the time direction as going from less probable to more probable states (the former will be the "past" and the latter the "future") and by virtue of this definition he will find that this small region, isolated from the rest of the universe, is "initially" always in an improbable state (Boltzmann, 2003, p. 416).

According to Boltzmann, we distinguish past/future directions in our region of space and during the time interval that we inhabit only because our region of space, over the relevant time interval, includes a consistent entropy gradient. For Boltzmann, we identify a specific direction as the past only because the entropy is low in that direction. This suggests that there is a temporal direction (the past) in which the entropy is a minimum. Creatures who live on the other side of the entropy minimum may regard our past direction as their future direction. Although there is a loose sense in which we might regard the minimum as the Cosmos's "beginning", the entropy minimum would not fundamentally be to the past of any other time and would not have a distinguished status. In that case, we should not say that the Cosmos truly began.

By way of illustration, consider a two-dimensional space-time. We can represent a two dimensional space-time with a piece of graph paper. Suppose, moreover, that there are local features of the space-time such that (for whatever reason) time could consistently point in the direction up the page. That is, one possible configuration of the space-time, and the matter-energy populating space-time, is such that future directed vectors can be drawn pointing from each point on the paper to the top of the page. Since all of the vectors can be drawn pointing to the top of the page, our two-dimensional space-time is temporally orientable and admits of a globally definable direction of time. However, merely *admitting* a global direction of time – that is, mere consistency with a global direction of time – is no guarantee that there *is* a global direction of time. If the direction of time varies from one region to another – as in Boltzmann's cosmology – then the direction in which the vectors point will smoothly vary from one point on the graph paper to another.

To ensure that all points in the space-time agree on the direction of time, consider any arbitrary future (past) directed vector $\vec{u}$ at p, whose direction is defined by the absolute direction of time p, parallel transport $\vec{u}$ to some point q, and compare $\vec{u}$ to $\vec{v}$, a future (past) directed vector at q whose direction is determined by the absolute direction of time at q. If $\vec{u}$ and $\vec{v}$ agree on temporal orientation for all future (past) directed vectors for all pairs of points in the space-time, then the space-time has a global direction of time.

6.3 Summary

In this section, I articulated the Direction Condition, i.e., that the Cosmos began only if the Cosmos has a global direction of time. A beginning requires a global direction of time because, intuitively, the Cosmos's beginning should be to the past of all non-initial space-time points. In turn, a space-time S has a global direction of time if and only if S satisfies three conditions (Matthews, 1979, p. 84, Castagnino et al., 2003). First, S is temporally orientable. Second, for any space-time point p, there is a locally defined direction of time at p. Third, for all pairs of points p and q in the space-time, the future (past) direction defined at p agrees with the future (past) direction defined at q.

7. THE BOUNDARY CONDITION

I turn to the third of the conditions for the Cosmos to have a beginning, i.e., the Boundary Condition. Intuitively, for the Cosmos to have a beginning, the Cosmos must have a past temporal boundary, such that the Cosmos did not exist before the boundary. There are two ways in which the Cosmos could be said to have a past boundary and so the Boundary Condition is defined disjunctively: either there is a closed boundary to the past of non-initial space-time points (the topological conception) or there is an initial objectively finite portion of the Cosmos's history (the metrical conception). Although the distinction between the topological conception and the metrical conception of a past temporal boundary was introduced by J. Brian Pitts (2008), I will argue that Pitts's distinction is not completely adequate. In this chapter, I develop a more sophisticated conception that improves upon Pitts's. Let's turn to examining the two conceptions, beginning with the topological conception.

7.1 The Topological Conception

Before motivating the topological conception, I need to first develop the notions of *closed*, *open*, and *clopen* sets. Here, I will forego providing a formal definition of the three notions in favor of providing some general intuitions. Consider a segment of the real line from -1 to 1. The segment is closed just in case the segment includes the points -1 and 1. The segment is open just in case the segment does not include -1 and 1. Lastly, the segment is clopen just in case the segment includes one of the end points but not the other. The complement of any open set is closed. The union of a collection of open sets is open. Now consider a point p in an n-dimensional space S. If there is *some* finite distance from p we can move in *any* direction while remaining within S, then p is not a point on a closed boundary. For example, consider a point on the left edge of a piece of paper. We say that the point is on the boundary of the piece of paper because we cannot move any distance further left while remaining on the piece of paper. Contrast the point on the left edge with a point q a distance ε to the right; no matter how small ε might be, so long as $\varepsilon > 0$, we can move ε to the right or the left of q and remain within the piece of paper.[1] Moreover, the notion of closed and open sets, and the related notions of closed and open boundaries, can be rigorously developed without appealing to any metrical notions, so that we can define the notion of a closed boundary without referring to the length of any curve.

In order to motivate the topological conception of a beginning of the Cosmos, let's turn to a consideration of a view in the metaphysical foundations of space-time theories called *metrical conventionalism*.[2] According to metrical conventionalism, there are no non-conventional facts concerning the space-time metric. The standard interpretation of relativity relativizes durations of time to reference frames. In this sense, relativity tells us that there is no fact about the duration of the temporal interval between two numerically distinct events independent of a choice of reference frame. The space-time conventionalist goes one step further; for the conventionalist, the length of a given temporal interval cannot be specified even after we've specified a particular reference frame. For the conventionalist, after we've picked out a reference frame, we can determine the temporal duration between numerically distinct events (or space-time points) only after selecting a specific convention for measuring temporal durations. If metrical conventionalism is true, there is no fact of the matter, independent of the adoption of a specific convention, as to the temporal duration that has passed so far in the Cosmos's history, including any fact about whether the temporal duration of the Cosmos's past history has been finite or infinite. Since, at the level of metaphysics, there are no conventional facts, metrical conventionalists say that there is no fact at all as to whether the Cosmos has a finite or an infinite past. As I discussed in chapter 5, some conceptions of the beginning of the Cosmos entail that the Cosmos had a beginning only if the Cosmos's past is finite. On that conception, metrical conventionalists would say that the Cosmos did not have a beginning, or at least did not have a beginning in any sense that has relevance for metaphysics.

However, there is a conception of the beginning of the Cosmos consistent with metrical conventionalism: the topological conception of the beginning of the Cosmos. In order to explicate the topological conception of a beginning, let's begin by considering the clopen

[1]↑I am assuming that the piece of paper is at least 2ε units wide.
[2]↑For defenses of metrical conventionalism, see Grünbaum, 1968; Poincaré, 2001a; Reichenbach, 1958, 1971.

interval $(0, 1]$. Using the standard Lebesgue measure defined over the real line, the interval $(0, 1]$ has a length of 1. But notice that $(0, 1]$ has the same set theoretic and topological features as $(-\infty, 1]$, that is, both intervals are continous, clopen intervals containing an uncountable infinity of points. If we set aside the Lebesgue measure – that is, if we set aside the metrical features of the interval – then there is no fact that distinguishes $(0, 1]$ from $(-\infty, 1]$ and so no fact distinguishing infinite from finite intervals. Likewise, suppose that the Cosmos has an open boundary to the past. In that case, the Cosmos's past history has the same topological features as a past eternal Cosmos.[3] If metrical conventionalism is true and the Cosmos has an open boundary to the past, then the Cosmos did not have a beginning.

Now consider the closed interval $[0, 1]$. The interval $[0, 1]$ differs topologically from $(-\infty, 1]$ in virtue of having a closed boundary at 0. Importantly, if we set aside all of the metrical features of the interval, we can still say that $[0, 1]$ has a closed boundary to the left at the point we've labeled '0'. For analogous reasons, if space-time conventionalism is true and the Cosmos has a closed boundary to the past of every observer, we can still say that the Cosmos has a past boundary, even though there is no fact concerning the temporal interval between the boundary and ourselves. To put this point into intuitive terms, if the Cosmos includes a first instant of time (and satisfies the other necessary conditions for having a beginning) then we should say that the Cosmos began to exist. Whether that first instant is finitely far, infinitely far, or indeterminately far into the past is irrelevant.

There is another closely related reason to prefer the topological conception over a conception that appeals to metrical information. Relativistic space-times are defined by a manifold **M** and a metric **g**. **M** is a collection of space-time points equipped with topological structure. The spatio-temporal distance between any two points in **M** can be defined in terms of **g**. There is no logical or mathematical inconsistency involved in defining a second distinct metric **g′** over the same members of **M**, in terms of which

[3] ↑To show that there is a topological distinction between an open and a closed boundary, consider that the topological properties of a surface (or space or whatever) are obtained by considering the full set of features of that surface (or whatever) invariant under continuous transformations. One can prove that compact (closed and bounded) sets can be mapped by continuous functions only to compact sets. Consequently, there is no continuous function mapping the compact set $[0, 1]$ to the non-compact set $(0, 1]$; hence, the two intervals are not topologically equivalent (Wapner, 2005, p. 121).

we can define a second set of spatio-temporal distance relations. Theories that postulate two metrics on a given manifold are called *bimetric* theories.[4] And, of course, nothing at the level of logical or mathematical consistency forbids us from defining more than two metrics on the members of **M**; theories that postulate n metrics on a given manifold can be called n-metric theories.

For an intuitive grasp of the notion of a bimetric theory, consider once more the clopen interval $(0, 1]$. Consider two points in that interval, for example, the points labeled by 0.5 and 0.7. On one way of defining the distance between the two points, the distance is the absolute value of the difference between their respective labels, i.e., $|0.7 - 0.5| = 0.2$. We can define another metric according to which the distance between any two points is the absolute value of the difference in the squares of the two labels, i.e., $|0.7^2 - 0.5^2| = 0.24$. We ordinarily think that the distance between two points has a unique value. But on a bimetric theory, there are two distances between any two points. In our example, the distance between the points labeled by 0.5 and 0.7 is *both* 0.2 *and* 0.24.

In theoretical physics, there are a variety of motivations for bimetric theories. Consider the following as a motivation that significantly problematizes the metrical conception of a beginning of the Cosmos. As Henri Poincaré (2001a, pp. 55–57) and Hans Reichenbach (e.g., 1958, pp. 30–34, 118–119) famously pointed out, any determination of chronogeometry will be systematically underdetermined. We can always save the hypothesis that space-time has some specific chronogeometry by introducing forces that universally act on measuring instruments and distort all measurements taken by rulers or clocks. Poincaré and Reichenbach argued that, given our inability to determine which effects are legitimately chronogeometrical, there is no fact of the matter as to which effects are due to forces and which are due to chronogeometry. Philosophers of science have since given up on verificationism and are less prone to infer from systematic underdetermination between two hypotheses h_1 and h_2 that there is no fact of the matter as to which of h_1 or h_2 are cor-

[4]↑Bimetric theories indistinguishable from standard General Relativity have been considered in Feynman et al., 2003; Pitts, 2019; Pitts and Schieve, 2003, 2004, 2007; Lockwood, 2007, pp. 335–336. A similar – though in principle observationally distinguishable – theory was considered in Pitts and Schieve, 2007 and Pitts, 2019; that theory approximates standard General Relativity arbitrarily well given a sufficiently small graviton mass.

rect. For that reason, we can rethink Poincaré's and Reichenbach's point; perhaps we can distinguish between the effective metric handed to us by our observations and whatever metric legitimately describes our space-time despite our observations. In that case, the true duration of past time could be systematically hidden from us precisely because the true metric would be epistemically inaccessible. In that case, we would have no right to infer from the Cosmos appearing to have a finite age that the Cosmos really does have a finite age. (Note that I am merely discussing this case as an epistemic possibility for the course of future inquiry and not endorsing it. There may be other extra-empirical theoretical virtues that would help us to distinguish hypotheses about physical chronogeometry, e.g., parsimony and the like.)

According to General Relativity, the distribution of matter-energy across space-time determines **g**. For that reason, insofar as **g** can be determined from observations, **g** is determined from the observed matter-energy distribution. (There is reason to think that **g** cannot be generically determined from the observations that would be available to any observer embedded within space-time, but set that aside until chapter 9.) But if a bimetric (or *n*-metric) theory turns out to be true, then the metric that can be constructed from observations may not have any fundamental significance for the duration of past time. Moreover, in the case that a bimetric theory does turn out to be true, perhaps we would be able to determine both metrics. However, suppose that one metric is useful for describing some class of phenomena and another metric for another class of phenomena. For example, in the previously discussed example, the distance between the points labeled by 0.5 and 0.7 is 0.2 with respect to one metric and 0.24 with respect to another. If both metrics are required by fundamental physical theory, where one metric is required to describe one set of physical phenomena and the other metric is required to describe another set of physical phenomena, then we should say that the points labeled by 0.5 and 0.7 are 0.2 distance units apart in one respect and 0.24 units apart in another respect.

Just as two points can be two distinct distances apart if fundamental physical theory includes two metrics, so, too, the Cosmos may be finitely old with respect to one metric and infinitely old with respect to another metric. In that case, even supposing that both metrics could be empirically determined, if a beginning of the Cosmos requires a finite past, there may not be a determinate fact as to whether the Cosmos began (see, for example, Swinburne and Bird, 1966, p. 128; Halvorson and Kragh, 2019; Milne, 1948; Misner, 1969; Roser, 2016; Roser and Valentini, 2017).

If we set aside the metric and focus only on **M**, then we have set aside all facts about the duration of past time. **M** is a point set that has set theoretic properties, such as cardinality, and topological properties, but not metrical properties. Since **M** does not come equipped with metrical properties in itself, we cannot, by focusing only on **M**, mathematically distinguish between whether **M** is a space-time with an open boundary in the finite past and a space-time with an open boundary in the infinite past. However, **M** is equipped, by construction, with topological structure. The distinction between an open and a closed boundary is a topological feature. Therefore, without appealing to any metrical facts, we can mathematically distinguish a space-time with a closed boundary – that is, a space-time with a topological beginning – from a space-time without a closed boundary – that is, a space-time without a topological beginning.

To complete my discussion of the topological conception, I turn to unpacking three distinct families of ways for the Cosmos to have a topological beginning. As we will see, two such ways are counterintuitive and surprising. The first family has a topological beginning in the most intuitive sense; that is, all members of the first family are such that there is a single closed boundary to the past of all non-initial space-time points. Consider, for example, flat (Minkowski) space-time. Let's define a system of coordinates with respect to a reference frame F and let's excise the portion of the space-time below the line $t = 0$. The resulting space-time has a closed boundary at $t = 0$ and so features a shared closed boundary to the past of all non-initial space-time points. If the space-time also satisfies the first two conditions for having a beginning, then, intuitively, the space-time's initial closed bounding surface is the space-time's beginning.

We can now turn to the second family. Let's first remind ourselves of the three conditions that are necessary for a topological beginning, i.e., that (i) at all of the closest possible worlds where time does not exist, the Cosmos does not exist; (ii) the Cosmos has a global direction of time; and (iii) there is a closed boundary to the past of every non-initial space-time point. A three-dimensional cross-section of the four-dimensional space-time block such that every pair of points in the cross-section are space-like related is said to be a *space-like surface*. Consider a space-time with space-like surface Σ and such that, in some specific reference frame F, Σ is a particular instant of time.[5] Let's suppose that the space-time is populated only by particles whose worldlines (or space-time worms) intersect Σ and that do not undergo any non-gravitational forces. (That is, space-time is populated only by particles traversing a time-like geodesic congruence.) Let's define the age of a particle according to F as the time that has elapsed since the particle's beginning in reference frame F. Suppose that for every particle whose age, in F, is a at Σ, there exists another particle whose age, in F, is $a + \varepsilon$, where $\varepsilon \in \mathbb{R}$ and $0 < \varepsilon < \infty$. In this case, even though every particle in the space-time had a beginning at some time in the finite past, so that every particle's worldline has a closed boundary to the past, there is no closed boundary shared by all worldlines in the entire space-time. Importantly, we can always trace the history of the space-time further back – according to time as measured in F – so that there is no specific time at which the Cosmos began. That is, there are examples of space-times where every object in the space-time began to exist, but there is no *one* time (or one space-like surface) at which the space-time, itself, began.

According to a now famous theorem due to philosopher David Malament (1977b), for temporally orientable space-times that possess a local past/future distinction, the space-time's topological, differential, and conformal structure can be completely determined by specifying a class of continuous time-like curves. Since all classical space-times with a topological beginning satisfy the Direction Condition and so have a global direction of time, Malament's theorem is applicable to all of the classical space-times we are considering. This suggests that we can construct all of the classical space-times with a topological beginning, up to but not including their metrical structure, by specifying a class of time-like curves. In the thought experiment in the previous paragraph, we considered the worldlines of particles piercing Σ. We can construct a space-time using the class of time-like curves that pierce Σ and, given Malament's theorem, that class of curves will suffice

[5]↑One consequence of the relativity of simultaneity is that different reference frames will disagree about which space-like surfaces correspond to instants of time, so Cosmos-wide instants can only be specified relative to a particular reference frame.

for determining the non-metrical structure of one family of classical space-times with a topological beginning. And this suggests a class of space-times with a "jagged" closed boundary, so that, in some sense, one's distance from the beginning of the Cosmos depends upon where one resides within the Cosmos. However old the Cosmos is in one's own "neck of the woods", there may be some other space-time point in Σ where, according to F, the Cosmos is older.[6]

Let's turn to a third family of classical space-times with a topological beginning. Once again, consider a classical space-time with a space-like surface Σ that corresponds to a particular instant of time according to the coordinates defined by reference frame F. And let's also suppose that the space-time is populated only by particles with worldlines that intersect Σ. This time, let's assign each particle the index ε, where ε is a real number between 0 and ∞ and such that there is a particle for each value of ε. Let's say that $a^F(\varepsilon)$ is the "age" of particle ε according to reference frame F at Σ. Now define the particles respective ages as a function of ε:

$$a^F(\varepsilon) = \frac{1}{1 + \mathrm{e}^{-\varepsilon}} \tag{7.1}$$

Notice that in the limit that ε increases without bound, $a^F(\varepsilon)$ approaches 1. That is, according to the coordinates defined by F, no particle has an age greater than 1, even though there is no oldest particle. In this case, there is no closed boundary shared by all particles, since each particle began at a distinct instant (relative to F), but the space-time is still bounded to the past because no part of the space-time is older (again, relative to

[6]↑A simple example of one such space-time can be explicitly constructed by modifying Minkowski space-time using the following procedure. Select some reference frame F. Consider the space-like surface $t = 0$ in frame F. Keep the portion of Minkowski space-time above $t = 0$ and throw away the portion below $t = 0$. The space-like surface $t = 0$ now forms a boundary to the space-time; let's call that boundary $\mathbb{B}$. Now perform a Lorentz boost into a frame F' in motion relative to F.

For simplicity sake, suppose that we performed the aforementioned operations on a two-dimensional space-time, where the t-axis runs up the page and the x-axis runs horizontally. In frame F, the boundary $\mathbb{B}$ is a horizontal line corresponding to the x-axis. In F', $\mathbb{B}$ is a diagonal line. Consider an observer, let's name them Albert, in frame F'. From Albert's perspective, Albert is a finite proper time – let's say T – from the closest point on $\mathbb{B}$ to Albert. T can be used to specify a simultaneity slice relative to F'. There is another point A on the T simultaneity slice some distance $\Delta x'$ away from Albert such that the shortest distance between $\mathbb{B}$ and A is $T + \varepsilon$. Since, relative to F', $\mathbb{B}$ is a diagonal line, we can always find a value of $\Delta x'$ such that ε is as large as we'd like. And since we can make ε arbitrarily large, there is no maximal value to the time, relative to F', between $\mathbb{B}$ and the corresponding closest point on T.

F) than 1. Malament's theorem suggests that we can define a class of time-like curves, all of which have a closed boundary in their respective pasts, even though there is no closed boundary shared by any two time-like curves. One may have the intuition that this family of space-times has a beginning in a stronger sense than the first family of classical space-times with a topological beginning that we examined. Indeed, this is so, because, in the sense to be explained below, this family of classical space-times has both a topological *and* a metrical beginning. But, contrary to our intutitions, the shared metrical beginning is an open boundary – since there is no time-like curve such that $a^F(\varepsilon) = 1$ – while the local and unshared "beginnings", i.e., the start of each time-like curve, is closed.

Unfortunately, although we can mathematically distinguish space-times with a topological beginning from space-times without a topological beginning, we cannot, in general, *empirically* distinguish the two. Again, the only features of space-time that can be empirically discovered are those related to the distribution of the matter-energy populating space-time. In the case of classical space-times, General Relativity ties a specific kind of boundary to space-time, i.e., curvature singularities, to the matter-energy distribution. However, curvature singularities are open boundaries. Thus, the only boundaries to classical space-times that are tied to the matter-energy distribution do not represent topological beginnings. Recall the criteria that I stated at the outset, i.e., that the three conditions I identify for the Cosmos to have a beginning be necessary for the Cosmos to have a beginning, that the criteria should be useful in determining whether the Cosmos had a beginning, and that the criteria should help to elucidate the concept of a beginning. Given that a topological beginning would not be tied to the matter-energy distribution and would, for that reason, not be empirically discoverable, and that there is little hope for discovering the Cosmos's beginning through non-empirical means, the topological conception of the Cosmos's beginning is not helpful in determining whether the Cosmos had a beginning. Nonetheless, there is another sense in which the Cosmos could have a past boundary, to which I now turn.

7.2 The Metrical Conception

Suppose that time is absolute and has an open boundary to the finite past such that there is no time before the boundary at which the Cosmos exists. Since the boundary is open, the topological conception would say that the Cosmos did not begin to exist. Nonetheless, there is a strong intuition that one way for the Cosmos to begin to exist would involve time having an open boundary in the finite past (or, if one endorses the Oxford School, there is a time interval with an open boundary such that the Cosmos does not exist before the boundary). And there is a strong intuition that if the Cosmos had another kind of open boundary – namely, an open boundary infinitely far to the past of all space-time points – then the Cosmos did not begin to exist. Since this intuition concerns the lapse (or total duration) of past time, following Pitts (2008), we can call the resulting conception of the Cosmos's beginning the *metrical conception*. Craig and Sinclair, following Smith (1985), endorse a metrical conception of the beginning of time:

> [...] we can say plausibly that time begins to exist if for any arbitrarily designated, non-zero, finite interval of time, there are only a finite number of isochronous intervals earlier than it; or, alternatively, time begins to exist if for some non-zero, finite temporal interval there is no isochronous interval earlier than it (Craig and Sinclair, 2012, p. 99).

Note that Craig and Sinclair's metrical conception is expressed disjunctively; while Craig and Sinclair mean for the second disjunct to be equivalent to the first, as I will show, the two disjuncts are not equivalent. Swinburne has endorsed a similar condition for a beginning of the Universe in the finite past, where, by 'Universe', Swinburne means roughly what I mean by 'Cosmos':

> [...] to say that the Universe began a finite time ago is to say that all physical objects spatially related to ourselves began to exist after a certain date, a finite time ago. To claim that the Universe is eternal is to deny that there is any date of which the last statement is true. [...] what does it mean to say that something had a beginning a finite time ago? [...] If the Universe can be shown

> to have begun *n* units of [a time scale defined by an ideal clock] ago, where *n* is a finite number, then the Universe can be said to have begun a finite time ago (Swinburne and Bird, 1966, pp. 127–128).

Elswhere, Swinburne (2004, p. 138) writes, “The interesting question about whether the universe is of finite age, or of infinite age, is the question about whether there has been a universe only for no more than a finite number of periods of equal length (for example, a finite number of years) or whether it has existed for an infinite number of such periods.”

Swinburne’s account of a finitely old Universe differs from Craig and Sinclair’s account of the Universe’s beginning for three reasons. First, Swinburne denies that the beginning of time is metaphysically possible and so claims that time existed before the Cosmos. For that reason, Swinburne’s account differs from accounts on which time began when the Cosmos began. Second, Craig and Sinclair’s first disjunct stipulates that when we pick out *any* arbitrarily specified finite interval of time, there are only a finite number of isochronous earlier intervals and Craig and Sinclair’s second disjunct stipulates that there exists *some* finite interval with a finite number of preceding isochronous intervals. In comparison, Swinburne’s conception demands that we pick out a particular instant as the present and that there are only a finite number of isochronous intervals earlier than the present. Third, while Swinburne offers a sufficient condition for the Cosmos (or the Universe) to have a beginning, Swinburne has elsewhere, e.g., (1996), argued that beginning a finite time ago is not necessary for the Cosmos to have begun.

In this section, I construct a new metrical conception of the beginning of the Cosmos. Swinburne (1996) offers a thought experiment from which he concludes that the Cosmos having an infinite past would not entail that the Cosmos is beginningless. Instead of reiterating Swinburne’s thought experiment, I offer three new thought experiments. The new metrical conception will fulfill three desiderata. First, the metrical conception should be consistent with the sufficiency of a finite past for establishing that the Cosmos had a beginning. Second, the new metrical conception should be consistent with the Cosmos, as described in the three thought experiments, having a beginning. Third, there should be cases where the new metrical conception agrees with our intuition that the Cosmos had no beginning. As we will see, the new metrical conception, in certain respects, resembles the second disjunct in Craig and Sinclair's conception.

7.2.1 Three Thought Experiments

I now turn to a consideration of the three thought experiments. Before explicating the three thought experiments, I briefly describe a collection of preliminary mathematical notions. Given two sets, e.g., $S_1 = \{a_1, b_1, ...\}$ and $S_2 = \{a_2, b_2, ...\}$, the *Cartesian product* of the two sets, denoted, e.g., $S_1 \times S_2$, is the set of all pairs taken from the two sets, e.g., $S_1 \times S_2 = \{(a1, a_2), (a1, b_2), ..., (b_1, a_2), ...\}$. That is, $S_1 \times S_2 \equiv \{(x, y) | x \in S_1 \& y \in S_2\}$. We can label all of the points in a given space by considering Cartesian products of the appropriate sets. For example, there is an isomorphism between a two-dimensional plane and the Cartesian product of the real line with itself, so that the set of points in the plane can be represented by $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$. We can represent the points in an n-dimensional space recursively, e.g., $\mathbb{R}^n \equiv \mathbb{R} \times$ ...(n-2 times)... $\times \mathbb{R}$. Describing a space's manifold in terms of Cartesian products of subsets of $\mathbb{R}$ allows us then to define the space's metrical properties in terms of functions over $\mathbb{R}$. Note that the collections of real numbers used to label the points in a given manifold do not carry any information about how far apart the two points are; to define the distance between two points in **M**, we need to define one or more metrical relations on **M** as well as the "lengths" of some appropriate set of curves connecting the two points.[7]

As I've said, relativistic space-times are a pair of objects, i.e., first, a set of points (the manifold) **M** and, second, the metric tensor **g**. We can provide an analogous, albeit anachronistic, description for pre-relativistic space-times. Newtonian and Galilean space-times are described by the manifold $\mathbb{R}^4$, a temporal metric **t**, describing the duration between any two instants of time, and a spatial metric **h**, describing the spatial distance between any two points in space. Newtonian/Galilean space-times can be subdivided into

[7]↑For example, in relativistic space-times, **g** is a rank 2 tensor, with components $g_{\mu\nu}$, from which we can compute the "distance" (that is, the interval) between points p and q by maximizing $\int_p^q \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$, where the integral is computed along a path from p to q. For philosophical discussion and elaboration, see Bricker, 1993.

three-dimensional spaces, where each three-dimensional space has a unique value of **t** – representing the space-time at a given value of absolute time – and in which **h** defines the standard Euclidean distance between any two points. In Newtonian space-time, points of space persist over time – which can be represented by re-identifying the same space-time points at successive times – whereas, in Galilean space-time, points do not persist over time.

Before continuing on to a discussion of the three thought experiments, I need to introduce a general principle that I will use to reach the lessons that I take from each of the thought experiments. Given any two observers A and B, if the Cosmos began for A then the Cosmos began for B and vice versa. If a version of the Boundary Condition entails that the Cosmos began relative to some observer and did not begin relative to some other observer, then that version of the Boundary Condition is inadequate.

Having laid out some mathematical foundations and stated a general principle, I continue on to a discussion of the three thought experiments.

The Partially Amorphous Cosmos

Some cosmological models include a space-time region where there are no metrical facts and another space-time region where there are metrical facts. Consider Bradford Skow's (2010) argument that an objective space-time metric might not be either an intrinsic feature of space-time or wholly the result of features intrinsic to space-time. Instead, Skow argues, space-time might have an objective, but extrinsic, temporal metric just in case there is some x that plays the functional role, in the physical laws, of determining the ratios between any two non-overlapping spatio-temporal intervals.[8] If metrical facts require a specific functional role to be fulfilled, then, in space-time regions where that functional role is not fulfilled, there might not be any metrical facts, even though metrical facts do obtain in other space-time regions.

For example, in Roger Penrose's (2012) Conformal Cyclic Cosmology, there are no facts about spatio-temporal scale, that is, no metrical facts, at early or late times in the history

[8]↑Skow cashes out his view in terms of absolute time, but indicates that he intends for his view to be generalizable to relativistic space-times.

of the observable universe.[9] A temporal (or spatio-temporal) interval for which there is no fact concerning the length of the interval – that is, an interval to which metrical facts are inapplicable – is said to be *amorphous*. To put the view another way, if space-time is metrically amorphous within some region, then there is no objective fact about the ratio of the durations of two non-overlapping intervals within that region. In relativistic space-times, lengths and temporal intervals depend upon the adoption of a specific reference frame. Amorphous time goes one step further in that if time is amorphous then, even within a given reference frame, there are no facts about how long a given temporal interval is. One example of amorphous time is time for which metrical facts are purely conventional, as already discussed, though amorphous time can also be such that one cannot even adopt a conventional metric. For the sake of simplicity, let's suppose that Newton and Galileo were correct that time is absolute.[10] Let's also suppose that there is a finitely long interval of non-amorphous time labeled A, followed by an interval of amorphous time labeled B, and then followed again by an interval of non-amorphous

[9]↑On some quantum gravity theories – such as causal set theory (Bombelli et al., 1987; Brightwell and Gregory, 1991; Dowker, 2006, 2013, 2017, 2020) – the space-time metric appears only in the theory's continuum limit, thereby allowing for the possibility that there are regions of the Cosmos where the space-time metric is inapplicable. However, we should not necessarily think of those regions as amorphous in the sense discussed in this section. Consider, for example, consider Brightwell and Gregory's (1991) construction of the continuum limit for a space-time interval spanned by a number of space-time atoms "linked" together in a chain. When the chain is sufficiently long, the space-time interval is proportional to the number of links in the chain. As causal set theorists like to say, in causal set theory, metrical facts are determined by counting. For that reason, supposing that there are only a small number of space-time atoms in some region, so that the continuum limit does not apply in the region, we need only consider a larger region to recover relevant metrical facts. In any case, recall that the Boundary Condition for the Cosmos to have a beginning is disjunctive. If the initial portion of the Cosmos is correctly described by causal set theory, then, since causal sets always have closed boundaries, the Cosmos would satisfy the first disjunct – by having a topological beginning – and so would have a beginning.

[10]↑Nothing crucial in this example hangs on whether time is absolute. The example can be reconstructed for relativistic space-times. To construct a relativistic space-time without metrical structure, first consider a space-time S with metric $g_{\mu\nu}$. And now consider the metric $\tilde{g}_{\mu\nu}$ produced from $\tilde{g}_{\mu\nu}$ by the conformal transformation $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ where Ω is a positive and smooth but otherwise arbitrary scalar function. For relativistic space-times, multiplication by Ω^2 leaves the space-time's light cone structure unaltered. Call the resulting space-time $\tilde{S}$. Two space-times that are related by such a transformation, e.g., S and $\tilde{S}$, are said to be *conformally equivalent*. A space-time *without* metrical structure can then be constructed by identifying all of the members of a given class of conformally equivalent space-times. Let's call the space-time that results from identifying all of the members of a given class of conformally equivalent space-times S_C. Since the conformal transformation left the light cone structure unaltered, S_C is equipped with light cone structure but not metrical structure and so S_C is an example of a relativistic amorphous space-time. For related technical details, see chapter 11 and references therein. To construct a relativistic space-time analogous to the space-time inhabited by Pam and Jim, one can "glue" a metrically amorphous space-time region R between two regions that are not metrically amorphous.

time labeled *C*. Formally, we are supposing that *A* is a Newtonian or Galilean space-time region with an objective temporal metric, that *B* has the topology of a Newtonian or Galilean space-time region without an objective metric, and that *C* is another Newtonian or Galilean space-time region. Suppose, further, that the Cosmos does not exist prior to *A*. Call this construction the *Partially Amorphous Cosmos*.

Suppose that Pam is an arbitrarily chosen observer in *A*. Pam should say that the Cosmos began in her finite past. Suppose that Jim is an observer in *C*. For Jim, since there is an interval of amorphous time between himself and the beginning identified by Pam, there is no fact concerning how far in the past the Cosmos began. Consequently, even though, intuitively, Jim should agree that time began, there is no fact about how many isochronous intervals can be placed into Jim's past. Since there is no fact about how many isochronous intervals can be placed into Jim's past, Craig and Sinclair's first disjunct entails the intuitively wrong conclusion that the Partially Amorphous Cosmos did not begin to exist. Swinburne's metrical conception entails the intuitively wrong conclusion that the Partially Amorphous Cosmos began to exist for Pam but not for Jim. A Newtonian or Galilean Cosmos with a beginning can have a non-initial segment in which there is no objective temporal metric. Instead of articulating the metrical conception in terms of there being a determinate number of isochronous intervals to the past of every temporal interval, as in Craig and Sinclair's first disjunct, or as indexed to some observer's present, as with Swinburne, the metrical conception should entail that, for space-times with a metrical beginning, time is not metrically amorphous in the initial segment of the Cosmos's history.[11]

[11]↑A similar point has been previously made in various places, but, in particular, see Earman, 1977, pp. 125–126, 131. For example, Hermann Weyl (1997) maintained that the choice of time scale is, is to a certain degree, conventional. In more technical terms, Weyl argued that there is gauge freedom in one's choice of metric tensor so that the metric tensor is determined only up to a conformal factor, as in footnote 10. Additional technical details for Weyl's theory can be found in Bell and Korté, 2016. The result, if Weyl were correct, is that there is gauge freedom in the proper time along any given trajectory. Weyl postulated that the gauge could be fixed for each trajectory, but only in such a way that the rate at which a given clock ticks depends the clock's specific trajectory. Someone who returns to Earth after having traveled at close to the speed of light would discover not only that their twin's clock read differently than their own – as in Einstein's relativity – but that their twin's clock ticks at a different *rate*.

In any case, were Weyl's theory correct, time scale would not correspond to any objective physical fact (Penrose, 2004, p. 451). Einstein objected that frequency and mass can be related through quantum mechanics (i.e., $E = hf$) and relativity (i.e., $E = mc^2$). Provided a frequency, one can construct a clock. So,

The Fractal Cosmos

There are a number of fractal curves whose arc length is infinite (von Koch, 2004, p. 38; Mandelbrot, 2004), even though they occupy a finite region of the plane. Consider, then, a fractal curve with infinite arc length that occupies a region of the $x - y$ plane with an end point at the left at $x = -1$ and an end point at the right at $x = 1$. We can "glue" finitely long line segments, parallel to the x-axis, to the curve's left end point and another to the curve's right end point. Call the line segment on the left L, the fractal curve C, and the line segment on the right R. Restricting ourselves to the resulting $L - C - R$ compound geometric object, notice that:

1. There is a finite distance between any two points in L.

2. There is a finite distance between any two points in R.

given that any observer with mass effectively carries a clock, there is an objective way for any observer with mass to fix a time-scale (Bell and Korté, 2016; Penrose, 2004, p. 453). Conversely, if Weyl's theory had been correct, then the masses of particles wouldn't be fixed and would vary with the history of a given particle. This would have violated the quantum mechanical principle that identical particles have identical masses. Thus, on Einstein's proposal, whether there are facts that distinguish finite and infinite temporal durations depends upon the local matter-energy distribution, i.e., in the absence of mass, there is no objective distinction between finite and infinite temporal intervals. One can adopt Einstein's proposal that the masses of objects determine the frequencies for locally fixing time scales without adopting Weyl's theory. If Einstein's proposal is correct and objective time scales depend on the local presence of mass then, if there were no masses in the initial segment of the Cosmos's history, there would be no objective distinction between the Cosmos having an infinitely or finitely long initial segment. In that case, unless the initial segment of the Cosmos has a closed boundary to the past of all other space-time points not on the boundary, the Cosmos would lack a beginning.

One set of authors has argued along independent and fairly different lines that in classic models of the Big Bang, which are often said to include a beginning, our universe's past should not be understood as objectively finite. Standard cosmological (i.e., Friedmann-Lemaître-Robertson-Walker or FLRW) space-times can be sliced into space-like hypersurfaces such that the mean extrinsic curvature is constant on each hypersurface. This is known as the Constant Mean (extrinsic) Curvature (CMC) foliation. We can label the hypersurfaces in the CMC foliation with the cosmic time, that is, time as recorded by observers who are locally at rest with respect to the universe's expansion. When cosmologists say that the universe has a finite age according to a given cosmological model, they typically mean that there is finite cosmic time to the past. Nonetheless, the hypersurfaces in the CMC foliation can be relabeled in such a way that the order of the hypersurfaces is preserved. For example, one can relabel the CMC hypersurfaces by the scale factor, a measure of a length-scale characteristic of the universe's expansion, or by various monotonic functions of the scale factor. Importantly, one can relabel the CMC hypersurfaces with parameters, e.g., the York time (York, 1972; Roser, 2016, p. 49), that map the singular surface bounding FLRW space-times to the infinite past. Various authors (Milne, 1948; Misner, 1969; Roser, 2016; Roser and Valentini, 2017) have argued that labelings that *do* map the singularity to the infinite past are more physically significant than the cosmic time. Consequently, showing that one inhabited an FLRW space-time bounded by a past singularity might not be sufficient for showing that the Cosmos had a beginning; one must also show that past time is objectively finite.

3. There is an infinite distance between any point in L and any point in R.

Once more, for the sake of simplicity, suppose that Newton and Galileo had been right that time is absolute.[12] Moreover, suppose that absolute time had the metrical structure of $L - C - R$. Suppose that Pam is an arbitrarily chosen observer in the L segment of history. By construction, L is a finitely long line segment; for that reason, there is only a finite period of absolute time to Pam's past (or, for the Oxford School, only a finite period of absolute time to Pam's past during which the Cosmos exists). Intuitively, supposing that the Modal Condition is satisfied, the metrical conception, when conjoined with the fact that there is only a finite period of absolute time to Pam's past, should strongly suggest that there was a beginning of absolute time. (Alternatively, so long as the Modal Condition is satisfied, the metrical conception and the fact that the Cosmos has only finite past temporal extension should strongly suggest that there was a beginning of the Cosmos.) However, for any arbitrarily chosen observer – call them Jim – in the C or R segments of history, the beginning suggested by Pam is located infinitely far in the past. For that reason, Craig and Sinclair's first disjunct clashes with intuition by entailing that the Fractal Cosmos has no beginning while Swinburne's version clashes with intuition by entailing that the Cosmos has a beginning for Pam but not for Jim. Again, we need a conception on which whether the Cosmos has a beginning is not observer relative.

The Partial Sum Cosmos

In this section, I'll consider two different mathematical constructions as thought experiments: first, a more "pedestrian" version and, second, a more technical version that draws upon the way in which partial sums relate to infinite convergent series. I offer the pedestrian version so that readers who cannot follow the technical version can at least draw the core points from the pedestrian version. In the pedestrian version, let's begin by considering the series of positive integers in increasing order: 1, 2, 3, ... Mathematicians say that the sequence has order type ω. Sequences of order type ω do not have a last member, but we can add in a last member z by defining z such that z comes after every

[12]↑An analogous construction can be produced using relativistic physics.

member in the sequence. We can then use z to define a new sequence: 1, 2, 3, ..., z. We can also consider the series of negative integers in increasing order: ..., −3, −2, −1. This sequence has order type ω^*. Sequences with order type ω^* have no first member – since the sequence of negative numbers has no start – but we can add in a first member a by defining a such that a comes before every member in the sequence. We can then use a to define a new sequence: a, ..., −3, −2, −1. Lastly, we can "glue" together the ω and $\omega*$ sequences by identifying z with a: 1, 2, 3, ..., z, ..., −3, −2, −1. Call this the $\omega - \omega^*$ sequence. Given a countably infinite set of points, we can identify each point in the set with one member of the $\omega - \omega^*$ sequence and we can define topological relations such that points labeled by sequential values in the $\omega - \omega^*$ sequence are neighbors, e.g., the point labeled 1 is to the right of the point labeled 2 and all of the points labeled by negative integers are to the right of the positive integers.

We can define a metric over the point set labeled by the $\omega - \omega^*$ sequence with the following properties: (i) the distance between any two points in the portion labeled by the positive integers is given by the absolute value of the difference between the corresponding two integers, (ii) the distance between any two points in the portion labeled by the negative integers is given by the absolute value of the difference between the corresponding two integers, (iii) the distance between z and any other point is infinite, and (iv) the distance between any of the points labeled by a positive integer and any point labeled by a negative integer is infinite. We've succeeded in defining a point set equipped with topological structure, that is, a manifold, and a metric. If time had the corresponding structure – so that −1 is used to label the present – then, even though there might be infinite time to our past, we should still intuitively say that time has a beginning. Time would have a first instant, namely, the point labeled by 1.

For the more technical version, let's begin by considering the infinite sum:

$$\frac{6}{\pi^2}\sum_{1}^{\infty}\frac{1}{n^2} = 1 \tag{7.2}$$

As with any convergent infinite series, the value of 7.2 is defined in terms of the limit of the corresponding partial sum:

$$q_N = \frac{6}{\pi^2} \sum_1^N \frac{1}{n^2}$$

In turn, since the generalized harmonic numbers $H_N^{(r)}$ are defined as $H_N^{(r)} \equiv \sum_1^N 1/n^r$, we can re-write the partial sum q_N in terms of $H_N^{(2)}$:

$$q_N = \frac{6}{\pi^2} H_N^{(2)}$$

Given the properties of the generalized harmonic numbers, the variable q_N can assume any one of the values in a set that is bounded from below by $6/\pi^2$ and from above by 1, i.e., $q_N \in \{\frac{6}{\pi^2}H_1^{(2)}, \frac{6}{\pi^2}H_2^{(2)}, ...\}$. Furthermore, define S_1 such that $S_1 \equiv \{q_1, q_2, ...\} \cup \{1\}$. Define a function $d(x_N, y_M) = |M - N|$ such that x_N and y_M are both possible values of q_N. $d(x_N, y_M)$ maps a pair of values in S_1 to the set of positive integers. Furthermore, define $d(x_N, 1) = d(1, y_M) \equiv \infty$. We can then think of $d(x_N, y_M)$ as a metric defining the distance between points labeled by x_N and y_M; moreover, we've defined the metric such that any one of the points labeled by values of $q_N < 1$ are infinitely far from 1.

Let's now "paste" a mirror image copy of this collection on to the points along the real line labeled by numbers between 1 and $2 - 6/\pi^2$, i.e., points labeled by numbers in the set $\{..., 2 - 6/\pi^2 H_2^{(2)}, 2 - 6/\pi^2 H_1^{(2)}\}$. We can do this by considering the sum

$$2 - \frac{6}{\pi^2} \sum_1^\infty \frac{1}{n^2} = 1$$

As well as the partial sum

$$q'_N = 2 - \frac{6}{\pi^2} \sum_1^N \frac{1}{n^2}$$

Define $S_2 \equiv \{..., 2 - 6/\pi^2 H_2^{(2)}, 2 - 6/\pi^2 H_1^{(2)}\} \cup \{1\}$. For the points labeled by values in S_2, we can define a corresponding metric as before, i.e., $d'(x'_N, y'_M) = |M - N|$. Using this metric, points labeled by values between 1 and $2 - 6/\pi^2$ are infinitely far from the point labeled by 1. We can now define a combined metric for the collection of points labeled by values between $6/\pi^2$ and $2 - 6/\pi^2$:

$$D(x_N, y_M) \equiv \begin{cases} |M-N| & x_N < 1 \ \& \ y_M < 1 \\ \infty & x_N < 1 \ \& \ y_M \geq 1 \\ \infty & x_N \geq 1 \ \& \ y_M < 1 \\ |M-N| & x_N > 1 \ \& \ y_M > 1 \end{cases}$$

Using $D(x_N, y_M)$, we can construct a space-time, featuring discrete time, in the following way; let's call this space-time the *Partial Sum Cosmos*. Suppose that time is discrete and, for simplicity, suppose, again, that time is absolute. Further, let us suppose that the whole of history consists of a set of temporal atoms labeled by values (as described in the construction above) between $6/\pi^2$ and $2 - 6/\pi^2$ with a temporal metric given by $D(x_N, y_M)$. Arbitrarily pick an observer situated at one of the temporal atoms a_P after the atom labeled $6/\pi^2$ and before the atom labeled 1. Once again, call this observer Pam. Because the temporal atom labeled $6/\pi^2$ is the first temporal atom, there is a boundary to absolute time located in Pam's past. Intuitively, since, according to the temporal metric $D(x_N, y_M)$, Pam is a finite distance from that boundary, i.e., $D(a_P, 6/\pi^2) < \infty$, Pam should conclude that time had a beginning in the finite past. However, for any of the observers located at one of the temporal atoms to the future of the temporal atom labeled by 1, the beginning identified by Pam is infinitely far to their past. This is analogous to the result we found for the Fractal Cosmos, where, for some set of observers, past time is infinite, even though we should intuitively say that time had a beginning.

Craig and Sinclair's first disjunct reaches conclusions for the Partial Sum Cosmos that clash with our intuitions for reasons parallel to those we identified for the Fractal and Partially Amorphous Cosmoses. That is, Craig and Sinclair's first disjunct would say that the Partial Sum Cosmos is beginningless. Swinburne's metrical conception also reaches conclusions for the Partial Sum Cosmos that clash with intuitions for reasons parallel to those we identified for the Fractal and Partially Amorphous Cosmoses. Swinburne's metrical conception would say that there is one set of observers in the Partial Sum Cosmos for whom the Cosmos began and another set of observers for whom the Cosmos did not begin. According to the new metrical conception that I develop below, supposing the

Partial Sum Cosmos satisfies the other conditions necessary for having a beginning, the Partial Sum Cosmos's beginning is not relative to any set of observers.

Drawing lessons from the three thought experiments

In the cases of the Fractal Cosmos and the Partial Sum Cosmos, one may object that infinity is not a number, in which case distances and temporal intervals cannot be infinite. Four replies can be offered. First, that there are space-time points between which the temporal interval is not well-defined would suffice for my purposes. For that reason, if we understand the temporal intervals involved not as infinite but as divergent – and so as not well-defined – similar conclusions follow. Second, while infinity is not a *real* number, there are well known geometrical constructions in which points are included that are at an infinite distance from other points. One family of constructions are the fractal curves already discussed. For another example, consider the projection of the Riemann sphere on to the complex plane, which allows one to identify complex infinity with the sphere's north pole. There is no recognized *mathematical* difficulty involved in including "points at infinity" in a given construction. As I've shown in chapter 2, whether there are metaphysical problems involved in such constructions has yet to be successfully shown. Third, there are solutions to the Einstein Field Equations – such as Malament-Hogarth space-times – that include observers who, in finite time, can observe the results of a computation that takes infinite time to perform (Earman and Norton, 1993; Etesi and Németi, 2002; Hogarth, 1966; Manchak and Roberts, 2016). (On a more technical level, what's crucial about Malament-Hogarth space-times is the feature that a time-like half-curve, along which there is infinite proper time, can "fit" inside some observer's past light cone, where the observer is not located at time-like infinity.) If we accept some standard solutions to the Einstein Field Equations, e.g., Kerr black holes or anti-De Sitter space-time, as legitimately metaphysically possible, then we need to allow for the metaphysical possibility of infinite arc lengths. Fourth, supposing that one considers the Fractal Cosmos and the Partial Sum Cosmos as metaphysically impossible, one is still left with the Partially Amorphous Cosmos as a viable epistemic possibility.

According to Craig and Sinclair's account, the Cosmos began only if the Cosmos's history includes no more than a finite number of isochronous intervals earlier than any arbitrarily chosen interval. In the three thought experiments, so long as the Cosmos satisfies the Modal Condition and the Direction Condition, there is a strong suggestion that the Cosmos began; nonetheless, in the three thought experiments, there are some temporal intervals such that there is no finite or determinate number of isochronous intervals to that interval's past. According to Swinburne's account, the Cosmos began just in case there are a finite number of isochronous intervals earlier than the present. But in the three thought experiments, there could be observers situated such that there there is no finite (or determinate) number of isochronous intervals before their present. Moreover, since Swinburne's version of the metrical conception is indexed to the present of a given observer, Swinburne's version yields inconsistent conclusions about whether a given Cosmos had a beginning. Craig, Sinclair, and Swinburne's accounts – like my three thought experiments – assume that time is absolute. In the case of absolute time, the new metrical account should (roughly) say that (i) there is a (closed or open) boundary B to the past of all space-time points and (ii) there exists *some* time T such that, according to the objective metric of absolute time, the span of time between B and T is finite.

A good conception of the beginning of the Cosmos should not depend on whether time is absolute and should at least be consistent with relativistic physics. (Ideally, the account should also be consistent with a future quantum gravity theory, but, given that we do not yet possess a successful quantum gravity theory, the account that I offer here will need to be provisional.) For that reason, a metrical conception of the beginning of the Cosmos that did not assume absolute time is desirable. In order to construct a new version of the metrical conception that does not assume absolute time, I need to first explicate the generalized affine parameter. But in order to motivate the generalized affine parameter, we need to take a brief detour through relativity. Readers already sufficiently familiar with relativity to know what the generalized affine parameter is can skip the detour.

7.2.2 A brief detour through relativity

In pre-relativistic physics, one could imagine releasing synchronized and properly functioning clocks, each of which has some arbitrary velocity, from numerically one point. (Nevermind worries about whether two clocks can occupy one point.) We could then synthesize all of the subsequent clock readings together to form an absolute time, where disparate parts of the Cosmos would be said to be located at objectively the same time T just in case they coincide with a clock that reads T. And then one could imagine checking that the clock readings were properly synthesized together by re-collecting the clocks at numerically one point and noting that they all remained synchronized. In relativistic physics, we cannot successfully perform this synthesis. After the clocks are released, the hypersurface on which all of the clocks record the same time should not be understood as a moment of time for, in that context, simultaneity becomes relativized to one's trajectory through space-time. Moreover, when the clocks are collected together at numerically one point, we would find that their readings were no longer synchronized; instead, their readings depend on the path that each clock has taken.

Instead of defining an absolute time for the whole of the Cosmos, relativistic physics introduces the notion of *proper time*. Proper time is analogous to the distance recorded by a car's odometer; in some sense, proper time records the distance that an object moves along a given path through space-time. Famously, a young person who shoots off in a rocket ship at close to the speed of light, turns around, and returns to Earth may find the twin they left behind in a nursing home. The difference in the twins' respective ages is explained by the fact that the twins traversed distinct paths through space-time. The twin who remained on Earth traversed an objectively longer trajectory than the twin who went away and came back; the fact that their trajectory is longer explains the fact that they experienced a longer duration of time. If time is a parameter marked off along the trajectory of an object then we should not think of time as a parameter describing the global chronogeometric structure of space-time. Instead, space-time is a collection of points, none of which should be considered specifically spatial or temporal. On Minkowski's metaphysics, there is a four dimensional space-time block, but the fourth dimension is not time. For this reason, the popular notion that time is the fourth dimension is either mistaken or at least misleading. Instead of thinking of time as a dimension spanning the block, from one end to the other, we should think of time as marked out along trajectories, just as odometers mark out distances along the trajectories traversed by cars in three dimensional space.

Consider a right triangle on a Euclidean plane, whose base has length a and whose height is b. According to the Pythagorean theorem, the hypotenuse h of the triangle is given by $a^2 + b^2 = h^2$. By specifying two orthogonal axes, we can use the Pythagorean theorem to express any length in terms of distances along the two axes. The Pythagorean theorem has a natural generalization for expressing intervals in four dimensional space-time. For the sake of simplicity, I will consider Minkowski space-time, that is, a relativistic space-time from which gravity and curvature are absent. In Minkowski space-time, the length of any four-dimensional interval I can be written in terms of the coordinates defined by a given reference frame F. Projecting the interval on three perpendicular spatial directions x, y, z and the temporal axis t, all defined by F, we have:

$$-c^2t^2 + x^2 + y^2 + z^2 = I^2$$

The parameter c is the speed of light and can be thought of as the conversion factor between space and time. Notice that, unlike the Pythagorean theorem, there is a negative sign on the first term. The negative sign introduces the possibility that I can be zero even though t, x, y, and z are non-zero.

To understand the situation in which $I = 0$, let's consider the distance r, in three dimensional space, that a beam of light traverses in a time t. Assuming that the light starts its journey at $x = 0$, $y = 0$, and $z = 0$, we can use the Pythagorean theorem for three dimensional space to find the distance the light has traveled when the light reaches the point (x, y, z):

$$r = \sqrt{x^2 + y^2 + z^2}$$

We know that the distance traversed by an object is given by the speed of the object multiplied by the time over which the object travels. Since light moves at the speed c, we have that $r = ct$, or, in other words,

$$\sqrt{x^2 + y^2 + z^2} = ct$$

Squaring both sides and re-arranging, we have:

$$x^2 + y^2 + z^2 - c^2t^2 = 0$$

In other words, along the trajectory traveled by a beam of light, $I = 0$. What is the significance of the fact that $I = 0$ along the trajectories that light travels? Even though t, x, y, and z individually vary between coordinate systems, I does not vary between coordinate systems. So, consider an interval marked out by t, x, y, and z in one coordinate system and t', x', y', and z' in a second coordinate system. Since I does not vary between coordinate systems, we have that:

$$-ct'^2 + x'^2 + y'^2 + z'^2 = -ct^2 + x^2 + y^2 + z^2$$

Since the variables on the left hand side come equipped with primes, we say that the observer whose coordinates are used on the left hand side is the *primed observer*. Suppose that I represents the interval along the primed observer's trajectory. Relative to oneself, one never moves, since one never becomes (for example) further away from oneself. Thus, relative to the primed observer's own coordinates, the primed observer does not move through space. For that reason, $x' = y' = z' = 0$. But any given observer will measure that, according to a clock that they carry, time passes, so that $t' \neq 0$. Consequently,

$$-ct'^2 = -ct^2 + x^2 + y^2 + z^2$$

Thus, the interval, when placed along one's own trajectory, measures one's own proper time. However, along the trajectories that light travels, $I = 0$. Thus, relative to light, time does not pass. More rigorously, we can say that, along the trajectories that light traverses, no proper time elapses.

This is a deeply counterintuitive result. Light traverses numerically distinct points and yet never records that time passes. One may argue that we made an error, though, if we did, the error is still more counterintuitive. We assumed that there is some reference frame relative to which light is at rest. If there were such a reference frame, then we could accelerate an object up to light speed, so that light was at rest relative to that object. Relativity forbids the existence of any reference frame from the perspective of which light is at rest. If so, light has no rest frame.

Recall the proposal that we started with, namely, that the Cosmos could be said to have a finite past just in case, according to any trajectory that an object could traverse, only finite proper time has transpired. We found that there is some sense in which time does not pass for light. If we take that result literally, then, even if for all observers moving slower than light, the Cosmos is eternal to the past, there is a sense in which, for light, zero time has passed in the Cosmos's history. What we need is a suitable alternative λ to proper time with two features. First, for bodies moving slower than light, λ should distinguish infinitely from finitely long trajectories. That is, for trajectories along which there is (in)finite proper time should be assigned (in)finite values of λ. Second, λ should parametrize the points along the trajectories followed by light in such a way that numerically distinct points are afforded distinct labels. There are a variety of parameters with these features that one could choose, but one standard choice is the *generalized affine parameter*, to be discussed below. If we accept the generalized affine parameter as the right choice for the job, we can say that two space-like surfaces are finitely separated one from another just in case all of the time-like and light-like curves between the two surfaces have finite generalized affine length. And then, to say that the Cosmos has a finite initial segment is just to say that there is a Cosmos-wide space-like surface Σ such that all of the time-like and light-like trajectories that can be traced backwards from Σ encounter a boundary at a finite value of the generalized affine parameter. In the next subsection, I will complete my articulation of the new metrical conception by articulating the generalized affine parameter.

7.2.3 The New Metrical Conception

Let's recall the problem that we are trying to resolve with the generalized affine parameter. We want to be able to distinguish finite from infinite space-time intervals in both time-like and light-like directions, but, due to the combination of positive and negative signs in the space-time interval, the proper time to any point whatsoever along light-like directions is zero. The combination of positive and negative signs in the space-time interval is a reflection of the fact that relativistic space-times are *hyperbolic* and not Euclidean. In a four dimensional Euclidean space, there is no negative sign, e.g., $I^2 = t^2 + x^2 + y^2 + z^2$. If we could map from hyperbolic space-time into a Euclidean space in a way that preserved finite time-like intervals as finite and infinite time-like intervals as infinite and that, along light-like directions, labeled numerically distinct points with distinct values, we would have constructed a parameter that satisfied the desiderata identified at the end of the last section. The generalized affine parameter makes use of precisely this trick.

I now turn to unpacking the technical details involved in constructing the generalized affine parameter. Readers unfamiliar with relativity may find the following exposition difficult to follow; my hope is that they will gather the general "gist". A *half-curve* is usually defined as a curve that starts somewhere in space-time and is inextendable. A classical space-time model S is said to be extendable just in case there is another larger space-time model S' into which S can be isometrically embedded; moreover, S is inextendable just in case S is not extendable. A typical assumption in relativistic physics is that space-times are "as large as they can be"; that is, that space-time is inextendable. A curve γ in S is inextendable just in case there is no larger space-time S' into which S can be isometrically embedded and in which γ is longer than γ was in S. Intuitively, an inextendable curve is a curve that encounters an impassible boundary to space-time. For the sake of complete generality in explicating the concept of the beginning of the Cosmos, I will not assume that the Cosmos is inexteendable and, for that reason, I will offer a modified definition of half-curves. For my purposes, a half-curve in a space-time S is a curve that begins somewhere in S and that has no further extension in S. Intuitively, if a half-curve γ in S has finite length, then γ encounters a boundary of S.

Consider a classical space-time $(\mathbf{M}, \mathbf{g})$. Without loss of generality, and utilizing the notation from Earman, 1995, p. 35, consider a time-like half curve $\gamma(v)$ defined on $[0, v_+) \to \mathbf{M}$, where v is a parametrization of γ and such that $v_+ \leq +\infty$. For each of the tangent spaces at each point in $\mathbf{M}$, we can choose a set of four orthonormal basis vectors; this is the so-called "frame field". In particular, let's denote the basis vectors defined at each of the tangent spaces at each point along $\gamma(v)$ as $\mathrm{e}_i^a(v)$, so that at $v = 0$, the basis vectors are given by $\mathrm{e}_i^a(0)$. Given $\mathrm{e}_i^a(0)$, we can define the other basis vectors in the tangent spaces at the other points along $\gamma(v)$ via parallel transport.

Now that we have defined orthonormal basis vectors for each of the tangent spaces along $\gamma(v)$, we can write the components of a tangent vector $\mathbf{V}$ in terms of the $\mathrm{e}_i^a(v)$ as:

$$V^a = \sum_{i=1}^{4} X^i(v)\mathrm{e}_i^a(v)$$

The Euclidean length of V^a is given by:

$$|\mathbf{V}| = \sqrt{\sum_{i=1}^{4}(X^i(v))^2}$$

And, thus, we have succeeded in expressing the tangent vectors along $\gamma(v)$ using the Euclidean signature. Given the components of this tangent vector, we can write the generalized affine parameter $\lambda(v)$ as

$$\lambda(v) = \int_0^v \sqrt{\sum_{i=1}^{4}(X^i(v^*))^2 dv^*}$$

Where v^* is a dummy variable replacing v inside the integral. Since the summation under the square root within this integral is defined using a positive definite signature, the generalized affine parameter can be thought of as the arc length of a curve in a four-dimensional space instead of a four-dimensional space-time. Using the generalized affine parameter, we can define a notion of *generalized affine length*. The generalized affine length g.a.l. is the total length of $\gamma(v)$, that is,

$$\text{g.a.l.} = \int_0^{v_+} \sqrt{\sum_{i=1}^{4} (X^i(v^*))^2 dv^*}$$

As Earman notes, the choice of a different set of basis vectors $e_i^a(v)$ for each tangent space leads to a different generalized affine parameter defined on $\gamma(v)$. (Of course, once a choice of basis has been made at $v = 0$, that choice is propagated to every other point along $\gamma(v)$ by parallel transport.) But if one choice of basis vectors leads to a finite generalized affine length, then any other choice of basis vectors will lead to a finite generalized affine length; likewise, if any choice of basis vectors leads to infinite generalized affine length, then any other choice will lead to infinite generalized affine length. For that reason, whether the generalized affine length is finite or infinite is independent of our choice of orthonormal basis vectors and satisfies the desiderata identified at the end of the previous section.

Recall the intuition that motivated this section. We can say that two space-like surfaces are finitely separated from each other just in case all of the time-like and light-like curves between the two surfaces have finite generalized affine length. Likewise, for the sake of intuition, imagine a closed or open boundary B where B is prior to all space-time points not included in B and a space-like surface Σ. Suppose, further, that the Cosmos satisfies the Modal Condition and the Direction Condition. If no other conditions are required for the Cosmos to have a beginning and if all of the time-like and light-like curves between B and Σ have finite generalized affine length, then, intuitively, B should count as the space-time's beginning. *Therefore, the Cosmos has a finite initial segment just in case there is a Cosmos-wide space-like surface* Σ *such that all of the time-like and light-like trajectories that can be traced backwards from* Σ *have finite generalized affine length.*

One may worry that I have made two implicit assumptions in explicating what it would mean for the initial segment of the Cosmos's history to be finite. First, one might worry that I have assumed that the time-like and light-like curves in the initial segment of the Cosmos's history are objectively comparable. Readers harboring this sort of worry are right to do so; if, for whatever reason, the generalized affine lengths of curves are incommensurate, then the fact that all of the curves have finite generalized affine length might not be meaningful. Second, one might worry that I have assumed that there is a meaningful distinction between finite and infinite space-time regions. For example, if the lengths of temporal durations are conventional or if time is amorphous in the initial segment, then there is no objective distinction between finite and infinite initial segments. But this worry is mistaken. On the one hand, if there is no objective finite/infinite distinction in the initial segment and the initial segment has a closed boundary, then the initial segment has a topological beginning. Since the third criterion for the Cosmos to have a beginning is disjunctive, we would be able to say that the Cosmos has a beginning. On the other hand, if there is no objective finite/infinite distinction in the initial segment and the initial segment has an open boundary, then the initial segment cannot be said to be finite in virtue of the generalized affine length; the generalized affine length, itself, wouldn't be either finite or infinite for any of the time-like or light-like curves in the segment. In that case, the Cosmos would not have a beginning.

7.3 Objections

In this section, I turn to two important objections to the view that I've presented in this chapter. According to the first objection, while the Boundary Condition, as I've stated it, captures two of the ways in which space-time could have a boundary, I haven't shown that there are no other ways in which space-time could have a boundary. According to the second objection, the metrical conception may be able to subsume the topological conception, in which case I wouldn't have to define the Boundary Condition disjunctively. In the following, I show that both objections are incorrect.

7.3.1 The First Objection: Uniqueness?

On my view, the Boundary Condition is a necessary condition for the Cosmos to have a beginning. While the reader might share my intuition that the Cosmos having a beginning requires that the Cosmos include a past boundary of some kind, the Boundary Condition – at least as I stated the condition – can be a necessary condition for the Cosmos to have a beginning only if the topological conception and the metrical conception exhaust all of the ways for the Cosmos to have a beginning. Why think that the topological conception and the metrical conception are the only two ways for the Cosmos to include a beginning? According to a standard mathematical procedure for constructing a space or space-time, we begin with a set of simples (or "points") which we can then endow with additional structure, e.g., Norton, 1999; Isham, 1994, pp. 10–11; Maudlin, 2010, Maudlin, 2012, pp. 5–8; DeLanda, 2013, pp. 14–18; North, 2021, pp. 40–51. The additional structure forms a hierarchy, that is,

1. The *set theoretic structure* describes the properties the point set has in virtue of being a set, e.g., the cardinality of the point set or whether a given entity is a member of the point set.

2. The *topological structure* describes the continuity or discontinuity of the space or space-time as well as whether the space or space-time has closed, open, or clopen boundaries.

3. The *affine structure* describes the primitive distinction between curves and straight lines.

4. The *metrical structure* describes the distance (or interval) between any two points.

5. The *differentiable structure* allows us to distinguish smooth curves from curves with sharp or broken edges.

Additional structure can be defined on any given point set as well. For example, A-theories of time define primitive temporal structure in terms of the monadic predicates of pastness, presentness, and futurity. Consequently, A-theories endow space-time with what I will call *monadic structure*. On B-theories of time, a binary relation – the B-relation – is defined between any two numerically distinct time-like related events α and β, in virtue of which we can say either that α is before β or β is before α. Likewise, on some – albeit outdated – metaphysical accounts of the nature of space (or of the nature of place), we should supplement space with additional structure. For example, Aristotle's view of the nature of place denies the homogeneity of space and defines the center of the Earth as the center of the Cosmos. For that reason, Aristotle's view includes fundamental relations of *up* and *down*. Let's call the additional structure added in the case of either B-theory or the Aristotelian conception of place *ordinal structure*, since, in either case, we are imposing an ordering relation on a given point set.

Plausibly, the Boundary Condition should be definable in terms of the formal structure out of which we can construct models of space-time. Intuitively, given the various formal structures described above, only two kinds of formal structure – that is, topological structure and metrical structure – are capable of capturing the notion of a boundary. For example, when we say that an ordinary object, e.g., a table, has a boundary, we might mean that, e.g., the table has an edge, that is, a topological boundary, or we might mean that the table has finite spatial extension, that is, a metrical boundary. We don't mean that the table has a boundary in virtue of our ability to define straight lines on the table, or our ability to distinguish smooth curves from curves with sharp edges, or in terms of some ordinal or monadic structure that we can define on the parts of the table.[13] Since there are only two ways of capturing the notion of a boundary in terms of the formal structure out of which we can construct models of space-time, I've defined the Boundary Condition disjunctively in terms of those two notions.

7.3.2 The Second Objection: Disjunctive or Atomic?

According to the second objection, the metrical conception is a broader family that includes all of the cases captured by the topological conception, in which case there is no need to define the Boundary Condition disjunctively. I'll begin by describing why someone might think that the metrical conception could subsume the topological conception. Consider a space-time S with a closed boundary ζ to the past of every non-initial point. Note that S satisfies the topological conception because there is a closed boundary to the past every non-initial space-time point. Suppose that ζ is a space-like surface.

[13]↑Perhaps the reader will object that one way that a series can have a boundary involves the series having a first member and having a first member has to do with the ordinal structure of the series. Nonetheless, having a first member is better described in terms of topological structure. Likewise, one might object that firstness is a monadic predicate; but, again, firstness is better understood in terms of topological structure.

Now consider any past directed half-curve originating on ζ. Since there are no points to the past of any point on ζ, the generalized affine length of any past directed half-curve originating on ζ is trivially zero. Consequently, ζ is a space-like surface such that all past directed half-curves have finite generalized affine length. Thus, S satisfies the metrical conception. If every space-time satisfying the topological conception is bounded by a space-like surface, then every space-time satisfying the topological conception also satisfies the metrical conception. (Note that the converse is not true, since a space-time could satisfy the metrical conception by including the appropriate kind of open boundary.)

One way to re-state the objection being considered in this section is as a challenge to produce a space-time that satisfies the topological conception but does not satisfy the metrical conception. As I've shown, every space-time that satisfies the topological conception by being bounded by a space-like surface will trivially satisfy both the topological conception and the metrical conception. Thus, any space-time that satisfies the topological conception without satisfying the metrical conception must be bounded by something other than a space-like surface. Recall the space-times with "jagged" boundaries considered earlier; for example, the space-time constructed from a congruence of time-like curves whose respective ages are given by the function $a^F(\varepsilon)$ in section 7.1. Call that space-time the a^F space-time. Space-times with a "jagged" boundary need not be bounded by a space-like surface. Nonetheless, the a^F space-time still satisfies the metrical conception; we need an additional criterion in order to identify space-times that satisfy the topological conception without satisfying the metrical conception.

The reason that the a^F space-time satisfies the metrical conception is that the a^F space-time includes an initial finite segment. Thus, any space-time satisfying the topological conception without satisfying the metrical conception must have a non-space-like "jagged" closed boundary without including an initial finite segment. There are at least two ways of constructing a space-time of that kind. First, consider a space-time S^* with a non-space-like "jagged" closed boundary that satisfies the metrical conception. Construct a new space-time by making the initial finite segment of S^* metrically amorphous. Since the initial segment of S^* is metrically amorphous, the initial segment of S^* is neither finite nor infinite. Given both that there is no finite initial segment of S^* and that the points on the boundary of S^* do not comprise a space-like surface, S^* does not satisfy the metrical conception. Nonetheless, S^* satisfies the topological conception, since S^* includes a closed boundary.

Here is a second way to construct a space-time satisfying the topological conception and not the metrical conception. Consider a space-time S^{**} that fails to satisfy the metrical conception by having infinite extension to the past of every space-time point. We can now construct a new space-time by "adding in" a non-space-like "jagged" boundary to the infinite past, analogous to the way in which the extended real line is constructed by adding points at positive and negative infinity to the standard real line. I'm not sure whether such a construction is reasonable, but such a construction at least seems logically possible. Given the logical possibility of such a construction, the Boundary Condition should be stated in such a way that allows for the construction's possibility.

7.4 Summary

In this chapter, I defended the last of my three necessary conditions for the Cosmos to have had a beginning. Intuitively, an entity begins to exist just in case there is a temporal boundary before which the entity did not exist. This intuition needs to be made more precise; as I argued, previous attempts to precisify the notion of a boundary to the Cosmos's history – as provided by Craig, Sinclair, and Swinburne – do not succeed. While Pitts (2008) previously offered a useful distinction between the topological and metrical senses of a beginning, I have shown that his version of the metrical conception is inadequate. The novel proposal that I offered in this chapter borrows Pitts's distinction, improves on the metrical conception, and, contrary to Pitts's rejection of the metrical conception, is defined in terms of a disjunction between the two. According to my proposal, the Cosmos had a beginning only if either the topological conception or the metrical conception are satisfied. According to the topological conception, there is a closed boundary to the past of non-initial space-time points. According to the metrical conception, there is an initial objectively finite portion of the Cosmos's history. In turn, there is an initial finite portion of the Cosmos's history just in case there is a Cosmos-wide space-like surface Σ such that all of the time-like and light-like trajectories that can be traced backwards from Σ have finite generalized affine length.

8. CLASSICAL BIG BANG MODELS AND THE DIRECTION/BOUNDARY CONDITIONS

8.1 Introduction

A variety of authors have expressed the intuitive idea that if classical Big Bang models were correct, then the Cosmos would have a beginning. I disagree; for example, even if classical Big Bang models were correct, and we (somehow) didn't need a quantum gravity theory, one would still need to show that the Cosmos satisfies the Modal Condition. However, the Modal Condition is one of the novel contributions made to the literature by this dissertation. For that reason, past authors have only had access to the Direction and Boundary Conditions. Consequently, we should interpret the claim that classical Big Bang models involve a beginning as a claim about classical Big Bang models satisfying the Direction and Boundary Conditions. As I prove in this chapter, if the Direction and Boundary Conditions were the only criteria needed for the Cosmos to have had a beginning, we assume (incorrectly) that General Relativity is a final theory of space-time, and we assume that space-time is maximally extended, then the Cosmos having a beginning would turn out to entail a technical criterion for a space-time to be singular, i.e., b-incompleteness. If we added the additional assumptions that the Cosmos is spatially homogeneous and isotropic, that is, the cosmological principle, then we would be able to *derive* classical, singular Big Bang models. The short theorem that I prove in this chapter precisifies the intuition that classical, singular Big Bang models include a beginning and, in doing so, provides evidence that I have provided the correct criteria for the Cosmos to have a beginning. The theorem also shows that the Direction and Boundary Conditions are more fundamental than the sort of "beginning" involved in classical Big Bang models.

I will first describe one way to more rigorously characterize space-time singularities than I have previously offered. Unfortunately, physicists, philosophers of physics, and mathematicians have yet to develop a fully satisfactory set of conditions for distinguishing singular from non-singular space-times. Given the deeply technical nature of this problem, the solution is beyond my current abilities and I will not attempt to resolve the problem here. Instead, I will summarize some of the relevant literature in order to offer one standard, if not fully satisfactory, conception of how singular and non-singular space-times differ. Having provided a more rigorous characterization of singular space-times – in terms of b-incompleteness, as described below – I will prove a theorem that states the precise relationship between singular space-times and my three conditions for a beginning of the Cosmos, namely, that all classical space-times satisfying the Direction and Boundary Conditions are b-incomplete and show how that result can figure into a derivation of the Big Bang singularity.

8.2 B-Incompleteness and Singular Space-times

Although singular FLRW space-times include a divergent Ricci scalar, divergences in the various curvature parameters are neither necessary nor sufficient for a classical space-time to be singular (Curiel, 1999, 2021; Earman, 1995; Joshi, 2014). For my purposes, we can utilize what John Earman (1995, p. 36) calls the "semi-official definition" and what elsewhere has been called the "most widely accepted solution" for defining singular space-times (Curiel, 2021). A classical space-time is said to be *b-complete* just in case every time-like and light-like half-curve has infinite generalized affine length. According to Earman's semi-official definition, a classical space-time is then said to be singular just in case the space-time is not b-complete. Arguably, one should add the condition that space-time is maximally extended (Lam, 2007, p. 715). Since this definition is not completely satisfactory,[1] I will not take up the position here that all and only singular space-times are b-incomplete. Moreover, I will not take up the debate, e.g., Earman, 1995, p. 32; Manchak, 2021, as to whether the space-time we inhabit is maximally extended. Instead, I will assume that space-time is maximally extended. In any case, b-incompleteness will allow us to see the precise sense in which my three conditions for the Cosmos to have a beginning relate to singular space-times. That is, as I prove in the next section, all classical space-times that are maximally extended and that satisfy the Direction and Boundary Conditions are b-incomplete. Consequently, if the Cosmos satisfies the Modal Condition

[1]↑For some of the problems involved with utilizing b-incompleteness as the definitive feature of singular space-times, see chapter 2 in Earman, 1995; also see Curiel, 2021.

and includes a classical space-time satisfying the Direction and Boundary Conditions – and so has a beginning – then space-time is b-incomplete.

8.3 All classical space-times satisfying the Direction and Boundary Conditions are b-incomplete

Before beginning the proof, two cautionary notes are in order. First, the converse of the result to be proved in this section does not hold, i.e., if the Cosmos is b-incomplete, it would not follow that the Cosmos satisfies the Direction and Boundary Conditions. By this point in the dissertation, the reason should be obvious. If the Cosmos were b-incomplete, this would tell us, at most, that the Cosmos satisfies the Boundary Condition, but would not tell us whether the Cosmos satisfies the Modal or Direction Conditions. Even if space-time were finite to the past, with no extension to the past of the Big Bang, the Cosmos might still fail to satisfy the Modal Condition and so fail to have a beginning.

Claim. All maximally extended classical space-times that satisfy the Direction and Boundary Conditions are b-incomplete.

Proof. To begin the proof, let's assume that the Cosmos includes a maximally extended classical space-time satisfying the Direction and Boundary Conditions. Recall that, according to the Boundary Condition, the Cosmos began to exist just in case either there is a Cosmos-wide closed boundary to the past of every non-initial space-time point or there is an initial objectively finite portion of the Cosmos's history. We can proceed to prove by cases.

Let's first suppose that space-time has a closed boundary $\mathcal{B}$ to the past of every non-initial space-time point. The proof for this case is trivial. Consider any time-like or light-like half-curve γ that originates at some point $p \in \mathcal{B}$. Any such curve will have zero extension backwards through the space-time.[2] Since the curve has zero backwards extension, the space-time is b-incomplete. Having established the first case, let's move to the second. Suppose that there is an initial objectively finite portion of the Cosmos's history.

[2]↑This result will not necessarily follow for any curve that is not located in $\mathcal{B}$. For example, suppose that space-time has a closed boundary but that the initial portion of the Cosmos has the "fractal" metrical properties discussed above. In that case, any time-like or light-like curve not located in $\mathcal{B}$ will have infinite backwards extension.

Now consider an arbitrary time-like or light-like half-curve originating on some space-like surface Σ in the initial objectively finite portion and that extends backwards through the Cosmos. By the definition of an objectively finite portion of the Cosmos's history established above, this half-curve must have finite generalized affine length. Therefore, if the Cosmos includes a classical space-time satisfying either of the two disjuncts – and so satisfying the Boundary Condition – the Cosmos is b-incomplete. Therefore, we have the desired result, i.e., if the Cosmos includes a classical space-time satisfying the Direction and Boundary Conditions, then space-time is b-incomplete.

Having proven the desired result, let's turn to considering how the result relates my three conditions for the beginning of the Cosmos to classical Big Bang cosmology. Classical Big Bang cosmology is modeled using FLRW space-times. The FLRW space-times that are sometimes claimed to be a model of the beginning of the universe include a curvature singularity and are b-incomplete. According to a result that has been proven elsewhere, all FLRW models (excluding those that have pathological features such as closed time-like curves) satisfy the Direction Condition (Castagnino et al., 2003; Matthews, 1979); as a consequence of the result that I've proven here, the non-pathological FLRW models satisfying the Boundary Condition are b-incomplete. In the case of maximally extended FLRW models, the converse holds as well, that is, all b-incomplete maximally extended FLRW models satisfy both the Direction Condition and the Boundary Condition.

Arguably, the Direction and Boundary Conditions are more fundamental than the Big Bang singularity because the Direction and Boundary Conditions figure into a derivation of the Big Bang singularity. Suppose (i) General Relativity is true, (ii) the cosmological principle is true, (iii) space-time is maximally extended, (iv) space-time satisfies the Direction Condition, and (v) space-time satisfies the Boundary Condition. The combination of those five deductively entails space-time is correctly modeled by one of the FLRW metrics with a Big Bang singularity. The assumption that General Relativity is true entails the Einstein Field Equations. The Einstein Field Equations together with the assumption that the cosmological principle is true – that is, that space-time can be "cut up" (or foliated) into space-like surfaces on which the matter-energy distribution is homogeneous and isotropic – entails that space-time is one of the FLRW models. Since we've assumed that space-time is maximally extended, we can make the further restriction to maximally extended FLRW models. Maximally extended FLRW models can be subdivided into two families: those that include a Big Bang type singularity and those that do not. Using the fact that the Direction and Boundary Conditions together entail that space-time is b-incomplete, we can eliminate the FLRW models that do not include a Big Bang type singularity. Thus, in the context of General Relativity, the Direction and Boundary Conditions, together with some additional assumptions about the global structure of space-time, can be used to derive the Big Bang singularity.

This result – that the Direction and Boundary Conditions can figure into a derivation of the Big Bang singularity – helps to show one sense in which the Direction and Boundary Conditions are more fundamental than Big Bang theory. The result also helps to clarify why, on the assumption that General Relativity is true, one still cannot infer that the Cosmos began to exist. I will argue in chapter 12 that we have no good reason for thinking that the cosmological principle is unrestricted in scope. The unrestricted cosmological principle is required for the derivation. Second, as I have mentioned, Manchak has challenged the notion that we can know space-time to be maximally extended, so that the assumption that space-time is maximally extended is at least controversial. Third, I will argue in chapter 9 that the conjunction of General Relativity and any set of observations that we are likely to have will not entail that the Cosmos satisfies the Direction or Boundary Conditions and I will argue in chapter 12 that inductive arguments for the view that the Direction or Boundary Conditions are satisfied do not succeed either. In order to know whether the Direction and Boundary Conditions are satisfied, we would need to know substantive details about the global distribution of matter-energy that we are not in an epistemic position to know.

8.4 Summary

In this chapter, I've completed two tasks. First, I provided a more rigorous characterization of what space-time singularities are in terms of b-incompleteness. Second, I showed what the relationship is between classical, singular Big Bang models and the

Direction and Boundary Conditions. I thereby proved a theorem explaining the intuition that classical, singular Big Bang models involve a beginning. According to the theorem, if the Cosmos satisfies the Direction and Boundary Conditions and space-time is maximally extended, then space-time is b-incomplete. I sketched a second theorem according to which if we add the cosmological principle, then singular FLRW models follow as a deductive consequence. Consequently, the Direction and Boundary Conditions are more fundamental than the Big Bang singularity.

Having established three necessary conditions for the Cosmos to have had a beginning and explicated how those three necessary conditions are connected to the mathematics of (classical) singular space-times, we have left to determine whether the Cosmos *in fact* satisfies the three conditions. That is the project that I take up in part III of this dissertation; as we will see, the current state of inquiry in physical cosmology provides us with strong reason to doubt that we know, or possibly even could know, whether the Cosmos satisfies the Modal, Direction, and Boundary Conditions.

Part III

COSMIC SKEPTICISM DEFENDED

9. OBSERVATIONALLY INDISTINGUISHABLE SPACE-TIMES AND THE BEGINNING OF THE COSMOS

Nature loves to hide.

— Heraclitus

Somewhere, something incredible is waiting to be known...

— Carl Sagan

9.1 Introduction to Part III

Cosmic Skepticism is the provisional thesis that the provinciality of our knowledge of the physical facts with respect to scale, spatio-temporal location, or energy prevents us from having empirical access to whether the Cosmos satisfies the Modal, Direction, and Boundary Conditions. If Cosmic Skepticism is true, then we do not have empirical access to either the formation of the Cosmos or whether there was such an event or process as the formation of the Cosmos. Cosmic Skepticism is a skeptical thesis not in the sense that we have an a priori in principle reason for thinking that we cannot know whether the Cosmos had a beginning but instead in the sense that *as empirical inquiry currently stands* we have reason to think that we cannot know whether the Cosmos had a beginning. Future inquiry may change our epistemic situation in radical ways that are impossible for us to foresee. My strategy for defending Cosmic Skepticism involves defending the following argument:

1. We know the Cosmos began to exist only if we know the Cosmos satisfies the three conditions introduced in part II, i.e., the Modal Condition, the Direction Condition, and the Boundary Condition.

2. We do not know whether the Cosmos satisfies the three conditions.

3. Therefore, we do not know whether the Cosmos began to exist.

In part II of this dissertation, I defended the view that the Cosmos began to exist only if the Cosmos satisfies the Modal, Direction, and Boundary Conditions. Thus, we can know that the Cosmos began to exist only if we know that the Cosmos satisfies the Modal, Direction, and Boundary Conditions. In the third part of this dissertation, I consider four arguments for the second premise.

First, whether the Cosmos satisfies the Boundary Condition is a bit of unobservable chronogeometric structure. According to a standard view in philosophy of science, we have reason to believe in an unobservable entity provided we have reason to believe a broader theory which entails that entity's existence. While we should expect General Relativity to be replaced in subsequent physical inquiry by a quantum gravity theory, General Relativity remains our best theory of chronogeometric structure. In the context of General Relativity, whether two space-times are observationally indistinguishable turns out to be a tractable and precise mathematical problem. As I will prove, no set of observations that we currently have, when conjoined with General Relativity, entails that the Cosmos satisfies the Direction or Boundary Conditions. That is, because of the provinciality of our knowledge of the Cosmos due to the relative scale of the Cosmos and our spatio-temporal location within the Cosmos, General Relativity suggests that our Cosmos is observationally indistinguishable from another very different space-time that fails to satisfy the Direction or Boundary Conditions.

Second, considerations in the philosophical foundations of statistical mechanics entail either that the Cosmos violates the Modal Condition or else that there is a transcendental condition on the possibility of our knowledge of the past that prevents us from having knowledge of states of affairs prior to a specific past boundary. Here we meet a warning from the nineteenth century: the fact that there is some past boundary beyond which we cannot make reliable inferences does not entail that the Cosmos satisfies the Boundary Condition. Instead, the existence of a past boundary beyond which we cannot make reliable inferences suggests that the provinciality of our knowledge of the physical facts with respect to spatio-temporal location prevents us from knowing whether the Cosmos satisfies the Boundary Condition.

Third, if a variety of live cosmological models are true, then the Cosmos does not satisfy the Boundary Condition. Due to the provinciality of our knowledge with respect to scale, time, space, and energy, we do not know whether any of those cosmological models are true, or at least true in sufficient detail to suggest on their basis whether the Cosmos satisfies the Boundary Condition. Nonetheless, we cannot rule the models out and so cannot rule out the possibility that the Cosmos was beginningless.

Fourth, I complete the case for Cosmic Skepticism by turning to confirmation theory. There are two families of inferences that could be used in arguing for the conclusion that the Cosmos satisfies the Modal, Direction, and Boundary Conditions: part-to-part inferences and part-to-whole inferences. Part-to-part inferences involve projecting an empirical regularity from an observable portion of the Cosmos into an unobservable portion of the Cosmos. Once the empirical regularity has been projected into the unobservable portion, the empirical regularity can be used to argue either that the Cosmos began to exist or that the unobservable portion includes features relevant to whether the Cosmos began to exist. I will show that part-to-part inferences fail because they rely upon a weak analogy between observable and unobservable portions of the Cosmos and because we have no good reason to think that the known physical facts are representative of all of the physical facts that there are.

Next, I turn to part-to-whole inferences. Part-to-whole inferences project an empirical regularity from an observable portion of the Cosmos to the whole Cosmos. I will show that part-to-whole inferences are poor inferences because, as with part-to-part inferences, we have no good reason for thinking that the known physical facts are representative of all of the physical facts that there are. However, part-to-whole inferences are also poor inferences for a more profound reason. Assuming that Paul Draper's account of intrinsic probability is correct, the intrinsic probability of a hypothesis is determined by the modesty of the hypothesis, that is, how much the hypothesis tells us about the world, the coherence of the hypothesis, that is, the degree to which the parts of the hypothesis are mutually supportive, and nothing else. I show that on the assumptions that Draper's account of intrinsic probability is correct and that induction is reliable, there is an as yet unresolved tension between the modesty and the coherence of hypotheses that is particularly acute for hypotheses about the totality of physical reality. As long as that tension remains unresolved, we are unable to judge the intrinsic probability of hypotheses about the entire Cosmos and thus ill-equipped to make part-to-whole inferences.

The four arguments collectively provide a strong case for the conclusion that we cannot know whether the Cosmos satisfies the Modal, Direction, or Boundary Conditions and so cannot know whether the Cosmos began to exist. And since we cannot know whether the Cosmos began to exist, we cannot know whether the second premise of the KCA is true. Ergo, the wholly a posteriori defense of the KCA fails.

9.2 Introduction to Chapter 8

Whether the Cosmos has a beginning – and so whether the Cosmos satisfies the Modal, Direction, and Boundary Conditions – is not directly observable. Nonetheless, according to a standard view in philosophy of science, we have reason to endorse the truth of an unobservable claim just in case a well supported scientific theory, in conjunction with some body of observations, entails the truth of the unobservable claim. In this chapter, I address whether some collection of observations, in conjunction with General Relativity, entails that physical reality includes a space-time satisfying the Direction and Boundary Conditions. General Relativity is not likely to be a final or complete theory of space-time, but, until we have a well supported quantum gravity successor theory, General Relativity is the best scientific theory of space-time that we have. For that reason, the conclusions that I reach in this chapter should be understood as only provisionally held. Moreover, even if no body of observations, in conjunction with General Relativity, *entails* that space-time satisfies the Direction and Boundary Conditions, there may be other reasons – such as extra-empirical theoretical virtues (simplicity, parsimony, fecundity, and the like) – that would support the view that space-time satisfies the Direction and Boundary Conditions. Whether or not we should think the Cosmos has a beginning ultimately depends on the a posteriori epistemic probability of the hypothesis that the Cosmos has a beginning; thus, a full discussion as to whether the Cosmos has a beginning will need to wait until we've discussed confirmation theory in chapter 12.

Fallowing Malament, 1977a, a given space-time S is said to be *observationally indistinguishable* from a distinct space-time S' just in case no collection of observations that a given observer in S or S' could make would allow them to determine whether they inhabited S or S'. S is said to be *weakly observationally indistinguishable* from a distinct space-time S' just in case no collection of observations that a given observer in S could make would allow them to distinguish the space-time they inhabit from S'. Lastly, I will say that S is *super weakly observationally indistinguishable* from S' just in case observers in a specific region (more precisely and rigorously specified below) cannot determine whether they inhabit S as opposed to S'. The Cosmos has a beginning only if the Cosmos includes a space-time satisfying both the Direction and Boundary Conditions. Let's call space-times that satisfy the Direction and Boundary Conditions DB space-times. Since observational indistinguishability (of whatever sort) as well as the Direction and Boundary Conditions are precisely specifiable mathematical conditions, whether an observer in a relativistic space-time could deduce that they inhabit a DB space-time turns out to be a precisely specifiable mathematical question. As I will prove, a variety of DB space-times are at least weakly or super weakly observationally indistinguishable from non-DB space-times. Supposing that we inhabit a DB space-time, no collection of observations that humans will ever be capable of making could be used to determine whether the Cosmos began to exist.

Philosophy of physics has long featured a niche literature devoted to space-time indistinguishability. As Enrico Cinti and Vincenzo Fano (2021) describe, philosophers of physics have generally understood a series of mathematical results established by J.B. Manchak (2009, 2011) to be the "the last word" and to have definitively established that cosmology is restricted "to the study of the so-called visible universe". Manchak's results, Cinti and Fano say, are generally taken (by philosophers of physics) to forbid cosmologists from successfully formulating "hypotheses regarding the universe as a whole", by which they mean space-time as a whole. Cinti and Fano are themselves critical of Manchak's results, but their quoted comments do accurately reflect the way in which philosophers of physics have generally received Manchak's results. For example, Jenann Ismael, e.g., (2018), and Claus Beisbart, e.g., (2022), signal that they take Manchak to have offered the final word when they put Manchak's results to work without raising or answering objections. Despite the fact that friends of the a posteriori defense of the KCA have paid some attention to models in physical cosmology, e.g., Craig and Sinclair, 2009, 2012, space-time indistinguishability has yet to catch their attention.[1] Many of the cosmological models developed by physicists were intended as toy models and were not necessarily intended to be realistic descriptions of the universe we inhabit. Thus, friends of the a posteriori defense of the KCA are better off addressing what might be taken to have much more significance, namely, what, if anything, we can justifiably say about global space-time structure on the basis of observations. This chapter advances the literature on the KCA by addressing whether DB space-times, and so space-times that might have a beginning, are observationally indistinguishable from non-DB space-times, and so space-times that certainly do not have a beginning.

I will first discuss results concerning the observational indistinguishability of relativistic space-times, beginning with the Malament-Manchak theorem and related results. Afterwards, I will introduce a new notion of observational indistinguishability – which I call *super weak observational indistinguishability* – and I will show that if our space-time is a DB space-time then, plausibly, our space-time is super weakly observationally indistinguishable from a non-DB space-time. In that case, unless our understanding of space-time is massively overturned in future inquiry, no set of observations that humans will ever make will allow us to decisively distinguish our space-time from a non-DB space-time. Lastly, I discuss how observational indistinguishability relates to the Borde-Guth-Vilenkin theorem, which has sometimes been claimed to provide strong support for the conclusion that the Cosmos began to exist.

[1] ↑I know of only two instances in which space-time indistinguishability has previously been discussed with some connection to the KCA. The first, i.e., Linford, 2021, is one of my own recent papers. The other, i.e., Beisbart, 2022, discusses space-time indistinguishability in connection with Kant's first antinomy, which has itself been interpreted as offering a version of the KCA.

9.3 Definitions

In this section, I define a number of technical terms that I will deploy in subsequent sections. In subsequent sections, I will offer both non-technical ("English" language) explanations of the results that I discuss or prove as well as technical versions. Readers who are only interested in the English language version, and not in the mathematical details, can skip or skim this section and then refer back to this section in cases where they might need a specific definition. Readers who are only interested in the mathematical results should read the definitions that I offer in this section and can feel free to skip or skim the English language explanations of the results that I subsequently offer.

A *relativistic space-time* is a pair $(\mathbf{M}, g_{\mu\nu})$, where $\mathbf{M}$ is a set of points equipped with, e.g., topological structure,[2] and $g_{\mu\nu}$ is a metric tensor defined on $\mathbf{M}$ with Lorentzian signature $(-,+,...,+)$ and with indices μ and ν ranging over $\{0,1,2,3\}$. In order for $(\mathbf{M}, g_{\mu\nu})$ to be consistent with General Relativity, $(\mathbf{M}, g_{\mu\nu})$ needs to satisfy a set of non-linear coupled partial differential equations called the *Einstein Field Equations* (EFE), i.e., $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, where $G_{\mu\nu}$ is the Einstein tensor, encoding the curvature of space-time, G is Newton's gravitational constant, and $T_{\mu\nu}$ is the energy-momentum tensor, encoding the distribution of mass, energy, and momentum throughout space-time. As Misner, Thorne, and Wheeler famously quipped, in virtue of the EFE, matter-energy tells space-time how to curve and space-time tells matter-energy how to move.

For the Cosmos to have a beginning requires that the Cosmos satisfies the Modal, Direction, and Boundary Conditions. In this chapter, I am focused on how results from General Relativity bear on whether we can know that the Cosmos had a beginning. Since General Relativity is a theory about space-time, I set aside the Modal Condition and focus on the Direction and Boundary Conditions. To review, $(\mathbf{M}, g_{\mu\nu})$ satisfies the *Direction Condition* just in case $(\mathbf{M}, g_{\mu\nu})$ satisfies the criteria discussed in chapter 6 and previously discussed in Matthews, 1979, p. 84 and Castagnino et al., 2003. That is, $(\mathbf{M}, g_{\mu\nu})$ satisfies the Direction Condition just in case (i) $(\mathbf{M}, g_{\mu\nu})$, is temporally orientable, (ii) for any point p in $(\mathbf{M}, g_{\mu\nu})$, there is a locally defined direction of time at p, and (iii) for all pairs of points

[2]↑More precisely, $\mathbf{M}$ is a C^∞, connected, Hausdorff, and paracompact manifold.

p and q in $(\mathbf{M}, g_{\mu\nu})$, the future (past) direction defined at p agrees with the future (past) direction defined at q. $(\mathbf{M}, g_{\mu\nu})$ satisfies the Boundary Condition just in case either there is a closed boundary to the absolute past of all non-initial points in $\mathbf{M}$ or $(\mathbf{M}, g_{\mu\nu})$ includes a finite initial segment, that is, there is a space-time-wide space-like surface Σ such that all of the time-like and light-like trajectories that can be traced backwards from Σ have finite generalized affine length. Σ is *space-time-wide* just in case one of three conditions is met:

1. Σ is not a boundary of space-time and Σ cuts space-time into three parts: those that are before Σ, those that are located on Σ, and those that are located after Σ.

2. Σ is a boundary of space-time and all of the space-time points that are not in Σ are to the absolute future of Σ.

3. Σ is a boundary of space-time and all of the space-time points that are in Σ are to the past of Σ.

As I've said, the current chapter deals with whether some collection of data obtainable by observers embedded in a relativistic space-time, in conjunction with General Relativity, entails that the space-time the observers inhabit satisfies the Direction and Boundary Conditions. To answer that question, we need two pieces of formal machinery: first, we need to be able to describe the data available to a given observer in some mathematically tractable way and, second, we need to be able to say when two space-times $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M}', g'_{\mu\nu})$ are either distinct or the same as one another.

First, how can we describe the data available to a given observer? Consider a point p in $\mathbf{M}$. Denote p's past light cone $I^{-}(p)$, so that $I^{-}(p)$ is the set of points from which information can reach p without exceeding the speed of light. Likewise, let's denote p's future light cone $I^{+}(p)$, so that $I^{+}(p)$ is the set of points to which information can travel from p without exceeding the speed of light. I will assume that the data available to an observer at p is exhausted by the points in $I^{-}(p)$ as well as the distribution of properties in $I^{-}(p)$. I am excluding the possibility that, e.g., observers can acquire information either from events to which they are space-like related or that are in their absolute future.

Second, what is the appropriate formalism for describing when $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M}', g'_{\mu\nu})$ are either distinct or the same as one another? I will say that $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M}', g'_{\mu\nu})$ are *isometric* if there exists a function that smoothly maps every point in $(\mathbf{M}, g_{\mu\nu})$ into a suitable counterpart in $(\mathbf{M}', g'_{\mu\nu})$. A counterpart is suitable just in case the smooth function mapping $(\mathbf{M}, g_{\mu\nu})$ into $(\mathbf{M}', g'_{\mu\nu})$ preserves the lengths of space-time intervals. A function of that kind can be rigorously defined and is called a *diffeomorphism*; thus, $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M}', g'_{\mu\nu})$ are said to be isometric just in case there exists a diffeomorphism $\phi : \mathbf{M} \to \mathbf{M}'$ such that $\phi(g_{\mu\nu}) = g'_{\mu\nu}$ (Manchak, 2020, pp. 9–10). We now possess a suitable notion of "distinct" space-times, that is, two space-times are distinct just in case they are not isometric. An *open neighborhood* around a space-time point p is a spatio-temporal region, with an open boundary, containing p. $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M}', g'_{\mu\nu})$ are *locally isometric* just in case for each point p in $\mathbf{M}$ there is an open neighborhood $O \subset \mathbf{M}$ containing p and an open neighborhood $O' \subset \mathbf{M}'$ such that $(O, g_{\mu\nu})$ and $(O', g'_{\mu\nu})$ are isometric and vice versa (Manchak, 2020, p. 11).

Time-like curves are the trajectories traced by particles moving through space-time at less than the speed of light. Consider a time-like curve γ with the following properties. An observer traversing γ always – as far as they are concerned – moves into the future; nonetheless, the observer eventually comes to a space-time point numerically identical to the point at which they began. This is a case of time travel. For any observer traversing such a trajectory, there is a space-time point in their future numerically identical to a space-time point to their past. γ is said to be a *closed time-like curve*. A physical system traversing a closed time-like curve must, for self-consistency, return to a physical state numerically identical to the physical state with which the system began; otherwise, a "grandfather paradox" takes place.[3] Whether closed time-like curves are metaphysically

[3]↑In the film *Groundhog Day*, Phil Connors (played by Bill Murray) repeatedly relives the same day. Although beginning each day in the same bed, Phil does not traverse a closed time-like curve because Phil does not return to the same physical state; Phil retains memories of the "previous" days and performs distinct actions "each" day. There are at least two possible interpretations of the film's events. Supposing that Phil relives numerically the same day over again, the film's events are contradictory. Since all solutions to the Einstein Field Equations are self-consistent, no solution to the Einstein Field Equations includes a trajectory like Phil's. On a second possible interpretation, Phil does not relive numerically the same day. Instead, there is some process that resets Phil's environment at the end of each day to a state qualitatively indistinguishable, but numerically distinct, from the state at the start of the day. In that case, Phil inhabits linear time instead

possible remains controversial, but there are self-consistent solutions to the Einstein Field Equations that contain closed time-like curves. A space-time S is said to be *causally bizarre* just in case S includes at least one closed time-like curve.

The limit of a time-like curve extended infinitely far into the future is said to be *future time-like infinity*. The limit of a light-like curve extended to infinite affine parameter is likewise said to be *future light-like infinity*. The union of the set of points in future time-like infinity and future light-like infinity is called the *future conformal boundary*. A pair of space-times $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M}', g'_{\mu\nu})$ is said to be *observationally indistinguishable* just in case $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M}', g'_{\mu\nu})$ satisfy the following two properties:

1. For every point $p_i \in \mathbf{M}$ with past light cone $I^-(p_i)$ and that is not on the future conformal boundary of $\mathbf{M}$, there exists a point $q_i \in \mathbf{M}'$ with past light cone $I^-(q_i)$ such that $I^-(p_i)$ and $I^-(q_i)$ are isometric.

2. For every point $q_i \in \mathbf{M}'$ with past light cone $I^-(q_i)$ and that is not on the future conformal boundary of $\mathbf{M}$, there exists a point $p_i \in \mathbf{M}$ with past light cone $I^-(p_i)$ such that $I^-(p_i)$ and $I^-(q_i)$ are isometric.

While this is the definition of observational indistinguishability originally offered in Malament, 1977a, a weaker condition suffices for my purposes. We need only to require that any observer in $(\mathbf{M}, g_{\mu\nu})$ cannot determine whether they inhabit $(\mathbf{M}, g_{\mu\nu})$ or $(\mathbf{M}', g'_{\mu\nu})$. $(\mathbf{M}, g_{\mu\nu})$ is said to be *weakly observationally indistinguishable* from $(\mathbf{M}', g'_{\mu\nu})$ just in case, for every point $p_i \in \mathbf{M}$ with past light cone $I^-(p_i)$ and that is not on the future conformal boundary of $\mathbf{M}$, there exists a point $q_i \in \mathbf{M}'$ with past light cone $I^-(q_i)$ such that $I^-(p_i)$ and $I^-(q_i)$ are isometric. As defined, the weak observational indistinguishability relation is asymmetric. The statement that $(\mathbf{M}, g_{\mu\nu})$ is weakly observationally indistinguishable from $(\mathbf{M}', g'_{\mu\nu})$ does not entail that $(\mathbf{M}', g'_{\mu\nu})$ is weakly observationally indistinguishable from $(\mathbf{M}, g_{\mu\nu})$. This should make intuitive sense. Consider that a function mapping from students to seats in a non-full classroom is injective, i.e., for every student there is a unique seat, but not surjective, i.e., there are some seats for which there are no students. Likewise, the fact

of a temporal loop. In that case, the film is self-consistent. And while a process that resets in the way that Phil experiences is improbable, such a process is not impossible.

that every past light cone in $(\mathbf{M}, g_{\mu\nu})$ has an isometric counterpart in $(\mathbf{M}', g'_{\mu\nu})$ does not entail that every past light in $(\mathbf{M}', g'_{\mu\nu})$ has an isometric counterpart in $(\mathbf{M}, g_{\mu\nu})$.

Lastly, I need to define some global properties of space-time that might vary between a space-time and its (weakly) observationally indistinguishable counterpart. $(\mathbf{M}, g_{\mu\nu})$ is *extendible* just in case there exists a space-time $(\mathbf{M}', g'_{\mu\nu})$ and an isometric embedding $\phi : \mathbf{M} \to \mathbf{M}'$ such that $\phi(\mathbf{M}) \subset \mathbf{M}'$ (Manchak, 2016, p. 268). Note that since ϕ maps $\mathbf{M}$ to a proper subset of $\mathbf{M}'$, this definition captures the intuitive idea that $\mathbf{M}'$ is an extension of $\mathbf{M}$ only if $\mathbf{M}'$ is "larger" than $\mathbf{M}$.

A space-time is *inextendible* only if the space-time has no extensions. That is, space-time is said to be inextendable just in case, relative to some background collection of models of space-time, space-time is as large as space-time can be. Of course, this condition has physical relevance only if the background collection coincides with the full collection of physically reasonable space-times (Manchak, 2020, pp. 41–42).

A space-time is *isotropic* just in case the space-time realizes the same properties in every direction from any space-time point. A space-time is *spatially* isotropic just in case, on each of a series of space-like surfaces that exhaust all of space-time, space-time realizes the same properties in every direction from a given point. A space-time is *homogenous* just in case space-time realizes the same properties at every point. A space-time is *spatially* homogenous just in case, on each of a series of space-like surfaces that exhaust all of space-time, the same properties are realized at every point. *Friedman-Lemaître-Robertson-Walker (FLRW) space-times* are those space-times that are both spatially homogenous and spatially isotropic and are the space-times used to traditionally model Big Bang cosmology, at least in the context of General Relativity.

Given some set A, a set of open sets $\{O_i\}$ is an *open cover* of A if and only if the union of all of the sets in $\{O_i\}$ contains A (Manchak, 2020, p. 57; Garrity, 2001, p. 64). Any subset of $\{O_i\}$ that also covers A is said to be a subcover Manchak, 2020, p. 57. A set A is *compact* just in case every open cover of A has a finite subcover (Garrity, 2001, p. 64). In the case that $\mathbf{M} = \mathbb{R}^n$, for some integer n, A is compact just in case A is closed and bounded (Garrity, 2001, p. 69). A is said to be *non*-compact just in case A is not compact. A space-time is *causally compact* if for all $p, q \in \mathbf{M}$, the region $J^-(p) \cap J^+(q)$ is compact (Manchak, 2009, p. 18).

A space-time is *globally hyperbolic* just in case the space-time is not causally bizarre and is causally compact (Manchak, 2020, p. 18). Alternatively, we can say that a space-time is globally hyperbolic just in case the space-time includes a Cauchy surface.

9.4 The Malament-Manchak Theorem and Related Results

Here is how Manchak states the Malament-Manchak Theorem (MMT):

> *MMT* := "Let (M, g_{ab}) be any spacetime which is not causally bizarre [and is temporally orientable]. There exists another spacetime (M', g'_{ab}) (one that is not isometric to (M, g_{ab})) such that (M, g_{ab}) is [weakly] observationally indistinguishable from (M', g'_{ab})" (Manchak, 2009, p. 54).

Since I will need the chronogeometric construction involved in Manchak's proof of the MMT for proving a result in a subsequent section, I turn to sketching Manchak's proof. I will first summarize how the proof works in quasi-ordinary English before providing a more rigorous description of the proof.

9.4.1 The "English" Version

Suppose that Pam is trying to determine what space-time she inhabits. The observational data available to Pam is contained entirely within Pam's past light cone. Pam's space-time S contains her past light cone. Suppose that another quite different space-time S' includes a space-time region qualitatively indistinguishable from Pam's past light cone. In that case, Pam will not be able to use the data that is observationally available to her, in conjunction with General Relativity, to determine whether she inhabits S or S'. More generally, all of the observational data available to any collection of observers within S are exhausted by the collection of their respective past light cones. If we can construct another quite different space-time S' that contains regions isometric to all of the past light cones in S, then no observer in S can determine whether they inhabit S or S'.

The MMT states that given any space-time S that is not causally bizarre and which is temporally orientable, there exists another space-time S', distinct from S, that is weakly observationally indistinguishable from S. In order to show that result, Manchak begins with a space-time S, which he assumes to be not causally bizarre and temporally orientable, and then explicitly constructs S' from S. To do so, Manchak considers a sequence of light cones in S; let's denote that sequence of light cones $C = \{c_1, c_2, ...\}$. For each light cone c_i in C, construct a space-time containing a region isometric to c_i; the result is a countably infinite collection of distinct space-times that we can denote $\{S_1, S_2, ...\}$. Now take one of those space-times S_i and place the mouth of a wormhole in that space-time outside of the region isometric to c_i. Since the wormhole is placed outside of the region isometric to c_i, the resulting space-time, which includes the wormhole mouth, still includes a region isometric to c_i. Connect that wormhole to another space-time – which we can call $\tilde{S}_i$ – which can have any collection of properties that we'd like. Now, construct another wormhole mouth in $\tilde{S}_i$ that connects to a wormhole in S_{i+1}, while ensuring that the wormhole in S_{i+1} is outside of the region isometric to c_{i+1}. Let's refer to each of the $\tilde{S}_i$ as "filler" space-times.

Since we can iterate through the entire sequence of space-times $\{S_1, S_2, ...\}$, placing a filler space-time "between" each of them, we've effectively constructed one giant space-time; the procedure used for constructing that giant space-time is referred to as the *clothesline construction*, since the space-times are analogous to clothes and the wormwholes between all of the space-times are analogous to a clothesline. Since the giant space-time that results from the clothesline construction includes all of the light cones from the original space-time, the original space-time is observationally indistinguishable from the giant space-time. And since the filler space-times can have nearly any set of properties that we'd like, the giant space-time can be almost arbitrarily different from the original space-time.

Supposing that the characters on the television show *Star Trek* inhabited a giant space-time resulting from a clothesline construction, we can imagine the following scenario. Suppose that the space-time region inhabited by the Milky Way galaxy has a finite past with a temporal boundary at the Big Bang. Since the *Enterprise* inhabits one of the giant space-times resulting from the clothesline construction, the *Enterprise* can be piloted through one of the wormholes. Exiting the wormhole, the *Enterprise* crew encounter a space-time region without a past temporal boundary. But from that space-time region, they can again encounter a wormhole. Piloting the *Enterprise* through that wormhole, they emerge to find a space-time region that closely resembles the space-time region they started in, but which is numerically distinct from their original space-time region. *That* space-time region has a finite past, originating in its own big bang. Again, the *Enterprise* can encounter a wormhole. Traversing the wormhole, the *Enterprise* crew could find a space-time region with radically different properties from any region they have encountered before. And so on; the *Enterprise* could continue traversing wormholes and encountering space-time regions with almost any set of characteristics that we could care to specify interleaved with space-times that are qualitatively similar to the space-time they originally inhabited.

One might worry that this construction will result in space-times that do not satisfy the laws of physics. For example, relativistic space-times satisfy the Einstein Field Equations. When we cut up space-times and then reconnect them, as in the clothesline construction, are we guaranteed that the resulting space-time satisfies the Einstein Field Equations? As I discuss in section 9.4.3, Manchak has previously proved that the space-times constructed using the clothesline construction will satisfy any set of local conditions, including the Einstein Field Equations. Moreover, while the MMT may be an interesting piece of mathematics, I am ultimately interested in the philosophical and scientific lessons that may be drawn from the MMT. As I discuss in section 9.4.4, Cinti and Fano have objected that the space-time resulting from the clothesline construction is unphysical, so that perhaps we can rule out clothesline-type space-times as live possibilities for the global structure of our space-time. While I concede that the clothesline construction might itself be an implausible candidate for the global structure of space-time, I suggest that qualitatively similar space-times may be constructed that are more realistic candidates for the global structure of our space-time.

9.4.2 The Technical Version

I now turn to a more technical sketch of Manchak's proof. Following Manchak (2009), suppose that $(\mathbf{M}, g_{\mu\nu})$ is a non-causally bizarre space-time. Let $\{p_i\}$ be a countable series of points in $\mathbf{M}$ such that i is a positive integer and $\cup_i I^-(p_i) = \mathbf{M}$. Since $(\mathbf{M}, g_{\mu\nu})$ is assumed to not be causally bizarre, we know that, for all i, $I^-(p_i) \neq \mathbf{M}$. For each point p_i, identify some other point $q_i \in \mathbf{M}$. Denote the neighborhood of q_i as O_i and choose O_i such that $O_i \cap I^-(p_i) = \emptyset$. Since, by definition, $\mathbf{M}$ satisfies the Hausdorff condition,[4] we can choose O_i and O_{i+1} such that $O_i \cap O_{i+1} = \emptyset$. Let $K_i^+ \subset O_i$ and $K_i^- \subset O_i$ be three dimensional space-like surfaces such that $K_i^+ \cap K_i^- = \emptyset$. For example, K_i^+ might be the interior and surface of a three-dimensional sphere.

We can now use the so-called "clothesline" construction to construct a space-time $(\mathbf{M}', g'_{\mu\nu})$ that is weakly observationally indistinguishable from $(\mathbf{M}, g_{\mu\nu})$. This construction involves a countably infinite collection of space-time manifolds – each denoted either $(\mathbf{M}(i,\alpha), g_{\mu\nu})$ or $(\mathbf{M}(i,\beta), g_{\mu\nu})$ – strung together by an intricate series of "wormholes". Define $(\mathbf{M}(i,\alpha), g_{\mu\nu})$ and $(\mathbf{M}(i,\beta), g_{\mu\nu})$ such that:

$$\mathbf{M}(i,j) = \begin{cases} \mathbf{M} - K_1^+ & i = 1 \qquad j = \alpha \\ \mathbf{M} - (K_i^+ \cup K_i^-) & i > 1 \qquad j = \alpha \\ \mathbf{M} - (K_i^+ \cup K_{i+1}^-) & i = 1, 2, 3, \ldots \quad j = \beta \end{cases}$$

Now let's string together the $\mathbf{M}(i, j)$. On a two-dimensional space-time diagram where time is represented by a vertical axis and we retain one dimension of space running along the horizontal, K_i^+ and K_i^- appear as line segments. For that reason, on the two-dimensional diagram, we can distinguish the *lower edge* and the *upper edge* of K_i^+ (for example), where the lower and upper edges correspond to the three dimensional interior volume of K_i^+. Trajectories entering the lower edge of K_i^+ are entering the interior of K_i^+ from the past while trajectories leaving the upper edge of K_i^+ are going from the interior of K_i^+ into the future. For all values of i, make the following identifications, where we exclude the boundary points:

[4]A space-time S is said to satisfy the Hausdorff condition just in case S satisfies the following condition. Given any two points p and q in S, there exists an open set P centered on p and an open set Q centered on q such that P and Q are disjoint.

$$
\begin{aligned}
\text{upper edge of } K_i^+ \text{ in } (\mathbf{M}(i,\alpha), g_{\mu\nu}) &\iff \text{lower edge of } K_i^+ \text{ in } (\mathbf{M}(i,\beta), g_{\mu\nu}) \\
\text{lower edge of } K_i^+ \text{ in } (\mathbf{M}(i,\alpha), g_{\mu\nu}) &\iff \text{upper edge of } K_i^+ \text{ in } (\mathbf{M}(i,\beta), g_{\mu\nu}) \\
\text{upper edge of } K_{i+1}^+ \text{ in } (\mathbf{M}(i,\beta), g_{\mu\nu}) &\iff \text{lower edge of } K_i^+ \text{ in } (\mathbf{M}(i+1,\alpha), g_{\mu\nu}) \\
\text{lower edge of } K_{i+1}^+ \text{ in } (\mathbf{M}(i,\beta), g_{\mu\nu}) &\iff \text{upper edge of } K_i^+ \text{ in } (\mathbf{M}(i+1,\alpha), g_{\mu\nu})
\end{aligned}
$$

We've now successfully strung together all of the $(\mathbf{M}(i,j), g_{\mu\nu})$; that is, $(\mathbf{M}(1,\alpha), g_{\mu\nu})$ is connected to $(\mathbf{M}(1,\beta), g_{\mu\nu})$, which is connected to $(\mathbf{M}(2,\alpha), g_{\mu\nu})$, which is connected to $(\mathbf{M}(2,\beta), g_{\mu\nu})$, and so on. The strung together series of space-times itself forms a space-time; denote that space-time $(\mathbf{M}', g'_{\mu\nu})$. Since each of the $(\mathbf{M}(i,\alpha), g_{\mu\nu})$ contains a copy of $I^-(p_i)$, $(\mathbf{M}', g'_{\mu\nu})$ contains all of the $I^-(p_i)$'s. Thus, $(\mathbf{M}, g_{\mu\nu})$ is weakly observationally indistinguishable from $(\mathbf{M}', g'_{\mu\nu})$. Since $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M}', g'_{\mu\nu})$ are not isometric, we have that all non-causally bizarre space-times are weakly observationally indistinguishable from some other space-times. (For a more rigorous statement, see Manchak, 2009.)

Manchak has provided a logically stronger variant of the MMT. Recall the definition of 'locally isometric', i.e., space-times $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M}', g'_{\mu\nu})$ are locally isometric just in case for each point p in $\mathbf{M}$ there is an open neighborhood $O \subset \mathbf{M}$ containing p and an open neighborhood $O' \subset \mathbf{M}'$ such that $(O, g_{\mu\nu})$ and $(O', g'_{\mu\nu})$ are isometric and vice versa. Using the definition of 'locally isometric', we can define the notion of a local property. A space-time property ρ is *local* if, given any pair of locally isometric space-times S and S', S has ρ if and only if S' has ρ (Manchak, 2020, p. 11). A property ρ is *global* if and only if ρ is not local (Manchak, 2020, p. 11). One implication of the MMT is that we cannot use the data in any collection of past light cones in $(\mathbf{M}, g_{\mu\nu})$ to infer the global properties of $(\mathbf{M}, g_{\mu\nu})$; but what are the global properties that we are unable to infer? Manchak (2011) describes four: inextendibility, isotropy, global hyperbolicity, and hole-freeness.[5] Thus, a logically stronger version of the MMT can be stated in terms of these four global properties. Let $(\mathbf{M}, g_{\mu\nu})$ be any spacetime which is not causally bizarre and is temporally

[5]↑For a definition and discussion of the property of "hole-freeness", see Manchak, 2016.

orientable; moreover, let *GP* be the set of properties { inextendibility, isotropy, globally hyperbolicity, hole-freeness }. Then the logically stronger version of the MMT states:

> *MMT-GP* := If $(\mathbf{M}, g_{\mu\nu})$ has all of the properties in the set *GP* then there exists another space-time $(\mathbf{M}', g'_{\mu\nu})$ such that (i) $(\mathbf{M}', g'_{\mu\nu})$ is not isometric to $(\mathbf{M}', g'_{\mu\nu})$, (ii) $(\mathbf{M}', g'_{\mu\nu})$ does not satisfy all of the properties in *GP*, and (iii) $(\mathbf{M}, g_{\mu\nu})$ is weakly observationally indistinguishable from $(\mathbf{M}', g'_{\mu\nu})$.

By de Morgan's law, MMT-GP entails that if $(\mathbf{M}, g_{\mu\nu})$ satisfies the four properties in *GP*, then $(\mathbf{M}, g_{\mu\nu})$ has a weakly observationally indistinguishable counterpart that is either not inextendable, not isotropic, not globally hyperbolic, or not hole-free. We can ask the further question as to whether, e.g., any isotropic space-time has a non-isotropic weakly observationally indistinguishable counterpart (and likewise for the other three global properties). While, as far as I know, no general set of theorems exists for all four global properties, Malament (1977a) defends the following conclusion. Following Malament (1977a, pp. 70–71), given a space-time S, a property Π of S is *invariant* under weak observational indistinguishability (WOI) just in case (i) there exists another space-time S' such that S and S' are non-isometric, (ii) S is weakly observationally indistinguishable from S', and (iii) S has Π only if S' has Π. Let GP^* be the set of global properties { temporal orientability, spatially orientability, orientability, non-compactness, having a global time function, having a Cauchy surface }. As Malament argues, none of the properties in GP^* are invariant under WOI. Consequently, that a space-time satisfies one of the properties in GP^* is consistent with that space-time having a weakly observationally indistinguishable counterpart that does not satisfy that property. We will return to this result below when we consider whether the Direction Condition is invariant under WOI.

9.4.3 Laws and the MMT

One might worry that two observationally indistinguishable space-times may be such that only one of them solves the EFE. If only one of the two solves the EFE, then, given that we have grounds to think that the EFE describe laws of nature that can be projected into unobservable regions, we have reason to prefer the space-time that solves the EFE over the one that does not. In that case, the epistemological predicament would have been considerably weakened. However, Manchak is able to prove a stronger result that *guarantees* that, beginning with a space-time satisfying the EFE, one can construct an observationally indistinguishable space-time that also satisfies the EFE. To consider Manchak's stronger version of the MMT, I first need to offer some definitions.

Recall that two space-times $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M'}, g'_{\mu\nu})$ are said to be locally isometric if, for every point p in $(\mathbf{M}, g_{\mu\nu})$, there exists an open neighborhood around p isometric to an open neighborhood around a corresponding point p' in $(\mathbf{M'}, g'_{\mu\nu})$. Recall, too, that the notion of a local property was defined in terms of local isometry. Analogous to the notion of a local property, we can define the notion of a collection of local conditions. A collection of conditions $\mathfrak{C}$ is considered *local* if, given any two locally isometric space-times $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M'}, g'_{\mu\nu})$, $(\mathbf{M}, g_{\mu\nu})$ satisfies $\mathfrak{C}$ if and only if $(\mathbf{M'}, g'_{\mu\nu})$ satisfies $\mathfrak{C}$. Manchak proves the following theorem:

> *MMT-Laws* := "Let (M, g_{ab}) be any [temporally orientable] spacetime which is not causally bizarre satisfying any set $\mathfrak{C}$ of local conditions. There exists another spacetime (M', g'_{ab}) (one that is not isometric to (M, g_{ab})) such that (i) (M', g'_{ab}) satisfies the set $\mathfrak{C}$ of local conditions and (ii) (M, g_{ab}) is [weakly] observationally indistinguishable from (M', g'_{ab})" (Manchak, 2009, p. 55).

There is, again, another more precise statement of this theorem in terms of four important global properties, i.e., inextendibility, isotropy, global hyperbolicity, and hole-freeness. The set $\mathfrak{C}$ of local conditions can include the EFE, so that, given that $(\mathbf{M}, g_{ab})$ satisfies the EFE, a corresponding $(\mathbf{M'}, g'_{ab})$ can be constructed that likewise solves the EFE. This stronger theorem takes care of an additional worry. Philosophers of science have long recognized that one reason we may have for being committed to the existence of unobservable entities is that we are more generally committed to a realistic interpretation of a scientific theory entailing the existence of those unobservable entities. The global properties of space-time might not be observable, but, if we are already committed, on independent grounds, to a broader theory that entails that space-time has specific global properties, then we have reason to be committed to the view that space-time has those specific global properties. But, given MMT-Laws, our commitment to a more general theory of space-time – for example, our commitment to General Relativity – in conjunction with any collection of observations that could be made by an observer within a non-causally bizarre space-time does not entail that their space-time has a specific set of global properties. We can always identify another space-time, weakly observationally indistinguishable from our own, that satisfies the same (local) physical laws.

Recall that the MMT is proved using the clothesline construction. As shown in Manchak, 2009, given that the EFE are a set of local conditions, the clothes-line construction can be done in such a way that, beginning with a space-time satisfying the EFE, the resultant space-time also satisfies the EFE. One may worry that there are other constraints – such as those due to the matter-energy content in space-time – that could delimit the collection of possible space-times that one might inhabit. But, as Malament has argued, the clothesline construction can be modified in such a way that beginning with a space-time with a particular matter-energy content, the newly constructed space-time will also have an appropriate matter-energy content (Malament, 1977a, pp. 75–76).

9.4.4 Cinti and Fano's Objection

Before moving on to a discussion of whether DB space-times are weakly observationally indistinguishable from non-DB space-times, I briefly turn to an important recent objection made by Enrico Cinti and Vincenzo Fano (2021). Let's call the space-times that result from the clothesline construction *clothesline space-times*. While Cinti and Fano admit that clothesline space-times are solutions to the Einstein Field Equations, they claim that clothesline space-times are physically unreasonable. As Cinti and Fano point out, clothesline space-times include naked singularities that do not result from any antecedent physical process. Since the naked singularities do not result from any antecedent physical process, the claim goes that clothesline space-times are physically unreasonable.

If clothesline space-times are physically unreasonable, then we can rule clothesline space-times out as live possibilities for the structure of the space-time we inhabit. In that case, Cinti and Fano argue, the MMT is not relevant for whether we can discern the global structure of our space-time. Several replies can be offered.

As a possible first response, one may wonder whether the criteria that Cinti and Fano pick out for determining whether a space-time is physically reasonable are good criteria. There is a live dispute concerning what features a space-time needs to have in order to be considered physically reasonable (Manchak, 2011, 2021). However, I'm inclined to agree with the criteria that Cinti and Fano pick out. For that reason, let's put this first reply to one side. If we accept Cinti and Fano's criteria, we are left with a question as to whether the MMT and allied results need to be constructed using the clothesline construction. For example, if an alternative, but qualitatively similar, construction can be provided, then Cinti and Fano's objection will turn out to rely on features that are idiosyncratic to a specific version of the clothesline construction. There are at least two ways that a qualitatively similar construction might be carried out.

First, the clothesline construction depends upon connecting a series of space-times via wormholes. The naked singularities appear because of the specific way in which the clothesline construction's wormholes are mathematically constructed. Instead of connecting a series of space-times via wormholes, the series of space-times could be isometrically embedded in a higher dimensional space-time. In that case, there will still be one large space-time with subregions from each of a series of space-times, as in the clothesline construction. But, instead of being connected by wormholes, the space-times will be connected to each other by being space-like related to one another in a higher dimension.

Second, while the cut-and-paste methodology standardly used in the clothesline construction results in wormholes featuring inexplicable naked singularities, wormholes can be constructed without naked singularities. For example, there could be a physical process in some distant region space-like connected to us that does result in a wormhole that bridges our space-time to another space-time. In that case, a series of space-time regions can, again, be strung together in a manner qualitatively similar to the clothesline construction. In fact, if the ER=EPR conjecture – that is, that every entangled pair of particles is connected by a wormhole – turns out to be correct, the Cosmos might be filled with vast numbers of wormholes.

So far, we've reviewed three important theorems (MMT, MMT-GP, and MMT-Laws) concerning the weak observational indistinguishability of non-isometric space-times and their consequences for making inferences about the global or cosmological properties of space-time. I've also responded to an important recent objection to the clothesline construction used in the three theorems. Next, I turn to discussing whether DB space-times can be observationally distinguished from non-DB space-times.

9.5 DB Space-Times and Observational Indistinguishability

The question as to whether DB space-times are weakly observationally indistinguishable from non-DB space-times can be answered by addressing whether the Direction and Boundary Conditions are invariant under WOI. A number of important results are already known that help to elucidate this question.

Here is a quasi-"English" language summary of the results that I prove in the subsequent subsections. In section 9.5.1, I discuss a series of results from Malament suggesting that space-times satisfying the Direction Condition – that is, D space-times – are weakly observationally indistinguishable from non-D space-times. However, I note that insofar as we have evidence for the A-theory of time, we have local evidence that would delimit the space of possible space-times to those that satisfy the Direction Condition. Whether we do have good local evidence for the A theory of time remains controversial.

In section 9.5.2, I discuss a series of results suggesting that space-times satisfying the Boundary Condition are weakly observationally indistinguishable from space-times that do not satisfy the Boundary Condition. With the exception of the subsection titled 'Open boundary finitely far to the past', I do not rely upon the clothesline construction for the results that I prove in this section. This is important, since, even if the objection offered by Cinti and Fano were successful, their objection applies only to results that make use of the clothesline construction. First, I discuss the hypothesis that our space-time could be embedded in a higher dimensional space. Compare the following two hypotheses: first, the Cosmos is such that our space-time is not embedded in a higher dimensional space, or, second, the Cosmos is such that our space-time is embedded in a higher dimensional space. Supposing that the Boundary Condition is satisfied in the former case does not entail that the Boundary Condition is satisfied in the latter case, even though the two cases are not observationally distinguishable. I introduce a new kind of observational indistinguishability that I call super weak observational indistinguishability. I then prove a series of results showing that space-times that include various kinds of past boundaries are at least super weakly observationally indistinguishable from space-times that do not include those past boundaries.

9.5.1 The Direction Condition

Consider three global properties: (i) temporal orientability, (ii) the existence of a global time function, and (iii) the existence of a Cauchy surface. We've already discussed temporal orientability in chapter 6, so I won't define that concept again here. A space-time is said to have a global time function (roughly) just in case there is at least one way to carve the entire space-time up into simultaneity slices that can be labeled with a time.[6] Lastly, a Cauchy surface is a space-like surface that is intersected no more than once by every (inextendable and differentiable) time-like curve. That a space-time includes a Cauchy surface turns out to be equivalent to the statement that the space-time is globally hyperbolic. In turn, the initial value problem is well-defined for the Einstein Field Equations only if space-time is globally hyperbolic. Thus, (i)-(iii) are closely related to the notion that space-time can be thought of as having a global development from past to future; in fact, violation of any one of (i)-(iii) would suffice for violating the Direction Condition. As previously discussed, Malament (1977a) argued that (i)-(iii) are not invariant under WOI. Consequently, the Direction Condition is not invariant under WOI. Therefore, DB space-times are weakly observationally indistinguishable from non-DB space-times. Taken at face value, this result suggests that observers living in DB space-times can never amass sufficient data to determine that they inhabit a DB space-time.

[6]↑More rigorously, a space-time is said to have a global time function just in case there exists a smooth map $t : M \to \mathbb{R}$ and $\forall x \forall y \in M$, ($x$ is before y and $x \neq y$) $\implies$ ($t(x) < t(y)$).

There is reason not to take this conclusion at face value. There may be reason to restrict the space of solutions to the EFE to those space-times that are in some sense physically reasonable. For example, let's call space-times that satisfy the Direction Condition D space-times. While there is, as yet, no agreement among philosophers or physicists as to which conditions physically reasonable space-times satisfy, if the *A*-theory of time is true, then, plausibly, the *A*-theory of time is necessarily true (or true in all possible worlds where time exists). In that case, since the *A*-theory of time plausibly requires temporal orientability, the existence of a global time function, and the existence of a Cauchy surface, the *A*-theory of time plausibly requires that the space-time we inhabit is a D space-time. Thus, if the *A*-theory of time is true, then, plausibly, non-D space-times are metaphysically impossible. Moreover, the most popular arguments for the *A*-theory of time are based on our phenomenological experience of temporal passage. Since experience is a local phenomenon – in that conscious observers have specific spatio-temporal locations – *A*-theorists endorse a view according to which we can know that we inhabit a D space-time on the basis of local evidence. Such arguments are deeply controversial and *B*- and *C*-theorists would likely not accept, at least on that basis, such a restriction to D space-times. This is a significant result: whether we have sufficient grounds for inferring from local evidence that our Cosmos includes a D space-time turns out to depend on how the metaphysical debate concerning the fundamental nature of time is ultimately decided. Nonetheless, there is more to be said; let's set aside the Direction Condition and focus on whether the Boundary Condition is invariant under WOI.

9.5.2 The Boundary Condition

There are several senses in which DB space-times might be weakly observationally indistinguishable from non-DB space-times. First, as proved by CJS Clarke (1970), any non-compact space-time can be globally embedded in a higher dimensional flat 89 dimensionsional space with arbitrarily high differentiability conditions. As George Ellis (1971, p. 9) notes, this entails that "the original concept of a manifold as a subspace of a flat space extends to the space-time manifold (M, g) of every reasonable cosmological model". Subsequently, Marc Lachièze-Rey (2000) showed that any FLRW space-time can be isometrically embedded in a flat five dimensional space-time.[7] This suggests the possibility that our space-time is embedded in a higher dimensional space. Supposing that our space-time is embedded in a higher dimensional space, could we observationally determine whether the higher dimensional space-time satisfies the Boundary Condition? A similar question arises if, as some authors (e.g., Alyssa Ney, David Albert, Jill North, Barry Loewer) have suggested, the space-time of our ordinary experience is functionally realized by the distribution of the universal wavefunction in a higher dimensional, fundamental space, e.g., configuration space or perhaps something more exotic, or if, as string theorists have suggested, our space-time is a brane in a higher dimensional space-time. Let's call these scenarios Higher Dimensional Scenarios.

Assuming that one of the Higher Dimensional Scenarios is correct, the Boundary Condition is plausibly not invariant under WOI. Let's denote our space-time S, the higher dimensional space-time S_{hd}, and suppose that S_{hd} satisfies the Boundary Condition. Let's use $\{p_i\}$ to denote the set of space-time points in S so that $\cup_i I^-(p_i)$ is the union of all of the past light cones of all possible observers in S. I see no reason to block the construction of another space-time S'_{hd} with the following properties: (i) S'_{hd} has the same dimensionality as S_{hd}, (ii) S is embedded in S'_{hd} or else S is functionally realized by some entities in S'_{hd}, and (iii) S'_{hd} does not satisfy the Boundary Condition. Consequently, if we allow for the possibility of Higher Dimensional Scenarios, then, plausibly, the Direction Condition is not invariant under WOI. Nonetheless, Higher Dimensional Scenarios are at least controversial, so let's set aside Higher Dimensional Scenarios for the remainder of this chapter. Supposing that space-time is restricted to four dimensions, could we have grounds for inferring that the space-time we inhabit satisfies the Boundary Condition?

For my purposes, I do not need a statement as strong as the MMT. Recall that a space-time $(\mathbf{M}, g_{\mu\nu})$ is *weakly observationally indistinguishable* (to use Malament's terminology; see, e.g., Malament, 1977a, p. 68) from $(\mathbf{M}', g'_{\mu\nu})$ just in case for every point p in $(\mathbf{M}, g_{\mu\nu})$ there exists a point p' in $(\mathbf{M}', g'_{\mu\nu})$ such that $I^-(p)$ and $I^-(p')$ are isometric. This condition would guarantee that there are no observers, at *any* location in $(\mathbf{M}, g_{\mu\nu})$, who could make

[7] ↑A short historical review is provided in Wesson, 2010.

an observation that would justify their saying that they are not in $(\mathbf{M}', g'_{\mu\nu})$. We do not need the condition that our space-time is indistinguishable from another for *any* possible observer in our space-time no matter where they are situated. All the space-time points that could ever be observed by members of our species, and any observer with whom humans will ever have two-way communication, presumably occupies a finite space-time hypervolume. Therefore, for members of our species to be unable to distinguish our space-time from another very different space-time is entailed by, but does not require, weak observational indistinguishability. In the next section, I develop an even weaker form of observational indistinguishability than those that were previously developed by Malament or by Manchak.

Super Weak Observational Indistinguishability

Let's define another form of observational indistinguishability that I call *super weak observational indistinguishability*. First, I need to offer some additional definitions. A space-time region is said to be space-like just in case all of the points in that region are space-like related to one another. A congruence is a "bundle" of curves through space-time; if the congruence includes only time-like curves (for example) then the congruence is said to be time-like. Intuitively, we can think of a time-like congruence as analogous to a bundle of uncooked spaghetti, where each uncooked spaghetti noodle is analogous to a time-like curve. Consider two space-like regions R_1 and R_2 in a space-time S such that there exists a time-like and light-like congruence C consisting of all of the time-like and light-like curves passing through both R_1 and R_2. Let U denote the space-time region formed by the set of events that can reach, without exceeding the speed of light, any point p_i on any curve in C, that is, $U = \cup_i I^-(p_i)$. Moreover, let's suppose that C is large enough that a collection of observers within C will never come into contact with observers whose past light cones include points outside of U. For example, C might be the observable universe. If there exists another space-time S' such that there is a region isometric to U in S', then S is said to be super weakly observationally indistinguishable from S'. The MMT guarantees that any temporally orientable and non-causally bizarre space-time will be super weakly observationally indistinguishable from some other very different space-time. But super weakly observationablly indistinguishable space-times are a larger collection because, unlike the MMT, there is no demand that observational indistinguishability hold for all points in space-time.

Are DB space-times super weakly observationally indistinguishable from non-DB space-times? To start, consider a non-causally bizarre DB space-time *S*. Since all DB space-times are temporally orientable, *S* is temporally orientable. Given the MMT, *S* is weakly observationally indistinguishable from some other distinct (i.e., non-isometric) space-time. And since weak observational indistinguishability is logically stronger than super weak observational indistingishuability, the conclusion follows that *S* is super weakly observationally indistinguishable from some distinct space-time.

So far, I have shown only that all non-causally bizarre DB space-times are super weakly observationally indistinguishable from some distinct space-time; this does not suffice for the stronger conclusion that all non-causally bizarre DB space-times are super weakly observationally indistinguishable from some non-DB space-time. For this section, I've set aside the question as to whether the Direction Condition is invariant under WOI or SWOI in order to investigate whether the Boundary Condition is invariant under WOI or SWOI. There are two ways that *S* can satisfy the Boundary Condition: either there is a past topological boundary to space-time or else there is a finite initial segment, as defined in chapter 7. Therefore, we can re-phrase our question as the following: are either a past topological boundary or a finite initial segment invariant under SWOI? I will investigate the invariance of the topological boundary and of a finite initial segment under SWOI in turn.

Past Topological Boundary

Suppose that *S* satisfies the Direction Condition and satisfies the Boundary Condition by including a closed boundary ζ to the past of every time-like and light-like curve.[8] Let *S* be denoted $(\mathbf{M}, g_{\mu\nu})$. In this case, ζ is a topological boundary. To show that $(\mathbf{M}, g_{\mu\nu})$

[8]↑To be more precise, ζ is the collection of all of the points bounding time-like and light-like curves in the past.

is super weakly observationally indistiguishable from some distinct non-DB space-time, our job will involve constructing a corresponding non-DB space-time $(\mathbf{M}', g'_{\mu\nu})$ that does not satisfy the Boundary Condition. Since we are considering the case that ζ is a closed boundary, I will allow for the possibility that ζ is infinitely far to the past (e.g., there is a point of ζ at past time-like infinity for every observer); I will say more about constructing this case below. I will also allow for the possibility that ζ is succeeded by a space-time wide metrically amorphous region so that there is no determinate fact about the space-time interval between ζ and any given observer not located on ζ; I will have more to say about how to construct this possibility below. There are then three sub-cases: first, ζ might be located finitely far to the past of any given observer, second, ζ is located infinitely far to the past, and, third, ζ is located indeterminately far to the past.

ζ is at finite proper time to the past

Suppose that ζ is located at some finite proper time to the past of any possible observer. Recall the notion of an extendable space-time previously defined, where the intuition is that an extendable space-time $(\mathbf{M}, g_{\mu\nu})$ can be made "larger" because there is a proper part of $(\mathbf{M}', g'_{\mu\nu})$ isometric to $(\mathbf{M}, g_{\mu\nu})$. Moreover, I will refer to $(\mathbf{M}', g'_{\mu\nu})$ as the *extension* of $(\mathbf{M}, g_{\mu\nu})$. If $(\mathbf{M}, g_{\mu\nu})$ is inextendable, then we say that $(\mathbf{M}, g_{\mu\nu})$ is *maximally extended*. Moreover, recall that $\phi : \mathbf{M} \to \mathbf{M}'$ is the diffeomorphism mapping points in $\mathbf{M}$ to their counterparts in $\mathbf{M}'$. Any relativistic space-time with a closed boundary ζ_0 has an extension in which $\phi\zeta_0$ is not a boundary; in particular, given that both $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M}', g'_{\mu\nu})$ satisfy the Direction Condition, the extension will include points to the past of $\phi\zeta_0$. We can therefore say that any relativistic space-time with a past space-time wide closed boundary ζ_0 has an extension to the past of ζ_0. Since any relativistic space-time with a specific closed boundary has an extension to the past of that boundary, S is weakly observationally indistinguishable from a space-time without S's closed boundary. And since weak observational indistinguishability is a logically stronger condition than super weak observational indistinguishability, S is super weakly observationally indistinguishable from a space-time without S's closed boundary.

This does not suffice for showing that a DB space-time with a closed boundary at finite proper time to the past is super weakly observationally indistinguishable from a non-DB space-time; for example, all of the extensions of S could themselves be DB space-times. For example, even if the closed boundary of S does not map to a closed boundary in S', S' could still include a space-time wide closed boundary to the past of all non-initial points in S'. If S, together with all of S's super weakly observationally indsitinguishable counterparts, have space-time wide closed boundaries, then observers in S could (in principle) use their data to infer that their space-time has a space-time wide closed boundary even if they couldn't infer how distant the boundary is to their past. Fortunately for my purposes, every space-time that includes closed boundary in the finite past has an extension without a closed boundary in the finite past.[9] This leaves us with the possibility that the space-time includes a closed boundary is located infinitely far to the past; I will consider that possibility below.

Perhaps the argument I've presented in this subsection has gone too fast. Whether $(\mathbf{M}, g_{\mu\nu})$ has an extension is always relative to whatever class of space-times are understood to be possible. There is an on-going philosophical debate concerning which features a space-time must possess in order to be considered physically reasonable, but we can likewise ask which features a space-time must possess in order to be considered metaphysically reasonable. By way of example, suppose that the A-theory of time is true. If the A-theory of time is true, then the A-theory of time likely describes some essential features of time, so that the A-theory of time is true in all metaphysically possible worlds where time exists. The A-theory of time – at least as traditionally understood – likely requires that space-time is globally hyperbolic. Let's say that $(\mathbf{M}, g_{\mu\nu})$ is *maximally GH-extended* just in case $(\mathbf{M}, g_{\mu\nu})$ has no globally hyperbolic extensions. If global hyperbolicity is a metaphysically necessary feature of space-time, then any space-time without a GH-extension has no metaphysically possible extensions.

[9]↑This follows from a clothesline construction similar to the one that I use below for showing that all space-times with a past Cosmos-wide singular boundary are observationally indistinguishable from a space-time without a past Cosmos-wide singular boundary.

Consider an illustrative example from Manchak (2021). There is a specific solution to the Einstein Field Equations, called *Misner space-time*, with the feature that in one region of the space-time, a closed time-like curve passes through every point and, in another region, there are no closed time-like curves. The specific details as to how one might mathematically construct Misner space-time are not important for my purposes; for my purposes, what is relevant is that Misner space-time is not globally hyperbolic because Misner space-time includes a region that includes closed time-like curves. So, supposing that only globally hyperbolic space-times are metaphysically possible, Misner space-time is not metaphysically possible. Nonetheless, Misner space-time includes a hyperbolic region – that is, the region in Misner space-time that does not include closed time-like curves. Consider a truncated Misner space-time that includes only the hyperbolic region. The resulting space-time is globally hyperbolic; thus, nothing I've said so far rules out that truncated Misner space-time as a legitimate metaphysical possibility. Moreover, the truncated Misner space-time is maximally GH-extended because, as Manchak has argued, the truncated Misner space-time has no GH-extensions. Similar conclusions follow given other restrictions on the space of possible space-times. Thus, whether a space-time with a closed boundary a finite proper time to the past of any possible observer has a metaphysically possible extension will depend on what sort of restrictions are implemented with respect to the space of metaphysically possible space-times.

There is then perhaps one way to infer that the Cosmos has a closed boundary in the finite past. If one could show that the Cosmos is one member of a collection of space-times with closed boundaries located in the finite past and that do not have metaphysically possible extensions without closed boundaries in the finite past, then we would have reason for thinking that the space-time we inhabit has a closed boundary in the finite past. Nonetheless, this strategy has, to my knowledge, never been pursued. Since I cannot see how to formulate such an argument and since the debate about what features a space-time must have in order to be metaphysically (or physically) reasonable is still on-going, I do not think this strategy has much merit.

ζ is infinitely far to the past

Suppose instead that ζ is a closed boundary located infinitely far to the past. In this section, I will assume that ζ is not succeeded by a finite initial segment and will postpone discussion of that possibility until a subsequent section. An immediate first objection is that relativistic space-times, as they are ordinarily considered, do not include points at past time-like or light-like infinity. However, as I have previously discussed in this dissertation, there are a number of mathematical procedures for "adding in" points at infinity, as in, e.g., the extended complex plane. A similar procedure can be carried out for relativistic space-times. Suppose that $(\mathbf{M}, g_{\mu\nu})$ is a standard inextendable and non-singular relativistic space-time that does not include points at infinity. Let's suppose that the pair $(\zeta, h_{\mu\nu})$ is a space-like surface that includes the point set A, equipped with, e.g., topological structure, and that h_{ab} is a spatial metric defined on A. We can now define a new space-time $(\mathbf{M} \cup \zeta, g'_{\mu\nu})$ such that:

1. Every point in ζ is to the absolute past of some point in $\mathbf{M}$.
2. $g'_{\mu\nu}$ is defined in such a way that the spatial metric on ζ is h_{ab} and all points in ζ are infinite affine parameter from any given point in $\mathbf{M}$.

Adding ζ to $\mathbf{M}$ amounts to including points infinitely far to the past of any point within $\mathbf{M}$. One natural choice for ζ is for ζ to simply "fill in" the past conformal boundary of $(\mathbf{M}, g_{\mu\nu})$.

Without loss of generality, consider any given observer within the portion of space-time not included in ζ. Let's say that the observer is situated at point p so that their past light cone is $I^{-}(p)$. There are two possibilities. Either $I^{-}(p)$ includes all of ζ or not. There are at least two ways for $I^{-}(p)$ to include all of ζ. First, space-time might be, e.g., Minkowski space. However, the only observers in Minkowski space whose past light cones include all of ζ are those sitting on the future conformal boundary. Second, space-time might undergo some kind of sufficiently rapid superluminal contraction. Space-times that undergo sufficiently rapid superluminal contraction develop an observational horizon. The observational horizons prevent any observer, even those on the future conformal boundary, from gathering data from the entire space-time, that is, $I^{-}(p) \neq \mathbf{M}$. Nonetheless, observers could conceivably gather data from the entirety of ζ, that is, $\zeta \subset I^{-}(p)$.

Suppose, instead, that, for any observer not located on ζ, $I^{-}(p)$ does not include all of ζ. In some sense, this is a more physically realistic scenario, since we know that the space-time we inhabit includes a cosmological horizon and may have undergone a period of inflationary expansion in the past that approximated de Sitter space. A simple construction can be used to show that the closed boundary is not invariant under SWOI. Denote space-time as $S = (\mathbf{M}, g_{\mu\nu})$ where M includes ζ. Recall that my definition of super weak observational indistinguishability involved U, the union of the past light cones of all of the points p_i on the portion of the congruence bounded between R_1 and R_2. Let's define V as the intersection between U and ζ, that is, $V = U \cap \zeta$. Now construct another space-time S' such that $S' = ((\mathbf{M} \setminus \zeta) \cup V, g_{\mu\nu})$. In this case, there is a closed boundary to the past of any point in C between R_1 and R_2 – namely V – but there is no past boundary to the points that are not to the future of V. In less technical language, there is a class of observers (namely, those on time-like curves between R_1 and R_2) for whom there is a closed boundary to their past even though there is no Cosmos-wide past boundary. Ergo, ζ is not invariant under SWOI.

Thus, we have mixed results: ζ is invariant under SWOI only if there are points whose past light cones do not include all of ζ. However, even if we set aside the fact that the space-time we inhabit includes an observational horizon, this is hardly a concession to those who hope that we can infer, from observations, whether we inhabit an MDB space-time. For example, suppose that $S = (\mathbf{M}, g_{\mu\nu})$ is a space-time where light cones do include all of ζ. In this case, we can construct another space-time $S' = (\mathbf{M} \setminus \zeta, g_{\mu\nu})$ that does not include a closed boundary infinitely far to the past. Since the two space-times are near duplicates – differing only in whether they include a set of boundary points infinitely far to the past – there is no observation that any observer could make that would distinguish the two.

We can easily prove that this is so. Let's begin by noting that any observer only "sees" the local distribution of matter-energy within their vicinity. Both S and S' are described by the same metric $g_{\mu\nu}$. The matter-energy distribution is described by the stress-energy tensor $T_{\mu\nu}$. According to the Einstein Field Equations, $T_{\mu\nu}$ is proportional to the Einstein Tensor $G_{\mu\nu}$. In turn, $G_{\mu\nu}$ can be computed from derivatives of $g_{\mu\nu}$. Thus, if two space-time regions are described by the same metric, then they have the same Einstein Tensor and, consequently, the same stress-energy tensor. Setting aside points on the boundary of S, for which there are no counterparts in S', S and S' are described by the same metric and thus the same stress-energy tensor. For that reason, observers in S encounter the same matter-energy distribution as their counterparts in S'; no observer in S could distinguish their space-time from S'.

ζ is indeterminately far to the past

In this section, I consider the possibility that ζ is indeterminately far to the past. That is, ζ is succeeded by a space-time wide region in which the space-time metric is amorphous. More rigorously, we can construct a space-time region with an amorphous metric through the following procedure. Consider a space-time $(\mathbf{M}, g_{\mu\nu})$ with a subregion $(\mathbf{R}, g_{\mu\nu})$. Note that $(\mathbf{R}, g_{\mu\nu})$ is, itself, a space-time, since $(\mathbf{R}, g_{\mu\nu})$ is a pair consisting of a manifold $\mathbf{R}$ and a metric tensor $g_{\mu\nu}$ defined on that manifold. We say that two space-times $(\mathbf{R}, g_{\mu\nu})$ and $(\mathbf{R}, g'_{\mu\nu})$ are *conformally equivalent* just in case there exists a smooth and everywhere positive scalar field $\Omega : \mathbb{R} \to \mathbb{R}$ – called the *Conformal Factor* – such that $g'_{\mu\nu} = \Omega^2 g_{\mu\nu}$ (Manchak, 2020, p. 13). To construct a metrically amorphous space-time region, take the full collection of space-times conformally equivalent to $(\mathbf{R}, g_{\mu\nu})$ and identify all members of that collection. The resulting space-time retains information about how points are connected together (for example, whether p is in the past light cone of q) but "forgets" all of the information about, e.g., the lengths of curves encoded in $g_{\mu\nu}$. Now that we've constructed a metrically amorphous space-time region, we can replace $\mathbf{R}$ in $(\mathbf{M}, g_{\mu\nu})$ with the metrically amorphous region.

Suppose that S is a space-time that satisfies the Direction Condition, includes a space-time wide past topological boundary ζ, and that S includes a space-time wide amorphous region $\mathcal{R}$. Recall that super weak observational indistinguishability is defined in terms of a set U such that $U = \cup_i I^-(p_i)$, where p_i are points within the congruence bounded by R_1 and R_2. Suppose that $\zeta \not\subset U$. We can then construct another space-time S' without a topological boundary and that is super weakly observationally indistinguishable from S'. Let $A = \zeta \cap U$. Then we can define another space-time by retaining A but removing the other points in ζ. The resulting space-time is super weakly observationally indistinguishable from the space-time with which we started.

Suppose, instead, that there is no delimited region of points p_i such that, for all i, $\zeta \not\subset I^-(p_i)$. That is, for any point p in S, $\zeta \subset I^-(p)$. In this case, since the entirety of ζ is located in the past light cone of every possible observer, that the space-time has a topological boundary is neither weakly nor super weakly observationally indistinguishable. Nonetheless, this is a Pyrrhic victory for those hoping to infer whether our space-time has a topological boundary to the past, for similar reasons as those previously expressed when ζ is located infinitely far to the past. That is, the mass-energy distribution is unaltered between S and one of S's observationally distinguishable counterparts so that local observations are unable to distinguish the two.

Finite Initial Segment

I turn now to the possibility that the Cosmos has a finite initial segment. There are four possibilities to consider. First, I postponed discussion of the possibility that a closed boundary located infinitely far to the past is succeeded by a finite initial segment; I will take up that possibility in this section. Second, there is the possibility that there is a finite initial segment located infinitely far to the past with an open boundary. Third, there is the possibility that the finite initial segment is located finitely far to the past with an open boundary. Fourth, there is the possibility that there is a finite initial segment located finitely far to the past with a closed boundary. I do not need to discuss the fourth possibility in this section because I have already discussed the fourth possibility in my discussion of a past topological boundary. Note that I do not need to discuss space-times with an initial metrically amorphous region with an open boundary because such space-times do not satisfy the Boundary Condition. This leaves us with the original three possibilities, which I will take up in turn.

Closed boundary infinitely far to the past

In this section, I consider the possibility that there is a closed boundary, again denoted ζ, located infinitely far to the past, that is succeeded by a finite initial segment of space-time. Recall that there is a finite initial segment just in case there is a space-time-wide space-like surface Σ such that all of the time-like and light-like trajectories that can be traced backwards from Σ have finite generalized affine length. Thus, in this section, the initial space-time segment is sandwiched by two space-like surfaces, ζ and Σ. Since the generalized affine length along all of the time-like and light-like curves between ζ and Σ is finite and we've assumed ζ to be located infinitely far to the past, Σ must be located infinitely far to the past.

There are two classes of observers that we should consider: first, observers who are located finitely far from ζ and, second, observers who are located infinitely far from ζ. Let's consider, first, observers who are located finitely far from ζ. Their epistemic situation is the same as the one that we previously considered in the section on the topological boundary when we considered a closed boundary finitely far to the past. Since I've already considered that case, let's move on to the second group, that is, the observers who are located infinitely far from ζ. There are two possibilities for this set of observers in that either the entirety of ζ is in their past light cone or not.

Consider the possibility that the entirety of ζ is in the past light cone of all possible observers located infinitely far to the future from ζ. Since ζ is a closed boundary, we again encounter the difficulty as to whether DB space-times with closed boundaries have non-DB extensions; in turn, as I previously argued, resolution of this issue depends on a philosophical controversy that has yet to be resolved, i.e., how to properly think about the class of metaphysically or physically reasonable space-times. But, even if we suppose that all space-times with closed boundaries have extensions with closed boundaries, there is little prospect for observers in any cosmologically realistic space-time to receive signals from infinitely far to their past. So, even if ζ turns out not to be invariant under weak observational indistinguishability, there would remain little hope of being able to detect significant information about the ζ-Σ region; thus, observers located infinitely far to the future of the ζ-Σ region are unlikely to be able to determine that the space-time they inhabit includes a finite initial portion.

Now consider the possibility that the entirety of ζ is not in the past light cone of all possible observers located infinitely far to the future from ζ. In some sense, this is a more physically realistic scenario, since (again) we know that the space-time we inhabit includes a cosmological horizon and that there may have been a period of inflationary expansion in the past that approximated de Sitter space. Here, we can modify the simple construction previously offered in the subsection titled 'ζ is infinitely far to the past' in order to show that ζ is not invariant under SWOI.

Let's begin as before. Denote the space-time of interest as $S = (\mathbf{M}, g_{\mu\nu})$, where $\mathbf{M}$ includes ζ. Recall, once more, that $U = \cup_i I^-(p_i)$, where p_i are points within the congruence bounded by R_1 and R_2 and where R_1 and R_2 are understood to be infinitely far from ζ. Let's use Δ to denote the finite initial segment that succeeds ζ. Recall how we constructed a space-time with an infinite past that includes a finite initial segment. We took a space-time with an infinite past and then joined on a finite segment in the infinite past. Therefore, if we remove the initial segment, then we are left with a space-time with an open boundary infinitely far to the past, which is just to say that if we remove the finite initial segment we are left with a non-DB space-time. In fact, there is no need to remove all of Δ to construct a non-DB space-time; removing all of the points in Δ that are in the past light cone of some point q would suffice for turning S into a non-DB space-time. This is the feature that we will exploit in this construction; we will construct a space-time retaining the points in Δ that are included in U while excluding all other points in Δ. Define V^* such that $V^* = (U \cap \zeta) \cup (U \cap \Delta)$. We now construct another space-time S' such that $S' = ((\mathbf{M} \setminus (\zeta \cup \Delta)) \cup V^*, g_{\mu\nu})$. Clearly, S' is not a DB space-time, since there are points space-like related to all of the points in the set $\{p_i\}$ such that there is no topological boundary or finite initial segment to their past. Thus, ζ is not invariant under super weak observational indistinguishability.

Open boundary infinitely far to the past

There are two possibilities. Either the open boundary is succeeded by a finite initial segment or not. If the boundary is not succeeded by a finite initial segment, then space-time is not a DB space-time – since all points in the space-time are located infinitely far to the future of the open boundary – and so lacks a beginning. Thus, in this section, I am concerned only with an open boundary located infinitely far to the past that is succeeded by a finite initial segment.

There is once more the difficulty that standard relativistic space-times do not include regions located infinitely far to the past. Nonetheless, we can utilize a similar procedure as the one previously discussed for adding in points at past time-like infinity, except that, in this case, we add a space-time region with some temporal "thickness" instead of a single space-like surface. Once we've constructed such a space-time, we can again divide the class of all possible observers into two sets. First, there is the set of possible observers who are located finitely far from the open boundary. Below, I consider the possibility that there is an open boundary finitely far to the past, and the considerations that I apply in that section apply to the group of possible observers located finitely far from the open boundary. Second, there is the group of possible observers who are located infinitely far from the open boundary. Here, we can use an argument nearly identical to the one presented at the end of the subsection titled 'Closed boundary infinitely far to the past'. That is, we can construct a non-DB space-time by retaining only those points in the finite initial segment that are in U. (In this case, there are no points in the boundary, so there are no points to retain from ζ.) The result is that, once again, the Boundary Condition is not invariant under SWOI.

Open boundary finitely far to the past

If there is an open boundary located in the finite past of all space-time points, then there are two possibilities. First, space-time might be truncated by an open boundary. This first possibility is similar to the situation considered in the subsection titled 'ζ is at finite proper time to the past'. If a space-time S is truncated by an open boundary $\mathfrak{B}$, then

S is not maximally extended, in the sense that there exists another space-time S' with a proper part isometric to S and without $\mathfrak{B}$. But, in that case, S is weakly observationally indistinguishable from S'.[10] Again, we can ask whether the extensions of S are all DB space-times and, again, whether we could determine, from some set of data, that we are inhabiting a DB space-time would be determined by whether all of the space-times weakly or super weakly observationally indistinguishable from our own are DB space-times.

The second possibility is that space-time is maximally extended but includes an open boundary in the finite past. This could be the case if, for example, space-time includes a curvature singularity to the past of every space-time point, as in many of the classic FLRW models of the Big Bang. For the sake of rigor, let's say a space-time S is *everywhere past b-incomplete* just in case there exists a Cosmos-wide space-like surface Σ such that every past-directed time-like and light-like half-curve originating on Σ has finite generalized affine length. In the case that the space-time is maximally extended and everywhere past b-incomplete, there is a curvature singularity that, in some sense, is to the past of every non-initial space-time point. (For example, every maximally extended geodesic would be past-incomplete.) Here, I will prove that any non-causally bizarre spacetime which is everywhere past b-incomplete is weakly observationally indistinguishable from some spacetime that fails to be everywhere past b-incomplete.[11]

To prove that any non-causally bizarre spacetime which is everywhere past b-incomplete is weakly observationally indistinguishable from some spacetime that fails to be everywhere past b-incomplete, consider some non-causally bizarre space-time $(\mathbf{M}, g_{\mu\nu})$. Using the clothesline construction previously discussed to prove the MMT, construct a space-time $(\mathbf{M}', g'_{\mu\nu})$ such that $(\mathbf{M}, g_{\mu\nu})$ is weakly observationally indistinguishable from $(\mathbf{M}', g'_{\mu\nu})$. In order to endow $(\mathbf{M}', g'_{\mu\nu})$ with various global properties distinct from those possessed by $(\mathbf{M}, g_{\mu\nu})$, we can make modifications to the $(\mathbf{M}(i, \beta), g_{\mu\nu})$'s; doing so will not affect the

[10]↑The reader may worry that this conclusion was reached too fast. As discussed in the aforementioned subsection, there may be metaphysical reasons to delimit the space of possible space-times. And if so, we wouldn't be able to draw the general conclusion that S has extensions without $\mathfrak{B}$. In any case, I do not have anything more to say on this issue than I already said in the subsection titled 'ζ is at finite proper time to the past'.

[11]↑Thanks to JB Manchak for his help in constructing this proof.

weak observational indistinguishability between $(\mathbf{M}, g_{\mu\nu})$ and $(\mathbf{M}', g'_{\mu\nu})$ because all of the $I^{-}(p_i)$'s are in the $(\mathbf{M}(i, \alpha), g_{\mu\nu})$'s.

Consider an open neighborhood O in the $(\mathbf{M}(1, \beta))$ portion of $(\mathbf{M}', g'_{\mu\nu})$. By lemma 1 from Manchak (2016, p. 1055),[12] there is an open neighborhood $\hat{O}$ in O with the following property. We can construct another space-time $(\mathbf{M}', g''_{\mu\nu})$ that is an exact copy of $(\mathbf{M}(1, \beta))$ outside of O but which is such that $g''_{\mu\nu}$ is flat inside $\hat{O}$. Moreover, let's re-define $(\mathbf{M}', g'_{\mu\nu})$ such that $(\mathbf{M}', g''_{\mu\nu})$ replaces $(\mathbf{M}(1, \beta))$. Now consider Minkowski space-time, which I will denote $(\mathbf{N}, h_{\mu\nu})$. Let σ be a three dimensional space-like surface in $\hat{O}$ in $(\mathbf{M}', g''_{\mu\nu})$ and let σ' be a three dimensional space-like surface in $(\mathbf{N}, h_{\mu\nu})$. Excluding boundary points, make the following identifications:

$$\text{upper edge of } \sigma \iff \text{lower edge of } \sigma'$$
$$\text{lower edge of } \sigma \iff \text{upper edge of } \sigma'$$

By combining $(\mathbf{M}', g'_{\mu\nu})$ and $(\mathbf{N}, h_{\mu\nu})$, we arrive at a new space-time. Let's denote our new space-time $(\mathbf{M}'', g''_{\mu\nu})$. Note that $(\mathbf{M}'', g''_{\mu\nu})$ is not everywhere past b-incomplete. To show that this is so, consider any maximally extended time-like geodesic γ in the $\mathbf{N}$ portion of $(\mathbf{M}'', g''_{\mu\nu})$ which does not intersect σ' or the boundary of σ'. Since all time-like geodesics are past-complete in Minkowski space-time, γ is likewise past-complete. $(\mathbf{M}, g_{\mu\nu})$ is weakly observationally indistinguishable from $(\mathbf{M}'', g''_{\mu\nu})$ for the same reasons that $(\mathbf{M}, g_{\mu\nu})$ is weakly observationally indistinguishable from $(\mathbf{M}', g'_{\mu\nu})$, that is, $(\mathbf{M}'', g''_{\mu\nu})$ contains a copy of each of the $I^{-}(p_i)$'s. Therefore, any non-causally bizarre space-time which is everywhere past b-incomplete is weakly observationally indistinguishable from some space-time that fails to be everywhere past b-incomplete.

One may worry that the construction I've provided in the preceding proof is somewhat artificial. One may also worry – as Manchak expressed to me in correspondence – about whether the $O - \hat{O}$ region of $(\mathbf{M}', g_{\mu\nu})$ satisfies a variety of local conditions. I will take two steps to at least somewhat alleviate that worry. First, I've shown that everywhere past sin-

[12]↑Lemma 1 from Manchak (2016, p. 1055) tells us: "Let (M, g_{ab}) be any space-time and let O be any [open neighborhood] in M. There is an [open neighborhood] $\hat{O}$ in O and a space-time (M, g'_{ab}) such that g'_{ab} is flat on $\hat{O}$ and $g'_{ab} = g_{ab}$ on $M - O$."

gular space-times are weakly observationally indistingiuishable from space-times that are not everywhere past singular. Since weak observational indistinguishability is a logically stronger condition than super weak observational indistinguishability, a trivial consequence is that everywhere past singular space-times are super weakly observationally indistingiuishable from space-times that are not everywhere past singular. However, we can easily construct a space-time that is super weakly observationally indistinguishable from a cosmologically relevant space-time but which is not as artificial as the space-time considered in the proof above. For example, recall the region U from the definition of super weak observational indistinguishability. Interpret U as the union of the past light cones of all the points inside the Milky Way galaxy throughout some vast but finite cosmological epoch. Now we can suppose that there exists a wormhole to a space-time without a past boundary located outside of U. In that case, even if there is a boundary to the past of U, there wouldn't be a boundary to the past of every space-time point.

Second, I turn to surveying a variety of results concerning how a region from a space-time with a finite initial segment can be "glued" together with another space-time region to construct a new space-time without a finite initial segment. One can isometrically embed a spatio-temporal region from an FLRW space-time in a space-time without a open boundary to the past of every space-time point. In Newtonian gravitation, the field of a point mass is given by the familiar inverse square relationship. Due to Gauss's Law, outside the surface of the Earth, the Earth's gravitational field is likewise given by the familiar inverse square relationship, as if the field were concentrated at the Earth's center of mass. The General Relativistic equivalent – the space-time of a point mass – is given by the space-time of a black hole. For example, if the point mass has no electric charge, magnetic field, or angular momentum, then the point mass's space-time is described by the Schwarzschild metric. Likewise, supposing that a mass-energy distribution is surrounded by vacuum, the space-time far from the mass-energy distribution is well approximated by the space-time of a black hole. This suggests one way to embed a spatio-temporal region from an FLRW space-time into a vacuum space-time would involve "gluing" the FLRW region's boundaries to the interior of a black hole's event horizon. That is, the interior of the event horizon would look like an FLRW space-time while the exterior would look like a Schwarzschild black hole. The black hole's space-time does not have a past boundary. Consequently, a space-time without a past boundary with an FLRW region might be constructed by gluing the FLRW region to the inside of a black hole's event horizon.

In the 1960s, Oskar Klein (1961), Igor Novikov (1963), and Yakov Zel'dovich (1963) showed how to isometrically embed a patch from a closed FLRW space-time within a black hole's event horizon. Werner Israel (1966) subsequently provided a general set of conditions – the Israel Junction Conditions – for when one can "glue" together space-time patches, each satisfying the Einstein Field Equations and with distinct metrics, so that the newly constructed space-time also satisfied the Einstein Field Equations. (Also see the discussion in Poisson, 2004, pp. 84–86.) Using the Israel Junction Conditions, one can isometrically embed a patch from either a flat or open FLRW space-time within a black hole event horizon (Geller et al., 2018). Klein, Novikov, and Zel'dovich's results led several Soviet physicists to speculate that the observable universe might be the interior of a black hole; in turn, black holes containing entire universes could appear to outside observers as subatomic particles called *friedmons* (Barashenkov, 1983; Markov, 1974; Markov and Frolov, 1971). More recently, friedmons have been proposed as a possible dark matter candidate (Dokuchaev and Eroshenko, 2014; Polishchuk, 2012, 2017). Other physicists have taken seriously the proposal that universes are born from astrophysical (and not subatomic) black holes (Oshita and Yokoyama, 2018; Popławski, 2010, 2016; Smolin, 1992, 2006). In inflationary cosmology, an embedding of a patch of FLRW space-time in a background Schwarzschild space-time has been considered as a model of a false vacuum bubble in flat space (Ansoldi and Guendelman, 2006; Blau et al., 1987; Farhi and Guth, 1987; Farhi et al., 1990; Haque and Underwood, 2017). Inflationary cosmologies also routinely involve gluing together multiple FLRW or approximately FLRW regions with distinct scale factors (e.g., Clough, 2018; Haque and Underwood, 2017). As is well known, one can glue together approximately FLRW regions within a background de Sitter space-time where the background de Sitter space-time is nowhere past singular.

Various results are known about space-times that include regions approximating FLRW space-times. FLRW space-times are spatially isotropic and spatially homogeneous. The space-time we inhabit is neither spatially homogeneous nor spatially isotropic. The matter-energy distribution varies locally; for example, the density of matter rises from points outside the Earth's surface to points inside the Earth's surface. Cosmologists utilize FLRW models because space-time approximates spatial isotropy and spatial homogeneity on the largest observable length-scales. However, cosmologists generally expect spatial homogeneity and spatial isotropy to fail on length-scales significantly larger than our observational horizon. For example, in inflationary cosmology, the inflationary epoch produced spatially isotropic and spatially homogeneous regions without producing global spatial isotropy or global spatial homogeneity.

A space-time is said to undergo *intermediate isotropisation* just in case the space-time features some epoch during which the space-time is well-approximated as spatially isotropic despite beginning or ending with epochs that are not well-approximated by spatial isotropy. One family of nearly FLRW space-times are the Bianchi space-times, which are spatially homogeneous but not spatially isotropic. George Ellis has offered an important result he calls the *Bianchi Evolution Theorem*. Consider a Bianchi space-time that undergoes intermediate isotropisation. We can define a state space to represent the solutions to the EFE. In that state space, define an ϵ-neighborhood of an FLRW space-time as a region in which all of the quantities characterizing space-times are closer than ϵ to their values in the FLRW space-time. As Ellis and van Elst describe the theorem, "Choose a time scale L. Then no matter how small ϵ and how large L, there is an open set of Bianchi models in the state space such that each model spends longer than L within the corresponding ϵ-neighbourhood of the FLRW model" (G. F. R. Ellis and van Elst, 1999, p. 51).

The Bianchi Evolution Theorem follows as a consequence of the fact that the FLRW space-times are saddle points in state space and that the saddle points are fixed points of the phase flow. In other words, the theorem follows because FLRW space-times are attractors in the state space for solutions of the EFE. While a variety of Bianchi space-times have been known to be singular since at least the 1970s (see the results summarized in Collins and Ellis, 1979), that FLRW space-times are an attractor in the space of solutions to the EFE suggests that there are non-singular nearly FLRW space-times with epochs that approximate a variety of singular FLRW space-times arbitrarily well. In fact, a variety of non-singular spatially anisotropic and inhomogeneous that approximate FLRW space-times have been known for several decades (Senovilla, 1996).

In sum, we have three reasons for thinking that cosmologically relevant and everywhere past singular space-times are weakly or super weakly observational indistinguishable from space-times that are not everywhere past singular. First, I've proven a general theorem according to which any non-causally bizarre spacetime which is everywhere past b-incomplete is weakly observationally indistinguishable from some spacetime that fails to be everywhere past b-incomplete. Second, various results have been known for several decades according to which one can isometrically embed a spatio-temporal region from an FLRW space-time into a larger spacetime that fails to be everywhere past b-incomplete. Third, there are results, such as the intermediate isotropisation theorems, about space-times that approximate FLRW space-times arbitrarily well over arbitrarily long periods of time; while such space-times are not observationally indistinguishable from FLRW space-times in the sense that I defined at the outset of this chapter, they are observationally indistinguishable from FLRW space-times in the weaker sense that observers embedded within space-times cannot determine whether they inhabit an FLRW space-time or some space-time that approximates FLRW space-time arbitrarily well. Since singular FLRW space-times are approximated arbitrarily well by non-singular FLRW space-times, observers cannot determine whether they inhabit a space-time that includes a past singularity.[13]

9.6 The Borde-Guth Vilenkin Theorem and Observational Indistinguishability

According to the Borde-Guth-Vilenkin (BVG) theorem, any congruence of time-like geodesics along which a certain generalization of the Hubble parameter, denoted H, has a positive average value is not geodesically complete to the past. That is, the BVG theorem shows that a space-time consisting only of a time-like geodesic congruence along which the generalization of the Hubble parameter is positive is everywhere past singular. Craig

[13]↑Of course, we can determine that our space-time only approximates FLRW space-time because we can measure local inhomogeneities in the matter-energy distribution. However, the point is that the solution space of the Einstein Field Equations includes space-times that closely approximate our own space-time but which are not past b-incomplete.

and Sinclair have argued that the BVG theorem, together with the evidence that the BVG theorem applies to the space-time we inhabit, provides compelling evidence for the conclusion that the Cosmos began to exist. As they interpret the BVG theorem, the BVG theorem shows that space-time could not have been expanding forever and must have begun in the finite past.

Nonetheless, our observational data at most allows us to deduce that space-time has been expanding in our past – or within our observational horizon – and not that the entirety of space-time has been expanding. And, as we've already seen, any space-time that is everywhere past singular is weakly and super weakly observationally indistinguishable from a space-time that is not everywhere past singular. Space-times to which the BVG theorem applies are no different. Consider a DB space-time S such that, as a consequence of the BVG theorem, S is everywhere past singular. We can again let U represent the space-time region formed by taking the union of all of the past light cones of any point in a time-like geodesic congruence bounded between two space-like regions R_1 and R_2. Once again, we can construct another very different space-time S' with a region isometric to U and we have enough freedom in constructing S' to ensure that there exists at least one time-like curve in S' that is not bounded to the past by Σ. We have enough freedom because we can isometrically embed U into a space-time that includes another time-like congruence along which the average of H is not positive. Consequently, there could be time-like geodesics in regions beyond our cosmological horizon that can be extended infinitely far into the past regardless of whatever data we might gather.

There is another reason related to observational indistinguishability for objecting to Craig and Sinclair's treatment of the BVG theorem. The BVG theorem tells us that a time-like geodesic congruence along which space-time, on average, expands cannot be continued indefinitely far into the past. Thus, at most, the BVG theorem tells us that any cosmological process during which space-time expands lasts for finite time. For example, as Delia Perlov and Alexander Vilenkin (2017, p. 331) have noted, while the theorem tells us the period of cosmic inflation must have only finite temporal extension, the theorem does not tell us anything about whether the Cosmos has a history prior to the period of cosmic inflation, let alone whether the Cosmos satisfies the Boundary Condition or has a beginning. For all that the theorem tells us, the beginning of the Cosmos is left mysterious.

Supposing that we did somehow know that space-time regions beyond our cosmological horizon are, on average, expanding, Guth has noted that the BGV theorem does not require the existence of a "unique beginning". For example, two time-like curves, along which space-time is, on average, expanding at all points on both curves, must either terminate in the past or else exit classical space-time.[14] However, they do not need to terminate at either a shared event or on a shared space-like surface. Furthermore, the BGV theorem provides no upper bound to the lengths of time-like or null geodesics (Guth, 2007, p. 6623). Relatedly, Andre Linde notes that if one picks some time-like geodesic on which inflation lasts for a proper time given by T, then one can always find another geodesic on which inflation lasts for a proper time t_i, such that $t_i > T$. That there is no upper bound to the past length of time-like or null curves in space-times to which the BGV theorem applies has led Andre Linde to argue that the universe can be past eternal. Linde (2008, p. 16) writes that, "If this upper bound [to the past length of geodesics] does not exist, then eternal inflation is eternal not only in the future but also in the past. [...] at present we do not have any reason to believe that there was a single beginning of the evolution of the whole universe at some moment $t = 0$, which was traditionally associated with the Big Bang".

We should be careful in evaluating the points made by Guth and Linde. The singular boundary associated with the BVG theorem is an open boundary. For that reason, the BVG theorem is relevant for whether the Cosmos satisfies the Boundary Condition only if the BVG theorem tells us that the Cosmos includes a finite initial segment. Recall that the Cosmos includes a finite initial segment just in case there exists a space-like surface Σ such that all past directed time-light and light-like half-curves starting at Σ have a finite generalized affine length. Supposing that there is a finite initial segment, there may nonetheless be no upper bound to the past lengths of time-like geodesics. That is, for any given time-like geodesic γ passing through Σ, there may be another geodesic γ' such that

[14]↑There is also the possibility that the expansion begins at some point on each of the time-like curves. In that case, space-time is not, on average, expanding everywhere along the curve.

the past of γ', at Σ, is longer than the past of γ, at Σ, so long as no geodesic passing through Σ has infinite past length. Nonetheless, what Guth and Linde have pointed out is that the BVG theorem does not, in itself, mandate that the Cosmos has a boundary located finitely far to the past.

Craig and Sinclair discuss the point made by Guth and Linde but wrongly interpret the point as an objection to the BGV theorem (see footnote 41 in Craig and Sinclair, 2009, p. 142). Guth and Linde are not arguing against the BGV theorem. After all, theorems are statements proved in mathematics and, if proved, cannot be false. However, theorems can fail to apply if their application conditions are not met and they can have different implications than those we at first expect them to have. Guth and Linde are pointing out an important collection of insights about what the BGV theorem entails, provided that we inhabit a space-time to which the theorem applies, namely, that the theorem does not entail that any space-time to which the theorem applies had a beginning located finitely far to the past.

In any case, there is a more substantive reason for doubting that the BVG theorem is relevant for thinking about whether space-time satisfies the Boundary Condition. Both Guth and Linde assume that the regions outside of the observable past are populated either by inflating regions or by regions that have left inflation. However, one of the lessons that we should learn from the literature on observationally indistinguishable space-times is that we can take any finite number of geodesic congruences along which inflation occurs and embed them into an observationally indistinguishable space-time that contains regions that were never inflating – that is, regions to which the BVG theorem simply does not apply. For that reason, even if we suppose that the BVG theorem tells us that any given inflating region must be bounded in the finite past, we cannot take this to entail that all parts of the Cosmos include a past boundary. A space-time that includes only an inflating region bounded to the past is weakly observationally indistinguishable from another space-time that includes regions that were never inflating; thus, space-times to which the BVG theorem has global application are weakly observationally indistinguishable from space-times to which the BVG theorem does not have global application. Hence, the BVG theorem is incapable of telling us whether the Cosmos satisfies the Boundary Condition.

9.7 Summary

As I have emphasized throughout, whether the Cosmos satisfies the Direction and Boundary Conditions is a bit of unobservable chronogeometric structure. As such, any wholly empirical argument for the conclusion that the Cosmos does satisfy the Direction or Boundary Conditions must show that an independently well-supported scientific theory, when conjoined with observational data, entails that the Cosmos satisfies the Direction and Boundary Conditions. A scientific theory, when conjoined with observational data, can entail that the Cosmos satisfies the Direction and Boundary Conditions only if the observational data allows us to distinguish whether space-time satisfies the Direction and Boundary Conditions.

General Relativity is our best theory of space-time. As I proved in this chapter, a broad class of relativistic space-times satisfying the Direction and Boundary Conditions are observationally indistinguishable from space-times that fail to satisfy either the Direction or Boundary Conditions. While General Relativity will likely be replaced in future physical inquiry, we do not yet know what the correct successor theory to General Relativity will turn out to be. Thus, this chapter leaves us with a disjunction. We can either look to General Relativity or to a future physical theory. Insofar as we look to General Relativity as our theory of space-time, the conjunction of theory and observational data do not entail that the Cosmos satisfies the Direction and Boundary Conditions. On the other hand, insofar as we look to a future physical theory as our theory of space-time, we do not know whether the conjunction of theory and observational data entails that the Cosmos satisfies the Direction and Boundary Conditions. More generally, no set of observations that we currently have, when conjoined with General Relativity, entails that the Cosmos satisfies the Modal, Direction, or Boundary Conditions.

We have one part of a case for Cosmic Skepticism. Due to the provinciality of our knowledge of the Cosmos due to the relative scale of the Cosmos and our spatio-temporal location within the Cosmos, General Relativity suggests that even if the Cosmos began to exist, the Cosmos is observationally indistinguishable from a beginningless Cosmos. Nonetheless, the case for Cosmic Skepticism is not yet complete. While we might not have a scientific theory, which, when conjoined with observational data, entails the conclusion that the Cosmos began to exist, there may be other theoretical virtues which narrow the range of possible space-times to those that do satisfy the Direction and Boundary Conditions. For example, if the epistemic probability that the Cosmos satisfies the Modal, Direction, and Boundary Conditions is higher than the epistemic probability that the Cosmos fails to satisfy at least one condition, we would have good reason to believe that the Cosmos began to exist. Thus, a full defense of Cosmic Skepticism will have to await a discussion of confirmation theory in chapter 12.

10. THE MENTACULUS, WILLIAM CLIFFORD, AND EPISTEMIC HORIZONS

10.1 Introduction

This chapter continues my defense of Cosmic Skepticism by turning to a number of epistemic constraints that may be imposed on our knowledge of cosmological history given contemporary physical cosmology. David Albert and Barry Loewer have developed a reductive account of the direction of time called the Mentaculus. One of the principles in the Mentaculus – the Past Hypothesis – provides a low entropy boundary condition for the universe. In defense of the Past Hypothesis, Albert offers a transcendental condition for the possibility of our knowledge of the past, including our knowledge of the history of the Cosmos. As I argue in this chapter, either (a) the direction of time is reducible, in which case the Cosmos does not satisfy the Modal Condition and so lacks a beginning, or else (b) the direction of time is not reducible. I argue that if the direction of time is not reducible, then the Past Hypothesis still provides a transcendental condition on the possibility of our past knowledge. Nonetheless, if the direction of time is not reducible, the possibility opens that there are physical states to which Albert's transcendental condition is inapplicable. If there are any such states, they are not empirically accessible to us. The upshot will be that either the Cosmos lacks a beginning, because the Cosmos does not satisfy the Modal Condition, or else we cannot know whether the Cosmos satisfies the Boundary Condition because there may be past states that are inaccessible to us.

After discussing Albert's transcendental argument, I discuss a related argument developed by William Clifford. According to various cosmological models, cosmological history can be traced back to some sui generis state of affairs and no further. The sui generis state of affairs may have been prepared by exogenous factors epistemically lost to us, so that we cannot fully determine the Cosmos's history. Here, the upshot is that the inability to trace the Cosmos beyond a specific period in the Cosmos's history does not entail that the Cosmos had no history prior to that period. The provinciality of our knowledge of the physical facts with respect to spatio-temporal location may prevent us from knowing whether the Cosmos satisfies the Boundary Condition. I briefly discuss how concerns like Clifford's apply to inflationary cosmology and to the Emergent Universe scenario and then use this chapter's arguments to reply to an objection to Cosmic Skepticism.

10.2 Cosmological Horizons

This chapter adds to our understanding of the epistemic limitations of cosmologically relevant horizons. *Horizons* – roughly, space-time regions through which information cannot be successfully transmitted – are a ubiquitous feature of relativistic space-times. For example, if a star reaches a sufficient mass-energy density, the escape velocity reaches or exceeds the speed of light. The resulting object is called a black hole. Given the prohibition on signals traveling faster than light, no signal can be transmitted from within a specific radius R_0 to observers located at radii greater than R_0. R_0 is then called the black hole's event horizon. Some cosmological models – e.g., inflationary cosmologies – involve a period of superluminal expansion, so that degrees of freedom important for describing space-time's global structure are located outside of the past light cones of subsequent observers. Horizons can also develop due to processes that systematically "scramble" or "erase" information. For example, in the early universe, electromagnetic radiation was absorbed almost immediately after having been released. The early universe underwent a transition – electromagnetic decoupling – after which electromagnetic radiation could, in principle, travel indefinitely far. (In technical verbiage, we say that, before electromagnetic decoupling, photons had a finite mean free path, whereas, after electromagnetic decoupling, photons had an infinite mean free path.) We cannot receive electromagnetic signals from before electromagnetic decoupling. There is a similar story concerning neutrino decoupling or gravitational wave decoupling, so that there are periods in the universe's early history from which we cannot receive neutrino or gravitational wave signals. Additionally, there is a so-called *physics horizon* in the early universe. Before the physics horizon – assuming the notion of 'before' continues to be a sensible notion – we have no agreed upon or well-confirmed physical theories.

All of the aforementioned horizons have been described in detail elsewhere and I have nothing novel to add to their description here. This chapter relates to traditional discussions concerning cosmologically relevant horizons in several ways. I argue below that one popular research program in the foundations of statistical mechanics – the Mentaculus project – introduces a cosmologically relevant horizon that threatens our ability to know whether a boundary is included in our past and so threatens our ability to know whether the Cosmos satisfies the Boundary Condition. Moreover, I discuss an argument due to Clifford that takes on new relevance in the context of cosmologically relevant horizons. Due to cosmologically relevant horizons, unless physical theory changes substantially, we will never be able to gather sufficient data to determine whether the Cosmos satisfies the Boundary Condition. And since, given present physical theory, we will never be able to gather sufficient data to determine whether the Cosmos satisfies the Boundary Condition, we will never be able to gather sufficient data to determine whether the Cosmos began to exist. Lastly, I will discuss how cosmologically relevant horizons provide me with a response to an important objection to the view that, given our current understanding of contemporary physical cosmology, we cannot know whether the Cosmos satisfies the Boundary Condition.

10.3 The Mentaculus

The first of my two philosophical arguments concerning the possibility of past knowledge was originally developed within the context of the Mentaculus project. The Mentaculus project is a research program devoted to providing a reductive explanation of the direction of time. In this section, my aim is to provide a brief discussion introducing the Mentaculus project, so that I can make reference to that project in this chapter and in the next. While I describe some arguments offered by proponents of the Mentaculus project, my aim is to familiarize readers with the project and not to convince readers who may be skeptical that the direction of time is reducible. According to proponents of the Mentaculus project, if there were a direction of time fundamental to the Cosmos then that direction should appear in fundamental physical theory; for friends of the Mentaculus, the purported fact that there is no time asymmetry in fundamental physics provides reason to think that the time asymmetry in our ordinary lives is, in some way, reducible to time symmetric phenomena.

I turn to summarizing two important arguments for the conclusion that the direction of time is reducible. First, our best theories of fundamental physics do not distinguish the way in which past states depend on future states from the way in which future states depend on past states. As a simple example, consider a ball shot out of a cannon at time $t = 0$ and that hits the ground at time $t = 1$. Given the initial state with which the ball is shot out of the cannon, we can calculate the ball's entire trajectory up to the moment when the ball impacts the ground. Assuming that there is no air resistance, we can instead use the ball's final state to calculate the ball's prior trajectory down to the time when the ball exited the cannon. The claim is not merely that the ball's final state *mathematically* depends on the ball's initial state in the same way that the ball's initial state mathematically depends on the ball's final state. Instead, the claim is that, insofar as physical theory encodes information about nomological dependence at all, the ball's initial state nomologically depends on the ball's final state in the same way that the ball's final state nomologically depends on the ball's initial state.

So, either our physical theories inadequately describe fundamental nomological dependence or else fundamental nomological dependence is time symmetric. There are two reasons for thinking that the latter – that fundamental nomological dependence is time symmetric – is more plausible. First, all else being equal, we should prefer more modest theories. The greater the number of metaphysical principles we add to our physical theories, the more immodest our theories become. Since our best fundamental physical theories do not postulate a time asymmetric dependencce relation, all else being equal, we should prefer a metaphysical interpretation of our best fundamental physical theories that avoids adding a time asymmetric dependence relation. Second, one might have independent reasons for accepting a naturalistic approach to metaphysical theorizing on which we should avoid, when possible, conjoining extra-empirical metaphysical postulates to our scientific theories. Thus, insofar as we have independent reason to endorse that kind of naturalism, we should avoid conjoining our best theories of fundamental physics with a time asymmetric nomological dependence relation.

I turn to a second argument for the view that the direction of time is reducible. The second argument makes use of the fact that the fundamental laws of physics are time reversal invariant. Consider a ball traveling at a fixed speed on a frictionless surface in a vacuum. Suppose that the ball rebounds off of a wall. Supposing that the wall is mounted atop frictionless rollers, in order to conserve momentum, when the ball collides with the wall, the wall will begin moving. We can break this process down into three stages: (i) the ball is moving while the wall is at rest, (ii) the ball hits the wall, and (iii) the ball and the wall are traveling in opposite directions. We can now consider the time reverse process, wherein (i*) the ball and the wall are traveling in opposite directions, (ii*) the ball and the wall collide, (iii*) the wall is at rest while the ball continues moving in the opposite direction. Newtonian mechanics is said to be time reversal invariant because, for all forward (reverse) sequences in Newtonian mechanics, the reverse (forward) sequence is nomologically permissible.

Newtonian mechanics has since been succeeded by other physical theories. There is a subtle issue about what, exactly, time reversal invariance comes to in electromagnetism, non-relativistic quantum mechanics, or in quantum field theory. Suppose that we are provided a list of sentences describing temporally sequential physics states, i.e., $S \equiv \{S_1, S_2, ..., S_N\}$. In our best fundamental physical theories, we can define an operation which, when fed S, will output $S^* \equiv \{S^*_N, S^*_{N-1}, ..., S^*_1\}$, where S^* is said to be the time reversal of S. And then to say that the fundamental dynamical laws are time reversal invariant is to say that S is nomologically permissible just in case S^* is nomologically permissible and vice versa. For example, in the standard model of particle physics, S^* is obtained by replacing every charge with the opposite charge, every system with its mirror image, and every instance of t with $-t$, that is, the standard model respects CPT symmetry and not T symmetry (Christenson et al., 1964; Kobayashi and Maskawa, 1973). Whether the fundamental dynamical laws involved in classical electromagnetism are time reversal invariant has been controversial because, to obtain the time reverse sequence, one must replace the magnetic field $\vec{B}$ with $-\vec{B}$ (Albert, 2000, p. 21; Earman, 2002b).

Moreover, whether quantum mechanics is truly time reversal invariant is difficult to address, both because there is a first order time derivative in the Schrödinger Equation and because there are specific interpretations of quantum mechanics (such as objective collapse theories) that are explicitly not time reversal invariant. I do not take up a position here as to whether the fundamental laws should actually be said to be time reversal invariant. Instead, I note merely that friends of the Mentaculus have been motivated by the notion that even if there are time asymmetries in the fundamental dynamical laws, those time asymmetries likely cannot explain the macrophysical time asymmetry in our ordinary experience; the explanation for the time asymmetry in our ordinary experience must be sought elsewhere.

Although the fundamental laws might be time reversal invariant, macrophysical phenomena are obviously not time reversal invariant. Eggs fry, but never unfry. Radiation is emitted from, but does not fall onto, stars. Temperature differences spontaneously decrease and do not spontaneously increase. In order to explain how fundamentally time symmetric dynamical laws can ultimately explain the world of our ordinary experience, I turn to introducing phase space. Phase space is the space of all of the microphysical configurations that a given physical system could have. Each point of phase space represents a specific microphysical configuration. Sets of macrophysical measurements carve up phase space into disjoint regions corresponding to distinct macrophysical states.

Suppose that we would like for a crowd of people to move a boulder and that no one person in the crowd is powerful enough to move the boulder themselves. If we command the crowd to charge the boulder, but do not command the members of the crowd to coordinate their efforts, then the boulder will, at best, "quiver" when, by chance, a larger number of people charge the boulder from one side than from other sides. The most effective way for the crowd to move the boulder would be for the members of the crowd to coordinate their efforts so that they, e.g., charge the boulder at a common angle. Assuming that we can approximate each person in the crowd as a point particle moving in a two-dimensional plane, each person has a position, e.g., for the ith person, $\vec{q_i} = (x_i, y_i)$, and each person has a momentum, e.g., $\vec{p_i} = (p_{i,x}, p_{i,y})$. Suppose that there are N people in the crowd. Then we can specify the "microphysical state" of the crowd by combining all of their positions and momenta, i.e., $ms = (x_1, y_1, x_2, y_2, ..., x_N, y_N, p_{1,x}, p_{1,y}, ..., p_{N,x}, p_{N,y})$. The crowd's phase space is a 4^N dimensional space, where *ms* labels a specific point. Since there are a larger number of uncoordinated configurations of the crowd than there are coordinated configurations of the crowd, the largest regions in the crowd's phase space correspond to states that will not move the boulder. Suppose that we begin with a coordinated crowd but that the crowd is subject to randomized "reshufflings". While the crowd could be reshuffled into another coordinated configuration, since the number of uncoordinated configurations vastly outnumber the number of coordinated configurations, the crowd will most likely be reshuffled into increasingly less coordinated configurations.

The phase space of a gas can be constructed analogously. Supposing that the gas is composed of point particles satisfying Newton's laws, the *i*th particle has position $\vec{q_i} = (x_i, y_i, z_i)$ and momentum $\vec{p_i} = (p_{i,x}, p_{i,y}, p_{i,z})$. The gas's phase space is a 6^N dimensional space where each point corresponds to a specific microphysical state, e.g., $(x_1, y_1, z_1, x_2, y_2, z_2, ..., x_N, y_N, p_{1,x}, p_{1,y}, p_{1,z}, ..., p_{N,x}, p_{N,y}, p_{N,z})$. Just as there are far fewer configurations of a crowd that cannot move a boulder than those that can move a boulder, there are far fewer configurations of a gas that can do work in pushing a piston than there are configurations of the gas that cannot move the piston.

Assuming a uniform measure over phase space, the entropy associated with a given macrophysical state measures the size of the corresponding phase space region. A coordinated system – which is to say a system in a low entropy macrophysical state – can perform work. In the phase space of a gas in a cylinder, there are a vastly greater number of trajectories from low entropy regions to high entropy regions than from low entropy regions to regions with even lower entropy. Thus, given that a system begins in a state of low entropy, the most probable evolution of the system is to states with greater entropy. As the entropy of the system increases, the amount of energy available to do work decreases, and the system approaches an equilibrium state where no macrophysical work can be accomplished.

Suppose that we begin with a coordinated gas but that the particles in the gas are subject to randomized "reshufflings". While the gas could be reshuffled into another coordinated configuration, since the number of uncoordinated configurations vastly outnumber the number of coordinated configurations, the gas will most likely be reshuffled into increasingly less coordinated configurations. The chance collisions between gas particles act to effectively reshuffle the gas's microphysical state. Since the particles are subject to time symmetric fundamental dynamical laws, we've managed to recover time asymmetric macrophysics from time symmetric microphysics.

Supposing that the observable universe once occupied an improbable and "coordinated" state – that is, a low entropy state – energy would have been available for doing macrophysical work. The subsequent history of the observable universe would be one according to which the observable universe followed some trajectory through phase space from less probable states, i.e., lower entropy macrostates, and to more probable states, i.e., higher entropy macrostates. During the nineteenth century, this observation led Ludwig Boltzmann to propose that the direction of time is reducible to the entropy gradient. On Boltzmann's view, the temporal direction we call "the past" is reduced to the direction in which the entropy of our cosmic environment is lower and the temporal direction we recognize as "the future" is the direction in which the entropy of our cosmic environment is greater. The Mentaculus project – as developed by David Albert (2000, 2015) and Barry Loewer (2007, 2013, 2020) – is a sophisticated descendent of Boltzmann's project. The Mentaculus is the conjunction of three postulates:

1. *The Fundamental Dynamical Laws*, whatever they ultimately turn out to be.

2. *The Statistical Postulate*. This is the principle that "the right probability-distribution to use for making inferences about the past and the future is the one that's uniform, on the standard measure, over those regions of phase space which are compatible with whatever other information – either in the form of *laws* or in the form of *contingent empirical* facts – we happen to have" (Albert, 2000, p. 96).

3. *The Past Hypothesis*. This is the cosmological hypothesis that "[...] the world first came into being in whatever particular low-entropy highly condensed big-bang sort of macrocondition it is that the normal inferential procedures of cosmology will eventually present to us" (Albert, 2000, p. 96).

For my purposes, we need to revise the statement of the Past Hypothesis. As Albert states the Past Hypothesis, the Mentaculus *entails* that the Cosmos began to exist. However, I doubt that Albert would endorse the view that the Mentaculus entails the Cosmos satisfies the Modal, Direction, or Boundary Conditions and so I doubt that Albert means for the Past Hypothesis to entail that the Cosmos began to exist in the sense I have defended in this dissertation. Moreover, if the direction of time is reducible, then what *makes* the "low-entropy highly condensed big-bang sort of macrocondition" a part of our past is just that the macrocondition lies at one end of the entropy gradient we find in our cosmological environment. Instead of understanding the Past Hypothesis as stipulating a low entropy state for the entire Cosmos, I will understand the Past Hypothesis as providing a low entropy boundary condition for a sufficiently large space-time region, of which the observable universe is a part. Stated that way, the Mentaculus does not, itself, entail whether (i) the Cosmos began to exist or (ii) the low entropy state postulated by the Past Hypothesis is to the past of all events in the Cosmos. For that reason, even if the low entropy state is a boundary, the low entropy state is not necessarily the sort of space-time-wide boundary required for satisfying the Boundary Condition.

According to friends of the Mentaculus, the direction of time is not reducible to the entropy gradient. Instead, their view is that the direction of time shares a reductive explanation with the entropy gradient. They argue that the Mentaculus provides a "probability map of the world" (e.g., Loewer, 2020), i.e., every formalizable proposition concerning the state of the physical world – and so nearly every statement that could ever be made in the sciences – is assigned an objective chance through conditionalization on the Mentaculus. The conditionalization of statements on the Mentaculus explains why, e.g., ice, left to its own devices, is vastly more probable to melt in one temporal direction – the direction we label as "the future" – than the other – the direction that we label as "the past".[1]

[1]↑For the sake of completeness, I include a brief discussion of the Mentaculus's relationship to a Neo-Humean approach to laws, chance, and causation originally developed by David Lewis. As I will explain, the Mentaculus project is compatible with Neo-Humeanism, but does not entail Neo-Humeanism. On Lewis's view, our world ultimately consists of a space-time manifold and a distribution of properties sprinkled across that manifold. The distribution of properties is called the *Humean Moasic*. All else – and, in particular, laws, chance, and causation – supervene on the Humean Mosaic. For example, laws are understood as theorems of whatever description of patterns in the Humean Mosaic provides the best trade-off between simplicity and generality. Counterfactuals are understood in terms of how the Humean Mosaic

10.3.1 A Transcendental Condition on Past Knowledge

Having completed a discussion of the Mentaculus, I turn to discussing a transcendental argument concerning the possibility of past knowledge, including the possibility of our having knowledge of cosmological history, that has been offered in defense of the Past Hypothesis. To examine that argument, I need to introduce the distinction between *retrodictions* and *records*. First, I turn to examining the notion of retrodiction. Retrodictions are the time reverse of predictions. For example, given the current state of a cup of water containing a single ice cube, we can predict subsequent states of the ice cube or retrodict prior states of the ice cube. In an experimental arrangement, only a specific set of macrophysical parameters characterizing the ice cube, e.g., ambient pressure, volume, temperature, etc, are epistemically available to us. Since we do not know the ice cube's specific microphysical state, we can at best say which macrophysical state the ice cube is most likely to evolve into. Likewise, in performing a retrodiction, we can at best say which macrophysical state the ice cube is most likely to have evolved from. In order

differs between possible worlds and causation is then analyzed in terms of counterfactuals. Lewis's view is *Neo-Humean* because there is an intuitive sense in which the view captures Hume's intuition that there are no necessary connexions between distinct existences. Albert and Loewer endorse their own versions of Lewis's original program; for example, on their view, the three statements in the Mentaculus are laws because they are theorems of the system providing the best trade-off between simplicity and generality. Moreover, there is a sense in which the Mentaculus and the Lewisian program are kindred spirits; the statistical reduction of thermodynamics and the reduction of nearly everything to patterns in the Humean Mosaic seem natural companions, even if neither entails the other.

Nonetheless, the Mentaculus is consistent with the denial of Lewis's project. There is no inconsistency between the view that there are necessary connexions between distinct existences and the view that time is fundamentally directionless. For example, consider that, for Lewis, the notion that there are no necessary connexions between distinct existences entails a modal recombination principle, according to which a possible world can be constructed from two others by "stitching" together a patch from either in a novel way. As I discussed in chapter 9, whether two space-time regions, both of which satisfy the Einstein Field Equations, can be patched together in such a way that their combination satisfies the Einstein Field Equations depends on whether their combination satisfies the Israel Junction Conditions. According to an oft-told story, when scientists discovered the chemical composition of water, they discovered an a posteriori necessity, that is, they discovered the essence of water. A similar story might one day be told about space-time, where some set of a posteriori claims reveals to us the essence of space-time. That story is likely to be told in terms of some future successor to General Relativity, but suppose – for the sake of the thought experiment – that we accept, as a necessary consequence of the essence of space-time, itself, that space-time is consistent with the Einstein Field Equations. In that case, as a matter of metaphysical necessity, two patches of space-time can be stitched together only if their combination satisfies the Israel Junction Conditions. Since the Israel Junction Conditions are time symmetric, one can endorse the view that there are necessary connexions between distinct existences, thereby denying Lewis's modal recombination principle, while accepting that physics is fundamentally time symmetric. We would then need a way to explain how the world of our ordinary experience could be time asymmetric; the Mentaculus could be invoked to do so.

to calculate the probabilities involved, both predictions and retrodictions make use of a suitably coarse grained measure over trajectories through phase space.

Consider a cup of water containing an ice cube. Let's call the combined cup, water, and ice cube system C. Suppose that C occupies a low entropy state at time t_0. At time t_1, where $t_1 > t_0$, we measure C's macrophysical state. At t_1, C has not yet reached thermodynamic equilibrium; for example, perhaps the ice cube is only half-melted. At t_1, C's entropy is higher than C's entropy at t_0. Suppose, further, that we are interested in making a prediction to C's macrophysical state at time t_2, such that $t_2 > t_1$. Since C has not yet reached thermodynamic equilibrium, we can predict that, at t_2, C will have a higher entropy macrophysical state. For example, perhaps we calculate that, with exceedingly high probability, the ice cube will be completely melted at t_2.

Given C's macrophysical state at t_1 and without any exogenous interactions to disturb the state of C from the outside, we can reliably predict C's future evolution, without conditionalizing on the Past Hypothesis, with such exceedingly high reliability that a counterexample to our prediction will not be observed on time scales longer than the history of the observable universe. However, we cannot reliably retrodict C's initial state at t_0 using C's macrophysical state at t_1 without conditionalizing on the Past Hypothesis. Suppose that we try to retrodict C's macrophysical state at t_0 from C's macrophysical state at t_1 without conditionalization on the Past Hypothesis and without any measurements from t_0. Supposing that we conditionalize on the Statistical Postulate and the fundamental dynamical laws but not on the Past Hypothesis, the suitably coarse grained measure of microphysical trajectories exiting some phase-space region R and entering some higher entropy region R^* is equal to the measure of microphysical trajectories entering R from R^*. Contrary to C's actual evolution from t_0 to t_1, a half melted ice cube is more likely to have been generated spontaneously from a cup of water, sans ice cube, than from a less melted ice cube.

Records provide us with better access to the past than do retrodictions because records utilize a fundamentally different mechanism for accessing the past than do retrodictions. A retrodiction is a relation between two times (a past time and the present) expressed in virtue of the fundamental dynamical laws of temporal evolution. In contrast, a record of an event requires a three-part relation between a moment before the record was created, the event, itself, and the present. To guarantee that some record we possess did not merely fluctuate into existence by chance from a higher entropy state, we must ensure that the corresponding recording device, in the moment before the creation of the record, occupied the appropriate state. Drawing on vocabulary from the quantum foundations literature, Albert (2000, p. 118) refers to that state as the device's *ready state*. For example, for an analogue camera to take a photograph, unexposed film must be properly loaded into the camera, the camera's shutter must be properly cocked, and so on. To ensure that the device was prepared in the ready state requires that we have a record of the device having been prepared in the ready state. We're off to the races on a regress of ready states, each one further to the past, until we reach the ultimate ready state, viz, a low entropy constraint for the entire observable universe. We cannot have a record of the ultimate ready state. If we did have a record of the ultimate ready state, then the ultimate ready state would not successfully halt the regress. Thus, just as the cause that halts the regress that appears in some cosmological arguments for theism must be uncaused, the ultimate ready state must be unrecorded. Consequently, unlike all of the other ready states appearing in the regress, the ultimate ready state cannot itself be available to ordinary empirical procedures.

Without conditionalizing on the Past Hypothesis, we would retrodict all sorts of anti-thermodynamic phenomena, e.g., that the ice cubes we observe were mostly likely spontaneously formed out of liquid water via the chance collisions of atoms. Likewise, without conditionalization on the Past Hypothesis, the records we believe ourselves to possess are vastly more likely to have fluctuated into existence by the chance collisions of atoms than to have been generated by a recording process. Friends of the Mentaculus defend the Past Hypothesis not on the grounds that there are empirical observations supporting the Past Hypothesis but instead because, without the Past Hypothesis, our knowledge of the past would not be possible. Without reliable records of the past, my memory of the beginning of this sentence is no longer reliable by the time I finish writing this sentence. The kind of wholesale global skepticism concerning past knowledge that results without the Past Hypothesis would undermine all of our scientific knowledge. Thus, insofar as scientific knowledge is possible at all, we must presuppose the Past Hypothesis. There can be no empirical cosmology without the Past Hypothesis. In other words, Albert has offered us a transcendental argument for the Past Hypothesis.[2]

What about events that are *prior* to the ultimate ready state? Recall that friends of the Mentaculus are pursuing a reductive explanation of the macrophysical direction of time. According to friends of the Mentaculus, there is a sense in which there can be no state prior to the ultimate ready state because friends of the Mentaculus endorse a view on which the direction of time shares a reductive explanation with the entropy gradient. Wherever the global entropy gradient begins must be considered the start of a macrophysical temporal series.[3] However, there are two reasons for thinking that the reducibility of the direction of time is incompatible with the Modal Condition. For the first reason, note that, if the direction of time is reducible, then the Cosmos has no fundamental temporal direction. Without a fundamental temporal direction, the Cosmos is fundamentally timeless. Perhaps one could object on the grounds that a macrophysical direction of time suffices for the existence of time. In that case, I turn to a second reason for thinking that the reducibility of the direction of time is incompatible with the Modal Condition. On a view according to which the direction of time is reducible, *B*-relations are reducible. Suppose that all numerically distinct time-like related macrophysical events could be placed into before or after relations with respect to one another. Given how the macrophysical *B*-relations are explained in terms of the mass-energy distribution throughout the Cosmos, plausibly, there is a nearby possible world *w* where macrophysical *B*-relations do not obtain between all time-like related macrophysical events. Without the macrophysical *B*-relations between all time-like related macrophysical events, *w* lacks a

[2]↑Albert (2000, p. 94) tells us that we know the Past Hypothesis in a different way from how we know "of straightforward everyday particular *empirical* facts". Subsequently, Albert (2000, p. 96) tells us that the Mentaculus consists of "three laws and one contingent empirical fact", where the "contingent empirical fact" is the present state of the world that we need to input to the Statistical Postulate. The low entropy condition of the early universe – and so the Past Hypothesis – is (apparently) *not* a matter of contingent empirical fact. Albert states that we know the Past Hypothesis more like how we know laws, that is, as a postulate that is enormously helpful in formulating predictions. But the situation seems to be a bit different from how we ordinarily infer laws as well. Prototypically, we infer laws by inductive generalization. Instead of an inductive generalization, Albert establishes the Past Hypothesis by way of a transcendental argument. In the case of the Past Hypothesis, once we know the fundamental laws and the statistical postulate, we *must* assume the Past Hypothesis in order to secure the possibility of various forms of knowledge.

[3]↑I say the "start" of a macrophysical temporal series – as opposed to the "beginning" of a macrophysical temporal series – in order to avoid confusion with a beginning in the sense that I defended in part II.

macrophysical direction of time. Thus, even if we supposed that a macrophysical direction of time sufficed for the existence of time, there would be a nearby possible world where the Cosmos exists but time does not. In that case, by definition, the Cosmos would not fulfill the Modal Condition.

Nonetheless, suppose that the direction of time is not reducible. For example, perhaps the direction of time is not a physical phenomenon and needs to be added, as a metaphysical postulate, to any complete description of our world. In that case, the fact would remain that time asymmetric macrophysical phenomena need to be understood in terms of time symmetric microphysics. And since most or all of the records we have of the past are physical objects, Albert's transcendental argument would still apply to whatever records we possess of the past. Since the direction of time would not be reducible, one could sensibly ask about states absolutely prior to the ultimate ready state. The consequence of Albert's transcendental argument is that all of the physical states of affairs of which we can have records must be located somewhere between our present state of affairs and the ultimate ready state. If the direction of time is irreducible, then there may be states of affairs temporally prior to the ultimate ready state. Nonetheless, there can be no records of any such state of affairs. Since there can be no records of any such state of affairs and, plausibly, reliable empirical access to the past is provided only by records, we cannot have reliable empirical access to whatever states there might have been prior to the ultimate ready state. Consequently, the ultimate ready state provides an epistemic horizon for our knowledge of cosmological history. Given an epistemic horizon for our knowledge of cosmological history, we would not be able to determine whether there is a boundary to our past and thus could not determine whether the Cosmos satisfies the Boundary Condition.

10.4 Clifford's Argument

In the nineteenth century, William Clifford offered a distinct, though related, set of arguments for the conclusion that there is a horizon for our knowledge of cosmological history. Physicists knew, by the latter nineteenth century, that thermodynamic processes are time asymmetric. The fact that the observable universe has yet to reach equilibrium was suggested as a reason for thinking that the Cosmos had a beginning (Kragh, 2008). William Clifford responded in an article published in *The Fortnightly Review* in 1875. By way of analogy, Clifford considers an iron poker that has been heated red hot by a fire:

> Suppose you put the end of a poker in the fire and make it red hot, that end is much hot than the other end, and if you take it out and let it cool, you will find that heat is traveling from the hot end to the cool end, and the amount of this travelling, and the temperature at either end of the poker can be calculated with great accuracy. That comes out of Fourier's theory. Now, suppose you try to go backwards, in time, and take the poker at any instant when it is about half cool, and say this equation,– does it give me the means of finding out what was happening to it before this time, in so far as that state had been produced by cooling? You will find that the equation will give you an account of the state of the poker before the time when it came into your hands, with great accuracy up to a certain point, but beyond that point it refuses to give you any more information, and it begins to talk nonsense. It is in the nature of the problem of the conduction of heat that, that it allows you to trace the forward history of it to any extent that you like; but it will not allow you to trace the history of it backward, beyond a certain point [...] you will find that the point where the equation begins to talk nonsense is the point where you took it out of the fire (Clifford, 1875, pp. 478–479).

Clifford considers how analogous results obtain for the mixing of two fluids; the mixture can be traced back to the unmixed state, but not beyond the unmixed state. In the case of the preparation of the hot poker or the preparation of two unmixed fluids, the trouble is that the equations of heat conduction or of diffusion do not provide information about the process in virtue of which the poker or mixture were prepared. But a similar problem arises if the poker became hot or the fluids unmixed through a statistical fluctuation in the motions of the atoms comprising the poker or the fluids. Applied to a portion of the Cosmos, one could at most trace that portion back to a specific state but not beyond. As if to presage the big bang cosmology that would develop in the next century, Clifford (1875, p. 482) argues that if we trace the history of the world backwards, we come to a "catastrophe" beyond "which we cannot make any further calculation".

We may have reason to endorse the view that our location in cosmic history hides from us a representative sample of all of the physical facts that there are and in such a way so as to prevent us from knowing physical facts pertinent for addressing whether the Cosmos began to exist. Information about the matter-energy contents important for reconstructing a full cosmological history may have been hopelessly and irretrievably lost. Importantly, the fact that we can identify a state of affairs beyond which we are unable to determine the Cosmos's history does not entail that the Cosmos satisfies the Boundary Condition.

Scenarios resembling the one envisioned by Clifford appear throughout contemporary physical cosmology. According to inflationary cosmological models, there was a primordial period during which the space-time underwent an exponential expansion driven by the so-called "inflaton" field. (Variations of cosmic inflation assume different behavior for or properties of the inflaton field, but the variations do not matter for my purposes.) According to inflation's proponents, the exponential expansion is supposed to account for the observed universe's flatness and homogeneity as well as the spectrum of fluctuations in the cosmic microwave background. In chapter 9, I discussed how Delia Perlov and Alexander Vilenkin (correctly) interpret the BVG theorem, that is, that any period of cosmic expansion – including inflation – has no more than finite temporal extension. As Perlov and Vilenkin argue, the BVG theorem leaves the origin of the Cosmos mysterious.

Likewise, in a previous chapter, I argued that Craig and Sinclair misinterpret the BVG theorem to suggest that if inflationary cosmology is correct, then the Cosmos must have had a boundary in the finite past. As I've argued, the BVG theorem does not imply a boundary in the finite past both because the inflationary period could have been preceded by space-time regions that fail to satisfy the Direction or Boundary Conditions and because an inflating region can be isometrically embedded in a space-time that includes regions that were never inflating. Moreover, an inflating space-time could be joined on to or exist within a non-spatio-temporal structure and, therefore, violate the Modal Condition. Thus, inflationary cosmology provides no indication that the Cosmos began to exist.

A closely related issue arises for the *Emergent Universe scenario*, as pioneered by George Ellis and co-authors (G. F. R. Ellis et al., 2004; G. Ellis and Maartens, 2004). The Einstein Static State (ESS) is a space-time region that has a finite size, but no boundary. Observers with sufficiently long lives who set off at some finite velocity will eventually find themselves coming back to their starting point, despite never having changed their direction of motion. This is analogous to an ant who, walking in a fixed direction on the surface of a ball, eventually makes their way back to their starting point. The observer – like the ant – never encounters a boundary, even though the space within which they live is finite in size.

The Emergent Universe scenario includes an inflationary phase, but the inflationary phase is preceded by an ESS. The physicists who developed the Emergent Universe scenario had initially hoped that the ESS could have persisted since eternity past. Their hopes seem to have been dashed in virtue of the fact that the ESS is unstable to quantum fluctuations. As David Mulryne (2005), and co-authors, note, "the instability of the [Einstein Static State] makes it extremely difficult to maintain such a state for an infinitely long time in the presence of such fluctuations, such as quantum fluctuations, that will inevitably arise".

In quantum mechanics, systems cannot typically be static in the way that they can be in the classical context. The Einstein Static State is balanced precariously. Quantum fluctuations would result in the universe losing its "grip" and transitioning away from the Einstein Static State. Though the universe could fluctuate back into the Einstein Static State, transitions to states even further away are exponentially more likely. Quantum systems which can be fixed in a single state without fluctuating away from that state do so because they *cannot* transition from that state, i.e., eigenstates of the Hamiltonian. But, if the universe were in a state *like that*, then the universe would never have exited into the inflationary epoch. As Anthony Aguirre and John Kehayias (2013) describe, "it is very difficult to devise a system – especially a quantum one – that does nothing 'forever,' then evolves. A truly stationary or periodic quantum state, which would last forever, would never evolve, whereas one with any instability will not endure for an indefinite time". Craig and Sinclair (Craig and Sinclair, 2009, p. 150) take the failure of the Emergent

Universe scenario to offer an infinite past history as a win for their view that the Cosmos began to exist at a finite time in the past. They are careful enough to admit, "Metastable states leave unexplained how they came to exist." But then they immediately follow with, "Universes with a metastable initial state must therefore have a beginning". Two points can be made here. First, to suppose that the metastable state was the *initial* state is question begging. Second, if we instead consider the statement that universes with a metastable state must therefore have a beginning, then we can see that this statement does not follow from the previous statement that metastable states leave unexplained how they came to exist. If some hypothesis leaves the origins of a particular cosmological epoch unexplained or postulates that some cosmic epoch probably existed only for finite time, that hypothesis does not entail that the Cosmos began to exist or even that the Cosmos satisfies any of my three conditions for the Cosmos to have had a beginning.

Craig and Sinclair (2009) argue that the conclusion they reach for the Emergent Universe scenario applies "across a wide array of model classes" that include a primordial meta-stable state. As a second example, the inflaton field might once have occupied a meta-stable state, in which the inflaton field's energy, i.e., the vacuum expectation value, was significantly higher than the field's subsequent value. The field may have decayed into some lower energy state, in which the vacuum expectation value is close to zero, and (perhaps) corresponds to dark energy in our current cosmological epoch. Like the meta-stable state in the Emergent Universe scenario, the higher energy state of the inflaton field would have been unstable and is unlikely to have existed for an infinite length of time. While Craig and Sinclair are right that a meta-stable state would probably not have persisted for an infinite length of time, all that we can conclude is that the meta-stable state probably had a finite lifetime. The inflaton field (for example) could have transitioned *between* any number of meta-stable states for some indefinite period of time – with no upper bound to the length of time – before exiting. Moreover, there may be a vast number of trajectories that pass through the meta-stable state, so that noting that the system was once in the meta-stable state is insufficent for determining how the system came to be in that state.

We can again recall Clifford's argument. We can use the heat conduction equation to trace the poker's temperature back to the time that the poker was inserted in the fire or the diffusion equation to trace the history of mixed fluids back to an unmixed state, but the inability to trace the poker or the fluids further back in time does not entail a beginning of either the poker or the fluids. Likewise, our inability to determine a prior history for some specific epoch in cosmological history does not indicate the Cosmos had a boundary that would satisfy the Boundary Condition. We may have simply reached a catastrophe beyond which our calculations cannot be successfully extended.

10.5 An Important Objection

So far, I've argued that our current understanding of physical cosmology prevents us from knowing whether the Cosmos satisfies the Boundary Condition. Unless physical cosmology changes substantially in specific ways in future physical inquiry, we are unlikely to be able to infer whether the Cosmos satisfies the Boundary Condition. In this section, I will highlight a potential objection to my dissertation's project and will show how the arguments that I've presented in this chapter overcome that objection.

Despite what I've argued in previous chapters, friends of the KCA's second premise might hope that, in future inquiry, physicists will determine that there is a boundary in our past. The question would remain open as to whether there is a boundary to the past of all space-time points. For example, one way to violate the Boundary Condition would involve the existence of points that are space-like separated from us but which are such that there is no boundary to their past. However, if one discovered that there is a boundary to one's past, perhaps one should use induction to project the feature of having a past boundary to all space-time points. In that case, while the presence of a boundary in one's past would not demonstrate that the Cosmos satisfies the Boundary Condition, the presence of a boundary in one's past would provide evidence that the Cosmos satisfies the Boundary Condition.

The considerations in this chapter cast doubt on such a possibility. To see why, let me grant that if one did know that there were a boundary to one's past, then one would have some evidence that the Cosmos satisfies the Boundary Condition. Nonetheless, given the epistemic horizons that I've considered in this chapter, we wouldn't know whether any candidate boundary really is a boundary. For that reason, one wouldn't be able to infer that one's past did include a boundary. Without being able to infer that one's past did include a boundary, one would not be able to employ an inductive generalization from the fact that one's own past includes a boundary.

10.6 Summary

This chapter adds to our understanding of the epistemic limitations of cosmologically relevant horizons and how those epistemic limitations might prohibit us knowing from whether the Cosmos satisfies the Modal or Boundary Conditions.

First, I have argued that the Mentaculus project provides us with reason to endorse a specific cosmologically relevant horizon. Either – as friends of the Mentaculus have argued – the direction of time is reducible, in which case the Cosmos does not satisfy the Modal Condition, or else the direction of time is not reducible. Albert's transcendental argument for the Past Hypothesis is consistent with the irreducibility of the direction of time. The consequence of Albert's transcendental argument is that all of the physical states of affairs of which we can have records must be located somewhere between our present state of affairs and the ultimate ready state. If the direction of time is irreducible, then there may be states of affairs prior in time to the ultimate ready state. Nonetheless, there can be no records of any such state of affairs. Since empirical access to the past is plausibly available only by way of records, we cannot have empirical access to whatever states of affairs there might have been before the ultimate ready state. For that reason, the ultimate ready state is an epistemic horizon to our knowledge of cosmological history.

Second, I discussed a related argument originally offered by William Clifford in the nineteenth century. According to various cosmological models, cosmological history can be traced back to some sui generis state of affairs and no further. The factors which prepared that sui generis state of affairs may be completely lost to present observers so that the sui generis state of affairs provides an epistemic horizon to our knowledge of cosmological history. The inability to trace the Cosmos's history beyond a specific sui generis state of affairs does not entail that the Cosmos had no prior history. As the Cosmic Skeptic claims, the provinciality of our knowledge of the physical facts with respect to spatio-temporal location may prevent us from knowing whether the Cosmos satisfies the Boundary Condition.

Lastly, I turned to considering how a primordial sui generis state of affairs appears in standard inflationary cosmology and in the Emergent Universe scenario and used the arguments developed in this chapter to reply to an important objection to Cosmic Skepticism.

11. BIG BOUNCE OR DOUBLE BANG?

11.1 Introduction

In this chapter, I turn to considering cosmological models on which some dynamical mechanism prepares the primordial observable universe in a low entropy macrophysical state and I will show that, even if we had strong evidence for one of those models, we would have no reason for thinking that the Cosmos began to exist. According to the standard big bang model, the observable universe began in a catastrophic event approximately fourteen billion years ago. Nonetheless, since the beginning of physical cosmology as a science in the first half of the twentieth century, physicists have explored "bounce" cosmologies (Kragh, 2009, 2018). According to the usual interpretation of bounce cosmologies, the observable universe originated when a pre-existing universe "bounced" through a highly compressed state; this could have happened in a variety of ways. Space-time, of which our present universe is one proper part, might cycle through multiple generations of universes, each reaching a maximum size before contracting and eventually giving birth to a subsequent universe (as in, e.g., Ijjas and Steinhardt, 2017, 2018, 2019; P. Steinhardt and Turok, 2007; P. J. Steinhardt and Turok, 2002). Or there could have been a single previous universe that "bounced" through a maximally dense state to give birth to our universe, which will expand indefinitely into the future. (For reviews of models in the former two families, see Battefeld and Peter, 2015; Brandenberger and Peter, 2017; Lilley and Peter, 2015; Novello and Bergliaffa, 2008.) Alternatively, each universe might give birth to offspring universes through the highly compressed state found within black holes (Popławski, 2010, 2016; Smolin, 1992, 2006).

Bounce cosmologies, if true, do not necessarily preclude the possibility that the Cosmos satisfies the Boundary Condition. For example, a chain of universes, each "bouncing" into the next, might be initiated with a finite initial segment, in which case the entire chain might be a DB-spacetime. Nonetheless, many bounce cosmologies, under the traditional interpretation and as they are traditionally discussed, explicitly violate the Boundary Condition. We do not currently know if any bounce cosmology is correct, but, so long as bounce cosmologies remain live hypotheses, we do not know whether the second premise of the KCA is true.

As part of their defense of the second premise of the KCA, William Lane Craig and James Sinclair (2009, 2012) disagree with the traditional interpretations of bounce cosmologies. Craig and Sinclair (2012, pp. 125–127) re-interpret the interface between the two universes to represent the *ex nihilo* birth of both universes – a "double big bang" . Setting aside questions about bounce cosmology's plausibility or about the compatibility of bounce cosmology with Albert's transcendental argument, I will show that bounce cosmologies have features which Craig and Sinclair's interpretation cannot plausibly explain. There are bounce cosmologies in which the features of one universe explain features of the other, which seems inconsistent with the interpretation that both universes were born simultaneously, and there are bounce cosmologies in which the thermodynamic arrow of time is continuous from one universe to the next. Thus, if various bounce cosmologies are correct, then the Cosmos does not satisfy the Boundary Condition. Due to the provinciality of our knowledge with respect to scale, time, space, and energy, we do not know whether any of the bounce cosmologies are correct, or at least correct in sufficient detail to suggest on their basis whether the Cosmos satisfies the Boundary Condition. We cannot rule bounce cosmologies out and so cannot rule out the possibility that the Cosmos was beginningless.

11.2 The BVG Theorem and Bounce Cosmologies

As I've discussed in chapter 9, according to the BVG theorem, in any classical space-time, any geodesic congruence of time-like and null curves along which the average of a specific generalization of the Hubble parameter is positive must be finite in temporal extension. While Borde, Guth, and Vilenkin interpret the result to indicate that our understanding of cosmological history is incomplete, Craig and Sinclair interpret the singular behavior as evidence that the Cosmos had a beginning in the finite past. Nonetheless, a variety of non-singular cosmologies – including bounce cosmologies – have been proposed. Bounce cosmologies avoid a past boundary to space-time because instead of postulating that the generalized Hubble parameter is always greater than zero, bounce cosmologies postulate that space-time can be smoothly continued – that is, without becoming singular – from our expanding phase into a past contracting phase, where the generalized Hubble parameter is negative. The interface at which the expanding and contracting phases smoothly join on to one another is termed the "bounce".

Recall that, for Craig and Sinclair, the Cosmos had a beginning just in case the Cosmos's past is finite. I've argued that a finite past does not suffice for the Cosmos to have had a beginning. Nonetheless, since Craig and Sinclair utilize a conception on which a finite past does suffice for a beginning, Craig and Sinclair have endeavored to show that the Cosmos has a finite history. Bounce cosmologies might be thought to avoid a finite history because, at least at first glance, they postulate that space-time has an infinite history. For my purposes in this chapter, the question will be whether the truth of a bounce cosmological model would entail that the Cosmos includes a space-time satisfying the Boundary Condition. There are two possibilities for the temporal location of the boundary required for the Boundary Condition, namely, that the boundary is located in the finite past or else the boundary is located in the infinite past. No current proposal for a bounce cosmology entails a boundary located infinitely far to the past; so, the question is whether bounce cosmologies should be interpreted to include a boundary located finitely far to the past.

To continue their defense of the *Kalām* argument in the light of non-singular cosmologies, Craig and Sinclair have sought to provide a typology of cosmological models that "evade" the Hawking-Penrose or Borde-Guth-Vilenkin singularity theorems (2009, p. 143; 2012, p. 111) and to show that either non-singular cosmological models suggest the universe did have a past boundary or that non-singular cosmologies are implausible. In their typology, Craig and Sinclair discuss bounce cosmologies in which the entropic arrow of time reverses at the interface between universes; Craig returns to this point in his (2016) debate with Sean Carroll and in discussion of Penrose's Conformal Cyclic Cosmology (Craig and Sinclair, 2012, p. 127; Craig, 2016). I turn to Craig and Sinclair's interpretation of bounce cosmologies in the next section.

11.3 The Interface and the Arrow of Time

I will refer to the space-like surface joining the two universes as the *interface*. On the orthodox interpretation of bounce cosmologies, from the perspective of the current universe – which I will call the *expanding universe* – the other universe – which I call the *contracting universe* – is located to our past. This interpretation can be defended from within the view that the direction of time shares a reductive explanation with the entropy gradient. To see that this is so, we need to distinguish bounce cosmologies in which there is an entropy minimum at the bounce from bounce cosmologies in which the bounce is not an entropy minimum.

First, let's turn to bounce cosmologies in which there is no entropy minimum at the bounce and, consequently, no reversal of the entropic arrow of time. I will have more to say about models of that type below, but note that, on the view that the direction of time is reducible, the interface is not a past boundary; according to the local direction of time, as indicated by the entropy gradient, there are states of affairs located before the interface. Thus, the interface is not the Cosmos's beginning. Second, let's consider bounce cosmologies in which there is an entropy minimum at the bounce. In that case, the entropic arrow of time reverses at the interface between the two universes. Since we've assumed that the direction of time shares a reductive explanation with the entropic arrow of time, observers located in our current universe would correctly regard the entropy minimum as being located to their past. For example, suppose that José is an observer in our present universe. We can trace a time-like geodesic through José's past light cone, to the entropy minimum, and beyond to another observer, located in the contracting universe, who I will call Mariana. José would correctly say that Mariana is located in his past light cone and in the direction that he regards to be past based on the entropy gradient that he observes in his surroundings. But equally so, Mariana can say that José is located in the direction that she regards to be past based on the entropy gradient that she observes in her surroundings. Since the entropic arrow of time and the direction of time share a reductive explanation, José and Mariana are equally correct to claim that the other is to their past. Their disagreement reflects the fact that if the direction of time is reducible, the Cosmos does not satisfy the Modal Condition and lacks a beginning. Thus, friends of the view that the direction of time shares a reductive explanation with the entropic arrow of time can endorse the orthodox interpretation of bounce cosmologies.

Craig and Sinclair disagree with the orthodox interpretation. Craig and Sinclair neglect the important fact that there are bounce cosmologies in which the interface is not an entropy minimum; I will object to their interpretation of bounce cosmologies on that basis below, but, for now, set it aside. As far as Craig and Sinclair are concerned, the entropic arrow of time reverses at the interface. Given the correlation between the direction of time and the entropic arrow of time, and that the entropic arrow points away from the interface in either direction, Craig and Sinclair argue that the interface should be understood as the birth of two universes (a "double Big Bang"). As Craig and Sinclair describe, "The boundary that formerly represented the 'bounce' will now [be interpreted to] bisect two symmetric, expanding universes on either side" (Craig and Sinclair, 2012, p. 122). Elsewhere, Craig and Sinclair write that, "The last gambit [in trying to avoid an absolute beginning], that of claiming that time reverses its arrow prior to the Big Bang, fails because the other side of the Big Bang is *not* the past of our universe" (Craig and Sinclair, 2009, p. 158). As Craig and Sinclair conclude, "Thus, the [universe on the other side of the interface] is not our past. This is just a case of a double Big Bang. Hence, the universe *still* has an origin" (Craig and Sinclair, 2009, pp. 180–181; also see Craig and Sinclair, 2012, pp. 125–127).[1]

In some sense, Craig and Sinclair's choice to analyze bounce cosmologies in terms of an entropy minimum is an odd one. There is a peculiarity involved in all of Craig and Sinclair's analyses of live cosmological models. All of the cosmological models Craig and Sinclair analyze are speculative. In many cases, the physicists who developed the model understood the model to demonstrate a specific principle as opposed to understanding the model as a live candidate for the global structure of space-time. For that reason, if Craig and Sinclair base their analysis on model features that are not necessary ingredients,

[1]↑Though much of the argumentation that Craig and Sinclair offer in their (2009, 2012) concerns the Aguirre-Gratton model (2002, 2003), Craig and Sinclair draw conclusions which Craig and Sinclair take to apply to *any* cosmological model in which there is an interface at which the entropic arrow of time reverses direction, e.g., Craig and Sinclair, 2009, p. 158.

their analysis might be provincial. We should instead be interested in developing general principles that are necessary, or at least highly probable, features of a broad variety of models.

Even if we set aside models in which the interface is not an entropy minimum, there is no entailment relation between including a bounce and including an entropy minimum. In fact, we can define cosmological models that include a bounce, e.g., de Sitter space, without including an entropy function at all. Recall that Craig and Sinclair understand bounce cosmologies as attempts to "evade" the BVG theorem. Since the relationship between a bounce and an entropy minimum is not a necessary feature, there is no general reason for thinking that models that "evade" the BVG theorem by including a bounce will also include an entropy minimum. For all that Craig and Sinclair have argued, their interpretation might only be a provincial interpretation of a specific collection of toy models.

Craig and Sinclair's interpretation is perplexing for another reason. On the one hand, if the direction of time does share a reductive explanation with the entropic arrow of time, there would be reason to think that the entropy gradient indicates the local direction of time. Setting aside the interpretations I've already offered on behalf of friends of the view that the direction of time is reducible, if the direction of time is locally indicated by the entropy gradient, there is at least some reason to think the interface initiates the histories of both universes. Nonetheless, the Cosmos would still fail to satisfy the Modal Condition. On the other hand, Craig and Sinclair deny that the direction of time is reducible and so deny that the direction of time can share a reductive explanation with the entropic arrow of time. Craig writes:

> From a theistic perspective [...] all such attempts [to reduce the direction of time] seem misconceived. For one can easily conceive of a possible world in which God creates a universe lacking any of the typical thermodynamic, cosmological or other arrows of time, and yet He experiences the successive states of the universe in accord with the lapse of His absolute time (Craig, 2001c, p. 162).

One can likewise imagine God experiencing the lapse of absolute time while the entropy of the universe decreases. In fact, one way that the universe *could* lack the typical thermodynamic arrow of time would be if the entropic arrow of time and the direction of absolute time were not consistently aligned. None of this requires God's creation; as Henri Poincaré wrote, "the atheists [can imagine] put[ting] themselves in the place where God would be if he existed" (Poincaré, 2001b, p. 217). Craig has argued that if the entropic arrow and the direction of time do not align then this entails "a non-reductionistic view of time [...] where the direction of entropy increase doesn't define the direction of time". Craig may object that the misalignment between the entropic arrrow and the direction of time "is physically impossible" because this would contradict the second law of thermodynamics (S. Carroll and Craig, 2016, p. 78). That is, that the alignment of the two is nomologically necessary. But Craig doesn't provide us with an account of why the alignment would be nomologically necessary.

Importantly, the second law of thermodynamics is already known to be a statistical regularity that admits of exceptions. As Craig has described, on his view, "the physical arrows [of time] are neither necessary nor sufficient for time's having a direction and/or anisotropy" (Craig, 1999, p. 352). One advantage that the non-reductive view has is that time consistently flows in a fixed direction even when, e.g., through a statistical fluke, the entropic arrow reverses. Craig agrees and has considered a thought experiment in which the universe is a vast equilibrium gas with small, localized fluctuations from equilibrium. As Craig notes, for his reductionist interlocutors, there may be no sense in which a fluctuation at one time is either before or after a fluctuation at another distinct time. Craig thinks that an advantage of his anti-reductionism is that there would be a fact about which fluctuation is first regardless of how the universe's entropy changes in the interim (Craig, 1999, p. 354). As Craig writes, "The fact that entropy states of a process range in value between higher and lower numbers tells us nothing about which values exist later" (Craig, 1999, p. 355).

Thus, on the anti-reductionist view, the alignment between the entropic arrow and the direction of time is not nomologically, metaphysically, or logically necessary. If Craig is right that the direction of time and the entropic arrow need not be aligned, then there is no reason to think the reversal of the entropic arrow of time, in those models where the entropic arrow of time reverses, suggests a double big bang. I think that this is a strong objection to Craig and Sinclair and I pursue a longer defense of this objection elsewhere (Linford, 2020). Nonetheless, I will set this worry aside so that I can pursue a different (and complementary) objection to Craig and Sinclair's interpretation of bounce cosmologies.

11.4 A Double Big Bang?

I can now turn to showing that Craig and Sinclair's interpretations of bounce cosmologies are implausible. If the interface were the *ex nihilo* origin of two universes, then features of the universe on one side of the bounce, particularly those features that develop out of late time evolution, cannot provide an explanation for features of the universe on the other side of the bounce. But, as I will show, features of one universe do explain features of the other universe. Moreover, I will show that there are models in which the entropy is "reset" without reversing the entropic arrow. For both reasons, authors who accept an anti-reductive view of the direction of time should not interpret bounce cosmologies in a way that entails the Cosmos includes a space-time satisfying the Boundary Condition.

11.4.1 Anti-Inflationary Bounce Cosmologies

The BGV theorem applies only to space-times with a positive average expansion rate. If space-time undergoes a contraction, space-time's average expansion rate might not be positive. For one example, we can consider Anna Ijjas and Paul Steinhardt's pedagogical introduction to their favored family of bounce cosmologies (2018). In the models that Ijjas and Steinhardt describe, a bounce between two universes is postulated in order to explain the features of our universe that inflationary cosmology had previously been meant to explain (e.g., the horizon, flatness, and smoothness problems); I will refer to these models as the anti-inflationary models. Consider the horizon problem. According to the horizon problem, in order for two regions, R_1 and R_2, of the Cosmic Microwave Background (CMB) to have reached a uniform temperature, signals traveling no faster than the speed of light would have had to have traveled from R_1 to R_2. But there are regions of the CMB that are further apart than signals could have traveled in the early universe according to the Standard Big Bang (SBB) model.

Let's recall some formal machinery. For a given point p in a relativistic space-time, the *causal future* of p is the set of points that a particle could reach from p without exceeding the speed of light. The *causal past* of p is the set of points that could reach p without having exceeded the speed of light. And the *absolute elsewhere* of p are all of those points which cannot be reached from, and cannot reach, p without exceeding the speed of light. Let's say that the *patch* for p at T is the set of all the points at some time T that are either in the causal future or causal past of p, where p can be located at some time other than T. So, for example, there is a patch that consists of all of those points five minutes in my past from which particles can reach me now without exceeding the speed of light; included among those points are all of the points occupied by my computer five minutes ago, the entirety of the apartment that I am writing in five minutes ago, and so on.

We've said that, on the SBB model, when a present day observer, at time t_1, looks back to the early universe at $t_0 \ll t_1$, she can measure regions of the CMB between which signals could not have propagated without exceeding the speed of light. In other words, on the SBB model, when a present day observer, at time t_1, looks back to the early universe at $t_0 \ll t_1$, her patch at t_0 exceeds the horizon size at t_0. Nonetheless, she would observe her patch to be uniform in temperature, suggesting that the parts of the patch at t_0 must somehow have come into contact with one another. Inflation proposed a modification to the SBB model in which the early universe underwent a period of exponential expansion. If the universe underwent a period of exponential expansion, then the exponentially fast expansion of space could have pushed space-time regions that are initially in contact outside of one another's horizons. The anti-inflationary bounce cosmologies resolve the horizon problem in a different way. For anti-inflationary bounce cosmologies, the horizon of a previous universe was significantly larger than our patch in that universe. This would allow regions of the patch to come into thermal equilibrium *before* our universe, so that the causally disconnected regions that produced the CMB would have uniform temperatures.

The anti-inflationary bounce cosmologies provide a natural explanation for the reduction in the entropy in the previous universe that allows a consistent arrow of time through the bounce. As Ijjas and Steinhardt describe, "the patch corresponding to our observable universe today was only an infinitesimal fraction of the horizon size long before the bounce. That means only the limited entropy in the pre-bounce phase that is contained within [the patch] contributes to what is in the observable universe at the beginning of the expanding phase" (Ijjas and Steinhardt, 2018). Elsewhere, in describing ekpyrotic cosmological models[2] – one kind of anti-inflationary bounce cosmology – Steinhardt and Turok write, "Globally, the total entropy in the Universe grows from cycle to cycle [...]. However, the entropy density, which is all any real observer would actually see, has perfect cyclic behavior with entropy density being created at each bounce, and subsequently being diluted to negligible levels before the next bounce" (P. J. Steinhardt and Turok, 2002, p. 1; also see P. Steinhardt and Turok, 2007, pp. 192–193). In stating that the "global" entropy grows from cycle to cycle, Steinhardt and Turok mean that the entropy generated within a given cosmological horizon during the previous cycle is not destroyed, but the entropy density is decreased exponentially, with an associated reduction of the degrees of freedom per horizon to nearly zero (P. J. Steinhardt and Turok, 2002, p. 17). Thus, the entropy reversal through the bounce is not contrary to the entropic arrow of time; instead, the entropy simply left our causal horizon, rapidly becoming too distant for signals to successfully propagate to us.[3]

Anti-inflationary bounce cosmologies are inconsistent with Craig and Sinclair's interpretation for two reasons. First, anti-inflationary bounce cosmologies resolve the horizon, smoothness, and flatness problems by invoking features of the late time evolution of an-

[2]↑Craig and Sinclair (2009, pp. 167–169) have objected that the ekpyrotic model is geodesically incomplete and, therefore, not past eternal. However, Ijjas and Steinhardt have recently proposed a new version of the ekpyrotic model (Ijjas and Steinhardt, 2017, 2019). Steinhardt confirmed via correspondence that the new model can be made geodesically complete (per. corr. June 24, 2019).

[3]↑In an argument that Steinhardt and Turok attribute to Richard Tolman the universe could not cycle through an eternity of contractions and expansions because entropy would build up in each cycle (P. Steinhardt and Turok, 2007, pp. 180–182). As Helge Kragh (2009, p. 606) has pointed out, "Tolman did not actually conclude that there had been only a finite number of earlier cycles" and did not think thermodynamic considerations made a good case for the universe having begun at some finite time in the past. In fact, Tolman (1934, p. 486) argued the universe might well extend infinitely far into the past and infinitely far into the future. Nonetheless, as Steinhardt and Turok point out, the objection does not depend upon an increase in the total entropy; instead, the objection depends upon an increase in the entropy density in each cycle. For this reason, Tolman's argument is inapplicable to models in which the entropy becomes dilute (2007, pp. 192–193) or becomes hidden behind a horizon.

other universe, which would be difficult to explain if one universe did not precede the other in time. As Steinhardt and Turok (2007, p. xiv) describe, "The events that occured before the big bang shaped the large scale structure of the universe observed today, and the events that are occuring today will determine the structure of the universe in the cycle to come". Second, the anti-inflationary bounce cosmologies explain the low entropy of the early universe in a way that consistently maintains the entropic arrow of time through the bounce. As anti-inflationary bounce cosmologies have (arguably) become the most popular bounce cosmologies, and Craig and Sinclair's interpretation is inconsistent with the anti-inflationary bounce cosmologies, we may already have reason to reject their interpretation altogether. But let's push forward.

11.4.2 Bouncing Through Black Holes

The BGV theorem applies only to classical space-times. Models that modify the Einstein Field Equations to produce a non-classical space-time can produce a non-singular "bounce" (Corda and Cuesta, 2011; Edholm, 2018; Ijjas and Steinhardt, 2017; Kehagias et al., 2014; Lilley and Peter, 2015; Popławski, 2010, 2016; Sotiriou and Faraoni, 2010; Starobinsky, 1980). In this section, I consider two models – Lee Smolin's evolving universe scenario (1992, 2006) and Nikodem Poplawski's model (2010, 2016) – in which Einstein's gravity is modified in ways that allow the interior of a black hole to "bounce" and produce a baby universe.[4] In Smolin and Poplawski's models, the thermodynamic arrow of time is continuous along geodesics that pass through the interface even though the entropy is "reset" at the interface.

Smolin's evolving universe hypothesis was developed to explain the so-called *anthropic coincidences*. That is, that the free parameters appearing in our best theories of fundamental physics (e.g., the cosmological constant, the coupling constants, and so on) are consistent with the existence of life – or large scale structures generally – only if the parameters assume values from a narrow range compared to the range of values that the parameters

[4]↑Quentin Smith (1990, 2000) has offered a different, but related, cosmology in which universes are born from black holes. I do not consider Smith's model in this chapter because his model is not a bounce cosmology and the parent/child universes do not bear a temporal relationship to each other.

could have had. That is, if the parameters are understood to be selected from a prior distribution uniform over all possible values of the parameters, then life is improbable. This problem can be resolved if one can provide a non ad hoc and plausible hypothesis according to which the probability distribution over the space of possible parameter values is not uniform. In other words, while the uniform distribution would poorly predict the existence of life, the existence of life is a prediction of the new distribution induced by the theory. Smolin's (1992, pp. 173–174) model is an attempt to provide one such explanation.

If the density of matter or energy within some volume is sufficiently large and the matter-energy density outside the region sufficiently low, the result is a black hole. As in many classical Big Bang models, General Relativistic models entail that black holes contain a curvature singularity. And just as with the Big Bang, physicists suspect that black hole curvature singularities will be replaced in a quantum mechanical description. Smolin's evolving universe hypothesis consists of two postulates:

1. The curvature singularities General Relativity predicts to reside inside black holes will be replaced by the beginning of a child universe within a complete quantum gravity theory.

2. In the creation of a child universe, the values of the free parameters in fundamental physical theories will slightly change (Smolin, 1992, p. 175; 2006, p. 6).

From these two postulates, given that large universes produce large numbers of black holes, large universes will have many more offspring than small universes. But the universes cannot be too large, or otherwise matter can never clump together to form black holes. The size of a given universe is determined by the rate at which the universe expands. Thus, the two postulates entail that space-time will come to be dominated by universes selected from a fixed range of expansion rates. In turn, the expansion rate is determined by the cosmological constant. So, a restriction on the range of expansion rates entails a restriction on the range of cosmological constants. The consequence will be that universes with values of the cosmological constant no larger than some maximum value would come to dominate space-time. Smolin has argued that his hypothesis affords an explanation of most of the other anthropic coincidences. For example, the hypothesis explains the difference in mass between the proton and neutron and provides an explanation for the gauge hierarchy problem (Smolin, 1992). Moreover, Smolin's hypothesis makes a falsifiable prediction that could be used, in principle, to rule out the hypothesis. Because universes with values of the free parameters that maximize the number of black holes would dominate space-time, we should predict that variations of the parameters characterizing our universe would result in universes with fewer black holes (Smolin, 1992, p. 176).

Smolin's model requires that at least two features of the offspring universes be explained in terms of features of the parent universes. First, the model's ability to deliver on the desideratum that the hypothesis provide a non ad hoc reason for thinking that the distribution on the range of possible parameter values is not uniform. The evolving universe hypothesis satisfies this desideratum by entailing that the majority of the distribution's mass is located within the range conducive to black hole production. If we begin with some population of n universes, such that $n \geq 1$, selected from a distribution uniform over the space of possible free parameter values – or, indeed, a variety of other distributions – this distribution will generically evolve to a situation in which most of the probability mass *is* located within the range of life-conducive values.[5]

Second, Smolin's model involves dynamics that maximize the number of black holes a given universe produces. Therefore, Smolin's model predicts that, if we vary the measured values of the free parameters characterizing our current universe, we should find that variations would result in hypothetical universes that would produce fewer black holes (Smolin, 1992, p. 176). This prediction is explained by a selection history of prior universes. Consequently, if the interface between parent and child universes is not

[5]↑Two caveats are in order. First, the reader should not take the language of "beginning" too seriously. Smolin's model is consistent with a space-time with an indefinitely long history and does not require that space-time ever *began* to exist. Second, if we begin the model with $n = 1$ universes and the cosmological constant selected for the initial universe is too large, the one universe could accelerate apart without producing any black holes. Therefore, in order for Smolin's model to work, either the initial universe must be sufficiently improbable so to produce some population of black holes or else we should consider a situation in which we begin with multiple universes. Presumably, the most sensible possibility would be that space-time is eternal into the past so that there has always been some network of universes connected by black holes.

understood as the birth of a child universe out of a parent universe, then Smolin's model cannot do the explanatory work the model sets out to do.

Given that at least two features of the offspring universes must be explained in terms of features of the parent universes, one should not interpret the interfaces between universes that appear in Smolin's model as the birth of two universes. Instead, one should favor Smolin's interpretation, in which parent universes give rise to offspring universes.

I now turn to discussing Poplawski's cosmology. Poplawski's cosmology is produced within the Einstein-Cartan framework. Einstein-Cartan is a modification to the Einstein Field Equations that results from coupling spin – an intrinsic property of some fundamental particles – to torsion – a geometric property of space-time. Coupling spin-to-torsion prevents the formation of singularities in black holes. Instead of forming a singularity, the black hole creates a child universe. The entropic arrow of time is continuous from the parent universe, through the black hole, and into the child universe (Popławski, 2010, 2016).[6] One might worry that bounce cosmologies in which the arrow of time is continuous through the interface do not avoid a beginning because the entropy could not have been increasing from eternity past. Consider, for example, counting backwards from ten. If one counts one number per second, then, after ten seconds, one must reach zero. So, if the entropy decreases into the past, shouldn't we hit some absolute zero on the entropy scale at some finite time in the past?

Here, the answer is no, and for two reasons Poplawski offers in his (2010). First, while the field equations for Einstein-Cartan gravity are time symmetric, the boundary conditions of black holes are not time symmetric. That is, objects can travel through the black hole's event horizon but cannot travel back out. For this reason, the boundary condition for the child universe would be temporally asymmetric. Second, while observers

[6]↑Poplawski's model has an advantage over some other bounce cosmologies, because Poplawski's model avoids one of the criticisms Craig and Sinclair leverage against bounce cosmologies. As Craig and Sinclair argue, models in which a previous universe collapses to some minimum size before expanding into our universe needs to be carefully fine-tuned from eternity past in order to successful collapse to the minimum size (Craig and Sinclair, 2012, pp. 111–2). But, like Smolin's model, Poplawski's model involves the creation of offspring universes from black holes. For this reason, neither Smolin's nor Poplawski's models require such fine-tuning. Between the two, Poplawski's model is more convincing because, unlike Smolin, Poplawski provides a mathematical model and a physical mechanism for the dynamical evolution of black holes within one universe into subsequent offspring universes.

outside the black hole will observe the horizon of the black hole maximizing the entropy, the entropy will not have been maximized for observers inside the black hole – that is, in the offspring universe – for whom the entropy can be increased still further. An entropy gradient requires only that the entropy on the interface be smaller than the present entropy, but not that the entropy has never been lower than the entropy on the interface. Indeed, there exist monotonically increasing functions $f(t)$ such that, for any time T_0, there will exist some $T_{-1} < T_0$, such that $f(T_{-1}) \leq f(T_0)$. For this reason, an entropy gradient can be established without postulating a beginning.[7]

11.4.3 Conformal Cyclic Cosmology

Roger Penrose has proposed a different modification to General Relativity that, again, avoids the BGV theorem by proposing a non-classical space-time.[8] Though the CCC is not typically considered a bounce cosmology, I will include the CCC in this chapter for two reasons. First, Craig and Sinclair offer an interpretation of the CCC that parallels their interpretation of bounce cosmologies. Importantly, Craig and Sinclair argue that the interface between universes that appears in the CCC should be interpreted as the birth of two universes because the interface is an entropy minimum (Craig and Sinclair, 2012, p. 127; also see Craig's blogpost (2016)). Second, several features of the CCC bear a significant resemblance to features of models traditionally considered bounce cosmologies (e.g., cyclic generations of universes, an entropy minimum on the interface between universes, one universe that results in a highly compressed state in order to produce a subsequent universe).

Penrose postulated the CCC to explain the low entropy of the early universe (Penrose, 2012, p. 144). As Penrose notes, the most probable way for a universe to evolve *into* a

[7]↑For a reply to a related argument originally offered by Tolman, see footnote 3. As in Steinhardt and Turok's model, in the offspring universe, the total entropy of the parent universe has become hidden behind a horizon and is not accessible to the offspring universe.

[8]↑According to Craig and Sinclair, Penrose's model is consistent with the BGV theorem because the average expansion rate of a cyclic universe is zero (Craig and Sinclair, 2012, p. 111). This is incorrect. In the CCC, the expansion or contraction of the universe is not a well-defined notion for every period of the universe's evolution. But, during those periods in which expansion/contraction are well-defined, the universe only expands and never contracts. The CCC is not singular because the CCC utilizes a space-time to which the BGV theorem cannot be applied.

cosmologically-relevant curvature singularity results in a highly disordered state because the evolution generically involves an amplification of any anisotropies or inhomogeneities. The anisotropries and inhomogeneities are amplified into black holes and then the black holes successively fuse. Penrose argues that this complex singularity structure attributes high entropy to the gravitational degrees of freedom. General Relativity is a time reversal invariant theory, so that the most probably way to evolve into a cosmological singularity should be the time reverse of the most probable way to have evolved from a cosmological singularity. Consequently, the most probable way to evolve from a cosmological singularity would again involve a complex singularity structure which attributes high entropy to the gravitational degrees of freedom (Penrose, 2012, pp. 124–125).[9] If a low entropy singularity is an improbable beginning on General Relativistic models, and we know that the universe began in a low entropy state, then some revision to the General Relativistic models is required in which the universe's beginning is not improbable. To produce a model like that, Penrose proposes a mechanism by which preceding physical states could dynamically produce the low entropy condition of the early universe.

As Penrose interprets current fundamental physical theories, length and time scales are determined by the presence of mass in the universe.[10] Penrose argues that length and temporal scales ultimately depend upon the existence of mass, so that in a universe in which there are no masses, length and temporal duration lose meaning. If length has lost its meaning, then an infinitely compressed point – that is, the low entropy initial singularity – cannot be distinguished from an infinitely large universe.[11] According to Penrose, mass

[9]↑This can be put more carefully. As I noted earlier in this chapter, curvature singularities are not points that General Relativity includes in the space-time manifold. Therefore, one should not say that, in General Relativistic models, the universe began with a low entropy singularity. However, one can accurately say that, when the universe is reversed in time in General Relativistic models, space-time tends towards a low entropy singularity. If one chooses any arbitrarily small value $\varepsilon > 0$ then there will exist some time t such that the scale factor $a(t) < \varepsilon$. The low entropy singularity corresponds to the limit in which $\varepsilon \to 0$.

[10]↑Penrose favorably cites Rugh and Zinkernagel, 2009 for their relationist view of space and time scales. Also see Rugh and Zinkernagel, 2017.

[11]↑I'm speaking loosely. In the absence of length and time scales, a single point and three dimensional space *do* differ, for example, in topological structure. A single point has the topology of $\mathbb{R}^0$ while a three dimensional space has the topology of $\mathbb{R}^3$. But note the qualifications that I made in footnote 9. For any $a(t) > 0$, three dimensional slices of space-time have the topology of $\mathbb{R}^3$. Penrose should be interpreted as arguing that when length loses its meaning, space-time loses length and time scales. For that reason, we can identify an arbitrarily "compressed" three dimensional space with an arbitrarily "expanded" three dimensional space.

can be expected to exit our universe for three reasons. First, the universe that we inhabit will expand indefinitely into the future. As our universe expands indefinitely into the future, the density of the universe will decrease and mass will leave our cosmological horizon. Second, some proportion of the mass will be swallowed by black holes and those black holes will decay. Third, Penrose postulates that all of the remaining massive particles will eventually decay into massless products (Penrose, 2012, p. 153). The mass-free homogeneous and isotropic universe towards which our universe tends in the infinitely far future will then be the smooth Big Bang of a subsequent universe. The beginning of a universe would involve a low entropy state because some of the processes that eliminate the masses within a given universe reduce the entropy of the universe.[12]

In CCC, events in the universe on one side of the interface explain events on the other side of the interface but not vice versa. First, like the anti-inflationary cyclic model discussed in section 11.4.1, CCC postulates that features of the Cosmic Microwave Background often solved through an inflationary phase in the present universe (e.g., the horizon problem) are instead solved by exponential expansion in a previous universe (Penrose, 2012, p. 210). Second, in addition to reproducing several of the predictions of inflationary cosmology, Penrose (and collaborators) have argued that the universe prior to ours should leave a signature in the Cosmic Microwave Background not predicted by inflation (Penrose, 2012, pp. 211–219; An et al., 2018; Gurzadyan and Penrose, 2010, 2013).

[12]↑For example, one of the processes Penrose discusses for eliminating masses from the universe involves massive particles being swallowed by black holes and then the black holes undergoing a non-unitary decay process (Penrose, 2012, pp. 186–188). The non-unitary decay process reduces entropy. Craig and Sinclair reply to this feature of CCC with the complaint that the late time evolution of the universe will be dominated by the entropy associated with the universe's horizon. This entropy is far larger than the entropy reduced through the decay of black holes (Craig and Sinclair, 2012, pp. 120–121). While Penrose preempts this objection, Craig and Sinclair (wrongly) complain that Penrose only offers an instrumentalist interpretation of the entropy associated with the universe's horizon. Importantly, Penrose provides a reply both to the realist and instrumentalist interpretations of the universe's horizon entropy; on Penrose's view, even if the entropy of the universe's horizon is real, that entropy can be ignored because it plays no role in the universe's dynamics (Penrose, 2012, p. 202). Moreover, Craig and Sinclair are inconsistent in their interpretation of the entropy associated with the universe's horizon. As Craig and Sinclair write in their 2009, p. 155, the universe's horizon differs from the horizon of a black hole because the former should not (in their view) be understood as objectively real. But if the universe's horizon is not objectively real, in what sense can the entropy associated with that horizon be understood as objectively real? In any case, the resolution of this debate is irrelevant for my purposes here because I am only concerned with how we ought to interpret CCC.

Photons and gravitons do not possess mass, so the elimination of all *mass* within a given universe would not result in the elimination of all particles. Because photons and gravitons do not have mass, photons and gravitons do not experience temporal duration. Consequently, the trajectory of a photon or a graviton can extend from the present universe to the birth of another universe in the infinitely far future. For example, Penrose argues that collisions between black holes in a previous universe should have left a signal detectable by us (Penrose, 2012, p. 215).

Philosophers of physics distinguish the entropic arrow of time from a variety of other arrows of time – for example, the temporal asymmetry of causation is referred to as the "causal arrow of time". The causal arrow of time has often been understood to align with the entropic arrow of time because both arrows can be afforded a reductive explanation within statistical mechanics (Albert, 2000, 2015; Loewer, 2007, 2012a, 2020; Papineau, 2013). However, the causal and entropic arrows of time come apart in Penrose's model, provided that what he, and co-authors, claim about the model is correct. That is, the causal arrow of time does not reverse at the interface, even though the entropic arrow of time does reverse.[13] Instead, the causal arrow of time is continuous through the interface – features of a prior universe explain features of a subsequent universe but not vice versa – and this can be taken to suggest that the direction of time is continuous through the interface.

Craig and Sinclair have provided another reason to think that, in CCC, the interface between universes is the beginning of our universe. As Craig and Sinclair point out, in Penrose's model, some of the mathematical structure usually attributed to time disappears at the interface between universes. For example, in the orthodox interpretation, time scales lose meaning in both the early and late universe. Craig and Sinclair interpret this aspect of the model to mean that the two universes cannot stand in relations of *before* and *after*

[13]↑Penrose has been careful to argue that while the total entropy of the universe is reduced prior to the bounce through non-unitary processes (see footnote 12), a thermodynamic arrow of time is nonetheless preserved and never reverses direction (Penrose, 2012, pp. 175–190). For this reason, Penrose argues that the entropy reduction in CCC is not a violation of the second law of thermodynamics and, thus, CCC might not contradict the Mentaculus. Be this as it may, if we understand the entropic arrow solely in terms of the entropy gradient, then, according to CCC, there is an epoch in which the entropy gradient points contrary to the causal arrow of time.

because to say that one universe preceded the other, when time has lost its meaning at the interface between the two, is incoherent. Instead, Craig and Sinclair claim that there are only topological relations – and not temporal relations – between the two universes (Craig and Sinclair, 2012, pp. 127–128). Here, Craig and Sinclair move too quickly.

In Penrose's model, there are null geodesics that connect the present universe to events in the past universe. Craig and Sinclair agree that the geodesics extend through both universes. But the existence of geodesics traversing the two universes entails that my causal past includes a patch in a previous universe. (For example, photons and gravitons traverse null geodesics between the two universes, e.g., Penrose, 2012, pp. 157–159.) This is precisely the sort of thing that needs to be invoked in order to explain the features that CCC predicts appear in the CMB. The order of events into the causal past, the causal future, and the absolute elsewhere are maintained even when length and time scales are lost because, as Penrose points out, the light cone structure is maintained through the interface (Penrose, 2012, pp. 139–147). So, the relations that exist between universes in Penrose's model allow one to make sense of the claim that one of the two universes temporally succeeds the other.

The fact that Craig and Sinclair have misconstrued the consequences of the breakdown of metrical structure is somewhat perplexing, for, elsewhere, Craig has offered a series of objections to the Oxford School. As discussed in chapter 4, according to the Oxford School, prior to creating the Cosmos, God existed in metrically amorphous time, that is, that although God's successive mental events (for example) were ordered into relations of before and after, there was no objective fact as to the ratio in the lengths of the non-overlapping temporal intervals occupied by God's distinct mental events (see chapter 9 in Craig, 2001b). Meanwhile, Penrose describes how Weyl maintained a view similar to metrically amorphous time, in which the choice of time scale is a choice of gauge (Penrose, 2004, p. 451). Because the choice of gauge is conventional and does not correspond to any physical fact, Weyl maintained that there are no facts about time scale. Einstein objected that the conjunction of quantum mechanics and relativity – by equating energy to both mass and frequency – suggests time scale is fixed for the rest frame of a given mass. For that reason, Einstein argued that time scale cannot be purely conventional (Penrose, 2004,

p. 453). Hence, Penrose's CCC endorses Einstein's notion that mass fixes the time scale while simultaneously endorsing the view that, without mass to fix the time scale, time would be metrically amorphous. Though Craig argues against the Oxford School, Craig admits that, unlike a timeless state, a metrically amorphous state *can* stand in the before relation with respect to the universe: "this [metrically amorphous] state exists literally before God's creation of the world and the inception of metric time" (Craig, 2001b, p. 270). So, just as a deity who is metrically amorphous prior to creation can stand in the before relation with respect to that deity's creation, so, too, can a metrically amorphous physical state stand in the before (or after) relation with respect to metric time.

This point is worth unpacking in some more technical detail. Readers who are satisfied with the qualitative description already given can safely skip to the next section. Any metric tensor $g_{\mu\nu}$ can be decomposed into a volume element (i.e, $|\det(g_{\mu\nu})|^{1/4}$) and a conformal metric density (i.e., $\tilde{g}_{\mu\nu}$) (as first worked out in detail in Thomas, 1925, 1932a, 1932b; also see Anderson, 1967, pp. 63–64; Anderson and Finkelstein, 1971). That is, $g_{\mu\nu} = \tilde{g}_{\mu\nu}|\det(g_{\mu\nu})|^{1/4}$. Penrose maintains the *Weyl Curvature Hypothesis* (WCH), according to which the Weyl curvature tensor exactly vanishes in the low entropy condition of the early universe (Penrose, 2012, pp. 132–135). If the Weyl curvature vanishes everywhere on a region of space-time, then that region is conformally flat, that is, there exists a conformal transformation to a diagonal (flat) metric (Anderson, 1967, p. 63; Penrose, 2004, p. 464). This follows as a consequence of the fact that the Weyl curvature tensor can be expressed entirely in terms of the conformal metric density and its inverse and is equal to the Riemann curvature tensor formed by substituting $g_{\mu\nu}$ with $\tilde{g}_{\mu\nu}$ (Anderson, 1967, pp. 63–64). For example, the Weyl curvature tensor vanishes in FLRW space-times and, as a result, there exists a conformal transformation from FLRW space-time to Minkowski space-time. Thus, if the WCH is true, then there exists a conformal transformation from the early universe to a flat space-time (Penrose, 2004, p. 464). According to CCC, in the far future, when all mass in the universe vanishes, the universe will again be conformally flat. Thus, the early universe, the late universe, and a corresponding region of flat space-time are all conformally equivalent; that there is no mass present at early or late times entails that all three are physically equivalent. So, Penrose concludes, conformal structure remains at early and late times even though the full metrical structure is lost.

At early times, the volume element vanishes – reflecting the singularity in classical models of the Big Bang – but the conformal metric density is perfectly well-behaved. Since CCC attributes physical significance only to those features invariant under conformal transformations in the early or late universe, the volume element – and so the associated singularity – has no physical significance. In turn, the conformal metric density can be used to smoothly continue time-like curves from one universe into the next. As is well known, conformal transformations are precisely those that leave the light cone structure invariant; importantly, this has the implication that, under the conformal transformation, future (past) directed tangent vectors are mapped to future (past) directed tangent vectors. Thus, the light cones along any given time-like curve encodes a conformally invariant temporal ordering of events. Because the conformal metric density allows one to continue time-like curves through the interface between universes, the two universes can be placed into before and after relations, as expected.

11.5 Summary

I've argued that Craig and Sinclair's interpretation of bounce cosmologies does not sit well with a number of features of bounce cosmologies. On the one hand, there are cyclic models, like those endorsed by Ijjas, Steinhardt, Turok, and Penrose, or models in which universes are born out of black holes, like Smolin's or Poplawski's, in which features of one universe explain features of a subsequent universe. We've also seen that there are a number of bounce cosmologies in which the entropy is "reset" at the interface even though the thermodynamic arrow of time is continuous along time-like and null geodesics piercing the interface. Whether any of the cosmological models I've discussed in this chapter are plausible continues to be discussed by physicists. Due to the provinciality of our knowledge with respect to scale, time, space, and energy, we do not know whether any of the cosmological models discussed in this chapter, or models appropriately similar, are correct, or at least correct in sufficient detail to suggest on their basis whether the

Cosmos satisfies the Boundary Condition. However, the models remain live possibilities that should be up for empirical investigation. Since the models remain live possibilities up for empirical investigation and, if correct, would entail that the Cosmos violates the Boundary Condition, we do not know whether the Cosmos began to exist. That is, live cosmological models provide us with additional reason for endorsing Cosmic Skepticism.

In the next chapter, I turn to completing the case for Cosmic Skepticism. The possibility remains that one could – somehow – either project locally available empirical regularities to unobserved portions of the Cosmos or to the Cosmos as a whole and so infer that the Cosmos satisfies the Modal, Direction, and Boundary Conditions or conditions relevant to whether the Cosmos satisfies the Modal, Direction, and Boundary Conditions. As I will show, both inferences are unsuccessful, at least at the present stage of scientific and philosophical inquiry, and we are left unable to infer whether the Cosmos began to exist.

12. COSMIC SKEPTICISM AND CONFIRMATION THEORY

12.1 Introduction

As Hume (or Philo) reminds his readers in *Dialogues Concerning Natural Religion*, only a small fragment of the Cosmos, during a very short time, has been very imperfectly discovered to us. For all we know, space-time is indefinitely larger than the portion available to us; the Cosmos may be larger still. *Cosmic Skepticism* is the thesis that the provinciality of our knowledge of the physical facts with respect to scale, spatio-temporal location, or energy prevents us from having empirical access to whether the Cosmos satisfies the Modal, Direction, and Boundary Conditions. If Cosmic Skepticism is true, then we cannot – at least in our present stage of philosophical and scientific inquiry – determine that the Cosmos began to exist.

So far, I've gathered a number of results which might *suggest* that we should adopt Cosmic Skepticism, but the case has been incomplete because I haven't paid adequate attention to confirmation theory. For example, I've shown that no set of observations that we currently have, when conjoined with General Relativity, entails that the Cosmos satisfies the Modal, Direction, or Boundary Conditions. I've shown that considerations in the philosophical foundations of statistical mechanics entail either that the Cosmos violates the Modal Condition or else that there is a transcendental condition on the possibility of our knowledge of the past that prevents us from having knowledge of states of affairs prior to a specific past boundary. I've taken note of a warning from the nineteenth century – the fact that there is some past boundary beyond which we cannot make reliable inferences does not entail that the Cosmos satisfies the Boundary Condition – and I've shown that if a variety of live cosmological models are true, then the Cosmos does not satisfy the Boundary Condition. While we do not have a physical theory which, when conjoined with any data we currently have, deductively entails that the Cosmos satisfies the Modal, Direction, or Boundary Conditions, why can't we use an inductive argument to somehow infer from conditions in our local cosmological environment whether the Cosmos satisfies the Modal, Direction, and Boundary Conditions?

In this chapter I complete the case for Cosmic Skepticism by turning to Confirmation Theory. I consider two distinct kinds of inductive arguments, that I call part-to-part inferences and part-to-whole inferences, that might be used in inferring that the Cosmos satisfies either the Modal, Direction, or Boundary Conditions. First, I consider part-to-part inferences, according to which we project some set of empirical regularities gathered from the portion of the Cosmos empirically accessible to us into a portion of the Cosmos that is not accessible to us. We then use the empirical regularities that we've projected into that domain to infer whether the Cosmos satisfies the Modal, Direction, or Boundary Conditions. I will show that, at least in our present stage of scientific and philosophical inquiry, part-to-part inferences fail because the portions of the Cosmos relevant for whether the Cosmos satisfies the Modal, Direction, and Boundary Conditions bare only a weak analogy to the portions that are empirically accessible to us and because we have no good reason to believe that the portion of the Cosmos empirically accessible to us is representative of the entire Cosmos.

Second, I consider part-to-whole inferences. Part-to-whole inferences project an empirical regularity from the portion of the Cosmos accessible to us to the Cosmos as a whole in order to infer whether the Cosmos satisfies the Modal, Direction, or Boundary Conditions. Part-to-whole inferences require an inductive generalization. I will argue that, at the present stage of philosophical and scientific inquiry, there is an unresolved tension between how to adjudicate the tension between the modesty and the coherency of a hypothesis. The tension between the modesty and the coherency of a hypothesis is particularly acute for large scale inductive generalizations; a fortiori for the inductive generalization involved in part-to-whole inferences, which requires an inductive generalization over the whole of physical reality.

After discussing the two kinds of inductive arguments that might be involved in inferring that the Cosmos satisfies the Modal, Direction, and Boundary Conditions, I briefly discuss a puzzle about whether any sort of inductive inference could tell us whether the Cosmos satisfies the Modal Condition. Conceivably, depending upon how future physical inquiry proceeds, there may be no way, even in principle, to tell whether the

Cosmos satisfies the Modal Condition and so no way to determine whether the Cosmos began to exist.

Lastly, I reply to four objections. First, I consider an objection according to which we should reason about cosmological hypotheses in terms of inference to the best explanation and not in terms of inductive arguments. On one hand, there are philosophers who have maintained that inference to the best explanation is reducible to induction. On the other hand, as I show, there are good reasons for thinking that inference to the best explanation over sufficiently large domains are beset by the tension between modesty and coherence. Second, I discuss an objection according to which our best scientific theories require laws of nature with global scope. I reply that the laws appearing in our best scientific theories should not be understood as having global scope. Third, I discuss an objection according to which there are successful inductive generalizations over infinite domains in mathematics. I discuss an explicit example and show that the example does not include an inductive generalization over an infinite domain. Fourth, I discuss an objection according to which natural theologians are able to make an inductive generalization over an infinite domain when they infer that God is omnipotent. I show that it's far from clear whether the inductive generalization utilized by natural theologians succeeds.

12.2 Part-to-Part Inferences

Long before the development of astronomy as a mature scientific discipline, our ancestors could infer that the Sun will rise on the following day. Our ancestors had observed that there is a strong correlation between mornings and Sun rises and they could project that correlation to the following day; that is, since the following day also includes a morning, the following day would also include a Sun rise. A structurally similar inference plays a central role in David Hume's *Dialogues Concerning Natural Religion*. Hume's character Cleanthes observes that there is a correlation between systems that exhibit a specific kind of order and systems that were designed. Since, according to Cleanthes, the universe also exhibits that kind of order, the universe was also the product of design. Both arguments are members of a family of arguments that have the following form (as described in Draper, 1991, p. 136):

P1) There is an empirical correlation between class A and class B.

P2) k is a member of A.

C) Therefore, k is a member of B.

Hume calls arguments with this form "arguments from experience"; since there are other kinds of arguments that likewise have their basis in experience, I will refer to arguments of this kind as *analogical arguments from experience*. The argument is said to be *analogical* because the argument is based on an analogy between As that were observed in the past and the new instance of an A. In Cleanthes's design argument, for example, there is an analogy between previous ordered entities that were observed to be designed, e.g., houses, and the new entity, e.g., the universe.

One family of arguments that might be used in arguing for the view that the Cosmos has a beginning, that I will call *part-to-part inferences*, involve an analogical argument from experience that projects empirical regularities from the observable portion of the Cosmos into a portion of the Cosmos to which we do not have direct observational access. Part-to-part inferences then use those empirical regularities to draw the conclusion that the portion of the Cosmos into which those empirical regularities have been projected either involves the Cosmos's beginning or features important for establishing the Cosmos's beginning. For illustrative purposes, set aside the fact that General Relativity is likely to be supplanted by a quantum gravity theory in future physical inquiry. How would physicists infer that some particular epoch in the history of the observable universe – for example, the portion of the observable universe approximately 14 or so billion years into our past – includes a temporal boundary before which nothing at all existed? Since we've observed other contexts in which the Einstein Field Equations are confirmed by our observations, we might project the Einstein Field Equations approximately 14 billion years into our past and conclude that there was a boundary before which nothing at all existed. In a subsequent section, I will discuss more sophisticated examples of part-to-part inferences.

In general, we can think about how we would evaluate any particular portion of the Cosmos as a candidate for including the Cosmos's beginning. The inference that any portion of the Cosmos's history includes the Cosmos's beginning would involve two stages. At the first stage, we project previously confirmed laws into a new domain. At this stage, we use an analogical argument from experience to project some set of empirical regularities into a new domain. At the second stage, we evaluate the probability that the set of empirical regularities confer on to the hypothesis that that portion of the Cosmos includes the Cosmos's beginning or features relevant for inferring whether the Cosmos has a beginning. I will argue that we have good reason for thinking that, given our current stage of physical inquiry, there is no way for us to produce a successful first stage of a part-to-part inference.

An argument is said to be *deductively valid* just in case there is no possible world where the argument's premises are true but the argument's conclusion is false. Analogical arguments from experience are not deductively valid; there are possible worlds where there is an empirical correlation between *A*s and *B*s, but the next observed instance of an *A* is not a *B*. For example, if powerful aliens destroy the Sun tonight, then the Sun will not rise tomorrow. Nonetheless, analogical arguments from experience confer a probability on their conclusions. If the probability an analogical argument from experience confers on the conclusion is high, then the analogical is *strong*; if the probability an analogical argument from experience confers on the conclusion is low, then the argument is *weak*. An analogical argument from experience is said to be *midling* just in case the argument is neither strong nor weak. In order to establish that we should be skeptical of part-to-part inferences, I will argue that the analogical argument from experience involved is at least not strong. And to do that, I need to first identify the criteria in virtue of which analogical arguments from experience can be said to be strong. In the next section, I turn to discussing the relationship that evidence generally bears to hypotheses.

12.2.1 The Strength of Analogical Arguments from Experience

In his *Treatise* (2005), Hume discusses two ways in which an analogical argument from experience might be weakened.[1] There are two corresponding ways that an analogical argument from experience can be strengthened. Recall that analogical arguments from experience depend upon an observed correlation between *A* and *B*. For example, in his design argument, Cleanthes appeals to the correlation between entities exhibiting a specific kind of order (*A*) and entities that were designed (*B*). Good analogical arguments from experience are based on a strong correlation. A correlation's strength is determined by how many confirming instances have been observed (that is, how many *A*s have been observed to be *B*s) and by how many disconfirming instances have been observed (that is, how many *A*s have been observed to be non-*B*s) (Hume, 2005, p. 90). For example, there is a strong correlation between periods of time that included mornings and periods of time that were observed to include Sun rises because no morning has been observed that did not include a Sun rise. The second way in which an analogical argument from experience might be strengthened is in terms of the analogy involved in the argument (Hume, 2005, p. 97). There is a strong analogy between previous periods of time that included mornings and tomorrow because we have no reason to think that tomorrow will differ in any relevant way from past mornings.

Hume's criteria do not exhaust all of the criteria that make for strong analogical arguments from experience (Draper, 1991). For example, arguments that use a sample to make an inference about unobserved members of the population confer a high probability on to their conclusion only if we have good reason to think that the sample is representative of the entire population. For example, an analogical argument from experience that draws its conclusion solely based on a biased sample cannot strongly support its conclusion. For that reason, the strength of an analogical argument from experience depends on whether we have good reason to believe that our past sample of *A*s is representative of all the *A*s that there are. Let's say that a confirming instance is an *A* that is observed to be a *B* and a disconfirming instance is an *A* that is observed to not be an *A*. Likewise, a positive

[1] ↑My interpretation of Hume in this chapter is based largely upon Paul Draper's (1991).

instance is an A that is a B, regardless of whatever we've observed, and a negative instance is an A that is not a B, again regardless of what we've observed. In that case, a sample is said to be representative just in case the number of disconfirming instances d divided by the number of confirming instances c is approximately equal to the number of negative instances n divided by the number of positive instances p, i.e., $d/c \approx n/p$. We have good reason to think that a sample is representative just in case have good reason to think that $d/c \approx n/p$.

The strength of the correlation, the degree of analogy, whether we have good reason to think the sample is representative, and possibly other criteria determine the degree to which the premises in an analogical argument from experience raise the probability of the conclusion. According to the perspective that I assume in this chapter, as inquirers gather data about their world, they update the epistemic probabilities that they assign to various hypotheses. Bayes's Theorem[2] can be used to express the probability of a hypothesis h in terms of the evidence e and background knowledge K:

$$Pr(h|\mathrm{e}\&K) = \frac{Pr(h|K)Pr(\mathrm{e}|h\&K)}{Pr(\mathrm{e}|K)} \tag{12.1}$$

Using Bayes's Theorem, we can partition $Pr(h|\mathrm{e}\&K)$ into two parts. First, there is the ratio $Pr(\mathrm{e}|h\&K)/Pr(\mathrm{e}|K)$, which we can think of as the contribution that our evidence makes to $Pr(h|\mathrm{e}\&K)$. Second, there is the *prior probability* $Pr(h|K)$, which expresses the probability of h solely in light of our background knowledge and independent of our evidence. According to a (perhaps overly) simplified model, as we learn about the world we inhabit, successive propositions are conjoined to our background knowledge. We can roughly think of the prior probability as the probability that a hypothesis has before we update the hypothesis in light of new evidence. Let's define the *intrinsic probability* of h as the probability that

[2]↑Bayes's Rule should be distinguished from Bayes's Theorem. Bayes's Theorem is a deductive consequence of the axioms describing orthodox probability theory (i.e., the Kolmogorov axioms). Bayes's Theorem states that, for any three sentences A, B, and C, $Pr(A|B) = Pr(A)Pr(B|A)/Pr(B)$. In contrast, Bayes's Rule is a statement about how we ought to update our epistemic probability in light of evidence relative to our background knowledge. Bayes's Rule tells us that the epistemic probability of h after we have gathered evidence e, $Pr_f(h)$, can be found from the initial probability of h, given e and K, i.e., $Pr_i(h|\mathrm{e}\&K)$, using Bayes's Theorem, i.e., $Pr_f(h) = Pr_i(h|\mathrm{e}\&K)$. I make use of the relevance criterion of confirmation and Bayes's Theorem in modeling scientific reasoning, and both principles are consistent with Bayes's Rule, but neither principle requires Bayes's Rule.

h has relative only to tautologous information. In that case, the intrinsic probability can roughly be thought of as the probability with which h begins prior to any empirical investigation whatsoever.

There are two additional criteria that determine whether we have good reason to believe the conclusion of an analogical argument from experience. First, since $Pr(h|e\&K)$ is determined, in part, by $Pr(h|K)$, $Pr(h|e\&K)$ will be high if either $Pr(h|K)$ is not too low or else the evidence is sufficiently surprising relative to our background knowledge.[3] Since the value of $Pr(h|K)$ is determined by the degree to which h comports with our background knowledge, $Pr(h|e\&K)$ is high only if h either comports with our background knowledge or the evidence for h is sufficiently strong so as to overcome the tension between h and K. Moreover, since $Pr(h|K)$ is determined, at least in part, by the intrinsic probability of h, $Pr(h|e\&K)$ will be high only if the intrinsic probability of h is not too low. Moreover, as I will discuss in a subsequent section, according to Draper's theory of intrinsic probability, the intrinsic probability of a hypothesis is determined by the modesty of the hypothesis, roughly, how much the hypothesis claims about the world, and the coherence of the hypothesis, roughly, the degree to which the parts of the hypothesis are mutually supportive.

Let's call the conjunction of the criteria that determine the strength of an analogical argument from experience the *Strength Criteria*. In sum, the Strength Criteria conjoin the strength of the correlation, the degree of analogy, whether we have good reason to think the sample is representative, how well the hypothesis comports with our background knowledge, how surprising the evidence is relative to our background knowledge, the modesty of the hypothesis, the coherence of the hypothesis, and possibly other criteria. I do not claim that any one criterion in the Strength Criteria is logically independent of the others. Instead, I claim only that we should think about the strength of analogical arguments from experience in terms of the Strength Criteria.

[3]↑According to Bayes's Theorem, $Pr(h|e\&K) = Pr(h|K)Pr(e|h\&K)/Pr(e|K)$. If $Pr(h|K)$ is low, then, in order for $Pr(h|e\&K)$ to be large, $Pr(e|h\&K)/Pr(e|K)$ must be large, which requires that $Pr(e|K)$ be low. Thus, if e is sufficiently surprising relative to our background knowledge, then, despite h having a low prior probability, h might have a large posterior probability.

12.2.2 What are Part-to-Part Inferences?

As I've said, a part-to-part inference involves two stages. The first stage involves projecting an empirical regularity into a novel domain using an analogical argument from experience. Recall my previous illustrative example involving classical relativistic cosmology. We might project the Einstein Field Equations into a portion R of the Cosmos's history. The second stage involves inferring from that empirical regularity that the novel domain includes the Cosmos's beginning. For example, we might use the Einstein Field Equations to deduce that R includes the Cosmos's temporal boundary.

Any given state of a physical system can be characterized by some number of parameters. Call the space of all of the possible states of a physical system a *parameter space*. For example, the state of an ideal gas can be completely specified by the pressure, the volume, and the temperature of the gas. If we construct a three-dimensional space where the three axes represent the pressure, the volume, and the pressure, then any ideal gas can be represented by a point in that space. When we project an empirical regularity into a novel domain, we are projecting that regularity from parts of a parameter space that we have previously studied into a portion of that parameter space that has not been previously studied; for example, in the case of an ideal gas, we might project a feature of the gas, e.g., the temperature, at one value of the pressure to another value of the pressure. In order to formalize the argument from the first stage in a part-to-part inference, suppose that we have a correlation between being a proper part of a parameter space and being such that some empirical regularity L applies. Call that correlation the Principle Correlation. Let h represent the statement that L applies in some novel domain F. Let e be the statement that F is a proper part of the parameter space. Let K be our background knowledge, which includes the Principle Correlation. Consequently, the first stage of any part-to-part inference can be represented as the following analogical argument from experience:

P1) There is an empirical correlation between being a proper part of a parameter space $\mathbb{P}$ and being such that L applies.

P2) F is a proper part of $\mathbb{P}$.

C) Therefore, *L* applies in *F*.

Whether any argument of this form is strong will depend upon the probability conferred by the premises on the conclusion, that is, $Pr(h|e\&K)$. In turn, $Pr(h|e\&K)$ will depend, among other criteria, upon whether the Strength Criteria are satisfied. In the next subsection, I will argue that we have good reason to think that the Strength Criteria are not satisfied for the first stage of Part-to-Part Inferences.

12.2.3 The Problem for Part-to-Part Inferences

In order to present an argument that, at the present stage of scientific and philosophical inquiry, no part-to-part inference succeeds, let's begin by considering a passage from Philo's response to Cleanthes's design argument. Philo states,

> Nature, we find, even from our limited experience, possesses an infinite number of springs and principles which incessantly discover themselves on every change of her position and situation. And what new and unknown principles would actuate her in so new and unknown a situation as that of the formation of a universe, we cannot, without the utmost temerity pretend to determine.
>
> [A very small part of this great system, during a very short time, is very imperfectly discovered to us: And do we then pronounce decisively concerning the origin of the whole?] (Hume, 2008, pp. 50–51)

In this passage, Philo is responding to an analogical argument from experience that Philo implicitly attributes to Cleanthes. The argument that Philo implicitly attributes to Cleanthes begins with some set of empirical regularities, i.e., principles, and then projects those principles into a situation involving the formation of the universe.

What does Philo mean by a *situation*? Philo seems to allow that the universe occupies a specific region in space and time; for example, as part of a parody of Cleanthes's design argument, Philo imagines that comets are analogous to seeds and can sprout in the "unformed elements" that "everywhere surround this universe" (Hume, 2008, p. 79).[4] So,

[4]↑Unlike contemporary authors who may be influenced by relativistic cosmology, a variety of early modern authors distinguished between the universe (or the material world) and the totality of space and time.

let's suppose that a situation is a space-time region where a universe might form. In that case, the argument that Philo implicitly attributes to Cleanthes is based on a correlation between being a situation such that principle P applies (A) and being a space-time region (B). Let's call this correlation the Principle Correlation. Moreover, the argument attempts to establish the hypothesis that P applies in situation S, uses the evidence that S is a space-time region, and assumes that our background knowledge includes the Principle Correlation.

Since the argument that Philo implicitly attributes to Cleanthes is an analogical argument from experience, we can evaluate the argument that Philo implicitly attributes to Cleanthes in light of the Strength Criteria. Since Hume recognized only two of the criteria – that is, the strength of the correlation and the degree of analogy – Philo's counterargument must be in terms of those two criteria. When Hume discusses *principles*, Hume is roughly referring to what we might call *laws of nature*.[5] And as Hume notes in his essay on miracles, i.e., Hume, 1992, pp. 107–131, our past experience consistently and strongly

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According to Edward Harrison (1986) and Edward Grant (1969), at the start of the seventeenth century, there were three live cosmological models: (i) an Aristotelian model, according to which there is a set of geocentric celestial spheres bounded by the fixed stars, (ii) a stoic model, according to which there are a finite collection of stars in one portion of an infinitely large space, and (iii) an Epicurean model, according to which there is infinite matter strewn throughout infinite space.

While the Aristotelian model collapsed during the seventeenth century, the notion of an infinitely large space or time beyond the material universe continued to be discussed into the eighteenth century (Grant, 1969). Immanuel Kant, Gottfried Leibniz, and Samuel Clarke explicitly discuss the possibility that there is infinite space or time beyond the material universe. In the antinomies, Kant (2009, p. 471) considers (and rejects) an argument that the "world" could not be finitely old or finite in size. As part of that argument, Kant assumes that if the world were finitely old and of finite size, then the world would be proceeded by empty time, that is, time without a material universe, and surrounded by empty space. In the Leibniz/Clarke correspondence, Leibniz (1956, p. 25) criticizes absolute space on the grounds that, while the material world was created by God, absolute space is co-eternal with God and uncreateable. Clarke (S. Clarke, 1956, p. 31) replies that absolute space is not an eternal being but instead an eternal consequence of God being infinite and eternal. That is, Clarke concedes that while the material world has a finite past, absolute space does not have a finite past. Leibniz (1956, p. 26) also criticizes absolute space on the grounds that if space were absolute then God couldn't have had a sufficient reason for creating the material world at a specific location in space. Likewise, Leibniz (1956, pp. 26–27) criticizes absolute time on the grounds that if time were absolute then God couldn't have had a sufficient reason for creating the material world at a specific moment in infinite time. Clarke (1956, p. 32) disagrees, but continues to insist that God created the material world at a specific location in space and at a specific moment in time.

[5]↑For example, Hume states that the "secret springs and principles" that explain the functioning of the mind are analogous to the "laws and forces" that Newton discovered to govern and direct the "revolutions of the planets" (Hume, 1992, p. 14). Elsewhere, Hume tells us that elasticity, gravity, the cohesion of parts, and the communication of motion by impulse are likely the most "ultimate" principles humans will discover (Hume, 1992, p. 30).

confirms the laws of nature and includes no exceptions to the laws of nature: "[...] as a firm and unalterable experience has established these laws, the proof against a miracle [that is, against a violation of a law of nature], from the very nature of the fact, is as entire as any argument from experience can possibly be imagined" (Hume, 1992, p. 114). Thus, the Principle Correlation includes a large number of confirming instances and no disconfirming instances. However, Philo rejects the argument that he implicitly attributes to Cleanthes. Since Hume judges the strength of analogical arguments from experience solely in terms of the strength of the correlation and the degree of analogy, Philo should be understood as rejecting the argument either for involving a poor correlation, or for involving a weak analogy, or for both reasons. Since the correlation is strong, Philo should be interpreted as rejecting the argument for including a poor analogy.

In the quoted passage, Philo points out that as we investigate nature, we find that new principles apply to domains that are further removed from our prior investigations, i.e., nature "possesses an infinite number of springs and principles which incessantly discover themselves on every change of her position and situation". Philo encourages us to look back on the history of physical inquiry. Suppose we look back to a time t hundreds of years into the past. In the intervening centuries, our ancestors investigated then new domains that had not yet been investigated and found that those then new domains were correctly described by principles that were unknown at t. Thus, according to Philo, when we investigate new domains, we should expect to discover phenomena described by principles that were previously unknown and that are consequently disanalogous to our previous confirming instances. Since a situation that involves the formation of the universe is so far removed from our prior experience, Philo argues that we have reason to think that situations involving the formation of a universe are disanalogous to situations with which we have prior experience. In that case, we have reason to doubt the projection of previously discovered principles into situations involving the formation of a universe.

We can strengthen Philo's argument. First, Philo presents a kind of meta-inductive argument over the course of past physical inquiry in order to establish that we should expect new principles in situations that are sufficiently far removed from the situations we've previously investigated. Philo's meta-inductive argument is philosophically contentious. For that reason, if Philo can construct his argument without the meta-inductive argument, then Philo's argument can do without a philosophically controversial premise. Happily, Philo doesn't need the meta-inductive argument in order to argue that situations involving the formation of a universe are far removed from previous situations that we've investigated. Instead, all that Philo needs to reach his conclusion is that situations involving the formation of a universe are disanalogous from previous situations that we've investigated in virtue of involving the formation of a universe. Since the strength of analogical arguments from experience is determined by the degree of analogy between the previous confirming instances and the new instance, Philo merely needs the fact that situations involving the formation of the universe are not analogous to any of the instances that previously confirmed the empirical regularities to which we have access.

Second, without the meta-inductive argument, Philo can point out that we have no good reason to think that situations involving the formation of a universe only involve the principles that have been previously discovered. In addition, we have no good reason for thinking our old principles are applicable to situations involving the formation of a universe.

Third, Philo is considering an argument according to which our old principles would apply to a situation involving the formation of a universe. However, we might ask how one would establish that the situation involves the formation of a universe in the first place. Presumably, whatever features of a situation would bring us to infer that a given situation involves the formation of a universe would have to be fairly exotic features that are not shared by other situations. Suppose that the old principles do apply to a situation that might involve the formation of a universe and suppose that the old principles, when applied to the situation in question, imply that the situation likely involved the formation of a universe. As I've discussed, there may be also be new principles in that situation. While the old principles, taken in isolation, might imply that the situation in question likely includes the formation of a universe, the old principles, when conjoined with a set of new principles, might render that situation unlikely to include the formation of a universe. Thus, any argument that establishes that a given situation includes the formation of a universe would need to establish both that old principles would apply to that situation and that there won't be new principles in that situation which, when in combination with the old principles, would render the formation of a universe unlikely. If we lack justification for saying that the old principles apply to the new situation and we lack justification for saying that there wouldn't be new principles, that in combination with the old principles, render the formation of a universe unlikely, we would lack justification for concluding that the situation does include the formation of a universe. Likewise, the old principles might not apply at all to the situation in question and, instead, an altogether different set of principles might apply. In that case, the new set of principles might, unlike the old set of principles, entail that the situation does not likely include the formation of a universe. Thus, not only should Philo be doubtful of the claim that we know which principles apply to a situation that includes the formation of a universe, but Philo should also be doubtful of the claim that any given situation likely does include the formation of a universe.

Fourth, recall that, in addition to the strength of the analogy, the Strength Criteria require that we have good reason for thinking that our sample of *A*s is representative of all the *A*s that there are. Philo has no good reason to think that the situations that were previously investigated are a representative sample of all of the situations that there are. For all that Philo knows, the principles that he has access to represent a provincial portion of the universe that would have no relevance for situations that include the formation of a universe.

So far, we've been discussing the issue in fairly abstract terms. In Hume's time, relativistic cosmology had not yet been discovered and few, if any, people could have imagined that a set of empirical regularities could themselves entail, or even make probable, that a given situation involves the formation of a universe. Moreover, while Philo seems to take the universe to be a proper part of physical reality – since Philo speculates that comets, like seeds, might somehow germinate into universes in the chaos between universes – I am interested in whether the totality of physical reality had a beginning. As I stated at the outset of this chapter, I endorse a view that I call *Cosmic Skepticism*, according to which the provinciality of our knowledge of the physical facts with respect to scale, spatio-temporal location, or energy prevents us from having empirical access to whether the Cosmos satisfies the Modal, Direction, and Boundary Conditions. If Cosmic Skepticism is true, then, since our knowledge of the physical world is provincial with respect to scale, spatio-temporal location, and energy, and the best candidates for portions of the Cosmos that (for example) include a space-time-wide temporal boundary – such as the hot, dense period in the history of the observable universe located approximately 14 billion years to our past – are exotic with respect to scale, spatio-temporal location, and energy, our best candidates are not analogous to the situations in which our strongest empirical regularities have been confirmed. In addition, since our knowledge of the physical facts is provincial with respect to scale, spatio-temporal location, and energy, we have no good reason for thinking that the domains in which we've previously investigated the physical facts are representative of all of the physical facts that there are. For example, physicists widely suspect that at sufficiently high energies, General Relativity needs to be replaced by a quantum gravity theory. In that case, we should not project General Relativity 14 billion or so years into our past and draw the inference that the observable universe includes a past temporal boundary. In the following two subsections, I consider in more detail why, at our present stage of philosophical and scientific inquiry, we should doubt the analogical argument from experience that features in specific part-to-part inferences.

Projections involving distant energy scales and densities

In this subsection, I examine a part-to-part inference that projects physical principles from observable energy scales or matter-energy densities to energy scales or matter-energy densities that vastly exceed the observable scales or matter-energy densities.

In order to have a beginning, the Cosmos must satisfy the Boundary Condition. As I've said, the only scientifically respectable candidate for the Cosmos's past boundary is the Big Bang, that is, the hot, dense epoch in the history of the observable universe located approximately 14 billion years in our past. When we turn back the clock and approach the boundary postulated in classical models of the Big Bang, we find that the matter-energy density within the observable portion of the universe grows without bound. Physicists strongly suspect that at sufficiently high energies, General Relativity will be supplanted by a still as yet undeveloped quantum gravity successor theory. For that reason, we should not project General Relativity to arbitrarily high energy scales or arbitrarily high matter-energy densities. But set that aside; there are three independent reasons why we should not project General Relativity to arbitrarily high energy scales or arbitrarily high matter-energy densities. In fact, the three independent reasons help to explain why physicists sometimes claim that General Relativity predicts its own demise (Burger et al., 2018; Cuttell, 2019, pp. 2, 14; Israel, 2018, p. 115) and specifically predicts its own inapplicability to the Big Bang.

First, the degree of analogy between observed matter-energy densities and unobserved matter-energy densities decreases with the difference between the two. As we approach the Big Bang, according to General Relativity, the matter-energy density grows without upper bound. Thus, as we approach the Big Bang, the degree of analogy approaches zero; for that reason, General Relativity, itself, provides us reason to doubt General Relativity's application to the Big Bang. Second, we have no reason to think that the physics that we know of is representative, with respect to the matter-energy density, of all of the physics that there is. Our sample of known physics is biased because we have only been able to sample the finite range of matter-energy densities available in terrestrial experiments or in astrophysical observations. If the matter-energy can grow without bound, then there is an infinite range of physical phenomena that we have not been able to sample; moreover, in order to understand the Big Bang, we require physics from the portion that we have not yet sampled. Thus, the only scientifically respectable candidate for the portion of the Cosmos that might include the Cosmos's boundary is shrouded in just those conditions for which we should exercise the greatest degree of skepticism.

Projections into future physical theories

In this subsection, I examine a family of part-to-part inferences that project features of past physical theories into future physical theories in the attempt to establish that the Cosmos satisfies the Modal Condition. In chapter 5, we saw that there were several distinct reasons for considering the possibility that the Cosmos violates the Modal Condition. For example, several current proposals for quantum gravity theories postulate that space-time is not fundamental. If space-time is not fundamental, then the Cosmos might turn out to be fundamentally timeless. If the Cosmos is fundamentally timeless then the Cosmos did not begin to exist. As I discussed, there are live candidates for quantum gravity theories that have been argued to have the consequence that the Cosmos is fundamentally timeless. In other words, if some specific candidates for quantum gravity theories, together with some specific interpretations of those theories, turn out to be correct, then the Cosmos violates the Modal Condition and is therefore beginningless.

Therefore, in order to evaluate whether the Cosmos is fundamentally timeless, we would need to know which features of our current scientific theories will survive into those theories that will supplant our current theories. We don't yet know what those features will be. Perhaps one could propose an analogical argument from experience using the features of past and current theories to the features of future theories, but any such projection seems doubtful. Alternatively, one could attempt to argue that all physical entities are fundamentally spatio-temporal by projecting from the collection of physical entities that have been subsumed under past and current theories to the class of physical entities that will be subsumed by our final theory at the end of physical inquiry. Recall again that analogical arguments from experience require, among other criteria, a strong degree of analogy and good reason for thinking that our sample is representative. We have no good reason to think that future physical theories will have a high degree of analogy, in relevant respects, to past and current physical theories nor do we have reason to think that our collection of past and current physical theories are representative of all of the physical theories that will ever be developed.

There is good reason to think that quantum gravitational effects become relevant at high energies. For that reason, there are quantum gravity theories which, if true, would entail that a spatio-temporal description of physical phenomena no longer applies at a some sufficiently large mass-energy density, e.g., the Plank energy. Friends of the view that the Cosmos had a beginning may try to use an analogical argument from experience to rule out the possibility of a mass-energy density at which a spatio-temporal description no longer applies. I'm not entirely sure how such an argument would work in detail, but, at our present stage of philosophical and scientific inquiry, we should be doubtful of any analogical argument from experience that attempts to rule out the possibility of a mass-energy density at which a spatio-temporal description no longer applies. There is no good reason to think that there is a high degree of analogy between previously observed physical phenomena and phenomena at the Planck energy. For example, the most energetic particle ever observed – the Oh-My-God particle – was a cosmic ray whose energy was approximately 10^9 times smaller than the Planck energy (Bird et al., 1995). Second, we have no reason to think that the physics we know of is representative of all of the physics that there is. Even if we supposed that the spatio-temporal description was applicable up through the Planck energy, there may be a larger energy scale at which the spatio-temporal description is no longer applicable.

Moreover, as I will subsequently argue, there is a tension between the coherence and the modesty of a hypothesis. While I will argue that the coherence of a hypothesis depends on the degree of objective uniformity the hypothesis attributes to the world, modesty depends upon the scope of a hypothesis. Thus, there is a tension between modesty and coherence because hypotheses that attribute uniformity over a large portion of the world have a large scope. The inference that a spatio-temporal description applies at a large range of energy scales attributes objective uniformity to the world but is also immodest. Since we currently lack a theory that would allow us to adjudicate the tension between modesty and coherence, we are not in a position to judge whether the inference that the spatio-temporal description applies at a large range of energy scales has a high or a low intrinsic probability.

One might try to project results into a future physical theory by appealing to the fact that our current physical theories will be approximations to future physical theories. For example, recall that, in chapter 3, I described Craig and Sinclair's view that, "There may be no such things as singularities per se in a future quantum gravity formalism, but the phenomena that [General Relativity] incompletely strives to describe must nonetheless be handled by the refined formalism, if that formalism has the ambition of describing our universe" (Craig and Sinclair, 2012, p. 106). Craig and Sinclair go on to assert that the singularities which appear in relativistic cosmology will be replaced by a suitable equivalent in a quantum gravity formalism, so that quantum gravity will continue to entail that space-time includes a past boundary.

However, as Jeffrey Barrett (2003, 2008) points out, if we grant that empirically successful scientific theories are approximations to their successors, then, at any given point in scientific inquiry, we don't know what the sense is in which our current scientific theories will approximate their successors. This is so for three reasons. First, as Barrett (2003) argues, we cannot identify the bridge principles that link current scientific theories to their successors until after the successors have been successfully identified. Barrett (2003) uses the example of Newtonian gravity *NG* and the general theory of relativity *GTR*; the theory that best captures the bridge principles between *NG* and *GTR* is another theory, namely, geometrized Newtonian gravity *GNG*. But *GNG* is constructible only in retrospect after having found *GTR*. Since *GNG* is available only in retrospect, while *GNG* now allows us to say, in precise terms, what is preserved from *NG* to *GTR*, no one prior to the development of *GTR* could have said what would be preserved in *NG*'s successor theory. Second, as Barrett (2008) argues using the Dirac/von Neumann formulation of quantum mechanics, before the successor theory to a current theory is established, there may be a variety of known mutually incompatible possible successor theories. Each of the mutually incompatible possible successor theories might retain different features of the original theory, so that, even if we accept that one of the known possible successor theories will eventually be established as the correct successor theory, the space of mutually incompatible possible successor theories might not provide any guidance as to which features will be retained in the successor theory. For example, some proposed successors to General Relativity, such as loop quantum gravity and string theory, have been interpreted to entail that the Cosmos is not fundamentally temporal, while other proposed successor theories, such as causal set theory (Dowker, 2020), have been interpreted as better accommodating temporal becoming, at the fundamental level, than General Relativity. Thus, whether the successor to General Relativity will entail that the Cosmos is fundamentally timeless will depend upon which quantum gravity theory is successful. As a second example, there a large number of proposed cosmological models utilizing quantum gravity theories; some proposals include a past boundary and others do not. Thus, whether a model that includes a past boundary comes to be adopted in a successor theory will depend, among other details, upon which successor theory is adopted.

Third, if we knew how to replace a current theory with a successor theory, then we would have done so; the fact that we have not yet done so at any given stage of scientific inquiry reflects the fact that we do not then know how to replace a current theory with a successor. In turn, we don't know which features of a current theory will be retained in successors. We don't know whether the feature of having, e.g., a temporal boundary or being fundamentally temporal (assuming that those are features of current theories) will be retained in successor theories.

12.3 Part-to-Whole Inferences

I said that there are two families of inferences that might be used to argue that the Cosmos has a beginning. The second family are part-to-whole inferences. In order to discuss part-to-whole inferences, I again begin by drawing inspiration from Hume's *Dialogues*. While responding to Cleanthes's design argument, Philo rhetorically asks, "But is a part of nature a rule for another part very wide of the former?" (Hume, 2008, p. 51) That is, can we project a principle from one part of nature to another distant part? Someone who answers yes and offers the corresponding inference is making the sort of inference I considered at the outset of section 12.2.3. Philo continues by rhetorically asking whether the principle is a "a rule for the whole? Is a very small part a rule for the universe?"[6] That is, can we use a portion of the universe to project an inference to the entire universe? Elsewhere, Philo states, "Our experience, so imperfect in itself, and so limited in extent and duration, can afford us no probable conjecture concerning the whole of things" (Hume, 2008, p. 79). Whereas the argument with which I began the section on part-to-part inferences concerned whether we can project from observable portions of the universe into a novel situation that involves the formation of the universe, Philo now makes a case that we cannot make a generalization from a part of the universe to the whole of the universe.

[6] ↑Whether Hume recognized a distinction between the two kinds of arguments is unclear to me. Philo asks his rhetorical questions as if they were variations on a single point instead of two wholly distinct points.

Part-to-whole inferences project an empirical regularity from a portion of the Cosmos to which we have empirical access to the Cosmos as a whole. Instead of using an analogical argument from experience, the first stage of a part-to-whole inference utilizes an *inductive generalization*. Inductive generalizations have the following form:

P1*) All observed members of class A have property Q.

C*) Therefore, all members of class A have property Q.

The first stage of a part-to-whole inference states:

P1**) All observed parts of the Cosmos are described by a set of natural laws $L_1, L_2, ...$

C**) Therefore, all parts of the Cosmos are described by a set of natural laws $L_1, L_2, ...$

In the second stage of a part-to-whole inference, the set of natural laws that are projected to the entire Cosmos are used to draw the inference that the Cosmos satisfies either the Modal, Direction, or Boundary Conditions and so as part of an argument that the Cosmos has a beginning.

For illustrative purposes, let's again set aside the fact that General Relativity will likely be replaced by a quantum gravity successor theory in future physical inquiry. We have observed that, on large scales, the observable universe is, to within a close approximation, described by the Friedmann-Lemaître-Robertson-Walker (FLRW) equations. If we could project the FLRW equations to the entirety of space-time, then we might be able to deduce from the FLRW equations that the Cosmos satisfies the Boundary Condition. Moreover, if we knew that there was a consistent direction of time throughout the entire observable universe and if we could project that consistent direction of time throughout the entirety of space-time, then we could know that the entire Cosmos satisfies the Direction Condition.

Good inductive generalizations confer a high probability on to their conclusions. In order for an inductive generalization to confer a high probability on some hypothesis h, we must have good reason for thinking that our sample of As is representative and our sample of As must be sufficiently large. For reasons similar to those already discussed in connection with analogical arguments from experience, we have no good reason for thinking that the portion of the physical world that has been investigated thus far is a representative sample of the totality of physical reality nor do we have good reason for thinking that our sample of physical reality is large relative to the totality of physical reality.[7] Moreover, there are two conditions on the probability of h imposed by the prior probability of h. First, h must either comport well with our background knowledge or else the evidence for h must be sufficiently surprisingly, relative to our background knowledge, so as to overcome the tension between our background knowledge and h. Second, since the prior probability of h depends, in part, on the intrinsic probability of h, h must not be too immodest or too incoherent. I will argue that we should be skeptical concerning the part-to-whole inferences that one might try to use as part of an argument for the Cosmos's beginning because of an unresolved tension between the modesty and the coherence of a sufficiently large scale hypothesis.[8] In the next subsection, I turn to further explicating the notion of intrinsic probability.

12.3.1 Intrinsic Probability

As a simple model, we can understand the prior probability as resulting from updating the intrinsic probability with respect to our background knowledge. Whether there are objective prior probabilities remains a matter of philosophical debate. A fortiori for objective *intrinsic* probabilities. We can strike a compromise by conceding that although *numerical* objective probabilities may be rare, hypotheses without numerical objective probabilities have objective comparative rankings that respect the probability calculus. In the following, I will use "intrinsic probabilities" as shorthand for referring to intrinsic comparative probabilities. If we accept that there are objective intrinsic probabilities, how do we assess those intrinsic probabilities? Draper's (e.g., 2015, 2017) theory of intrinsic probabilities is the conjunction of three principles:

1. The intrinsic probability of a hypothesis depends on the modesty of the hypothesis.

[7] ↑By 'portion', I do not merely mean a spatio-temporal portion. Instead, I mean the portion of the Cosmos's parameter space.

[8] ↑Draper (2015, pp. 61–63) has previously noted that what sort of conclusions one reaches in natural theology will depend upon how one thinks about the balance between modesty and coherence.

2. The intrinsic probability of a hypothesis depends on the coherence of the hypothesis.

3. The intrinsic probability of a hypothesis depends on nothing else.

In other words, in terms of intrinsic comparative probability, all else being equal, a less modest hypothesis has a lower ranking than a more modest hypothesis, and, again all else being equal, a less coherent hypothesis has a lower ranking than a more coherent hypothesis.

Why think that a hypothesis's intrinsic probability should depend only upon the modesty and coherence of the hypothesis? Draper (2017, p. 73) tells us that he has difficulty imagining what else the intrinsic probability of a hypothesis could depend upon: "If we abstract from all factors that are extrinsic to a hypothesis (e.g., all confirming and disconfirming data, arguments, perceptions, etc.), focusing only on what is intrinsic to the hypothesis, then what else could its probability depend on other than how little it says and how well what it says fits together?"

Of course, there are other philosophers who have other accounts of intrinsic probability, or at least notions close to intrinsic probability. For example, Richard Swinburne (2001, pp. 80–83; 1997, pp. 24–26) has argued that the intrinsic probability of a hypothesis depends upon the scope (i.e., the modesty) and the simplicity of the hypothesis. In reply, Draper (2015) points out that simplicity, as characterized by Swinburne, is a complex, multifaceted notion that should be reduced to coherence. Likewise, Sean Carroll (2005) has argued that simplicity should be understood in terms of a simpler notion than Swinburne's, i.e., the Kolmogorov complexity. But the Kolmogorov complexity is capable of addressing only the syntactic features of a hypothesis. As I will show in subsection 12.3.2, two hypotheses that differ with respect to whether they postulate objective uniformity or objective variety can be stated in ways that are equally syntactically complex. Thus, modesty and coherence are better conceptions of how a given hypothesis should be evaluated independent of our evidence or background knowledge. In the next two subsections, I turn to explicating modesty and coherence.

Modesty

Roughly, the modesty of a hypothesis tells us how much the hypothesis asserts about the world beyond what we know by rational intuition alone (Draper, 2017, p. 70). For example, hypotheses with a narrower scope are more modest than hypotheses with broader scope. Intuitively, the more that a given hypothesis claims about the world, the greater the number of ways there are for the hypothesis to be false. For example, all else being equal, the hypothesis that all bank tellers are red haired men is more immodest than the claim that all bank tellers are men. This is so because the claim that all bank tellers are red haired men is more specific than the claim that all bank tellers are men. All else being equal, more immodest hypotheses are less intrinsically probable.

Coherence

The coherence of a hypothesis is how well the parts of the hypothesis intrinsically fit with one another (Draper, 2015, p. 53, Draper, 2017, pp. 70–71). The greater the degree to which the parts of a hypothesis support each other, relative to rational intuition, the greater the coherence of the hypothesis. For example, if h is the hypothesis that $A\&B$, where, purely by rational intuition, we can see that A entails B and that B entails A, then no modification to h, short of removing a conjunct, would render h more coherent. In the case that A does not entail B and B does not entail A, the coherence of $A\&B$ depends on the degree to which, relative to rational intuition alone, B predicts A and A predicts B. Thus, the coherence of $A\&B$ is determined by the degree to which, relative to rational intuition alone, A would provide evidence for B – that is, the degree to which A would raise the probability of B – and vice versa.

12.3.2 The Tension Between Modesty and Coherence

In this section, I introduce the problem that intrinsic probability poses for part-to-whole inferences. Recall that the coherence of a hypothesis is determined by the degree to which the parts of the hypothesis support each other. When the fact that one part of nature has some feature F raises the probability that another part of nature also has feature F, I will say that there is an inductive support relation between the two parts. If there is an inductive support relation between two parts of nature, then the hypothesis that both parts have that feature will be more coherent than the hypothesis that only one part has that feature. I will argue that, assuming Draper's account of intrinsic probability is correct and that inductive generalization is reliable, each part of nature has an inductive support relation with neighboring parts of nature. If each part of nature does have an inductive support relation with neighboring parts of nature, then hypotheses that postulate objective uniformity extending over the entire Cosmos are as coherent as hypotheses about physical reality could possibly be. On the other hand, hypotheses concerning the entirety of the Cosmos are as immodest as hypotheses about physical reality could possibly be. Thus, asssuming Draper's account of intrinsic probability and that inductive generalization is reliable, there is a tension between modesty and coherence that grows with the scope of a given hypothesis and which is maximized for hypotheses about the whole of the Cosmos. Part-to-whole inferences require us to project a set of empirical regularities from a specific portion of the Cosmos to the Cosmos as a whole. Thus, while the hypotheses that feature in part-to-whole inferences are as coherent as hypotheses about physical reality could be, they are also as immodest as hypotheses about physical reality could be. There is, as yet, no known way to adjudicate the tension between modesty and coherence. And since the tension between modesty and coherence is particularly acute for part-to-whole inferences, at our present stage of philosophical and scientific inquiry, we have reason to be skeptical of part-to-whole inferences.

In order to show that the reliability of inductive generalization depends on the existence of inductive support relations between distinct parts of nature, I turn to considering a thought experiment. (Similar thought experiments are provided in Draper, 2015 and Draper, unpublished.) Let's suppose that we are in the epistemic situation of our Neolithic ancestors with respect to Sun rises – that is, we do not have any sophisticated astronomical knowledge – and consider the following two hypotheses:

1. RISES := All mornings include a Sun rise.

2. NISES := All mornings before 2050 include a Sun rise and all mornings after 2050 will not include a Sun rise.

Let a Sun nise be an event before 2050 in which the Sun rose or an event after 2050 that will not include a Sun rise. Then NISES can be re-written as:

2.* NISES := All mornings include a Sun nise.

We can construct an inductive generalizations whose conclusion is RISES.

R1) All observed mornings included a Sun rise.

RC) Therefore, all mornings include a Sun rise.

We can likewise construct an inductive generalization whose conclusion is NISES:

N1) All observed mornings included a Sun nise.

NC) Therefore, all mornings include a Sun nise.

Both inductive generalizations have true premises, but, if inductive generalization is reliable, then the conclusion of the first argument must be more probable than the conclusion of the second argument. The challenge is to determine why we should think that the conclusion of the first inductive generalization is more probable than the conclusion of the second inductive generalization. As I've said previously, the probability of a hypothesis in light of the evidence is determined by Bayes's Theorem:

$$Pr(h|\mathrm{e}\&K) = \frac{Pr(h|K)Pr(\mathrm{e}|h\&K)}{Pr(\mathrm{e}|K)} \tag{12.2}$$

As I noted previously, we can partition $Pr(h|\mathrm{e}\&K)$ into two parts. We can think of the ratio $Pr(\mathrm{e}|h\&K)/Pr(\mathrm{e}|K)$ as the contribution that our evidence makes to $Pr(h|\mathrm{e}\&K)$ and we can think of the *prior probability* $Pr(h|K)$ as the probability of h relative only to our background knowledge and independent of our evidence. Since the evidence is predicted equally well by RISES and NISES, we have that:

$$Pr(\text{e}|\text{RISES}\&K) = Pr(\text{e}|\text{NISES}\&K) \tag{12.3}$$

But since I've assumed that RISES is more probable, given the evidence, than is NISES, we have that:

$$Pr(\text{RISES}|\text{e}\&K) > Pr(\text{NISES}|\text{e}\&K) \tag{12.4}$$

If we apply Bayes's Theorem to both sides of inequality 12.4 and then utilize equation 12.3, we arrive at:

$$Pr(\text{RISES}|K) > Pr(\text{NISES}|K) \tag{12.5}$$

That is, the reason RISES has a greater epistemic probability than NISES must be due to RISES's prior probability.

The prior probability is determined by our background knowledge and by the intrinsic probability. Since the difference in the prior probabilities is unlikely to be due to our background knowledge – in the absence of astronomical knowledge, what information could our background knowledge contain that would distinguish between RISES and NISES? – the difference must be explained by the difference in the intrinsic probabilities of the two hypotheses. That is, RISES has a greater prior probability than NISES because RISES has a greater intrinsic probability than NISES. As I've said, according to Draper's account of intrinsic probability, the relative intrinsic probability of the two hypotheses are determined by their relative modesty and coherence. Recall that the coherence of a hypothesis is determined how well the parts of a hypothesis support one another. Suppose that, all else being equal, one part of nature having a specific feature raises the probability that other parts of nature also have that feature. In that case, all else being equal, the hypothesis that distinct parts of nature share a specific feature – that is, the hypothesis of objective uniformity – will be a more coherent hypothesis than the hypothesis that the two parts do not share that feature – that is, the hypothesis of objective variety.

Although RISES and NISES can be expressed in ways that are equally syntactically complex, if we assume that there are natural kinds, RISES is stated in terms of natural kinds and NISES is not. Thus, despite how the two hypotheses might be expressed, RISES postulates an objective uniformity whereas NISES does not. (Draper (2015, p. 55) has made a similar point.) Therefore, if, all else being equal, objective uniformity is more intrinsically probable than objective variety, RISES would have a greater intrinsic probability than does NISES. And, thus, the reliability of inductive generalization requires that, all else being equal, objective uniformity is more intrinsically probable than objective variety.

I will not attempt to *prove* that there are inductive support relations between distinct parts of nature. Doing so would likely require an entire dissertation unto itself. Instead, I claim only that insofar as Draper's account of intrinsic probability is correct and insofar as inductive generalization is reliable, there are inductive support relations between distinct neighboring parts of nature. In the next subsection, I further explicate the notion of inductive support relations and the related notion of friendliness to the scientific project.

Inductive Support Relations and the Scientific Project

We can distinguish possible worlds that are friendly to the scientific project from worlds that are unfriendly to the scientific project. Worlds that are friendly to the scientific project are worlds in which there is an inductive support relation between neighboring parts of the space of parameters describing that world. Consider Conway's Game of Life.[9] The Game of Life consists of an infinite array of squares, resembling an infinite chess board, where each square can be in one of two states, i.e., {Black, White}. At each time, the state of the array is updated by a set of rules to produce a subsequent state. If a square is in the black state and is neighbored by either zero or one squares in the black state, then, at the next time, the square transitions to the white state. If a square is in the black state and is neighbored by four or more squares in the black state, then the square transitions to the white state at the next time. If a square is in the black state with two or three neighboring

[9]↑My use of this example is inspired by the discussion in chapter 2 of Daniel Dennett's (2003).

squares that are also in the black state, then the square persists in the black state to the next time. If a square is in the white state with three or more neighbors, then the square transitions to the black state. Given some initial state, the array of squares results in a surprisingly complex pattern. In fact, despite the fact that there are simple rules for updating each square, one can assemble a universal Turing machine in the Game of Life (Dennett, 2003, p. 46); consequently, any computer program, of any degree of complexity, can be functionally realized by the Game of Life so long as the array's initial state – that is, which squares are in the black state and which squares are in the white state – is carefully chosen.

Suppose that some possible worlds consist only of an instance of the Game of Life (or of some structure isomorphic to an instance of the Game of Life) and call such a world a Life World. The state of a Life World at a given time can be specified by specifying the squares that have the state 'Black'. Likewise, the state of the Life world at all times can be specified by indicating the state of the squares at all times. We can define an inductive support relation between two squares at a time or between one time and another.

Friendliness to the scientific project is a degreed quantity. A world is said to be friendlier to the scientific project the greater the degree to which one part of the parameter space describing that world raises the probability that an adjacent part of the parameter space has a corresponding state. In Life Worlds, the state of any square at a time is independent of the states of the other squares at the same time so that there are no inductive support relations between the squares at a fixed time. However, the state of a Life World at time t, together with the rules, deductively entails the state of that same world at time $t + 1$, so that there is an inductive support relation between the state of a Life World at a time and the state of a Life World at any subsequent time. To put the point another way, the state at t, in conjunction with the rules, provides the best sort of evidence for the state at subsequent times. Moreover, since the state of a Life World at t is inconsistent with all but a specific subset of possible past states, given the rules, there is an inductive support relation between the state at t and states at previous times. That is, while the state at t does not deductively entail the state at all prior times, the state at t can support inferences about the states at prior times by ruling out states that are inconsistent with the state at t.

Unlike Life Worlds, our world contains an approximately relativistic space-time that might not be decomposable into an array of discrete elements and in which there may not be such things as Cosmos-wide instants of time. However, friendliness to the scientific project can be suitably generalized to any parameter space, regardless of whether that parameter space is discrete or continuous. Our world appears to be sufficiently friendly to the scientific project that there are inductive support relations between space-time events that are sufficiently close to each other, e.g., the space-time interval between the two events is sufficiently small.[10] That is, if space-time point a has feature F,[11] then, all else being equal, the fact that a is F raises the probability that a nearby space-time point b is also F, so long as a and b are sufficiently close:

$$Pr(Fb|Fa) > Pr(Fb) \tag{12.6}$$

As I've explained, coherence is the degree to which the parts of a hypothesis are mutually supportive. Intuitively, one might expect that the coherence of the hypothesis $A\&B$ increases as $Pr(A|B)/Pr(A)$ increases.[12] If equation 12.6 applies to two events, then the

[10]↑Thanks to David Albert for offering some casual comments that aided me in an early formulation of this notion. On Albert's formulation, inductive support relations are relations between two neighboring spatio-temporal regions. Unfortunately, that formulation won't do, in part because the size of a spatio-temporal region is not a relativistic invariant. The space-time interval is not a good parameter for this purpose either because, in light-like directions, the space-time interval between numerically distinct points is always zero. The affine parameter is not a good parameter because the affine parameter is defined only up to an arbitrary constant. The generalized affine parameter is not a good parameter either because the generalized affine parameter (i) depends on an arbitrary choice of basis vectors and (ii) because the generalized affine parameter is a Euclidean parameter, the parameter does not scale appropriately with spatio-temporal distances, e.g., along time-like directions, larger generalized affine parameters correspond to smaller time-like distances. However, there is a natural parameter for performing inductive inferences in the contexts ordinarily considered by physical cosmologists, namely, the average galactic red-shift. Given Hubble's law, this parameter *does* tell us important information about whether we should project an inductive inference to a specific cosmological context.

[11]↑Feature F can be fairly complex. For example, Fa might be the statement that a particle passing through a satisfies a particular equation of motion. In that case, Fb is the statement that a particle passing through b also satisfies that equation of motion, even though, e.g., the particle's velocity might be different at a than at b.

[12]↑I don't mean to *define* the intrinsic coherence of a hypothesis in terms of $Pr(A|B)/Pr(A)$ or in terms of any other probabilistic measure. There is a sizeable literature on probabilistic measures of coherence that deals with similar issues as Draper's account (for recent overviews, see Hansson, 2018; Koscholke, 2016; Olsson,

hypothesis that there is uniformity between the two events – that is, that the same predicate function F applies to a and b – is more coherent than the hypothesis that there is variety between the two events – that is, that the predicate apples to a and not to b. This is defeasible, e.g., the evidence could show that two events *are* different in important respects.

I've defined the inductive support relation in terms of the spatio-temporal distance between two points, but we can more generally define the inductive support relation in terms of the distance between any two parts of a parameter space. For example, in the case of an ideal gas, there is an inductive support relation between the characteristics of a gas at one value of the pressure and the characteristics of the gas at another value of the pressure; the further apart the two pressures, the weaker the inferences we can make about the second gas on the basis of the first. Consider a discrete parameter space $\mathbb{S}$. Let Fu be a predicate function that evaluates to true if u is F and is otherwise false, where u is an element of $\mathbb{S}$. Then let $Pr(Fu)$ be the epistemic probability that u is F. Finally, we can define the requisite inductive support relation as follows. There exists an inductive support relation between v and w, where v and w are elements of $\mathbb{S}$, iff

$$Pr(Fw|Fv) > Pr(Fw) \tag{12.7}$$

In other words, there is an inductive support relation between two elements of a given parameter space if the probability that a given element is F, given that the other element is known to be F, is greater than if the other element were not known to be F. To say that a discrete parameter space is friendly to the scientific project is then just to say that there is an inductive support relation between neighboring elements. I will assume that there are

2017), but no probabilistic account of coherence can offer an adequate articulation of Draper's intrinsic coherence. Probabilistic accounts of coherence define coherence measures as functions over probability distributions that are themselves defined on sets of propositions. According to Draper's account of intrinsic coherence, the intrinsic probability of a hypothesis is determined (in part) by the coherence. To compare Draper's account with probabilistic accounts of coherentist justification, we can think about hypotheses as the conjunction of the propositions in a set. In that case, supposing that some probabilistic account of coherence could be used to articulate Draper's view, one would conclude that the intrinsic coherence is determined by the intrinsic probability and the intrinsic probability is determined by the intrinsic coherence; we'd be left with a tremendous bootstrapping problem from which, as far as I can tell, there could be no escape.

similar inductive support relations for all of the dimensions of the parameter space of any given hypothesis. (If there is a dimension of the parameter space of a given hypothesis on which there are no inductive support relations then that dimension is not friendly to the scientific project.)

Why do I make the assumption that there are inductive support relations between neighboring parts of a parameter space but not between arbitrarily chosen points in a parameter space? Suppose that the state of some physical system S_1 is described by a point a in a parameter space, the state of another physical system S_2 is described by a neighboring point b, and a third physical system S_3 is described by a point c that is distant from a and b. In that case, there is a greater degree of analogy between the systems described by a and b than there is between either a or b and c. Given the greater degree of analogy between a and b, then is more reason to think that a resembles b (for example) then there is for thinking that a resembles c. This intuitive notion can be captured by stipulating that the strength of the inductive support relation decreases monotonically with distance.

As I've said, part-to-whole inferences require that there are inductive support relations between parts of nature that are arbitrarily far apart; if so, then the known physical facts could be extended to parts of nature infinitely distant (i.e., inequality 12.7 continues to hold in the limit that the two points are infinitely far apart). There are two cases to consider. First, suppose that we take two points, a and b, located arbitrarily far apart and such that a has feature F. And now we ask whether the fact that a has feature F raises the probability that b has feature F. Due to the fact that the inductive support relation decreases with distance, the degree to which a being F is evidence for the hypothesis that b is F will generally depend upon the distance between a and b. We don't have a mathematical function that describes how rapidly the inductive support relation would decrease and the details of such a function are likely dependent upon specific details about the system, feature, or parameter space in question. Nonetheless, given that the inductive support relation decreases with distance, the further that b is from a, the more skeptical we should be concerning inferences about b given information about a.

In the second case, consider a situation in which we are investigating a region in a parameter space; I will use R to represent the set of points in that region. And suppose that we have information about a sample of points from a subregion of R whose points I will denote S, namely, that all of the points in S have feature F. Can we infer that the rest of R – that is, the set of points in $R - S$ – probably has feature F so that there is an objective uniformity throughout the entirety of R? As I've discussed, on the assumption that there are inductive support relations between neighboring parts of a parameter space, objective uniformity is more coherent than objective variety. So, the hypothesis h that all of the points in R have feature F is coherent; moreover, the larger R is, the more coherent h is. However, the larger R is, the more h claims about the world. Consequently, the hypothesis that there is objective uniformity over a large region of a parameter space is an immodest hypothesis.

Part-to-whole inferences purport to project a feature from the portion of the Cosmos's parameter space available to us to the entire parameter space. The resulting hypothesis is as coherent as a hypothesis about physical reality could possibly be while simultaneously being as immodest as a hypothesis about physical reality could possibly be. How should the tension between modesty and coherence be resolved? This question has, thus far, gone unaddressed in the literature. Without criteria that allow us to adjudicate the tension between modesty and coherence, at the present stage of scientific and philosophical inquiry, part-to-whole inferences are unjustified. The trouble is not that we know that modesty will win out over coherence for any particular parameter space, but instead that, given the current state of our knowledge with respect to the nature of induction, we have no reason to think that modesty will not win out over coherence for large scale universal generalizations.

Without a fully articulated theory resolving the tension between modesty and coherence, we lack the grounds on which to make inferences about portions of the Cosmos's parameter space that are sufficiently exotic, including the scales, spatio-temporal locations, or energies relevant to the formation of the Cosmos or whether there was such an event or process as the formation of the Cosmos. As things stand at the current stage of inquiry, we should endorse Cosmic Skepticism, because our knowledge of the physical facts is provincial with respect to scale, spatio-temporal location, or energy in such a way that blocks empirical access to either the formation of the Cosmos or whether there was such an event or process as the formation of the Cosmos.

Any inference from empirically accessible features of the world that the Cosmos satisfies the Modal, Direction, and Boundary Conditions would need to project natural laws to domains far removed from observations in terms of the relevant scales, places, times, and energies. The tension between modesty and coherence provides us with reason to be skeptical that any known law has universal – or arbitrarily large – scope and so we have reason to be suspicious of projections to sufficiently exotic scales, places, times, and energies. Thus, we have reason to be suspicious that we can presently make the inductive inferences needed to infer whether the Cosmos had a beginning. In the next subsection, I turn to considering a specific family of part-to-whole inferences that might be used to argue that the Cosmos satisfies the Direction and Boundary Conditions. In light of the tension between modesty and coherence, the members of the family of arguments described are left without adequate justification.

Projections involving distant space-time domains

In this subsection, I will consider projections from the portion of space-time we can observe to space-time as a whole. We don't know how much larger the Cosmos is than our space-time, but we do know that all of space-time is included in the Cosmos. If our space-time is significantly larger than the observed portion of space-time, we know that the Cosmos is much larger than the observed portion of the Cosmos. We have several reasons for thinking that our space-time is significantly larger than the portion that we can observe. Our observational data – both from the Cosmic Microwave Background and from the observed distribution of galaxies – indicates that the observable universe is very close to being flat. So close, in fact, that, given current instrumentation, the observable universe is observationally indistinguishable from being flat. There is the possibility that space-time is truncated not far beyond our cosmological horizon; call this hypothesis TRUNCAT. In present physical theory, there is no principled reason as to why TRUNCAT should be true. And, importantly, if TRUNCAT is true, the inductive inferences we ordinarily take ourselves to be licensed to make to regions immediately neighboring our cosmological horizon would systematically result in incorrect conclusions.

Assuming that we are licensed to make inductive inferences to regions immediately neighboring our cosmological horizon, we should set TRUNCAT aside. This leaves us with three possibilities; in all three possibilities, space-time is large enough that the tension between modesty and coherence should make us suspicious as to whether our inductive inferences can be extended to the totality of space-time, let alone the totality of the Cosmos:

1. FLAT: Space-time globally approximates a flat FLRW space-time. In this case, space-time is infinitely spatially extended, so that the observable portion of space-time is infinitely smaller than space-time as a whole.

2. A-FLRW: The observable universe approximates a flat FLRW space-time, space-time is not globally flat, but space-time does globally approximate an FLRW space-time. In this case, space-time has some global curvature, but space-time is so large that the global curvature is not currently detectable. In that case, space-time would need to be sufficiently large – possibly infinitely large – in order to accommodate the global curvature.

3. OTHER: The observable universe approximates a flat FLRW space-time, space-time is not globally flat, and space-time does not globally approximate an FLRW space-time. In this case, all bets are off; as I showed in a previous chapter, supposing that the space-time does satisfy the Direction and Boundary Conditions, the observable portion of our space-time could be isometrically embedded in a variety of space-times that do not satisfy the Direction or Boundary Conditions.

In sum, insofar as our inductive procedures are reliable and we can set aside TRUNCAT, either FLAT, A-FLRW, or OTHER are true. But if FLAT, A-FLRW, or OTHER are true, then space-time must be significantly larger than our cosmological horizon. Given how large space-time must be if our inductive procedures are reliable, the tension between modesty and coherence provides us reason to be suspicious of the kind of large scale inductive inferences that one could make from the observable universe to space-time as a whole.

Inductive inferences regarding whether the Cosmos satisfies the Direction or Boundary Conditions require us to make a projection from the observed portion of the Cosmos to the unobserved portion. For example, consider that even if we somehow knew that the space-time region in our absolute past terminates with a boundary, there may be space-time points space-like separated from us whose pasts do not include a boundary, as I discussed in chapter 9. In that case, the Cosmos would not satisfy the Boundary Condition. To rule out that possibility would require us to project from our past to space-time points indefinitely far from us and, thus, would require us to make projections into domains where the tension between modesty and coherence is not adequately understood.

Moreover, supposing that we could establish that there is a fixed and uniform direction of time throughout the entire observable universe, one might try to establish that the entire Cosmos satisfies the Direction Condition by projecting that direction of time to the entire Cosmos. Again, there could be distant regions, space-like separated from ourselves, where the direction of time is inconsistent with the direction of time we observe throughout the observable universe. Thus, without resolving the tension between modesty and coherence, we do not know whether the Cosmos satisfies the Direction Condition.

The Copernican Principle

One might try to use a part-to-whole inference to evade the problems that I raised in chapter 9. If one could restrict the collection of space-times that are observationally indistinguishable from our own to a collection of space-times that satisfy the Boundary Condition, then one might be able to infer that the Cosmos satisfies the Boundary Condition. The Copernican Principle states, roughly, that our location within the Cosmos is not distinguished. Since the early modern period, physicists have tended to assume that the Copernican Principle is true.

As Chris Smeenk (unpublished, pp. 13–14) points out, if we interpret the Copernican Principle to entail that all free-falling observers see the Cosmic Microwave Background as isotropic, then the Ehlers-Geren-Sachs (EGS) theorem (1968) can be utilized to restrict the space of observationally indistinguishable space-times to FLRW space-times, since, in that case, the EGS theorem entails that the Cosmos *is* exactly FLRW. The Cosmic Microwave Background is known to be only approximately isotropic, but a generalization of the EGS theorem by Stoeger, Maartens, and Ellis (1995) ensures that in space-time regions where the Cosmic Microwave Background appears nearly isotropic for all free-falling observers, space-time can be well approximated as FLRW. So, if local conditions can be successfully projected to other parts of the Cosmos through the use of a part-to-whole inference, then perhaps we can know much more about the global distribution of matter-energy-momentum – and so be in a much better position to infer whether the Cosmos satisfies the Boundary Condition and possibly the Direction Condition, both of which depend upon the global matter-energy distribution.f

We have reason to doubt that this inference is successful. First, we can understand the Copernican Principle as having large scope – or at least large with respect to our observational capacities – without defining the Copernican Principle as having universal (or arbitrarily large) scope. As Chris Smeenk (unpublished, p. 14) points out, there are live cosmological hypotheses, such as inflation, that entail that the projection fails. Since we should not make the brash step of ruling out live cosmological hypotheses from the arm chair, we should not be confident that the projection succeeds. Moreover, the inference requires us to project a set of principles (e.g., the Copernican Principle, the isotropy of the microwave background, etc) to the whole of the Cosmos. Again, while the application of a set of principles to the entire Cosmos is as coherent as a hypothesis about physical reality could be, the hypothesis is simultaneously as immodest as a hypothesis about physical reality could possibly be.

12.4 Inductive Inferences and the Modal Condition

I've argued that two families of inductive inferences that might be used to establish that the Cosmos satisfies the Modal, Direction, and Boundary Conditions are unsuccessful. There is another set of considerations that suggest that establishing whether the Cosmos satisfies the Modal Condition might not be possible, even in principle, regardless of which form of inductive inference we attempt to use.

Recall that when I introduced the notion that the Cosmos could be fundamentally timeless in chapter 5, I drew an analogy with the Matrix. The Matrix might functionally realize space-time. If so, the space-time that we have experience with may have an altogether different structure than the space-time inhabited by the computers running the Matrix. Suppose that we had good evidence, from, e.g., a quantum gravity theory, that the space-time we have experience with, which I will call *ordinary space-time*, is functionally realized by some more fundamental substructure, e.g., a spin network or the universal wavefunction or whatever. In that case, just as the computers running the Matrix themselves enjoy a spatio-temporal existence, perhaps the substructure to which ordinary space-time is reducible itself enjoys a spatio-temporal existence. On what grounds could we distinguish between a timeless substructure and a spatio-temporal substructure? I'm not sure that we could; presumably, whether we could depends upon the specific details about the structure that we discover to functionally realize ordinary space-time and how we discover the fact that the structure functionally realizes space-time.

12.5 Objections

In this last section, I consider four objections to the arguments that I've offered in this chapter. First, I consider an objection according to which large scale cosmological inferences should proceed by inference to the best explanation instead of by inductive generalization. Second, I turn to an objection involving the fact that laws of nature are often thought to have universal scope. If laws of nature do have universal scope and our best scientific theories include laws of nature, then, one might object, there must be something wrong with the worries that I raised concerning part-to-whole inferences. Third, I turn to an objection concerning inductive generalizations in mathematics. Allegedly, mathematicians sometimes perform inductive generalizations over infinite domains. If mathematicians do perform inductive generalizations over infinitely large domains, then, one might object, there must be something wrong with my claim that we should be skeptical of universal generalizations performed over the entire Cosmos. Fourth, I turn to an inductive generalization performed by some natural theologians. According to some natural theologians, we can use an inductive generalization to infer that God (or a God-like being) is omnipotent. If so, then, one might object, perhaps we shouldn't be skeptical of universal generalizations performed over the entire Cosmos. I will show that none of these objections are successful.

12.5.1 Modesty, Coherence, and Inference to the Best Explanation

I've argued that large scale inductive generalizations are beset by an as yet unresolved tension between modesty and coherence. Perhaps the reader will be tempted to reply that instead of deploying induction over large scale domains, we should deploy Inference to the Best Explanation (IBE). Since IBE does not depend on inductive generalization, or so the claim goes, IBE does not meet the same objections as large scale inductive generalizations do. Some authors have argued that inferences to the best explanation are hidden inductive inferences, perhaps supplemented with deduction. For example, Fumerton (1980) argues that IBE depends upon a hidden analogical argument from experience. If so, inferences to the best explanation, when deployed over sufficiently large domains, will have to contend with the tension between modesty and coherence.

Regardless of whether we accept that induction is more fundamental than IBE, both inductive generalization and IBE depend on our theory of intrinsic probability. IBE is often utilized when we are faced with a situation where a set of mutually exclusive hypotheses predicts the data equally well. Compare the problem that I raised in subsection 12.3.2 concerning RISES and NISES. Even though RISES and NISES predicted the available evidence equally well, the reliability of induction required that RISES is more probable than NISES. At first glance, both the inference schema for IBE and the puzzle I posed using RISES and NISES ask us to perform the same task, viz, how should we choose the "best hypothesis" out of a collection of hypotheses all of which are consistent with our total evidence? According to standard accounts of IBE, the best hypothesis needs to be selected on the basis of a set of theoretical virtues, e.g., simplicity, parsimony, and the like. As I discussed in subsection 12.3.1, insofar as the theoretical virtues can be considered independent of our total evidence or background knowledge, on Draper's view, the theoretical virtues are reducible to modesty and coherence (Draper, 2015, 2017). Consequently, Draper's theory of intrinsic probability should tell us that, in a wide variety of cases, IBE is either reducible to or replaceable by a comparison of intrinsic probability.

Nonetheless, the theoretical virtues in terms of which a hypothesis may be considered the best explanation are not always independent of our background knowledge or total evidence. Jonah Schupback and co-author Jan Sprenger (2011) consider what they call the *explanatory power* of a hypothesis, which I will refer to as the power of the hypothesis; given that a hypothesis explains some datum, the power is the degree to which a hypothesis renders that datum explicable as opposed to the contrary datum. For example, if postulating an ancient earthquake renders the deformation of some portion of bedrock less surprising than the deformation would otherwise be, the power is the *degree* to which the deformation is less surprising than the deformation would otherwise be (J. N. Schupbach and Sprenger, 2011, p. 108). Shupback and Sprenger develop a set of desiderata that any candidate measure of power should satisfy and then show that there is a mathematical function which uniquely satisfies their desiderata (J. N. Schupbach and Sprenger, 2011; J. Schupbach, 2017, pp. 40–46, J. Schupbach, 2022, pp. 75–77, 56–60). As Shupback and Sprenger show, the explanatory power of a hypothesis h, given evidence e and background knowledge K, should be mathematically expressed by the following measure:

$$\varepsilon_{SS}(\mathrm{e}, h, K) = \frac{Pr(h|\mathrm{e}\&K) - Pr(h|\neg\mathrm{e}\&K)}{Pr(h|\mathrm{e}\&K) + Pr(h|\neg\mathrm{e}\&K)}$$

I will call ε_{SS} the Schupback-Sprenger power measure. Shupback shows that some hypothesis h_1 has greater explanatory power than some alternative and mutually incompatible hypothesis h_2, i.e., $\varepsilon_{SS}(\mathrm{e}, h_1) > \varepsilon_{SS}(\mathrm{e}, h_2)$, if and only if $Pr(\mathrm{e}|h_1\&K) > Pr(\mathrm{e}|h_2\&K)$. In other words, according to the Schupback-Sprenger power measure, h_1 has greater power than h_2, relative to evidence e and our background knowledge, only if h_1 predicts e better than h_2 predicts e. Of course, that h_1 has a greater ability to predict e than does h_2 does not render h_1 more probable than h_2. For example, if I hear indiscriminate banging sounds coming from my attic, we can hypothesize gremlins that produce precisely the sounds that I hear; while the gremlin hypothesis may predict the sounds that I hear better than any other available hypothesis, the gremlin hypothesis has a low posterior probability precisely because the gremlin hypothesis has a low prior probability relative to our background knowledge (Sober, 2008, p. 10; also see J. Schupbach, 2017, pp. 51–52). In fact, one can show that the fact that h_1 predicts the data better than h_2 entails that the posterior probability of h_1 is greater than the posterior probability of h_2 only on the condition that their prior probabilities are equal. Thus, the fact that h_1 predicts the data better than h_2 is no guarantee that h_1 is more probable, all things considered, than h_2. Nonetheless, Shupback uses computer studies to show that, in a variety of statistical tasks, selecting a hypothesis based on Shupback's power measure outperforms the selection of a hypothesis based on the prior probability (J. Schupbach, 2022, pp. 80–84). Shupback concludes that, all else being equal, we have defeasible reason to favor hypotheses that are ranked higher according to the Schupback-Sprenger power measure.

We can ask whether, in the case of inferences over large domains, the Schupback-Sprenger power measure allows us to overcome the tension between modesty and coherence. Note, once more, that the ranking of hypotheses in terms of power is equivalent to the ranking of hypotheses in terms of their respective ability to predict our data relative to our background knowledge. There are then two possibilities. First, as with the gremlin hypothesis, if the prior probability of a hypothesis h with a large domain is sufficiently low, then, in the absence of sufficiently strong evidence, the posterior probability of h will be low. Since the prior probability of h is determined – in part – by the intrinsic probability of h, the question then becomes how low the immodesty of h drives the intrinsic probability of h. In other words, we return to the unresolved tension between modesty and coherence. Second, supposing that the prior probability of h is not too low, we need to ask how well h predicts some body of evidence over the contrary. In the case of part-to-whole inferences, the question will be whether the hypothesis that the Cosmos satisfies the Modal, Direction, or Boundary Conditions has a high probability. In that context, one can compare the disjunction of hypotheses in which the Cosmos does satisfy the Modal, Direction, and Boundary Conditions against the disjunction of hypotheses in which the Cosmos does not satisfy the Modal, Direction, and Boundary Conditions. As far as inquiry has proceeded thus far, there is no clear reason for thinking that the former disjunction better predicts our total evidence than does the latter disjunction. Thus, there is no reason to think that the Schupback-Sprenger power measure saves the kind of large scale inferences needed for inferring that the Cosmos had a beginning.

There is also a sense in which Schupback's computational results support the argument that I have presented in this chapter. Schupback's simulations model an ideal agent that, given randomly generated data, selects a hypothesis based either on chance, standard probabilistic (e.g., Bayesian) reasoning, or via Schupback's power measure. Schupback scores each according to their respective *accuracy*, that is, out of a million trials, the number of occasions on which an ideal agent would choose the true hypothesis. As the number of hypotheses increase, the accuracy of all three methods decrease (J. Schupbach, 2017, pp. 52–53). As Schupback observes, this makes intuitive sense, viz, the accuracy decreases as the number of ways that one could be wrong increases. David Glass (2012) has shown that similar results follow for a variety of distinct formal models of inference to the best explanation.[13] But notice that this is equivalent to the statement that as one's hypothesis selection becomes increasingly immodest, one is more likely to select an incorrect hypothesis. Thus, the results of Schupback's computer studies suggest that even if we did utilize Schupback's power measure in place of standard methods, we would likely still need to contend with the tension between modesty and coherence in the case of hypotheses ranging over the totality of physical reality.

12.5.2 Laws with Global Scope

I've argued that there is a tension between modesty and coherence in virtue of which we should be skeptical about universal generalizations whose conclusions have large or universal scope. In this section, I consider an important objection involving the laws of nature. According to the objection, one could argue that scientific explanation requires

[13] ↑According to the results of Glass's computer studies, the effect is minimized when the hypothesis chosen is the one that maximizes the posterior probability.

laws of nature and that laws of nature have universal scope. If scientific explanation requires laws of nature, laws of nature have universal scope, and inferences involving universal scope are unavailable to us, then scientific explanation is unavailable to us. But scientific explanation is available to us, so we must be able to make successful empirical inferences involving universal scope after all, or so the objection goes.

To the contrary, science does not require laws of nature with universal scope. There are a variety of conceptions of laws of nature. On one conception, laws of nature are principles of physical necessitation with universal scope (e.g., Armstrong, 1983; Dretske, 1977; Hildebrand, 2020; Tooley, 1977). Insofar as there are true principles of physical necessitation with universal scope, they are likely unknown to us, and insofar as there are physical laws known to us at the present stage of physical inquiry, they are likely only approximate. And while principles that are only approximately (and not actually) true may be empirically adequate, might figure into successful scientific theories,[14] might be pragmatically useful in engineering and technological development, might be explanatory, and might even be enlightening for metaphysical inquiry and tell us partial information about the nature of reality, such principles, if interpreted to have universal scope, are literally false.

As Michael Scriven stated in a talk in 1959, "The most interesting fact about laws of nature is that they are virtually all known to be in error. And the few exceptions not only seem quite likely to become casualties before long, but their defection seems to be a matter for small mourning" (1961, p. 91). In a concluding paragraph, Scriven (Scriven, 1961, p. 101) tells us, "Laws are usually inaccurate; but they represent great truths so we forgive them their errors". Similar accounts have been defended by Eric Winsberg (2004, 2006) and Ronald Giere (1999). For Scriven, Winsberg, and Giere, laws have broad, but not universal, domains of application. For Scriven, laws are non-accidental generalizations that support counterfactuals within a specific domain, that acquire the status they have from playing a specific theoretical role, and that serve as useful approximations within a

[14]↑The pessimistic meta-induction and its relatives help to establish that though the laws which appear in our current best physical theories, if interpreted as principles with universal scope, are likely false, successful scientific practice can utilizes those principles all the same.

specific range of applications.[15] Ronald Giere has similarly offered an account of scientific practice that revises the traditional notion of laws of nature. Giere replaces laws by what he calls principles and which I will call Giere-principles. Giere-principles can be understood in one of two ways: either as prescriptions for the construction of models or as descriptions of models (Giere, 1999, pp. 5–6, 94).[16] Winsberg offers what he calls the "framework" conception of laws. In agreement with Scriven and Giere, Winsberg understands laws to be principles that enable us to build models, plan experiments, and to represent target phenomena of interest. As Winsberg (2004, p. 716) describes, "Rather than taking laws to be universally true and delimiting the character of all possible worlds, the proponent of the framework conception takes laws to be broadly reliable for a wide array of practical and epistemic tasks".

While I do not endorse all of the claims made by Scriven, Winsberg, and Giere, I agree with several of their shared conclusions. For example, Scriven tells us that the usefulness of an approximation depends on one's purposes while Giere (1999, p. 93) tells us that the fit of a model to a target phenomenon depends on one's purposes. I agree that how well a theory, hypothesis, model, or approximation partially describes reality depends on what features of reality we are interested in capturing. More important for my purposes in this chapter, I agree that laws are, at best, only approximate (Scriven, 1961, p. 92) and have a limited range of application (Scriven, 1961, pp. 92–93; Winsberg, 2004, p. 716; Giere, 1999, pp. 92–93). For that reason, we should not understand laws as having universal scope and should not understand laws as applying to an arbitrarily large domain. Since laws

[15]↑For Scriven, the usefulness of an approximation is determined by (1) accuracy, (2) the range of application, and (3) one's purposes, while theoretical tractability is determined either by the approximation's fit with established theory or by whether the approximation can serve as a "good basis" for the development of a new theory.

[16]↑According to Giere (1999, p. 5), scientific practice should be fundamentally understood in terms of models, where models are intermediate entities (e.g., Giere, 1999, p. 92) between theories/hypotheses and the objects of scientific study; the key role of models is to represent the objects of scientific study. Instead of being true representations of the target phenomena, models are similar to or fit the target phenomena (Giere, 1999, pp. 5, 73, 92–93). Moreover, the model bears a degree of similarity to a target system only in "specified respects" and "to limited degrees of accuracy" (Giere, 1999, pp. 92–93). Furthermore, fit is a matter of one's purposes; fit "requires a specification of which aspects of the world are important to represent and, for those aspects, how close a fit is desirable" (Giere, 1999, p. 93). Models fit a target only to "within the limits of what can be detected using existing experimental techniques" (Giere, 1999, p. 95). When scientists compare a model to a target phenomenon, they do so in virtue of data (Giere, 1999, p. 74). A model can be ruled out if, among other reasons, the model is a poor fit to the data (Giere, 1999, pp. 74–75).

do not have universal scope and do not apply to arbitrarily large domains, the objection with which I began this chapter evaporates.

Some philosophers would be scandalized by the suggestion that laws of nature are something other than true principles of physical necessitation with universal scope. For example, Swartz (n.d.) interprets Scriven's essay as implicitly drawing a distinction between Laws of Science (which Scriven calls "physical laws") and laws of nature. According to Swartz, the laws of nature are the true principles, even if unknown to us, that provide an exact – and not merely approximate – description of nature. Swartz tells us that when he and other philosophers are investigating the metaphysics of laws – e.g., agonizing over whether physical determinism is compatible with free-will – they are simply not interested in the "approximate truths" of science. The trouble is two-fold. First, the only laws known to us are the Laws of Science. Second, the only impetus philosophers have had for investigating the notion of laws of nature was the role that such laws apparently play in scientific practice or in scientific theories (Giere, 1999, p. 86).[17] The foundational principles included in our best physical theories are broadly reliable for a wide variety of practical and epistemic tasks; even if we accept that such principles need to be underwritten by more fundamental principles of physical or metaphysical necessitation, Scriven's, Winsberg's, and Giere's accounts land us closer to the truth about scientific practice.

Do Scriven's, Winsberg's, or Giere's accounts require that we accept some form of scientific anti-realism? Giere tells us that he is not an anti-realist and that, on his view, scientific theories are not merely empirically adequate. Instead, Giere tells us that models can successfully capture reality, so that, in virtue of models, we can come to have a partial understanding of reality (Giere, 1999, pp. 79–82). For my purposes in this chapter, it suffices that scientific theories provide us with partial access to the truth and that the principles fundamental to scientific theories, i.e., laws, can be understood as having a limited – and so not universal – scope. If laws are admitted to have only a limited, and

[17]↑One might object that some philosophers have had a different impetus, e.g., Swartz is motivated by questions about whether physical determinism is compatible with free-will. But the view that the physical world is deterministic, or at any rate might be deterministic, is a view that arises out of the role that laws have played in specific scientific theories.

not necessarily universal, scope, then no objection can be made that science requires laws with universal scope.

12.5.3 Inductive Inferences in Mathematics

I've been considering an argument according to which we cannot generally be confident in an extension of inductive inferences over arbitrarily large portions of a parameter space. One might object by noting that there are cases where inductions appear to be offered for infinite domains and where the induction is generally considered reliable. If there are legitimate examples of non-spurious inductive inferences over infinitely large domains, then there must be something wrong with the argument I've offered. Let's put one such example on the table.

Consider Goldbach's Conjecture.[18] According to Goldbach's Conjecture, for any positive even integer $2N$, $2N$ can be written as the sum of two primes (Weisstein, n.d.; Echeverría, 1996; Baker, 2008, p. 335). Though Goldbach's Conjecture has yet to be proven, mathematicians think that there is a strong case for the truth of Goldbach's Conjecture (Baker, 2007, 2008, 2017; Echeverría, 1996), and this case is supported, in part, by what, at least prima facie, appears to be a universal generalization. Mathematicians have searched the first 4×10^{18} integers, extended (by way of a theorem) from those integers to the first 8.37×10^{26} integers, and failed to find a counterexample (e Silva et al., 2014). Given that mathematicians have not found a counterexample among the first 8.37×10^{26} integers, are they justified in inferring that, probably, there are no counterexamples?

As it happens, on the assumption that Goldbach's Conjecture is false – that is, given that a counterexample exists – the counterexample is overwhelmingly likely to occur among the first 8.37×10^{26} integers. Consequently, the fact that a counterexample does not exist among the first 8.37×10^{26} integers provides overwhelmingly good evidence that Goldbach's Conjecture is true. To see how this result comes about, let's approximate the intrinsic probability that any given positive even integer *cannot* be written as the sum of two primes. Define G_{2N} as the sentence that $2N$ is a positive even integer that is the sum

[18] ↑Thanks to Aaron Nung Kwan Yip and Levi Greenwood for help in developing this example.

of two primes, $p = 2N - k$ and $q = k$. Consequently, $\neg G_{2N}$ is the sentence that $2N$ is not the sum of two primes, so that $Pr(\neg G_{2N})$ is the probability of a counterexample to Goldbach's Conjecture.

Consider all of the ways of summing the primes less than $2N$. By the prime number theorem, the probability that a randomly chosen number less than $2N$ is prime is $1/\log(2N)$. Denote the number of primes less than $2N$ as P_{2N}. Then the probability that a randomly chosen positive integer less than $2N$ is prime can be written as $P_{2N}/(2N)$. Thus, we have that $P_{2N}/(2N) \approx 1/\log(2N)$, so that $P_{2N} = 2N/\log(2N)$. The number of ways of summing the prime numbers less than $2N$ is the number of ways that prime numbers less than $2N$ can be paired:

$$(\text{ Number of sums }) \equiv a = \frac{1}{2}\left(\frac{2N}{\log(2N)}\right)^2 = 2\left(\frac{N}{\log(2N)}\right)^2$$

Let's use S_1, S_2, ..., S_a to denote the sums of distinct primes less than $2N$. We are trying to find $Pr(\neg G_{2N})$. Note that $\neg G_{2N}$ is equivalent to the sentence that $2N$ is not equal to the sum of any of the primes less than $2N$. That is,

$$Pr(\neg G_{2N}) = Pr[(S_1 \neq 2N)\&(S_1 \neq 2N)\& \ldots \&(S_a \neq 2N)]$$

Under the assumption that $S_i \neq 2N$ and $S_j \neq 2N$, for $i \neq j$, are probabilistically independent, i.e., $Pr(S_i \neq 2N | S_j \neq 2N) = Pr(S_i \neq 2N)$, and using the restricted conjunction rule, we have that:

$$Pr(\neg G_{2N}) = \Pi_{n=1}^{a} Pr(S_n \neq 2N)$$

Moreover, we also know that, for any i, $Pr(S_i \neq 2N) = 1 - \frac{1}{2N}$, so that:

$$Pr(\neg G_{2N}) = \Pi_{n=1}^{a}\left(1 - \frac{1}{2N}\right) = \left(1 - \frac{1}{2N}\right)^a$$

Using the probabilistic independence condition previously mentioned, the probability that there is a counterexample to Goldbach's Conjecture greater than $2M$ can then be computed using:

$$
\begin{aligned}
Pr(\exists x.(x > 2M)\&\neg G_x) &= Pr(\neg G_{2M} \vee \neg G_{2(M+1)} \vee \ \dots\ ) \\
&= Pr(\neg(G_{2M}\&G_{2(M+1)}\& \dots\ )) \\
&= 1 - Pr(G_{2M}\&G_{2(M+1)}\& \dots\ ) \\
&= 1 - \Pi_{i=0}^{\infty} Pr(G_{2(M+i)}) \\
&= 1 - \Pi_{i=0}^{\infty}\left[1 - \left(1 - \frac{1}{2(M+i)}\right)^{2\left(\frac{M+i}{\log(2(M+i))}\right)^2}\right]
\end{aligned} \tag{12.8}
$$

Likewise, we can find the probability that Goldbach's Conjecture is true given that all even integers less than $2M$ satisfy Goldbach's Conjecture:

$$
\begin{aligned}
Pr(\neg(\exists x.(x > 2M)\&\neg G_x)) &= Pr(\forall x.(x \leq 2M) \vee G_x) \\
&= Pr(\forall x.Gx|x > 2M) \\
&= \Pi_{i=0}^{\infty}\left[1 - \left(1 - \frac{1}{2(M+i)}\right)^{2\left(\frac{M+i}{\log(2(M+i))}\right)^2}\right]
\end{aligned} \tag{12.9}
$$

This quantity can be numerically computed. For example, for $2M = 1000$, $1 - Pr(\forall x.Gx|x > 2M) \approx 2 \times 10^{-3}$ and, for $2M = 5000$, $1 - Pr(\forall x.Gx|x > 2M) \approx 10^{-13}$. Since the majority of the probability for the occurrence of a counterexample obtains for small values of M (see figure 12.1), an exhaustive computer search of small values of M comes close to ruling out the possibility that there could be *any* counterexamples. That is, most of the probability mass associated with possible counterexamples to Goldbach's Conjecture appears in precisely the region of parameter space that can be searched using computer studies. The fact that we can see, by rational intuition alone, considerations that restrict the majority of the probability mass to low values of $2M$ explains why a counterexample is unlikely to appear for higher values of $2M$. No appeal was made to the modesty or to the coherence of Golbach's Conjecture and no inductive generalization was carried out. In the case of universal generalizations, there is typically no corresponding reason to think that the portion of parameter space most likely to include counterexamples to a given hypothesis of interest is restricted to the observable portion of parameter space.

Furthermore, let's consider the *kind* of reasons that allow us to restrict the majority of the mass of the probability distribution to the observable portion of parameter space in the case of Goldbach's Conjecture. (For a similar argument, see Paseau, 2021, pp. 9166–9167.) In the case of Goldbach's Conjecture, we ask whether each positive even integer is the sum of two primes; by way of rational intuition, we know substantive facts about the entire collection of positive even integers and the fact that we do know substantive facts about the entire collection of positive even integers by way of rational intuition plays a substantial role in the derivation of the probability distribution that I considered above. We are likely to know substantive facts about the global features of a given parameter space purely by way of rational intuition only in the case of wholly formal disciplines such as pure mathematics or logic. The sciences are not wholly formal disciplines and so are unlikely to include any analogous cases; thus, a part-to-whole inference that had a feature of that kind is implausible.

12.5.4 Inductive Inferences and Omnipotence

In the previous subsection, I considered an objection according to which mathematicians have successfully made inductive generalizations over infinitely large domains. I showed that mathematicians have not actually made an inductive generalization over an infinitely large domain. In this subsection, I turn to a claim that is sometimes made by natural theologians. Some natural theologians have claimed to be able to infer that God (or a God-like being) is omnipotent in virtue of the fact that a being that can perform every possible task is more intrinsically probable than a being that can perform some, but not all possible, tasks. If natural theologians can successfully perform such an inference, then they must be able to resolve the tension between modesty and coherence in favor of coherence. In the following, I will show that whether natural theologians can perform such an inference is far from obvious. While the hypothesis that there is a being that can perform all possible tasks is highly coherent, the hypothesis is profoundly immodest. And we can understand the immodesty of the hypothesis in virtue of the fact that there are a vast number of ways in which a being can fail to be omnipotent. (A similar argument, but in much less mathematical detail, was previously offered in the concluding section of Draper, 2015.)

Natural theologians purport to be able to surmise the existence of a powerful being from the existence of the Cosmos, the appearance of complex order in our world, or from other empirical observations; nonetheless, there are an infinity of powerful, but not omnipotent, beings that are consistent with the same data. How does one infer the existence of an infinitely powerful being from observing a series of finite effects? The strategy often pursued by natural theologians has involved the observation that the hypothesis that a being can do all (compossible) tasks is simpler or more coherent than the hypothesis that a being can do some specific delimited number of tasks.

To model the intrinsic probability of an omnipotent being, my strategy will be to first consider hypotheses about the tasks that may be accomplished by a finite being and then to study how the intrinsic probability changes as the number of tasks increases to infinity. This will allow me to develop a toy model that can be used to think about the extension of any hypothesis beyond a known domain of application. I will call the (in)finite being Taquesha. Let ϕ_1 be the hypothesis that Taquesha can perform task 1, ϕ_2 be the hypothesis that Taquesha can perform task 2, etc, up to ϕ_M. I will use A to represent the hypothesis that Taquesha can perform all M tasks. The hypothesis that Taquesha is omnipotent is recovered in the limit that M becomes infinitely large; I claim that we can expect the tension between modesty and coherence to grow as M increases. Omnipotence – i.e., the hypothesis that Taquesha can perform an infinity of tasks – is maximally coherent, but immodest – i.e., omnipotence rules out a large disjunction of alternatives according to which Taquesha can perform some, but not all, tasks. Let's begin with a simple example where there are four tasks, i.e., $\{\phi_1, \phi_2, \phi_3, \phi_4\}$. In this case, there are $2^4 = 16$ possible combinations:

$$\begin{array}{ccc} \phi_1\&\phi_2\&\phi_3\&\phi_4 & \phi_1\&\phi_2\&\phi_3\&\neg\phi_4 & \phi_1\&\phi_2\&\neg\phi_3\&\phi_4 \\ \phi_1\&\phi_2\&\neg\phi_3\&\neg\phi_4 & \phi_1\&\neg\phi_2\&\phi_3\&\phi_4 & \phi_1\&\neg\phi_2\&\phi_3\&\neg\phi_4 \\ \phi_1\&\neg\phi_2\&\neg\phi_3\&\phi_4 & \phi_1\&\neg\phi_2\&\neg\phi_3\&\neg\phi_4 & \neg\phi_1\&\phi_2\&\phi_3\&\phi_4 \\ \neg\phi_1\&\phi_2\&\phi_3\&\neg\phi_4 & \neg\phi_1\&\phi_2\&\neg\phi_3\&\phi_4 & \neg\phi_1\&\phi_2\&\neg\phi_3\&\neg\phi_4 \\ \neg\phi_1\&\neg\phi_2\&\phi_3\&\phi_4 & \neg\phi_1\&\neg\phi_2\&\phi_3\&\neg\phi_4 & \neg\phi_1\&\neg\phi_2\&\neg\phi_3\&\phi_4 \\ \neg\phi_1\&\neg\phi_2\&\neg\phi_3\&\neg\phi_4 & & \end{array} \tag{12.10}$$

We can divide the sixteen possibilities into four sets. The first set, which I will denote A, contains the single hypothesis:

$$\phi_1\&\phi_2\&\phi_3\&\phi_4 \tag{12.11}$$

The second set contains seven members:

$$\begin{array}{ccc} \phi_1\&\phi_2\&\phi_3\&\neg\phi_4 & \phi_1\&\phi_2\&\neg\phi_3\&\phi_4 & \phi_1\&\phi_2\&\neg\phi_3\&\neg\phi_4 \\ \phi_1\&\neg\phi_2\&\phi_3\&\phi_4 & \phi_1\&\neg\phi_2\&\phi_3\&\neg\phi_4 & \phi_1\&\neg\phi_2\&\neg\phi_3\&\phi_4 \\ \phi_1\&\neg\phi_2\&\neg\phi_3\&\neg\phi_4 & & \end{array} \tag{12.12}$$

I will use B to represent the hypothesis reporting the disjunction of the seven members of this set. Note that B is logically equivalent to $\phi_1\&\neg(\phi_2\&\phi_3\&\phi_4)$. That is, B consists of all of the possible combinations in which Taquesha can do task 1 but cannot perform one or more of the other tasks. The third set also contains seven members:

$$\begin{array}{ccc} \neg\phi_1\&\phi_2\&\phi_3\&\phi_4 & \neg\phi_1\&\phi_2\&\phi_3\&\neg\phi_4 & \neg\phi_1\&\phi_2\&\neg\phi_3\&\phi_4 \\ \neg\phi_1\&\phi_2\&\neg\phi_3\&\neg\phi_4 & \neg\phi_1\&\neg\phi_2\&\phi_3\&\phi_4 & \neg\phi_1\&\neg\phi_2\&\phi_3\&\neg\phi_4 \\ \neg\phi_1\&\neg\phi_2\&\neg\phi_3\&\phi_4 & & \end{array} \tag{12.13}$$

I will use C to represent the disjunction of these seven statements. Note the symmetry between B and C; whereas B is logically equivalent to $\phi_1\&\neg(\phi_2\&\phi_3\&\phi_4)$, C is logically equivalent to $\neg\phi_1\&\neg(\neg\phi_2\&\neg\phi_3\&\neg\phi_4)$. That is, C is the hypothesis that Taquesha cannot do task 1 but can do at least one task. The fourth set, like the first set, contains the single member:

$$\neg\phi_1 \& \neg\phi_2 \& \neg\phi_3 \& \neg\phi_4 \tag{12.14}$$

I will call the single member of this last set D. D is the hypothesis that Taquesha can perform none of the tasks. The division into four hypotheses is naturally generalized to the case where there are M possible tasks and $N = 2^M$ possible combinations. That is, A is the hypothesis that Taquesha can perform all M tasks, B is the hypothesis that Taquesha can perform task 1 and one or more other tasks, C is the hypothesis that Taquesha cannot perform task 1 but can perform one or more other tasks, and D is the hypothesis that Taquesha cannot perform any tasks.

At this stage, I need to introduce some additional terminology. Let's call each of the ϕ_i a *microhypothesis*. A, B, C, and D are *macrohypotheses*. The members of the above four sets, formed by conjoining microhypotheses, are *midhypotheses*, so-called because they are mid-level between microhypotheses and macrohypotheses.

Let P_n denote the probability of hypothesis n, e.g., P_A is the probability of A. I am going to suppose that all N midhypotheses are equally modest. Therefore, if there were no inductive support relations between ϕ_i and ϕ_j, then $P_A = 1/N$. However, since we are interested in understanding the relationship between modesty and coherence, I will suppose that there are inductive support relations between ϕ_i and ϕ_j, for any i and any j.[19] That is, P_A receives a boost, from the uniformity in the hypothesis, above the value that P_A would have been attributed when we only consider the modesty of P_A. Therefore, $P_A > 1/N$. Let's define ε as the boost that P_A receives due to coherence, i.e., $\varepsilon \equiv P_A - 1/N$.

That is, just as there is an inductive support relation between ϕ_i and ϕ_j, for any i and any j, I will suppose that there is an inductive support relation between $\neg\phi_i$ and $\neg\phi_j$ for any i and any j. In other words, just as the fact that Taquesha can perform task i is

[19]↑Parameter spaces have an underlying topology and metric that tells us how each part of the parameter space is connected to any other. In the case of a parameter space with a discrete topology, the metric can be "read off" the topology. (This is not so in the case of a continuous topology, where a single topology is equally consistent with multiple metrics.) For example, the squares in a Life World have a discrete topology and the distance between any two squares is found by counting the number of intervening squares. Generically, we can expect that the inductive support relation falls off with distance. In Taquesha's case, I will assume that the tasks form a complete graph, so that every task is a neighbor to every other task, i.e., there is an inductive support relation between ϕ_i and ϕ_j, for any i and any j, and an inductive support relation between $\neg\phi_i$ and $\neg\phi_j$ for any i and any j.

evidence that Taquesha can perform task j, so, too, Taquesha's inability to perform task i is evidence for Taquesha's inability to perform task j. Given the symmetry between A and D, we can reasonably suppose that $P_A = P_D$. There is also symmetry between B and C, i.e., for every disjunct in B, there is a corresponding disjunct in C, so that $P_B = P_C$. We have the normalization condition that $P_A + P_B + P_C + P_D = 1$, which, together with the aforementioned symmetries, entails:

$$P_A + P_B = \frac{1}{2} \tag{12.15}$$

From the definition of ε, 12.15 entails that we can re-write P_B in terms of ε:

$$P_B = \frac{1}{2} - \frac{1}{N} - \varepsilon \tag{12.16}$$

We know that $P_B \geq 0$; together with 12.15, this implies that P_A is bounded from above by $1/2$. Using the definition of ε, we then have that $1/N + \varepsilon \leq 1/2$. So, we have lower and upper bounds for ε:

$$0 \leq \varepsilon \leq \frac{1}{2} - \frac{1}{N} \tag{12.17}$$

Without an explicit theory of the degree to which coherence increases P_A, and so the size of ε, we cannot say anything definite about how P_A and P_B compare other than in the case where there are exactly two microhypotheses. In that case, $P_A = 1/4 + \varepsilon$ and $P_B = 1/4 - \varepsilon$; ergo, $P_A > P_B$. However, when the number of microhypotheses is greater than 2, we need a theory that would allow us to rank hypotheses in terms of the degree to which their internal inductive support relations boost ε and that would allow us to compare the intrinsic probabilities of two macrohypotheses. Notice, too, that as N increases, the upper bound on ε increases, so that there is a sense in which our uncertainty about the magnitude of ε generically grows with N. In other words, while coherence may be a good guide to intrinsic probability when there are very few microhypotheses, e.g., when we are considering the inductive support relations between two specific parts of a parameter space, we should be skeptical that coherence is a good guide to intrinsic probability when we consider large portions of a parameter space consisting of a large number of microhypotheses.

The bounds that I've identified on ε are consistent with both $P_A < P_B$ and $P_A > P_B$ and they remain consistent for arbitrarily large values of N. However, we *can* examine the space of possible values of ε if we treat ε as a random variable and assign a probability density to the space of possible values of ε, i.e., $\rho(\varepsilon)$. If we had an explicit functional form for $\rho(\varepsilon)$, we could then compute $Pr(P_A > P_B)$, that is, the probability that ε is large enough for P_A to exceed P_B.

There are three desiderata that $\rho(\varepsilon)$ should satisfy. First, comparatively more weight should be assigned to smaller values of ε, that is, $\rho(\varepsilon)$ should be a monotonically decreasing function. At best, $\rho(\varepsilon)$ could be a uniform distribution with support restricted to $[0, 1/2 - 1/N]$, which could, perhaps, be motivated through some version of the principle of indifference. Second, it would be preferable to have a single parameter λ that can be varied to study deviations from the uniform distribution, e.g., $\lambda = 0$ represents the uniform distribution and larger values represent deviations therefrom. Third, $\rho(\varepsilon)$ should have finite support only from 0 to $1/2 - 1/N$, since ε cannot assume values outside that interval. Unfortunately, there is no unique choice that satisfies all three desiderata. However, as it will turn out, the conclusions that I reach on the basis of $\rho(\varepsilon)$ will hold for any distribution that satisfies the three desiderata. One possible choice is the truncated exponential distribution:

$$\rho(\varepsilon) = \frac{\lambda \mathrm{e}^{-\lambda\varepsilon}}{1 - \mathrm{e}^{-\lambda(1/2 - 1/N)}} \tag{12.18}$$

The distribution needs to be truncated because, as discussed, ε can only assume values in the interval $[0, 1/2 - 1/N]$. Using the facts that $P_A = 1/N + \varepsilon$ and $P_B = 1/2 - 1/N - \varepsilon$, one can show that:

$$Pr(P_A > P_B) = Pr\left(\frac{1}{4} - \frac{1}{N} < \varepsilon \leq \frac{1}{2} - \frac{1}{N}\right) \tag{12.19}$$

In turn, this probability can be evaluated by integrating $\rho(\varepsilon)$:

$$Pr\left(\frac{1}{4}-\frac{1}{N}<\varepsilon\leq\frac{1}{2}-\frac{1}{N}\right)=\int_{\frac{1}{4}-\frac{1}{N}}^{\frac{1}{2}-\frac{1}{N}}\frac{\lambda\mathrm{e}^{-\lambda\varepsilon}}{1-\mathrm{e}^{-\lambda(1/2-1/N)}}d\varepsilon=\frac{1-\mathrm{e}^{\lambda/4}}{1-\mathrm{e}^{\lambda(1/2-1/N)}} \tag{12.20}$$

Having computed the integral, we can observe that $Pr(P_A > P_B)$ monotonically decreases as N increases; see figures 12.2 and 12.3. Moreover, $Pr(P_A > P_B)$ is maximized in the limit that λ becomes zero, that is, for the uniform distribution. In that limit, and for large N, one can show that $Pr(P_A > P_B) = 1/2$. This result should make intuitive sense. In the limit that $N \to \infty$, the upper bound on ε is $1/2$ and the lower bound on ε is 0. Moreover, for P_A to exceed P_B, ε must be at least $1/4$. Therefore, half of the values drawn from the uniform distribution restricted to $[0, 1/2]$ are such that P_A exceeds P_B. Ergo, assuming ε is drawn from a uniform distribution, $Pr(P_A > P_B) = 1/2$. For any other value of λ, for large N, $Pr(P_A > P_B)$ is less than $1/2$. A similar verdict can be reached for any other choice of $\rho(\varepsilon)$ that monotonically decreases with ε, since all possible choices of $\rho(\varepsilon)$ that monotonically decrease with ε will be bounded from above by the uniform distribution. For example, we can consider a truncated normal distribution with mean μ less than or equal to $1/4 - 1/N$ and study how $Pr(P_A > P_B)$ changes with the standard deviation σ. For the truncated normal distribution, one finds that:

$$Pr\left(\frac{1}{4}-\frac{1}{N}<\varepsilon\leq\frac{1}{2}-\frac{1}{N}\right)=\frac{\int_{\frac{1}{4}-\frac{1}{N}}^{\frac{1}{2}-\frac{1}{N}}\exp(-\frac{1}{2}(\frac{\varepsilon-\mu}{\sigma})^2)d\varepsilon}{\int_{0}^{\frac{1}{2}-\frac{1}{N}}\exp(-\frac{1}{2}(\frac{\varepsilon-\mu}{\sigma})^2)d\varepsilon}=\frac{\mathrm{erf}(\frac{1/2-1/N-\mu}{\sqrt{2}\sigma})-\mathrm{erf}(\frac{1/4-1/N-\mu}{\sqrt{2}\sigma})}{\mathrm{erf}(\frac{1/2-1/N-\mu}{\sqrt{2}\sigma})-\mathrm{erf}(\frac{-\mu}{\sqrt{2}\sigma})} \tag{12.21}$$

where $\mathrm{erf}(x)$ is the error function, defined as the integral of the Gaussian from 0 up to x. I wrote a short program to numerically solve this expression as a function of μ and σ in the limit that N goes to infinity; for results, see figure 12.4. As expected, when $\mu \leq 1/4$, $Pr(P_A > P_B) \leq 1/2$. Somewhat surprisingly, for values of μ larger than $1/4$, most values of σ yield a value for $Pr(P_A > P_B)$ that is not appreciably large. Nonetheless, this is consistent with the expectation that as σ becomes large as compared with $1/2 - 1/N$, that is, the size of the interval on which ε has support, the results for the truncated normal distribution should reproduce the results from the uniform distribution independent of the value of μ.

In the limit that we consider an infinity of possible tasks, the probability that P_A exceeds P_B is no greater than 1/2. The verdict that we can surmise from this discussion is that unless we have a well-motivated reason to think that ε has a large value, we cannot conclude that coherence saves the omnipotence hypothesis. More generally, when considering the extension of a hypothesis to an indefinitely large domain, unless we have a good reason to think otherwise, we should not be confident that coherence will make uniformity more probable than the alternative hypothesis that there is *some* variety, given all of the ways in which there could be variety. In application to hypotheses concerning the totality of physical reality, we should be skeptical concerning any confident claim that known physical principles extend to domains arbitrarily distant from those that are well understood.

12.6 Summary

This chapter completed my case for Cosmic Skepticism by turning to confirmation theory. I've previously shown that no widely agreed upon and empirically well supported theory demonstrates that the Cosmos began to exist. Nonetheless, scientific inferences are ampliative. One might have hoped that science could provide us reason to think that a beginning of the Cosmos is more probable than the contrary. In this chapter, I showed that, at least as philosophical and scientific inquiry currently stands, our hopes for an ampliative inference to the beginning of the Cosmos are dashed.

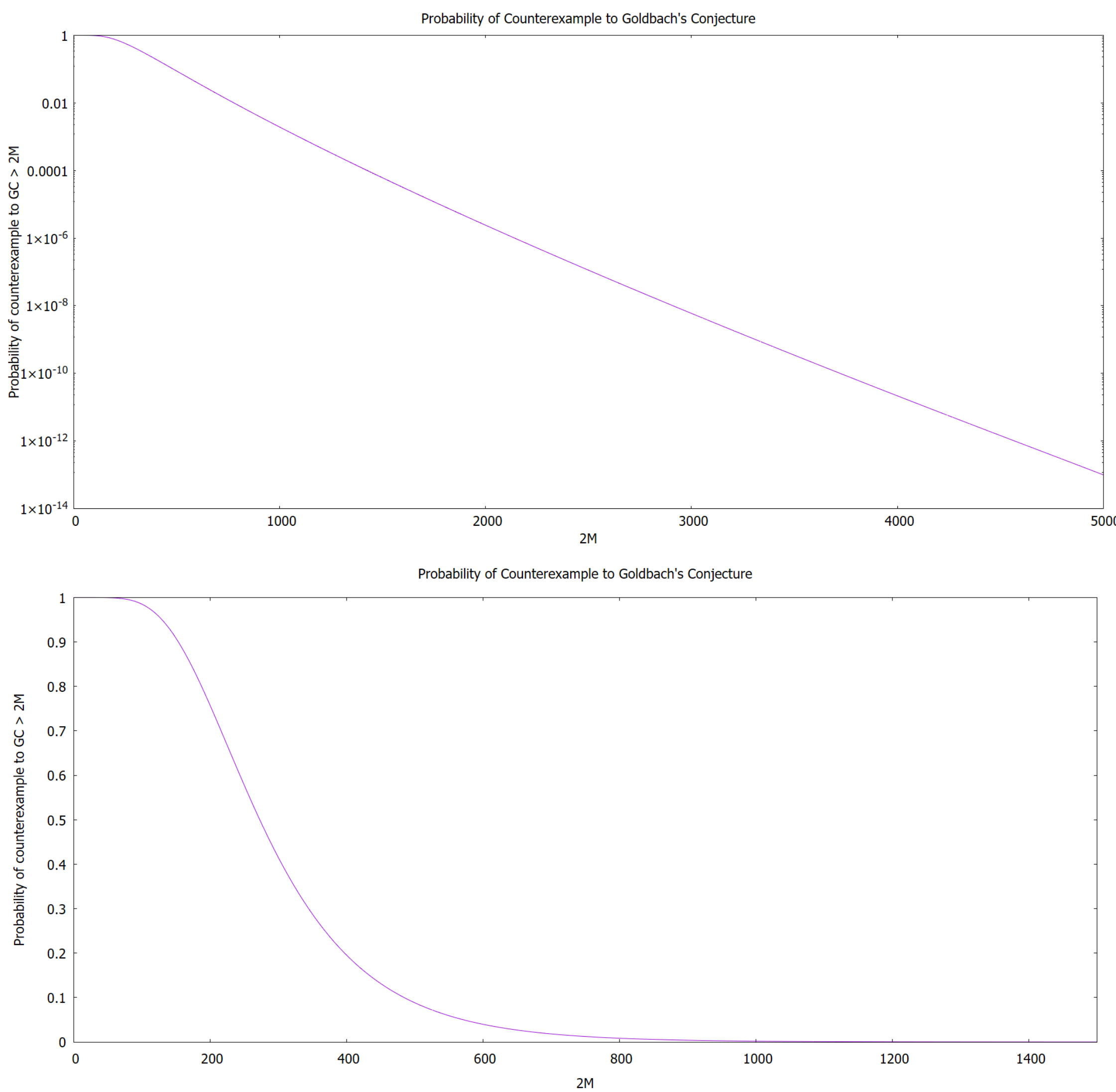

Figure 12.1. The probability that some number greater than $2M$ is a counterexample to Goldbach's Conjecture for various values of $2M$. Note that the majority of the probability mass is "clustered" around small values of $2M$ so that computational searches for a counterexample to the Goldbach Conjecture comes close to ruling out the possibility of a counterexample. Goldbach's Conjecture is a special case because there is no reason for the probability of a counterexample to a hypothesis to assume most of its mass within the accessible portion of a parameter space.

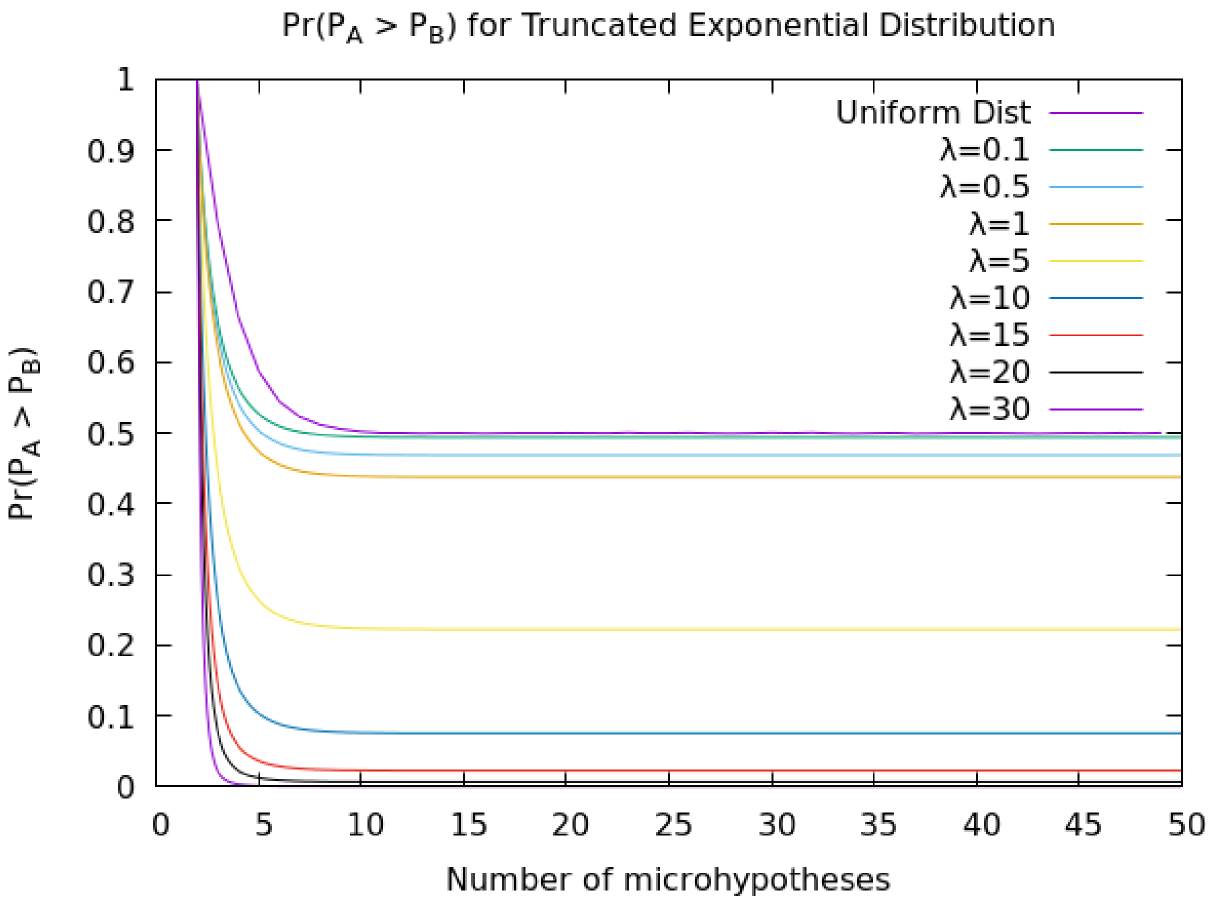

Figure 12.2. The probability that P_A exceeds P_B, as a function of the number of microhypotheses, assuming that ε is drawn from an exponential distribution restricted to $[0, 1/2 - 1/N]$.

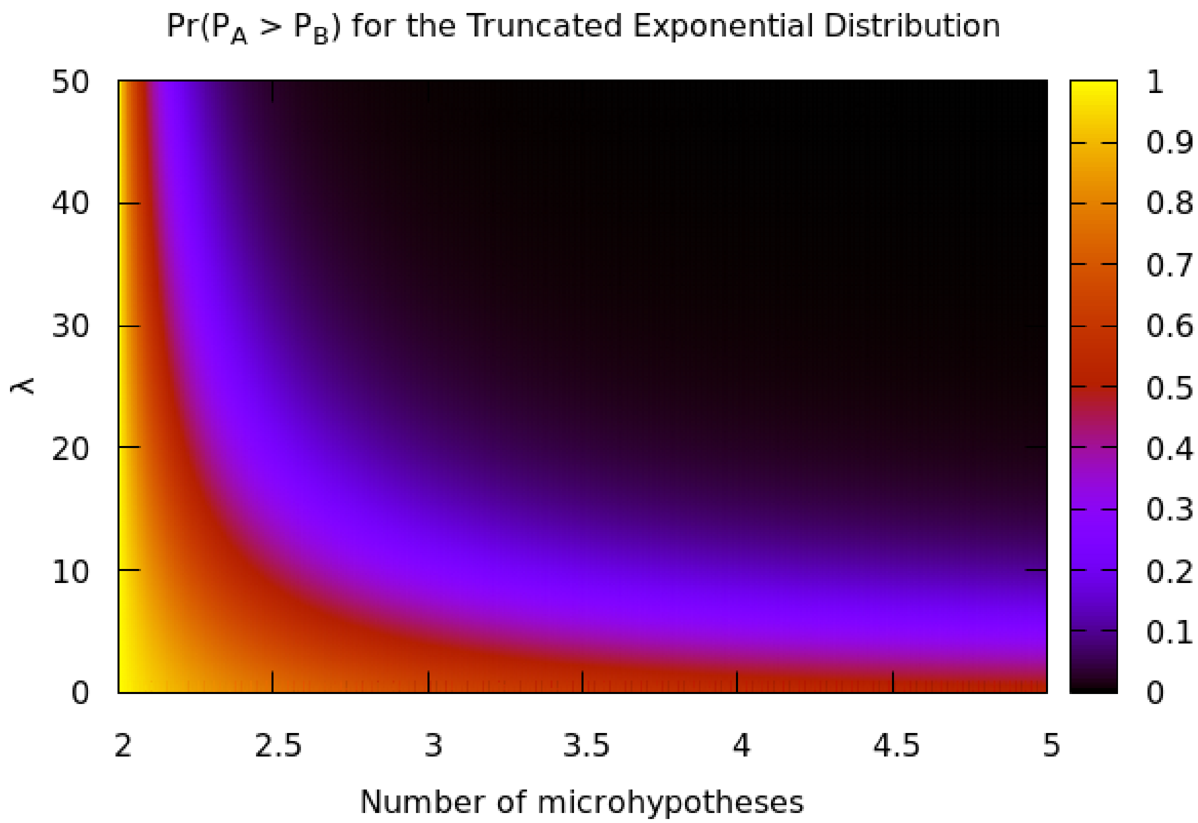

Figure 12.3. "Heat map" illustrating the probability that P_A exceeds P_B as a function of the number of microhypotheses on the assumption that ε is drawn from an exponential distribution restricted to $[0, 1/2 - 1/N]$. Note that although the number of microhypotheses is restricted to integer values greater than 2, the heat map was constructed using continuous values of the number of microhypotheses.

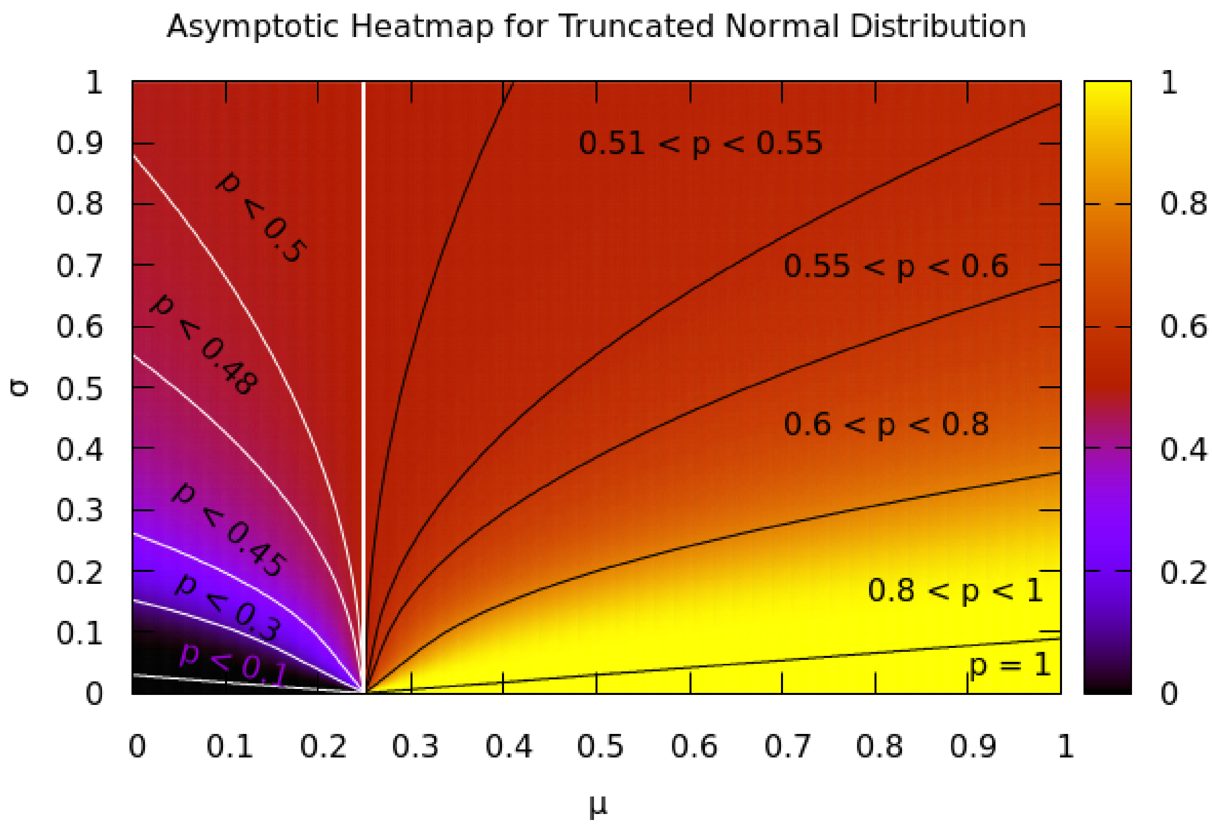

Figure 12.4. "Heat map" illustrating the behavior of $Pr(P_A > P_B)$ assuming that ε is drawn from a truncated normal distribution in the limit that $N \to \infty$. The variable p is an abbreviation for $Pr(P_A > P_B)$. Note that $Pr(P_A > P_B) = 1/2$ occurs only along the vertical line $\mu = 1/4$.

13. CONCLUSION

This dissertation has been a long argument against wholly empirical defenses of the second premise of the KCA, that is, that the Cosmos – the whole of physical reality – began to exist. The project was motivated in part I by the observation that a compelling case against the a priori defense of the second premise has already been provided in the literature. Given that a compelling case against the a priori defense of the second premise has already been provided in the literature, I devoted the rest of the dissertation to a consideration of the a posteriori (or wholly empirical) defense of the second premise. In part II, I turned to clarifying the concept that the Cosmos began to exist; unfortunately, while clarifying the concept that the Cosmos began to exist should be fundamental to any consideration of the KCA, few authors have attempted to clarify the concept. I develop three necessary (but not necessarily sufficient) conditions for the Cosmos to have had a beginning. The first condition encapsulates the notion that the Cosmos is fundamentally spatio-temporal:

1. Necessarily, if the Cosmos began to exist, then the Modal Condition is true. The Modal Condition states that at the closest possible worlds where time does not exist, the Cosmos does not exist.

In other words, the Cosmos began to exist only if there is nothing that suffices for the Cosmos's existence and which might have existed if time had not existed. The Modal Condition requires that the Cosmos is not fundamentally timeless. Given that the Cosmos is not fundamentally timeless, the second and third condition for the Cosmos to have had a beginning are conditions on the chronogeometric structure of the space-time S that the Cosmos includes:

2. Necessarily, if the Cosmos began to exist, then the Direction Condition is true. The Direction Condition states that here is a global direction of time throughout spacetime. There is a global direction of time throughout spacetime only if (i) S is temporally orientable, (ii) for any space-time point p, there is a locally defined

direction of time at p, (iii) for all pairs of points p and q in the space-time, the future (past) direction defined at p agrees with the future (past) direction defined at q.

3. Necessarily, if the Cosmos began to exist, then the Boundary Condition is true. The Boundary Condition states that either S includes a closed boundary to the past of all non-initial points in S (the topological conception) or S includes a finite initial segment (the metrical conception). S includes a finite initial segment just in case there is a space-like surface Σ such that all of the past directed half-curves originating on Σ have finite generalized affine length.

Having provided three necessary conditions for the Cosmos to have had a beginning, I turned to the relationship between my three conditions and classical models of the Big Bang in General Relativity. Given that Big Bang cosmology does not involve a discussion of the Modal Condition, on the conceptual analysis that I have offered for the beginning of the Cosmos, Big Bang cosmology cannot be a theory about the beginning of the Cosmos. The aims of Big Bang cosmology are narrower than my aims; Big Bang cosmology can perhaps be interpreted as a theory about the history of the observable universe and not a theory about the origins of the totality of physical reality. Nonetheless, if one assumes (i) the Direction and Boundary Conditions, (ii) that General Relativity is true, and (iii) the Cosmological Principle, one can derive Big Bang cosmology as a deductive consequence.

In part III, I turned to a defense of Cosmic Skepticism, the thesis that the provinciality of our current knowledge of the physical facts with respect to scale, spatio-temporal location, or energy prevents us from having empirical access to whether the Cosmos satisfies the Modal, Direction, and Boundary Conditions. In defense of Cosmic Skepticism, I offered the following argument:

4. We know the Cosmos began to exist only if we know the Cosmos satisfies the Modal Condition, the Direction Condition, and the Boundary Condition.

5. We do not know whether the Cosmos satisfies the three conditions.

6. Therefore, we do not know whether the Cosmos began to exist.

I developed three arguments to defend Cosmic Skepticism and thereby the view that we do not know whether the Cosmos satisfies the Modal Condition, the Direction Condition, and the Boundary Condition.

First, according to a widely held view in philosophy of science, we have reason to endorse unobservable entities if we have independent reason to endorse a broader theory that entails the existence of the unobservable entity. Chronogeometric structure is unobservable. For that reason, whether the Cosmos satisfies the Direction and Boundary Conditions is unobservable. My first argument addresses whether General Relativity, when conjoined with observational data, entails that the Cosmos satisfies the Direction and Boundary Conditions. General Relativity, conjoined with observational data, can entail that the Cosmos satisfies the Direction and Boundary Conditions only if cosmologically relevant space-times satisfying the Direction and Boundary Conditions are observationally distinguishable from space-times that do not satisfy the Direction and Boundary Conditions.

In the context of General Relativity, whether space-times having some precisely specifiable feature are observationally distinguishable from space-times lacking that feature turns out to be a tractable mathematical question that can be resolved using tools standardly employed (by mathematical and theoretical physicists) when studying the topology and large scale structure of relativistic space-times. I proved that a broad class of space-times – including cosmologically relevant space-times – that satisfy the Direction and Boundary Conditions are observationally indistinguishable from space-times that do not satisfy the Direction and Boundary Conditions.

Thus, General Relativity, conjoined with observational data, does not entail whether the Cosmos satisfies the Direction and Boundary Conditions. While General Relativity will likely be supplanted in future physical inquiry, we do not yet know what features General Relativity's successor theory will include. For that reason, we cannot yet use General Relativity's successor to determine whether the Cosmos satisfies the Direction and Boundary Conditions.

My discussion of observationally indistinguishable space-times makes a contribution to the literature on the KCA in an additional respect. While proponents of the KCA have analyzed specific cosmological models, e.g., Craig and Sinclair, 2009, 2012, there are a large number of live mutually exclusive cosmological models that are consistent with all of our observational data. Many of those models were developed as toy models or to explore physical possibilities and so were not intended as probable descriptions of the Cosmos as a whole. Moreover, many of the best models appear to be equally well supported by the data. Assuming that the model with the greatest epistemic probability is not significantly more probable than at least one other model, since the epistemic probabilities of mutually exclusive cosmological models must sum to 1, the epistemic probability of the most probable model is no greater than about 0.5. Even if suppose that *some* cosmological model is much more probable than any other cosmological model, no more than one cosmological model can have an epistemic probability greater than 0.5. Thus, most cosmological models are improbable. We should not be surprised if friends of the KCA are able to show that most cosmological models that lack a beginning are improbable. We should instead be interested in what, if anything, we can say about the large scale structure of space-time using the observational data empirically accessible to us. That question – what can we say about the global structure of space-time on the basis of our observations? – is the central question that has been investigated in the literature on observationally indistinguishable space-times. And while physicists and philosophers of physics have been discussing observationally indistinguishable space-times and global space-time structure for several decades, authors interested in the KCA have not yet taken notice. I have explicitly shown how results in that literature might be brought to bear on the KCA.

Next, I discuss some constraints imposed on our knowledge of cosmological history by foundational work in statistical mechanics. Either the Cosmos violates the Modal Condition or else that there is a transcendental condition on the possibility of our knowledge of the past that prevents us from having knowledge of states of affairs prior to a specific past boundary. Since the Cosmos began to exist only if the Modal Condition is satisfied, the Cosmos began to exist only if we do not have knowledge of cosmological history prior to a specific past boundary. Here we met a warning from the nineteenth century: the fact that there is some past boundary beyond which we cannot make reliable inferences does not entail that the Cosmos satisfies the Boundary Condition. And so we meet a second argument in defense of Cosmic Skepticism, viz, that there is a past boundary beyond which we cannot make reliable inferences suggests that the provinciality of our knowledge of the physical facts with respect to spatio-temporal location prevents us from knowing whether the Cosmos satisfies the Boundary Condition.

I turned to a broad family of live cosmological models – bounce cosmologies – according to which a past dynamical process prepared the compressed state that initiated the observable universe in the Big Bang. While friends of the KCA have interpreted bounce cosmologies to satisfy the Boundary Condition, I defended the traditional interpretation of bounce cosmologies according to which bounce cosmologies violate the Boundary Condition. We don't know whether any bounce cosmology is correct, much less whether any bounce cosmology is correct in sufficient detail to infer whether the Cosmos satisfies the Boundary Condition. Nonetheless, given that bounce cosmologies are empirically live options, we should not rule out bounce cosmologies prematurely. Again, this argument adds to my case for Cosmic Skepticism. Due to the provinciality of our knowledge with respect to scale, time, space, and energy, we do not know whether one of the bounce cosmologies is correct, or at least correct in sufficient detail to suggest on their basis whether the Cosmos satisfies the Boundary Condition. For that reason, we do not know whether the Cosmos began to exist.

Fourth, I completed my case for Cosmic Skepticism by turning to confirmation theory. There are two families of inferences that might be used as part of an argument for the conclusion that the Cosmos satisfies the Modal, Direction, and Boundary Conditions and so began to exist: part-to-part inferences and part-to-whole inferences. Part-to-part inferences first project empirical regularities from an observable portion of the Cosmos to an unobservable portion and then, using that empirical regularity in the unobservable portion, infer either that the Cosmos began to exist or that the Cosmos has features relevant for determining whether the Cosmos began to exist. I argued that part-to-part inferences fail for three reasons. First, part-to-part inferences rely upon a weak analogy between observable and unobservable portions of the Cosmos. Second, we have no good reason for thinking that the known physical facts are representative of all of the physical facts that there are. Our knowledge of the physical facts is provincial with respect to scale, spatio-temporal location, and energy. For that reason, part-to-part inferences are poor inferences.

In contrast, part-to-whole inferences first project empirical regularities from the observable portion of the Cosmos to the whole Cosmos and, second, use the unrestricted version of the empirical regularities to infer that the Cosmos began to exist or that the Cosmos has features relevant for determining whether the Cosmos began to exist. Part-to-whole inferences similarly fail because we have no good reason for thinking that the known physical facts are representative of all of the physical facts that there are. Our knowledge of the physical facts is provincial with respect to scale, spatio-temporal location, and energy. Thus, part-to-whole inferences are also poor inferences. But part-to-whole inferences face another problem with respect to their intrinsic probability. Assuming that Draper's account of intrinsic probability is correct, the intrinsic probability of a hypothesis is determined by the modesty of the hypothesis, the coherence of the hypothesis, and nothing else. As I have argued, there is a tension between modesty and coherence that grows as the scope of a hypothesis is increased. Hypotheses concerning the Cosmos – the totality of physical reality – have the broadest scope that hypotheses about physical reality can possibly have. Since there is, as yet, no widely agreed upon or well-supported theory for resolving the tension between modesty and coherence, we are ill-equipped to judge the intrinsic probability of hypotheses concerning the Cosmos. Therefore, at the present stage of inquiry, we are ill-equipped to judge whether the Cosmos satisfies the Modal, Direction, and Boundary Conditions. Consequently, we are ill-equipped to judge whether the Cosmos began to exist.

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