UNIVERSIDADE ESTADUAL DE CAMPINAS
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An axiomatic approach to theodicy via formal applied systems

Uma abordagem axiomática à teodiceia via sistemas formais aplicados

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Dissertação de Mestrado apresentada ao Instituto de Filosofia e Ciências Humanas da Universidade Estadual de Campinas como parte dos requisitos para a obtenção do título de Mestre em Filosofia.

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A Comissão Julgadora dos trabalhos de Defesa de Dissertação de Mestrado, composta pelos Professores Doutores a seguir descritos, em sessão pública realizada em 22 de dezembro de 2020, considerou o candidato Gesiel Borges da Silva aprovado.

Prof. Dr. Fábio Maia Bertato
Presidente da Comissão Julgadora

Prof. Dr. Marcelo Esteban Coníglio
IFCH/UNICAMP

Prof. Dr. Evandro Luís Gomes
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A Ata de Defesa com as respectivas assinaturas dos membros encontra-se no SIGA/Sistema de Fluxo de Dissertações/Teses e na Secretaria do Programa de Pós-Graduação em Filosofia do Instituto de Filosofia e Ciências Humanas.

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mentor, colleague, friend, and brother.

The disorders of the mind, continued Demea, though more secret, are not perhaps less dismal and vexatious. Remorse, shame, anguish, rage, disappointment, anxiety, fear, dejection, despair; who has ever passed through life without cruel inroads from these tormentors? How many have scarcely ever felt any better sensations? Labour and poverty, so abhorred by every one, are the certain lot of the far greater number; and those few privileged persons, who enjoy ease and opulence, never reach contentment or true felicity. All the goods of life united would not make a very happy man; but all the ills united would make a wretch indeed; and any one of them almost (and who can be free from every one?) nay often the absence of one good (and who can possess all?) is sufficient to render life ineligible.

— David Hume,
Dialogues concerning natural religion, X

We all believe in our hearts
and confess with our mouths
that there is a single
and simple
spiritual being,
whom we call God —

eternal,
incomprehensible,
invisible,
unchangeable,
infinite,
almighty;
completely wise,
just,
and good,
and the overflowing source
of all good.

— Guido de Brès,
Confessio Belgica, Article 1.

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Abstract

Edward Nieznański developed two logical systems in order to deal with a version of the problem of evil associated with two formulations of religious determinism. The aim of this research was to revisit these systems, providing them with a more appropriate formalization. The new resulting systems, namely, **N1** and **N2**, were reformulated in first-order modal logic; they retain much of their original basic structures, but some additional results were obtained. Furthermore, our research found that an underlying minimal set of axioms is enough to settle the questions proposed. Thus, we developed a minimal system, called **N3**, that solves the same issues tackled by **N1** and **N2**, but with less assumptions than these systems. All of the systems developed here are proposed as solutions to the logical problem of evil through the refutation of two versions of religious determinism, showing that the attributes of God in Classical Theism, namely, those of omniscience, omnipotence, infallibility, and omnibenevolence, when formalized, are consistent with the existence of evil, providing one more response to this traditional issue.

Keywords: Theodicy; problem of evil; modal logic; divine attributes; philosophy of religion.

Resumo

Edward Nieznański desenvolveu dois sistemas lógicos com o fim de lidar com uma versão do problema do mal associada a duas formulações de determinismo religioso. O objetivo desta pesquisa foi revisitar esses sistemas, proporcionando-lhes uma formalização mais adequada. Os novos sistemas resultantes, denominados **N1** e **N2**, foram reformulados em lógica modal de primeira ordem; ambos têm muito de suas estruturas básicas originais, mas alguns resultados adicionais são obtidos. Além disso, nossa pesquisa descobriu que um conjunto mínimo de axiomas subjacentes é suficiente para resolver as questões propostas. Assim, desenvolvemos um sistema minimal, denominado **N3**, que resolve os mesmos problemas tratados por **N1** e **N2**, mas com menos suposições do que estes sistemas. Todos os sistemas aqui desenvolvidos são propostos como soluções ao problema lógico do mal através da refutação de duas versões do determinismo religioso, mostrando que os atributos de Deus no Teísmo Clássico, a saber, os de onisciência, onipotência, infalibilidade e onibenevolência, quando formalizados, são consistentes com a existência do mal, fornecendo mais uma resposta a essa tradicional questão.

Palavras-chave: Teodiceia; problema do mal; lógica modal; atributos divinos; filosofia da religião.

Contents

Introduction

The application of logic to philosophical issues has a long and fruitful tradition in the history of philosophy. The last century, in particular, exhibited the growth of many fields within philosophical investigation in which logic and other areas of philosophy are inextricably intertwined, such that many philosophical issues intrinsically rely upon logical concerns.[1] Thus, in this sense, it is not a very easy task to deal with many relevant philosophical questions nowadays without appealing to formal logic and its benefits.

The aim of this research is to establish an axiomatic approach to deal with the problem of evil, one of the most relevant questions in the history of philosophy. This problem is proposed in contemporary philosophy in many ways, but one of the foremost versions of it is the logical problem of evil: the claim that the propositions "God exists" and "evil exists" are mutually inconsistent. While many works regarding this problem have been developed, this research intends to accomplish an original proposal, one that holds value for itself as an answer to the logical problem of evil and to religious determinism as well.

In the first chapter, an overview on the problem of evil is provided. Some historical formulations of the problem, from the Book of Job to contemporary philosophy, are presented. Among contemporary formulations, the logical problem of evil has been presented as a *logical* issue, proposed initially by John Mackie, as an allegation related to the rationality of classical theism. The problem raised by Mackie, as well as one of

[1]Many examples could be given here in order to support this affirmation. For instance, during the twentieth century, logical positivists defended that philosophical issues could be reduced to logical and scientific concerns (UEBEL, 2020); the Cracow Circle, on its turn, defended the application of formal tools to philosophical and theological questions (NIEZNAŃSKI, 1987). Some decades later, Alvin Plantinga (1974; 1977) applied possible worlds semantics to deal with the logical problem of evil. More recently, examples of groups that have developed many application of formal tools to philosophical investigation include the Brazilian School of Logic and the Polish School of Argumentation; see Agazzi, D'Ottaviano, and Mundici (2011) and Budzynska et al. (2014) for relevant summaries. Finally, on the application of logic to religious issues nowadays, see Beziau and Silvestre (2017), Da Silva and Bertato (2019), and Bertato (2020).

the main solutions to this problem – the free will defense proposed by Alvin Plantinga – are also presented. Yet, among the main tenets of this work is the belief that logical questions deserve logical answers. As such, this research looked for a different proposal that would suit that belief, and we found it in the two axiomatic systems proposed by Edward Nieznański to deal with the problem of evil. While many merits of these systems are to be recognized, other details required a careful treatment. We present some of these issues before properly revisiting his proposals.

The second chapter presents **N1**, the revisiting of Nieznański's first system (2007), and the third chapter presents **N2**, the revisiting of Nieznanski's second system (2008). These new resulting systems were reformulated in first-order modal logic; their new sets of axioms, definitions and theorems are presented. As we will explore, they have much in common with Nieznański's systems, but some different achievements are obtained.

Finally, in chapter 4, a new system is presented. This system, called **N3**, results of the two previous systems: it addresses many of the central questions tackled by **N1** and **N2**, but with less assumptions than these systems. As an outcome, **N1**, **N2**, and principally **N3** are proposed as solutions to the logical problem of evil through the refutation of two versions of religious determinism, showing that the attributes of God are consistent with the existence of evil.

It is clear to us that proposing such systems does not to solve the problem of evil as a whole. We should be aware that, in many cases, to present a logical system to answer such difficult questions can not only be improper, but even a lack of empathy. Facing evil in the world can be difficult, and it leads to many philosophical, religious, and personal challenges. However, the intention of this work is much more modest: its aim is to solve a philosophical question, stated as a logical problem, through logical tools, answering whether it is rational or not to believe in God given that evil exists, or at least providing some clarification on this issue. As an outcome, we hope this research may inspire both philosophers and logicians to search for more applications of formal tools to philosophical investigation.

Chapter 1

The problem of evil: why does the debate continue?

The aim of this chapter is to provide a brief account of the logical problem of evil and the issue on religious determinism. In the first section, some traditional formulations of the problem are presented; both older and contemporary formulations are described to introduce the current discussion to the reader. The second section describes the logical problem of evil, as well as Plantinga's free will defense, regarded as one of the most influential solutions to such problem. Finally, Nieznański's approaches to the problem of evil are presented in the last section of the chapter, and the main proposal delineated in the work is established.

On dealing with such an old and multifaceted question, many choices had to be made; thus, the accounts here described are far from exhaustive. Still, I think that many readers may find this approach a comprehensive one, and the references may serve as a guideline to further inquiries for those who are interested.

1.1 Traditional formulations of the problem

1.1.1 From the Book of Job to early modern philosophy

The problem of evil is arguably one of the most relevant questions in the history of human thought. It would be hard to give a detailed outline of all of the accounts of this issue ever formulated, and it is not the purpose of this work to provide such an overview, but back in ancient times we already see many instances of this problem, or at least some glimpses of it. For instance, the Book of Job is perhaps one of the most ancient books of both the Jewish Tanakh and the Christian Bible, and its main theme is Job's faithfulness to God in face of evil and suffering. In the book, many depictions of Job's hardships are portrayed in order to provide a wide existential reflection on the issue. For instance, in one of the first scenes of the narrative, God declares how Job is still faithful to Him, despite passing through many sufferings in his life. Then, Satan challenges God: "A man will give all he has for his own life. But now stretch out your hand and strike his flesh and bones, and he will surely curse you to your face" (Job 2:4).[1] Job is questioned about his faith in God by his own partner: "His wife said to him, 'Are you still maintaining your integrity? Curse God and die!' He replied, 'You are talking like a foolish woman. Shall we accept good from God, and not trouble?' In all this, Job did not sin in what he said" (Job 2:9-10). Later, his friends try to explain how could a good person like him pass through such dreadful suffering, including loss of his offspring, diseases, and hardship and sorrow of many kinds. In the end, according to the narrative, Job receives the best answer – a direct encounter with God Himself, who reveals Himself to Job and declares His sovereignty over the world.[2]

When it comes to Western philosophical tradition, the problem of evil appears early in Ancient Greece. However, as it will become clear, the many forms in which this problem appear then are far from our current interpretation of the issue. According to Michael Hickson (2013), the problem "lurks in the background" of some passages in the works of Plato, including the *Republic* (379c), *Timaeus* (48a, 29e, 30b, 86d–e), and *Laws* (X, 899b, 896c, 896e, 901d–903a). Plato considers evil in his account for the creation of the world, and how evil could be related to eternal primary and secondary causes. However, as Hickson goes on to argue, Plato does not explicitly proposes the question, and moreover, he is not concerned with the account on how could an almighty and good being allow evil; instead, Plato's concerns rely much more on how a dualistic account of reality could provide an explanation of such evil in the world (HICKSON, p. 4-6).

Epicurus is one of the ancient Greek philosophers to which the problem of evil is more frequently attributed. The tradition of attributing the origins of this problem to Epicurus has become stronger after David Hume's well-known statement of the problem

[1]The biblical passages cited are from the New International Version (1984).

[2]Job, 38-41. Job speaks on the experience: "My ears had heard of you, but now my eyes have seen you" (Job 42:5).

(see p. 20), and it has been taken by many authors as one of the first statements of an argument from evil, understood not as an existential claim, but much more as a statement against belief in a deity similar to that of classical theism.[3] The problem of evil is even called "Epicurean paradox" or "Paradox of Epicurus" by some authors, since it seems to be paradoxical that an omniscient, omnipotent, and omnibenevolent being could be compatible with the existence of evil.[4] However, as Hickson notes, it is difficult to know for sure whether Epicurus really stated the "Epicurean paradox". As he says:

> Perhaps Epicurus was the author of the "old questions" made famous by Hume, but perhaps he was not – not enough evidence exists to decide the question. There is evidence, however, that the questions were posed by early Skeptics, and/or perhaps Gnostics [...]. While the trilemma attributed to Epicurus is found in none of his extant works, or in those of his earliest followers, an expanded version of the old questions is found in the third book of the *Outlines of Scepticism* by Sextus Empiricus (circa 160–210) (HICKSON, 2013, p. 7).

Thus, while the problem of evil may have not been stated by Epicurus, Skeptics and Gnostics have established allegations related to the question. But Hickson defends that Skeptics have not done it as a challenge to belief in God, and, therefore, their stances do not consist in the problem of evil as we know. Sextus Empiricus, for instance, seems to affirm that evil is a threat to different beliefs in gods just to propose that we should suspend judgment concerning the existence of deities – and we should maintain the same attitude towards many subjects: neither believe, nor disbelieve them.[5] On the other hand, Gnostics, and more specifically, Manicheans, are among the groups that assumed a dualistic framework and opposed Christian philosophers and theologians concerning the problem of evil. In his works *On free choice of the will* (1993) and *Confessions* (2002, especially book V), Saint Augustine shows how he initially assumed the dualistic account of Manicheans and rejected Christian view of God and His Providence. They believed that two deities ruled in the world – one related to the light, and other, to the darkness; as a consequence, human beings had two souls, governed by each of those principles.

[3]One of such authors is Alvin Plantinga (2004, p. 3).

[4]See, for instance, Cudby (2005).

[5]Hickson resumes his position as follows: "Is Sextus Empiricus the first to offer an argument from evil? If arguments from evil are ultimately meant to disprove the existence of a good and powerful God, then the answer is "no." As a Pyrrhonian skeptic, Sextus does not argue for or against the existence of gods, but recommends suspension of judgment on the question" (2013, p. 8).

Thus, Manicheans concluded that human beings are neither free, nor responsible for their choices.[6]

Among medieval formulations of the problem of evil we see that of Saint Thomas Aquinas. In his *Summa Theologica*, Aquinas proposes it as one of the objections to the existence of God:

> Whether God Exists?
>
> Objection 1: It seems that God does not exist; because if one of two contraries be infinite, the other would be altogether destroyed. But the word "God" means that He is infinite goodness. If, therefore, God existed, there would be no evil discoverable; but there is evil in the world. Therefore God does not exist. (*Summa Theologica*, I, Q. 2, Art. 3. Cf. Saint Thomas Aquinas (1947).)

Of course, Aquinas probably did not mean to affirm that the problem is a real challenge to belief in God, in the sense of a debunking argument. In the passage, no response is given to the problem of evil; actually, this objection appears right before the statement of his famous Five Ways as a cumulative proof to the existence of God. This might be an indicative that one should first consider the positive reasons for the existence of God, as classical theists usually conceive it, before supporting seriously an argument from evil (if such an argument is possible).

Many other philosophers and theologians deal with the question in the medieval and early modern periods. For instance, Hickson mentions the Arminian controversy on the doctrine of predestination, Descartes's concerns regarding skepticism and divine benevolence, and Pierre Bayle's formulation of the problem of evil, which was very influential among philosophers during the Enlightenment (HICKSON, 2013, p. 11–14). Now, we turn our focus to Hume's statement on the problem, one of the most influential versions of it until contemporary debates.

1.1.2 Hume and the period of Enlightenment

The reader may have realized that, while versions of problems of evil are abundant in the history of philosophy, the problem of evil as a challenge to classical

[6]Cf. Oliveira (1996, p. 15) for a brief summary of these doctrines. He notes that Augustine's theodicy of free will was especially written as a response to Manicheans.

theism is quite recent in historical terms. Actually, even responses to it are now proposed in order to justify belief in God and to provide reasons against disbelief. It is not the focus of this section to deal with such responses, but one good example is the famous *Theodicy* of Gottfried Wilhelm Leibniz (1710/1985).[7] The work was written in response to many objections, casting optimistic reasons for believing in God. Leibniz affirms that, as God is infinitely good, He knows what is the best of the possible worlds; as He is almighty, He can create such a world. Therefore, our world should be the best of the possible worlds, despite evils existing in it.[8] This sparked many reactions during the period of "Enlightenment". Voltaire, for instance, wrote his book *Candide* in response to Leibniz. In the story, Candide is a very optimistic man who believes that our world is the best possible, and this belief is held amid many misfortunes and disasters (1758/2012). The book was written by the time of an earthquake in Lisbon (1755), an event that, on its turn, led some philosophers to consider seriously the problem of evil and the possible accounts on Divine Providence.[9]

However, it is David Hume who provided one of the most famous formulations of the problem of evil of that period. In his work *Dialogues concerning natural religion*, the philosopher lists a number of evils, diseases and horrible situations, and gives some emphasis to how human life is full of pain, suffering and evil of many kinds. The characters of the dialogue consider that, in face of such evils, the value of human life seems to completely vanish:

> [...] Remorse, shame, anguish, rage, disappointment, anxiety, fear, dejection, despair; who has ever passed through life without cruel inroads from these tormentors? How many have scarcely ever felt any better sensations? Labour and poverty, so abhorred by everyone, are the certain lot of the far greater number: And those few privileged persons, who enjoy ease and opulence, never reach contentment or true felicity. All the goods of life united would not make a very happy man: But all the ills united would make a wretch indeed; and any one of them almost (and who can be free from everyone?) nay often the

[7] *Theodicy* comes from two Greek words, Θεος ("God") and δικη ("justice"). It is widely affirmed that this was the very first time that the word has been used (WOUDENBERG, 2013, p. 181)

[8] In contemporary debate, this allegation is known as the "Lapse of Leibniz". See p. 27 of this work.

[9] On this subject, see Marques (2005).

> absence of one good (and who can possess all?) is sufficient to render life ineligible (HUME, 1779/2007, p. 71).

As human life is cruel and painful, so the reasoning goes, one might question whether such a person like God really controls the ends of life. Hume compares the attributes of God to "virtues of human creatures": if virtuous persons would conduct human life to happiness, then what should we expect from God, given that He is omnipotent, omniscient and totally good?[10] However, in light of human misery, one may question whether there is such a God. This entails one of the most famous formulations of the problem of evil:

> *Epicurus*' old questions are yet unanswered. Is he willing to prevent evil, but not able? then is he impotent. Is he able, but not willing? then is he malevolent. Is he both able and willing? Whence then is evil? (1779/2007, p. 74.)[11]

Questions like those seem to be difficult to answer. If God is omnibenevolent, *i.e.*, He wills evil situations not to happen, but He cannot prevent them to happen, then He is not omnipotent; if God is omnipotent, *i.e.*, He can prevent evil situations, but does not will to do that, then He is not omnibenevolent. If God is omnibenevolent and omnipotent, should not we expect no evil to exist in the world? Given the previous passage, one may come to the conclusion that Hume tends to think that the existence of evil requires teleological explanation, but no explanation is found by just comparing the attributes of God to our ordinary sense of goodness and happiness. This issue becomes more explicit in the passage below, another famous passage asserting the modern problem of evil, as recognized nowadays:

> [...] Why is there any misery at all in the world? Not by chance surely. From some cause then. Is it from the intention of the deity? But he is perfectly benevolent. Is it contrary to his intention? But he is almighty. Nothing can

[10]This could be seen as an irony, since classical theists usually compare human beings to God, and not the opposite. Here, as in other works, Hume seems to assume a skeptical position, playing the "devil's advocate".

[11]Note that Hume establishes Epicurus as a reference for his statement. Nonetheless, as affirmed in the last section, this reference is uncertain; it seems much more that such old skeptical tradition is recovered by Hume and, further, it is interpreted as an atheological challenge to belief in God. Meanwhile, Epicurus himself did not think in the contemporary categories we usually do.

> shake the solidity of this reasoning, so short, so clear, so decisive; except we assert, that these subjects exceed all human capacity, and that our common measures of truth and falsehood are not applicable to them. (1779/2007, p. 76)

Much could be said about those passages, but for our purposes, they state, together, a number of questions that are historically recovered as challenges to belief in the God of classical theism. From that period until nowadays, his formulation is widely mentioned, and remains as a kind of "standard" formulation of the problem of evil.

Nevertheless, as we will see in the following, some philosophers think that these questions need no answer for many reasons. Other formulations came to be the focus of contemporary debate, and one of them is the focus of our work.

1.2 The logical problem of evil and Plantinga's free will defense

In the first half of the last century, philosophical debates on religion were almost absent. At that time, Logical Positivism was very influential in the Anglo-Saxon tradition, and their so-called "principle of verification" was accepted by many philosophers as a criterion for justification: beliefs can only be accepted on the basis of either empirical or logical verification. Thus, metaphysics, aesthetics, value theory, theology, and other areas of human thought were completely rejected (UEBEL, 2020). As a consequence, religious propositions like "God exists" or "God created the world" had no space in mainstream philosophical circles.[12]

Later, in the decades of 1950 and 1960, Logical Positivism and its verificationist agenda was not widely held among philosophers. However, although belief in God was not considered meaningless anymore, many philosophers claimed that belief in God was irrational, and one of the objections raised against religious belief was the allegation that evil is inconsistent with the existence of God. H. J. McCloskey, for example, said: "Evil

[12] Perhaps, among philosophers in the "analytic stream" of philosophy at that time (in the decade of 1930), the only exceptions are those from the "Cracow Circle", like Jan Salamucha, Jan Franciszek Drewnowski and Józef Maria Bocheński. Their works are remarkable for the application of formal tools to philosophical and theological problems, but much of the production of these philosophers remains untouched (WOLEŃSKI, 2013; MURAWSKI, 2015)

is a problem for the theist in that a contradiction is involved in the fact of evil, on the one hand, and the belief in the omnipotence and perfection of God on the other" (MCCLOSKEY, 1960, p. 97). Aiken (1957) also suggested this very idea, although in a much more subtle way.

1.2.1 John Mackie on the logical problem of evil

In this context, it is widely attributed to John Mackie the merits of formulating the objection that came to be known as the *logical problem of evil.* In his article *Evil and Omnipotence*, the philosopher affirms:

> In its simplest form the problem is this: God is omnipotent; God is wholly good; yet evil exists. There seems to be some contradiction between these three propositions, so that if any two of them were true the third would be false. But at the same time all three are essential parts of most theological positions; the theologian, it seems, at once must adhere and cannot consistently adhere to all three. (MACKIE, 1955, p. 92-93)

In other words, Mackie affirms that there is a logical contradiction between belief in the God of classical theism and the existence of evil. On the one hand, the "theologian", representing classical theists in general, must adhere to propositions that state the attributes of God and the existence of evil; but, on the other hand, they are all contradictory. Thus, the "theologian" finds himself in a dilemma: he must *either* hold a consistent set of propositions in order to be rational *or* be a theist.

Despite his strong claim, Mackie recognizes that those propositions alone are not sufficient to lead to a logical contradiction,[13] and he goes on to explain some "principles" such that an "incompatibility" (as he puts it) is obtained:

> However, the contradiction does not arise immediately; to show it we need some additional premises, or perhaps some quasi-logical rules connecting the terms "good" and "evil" and "omnipotent" These additional principles are that good is opposed to evil, in such a way that a good thing always eliminates evil as far as it can, and that there are no limits to what an omnipotent thing

[13] As Alvin Plantinga noted in an exhaustive analysis of Mackie's claims, there is no explicit or implicit contradiction between the propositions stated. (PLANTINGA, 1977, p. 9-16.)

> can do. From these it follows that a good omnipotent thing eliminates evil completely, and then the propositions that a good omnipotent thing exists, and that evil exists, are incompatible (MACKIE, 1955, p. 93)

It is possible to summarize Mackie's objection as follows. Let g denote the proposition "God is omnipotent and wholly good", and e the proposition "Evil exists"; then, one can maintain that the logical problem of evil is the allegation that the set $\{g, e\}$ is contradictory, or that $\{g, e\} \vdash \bot$. However, as Mackie says, it is not possible to obtain the contradiction immediately; it is necessary to connect the terms "good", "evil" and "omnipotent" in order to advance the claim that the existence of God is contradictory with that of evil. His suggestion is to include the following propositions: "a good thing always eliminates evil as far as it can", denoted here by o, and "there are no limits to what an omnipotent thing can do", denoted here by l. Thus, one may synthesize Mackie's objection as the claim that $\{g, e, o, l\} \vdash \bot$.

But is it possible for a contradiction to obtain here? And if so, how?

1.2.2 Plantinga's free will defense

One of the most relevant and sophisticated responses to Mackie's claims was that of Alvin Plantinga. In a series of works, he argues that Mackie's logical objection to theism is not successful, elaborating his free will defense in response to the logical problem of evil (PLANTINGA, 1965, 1967, 1974, 1977). Here, I combine the versions of his argument developed in the books *God, freedom and evil* (originally published in 1974, and republished in 1977) and *The nature of necessity* (1974), in order to give a brief summary of the discussion, and to show how other proposals might be relevant to philosophical debate.

In the first place, Plantinga recognizes that the problem of evil is the most telling criticism to theism. But he also affirms that even if a theist does not have a good explanation on why or how God permits evil in the world, this does not mean that theism is irrational. Due to many reasons (for instance, human epistemic limitations), a theist may read to a claim like that of David Hume (see p. 20) and not feel compelled to abandon belief in God, and even may be violating no epistemic duty or norm on doing so. As he says,

> [...] suppose none of the suggested theodicies is very satisfactory. Or suppose that the theist admits he just doesn't know why God permits evil. What follows from that? Very little of interest. Why suppose that if God does have a good reason for permitting evil, the theist would be the first to know? Perhaps God has a good reason, but that reason is too complicated for us to understand. Or perhaps He has not revealed it for some other reason. The fact that the theist doesn't know why God permits evil is, perhaps, an interesting fact about the theist, but by itself it shows little or nothing relevant to the rationality of belief in God. Much more is needed for the atheological argument even to get off the ground. (PLANTINGA, 1977, p. 10.)

Thus, it may be the case that the theist does not possess a good answer to Hume's questions. Furthermore, it is not enough to just put forth a set of difficult questions to theists, for there are many difficult questions to which we have no answer, and that does not mean that we are irrational. Thus, an argument from evil should be a positive claim regarding the rationality of theism, as it is the case of Mackie's objection (PLANTINGA, 1977, p. 11).

However, Plantinga analyses the claims made by Mackie and shows that they are unsound. It is not difficult to see how. Consider, for instance, one of Mackie's "quasi-logical rules", restated below:

> **(1)** A good thing always eliminates evil as far as it can.

Clearly, **(1)** is not necessarily true: it is always possible that a good thing can eliminate evil, but does not know such evil, or that it can eliminate evil, but cannot do so without eliminating a greater good, etc. Thus, Plantinga considers a number of alternative suggestions:

> **(1a)** Every good thing always eliminates every evil that it knows about and can eliminate.
>
> **(1b)** A good being eliminates every evil E that it knows about and that it can eliminate without either bringing about a greater evil or eliminating a good state of affairs that outweighs E.
>
> **(1c)** An omnipotent and omniscient good being eliminates every evil that it can properly eliminate.

Plantinga analyses all of these alternatives in detail and concludes that none of them is necessarily true or, at any rate, can conduct to a successful objection to theism.[14] For instance, consider that there is an omnipotent and omniscient being, who knows and can properly eliminate two evil situations, without either bringing about a greater evil or eliminating a good state of affairs that outweighs such situation, but in any case, there is some evil which for some other reason this being cannot properly eliminate; in this case, **(1c)** is false, as well as the weaker statements **(1a)** and **(1b)**.

But are there situations, or states of affairs, which an omnipotent being cannot eliminate? Is there any limit to what an omnipotent being can do? Let us consider the other of Mackie's statements:

(2) There are no limits to what an omnipotent thing can do.

Plantinga argues that this is false, and to understand the many forms by which an omnipotent being cannot do anything is relevant for his response to the logical problem of evil, as well as for the development of our work. Most theists affirm that God, although omnipotent, is limited to the rules of logic and cannot commit logical or, more specifically, metaphysical contradictions. Among those theists are, for instance, Maimonides and Saint Thomas Aquinas.[15] He admits that there are theists in the history of philosophy that believed that God could create contradictory states of affairs; but for them, the logical problem of evil seems to be uninteresting. If God can create contradictions, His existence can be contradictory with the existence of evil, and these theists are ready to accept so.[16] However, most theists maintain that God does not do so, and interpret their versions

[14]See Plantinga (1977, p. 9-16) for the complete argument.

[15]As Hoffman and Rosenkrantz also affirm, on the concept of omnipotence and some different positions on the debate:

"One sense of 'omnipotence' is, literally, that of having the power to bring about any state of affairs whatsoever, including necessary and impossible states of affairs. Descartes seems to have had such a notion (Meditations, Section 1). Yet, Aquinas and Maimonides held the view that this sense of 'omnipotence' is incoherent. Their view can be defended as follows. It is not possible for an agent to bring about an impossible state of affairs (e.g., that there is a shapeless cube), since if it were, it would be possible for an impossible state of affairs to obtain, which is a contradiction (see Aquinas, Summa Theologiae, Ia, 25, 3; and Maimonides, Guide for the Perplexed, Part I, Ch. 15)" (HOFFMAN; ROSENKRANTZ, 2020).

[16]Not gratuitously, of course. It seems to me that if one considers that God can create or actualize contradictory states of affairs, one should be committed to the idea that dialetheias can be actual. But contemporary epistemologists have claimed that there are three possible doxastic states regarding one proposition: to believe *p*, to believe *not-p*, and to suspend judgment – see, for instance, Friedman (2017) McGrath (forthcoming). To believe *p & not-p* is an option rarely defended, if defended at all in mainstream epistemology; and to defend that *p & not-p* can be *actual* seems to be even more strange. One may reply that belief revision includes contradictions; however, one should also realize that epistemic rationality includes a vision of truth that is still a classical one.

of classical theism around this statement. In any case, theists who aim at providing an answer to the logical problem of evil should recognize that this is a question that can be only settled if one admits that God is consistent (PLANTINGA, 1977, p. 17).

Hence, in classical theism, God cannot contradict Himself. But there are several other things that, according to classical theists, God is not able to do. James Beebe (2003) gives us a list: God cannot lie, cheat, steal, be unjust, be envious, fail to know what is right, have false beliefs about anything, be ignorant, be unwise, cease to exist, and make a mistake of any kind. Furthermore, none of these "limitations" diminish the power or greatness of God; Beebe affirms that it is quite the opposite:

> According to classical theism, the fact that God cannot do any of these things is not a sign of weakness. On the contrary, theists claim, it is an indication of his supremacy and uniqueness. These facts reveal that God is, in St. Anselm's (1033-1109 A.D.) words, "that being than which none greater can be conceived". (BEEBE, 2003)

This characterization brought by Plantinga, in line with classical theists in general, is fundamental for his free will defense: God cannot create any possible world.

Now, according to Plantinga, both Leibniz and Mackie defended exactly the opposite, and the characterization above supports one of his criticisms to their ideas. Leibniz believed that, if God is almighty and wholly good, he would create (actualize) the best of the possible worlds. Mackie seems to agree with Leibniz on this point (MACKIE, 1955) But Leibniz believes that God is almighty and omnipotent; then, by *modus ponens*, He created (actualized) the best of the possible worlds. However, Mackie affirms that our world is not the best of the possible worlds. Thus, by *modus tollens*, God is not almighty and wholly good – *i.e.*, there is no God. This seems ironic, since Mackie and Leibniz seem to defend quite different positions; however, they do so from one same underlying agreement.[17]

Plantinga calls this confusion "Leibniz Lapse" and rejects this affirmation. He not only supports that God could not have created any possible world (as both Leibniz

[17] As Duncan Pritchard says, regarding another debate, but quite fitting to the case on Leibniz and Mackie: "one philosopher's *modus ponens* is another philosopher's *modus tollens*" (PRITCHARD, 2012, p. 114). Many debates in philosophy of religion can be settled in terms of what is considered to be good evidence for a certain position, or what its starting point should be. For instance, Leibniz probably would support that there is a lot of evidence for God; Mackie, on his turn, insists more on the evidence *against* God.

and Mackie seem to suppose), but also, he pressures the very concept of "best of all possible worlds". As he says,

> Just as there is no greatest prime number, so perhaps there is no best of all possible worlds. Perhaps for any world you mention, replete with dancing girls and deliriously happy sentient creatures, there is an even better world, containing even more dancing girls and deliriously happy sentient creatures (PLANTINGA, 1977, p. 61.)

Nonetheless, the logical problem of evil still remains. As a "last card", Mackie seems to believe that God could have created a world in which there is only good and no evil, and in which humans would still be free. As he asserts,

> [...] if God has made men such that in their free choices they sometimes prefer what is good and sometimes what is evil, why could he not have made men such that they always freely choose the good ? If there is no logical impossibility in a man's freely choosing the good on one, or on several, occasions, there cannot be a logical impossibility in his freely choosing the good on every occasion. God was not, then, faced with a choice between making innocent automata and making beings who, in acting freely, would sometimes go wrong: there was open to him the obviously better possibility of making beings who would act freely but always go right. Clearly, his failure to avail himself of this possibility is inconsistent with his being both omnipotent and wholly good. (MACKIE, 1955, p. 209.)

Plantinga considers that this claim has a central value for the logical problem of evil, and this is the central question to which his proposal is directed. If God is omnipotent and wholly good, Mackie says, He could have created a world where there are free creatures who never do wrong. For short, his claim is that

(3) God could have created a world containing moral good but no moral evil.

But is **(3)** true? For Plantinga, the answer is: *possibly no.*

Now, before unfolding his proposal, just one more differentiation is made, one that came to be very relevant to contemporary debate: the distinction between *theodicies* and *defenses*. According to Plantinga, a *theodicy* is an answer that includes the actual (or, at any rate, the most plausible) reasons why God permits evil. Classical theodicies are those from Saint Augustine (1993) and of Leibniz (1710/1985); they have both descriptive and explanatory concerns. On its turn, a *defense* is an answer concerned with the possible reasons why God could have permitted evil in the world. Different from a theodicy, a defense does not need to be actual or even plausible concerning the correspondence to reality; it is a weaker response, a logically consistent one, sufficient to debunk an argument from evil. Thus, the free will defense is different of a free will theodicy (like that of Saint Augustine), and it is proposed as an answer to Mackie's claims (PLANTINGA, 1974, p. 192).

As the free will defense is a *defense*, the free will defender should look for some proposition p that, together with the proposition g = "God is omnipotent and wholly good" would entail e = "evil exists", where the proposition p needs only to be *possible*. But as it is related to *free will*, it appeals to the claim that human beings have free will. As Plantinga says, "If a person S is free with respect to a given action, then he is free to perform that action and free to refrain; no causal laws and antecedent conditions determine either that he will perform the action, or that he will not" (1974, p. 165-166). Appealing to the metaphysics of modality, semantics of possible worlds and other contributions developed in his works,[18], he goes on to deal with the central question of the free will defense

Plantinga characterizes free will in terms of counterfactuals or subjunctive conditionals. He gives many examples of these conditionals in his books, and for the purposes of this explanation, one is provided here, with some adaptations. Let Edward be a man who goes on to fish in the River Tietê on a hot Sunday (by the northern region of São Paulo state, where fishes and other living creatures can enjoy the clean waters of this amazing river), and, unexpectedly, catches a tucunaré, a very big and tasty fish. When Edward returns home, his neighbour Alvin, who loves fishing but spent some time sleeping during that afternoon, sees Edward's tucunaré. Alvin becomes eager to buy the fish and offers $ 20 in order to obtain such a good meal for dinner. Edward does not accept, and later, at night, Alvin questions himself: "what would have happened had I offered $ 100"?

[18]Notably the book *The nature of necessity (1974).*

Let us call A the state of affairs in which Alvin offers to Edward the value of \$ 50 dollars. There are two counterfactuals associated with A:

(4) If the state of affairs A had obtained, Edward would have accepted the offer.

(5) If the state of affairs A had obtained, Edward would not have accepted the offer.

Either **(4)** or **(5)** is true; both cannot be true. However, as Edward has free will, A does not entail that he would accept the offer, nor entail that he would reject it. More specifically, the state of affairs A does not entail either

(6) Edward accepts the offer.

or

(7) Edward rejects the offer.

Many details are omitted here for the sake of brevity. But for our purposes, let us suppose now that **(6)** is true in our world. Could God have created a world in which **(7)** is true? The answer is *no*: because Edward is free in our world, God could have not caused him to reject the offer he accepted. Although being omnipotent, the actualization of either **(6)** or **(7)** depends only on the choice of Edward, who is free with respect to his action.[19]

But that being said, the main point of the free will defense goes: concerning morally significant situations, it is possible that God could have not created a world containing moral good and no moral evil. For even if there is a possible world in which every free creature chooses only to do what is good, this world is not the actual, and in the actual, they have counterfactual freedom to act differently; they are not bound by actions they could have performed in another world. Now, suppose that human beings

[19] There are two kinds of actualization with respect to a state of affairs S: one is the *strong actualization of* S, when God causes S to be actual and causes to be actual every contingent state of affairs S^* such that S includes S^*; and the other is the *weak actualization of* S, when God strongly actualizes a state of affairs S^* that counterfactually implies S (PLANTINGA, 1985, p. 49). For instance, when God actualizes a state of affairs involving the action of a free creature, the actualization depends on the action of such creature; thus, God only weakly actualizes such state of affairs – and this is the case of Edward in the example above.

suffer of a special property: in a possible world, all creatures only choose to do good with respect to an action *A*, but as they are free, no antecedent conditions determine their actions; and if God chooses to actualize this world, they actually choose to do what is wrong: then, God is omnipotent, but He cannot create a world in which they act freely, but do only what is good.

Plantinga calls the property above "transworld depravity". He provides an explicit definition. Let *S'* be a maximal world segment – a state of affairs equal to a possible world, including the antecedent of some counterfactual, but not including the consequent of it.[20] Thus,

> A person *P suffers from transworld depravity* if and only if the following holds: for every world *W* such that *P* is significantly free in *W* and *P* does only what is right in *W*, there is an action *A* and a maximal world segment *S'* such that
>
> (1) *S'* includes *A*'s being morally significant for *P*;
>
> (2) *S'* includes *P*'s being free with respect to *A*;
>
> (3) *S'* is included in *W* and includes neither *P* performing *A* nor *P*'s refraining from performing *A*;
>
> (4) If *S'* were actual, *P* would go wrong with respect to *A* (PLANTINGA, 1967).

Then, Plantinga reasons as follows: let us suppose now that every person suffers of transworld depravity,[21] and let us suppose there is a possible world *W* in which they are both significantly free and all of them do what is right, as Mackie suggests. Thus, they always do what is right concerning some specific action *A* in *W*. On its turn, *S'* is a segment of *W*, and as such, it is a possible state of affairs that does not include any decision of persons with respect to *A*. *S'* is included in *W*, so people choose to do what is right; however, if God decided to actualize *W*, He would have to actualize *S'*; but as people suffer of transworld depravity, by condition (4), they would go wrong with respect to *A*.

[20] More rigorously speaking, a *maximal segment of world S'* is a state of affairs such that the inclusion of any state of affairs compatible with *S'*, but which is not included in *S'*, would be a possible world. Another way of understanding the concept is provided in Plantinga (1974), and it is an even more rigorous definition, debated until nowadays; but for the purposes established here, it won't be necessary.

[21] Actually, it is their essences that include such a property, and the persons are the instantiations of their essences. See Plantinga (1974, p. 187).

But if this is the case, so the argument goes, then Mackie's claim is false: God could have created a world containing moral good but no moral evil. Now, of course the property of transworld depravity is quite strange. But let us remember that the free will defense does not aim at finding the actual reasons for God to create a world containing evil; instead, it requires only a possible statement. On its turn, it is possible that every person suffers from transworld depravity; if this is so, then the following is possible:

> **(8)** It was not within God's power to create a world containing moral good but no moral evil.

This is consistent with g = "God is omnipotent and wholly good" and with c = "God created a world containing moral good". Together, they entail e = "evil exists". Since **(7)** is possible, and $\{g, c, (8)\} \vdash e$, then, the free will defense provides an answer to the logical problem of evil.[22]

1.2.3 Summing up

The free will defense is still considered the most influent answer to the logical problem of evil: it sharpened the contemporary debate on this question so deeply that most philosophers rely upon his approach to this day. Philosophers like Robert Adams, William Alston (theists), and William Rowe (an atheist) recognized that the Defense has shown the consistency between God and evil.[23] Some even maintain that the Defense provided an answer so relevant to the problem that it can be considered as responsible for the shift in contemporary debate on the problem of evil, for since Plantinga's contribution, the debate on the evidential problem of evil got stronger. The basic difference between them is that, while the logical problem is an alleged inconsistency between Classical Theism and the existence of evil, the evidential problem is much more a question on the improbability of the existence of God, given the amount, diversity and apparent gratuitousness of evil in the world (TRAKAKIS, 2005).[24]

[22] Other related questions are answered through the free will defense. Plantinga considers an analogous strategy to provide answers to questions concerning the amount of moral evil and the occurrence of natural evil. See Plantinga (1977, p. 55-64) for a more accessible account, or Plantinga (1974, p. 190-195) for a more rigorous one.

[23] See Howard-Snyder and O'Leary-Hawthorne (1998).

[24] The reader may have realized that the evidential version is more complex to deal with (for, *prima facie*, what should be considered evidence? and how should we give it a formal treatment?), and also, weaker than the logical one (although rationality can be attached to improbability, the logical version can undermine rationality in a more direct way).

However, it might be said that almost no philosophical debate is completely solved since philosophy began in Ancient Greece, and this is so for the logical problem of evil as well. During the 1990's, Howard-Snyder and O'Leary-Hawthorne (1998) offered a criticism of the concept of transworld depravity, affirming that the free will defense was not successful.[25] Later, Richard Otte (2009) contested the idea that it is possible that all of human beings have the property of transworld depravity, and has shown that the definition provided in Plantinga (1974) is *necessarily false.*[26] Otte suggested a new definition of the concept that might work, and Plantinga himself recognized that the new definition was better than his own (PLANTINGA, 2009). However, there still remains a concern. Otte said that Plantinga's definition was not possible; but how could one *show* that his proposal is definitely possible, in the logical sense? Perhaps this is not completely needed; but while it remains a philosophical definition, it should not remain untouched forever. This can be shown by recent works which criticize or choose other ways. For instance, Pruss (2012) provided a counterexample to the free will defense, while Pruss himself (2003), as well as Bernstein and Helms (2015), have tried to provide simpler defenses.

Plantinga was also criticized for not providing a theodicy in the first place.[27] Actually, he defended that perhaps a theodicy was not possible or necessary to show that theists are rational,[28] but later, he provided his own *o felix culpa* theodicy in Plantinga (2004). This was a very good attempt to deal with wider questions on the problem of evil, and much could be said about this topic; but Davis and Franks (2018) have argued that his theodicy and his defense are "incompatible" for, as they argue, the property of transworld depravity contradicts the core assumptions of his theodicy.[29]

As it is possible to see, the debate is very rich; nevertheless, one should recognize that the debate on this formulation may not be the only option for a theist to hold. There are philosophers who hold, however, that the debate is closed, and that the

[25]In the meantime, Rowe (1998) supported the free will defense *contra* Howard-Snyder and Hawthorne.

[26]As a consequence, the definition stated in p. 30 is not sufficient. The reasons are out of the scope of this work; the reader is invited to read to Otte's article in order to understand why.

[27]See Walls (1991).

[28]See quote in p. 23.

[29]In Plantinga's theodicy, the best worlds include the *good-making* properties of Incarnation and Atonement of Christ, and these properties require sin and evil in these worlds. However, as Davis and Franks argue, if the defense is true (and it is possible that all of the humans suffer from transworld depravity), then the theodicy is false, for Jesus could not be in such a position such that Incarnation and Atonement could occur (2018, p. 220).

free will defense is the only option to be followed; but this is not true. Consider the following example. Sophia is a philosopher and a theist who carries her graduate research in a centre for logic and interdisciplinary research on a great university. Sophia, as a theist, could disagree on Plantinga's defense, or even on Otte's reformulation; although she did not, she is not constrained to rely upon previous treatments on the issue. She could, otherwise, agree that the defense works, but for a number of other reasons, she may find another path to reconsider the issue. She may find the debate confusing and misleading;[30] she may think that the definition of transworld depravity, although interesting, is too vulnerable to defeaters; she may be inspired by Plantinga's work, but in order to develop her own ways of thinking about the issue, she may appeal to other tools, others than metaphysics of modality, or even such a libertarian account. Sophia may think on the logical problem of evil and other correlate problems (for instance, religious determinism, the idea that evil is determined by the attributes of God) as problems that require logical treatment – not only semantics of possible worlds in a semi-formal way, but *full-blown* formal logic; and as she works on a centre for logic, she does not think of logic as a subject to be feared. Finally, she may find her own ways of presenting a defense or a theodicy to give a response in formal terms, presenting a different response. This response, on its turn, may have no direct parity, *prima facie*, with other previous debates, but it may open other strands to deal with the problem of evil.

And this is what this research is all about. In the next section, the work of Nieznański is presented, in order to introduce the problems and the aim of such an enterprise – to provide an axiomatic approach to theodicy via formal applied systems.

1.3 Nieznański, religious determinism, and the problem of evil

In what follows, a brief presentation of Nieznański's approaches to theodicy is exposed, as well as some questions related to religious determinism and the problem of evil.

[30] For instance, she may have read the recent work of Silvestre (2020), who argues that many terms of this debate are not well established in natural language, and provides a more rigorous characterization for the logical problem of evil.

1.3.1 On some Polish roots of philosophy of religion

The use of logic in philosophy, and more particularly, in the analysis of religious discourse, can be identified since ancient times. More recently, in the 20th century, some of the main philosophers that appealed to the methods of formal logic to deal with religious issues were Józef Maria Bocheński, Jan Salamucha, and Jan Drewnowski, who formed the so-called Cracow Circle (1934-1944). They aimed at employing the most current tools of mathematical logic in matters of philosophy of religion and theology. Their ideas and achievements were so important that, according to Roger Pouivet, the philosophers of religion of the Cracow Circle are "the principal precursors of what is now called the analytic philosophy of religion" (POUIVET, 2011, p. 1). This is greatly debatable, but it is noteworthy that the contributions of these philosophers took place decades before the mainstream philosophy of religion appeared (see p. 21). During these times, they rejected the verification agenda of logical positivists and assumed that religious questions were still important; moreover, they defended that logic could provide many useful tools to deal with such questions.

Bocheński, in particular, was one of such philosophers from the Cracow Circle and a recognized logician and historian of logic. He argues that the main concern of the group was methodological, and concerning philosophy and theology, he states that the circle defended three postulates: "(1) the language of philosophers and theologians should exhibit the same standard of clarity and precision as the language of science; (2) in their scholarly practice they should replace scholastic concepts by new notions now in use by logicians, semioticians, and methodologists; (3) they should not shun occasional use of symbolic language" (BOCHEŃSKI, 1989). Among the main outcomes of this group of researchers are the formalization and the analysis of many arguments in classical theism, including an Aquinas' proof for the existence of God, an Aquinas' proof of the immortality of the soul, the analysis of the scholastic concept of analogy, and a number of other contributions (1989, p. 14).

Despite their efforts, and due to many reasons,[31] their project of dealing with problems in philosophy of religion through formal methods did not remain. However, some contemporary philosophers have recovered such a tradition, and a movement inspired on Cracow Circle's postulates has grown in some places, mainly in Poland.

[31] The interested reader is invited to see Bocheński (1989) for the many details omitted here.

1.3.2 Edward Nieznański's approaches to the problem of evil

In the environment in which these seeds were sowed, some recent contributions to the interaction between logic and religion have began to flourish. Among others, Edward Nieznański is a Polish philosopher who works on formal logic, history of logic, metalogic, methodology of sciences and formalization of philosophical arguments, particularly of questions in philosophy of religion. One of his works, published some years ago, is the book *Towards a formalization of Thomistic theodicy*, which develops a theory of relations in order to reconstruct the argument for the existence of God as formalized by Salamucha, a member of the Cracow Circle, besides developing his own argument (NIEZNAŃSKI, 2014).[32]

Particularly, Nieznański developed two different systems in formal logic to deal with the problem of evil. He exposes in the abstract of his 2007 work – the only part written in English – the project of establishing a formal theodicy that aimed at showing that the existence of God is logically compatible with the existence of evil:

> The problem of justification of the almighty and perfect Creator in the face of the fact that there is evil in the world was posed as early as the 3rd century BC by Epicureans and Stoics. The author of the article uses St. Thomas Aquinas' and G.W. Leibniz's philosophical inspirations to demonstrate by means of formal-logical means that inferring non-existence of evil from existence of God, as well as non-existence of God from existence of evil is a logical error (NIEZNAŃSKI, 2007, p. 217).

To infer that God exists if and only if evil does not exist is equivalent to say that the existence of God and the existence of evil are mutually inconsistent. Thus, despite he does not say it clearly, he is concerned fundamentally with the logical problem of evil. Moreover, although inspired by the works of Saint Thomas Aquinas and G. W. Leibniz, Nieznański affirms that the problem should be tackled with the tools of formal logic, since that it consists in a "logical error".

He continues to establish the proposal:

[32] Concerning the term "theodicy", Nieznański seems to use it in more than one sense: sometimes meaning the classical one – an answer to the problem of evil, as in Nieznański (2007, 2008) –, but sometimes, referring to a justification in another sense – generally, proofs for the existence of God, as in Nieznański (2011, 2014).

> The analysis begins with the theory of an omniscient, infallible and omnipotent being, identified with God. “Will”, “allowance” and “objection” with respect to facts are differentiated and the law of logical squares with respect to acts of will and the iteration of states of God’s will are presented. A theistic axiology is suggested, religious fatalism and the superstition of predestination are refuted. The whole of the axiomatic calculus tends to the conclusion that evil in the world of the omnipotent Creator results from the purposefully established fortuitousness within the laws of nature, in the name of man’s freedom of choice and possibilities of development. (2007, p. 217.)

Thus, the goals of Nieznański’s project can be summarized as follows: first, to characterize some of the classic attributes of God; second, to establish a formal axiology, and third, to refute religious fatalism or determinism, in order to show that evil is justified by the “purposefully established fortuitousness within the laws of nature”. Thus, in order to deal with the problem of evil, Nieznański seems to suggest that one should deal with the issue on religious fatalism or determinism. His second system consists in a different approach, but the purpose is the same: to show that evil is consistent with the existence of God, through an approach in formal logic (NIEZNAŃSKI, 2008).

But why is religious determinism so problematic, when it comes to the problem of evil? Let us consider this in some detail, before we back to Nieznański’s approaches.

1.3.3 Religious determinism

When it comes to philosophy of religion, determinism is a difficult issue, and not only because of the problem of evil. One of the sources of this issue is that some of the attributes of God seem to be such that there is no free will or contingency. For instance, there is the very debated problem of divine foreknowledge and free will: since God knows every situation, does that mean that we are not free from acting otherwise? This is a really difficult question, and the challenge comes in many forms: logical determinism, epistemic determinism, and so on.[33] Thus, this is the first obstacle to deal with: what should be labeled as “religious determinism”?

The second problem is that many positions can be regarded as deterministic. For instance, Leigh Vicens says that theological determinism “is the view that God

[33]See Swartz (2004).

determines every event that occurs in the history of the world". He affirms that "St. Augustine, Thomas Aquinas, John Calvin, and Gottfried Leibniz all seemed to espouse the view at least at certain points in their illustrious careers" (VICENS, 2014). One may realize that this list of philosophers and theologians encompasses some different positions concerning determinism and free will; but what it means to be determinist, in religious or theological terms? What does it mean, exactly, the sentence "God determines every event that occurs in the history of the world"?

In this work, religious determinism is related to the position above, but it is expressed in formal terms. To say that God determines every situation in the world is the same as to say that every situation in the world is determined by God. In terms of will, this is close to say that, for all situations, if a situation is the case, then God wills it to be the case. In semi-formal terms, if p is a situation in the world:

(det) $\quad p$ is the case $\rightarrow$ God wills p to be the case

According to Nieznański (1987, p. 152-153), a similar characterization of religious determinism has been made before by Paul Weingartner in 1974. He calls "religious fatalism" the formula below:

$$\bigwedge p\ (p \rightarrow WLgp),$$

where p is a variable for situations and $WLgp$ stands for "God wants p".

Note, first, that the implication above (either in **det** or in Weingartner's formulation) does not establish a causal relation between the antecedent and the consequent. Actually, it is much closer to a relation of pertinence: if the implication is true, whenever the antecedent holds, the consequent also holds. Second, religious determinism in the sense above is quite different from saying that God is omnipotent: to affirm that God *determines* every event or situation that occurs in the world is not the same as affirming that He *can* do anything that does not involve a logical or metaphysical contradiction, or that his "overall power is not possibly exceeded by any being" (HOFFMAN; ROSENKRANTZ, 2020). The difference is subtle, but relevant. Just as theists very often do not affirm that God can create contradictory states of affairs, they also affirm that God is omnipotent, but this does not mean that if some situation is the case in the world, then God wills such situation. Rather, a minimal statement, consistent with many positions on the concept of omnipotence, is that, if God wills some situation to be the case, then it is the case. In semi-formal terms,

(op) God wills p to be the case $\rightarrow$ p is the case

This statement is similar to the statement of omnipotence provided by many authors. The first formal definition of it is attributed to Curt Christian, according to Nieznański (1987, p. 152-153):

$$AM_l x :\leftrightarrow \bigwedge p(WLxp \rightarrow p),$$

Where AWx =: "x is omniscient" and $WLxp$ =: "x wants p".[34] Nieznański, on his turn, defines omnipotence along similar lines:

$$x \varepsilon WM :\leftrightarrow \forall p(xCp \rightarrow p) \text{ (NIEZNAŃSKI, 2007, p. 204.)}$$

$$WMx :\leftrightarrow \forall p(Cxp \rightarrow p) \text{ (NIEZNAŃSKI, 2008, p. 255.)}$$

Where both WMx and $x \varepsilon WM$ stands for "x is omnipotent", and both xCp and Cxp stand for "x wants p".

Thus, both Christian and Nieznański agree on the characterization **op** above; furthermore, such characterization is compatible with many definitions of omnipotence in classical theism. A possible criticism is that it does not encompasses a full account of omnipotence. This is a relevant point. However, it is not the purpose of this work to give such a full account. Rather, our project is to establish a formal theodicy, and not a definitive description of the nature of God in formal terms (who are we to do so?). Furthermore, one should realize that the statement above is at least part of the *intension* of the concept of omnipotence of God: most classical theists would believe that, if God is omnipotent, it means *at least* that whenever He wills something to occur, this occurs. This may be enough for a while.

Finally, there is a relevant way to think about the problem of evil through religious determinism. Granted that God must satisfy all of the divine attributes, one may address the question as follows: God knows everything, for He is omniscient. God is also omnibenevolent, thus, everything He wills is good. But if God is omnipotent, then everything happens because God wills it all. Therefore, it is contradictory to believe in

[34] Świętorzecka (2011, p. 310-314) also provides some comments on Christian's definition.

the existence an omniscient, omnipotent and omnibenevolent God *and* in the existence of evil. Thus, one cannot believe that God exists *and* that evil exists; for if God exists, then, all of the situations, including the evil ones, should be attributed to Him, for He is omnipotent and omnibenevolent. Therefore, the logical problem of evil can be settled as a possible inconsistency between the existence of God and the existence of evil *given* a determinist definition of omnipotence.[35]

Given that religious determinism is relevant for the success of a logical argument from evil, let us now reconsider **det** in a more rigorous way. Such claim can be more precisely stated as "For all situations, if some situation is the case, then God wills such situation to be the case". Let $P(p)$ denote the expression "p is the case", $\mathcal{C}_\theta$ denote "God wills", $\forall p$ denote the expression "for all situations", and $\rightarrow$ be the material implication. Thus, **det** can be formalized as the formula **DET1** below:

$$\textbf{(DET1)} \qquad \forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$$

But there is one more question for us to deal with. Another related claim is the following: "If all situations are such that God knows abut it, then He wills these situations". The relation between knowledge and will is intricate; how do we relate God's knowledge and God's will? This is a hard question. However, as some suggest,[36] God being omniscient means the following: if some situation is the case, then He knows it. Of course the opposite is also true: if God knows that some situation is the case, then it is the case (because knowledge always includes truth). Thus, God knows that p *iff* p is the case; and in this sense, one may question: does God wills everything He knows? "Whence, then, is evil?"

Therefore, the proposition "For all situations, if God knows some situation to be the case, then God wills such situation to be the case" requires an answer; it is called here **(DET2)** [37]. If $\mathcal{W}_\theta$ denotes "God knows", and the other symbols are interpreted as before, it is possible to formalize such claim as follows:

[35]Mackie seems to assume a deterministic account on both free will and on omnipotence; this may be one of the reasons why he suggests that evil is inconsistent with the existence of God. See Mackie (1955, p. 209-210).

[36]See, for instance, Nieznański (1987, p. 151-154) for a detailed list of authors. See, also, Nieznański (p. 204 2007, 2008, p. 208) and Weingartner (2008, p. 48).

[37]Nieznański calls this claim "the fallacy of predestination", probably inspired by Czesław Oleksy (NIEZNAŃSKI, 1987, p. 153-154). But "predestination" is a term that does not help us to clarify the debate: it evokes many complex theological discussions we do not have to concern ourselves with.

$$\textbf{(DET2)} \quad \forall p(\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p))$$

These claims are certainly relevant to be answered. To provide such an answer is one of the goals of this work; hence, let us now go back to the axiomatic approaches that deal with them.

1.3.4 Back to Nieznański approaches: some technical notes

As previously exposed, in order to approach the problem of evil, Nieznański constructs two systems in which he characterizes the divine attributes, the situations of the world and shows the relation between God and these situations. However, although his philosophical insights are penetrating and inventive, and his general methodology of formalization is very inspiring, some issues led us to revisit his systems, proposing some changes. Some of these issues are explained as follows. In the first system (2007), Nieznański does not characterize explicitly the underlying logic by which he develops his formal approach. Consider, for example, the following list of formulas, the original axioms of the system. In these formulas, β stands for "God", x is a variable that stand for persons, p and q are variables that stand for situations, W is a symbol for "knows that", C is a symbol for "wills that",[38] D a symbol for 'permits that', $p \varepsilon d$ means 'the situation p is good', and finally, P stands for 'to be the cause of':

A1: $\forall x(\exists p\, xWp \wedge \exists q\, xCq)$

A2: $\forall p[\beta Cp \rightarrow \beta C(\beta Cp)]$

A3: $\forall p[\beta Dp \rightarrow \beta C(\beta Dp)]$

A4: $\forall p[\exists x\, \beta C(xCp) \rightarrow \forall x\, \beta D(xDp)]$

A5: $\sim \forall p(p \rightarrow p\, \varepsilon\, d)$

A6: $\forall p(\beta Pp \leftrightarrow \beta Cp)$

Just to take a look at it is not enough to recognize what logic is being employed, as there are no explicit formation rules to the formulas. Regarding the underlying logic used in the system, the author does not give any specific hints; the best assessment is that it consists of something between classical first-order logic and classical first-order modal

[38]The original term that interprets this symbol in Polish is *chcieć*, and the best possible translations are "to will" or "would like" (CAMBRIDGE POLISH-ENGLISH DICTIONARY. . . , 2019). The first of these options seems to fit better on the purposes of his works and the present approaches.

logic, but none of these options seem to be suitable. Consider, for instance, the formula **A2**; it can be read as *for all situations, if God wills a situation, then He wills to will that situation*. This formula involves a combination of C's, the symbol for "wills that". If C is a binary predicate, then the logical structure would be problematic, for it is not allowed to combine predicates in classical first-order logic in such way.[39]. To consider C as a modal operator is the most feasible way to combine such a sequence of symbols.

But this opens up another question: the quantifiers operate over symbols of variables that are not under the scope of a predicate. This can be seen in all axiom schemas and theorems of the original system[40]. Furthermore, there is a quantification over individuals (see **A1** above) that, in the case of being associated with what would be modal operators (such as W and C), it would create other problems. There is no literature that supports such a quantification over individuals that serve as indexes for modal operators[41]. In the end, it would not be necessary to quantify over individuals, unless with the intention of including more persons – and this is not neccessary in principle, not if the aim is to refute **DET1** and **DET2**.

A more robust account is provided in his second system (2008). This system is a modal account, and many details are more clearly characterized than in the first one. But some of the questions above can also be applied to this second approach as well, mainly the quantifications over modal symbols – something that, to the best of our knowledge, has no clear treatment in the respective literature.

Nevertheless, Nieznański's insights are penetrating and inventive, and his general methodology of formalization is very inspiring. For this reason, the issues presented in this section, among others, led us to work for a detailed treatment of his systems.

[39]See Margaris (1990, p. 48). One could argue that this symbol could stand for a symbol of function, but then we would have no formulas, just terms. To have formulas it must be involved at least one symbol of predicate; see, again, Margaris (1990, p. 48-49).

[40]The author seems to suggest the use of p as a metavariable: something that can be substituted by other formulas (NIEZNAŃSKI, 2007, p. 205).

[41]Gennady Shtakser, in two recent papers, proposes some families of propositional modal (epistemic) logics with quantification over agents of knowledge (SHTAKSER, 2018, 2019), but I have not found an analogous work for predicate logic.

1.4 Aims of this work

Thus, in this work, two systems are provided, as revisiting (or remaking) of Nieznański's approaches; the first of these systems is **N1**, published in an article (DA SILVA; BERTATO, 2019), exposed extensively in the chapter 2; and the second of these systems is **N2**, exposed in the chapter 3 of this work. The resulting systems have much of the original basic structure, but some new results are obtained. The same proceeding was done with both systems, and it can be synthesized as follows: firstly, we reestablished the formal language to one that, according to the issues aforementioned, is more adequate to attain Nieznański's tasks: both **N1** and **N2** are first-order modal systems, with two modal operators: $\mathcal{W}_\theta$ ("God knows") and $\mathcal{C}_\theta$ ("God wills"). These operators seem to be sufficient for dealing with some of the main subjects that Nieznański is concerned with in his approaches. Furthermore, the formal language is established, as well as the rules of inference and other features. Then, a new set of axiom schemes is defined, many of them inspired in the work of Nieznański, but with new formulations. Finally, some theorems were proved.

Among the outcomes, both **N1** and **N2** designate the attributes of God, describe formal axiologies, and refute **DET1** and **DET2**. All of these results are also obtained from their distinct sets of axioms.

However, as explained in the chapter 4, when **N1** and **N2** are considered together, there is a set of axioms enough to provide a new axiomatic system. The system, called **N3**, is based in **N1** and in **N2**, but has only three axioms (much less than the previous ones). These axioms are sufficient to prove the most relevant outcomes of **N1** and **N2**, as well as the results aimed by Nieznański: it proposes an answer to the problem of evil through the refutation of **DET1** and **DET2**, showing that those attributes of God are formally consistent with the existence of evil in the world.

Let us now present these approaches.

Chapter 2

N1: A first approach to a formal theodicy

In the first chapter, the current debate on the logical problem of evil was described, as well as the "Formal Theodicies" developed by the Polish philosopher and logician Edward Nieznański. In two articles (NIEZNAŃSKI, 2007, 2008), he developed two different approaches that aim at denying a version of the logical problem of evil associated with a form of religious determinism. In order to do this, in both systems, Nieznański states the "constitutive properties of God", in which the attributes of omniscience, infallibility and omnipotence are formally described, as well as some other attitudes of God regarding situations (*to permit* and *to oppose*). Then, he develops a formal axiology relating good, evil and neutral situations to finally refute his version of religious determinism and to solve a version of the logical problem of evil.

In this chapter, a revisiting of Nieznański's first system is provided. The system was originally published in Polish language (NIEZNAŃSKI, 2007). It is worth saying that reading this was challenging. Although the axioms and theorems had commentaries, I was not able to understand them without dictionaries and translation tools, for I am not fluent in Polish. In the end, what really helped me to understand the article was the logical language – and this can be counted as one more of the advantages of learning logic: it is an universal language, surpassing idioms, cultures, and ways of living; it expresses thoughts with clarity and precision, and finally, it is accessible to whoever grasps its structure, available to everyone who seeks to capture the basic rules of human thought.

These first efforts to make Nieznański's work available in English were presented years ago on two congresses (DA SILVA, 2017; DA SILVA; BERTATO, 2017). But many other issues remained untouched, until they received a proper treatment during this research. Some of the results presented here were published before in the *South American Journal of Logic* (DA SILVA; BERTATO, 2019). Both the system published there and the system here presented are called the same way, for despite their sets of axioms being different, they are equivalent – each system is enough to deduce the other one.

In what follows, we present the system **N1**.[1]

2.1 The system N1: language, rules, and axioms

In this section, the logical structure of **N1** is presented. The basis of **N1** is a first-order modal logic, *i.e.*, a first-order classical logic with the addition of two modal operators, $\mathcal{W}_\theta$ and $\mathcal{C}_\theta$.[2] The choice for a first-order modal framework is justified by previous discussion in chapter 1. However, it is relevant to emphasize, again, that the discussion of the logical problem of evil presupposes classical logic. Other non-classical approaches to the question are certainly welcome; such approaches may be fruitful and provide many interesting new results. Nevertheless, in the debate on the logical problem of evil, classical logic is still maintained as a basic framework – in this case, with the appropriate extensions, like classical modal logic and classical first-order modal logic.[3]

In the following, the language, the formation rules, and the deduction rules are presented, as well as some definitions of symbols. Furthermore, the proper axioms of the system will be also established.

2.1.1 Language and rules

The language $\mathcal{L}_{N1}$ of **N1** has the following symbols as primitives:

[1]The choice for an **N** followed by a number (**N1**, **N2** and **N3**) as names for these systems has been made as an acknowledgement to Edward Nieznański for his formal approaches to theodicy.

[2]Among the works consulted are: the book written by Walter Carnielli and Claudio Pizzi about modal logics and Modalities (CARNIELLI; PIZZI, 2008), the widely-known introductory book of George Hughes and Max Cresswell on modal logic, specially chapter 13 (HUGHES; CRESSWELL, 1996), and Fitting and Mendelsohn's book on first-order modal logic (FITTING; MENDELSOHN, 2012).

[3]See p. 25-26 for a brief discussion on the status of classical logic for classical theism.

(i) Symbols for unary predicates: B, P, d, z, n;

(ii) A symbol for a binary predicate: Op;

(iii) A symbol of constant: θ;

(iv) Variables for situations: p, q, r, possibly with indexes;

(v) The symbols for connectives: $\neg, \rightarrow$;

(vi) The symbol of universal operator: $\forall$;

(vii) Two symbols for specific modal operators: $\mathcal{C}_\theta, \mathcal{W}_\theta$.

The definition of a well-formed formula (abbreviated as *wff*) and the use of parentheses are the usual, with the expected extensions. The formation rules are the following:

(FR1) Any sequence of symbols consisting of an n-ary predicate followed by n individual variables is a *wff*.
(FR2) If ϕ is a *wff*, so are $\neg\phi$, $\mathcal{W}_\theta\phi$, and $\mathcal{C}_\theta\phi$.
(FR3) If ϕ and ψ are *wff*, so is $(\phi \rightarrow \psi)$.
(FR4) If ϕ is a *wff* and v is a variable that stands for situations, then $\forall v\phi(v)$ is a *wff*.

Some rules of deduction of **N1** are: *Modus Ponens* (MP), *Uniform Substitution* (US), *Rule of Necessitation* (Nec) and *Substitution of Equivalents* (Eq). They are stated below:

(MP) $\phi, \phi \rightarrow \psi \vdash_{N1} \psi$.
(US) (HUGHES; CRESSWELL, 1996, p. 25) The result of uniformly replacing any variable or variables $v_1, ..., v_n$ in a theorem by any *wff* $\phi_1, ..., \phi_n$, respectively, is itself a theorem.
(Nec) If $\vdash_{N1} \phi$, then $\vdash_{N1} \mathcal{W}_\theta\phi$ and $\vdash_{N1} \mathcal{C}_\theta\phi$.
(Eq) (HUGHES; CRESSWELL, 1996, p. 32) If ϕ is a theorem and ψ differs from ϕ in having some *wff* δ as a subformula at one or more places where ϕ has a *wff* γ as a subformula, then if $\gamma \leftrightarrow \delta$ is a theorem, ψ is also a theorem.[4]

[4]**Eq** can be deduced from the other rules and axioms of **N1**. Hughes and Cresswell present a deduction of **Eq** in K, a system that is weaker than that of **N1** (HUGHES; CRESSWELL, 1996, p. 32).

The Deduction Theorem **(DT)** is valid in the system:[5]

Theorem 1 (Deduction Theorem). *If* $\Gamma, \phi \vdash_{N1} \psi$*, then* $\Gamma \vdash_{N1} \phi \rightarrow \psi$.[6]

Other symbols of the language are defined as follows (ϕ and ψ are *wff*s):

Def. 2.1 ($\exists$). $\exists v\phi :\leftrightarrow \neg\forall v\neg\phi$

Def. 2.2 ($\leftrightarrow$). $(\phi \leftrightarrow \psi) :\leftrightarrow (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$

Def. 2.3 ($\vee$). $(\phi \vee \psi) :\leftrightarrow (\neg\phi \rightarrow \psi)$

Def. 2.4 ($\wedge$). $(\phi \wedge \psi) :\leftrightarrow \neg(\phi \rightarrow \neg\psi)$

As a convention, $\alpha(p)$ stands for any *wff* that involves *only* the variable p, where p is free.

Thus, in **N1**, a *wff* α that involves only a particular situation p is referred to as the 'state of affairs' $\alpha(p)$. The term 'state of affairs' is used here to indicate circumstances (possibly a fact) about a given situation.[7] So, any situation denoted by p is such that there are many states of affairs involving it. For instance, the formula $\alpha(p) \equiv P(p) \wedge \neg P(p)$ represents a state of affairs that does not occur, for it is contradictory.

It is necessary to concede that terms should not be used arbitrarily; but this is not the case here. In fact, there is is a philosophical debate on the relation between situations and states of affairs. For instance, while Jon Barwise and John Perry affirm that situations as portions of a world, and thus are not states of affairs, Edward Zalta affirm that situations are abstract objects that "encode" properties of state of affairs, and can even be possible worlds (TEXTOR, 2020). In this work, however, the distinction between "situation" and "state of affairs" serves more to avoid confusion, rather than to establish a framework to evaluate the application of these concepts in the wider philosophical debate. As the debate does not seem as strong and normative as one might affirm, there is some

[5]Maybe it is possible to prove the Deduction Theorem for the three systems presented in this dissertation. It seems to us reasonable to admit its validity, given that it consists of an inference which is very natural and usual. However, to prove it is not a task to which we will give attention here. However, theorems which depend on **DT** may be demonstrated otherwise or, in any case, left out, without affecting significantly the purpose of the systems described.

[6]Hakli and Negri establish the conditions for using this theorem in modal logic: through defining a formal notion of derivation from assumptions, it is possible to prove the theorem for modal logics as stated above (HAKLI; NEGRI, 2012, p. 859-861).

[7]Naturally, those possible facts that are expressible in the language of **N1**.

freedom to establish the distinction made here: situations are denoted by p, q, possibly with indexes; states of affairs are denoted by $\alpha(r), \beta(r)$, where r is a term for situations. Therefore, situations are simpler than states of affairs; while states of affairs encompass situations, the opposite is not true.[8]

For the sake of reading, the standard interpretations for each *wff* is included in parentheses. The following shall be considered as abbreviations or standard semantics in natural language:

θ := 'God';
$P(p)$:= 'p is the case';[9]
$B(\theta)$:= 'θ is divine'.[10]
$d(p)$:= 'p is good';
$z(p)$:= 'p is evil';
$n(p)$:= 'p is neutral';
$K(p)$:= 'p is contingent';
$Op(p, q)$:= 'p is opposed to q';[11]
$\mathcal{C}_\theta \alpha(p)$:= 'God wills the state of affairs $\alpha(p)$';
$\mathcal{W}_\theta \alpha(p)$:= 'God knows the state of affairs $\alpha(p)$'.

The standard interpretations of other symbols in **N1** are provided as they appear.

As usual, all theorems, rules and laws of Propositional Calculus are axioms, rules and laws in our system, respectively. The abbreviation **PC** denotes steps in the proofs that are based on rules and laws in Propositional Calculus, like contraposition, De Morgan, hypothetical syllogism, etc.; and the abbreviation **PC-Theorem** is used

[8] This distinction is used for **N2** and **N3** as well; see the next chapters.

[9] It is not needed to assume here that 'to be the case' is the same as 'to be actual'. To say that 'p is the case' is close to saying that 'p occurs' or that 'p has correspondence in reality' in a considered possible world.

[10] The letter 'B' here comes from the Polish *Bóg*, "God". Nieznański made use of this term both as the name of an individual and as a predicate, characterizing both with the same symbol β. Thus, there is a distinction between 'to be God' and 'to be divine': while θ stands for God, the agent or person, B is a predicate of divinity.

[11] Two situations are opposite if they are contrary. In other words, two opposite situations may at the same time both not be the case, but cannot at the same time both be the case.

whenever a valid **PC**-schema is evoked. Similarly, **FOL** denotes steps involving first-order logic, and **FOL**-Theorem denotes a valid **FOL**-schema.

2.1.2 Axioms of N1

The proper axioms of **N1** are presented in the following. For the sake of simplicity, both axioms properly speaking and axiom schemes are simply called "axioms".

The first axiom establishes that the distinguished element θ satisfies the primitive predicate B:

A2.1. $B(\theta)$

(God is divine.)

To characterize **N1** as a normal modal logic, the following formula is assumed as an axiom. It corresponds to a quantification over the famous modal axiom **K**, with $\mathcal{C}_\theta$ in the place of $\Box$. From a philosophical point of view, it is not difficult to assume it, as well:

A2.2. $\forall p(\mathcal{C}_\theta(\alpha(p) \to \beta(p)) \to (\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\beta(p)))$

(For all situations, if God wills a state of affairs to imply another state of affairs, then God wills the latter, provided that He wills the former.)

The following axiom is, similarly, associated with the formula **5**, the characteristic axiom schema of S5 system. One can see easily that there is an analogy between $\Diamond$, the operator of possibility in alethic modal logic, and $\mathcal{D}_\theta$, the operator of permission in **N1**:

A2.3. $\forall p(\mathcal{D}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p))$

(For all situations, if God permits a state of affairs, then He wills to permit such a state of affairs.)

Therefore, **N1** can be described as a first-order logic *plus* a S5 modal logic.

Another axiom here establishes something relevant, and easy to assume, in the context of the problem of evil:

A2.4. $\neg\forall p(P(p) \rightarrow d(p))$

(Not all the situations that are the case are good.)

Regarding the axiology or the evaluation of situations, there are three axioms that are based on Nieznański's system, but just implicitly, through definitions and symbols of the language. Just to give an example, in order to establish relations between situations with axiological significance, Nieznański originally defined a symbol, $(-)$, to denote the relation between situations that are *opposite*. However, this could lead some readers to think that it is a different type of negation. In a certain way, it really is: philosophically, one can say that a situation or state of affairs ω is opposite to another situation or state of affairs σ when ω is the case but σ is not the case, and vice versa.[12] Nevertheless, it works in a confusing way, for terms, propositional variables and the like are as much as the same as predicates in the language of his system, so the operator behaves sometimes as a negation, sometimes denoting a class of variables.

There are theorems that can be translated in the language of **N1** like the following formula (maintaining the operator "$-$"):

$$n(p) \leftrightarrow n(-p)$$

The formula says that, if a situation is neutral, then its opposite is neutral as well. But in syntactical terms, how should we deal with that? It is not formally adequate to apply a symbol of negation to a variable of a situation inside a predicate. The first attempt was to define a set of opposed variables, denoting them by $\bar{p}$, and then relate them to the "normal" situations through a definition. In this attempt of defining opposite situations, where $\mathcal{P}$ is any of the symbols of predicate d, z or n:

$$\mathcal{P}(\bar{p}) :\leftrightarrow \neg\mathcal{P}(p)$$

But that did not work, for this would lead to a contradiction: $n(p) \leftrightarrow n(-p)$ would be equivalent to $n(p) \leftrightarrow \neg n(p)$, and that is equivalent to $n(p) \wedge \neg n(p)$.[13]

[12] This definition was inspired by the definition of *complement* of a state of affairs given by Alvin Plantinga: "A *complement* of a state of affairs is the state of affairs that obtains just in case A does not obtain. [Or we might say that the complement (call it $\bar{A}$) of A is the state of affairs corresponding to the *denial* or *negation* of the proposition corresponding to A.]" (PLANTINGA, 1977, p. 36.)

[13] I thank Professor Marcelo Coniglio for showing me that.

After that, I realized that the most economical way to formalize Nieznański's intuition would be to define a primitive predicate for opposite situations, and to establish three new axioms that would regulate the axiology.[14] Let us begin with the axiological axioms:

A2.5. $\forall p(\mathcal{C}_\theta P(p) \leftrightarrow d(p))$

(For all situations, God wills some situation to be the case *iff* it is good.)

Originally, the axiom above was a definition, and it was given by $d(p) :\leftrightarrow \exists x \mathcal{C}_\theta(\mathcal{C}_x P(p))$ (adapted to **N1**'s notation), but as said before, **N1** was structured avoiding quantification over variables for individuals which serve as indexes for modal operators. Indeed, what we have now is an axiom that establishes the goodness of God, or the attribute of omnibenevolence. It is not so difficult to accept that in the present context; it is exactly because we are dealing with the logical problem of evil, an alleged inconsistency between evil and the divine attributes, that it is plausible to accept such an axiom. Furthermore, d is a primitive predicate; so A2.5 is the proper insertion of d in the system.

The next axiom is one that relates good to evil situations and inserts z, the predicate for evil situations:

A2.6. $\forall p(z(p) \rightarrow \neg d(p))$

(For all situations, if some situation is evil, then it is not good.)

A2.7 below states the relation between good and bad situations when they are opposite – and, of course, the primitive predicate of opposition:[15]

A2.7. $\forall p \forall q(Op(p,q) \rightarrow (d(p) \leftrightarrow z(q)))$

(For all situations, if two situations are opposed, then if one is good, the other is evil.)

The following axioms A2.8 and A2.9 (without quantification) were originally stated as definitions. Here they are established as proper axioms that introduce relations between will, opposition and permission of God regarding states of affairs:

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[14] I thank my supervisor, Professor Fábio Bertato, for this suggestion.

[15] I thank my supervisor, Professor Fábio Bertato, for the useful idea of establishing this predicate.

A2.8. $\forall p(\mathcal{S}_\theta \alpha(p) \leftrightarrow \mathcal{C}_\theta \neg \alpha(p))$

(For all situations, God opposes a state of affairs *iff* He wills the opposite.)

A2.9. $\forall p(\mathcal{D}_\theta \alpha(p) \leftrightarrow \neg \mathcal{S}_\theta \alpha(p))$

(For all situations, God permits a state of affairs *iff* He does not oppose it.)

The next axiom states the relation between the opposition of God and neutral situations. Intuitively, a situation is neutral if, and only if, God does not will it and is not opposed to it. Therefore, if God is opposed to a situation, we can assume that such a situation is not neutral.

A2.10. $\forall p(\mathcal{S}_\theta P(p) \rightarrow \neg n(p))$

(For all situations, if God opposes a situation to be the case, then the situation is not neutral.)

Thus, each situation admits one of three possible axiological values. In this sense, the next axiom establishes that neutral situations are neither good nor evil.

A2.11. $\forall p(n(p) \leftrightarrow (\neg d(p) \wedge \neg z(p)))$

(For all situations, a situation is neutral *iff* it is neither good nor evil.)

The system **N1** has eleven axioms. The axiom A2.1 establishes that our distinguished element θ ('God') is divine. The axioms A2.2 and A2.3 govern the iteration and the composition of the operators $\mathcal{C}_\theta$ and $\mathcal{D}_\theta$, which clearly show a modal character. The axiom A2.4 guarantees that there is at least one evil situation. The axiom A2.5 expresses that the will of God is the criterion for good. Axioms A2.6 and A2.7 provide a type of opposition between good and evil. Axioms A2.8 and A2.9 establish relations between the will and the opposition of God, and between the permission and the opposition of God with respect to states of affairs, while A2.10 establishes the relation between the opposition of God and neutral situations. Finally, the axiom A2.11 establishes that a situation is neutral if, and only if, such a situation is neither good nor evil.

The following formula was an axiom in the original approach (NIEZNAŃSKI, 2007, p. 208). It was called by Nieznański "axiom of justice"[16]:

$$\forall p(\exists x \mathcal{C}_\theta(\mathcal{C}_x \alpha(p)) \to \mathcal{D}_\theta(\mathcal{D}_x \alpha(p)))$$

(For all situations, if there is a person such that God wills this person to will some state of affairs, then, for all the persons, God permits that the persons permit it.) [17].

Nevertheless, for reasons aforementioned, quantification over individuals when they are associated with modal operators is not permitted here. The readjustment of this formula to the current system is given below.

$$\forall p(\mathcal{C}_\theta(\mathcal{C}_\theta \alpha(p)) \to \mathcal{D}_\theta(\mathcal{D}_\theta \alpha(p)))$$

(For all situations, if God wills to will some state of affairs, then God permits to permit it.)

However although this change would not result in loss of generality or philosophical content, it turned the "axiom of justice" unnecessary, for it can be proved from some of the theorems in the system.

In the following, some precise definitions of the divine attributes are presented, and a series of theorems relevant to the solution of the logical problem of evil and for the constitution of a formal theodicy are deduced.

2.2 The attributes of God

The following definitions delineate some attributes of God: omniscience, infallibility and omnipotence.

Def. 2.5 (Omniscience of God)**.** $WW :\leftrightarrow \forall p(\alpha(p) \to \mathcal{W}_\theta \alpha(p))$

(God is omniscient *iff*, for all situations, if a state of affairs is the case, then God knows it.)

Def. 2.6 (Infallibility of God)**.** $NM :\leftrightarrow \forall p(\mathcal{W}_\theta \alpha(p) \to \alpha(p))$

[16] In Polish, "Aksjomat o sprawiedliwości".

[17] The intuition behind this "axiom" is that God gives freedom to persons to permit what He wills they to will. The explanation of the formula in Polish gives us a hint: "Bóg jest konsekwentny, bezstronny i sprawiedliwy", *e.g.*, "God is consistent, impartial and just" (NIEZNAŃSKI, 2007, p. 208).

(God is infallible *iff*, for all situations, if God knows a state of affairs, then it is the case.)

Def. 2.7 (Omnipotence of God). $WM :\leftrightarrow \forall p(\mathcal{C}_{\theta}\alpha(p) \to \alpha(p))$

(God is omnipotent *iff*, for all situations, if God wills a state of affairs, then it is the case.)

Such definitions try to capture the historical conceptions and intuitions of the great religions that helped to shape an entire concept apparatus for the classical theism. It is not difficult to find the foundations for these definitions in such religious and philosophical traditions, but such a task goes beyond the scope of this work. Furthermore, the interested reader may find the discussion on logical characterizations of the divine attributes in the first chapter of this work (p. 36-40).

The following definition sets what it means to be 'divine' in the context of the system **N1**, according to the standard interpretation:

Def. 2.8 (God). $B(\theta) :\leftrightarrow WW \wedge NM \wedge WM$

(God is divine *iff* He is omniscient, infallible, and omnipotent.)

As God satisfies the predicate of divinity, we have theorem T2.1:

T2.1. $WW \wedge NM \wedge WM$

(God is omniscient, infallible, and omnipotent.)

Proof.

1. $B(\theta)$ [A2.1]
2. $B(\theta) :\leftrightarrow WW \wedge NM \wedge WM$ [Def. 2.8]
3. $WW \wedge NM \wedge WM$ [**PC**, 1, 2]

□

Theorems T2.2, T2.3, and T2.4 are also easily deduced from T2.1, and describe God's attitudes towards states of affairs. As Nieznański observes regarding the corresponding theorems in his system, theorems T2.2 and T**??** formalize a fact that is in agreement with the observation of Thomas Aquinas, who says that "God knows all things whatsoever that in any way are" (NIEZNAŃSKI, 2007, p. 204).[18]

T2.2. $\forall p(\alpha(p) \to \mathcal{W}_\theta \alpha(p))$ □

(For all situations, if a state of affairs is the case, then God knows it.)

Proof.

1. $WW \leftrightarrow \forall p(\alpha(p) \to \mathcal{W}_\theta \alpha(p))$ [Def. 2.5]
2. $WW \wedge NM \wedge WM$ [T2.1]
3. WW [**PC**, 2]
4. $\forall p(\alpha(p) \to \mathcal{W}_\theta \alpha(p))$ [**PC**, 1, 3]

□

T2.3. $\forall p(\mathcal{W}_\theta \alpha(p) \to \alpha(p))$ □

(For all situations, if God knows a state of affairs, then it is the case.)

Proof.

1. $NM \leftrightarrow \forall p(\mathcal{W}_\theta \alpha(p) \to \alpha(p))$ [Def. 2.6]
2. $WW \wedge NM \wedge WM$ [T2.1]
3. NM [**PC**, 2]
4. $\forall p(\mathcal{W}_\theta \alpha(p) \to \alpha(p))$ [**PC**, 1, 3]

□

T2.4. $\forall p(\mathcal{C}_\theta \alpha(p) \to \alpha(p))$ □

(For all situations, if God wills a state of affairs, then it is the case.)

Proof.

1. $WM \leftrightarrow \forall p(\mathcal{C}_\theta \alpha(p) \to \alpha(p))$ [Def. 2.7]
2. $WW \wedge NM \wedge WM$ [T2.1]
3. WM [**PC**, 2]
4. $\forall p(\mathcal{C}_\theta \alpha(p) \to \alpha(p))$ [**PC**, 1, 3]

□

[18] "*Deus scit omnia quaecumque sunt quocumque modo*" (Thomas Aquinas, *Summa Theologiae*, I, q. 14, a. 9 co.).

The following theorem states that God could not will contradictions, and it follows immediately from the underlying classical logic.

T2.5. $\neg\exists p(\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p)))$

(There is no situation such that God wills a contradiction.)

Proof.

1. $\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p)) \rightarrow (\alpha(p) \wedge \neg\alpha(p))$ [T2.4, $\alpha(p)/(\alpha(p) \wedge \neg\alpha(p))$, Spec]
2. $\neg(\alpha(p) \wedge \neg\alpha(p)) \rightarrow \neg\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [**PC**, 1]
3. $\neg(\alpha(p) \wedge \neg\alpha(p))$ [**PC**-Theorem]
4. $\neg\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [MP, 2, 3]
5. $\forall p\neg\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [Gen, 4]
6. $\neg\exists p\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [**FOL**, 5]

□

A similar result is derived below, this time using **Nec**:

T2.6. $\forall p(\mathcal{C}_\theta\neg(\alpha(p) \wedge \neg\alpha(p)))$

(For all situations, God wills contradictory states of affairs not to be the case.)

Proof.

1. $\neg(\alpha(p) \wedge \neg\alpha(p))$ [**PC**-Theorem]
2. $\mathcal{C}_\theta\neg(\alpha(p) \wedge \neg\alpha(p))$ [Nec, 1]
3. $\forall p(\mathcal{C}_\theta\neg(\alpha(p) \wedge \neg\alpha(p)))$ [Gen, 2]

□

The same can be stated about the knowledge of God, as follows:

T2.7. $\neg\exists p(\mathcal{W}_\theta(\alpha(p) \wedge \neg\alpha(p)))$

(There is no situation such that God knows a contradiction.)

Proof.

1. $\mathcal{W}_\theta(\alpha(p) \wedge \neg\alpha(p)) \rightarrow (\alpha(p) \wedge \neg\alpha(p))$ [T2.3, $\alpha(p)/(\alpha(p) \wedge \neg\alpha(p))$, Spec]
2. $\neg(\alpha(p) \wedge \neg\alpha(p)) \rightarrow \neg\mathcal{W}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [**PC**, 1]
3. $\neg(\alpha(p) \wedge \neg\alpha(p))$ [**PC**-Theorem]
4. $\neg\mathcal{W}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [MP, 2, 3]

5. $\forall p \neg \mathcal{W}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [Gen, 4]

6. $\neg\exists p \mathcal{W}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [Def 2.1, 5]

□

T2.8. $\forall p(\mathcal{W}_\theta \neg(\alpha(p) \wedge \neg\alpha(p)))$

(For all situations, God wills contradictory states of affairs not to be the case.)

Proof.

1. $\neg(\alpha(p) \wedge \neg\alpha(p))$ [**PC**-Theorem]
2. $\mathcal{W}_\theta \neg(\alpha(p) \wedge \neg\alpha(p))$ [Nec, 1]
3. $\forall p(\mathcal{W}_\theta \neg(\alpha(p) \wedge \neg\alpha(p)))$ [Gen, 2]

□

The following definition states what it means for God to be *coherent* regarding a situation, and T2.9 states another result about the will of God. This definition finds a similar in Nieznański's system. However, while he describes the coherence between two variables for individuals connected to operators (and thus lead again to the issue on quantification of variables for indexes), our approach defines coherence as a property that God holds with respect to Himself concerning *situations*.[19]

Def. 2.9 (Coherence)**.** $coherent_\theta(p) :\leftrightarrow (\mathcal{C}_\theta\alpha(p) \rightarrow \neg\mathcal{C}_\theta\neg\alpha(p))$

(God is said to be "coherent with Himself regarding a situation" whenever the following occurs: if He wills a state of affairs involving that situation, then He does not will the opposite.)

T2.9. $\forall p(\mathcal{C}_\theta\alpha(p) \rightarrow \neg\mathcal{C}_\theta\neg\alpha(p))$

(For all situations, if God wills a state of affairs, then it is not the case that He wills the opposite.)

Proof.

1. $\mathcal{C}_\theta\neg\alpha(p) \rightarrow \neg\alpha(p)$ [T2.4, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\neg\neg\alpha(p) \rightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [**PC**, 1]
3. $\alpha(p) \rightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [**PC**, 2]

[19] See Nieznański (2007, p. 205)

4. $\mathcal{C}_\theta\alpha(p) \to \alpha(p)$ [T2.4, Spec]

5. $\mathcal{C}_\theta\alpha(p) \to \neg\mathcal{C}_\theta\neg\alpha(p)$ [**PC**, 4, 3]

6. $\forall p(\mathcal{C}_\theta\alpha(p) \to \neg\mathcal{C}_\theta\neg\alpha(p))$ [Gen, 5]

□

By definition, it follows from T2.9:

T2.10. $\forall p(coherent_\theta(p))$

(God is coherent with Himself regarding all situations.)

Proof.

1. $\mathcal{C}_\theta\alpha(p) \to \neg\mathcal{C}_\theta\neg\alpha(p)$ [T2.9, Spec.]
2. $coherent_\theta(p)$ [Def. 2.9, 1]
3. $\forall p(coherent_\theta(p))$ [Gen, 2]

□

Next, some theorems are stated in order to explore the relations between "attitudes" of God towards states of affairs.

T2.11. $\forall p(\mathcal{S}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p))$

(For all situations, God is opposed to a state of affairs *iff* He does not permit it.)

Proof.

1. $\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{S}_\theta\alpha(p)$ [A2.9, Spec.]
2. $\neg\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{S}_\theta\alpha(p)$ [**PC**, 1]
3. $\forall p(\mathcal{S}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p))$ [**PC**, Gen, 2]

□

The theorem below states the dual relation between $\mathcal{C}_\theta$ and $\mathcal{D}_\theta$.

T2.12. $\forall p(\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p))$

(For all situations, God permits a state of affairs *iff* He does not will the opposite.)

Proof.

1. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [A2.8, Spec]
2. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p)$ [T2.11, Spec]

3. $\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [**PC**, 1, 2]
4. $\forall p(\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p))$ [Gen, 3]

$\square$

From T2.12 it is easy to recognize the analogy between alethic modal operators $\square$ and $\lozenge$ and **N1** modal operators $\mathcal{C}_\theta$ and $\mathcal{D}_\theta$, respectively. The following theorems can be deduced from T2.12:

T2.13. $\forall p(\neg\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p))$ $\square$

T2.14. $\forall p(\mathcal{D}_\theta\neg\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\alpha(p))$ $\square$

T2.15. $\forall p(\neg\mathcal{D}_\theta\neg\alpha(p) \leftrightarrow \mathcal{C}_\theta\alpha(p))$ $\square$

One more analogy between **N1** and normal modal systems emerges here: T2.16 below is related to the formula $\mathbf{T}^\lozenge$, valid in KT modal logic. Furthermore, it is philosophically meaningful, for it states the relation between the permission of God and the states of affairs:

T2.16. $\forall p(\alpha(p) \rightarrow \mathcal{D}_\theta\alpha(p))$

(For all situations, if a state of affairs is the case, then it is permitted by God.)

Proof.
1. $\mathcal{C}_\theta\neg\alpha(p) \rightarrow \neg\alpha(p)$ [T2.4, $\alpha(p)/\neg\alpha(p)$, Spec.]
2. $\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [T2.12, Spec]
3. $\alpha(p) \rightarrow \mathcal{D}_\theta\alpha(p)$ [**PC**, 1, 2]
4. $\forall p(\alpha(p) \rightarrow \mathcal{D}_\theta\alpha(p))$ [Gen, 3]

$\square$

The following theorem is a first-order version of the scheme known in modal logic literature as **D**, which characterizes a KD system. It states an interesting property concerning the relation between will and permission of God:

T2.17. $\forall p(\mathcal{C}_\theta\alpha(p) \rightarrow \mathcal{D}_\theta\alpha(p))$

(For all situations, if God wills a state of affairs, then God permits such a state of affairs.)

Proof.

1. $\mathcal{C}_\theta\alpha(p) \to \alpha(p)$ [T2.4, Spec.]
2. $\alpha(p) \to \mathcal{D}_\theta\alpha(p)$ [T2.16, Spec]
3. $\mathcal{C}_\theta\alpha(p) \to \mathcal{D}_\theta\alpha(p)$ [**PC**, 1, 2]
4. $\forall p(\mathcal{C}_\theta\alpha(p) \to \mathcal{D}_\theta\alpha(p))$ [Gen, 3]

□

Theorems T2.18 and T2.19, on the other hand, characterize the relation between God's opposition regarding states of affairs:

T2.18. $\forall p(\mathcal{S}_\theta\alpha(p) \to \neg\alpha(p))$

(For all situations, if God is opposed to a state of affairs, then such a state of affairs is not the case.)

Proof.

1. $\mathcal{C}_\theta\neg\alpha(p) \to \neg\alpha(p)$ [T2.4, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [A2.8, Spec]
3. $\mathcal{S}_\theta\alpha(p) \to \neg\alpha(p)$ [**PC**, 1, 2]
4. $\forall p(\mathcal{S}_\theta\alpha(p) \to \neg\alpha(p))$ [Gen, 3]

□

T2.19. $\forall p(\mathcal{S}_\theta\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p))$

(For all situations, if God is opposed to a state of affairs, then He permits the opposite.)

Proof.

1. $\mathcal{S}_\theta\alpha(p) \to \neg\alpha(p)$ [T2.18, Spec]
2. $\neg\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p)$ [T2.16, $\alpha(p)/\neg\alpha(p)$, Spec]
3. $\mathcal{S}_\theta\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p)$ [**PC**, 1, 2]
4. $\forall p(\mathcal{S}_\theta\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p))$ [Gen, 3]

□

The theorem below is equivalent to the axiom A2.3. In fact, a S5 system can be characterized by this schema, rather than the usual:

T2.20. $\forall p(\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\alpha(p))$

(For all situations, if God permits to will a state of affairs, then He wills such a state of affairs.)

Proof.

1. $\mathcal{D}_\theta\neg\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\neg\alpha(p)$ [A2.3, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\neg\mathcal{C}_\theta\mathcal{D}_\theta\neg\alpha(p) \to \neg\mathcal{D}_\theta\neg\alpha(p)$ [**PC**, 1]
3. $\mathcal{C}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\neg\alpha(p)$ [T2.12, Spec]
4. $\neg\mathcal{C}_\theta\mathcal{D}_\theta\neg\alpha(p) \to \mathcal{C}_\theta\alpha(p)$ [Eq, 3 in 2]
5. $\neg\mathcal{C}_\theta\mathcal{D}_\theta\neg\alpha(p) \leftrightarrow \mathcal{D}_\theta\neg\mathcal{D}_\theta\neg\alpha(p)$ [T2.13, $\alpha(p)/\mathcal{D}_\theta\neg\alpha(p)$, Spec]
6. $\mathcal{D}_\theta\neg\mathcal{D}_\theta\neg\alpha(p) \to \mathcal{C}_\theta\alpha(p)$ [Eq, 5 in 4]
7. $\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\alpha(p)$ [Eq, 3 in 6]
8. $\forall p(\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\alpha(p))$ [Gen, 7]

□

Theorem T2.21 is a first-order version of the formula well known in modal logics literature as **B**, the characteristic formula of system KTB or just B. Theorem T2.22, on its turn, is the converse of T2.21.[20] Both state relations between states of affairs that are the case and what God wills to permit or permits to will.

T2.21. $\forall p(\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p))$

(For all situations, if a state of affairs is the case, then God wills to permit such a state of affairs.)

Proof.

1. $\alpha(p) \to \mathcal{D}_\theta\alpha(p)$ [T2.17, Spec]
2. $\mathcal{D}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p)$ [A2.3, Spec]
3. $\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p)$ [**PC**, 1, 2]
4. $\forall p(\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p))$ [Gen, 3]

□

T2.22. $\forall p(\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \alpha(p))$

(For all situations, if God permits to will a state of affairs, then such a state of affairs is the case.)

[20] System B of propositional modal logic can be characterized either by the scheme $p \to \Box\Diamond p$, or by its converse $\Diamond\Box p \to p$.

Proof.

1. $\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\alpha(p)$ [T2.20, Spec]
2. $\mathcal{C}_\theta\alpha(p) \to \alpha(p)$ [T2.4, Spec]
3. $\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \alpha(p)$ [**PC**, 1, 2]
4. $\forall p(\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \alpha(p))$ [Gen, 3]

□

Similarly, the theorem below corresponds to a quantification over the well known scheme **4** of modal logic which characterizes the S4 system. In Nieznański's corresponding system, this formula is an axiom; however, due to the differences in the axiomatization of **N1** (see p. 48), T2.23 is a theorem of **N1**:

T2.23. $\forall p(\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{C}_\theta\alpha(p))$

(For all situations, if God wills a state of affairs, then He wills to will such a state of affairs.)

Proof.

1. $\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\alpha(p)$ [T2.20, Spec]
2. $\mathcal{C}_\theta(\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\alpha(p))$ [Nec, 1]
3. $\mathcal{C}_\theta(\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\alpha(p)) \to (\mathcal{C}_\theta\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{C}_\theta\alpha(p))$ [A2.2,[21] Spec]
4. $(\mathcal{C}_\theta\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{C}_\theta\alpha(p))$ [MP, 3, 4]
5. $\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p)$ [T2.21, $\alpha(p)/\mathcal{C}_\theta\alpha(p)$, Spec]
6. $\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{C}_\theta\alpha(p)$ [**PC**, 4, 5]
7. $\forall p(\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{C}_\theta\alpha(p))$ [Gen, 6]

□

Theorems from T2.24 to T2.34 state some inner relations between will, opposition, and permission of God regarding states of affairs.

T2.24. $\forall p(\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\alpha(p))$

(God wills to will a state of affairs *iff* He wills such a state of affairs.)

Proof.

1. $\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\alpha(p)$ [T2.4, $\alpha(p)/\mathcal{C}_\theta\alpha(p)$, Spec]

[21] The substitutions made here are the following: $\alpha(p)/\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p)$ and $\beta(p)/\mathcal{C}_\theta\alpha(p)$

2. $\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{C}_\theta\alpha(p)$ [T2.23, Spec]

3. $\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\alpha(p)$ [**PC**, 1, 2]

4. $\forall p(\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\alpha(p))$ [Gen, 3]

□

T2.25. $\forall p(\mathcal{C}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p))$

(For all situations, God wills to permit a state of affairs *iff* He permits such a state of affairs.)

Proof.

1. $\mathcal{C}_\theta\mathcal{D}_\theta\alpha(p) \to \mathcal{D}_\theta\alpha(p)$ [T2.4, $\alpha(p)/\mathcal{D}_\theta\alpha(p)$, Spec]

2. $\mathcal{D}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p)$ [A2.3, Spec]

3. $\mathcal{C}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p)$ [**PC**, 1, 2]

3. $\forall p(\mathcal{C}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p))$ [Gen, 3]

□

T2.26. $\forall p(\mathcal{D}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p))$

(For all situations, God permits to permit a state of affairs *iff* He permits such a state of affairs.)

Proof.

1. $\mathcal{C}_\theta\mathcal{C}_\theta\neg\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [T2.24, $\alpha(p)/\neg\alpha(p)$, Spec]

2. $\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [T2.12, Spec]

3. $\neg\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [**PC**, 2]

4. $\mathcal{C}_\theta\neg\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p)$ [Eq, 3 in 1]

5. $\neg\mathcal{D}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p)$ [Eq, 3 in 4]

6. $\mathcal{D}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p)$ [**PC**, 5]

7. $\forall p(\mathcal{D}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p))$ [Gen, 6]

□

The formula $\forall p(\mathcal{C}_\theta(\mathcal{C}_\theta\alpha(p)) \to \mathcal{D}_\theta(\mathcal{D}_\theta\alpha(p)))$ corresponds to a formula which is originally an axiom in Nieznański's system (see p. 52). It is easily demonstrated from T2.9, A2.9, T2.24, and T2.26:

T2.27. $\forall p(\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{D}_\theta\mathcal{D}_\theta\alpha(p))$

(For all situations, if God wills to will a state of affairs, then God permits to permit such a state of affairs.)

Proof.

1. $\mathcal{C}_\theta\alpha(p) \to \neg\mathcal{C}_\theta\neg\alpha(p)$ [T2.9, Spec]
2. $\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [T2.12, Spec]
3. $\mathcal{C}_\theta\alpha(p) \to \mathcal{D}_\theta\alpha(p)$ [Eq, 2 in 1]
4. $\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\alpha(p)$ [T2.24, Spec]
5. $\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{D}_\theta\alpha(p)$ [Eq, 4 in 3]
6. $\mathcal{D}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p)$ [T2.26, Spec]
7. $\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{D}_\theta\mathcal{D}_\theta\alpha(p)$ [Eq, 6 in 5]
8. $\forall p(\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{D}_\theta\mathcal{D}_\theta\alpha(p))$ [Gen, 7]

□

T2.28. $\forall p(\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\alpha(p))$

(For all situations, God permits to will a state of affairs *iff* He wills such a state of affairs.)

Proof.

1. $\mathcal{C}_\theta\mathcal{D}_\theta\neg\alpha(p) \leftrightarrow \mathcal{D}_\theta\neg\alpha(p)$ [T2.25, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\mathcal{D}_\theta\neg\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\alpha(p))$ [T2.12.2, Spec]
3. $\mathcal{C}_\theta\neg\mathcal{C}_\theta\alpha \leftrightarrow \neg\mathcal{C}_\theta\alpha$ [Eq, 2 in 1]
4. $\neg\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\mathcal{C}_\theta\alpha(p))$ [T2.12.1, $\alpha(p)/\mathcal{C}_\theta\alpha(p)$, Spec]
5. $\neg\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\alpha(p))$ [Eq, 3 in 4]
6. $\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\alpha(p)$ [**PC**, 5]
7. $\forall p(\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\alpha(p))$ [Gen, 6]

□

T2.29. $\forall p(\mathcal{C}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{S}_\theta\alpha(p))$

(For all situations, God wills to oppose a state of affairs *iff* He is opposed to such a state of affairs.)

Proof.

1. $\mathcal{C}_\theta\mathcal{C}_\theta\neg\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [T2.24, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [A2.8, Spec]
3. $\mathcal{C}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{S}_\theta\alpha(p)$ [Eq, 2 in 1]

4. $\forall p(\mathcal{C}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{S}_\theta\alpha(p))$ [Gen, 3]

□

T2.30. $\forall p(\mathcal{D}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{S}_\theta\alpha(p))$

(For all situations, God permits to oppose a state of affairs *iff* He opposes such a state of affairs.)

Proof.

1. $\mathcal{D}_\theta\mathcal{C}_\theta\neg\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [T2.28, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\alpha(p)$ [A2.8, Spec]
3. $\mathcal{D}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{S}_\theta\alpha(p)$ [Eq, 2 in 1]
4. $\forall p(\mathcal{D}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{S}_\theta\alpha(p))$ [Gen, 3]

□

T2.31. $\forall p(\mathcal{S}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{S}_\theta\alpha(p))$

(For all situations, God opposes to permit a state of affairs *iff* He opposes such a state of affairs.)

Proof.

1. $\mathcal{D}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p))$ [T2.26, Spec]
2. $\neg\mathcal{D}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p)$ [**PC**, 1]
3. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p)$ [T2.11, Spec]
4. $\mathcal{S}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{S}_\theta\alpha(p)$ [Eq, 3 in 2]
5. $\forall p(\mathcal{S}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{S}_\theta\alpha(p))$ [Gen, 4]

□

T2.32. $\forall p(\mathcal{S}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\neg\alpha(p))$

(For all situations, God opposes to will a state of affairs *iff* He permits the state of affairs not to be the case.)

Proof.

1. $\mathcal{C}_\theta\mathcal{D}_\theta\neg\alpha(p) \leftrightarrow \mathcal{D}_\theta\neg\alpha(p)$ [T2.25, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\mathcal{D}_\theta\neg\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\alpha(p)$ [T2.12.2, Spec.]
3. $\mathcal{C}_\theta\neg\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\neg\alpha(p)$ [Eq, 2 in 1]
4. $\mathcal{S}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\mathcal{C}_\theta\alpha(p)$ [A2.11, $\alpha(p)/\mathcal{C}_\theta\alpha(p)$, Spec]

5. $\mathcal{S}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\neg\alpha(p)$ [Eq, 4 in 3]

6. $\forall p(\mathcal{S}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\neg\alpha(p))$ [Gen, 5]

□

T2.33. $\forall p(\mathcal{S}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p))$

(For all situations, God opposes to oppose a state of affairs *iff* He permits such a state of affairs.)

Proof.

1. $\mathcal{S}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\mathcal{S}_\theta\alpha(p)$ [A2.8, $\alpha(p)/\mathcal{S}_\theta\alpha(p)$, Spec]
2. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p)$ [T2.11, Spec]
3. $\mathcal{S}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\neg\mathcal{D}_\theta\alpha(p)$ [Eq, 2 in 1]
4. $\mathcal{S}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p)$ [**PC**, 3]
5. $\mathcal{C}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p)$ [T2.25, Spec]
6. $\mathcal{S}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p)$ [Eq, 5 in 4]
7. $\forall p(\mathcal{S}_\theta\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p))$ [Gen, 6]

□

T2.34. $\forall p(\mathcal{C}_\theta\alpha(p) \rightarrow \neg\mathcal{C}_\theta\mathcal{C}_\theta\neg\alpha(p))$

(For all situations, if God wills a state of affairs, then He does not will to will the opposite.)

Proof.

1. $\mathcal{C}_\theta\alpha(p) \rightarrow \alpha(p)$ [T2.4, Spec]
2. $\alpha(p) \rightarrow \mathcal{D}_\theta\alpha(p)$ [T2.16, Spec]
3. $\mathcal{C}_\theta\alpha(p) \rightarrow \mathcal{D}_\theta\alpha(p)$ [**PC**, 1, 2]
4. $\mathcal{D}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p)$ [T2.26, Spec]
5. $\neg\mathcal{C}_\theta\neg\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\mathcal{D}_\theta\alpha(p)$ [T2.12, $\alpha(p)/\mathcal{D}_\theta\alpha(p)$, Spec]
6. $\neg\mathcal{C}_\theta\neg\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p)$ [Eq, 5 in 4]
7. $\neg\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [T2.12.1, Spec]
8. $\neg\mathcal{C}_\theta\mathcal{C}_\theta\neg\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p)$ [Eq, 7 in 6]
9. $\mathcal{C}_\theta\alpha(p) \rightarrow \neg\mathcal{C}_\theta\mathcal{C}_\theta\neg\alpha(p)$ [Eq, 8 in 3]
10. $\forall p(\mathcal{C}_\theta\alpha(p) \rightarrow \neg\mathcal{C}_\theta\mathcal{C}_\theta\neg\alpha(p))$ [Gen, 9]

□

Thus, in this section, many consequences of the attributes of God have been derived. From a logical perspective, they explicit the inner relation between the modal operations, providing a strong explanation on how they relate to each other; as $\mathcal{C}_\theta$ is regulated by a first-order version of $\mathsf{S5}$ and $\mathcal{W}_\theta$ is trivial, many modalities are reduced to only one or two combinations. From a philosophical point of view, the theorems deduced propose a strong framework to consider how knowledge, will, permission and opposition of God concerning states of affairs are inter-related.

Now, let us apply these notions, in order to approach the case for a formal theodicy.

2.3 God and values: a theistic axiology

In this section, we deal with a formal axiology, *i.e.*, a formal treatment of our ordinary notions of "good", "evil", and "neutral" situations. Good, evil, and neutral situations are established in axioms A2.5, A2.6, A2.10, and A2.11 (see p. 50–51).

Theorems from T2.35 to T2.42 are consequences of such axioms. They show some of the relations between good, evil, and neutral situations and their opposites.

T2.35. $\forall p(\neg n(p) \leftrightarrow (d(p) \vee z(p)))$

(A situation is not neutral *iff* it is either good or evil.)

Proof.

1. $n(p) \leftrightarrow (\neg d(p) \wedge \neg z(p))$ [A2.11, Spec]
2. $\neg n(p) \leftrightarrow \neg(\neg d(p) \wedge \neg z(p))$ [**PC**, 1]
3. $\neg n(p) \leftrightarrow (\neg\neg d(p) \vee \neg\neg z(p))$ [**PC**, 2]
4. $\neg n(p) \leftrightarrow (d(p) \vee z(p))$ [**PC**, 3]
5. $\forall p(\neg n(p) \leftrightarrow (d(p) \vee z(p)))$ [Gen, 4]

□

T2.36. $\forall p(n(p) \vee d(p) \vee z(p))$

(Every situation is neutral, good, or evil.)

Proof.

1. $\neg n(p) \leftrightarrow (d(p) \vee z(p))$ [T2.35, Spec]

2. $\neg n(p) \to (d(p) \lor z(p))$ [**PC**, 1]

3. $n(p) \lor (d(p) \lor z(p))$ [Def. 2.3, 2]

4. $n(p) \lor d(p) \lor z(p)$ [**PC**, 3]

5. $\forall p(n(p) \lor d(p) \lor z(p))$ [Gen, 4]

□

T2.37. $\forall p \forall q(Op(p, q) \to (d(p) \to \neg d(q)))$

(For all situations, if two situations are opposite, then if one is good, the other is not good.)

Proof.

1. $Op(p, q)$ [Hip.]

2. $Op(p, q) \to (d(p) \leftrightarrow z(q))$ [A2.7, Spec]

3. $d(p) \leftrightarrow z(q)$ [MP, 1, 2]

4. $z(q) \to \neg d(q)$ [A2.6, Spec]

5. $d(p) \to \neg d(q)$ [Eq, 3 in 4]

6. $Op(p, q) \to (d(p) \to \neg d(q))$ [DT, 1–5]

7. $\forall p \forall q(Op(p, q) \to (d(p) \to \neg d(q)))$ [Gen, 6]

□

T2.38. $\forall p(d(p) \to \neg z(p))$

(For all situations, if a situation is good, then it is not evil.)

Proof.

1. $z(p) \to \neg d(p)$ [A2.6, Spec]

2. $d(p) \to \neg z(p)$ [**PC**, 1]

3. $\forall p(d(p) \to \neg z(p))$ [Gen, 2]

□

T2.39. $\forall p(\neg d(p) \to (n(p) \lor z(p)))$

(For all situations, if a situation is not good, then it is neutral or evil.)

Proof.

1. $n(p) \lor d(p) \lor z(p)$ [T2.36, Spec]

2. $d(p) \lor (n(p) \lor z(p))$ [**PC**, 1]

3. $\neg d(p) \to (n(p) \lor z(p))$ [**PC**, 2]

4. $\forall p(\neg d(p) \to (n(p) \lor z(p)))$ [Gen, 3]

□

T2.40. $\forall p(((n(p) \lor z(p)) \to \neg d(p)))$

(For all situations, if a situation is neutral or evil, then it is not good.)

Proof.

1. $\neg((n(p) \lor z(p)) \to \neg d(p))$ [Hip.]
2. $(n(p) \lor z(p)) \land \neg\neg d(p)$ [**PC**, 1]
3. $n(p) \lor z(p)$ [**PC**, 2]
4. $d(p)$ [**PC**, 2]
5. $z(p)$ [Hip., 3]
6. $z(p) \to \neg d(p)$ [A2.6, Spec]
7. $\neg d(p)$ [MP, 5, 6]
8. $n(p)$ [Hip., 3]
9. $n(p) \leftrightarrow (\neg d(p) \land \neg z(p))$ [A2.11, Spec]
10. $(\neg d(p) \land \neg z(p))$ [MP, 8, 9]
11. $\neg d(p)$ [**PC**, 10]
12. $\neg d(p)$ [5-11]
13. $\neg\neg((n(p) \lor z(p)) \to \neg d(p))$ [¬ Hip, 1, 4, 12]
14. $((n(p) \lor z(p)) \to \neg d(p))$ [**PC**, 13]
15. $\forall p((n(p) \lor z(p)) \to \neg d(p))$ [Gen, 14]

□

T2.41. $\forall p(\neg d(p) \leftrightarrow (n(p) \lor z(p)))$

(A situation is not good *iff* it is either neutral or evil.)

Proof.

1. $\neg d(p) \to (n(p) \lor z(p))$ [T2.39]
2. $((n(p) \lor z(p)) \to \neg d(p))$ [T2.40]
3. $\neg d(p) \leftrightarrow (n(p) \lor z(p))$ [**PC**, 1, 2]
4. $\forall p(\neg d(p) \leftrightarrow (n(p) \lor z(p)))$ [Gen, 3]

□

T2.42. $\forall p \forall q(Op(p,q) \rightarrow (n(p) \leftrightarrow n(q)))$

(If two situations are opposite, then one of them is neutral *iff* the other is also neutral.)

Proof.

1. $Op(p,q)$ [Hip]
2. $Op(p,q) \rightarrow (d(p) \leftrightarrow z(q))$ [A2.7, Spec]
3. $d(p) \leftrightarrow z(q)$ [MP, 1, 2]
4. $n(p) \leftrightarrow (\neg d(p) \wedge \neg z(p))$ [A2.11, Spec]
5. $n(q) \leftrightarrow (\neg d(q) \wedge \neg z(q))$ [A2.11, p/q, Spec]
6. $d(q) \leftrightarrow z(p)$ [US, 3]
7. $n(p) \leftrightarrow (\neg z(q) \wedge \neg d(q))$ [Eq, 3 & 6 in 4]
8. $n(p) \leftrightarrow (\neg d(q) \wedge \neg z(q))$ [**PC**, 7]
9. $n(p) \leftrightarrow n(q)$ [**PC**, 5, 8]
10. $Op(p,q) \rightarrow (n(p) \leftrightarrow n(q))$ [DT, 1–9]
11. $\forall p \forall q(Op(p,q) \rightarrow (n(p) \leftrightarrow n(q)))$ [Gen, 10]

□

T2.43. $\forall p(\mathcal{C}_\theta P(p) \rightarrow d(p))$

(For all situations, if God wills a situation to be the case, then such a situation is good.)

Proof.

1. $d(p) \leftrightarrow \mathcal{C}_\theta P(p)$ [A2.5, Spec]
2. $\mathcal{C}_\theta P(p) \rightarrow d(p)$ [**PC**, 1]
3. $\forall p(\mathcal{C}_\theta P(p) \rightarrow d(p))$ [Gen, 2]

□

T2.44. $\forall p(\mathcal{S}_\theta P(p) \rightarrow \neg d(p))$

(For all situations, if God opposes to a situation that is the case, then such a situation is not good.)

Proof.

1. $d(p) \leftrightarrow \mathcal{C}_\theta P(p)$ [A2.5]
2. $\neg d(p) \leftrightarrow \neg \mathcal{C}_\theta P(p)$ [**PC**, 1]
3. $\mathcal{C}_\theta P(p) \rightarrow \neg \mathcal{C}_\theta \neg P(p)$ [T2.9, $\alpha(p)/P(p)$), Spec]

4. $\mathcal{C}_\theta \neg P(p) \rightarrow \neg \mathcal{C}_\theta P(p)$ [**PC**, 3]

5. $\mathcal{C}_\theta \neg P(p) \rightarrow \neg d(p)$ [Eq, 2 in 4]

6. $\mathcal{S}_\theta P(p) \leftrightarrow \mathcal{C}_\theta \neg P(p)$ [A2.8, $\alpha(p)/P(p)$, Spec]

7. $\mathcal{S}_\theta P(p) \rightarrow \neg d(p)$ [Eq, 6 in 5]

8. $\forall p(\mathcal{S}_\theta P(p) \rightarrow \neg d(p))$ [Gen, 7]

□

It is easy to prove the following theorem from A2.10, A2.11, and T2.44:

T2.45. $\forall p(\mathcal{S}_\theta P(p) \rightarrow z(p))$

(For all situations, if God opposes to a situation to be the case, then the situation is evil.)

The following theorems T2.46, T2.47, and T2.48 establish the relation between the axiological values of situations and the permission of God.

T2.46. $\forall p(d(p) \rightarrow \mathcal{D}_\theta P(p))$

(For all situations, if a situation is good, then God permits it to be the case.)

Proof.

1. $\mathcal{S}_\theta P(p) \rightarrow \neg d(p)$ [T2.44, Spec.]

2. $d(p) \rightarrow \neg \mathcal{S}_\theta P(p)$ [**PC**, 1]

3. $\mathcal{D}_\theta P(p) \leftrightarrow \neg \mathcal{S}_\theta P(p)$ [A2.9, $\alpha(p)/\alpha(p)$, Spec]

4. $d(p) \rightarrow \mathcal{D}_\theta P(p)$ [Eq, 3 in 2]

5. $\forall p(d(p) \rightarrow \mathcal{D}_\theta P(p))$ [Gen, 4]

□

T2.47. $\forall p(\neg d(p) \rightarrow \mathcal{D}_\theta \neg P(p))$

(For all situations, if a situation is not good, then God permits it not to be the case.)

Proof.

1. $\mathcal{C}_\theta \alpha(p) \rightarrow d(p)$ [T2.43, Spec]

2. $\neg d(p) \rightarrow \neg \mathcal{C}_\theta P(p)$ [**PC**, 1]

4. $\neg \mathcal{C}_\theta P(p) \leftrightarrow \mathcal{D}_\theta \neg P(p)$ [T2.12.2, $\alpha(p)/P(p)$, Spec]

5. $\neg d(p) \rightarrow \mathcal{D}_\theta \neg P(p)$ [Eq, 4 in 3]

6. $\forall p(\neg d(p) \to \mathcal{D}_\theta \neg P)$ [Gen, 5]

□

T2.48. $\forall p(n(p) \to (\mathcal{D}_\theta P(p) \land \mathcal{D}_\theta \neg P(p)))$

(For all situations, if a situation is neutral, then God permits it to be or not to be the case.)

Proof.

1. $n(p) \leftrightarrow (\neg d(p) \land \neg z(p))$ [A2.11, Spec]
2. $\neg z(p) \to \neg \mathcal{S}_\theta P(p)$ [T2.46, Spec, **PC**]
3. $\mathcal{S}_\theta P(p) \leftrightarrow \neg \mathcal{D}_\theta P(p)$ [A2.9, $\alpha(p)/P(p)$, Spec]
4. $\neg z(p) \to \mathcal{D}_\theta P(p)$ [Eq, 2 in 3, **PC**]
5. $\neg d(p) \to \mathcal{D}_\theta \neg P(p)$ [T2.47, Spec]
6. $n(p) \to (\mathcal{D}_\theta P(p) \land \mathcal{D}_\theta \neg P(p))$ [**PC**, 1, 4, 5]
7. $\forall p(n(p) \to (\mathcal{D}_\theta P(p) \land \mathcal{D}_\theta \neg P(p)))$ [Gen, 6]

□

The latter theorem states that some situations, namely neutral situations, are such that both their occurrence and non-occurrence are permitted by God.

The results established so far allow us to address the problem of determinism.

2.4 Refutation of determinism

Let us now present the central theorems of **N1**, which provide an answer to determinist claims and establish, properly, the core of a formal theodicy. As described in chapter 1, one of the determinist claims is **DET1**:

$$\textbf{(DET1)}\ \forall p(P(p) \to \mathcal{C}_\theta P(p))$$

We are now in position to answer this claim through formal means. The axiom A2.4 can be informally interpreted as saying that “nothing is all roses” in the world, or, as stated by Nieznański, “not all events are good” (NIEZNAŃSKI, 2007, p. 211.):

A2.4. $\neg\forall p(P(p) \rightarrow d(p))$

In **N1**, however, we derive T2.49, a very important theorem, since it is the negation of **DET1**:

T2.49 (¬**DET1**)**.** $\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$

(Not all situations are such that, if a situation is the case, then God wills such a situation to be the case.)

Proof.

1. $\neg\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [Hip]
2. $\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [**PC**, 1]
3. $P(p) \rightarrow \mathcal{C}_\theta P(p)$ [2, Spec]
4. $\forall p(\mathcal{C}_\theta P(p) \rightarrow d(p))$ [T2.43]
5. $\mathcal{C}_\theta P(p) \rightarrow d(p)$ [Spec, 4]
6. $P(p) \rightarrow d(p)$ [**PC**, 3, 5]
7. $\forall p(P(p) \rightarrow d(p))$ [Gen, 6]
8. $\neg\forall p(P(p) \rightarrow d(p))$ [A2.4]
9. $\neg\neg\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [¬ Hip, 7, 8]
10. $\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [**PC**, 9]

□

In what follows, it is defined what it means for God to be a 'will-it-all', the kind of person that always wills some state of affairs.

Def. 2.10 (will-it-all)**.** $OW :\leftrightarrow \forall p(\mathcal{C}_\theta \alpha(p) \vee \mathcal{C}_\theta \neg\alpha(p))$

(God is a 'will-it-all' regarding situations *iff* for all situations God wills a state of affairs or its opposite.)

The following theorem shows that, in **N1**, God is not a 'will-it-all'.

T2.50. $\neg OW$

(God is not a 'will-it-all'.)

Proof.

1. $\forall p(\mathcal{C}_\theta\alpha(p) \vee \mathcal{C}_\theta\neg\alpha(p))$ [Hip]
2. $\quad\forall p(\neg\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\neg\alpha(p))$ [**PC**, 1]
3. $\quad\neg\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\neg\alpha(p)$ [Spec, 2]
4. $\quad\mathcal{C}_\theta\neg\alpha(p) \to \neg\alpha(p)$ [T2.4, $\alpha(p)/\neg\alpha(p)$]
5. $\quad\neg\mathcal{C}_\theta\alpha(p) \to \neg\alpha(p)$ [**PC**, 3, 4]
6. $\quad\forall p(\neg\mathcal{C}_\theta\alpha(p) \to \neg\alpha(p))$ [Gen, 5]
7. $\quad\forall p(\alpha(p) \to \mathcal{C}_\theta\alpha(p))$ [**PC**, 6]
8. $\quad\neg\forall p(\alpha(p) \to \mathcal{C}_\theta\alpha(p))$ [T2.49]
9. $\neg\forall p(\mathcal{C}_\theta\alpha(p) \vee \mathcal{C}_\theta\neg\alpha(p))$ [¬ Hip, 1]
10. $\neg OW$ [Def. 2.10, 9]

□

Another statement of interest here is **DET2**:

$$\textbf{(DET2)}\ \forall p(\mathcal{W}_\theta P(p) \to \mathcal{C}_\theta P(p))$$

It is a remarkable fact that, in **N1**, **DET1** and **DET2** are equivalent, as T2.51 shows:

T2.51 (DET2 $\leftrightarrow$ DET1). $\forall p(\mathcal{W}_\theta P(p) \to \mathcal{C}_\theta P(p)) \leftrightarrow \forall p(P(p) \to \mathcal{C}_\theta P(p))$

(For all situations, to affirm that if God knows a situation to be the case, then God wills such a situation to be the case, is equivalent to affirm that if a situation is the case, then God wills it to be the case.)

Proof.

1. $P(p) \to \mathcal{W}_\theta P(p)$ [T2.2, $\alpha(p)/P(p)$, Spec]
2. $\mathcal{W}_\theta P(p) \to P(p))$ [T2.3, $\alpha(p)/P(p)$, Spec]
3. $\mathcal{W}_\theta P(p) \leftrightarrow P(p)$ [**PC**, 1, 2]
4. $(P(p) \to \mathcal{C}_\theta P(p)) \leftrightarrow (P(p) \to \mathcal{C}_\theta P(p))$ [**PC**-Theorem]
5. $(\mathcal{W}_\theta P(p) \to \mathcal{C}_\theta P(p)) \leftrightarrow (P(p) \to \mathcal{C}_\theta P(p))$ [Eq, 3 in 4]
6. $\forall p(\mathcal{W}_\theta P(p) \to \mathcal{C}_\theta P(p)) \leftrightarrow \forall p(P(p) \to \mathcal{C}_\theta P(p))$ [**FOL**, 5]

□

But **DET1** is false, thus **DET2** is also false:

T2.52 (¬ **DET2**)**.** $\neg\forall p(\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p))$ □

(Not all situations are such that if God knows a situation to be the case, then God wills such a situation to be the case.)

Proof.

1. $\forall p(\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p)) \leftrightarrow \forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [T2.52]
2. $\neg\forall p(\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p)) \leftrightarrow \neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [PC, 1]
3. $\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [T2.49]
4. $\neg\forall p(\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p))$ [MP, 2, 3]

□

The definition that follows sets up a new operator, and the following theorems extend the meaning of some results just stated above. We interpret it as 'God is the cause of':[22]

Def. 2.11 (God is the direct cause of)**.** $(\mathcal{A}_\theta\alpha(p) :\leftrightarrow \mathcal{C}_\theta\alpha(p))$

(God is the direct cause of a state of affairs *iff* He wills such a state of affairs.)

The following two theorems establish the relation between God as direct cause of situations and situations that are the case.

T2.53. $\forall p(\mathcal{A}_\theta\alpha(p) \rightarrow \alpha(p))$

(For all situations, if God is the direct cause of a state of affairs, then such a state of affairs is the case.)

Proof.

1. $\mathcal{A}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\alpha(p)$ [Def. 2.11]
2. $\mathcal{C}_\theta\alpha(p) \rightarrow \alpha(p)$ [T2.4, Spec]
3. $\mathcal{A}_\theta\alpha(p) \rightarrow \alpha(p)$ [Eq, 1 in 2]
4. $\forall p(\mathcal{A}_\theta\alpha(p) \rightarrow \alpha(p))$ [Gen, 3]

□

[22] Despite recognizing Nieznański's merit on defining this operator and its meaning in the context of a formal theodicy (as an attempt to deal with the will of God, His responsibility and the fact that He is the cause of everything in some way), the operator is interpreted it in a different way: instead of interpreting the operator defined as 'God is the cause of', in **N1**, the operator is interpreted as 'God is the *direct* cause of', for God's will is effective. Another relevant difference is that, in **N1**, the only person explicitly involved is God, and by doing this we avoid problems with quantifiers and multi-modalities – for instance, the definition above in his system would be stated as $\mathcal{A}_x\alpha(p) :\leftrightarrow \mathcal{C}_x\alpha(p)$, where x can be quantified.

T2.54. $\neg\forall p(P(p) \rightarrow \mathcal{A}_\theta P(p))$

(God is not the direct cause of every situation that is the case.)

Proof.

1. $\forall p(P(p) \rightarrow \mathcal{A}_\theta P(p))$	[Hip]
2. $\quad P(p) \rightarrow \mathcal{A}_\theta P(p)$	[Spec, 1]
3. $\quad \mathcal{A}_\theta P(p) \leftrightarrow \mathcal{C}_\theta P(p)$	[Def. 2.11, $\alpha(p)/P(p)$]
4. $\quad P(p) \rightarrow \mathcal{C}_\theta P(p)$	[Eq, 3 in 2]
5. $\quad \forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$	[Gen, 4]
6. $\quad \neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$	[T2.49]
7. $\neg\forall p(P(p) \rightarrow \mathcal{A}_\theta P(p))$	[$\neg$ Hip, 5, 6]

□

Next, we introduce the definition of contingent situation, that is a situation such that God permits it to be the case or not to be the case. Theorems from T2.55 to T2.60 show the relation between the will of God and contingent situations, and as a result, they show that there are contingent situations:

Def. 2.12 (Contingency)**.** $K(p) :\leftrightarrow (\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p))$

(A situation is contingent *iff* God permits it to be or not to be the case.)

T2.55. $\forall p(K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{C}_\theta \neg P(p)))$

(For all situations, a situation is contingent *iff* God neither wills that situation to be the case, nor wills its opposite to be the case.)

Proof.

1. $K(p) \leftrightarrow (\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p))$	[Def. 2.12]
2. $\mathcal{D}_\theta P(p) \leftrightarrow \neg\mathcal{C}_\theta \neg P(p)$	[T2.12, Spec]
3. $\mathcal{D}_\theta \neg P(p) \leftrightarrow \neg\mathcal{C}_\theta P(p)$	[T2.12.2, Spec]
4. $K(p) \leftrightarrow (\neg\mathcal{C}_\theta \neg P(p) \wedge \neg\mathcal{C}_\theta P(p))$	[Eq, 2 & 3 in 1]
5. $K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{C}_\theta \neg P(p))$	[**PC**, 4]
6. $\forall p(K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{C}_\theta \neg P(p)))$	[Gen, 5]

□

T2.56. $\forall p(K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{S}_\theta P(p)))$

(For all situations, a situation is contingent *iff* neither God wills that situation to be the case, nor is opposed to that.)

Proof.

1. $K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{C}_\theta\neg P(p))$ [T2.55, Spec]
2. $\mathcal{S}_\theta P(p) \leftrightarrow \mathcal{C}_\theta\neg P(p)$ [A2.8, Spec]
3. $\neg\mathcal{C}_\theta\neg P(p) \leftrightarrow \neg\mathcal{S}_\theta P(p)$ [**PC**, 2]
4. $K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{S}_\theta P(p))$ [Eq, 3 in 1]
5. $\forall p(K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{S}_\theta P(p)))$ [Gen, 4]

□

T2.57. $\forall p(K(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \vee \mathcal{S}_\theta P(p)))$

(For all situations, a situation is contingent *iff* it is not the case that God wills that situation to be the case or He is opposed to that.)

Proof.

1. $K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{S}_\theta P(p))$ [T2.56, Spec]
2. $K(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \vee \mathcal{S}_\theta P(p))$ [**PC**, 1]
3. $\forall p(K(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \vee \mathcal{S}_\theta P(p)))$ [Gen, 2]

□

T2.58. $\exists pK(p) \leftrightarrow \neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{S}_\theta P(p))$

(There is a contingent situation *iff* it is not the case that, for all situations, God wills that situation to be the case or He is opposed to that.)

Proof.

1. $K(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \vee \mathcal{S}_\theta P(p))$ [T2.57, Spec]
2. $\exists pK(p) \leftrightarrow \exists p\neg(\mathcal{C}_\theta P(p) \vee \mathcal{S}_\theta P(p))$ [**FOL**, 1]
3. $\exists pK(p) \leftrightarrow \neg\forall p\neg\neg(\mathcal{C}_\theta P(p) \vee \mathcal{S}_\theta P(p))$ [Def. 2.1]
4. $\exists pK(p) \leftrightarrow \neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{S}_\theta P(p))$ [**PC**, 3]

□

T2.59. $\exists pK(p) \leftrightarrow \neg OW$

(There is a contingent situation *iff* God is not a 'will-it-all'.)

Proof.

1. $\exists pK(p) \leftrightarrow \neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{S}_\theta P(p))$ [T2.58]
2. $\mathcal{S}_\theta P(p) \leftrightarrow \mathcal{C}_\theta \neg P(p)$ [A2.8, $\alpha(p)/P(p)$]
3. $\exists pK(p) \leftrightarrow \neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [Eq, 2 in 1]
4. $OW \leftrightarrow \forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [Def. 2.10]
5. $\exists pK(p) \leftrightarrow \neg OW$ [Eq, 4 in 3]

□

T2.60. $\exists pK(p)$

(There is at least one situation that is contingent.)

Proof.

1. $\exists pK(p) \leftrightarrow \neg OW$ [T2.59]
2. $\neg OW$ [T2.50]
3. $\exists pK(p)$ [MP, 1, 2]

□

An attempt to formalize the intuitive notion of responsibility is made below, where 'to be responsible for' is defined as an operator, $\mathcal{A}_\theta$.[23]

Def. 2.13 (Responsibility)**.** $\mathcal{O}_\theta \alpha(p) :\leftrightarrow \mathcal{A}_\theta \alpha(p)$

(God is responsible for a state of affairs *iff* He is the direct cause of that.)

Theorem T2.61 is simply the generalization of definition above:

[23]Originally, definition 2.13 was stated by Nieznański as $\mathcal{O}_x \alpha(p) :\leftrightarrow (\mathcal{A}_x \alpha(p) \vee (\neg \mathcal{S}_x \alpha(p) \wedge \mathcal{W}_x \mathcal{C}_\theta \mathcal{S}_x \alpha(p)))$, in the notation of this work. I recognize the merits of Nieznański's intuition: according to his line of thought, some person can be said "responsible" for some state of affairs whenever this person is the cause of that, or the person is not opposed to it, although knowing that God wills that person to be opposed to this state of affairs (NIEZNAŃSKI, 2007, p. 213). But here, changing to θ all occurrences of x avoided problems with multi-modalities. This led to a simplification of the definition of responsibility, for God is "the only person formalized" in the system.
Nevertheless, the second part of Nieznański's original definition is not valid regarding θ, for it is contradictory (perhaps intentionally): from $\mathcal{W}_\theta \mathcal{C}_\theta \mathcal{S}_\theta \alpha(p)$ one can deduce $\mathcal{S}_\theta \alpha(p)$ using T2.2, T2.3 and T2.4, and then we would have $\neg \mathcal{S}_\theta \alpha(p) \wedge \mathcal{S}_\theta \alpha(p)$. Therefore, this subformula is not necessary for the present purposes.
Some could argue that these changes resulted in a reduction of Nieznański's original intention, but **N1** was elaborated in first-order modal logic with basic modal operators, without indexes. Further works can be elaborated in order to handle of concepts like "responsibility" more adequately.

T2.61. $\forall p(\mathcal{O}_\theta \alpha(p) \leftrightarrow \mathcal{A}_\theta \alpha(p))$

(For all situations, God is responsible for a state of affairs *iff* He is the direct cause of such a state of affairs.)

In the following last three theorems of **N1**, it is shown that if God is responsible for some situation, then it is good. But if some situation is evil, God is not responsible for it. And, finally, if some evil happens, but God does not oppose to it (which would imply that it would not be the case), then the situation is contingent.

T2.62. $\forall p(\mathcal{O}_\theta P(p) \rightarrow d(p))$

(For all situations, if God is responsible for a situation that is the case, then the situation is good.)

Proof.

1. $\mathcal{C}_\theta P(p) \rightarrow d(p))$ [T2.43, Spec]
2. $\mathcal{A}_\theta P(p) \leftrightarrow \mathcal{C}_\theta P$ [Def. 2.11, $\alpha(p)/P(p)$]
3. $\mathcal{O}_\theta P(p) \leftrightarrow \mathcal{A}_\theta P(p)$ [T51, Spec]
4. $\mathcal{O}_\theta P(p) \leftrightarrow \mathcal{C}_\theta P(p)$ [Eq, 3 in 2]
5. $\mathcal{O}_\theta P(p) \rightarrow d(p)$ [Eq, 4 in 1]
6. $\forall p(\mathcal{O}_\theta P(p) \rightarrow d(p))$ [Gen, 5]

□

T2.63. $\forall p(z(p) \rightarrow \neg \mathcal{O}_\theta P(p))$

(For all situations, if a situation is evil, then God is not responsible for such a situation.)

Proof.

1. $z(p)$ [Hip]
2. $\quad \mathcal{O}_\theta P(p) \rightarrow d(p)$ [T2.61, Spec]
3. $\quad \neg d(p) \rightarrow \neg \mathcal{O}_\theta P(p)$ [**PC**, 2]
4. $\quad z(p) \rightarrow \neg d(p)$ [A2.6, Spec]
5. $\quad \neg d(p)$ [MP, 1, 4]
6. $\quad \neg \mathcal{O}_\theta P(p)$ [MP, 5, 3]
7. $z(p) \rightarrow \neg \mathcal{O}_\theta P(p)$ [DT, 1-6]

8. $\forall p(z(p) \rightarrow \neg\mathcal{O}_\theta P(p))$ [Gen, 7]

$\square$

T2.64. $\forall p((z(p) \wedge \neg\mathcal{S}_\theta P(p)) \rightarrow K(p))$

(For all situations, if a situation is evil, and God is not opposed to it, then the situation is contingent.)

Proof.

1. $z(p) \wedge \neg\mathcal{S}_\theta P(p)$ [Hip.]
2. $z(p)$ [**PC**, 1]
3. $\neg\mathcal{S}_\theta P(p)$ [**PC**, 1]
4. $z(p) \rightarrow \neg\mathcal{O}_\theta P(p)$ [T2.63, Spec]
5. $\neg\mathcal{O}_\theta P(p)$ [MP, 2, 4]
6. $\mathcal{O}_\theta P(p) \leftrightarrow \mathcal{A}_\theta P(p)$ [T2.61, Spec.]
7. $\neg\mathcal{O}_\theta P(p) \leftrightarrow \neg\mathcal{A}_\theta P(p)$ [**PC**, 6]
8. $\neg\mathcal{A}_\theta P(p)$ [MP, 5, 7]
9. $\mathcal{A}_\theta P(p) \leftrightarrow \mathcal{C}_\theta P(p)$ [Def 2.11, $\alpha(p)/P(p)$]
10. $\neg\mathcal{A}_\theta P(p) \leftrightarrow \neg\mathcal{C}_\theta P(p)$ [**PC**, 9]
11. $\neg\mathcal{C}_\theta P(p)$ [**PC**, 8, 10]
12. $\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{S}_\theta P(p)$ [**PC**, 11, 3]
13. $K(p) \leftrightarrow \neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{S}_\theta P(p)$ [T2.56, Spec]
14. $K(p)$ [Eq, 13 in 12]
15. $(z(p) \wedge \neg\mathcal{S}_\theta P(p)) \rightarrow K(p)$ [DT, 1–14]
16. $\forall p((z(p) \wedge \neg\mathcal{S}_\theta P(p)) \rightarrow K(p))$ [Gen, 15]

$\square$

Thus, T2.64 ends with a very relevant conclusion: if a situation is evil, and God is not opposed to it, then the situation is contingent. This theorem, together with some of the previous results, leads us to consider that evil situations cannot be attributed to God; for He is still omnipotent and omnibenevolent in face of evil situations.

2.5 Further considerations

In this chapter, a revisiting of Nieznański's first system, called **N1**, was provided. In the first part, the attributes of God were stated, and the relation between the will, permission and opposition of God concerning states of affairs was established. A formal axiology was described, in order to distinguish and relate good, evil and neutral situations to one another, as well as some of the relations between God and these situations. Finally, the claims **DET1**and **DET2** were refuted, and the theorems showed, as an outcome, that God is not responsible for every situation in the world; in particular, T2.63 and T2.64 showed that, if the situation is evil, then God is not responsible for it; furthermore, if the situation is evil and God does not oppose it to be the case, then such situation is contingent.

N1 is described here as a remaking of Nieznański's original system, and this fact is a key point to understanding some of the choices made here. First, some of the basic assumptions have been maintained, in order to conserve some parity with Nieznański's insights. Second, as **N1** is built as a first-order modal system, some of Nieznański's formulations could be maintained, but others could not. Whenever possible, the original formulation was preserved with the required adaptations, but when required, the simplest choice, or, at any rate, a simpler one, has been made.

These two points justify why such a system has lots of theorems that do not seem to explore religious determinism and the problem of evil. In fact, as we will see in the next chapters, these central questions can be approached by different formalizations.

The next chapter presents a second approach to theodicy, inspired by Nieznański's second proposal.

Chapter 3

N2: a second proposal

In this chapter, I revisit another system that Edward Nieznański has formulated as a response to the problem of evil. As I said before, this system was originally published in an article (NIEZNAŃSKI, 2008).

There are some differences between **N1** and the system developed in this chapter (henceforward called **N2**): the former is based upon a system proposed in an article in Polish, with no explicit subjacent logical system; the latter is based upon a system that was explored in an article written in English, elaborated explicitly in a Classical Modal Logic, with two modal operators: C_b is an operator for "God wills" and W_b an operator for "God knows", and this was maintained in the version hereby presented (**N2**). Some axioms are different, thus many results are deducible only in one system but not in the other, and vice versa. **N1** and **N2** carry on some of these differences.

Although different in many parts, however, the problem addressed by Nieznański is the same in both articles: The logical problem of evil and the determinist claims **DET1** and **DET2** (see p. 1.3.3). The spirit in which these proposals are addressed is the same, and the general logical structure of both approaches is also similar – including the ambiguities and many other slight logical imprecisions. Thus, **N2** is elaborated by revisiting carefully each axiom, definition and theorem, and providing a more detailed account of Nieznański's approach, similarly to what has been done for **N1**.[1]

[1] The system will be published soon as an article; see Da Silva and Bertato (forthcoming).

3.1 Formal structure of N2

Regarding the formal language and rules of **N2**, many of them are similar to the previous system. They are restated in this section.

(i) Unary predicate symbols: B, P, d, z, n;
(ii) A binary predicate symbol: Op;
(iii) A symbol of constant (a distinguished element): θ;
(iv) Variables for situations: p, q, r, possibly with indexes;
(v) The symbols for connectives: $\neg, \rightarrow$;
(vi) The symbol of universal operator: $\forall$;
(vii) Two symbols for specific modal operators: $\mathcal{C}_\theta, \mathcal{W}_\theta$.

The definition of a well-formed formula (abbreviated as *wff*) and the use of parentheses are the usual, with the expected extensions. The formation rules are stated below:

(FR1) Any sequence of symbols consisting of an n-ary predicate followed by n individual variables is a *wff*.
(FR2) If ϕ is a *wff*, so are $\neg\phi$, $\mathcal{W}_\theta\phi$, and $\mathcal{C}_\theta\phi$.
(FR3) If ϕ and ψ are *wff*, so is $(\phi \rightarrow \psi)$.
(FR4) If ϕ is a *wff* and v is a variable that stands for situations, then $\forall v\phi(v)$ is a *wff*.

Some rules of deduction of **N2** are: *Modus Ponens* (MP), *Uniform Substitution* (US), *Rule of Necessitation* (Nec) and *Substitution of Equivalents* (Eq). They are stated below:

(MP) $\phi, \phi \rightarrow \psi \vdash_{N2} \psi$.
(US) (HUGHES; CRESSWELL, 1996, p. 25) The result of uniformly replacing any variable or variables $v_1, ..., v_n$ in a theorem by any *wff* $\phi_1, ..., \phi_n$, respectively, is itself a theorem.
(Nec) If $\vdash_{N2} \phi$, then $\vdash_{N2} \mathcal{W}_\theta\phi$ and $\vdash_{N2} \mathcal{C}_\theta\phi$.
(Eq) (HUGHES; CRESSWELL, 1996, p. 32) If ϕ is a theorem and ψ differs from ϕ in

having some *wff* δ as a subformula at one or more places where ϕ has a *wff* γ as a subformula, then if $\gamma \leftrightarrow \delta$ is a theorem, ψ is also a theorem.[2]

The Deduction Theorem **(DT)** is valid in the system:

Theorem 2 (Deduction Theorem). *If* $\Gamma, \phi \vdash_{N1} \psi$, *then* $\Gamma \vdash_{N1} \phi \rightarrow \psi$.[3]

The definitions of auxiliary connectives are given below (ϕ and ψ are *wff*s):

Def. 3.1. $\exists v\phi :\leftrightarrow \neg\forall v\neg\phi$

Def. 3.2. $(\phi \leftrightarrow \psi) :\leftrightarrow (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$

Def. 3.3. $(\phi \vee \psi) :\leftrightarrow (\neg\phi \rightarrow \psi)$

Def. 3.4. $(\phi \wedge \psi) :\leftrightarrow \neg(\phi \rightarrow \neg\psi)$

Similarly to **N1**, as a convention, $\alpha(p)$ stands for any *wff* that involves *only* the variable p, where p is free.

Also, similarly to what has been done in **N1**, I will refer to a *wff* α that involves only a particular situation p as the 'state of affairs' $\alpha(p)$. The term 'state of affairs' indicates circumstances (possibly a fact) about a given situation. So, any situation denoted by p is such that there are many states of affairs involving it. For instance, the formula $\alpha(p) \equiv P(p) \wedge \neg P(p)$ represents a state of affairs that does not occur, for it is contradictory. Thus, as before, there is a distinction between 'situation' and 'state of affairs' here, but this differentiation serves more to avoid confusion.

For the sake of reading, throughout the system the standard interpretations for each *wff* are given in parentheses. The following shall be considered as abbreviations or standard semantics in natural language:

$\theta :=$ 'God';

$P(p) :=$ 'p is the case';[4]

[2]**Eq** can be deduced in a **K** modal system (HUGHES; CRESSWELL, 1996, p. 32). As **N2** includes **K**, it is presumable that **Eq** can be also derived from **N2** axioms and rules.

[3]See note 5, p. 46.

[4]Like in **N1**, to say that 'p is the case' here in **N2** is close to saying that 'p occurs' or that 'p has correspondence in reality' in a considered possible world.

$B(\theta)$:= 'θ is divine'.

$\delta(p)$:= 'p is good';

$\xi(p)$:= 'p is evil';

$\nu(p)$:= 'p is neutral';

$K(p)$:= 'p is contingent';

$Op(p, q)$:= 'p is opposed to q';

As usual, all valid *wff*s of Propositional Calculus (PC) are axioms in our system, as well as rules and laws of PC. The abbreviation **PC** denotes steps in the proofs that are based on rules and laws in propositional calculus, and the abbreviation **FOL** stands for any step in proofs that is based on first-order logic.

The specific axioms of **N2** are stated below. Some of them are equal to those of **N1**, but there are also some relevant differences.

The axiom below establishes that the distinguished element for God is divine in the system:

A3.1. $B(\theta)$

(God is divine.)

The next axiom is a quantification over the modal axiom of system K:

A3.2. $\forall p \forall q(\mathcal{C}_\theta(\alpha(p) \to \beta(q)) \to (\mathcal{C}_\theta \alpha(p) \to \mathcal{C}_\theta \beta(q)))$ [5]

(For all situations, if God wills a state of affairs to imply another state of affairs, then God wills the latter, provided that He wills the former.)

The axiom A3.3, on its turn, is a first-order version of the axiom **4** of $\mathsf{S4}$:

A3.3. $\forall p(\mathcal{C}_\theta \alpha(p) \to \mathcal{C}_\theta \mathcal{C}_\theta \alpha(p))$ [6]

(For all situations, if God wills some state of affairs, then He wills to will it.)

Thus, **N2** can be characterized as a $\mathsf{S4}$ modal system, plus quantification, plus other proper axioms of the theory.

In **N2**, the relation between good and evil situations is also established:

A3.4. $\forall p(\delta(p) \to \neg \xi(p))$

[5]In the original approach, A3.2 was written as $\mathcal{C}_b(p \to q) \to (\mathcal{C}_b p \to \mathcal{C}_b q)$. Nieznański explains the axiom by saying that "Gods will is monotonic" (NIEZNAŃSKI, 2008, p. 256).

[6]The original axiom was written as $\mathcal{C}_b p \to \mathcal{C}_b \mathcal{C}_b p$ (NIEZNAŃSKI, 2008, p. 257).

(If some situation is good, then it is not evil.)

One of the most relevant axioms of **N2**, one that brings a strong distinction from **N1**, is the following:

A3.5. $\neg\forall p(\mathcal{D}_\theta P(p) \to \mathcal{C}_\theta \mathcal{D}_\theta P(p))$

(Not all situations are such that, if God permits some situation to be the case, then He wills to permit such situation to be the case.)

The formula inside the universal quantifier is related to **5**, the characteristic formula of S5. One of the most surprising features of the present approach is the following: A3.5, the axiom just presented here in **N2**, is exactly the *negation* of A2.3, one of the axioms of **N1** (cf. p. 48). As it will be shown, A3.5 is strategically used to prove some theorems about contingency, and some of these theorems are analogous to some in **N1**, but A3.5 contradicts an axiom of **N1**. Further on, this issue will be dealt in more details.

In **N2**, the definition of permission has a direct analogy with $\Diamond$ and $\Box$, the modal operators that can be found in the literature of modal logic.

A3.6. $\forall p(\mathcal{D}_\theta \alpha(p) \leftrightarrow \neg\mathcal{C}_\theta \neg\alpha(p)))$

(God permits some state of affairs *iff* it is not the case that He wills its negation.)

The opposition of God concerning states of affairs is also defined here in terms of God's will:

A3.7. $\forall p(\mathcal{S}_\theta \alpha(p) \leftrightarrow \mathcal{C}_\theta \neg\alpha(p)))$

(God is opposed to some state of affairs *iff* He wills its negation.)

In Nieznański's proposal, the relation between opposite situations is made with the same operator for logical negation $\sim$. For instance, the definition for the predicate of evil situation is the following:

$$\text{Df. } \xi\text{: } \xi p \leftrightarrow \delta \sim p,$$ [7]

where $\sim$ also stands for the usual logical negation, as in the formula below (NIEZNAŃSKI, 2008, p. 259):

$$\text{T31. } \exists x CbCxp \rightarrow\sim \exists x CbCx \sim p$$

[7] Nieznański (2008, p. 258.).

However, according to first-order modal logic, it is not possible to apply a symbol for negation to a term, since terms are not *wff*s, and thus, are not atomic formulas. Atomic formulas are terms inside predicates: if δ is a symbol for predicate, and p is a term, then $\delta(p)$ or δp is a *wff*. This situation is similar to the one described in the second chapter (see p. 49-50).

Since the concept of proper opposition between situations is relevant for the axiology of **N2**, it is also established here as a predicate.

A3.8. $\forall p \forall q(Op(p,q) \to (\delta(p) \leftrightarrow \xi(q)))$

(For all situations, if two situations are opposed, then if one is good, the other is evil.)

The following axiom formalizes the relation between God's will and good situations:

As in **N1**, not all situations that are the case are good.

A3.9. $\forall p(\delta(p) \leftrightarrow \mathcal{C}_\theta P(p))$

(For all situations, a situation is good *iff* God wills it to be the case.)

The axiom A3.10 below has no corresponding formula in Nieznański's system. Due to the way he formalizes his approach, it is not necessary to assume that not all situations are good. Instead, he assumes an axiom analogous to the axiom A3.5, and with such axiom, he provides a refutation of the equivalent of **DET1**. On the one hand, in **N2**, A3.5 does not serve to prove that **DET1** is false, but to prove other features. On the other hand, it is not difficult to assume such an axiom in a discussion on the problem of evil. It is possible to affirm even that it is easier to assume the following theorem than to assume A3.5. Thus, let us assume the following axiom:

A3.10. $\neg\forall p(P(p) \to \delta(p))$

(Not all situations are such that, if a situation is the case, then it is good.)

Finally, A3.11 below establishes the relation between neutral, good and evil situations:

A3.11. $\forall p(\nu(p) \leftrightarrow (\neg\delta(p) \land \neg\xi(p)))$

(The situation is neutral *iff* it is neither good nor evil.)

Many axioms are quite similar to the axiomatization of **N1** (see p. 48-51 for details). The most relevant difference is that, while **N1** assumes a first-order version of **5** (A2.3, p. 48), **N2** assumes a negation of an analogous formula (A3.5, p. 85).

Let us now present the definitions and theorems of **N2**.

3.2 The divine attributes

In this subsection, the formalization of the attributes of God in **N2** is provided. Def. 3.5, 3.6 and 3.7 state some fundamental attributes of God in the system. On the other hand, Def. 3.8 state some properties satisfied by a person who is divine.

Def. 3.5 (Omniscience of God)**.** $WW :\leftrightarrow \forall p(\alpha(p) \rightarrow \mathcal{W}_\theta \alpha(p))$

(God is omniscient *iff* for all situations, if some state of affairs is the case, then God knows such state of affairs.)

Def. 3.6 (Infallibility of God)**.** $NM :\leftrightarrow \forall p(\mathcal{W}_\theta \alpha(p) \rightarrow \alpha(p))$

(God is infallible *iff*, for all situations, if God knows a state of affairs, then such state of affairs is the case.)

Def. 3.7 (Omnipotence of God)**.** $WM :\leftrightarrow \forall p(\mathcal{C}_\theta \alpha(p) \rightarrow \alpha(p))$

(God is omnipotent *iff*, for all situations, if God wills a state of affairs, then such state of affairs is the case.)

Def. 3.8 (God)**.** $B(\theta) :\leftrightarrow (WW \wedge NM \wedge WM)$

(God is divine *iff* He is omniscient, infallible and omnipotent.)

As a result, God satisfies the divine attributes:

T3.1. $WW \wedge NM \wedge WM$

(God is omniscient, infallible and omnipotent.)

Proof.

1. $B(\theta) \leftrightarrow WW \wedge NM \wedge WM$ [Def. 3.8]
2. $B(\theta)$ [A3.1]
3. $WW \wedge NM \wedge WM$ [**PC**, 1, 2]

□

The theorems from T3.2 to T3.9 show some relations between the knowledge of God and the states of affairs. It is noteworthy that the theorem T3.2 is the axiom **T** for the operator $\mathcal{W}_\theta$, and T3.4 is a first-order version of the formula **Triv** of modal logic. Thus, $\mathcal{W}_\theta$ is regulated by a Trivial system; as a consequence, from T3.4 one can derive any modal axiom, as, for example, the axiom **K** for the operator $\mathcal{W}_\theta$:

T3.2. $\forall p(\mathcal{W}_\theta \alpha(p) \rightarrow \alpha(p))$

(For all situations, if God knows a state of affairs, then such state of affairs is the case.)

Proof.

1. $NM \leftrightarrow \forall p(\mathcal{W}_\theta \alpha(p) \rightarrow \alpha(p))$ [Def. 3.6]
2. $WW \wedge NM \wedge WM$ [T3.1]
3. NM [PC, 2]
4. $\forall p(\mathcal{W}_\theta \alpha(p) \rightarrow \alpha(p))$ [**PC**, 1, 3]

$\square$

T3.3. $\forall p(\alpha(p) \rightarrow \mathcal{W}_\theta \alpha(p))$

(For all situations, if some state of affairs is the case, then God knows such state of affairs.)

Proof.

1. $WW \leftrightarrow \forall p(\alpha(p) \rightarrow \mathcal{W}_x \alpha(p))$ [Def. 3.7]
2. $WW \wedge NM \wedge WM$ [T3.1]
3. WW [PC, 2]
4. $\forall p(\alpha(p) \rightarrow \mathcal{W}_\theta \alpha(p))$ [**PC**, 1, 3]

$\square$

T3.4. $\forall p(\mathcal{W}_\theta \alpha(p) \leftrightarrow \alpha(p))$

(For all situations, God knows some state of affairs *iff* this state of affairs is the case.)

Proof.

1. $\mathcal{W}_\theta \alpha(p) \rightarrow \alpha(p)$ [T3.2, Spec]
2. $\alpha(p) \rightarrow \mathcal{W}_\theta \alpha(p)$ [T3.3, Spec]
3. $\mathcal{W}_\theta \alpha(p) \leftrightarrow \alpha(p)$ [**PC**, 1, 2]
4. $\forall p(\mathcal{W}_\theta \alpha(p) \leftrightarrow \alpha(p))$ [Gen, 3]

$\square$

T3.5. $\forall p \forall q(\mathcal{W}_\theta(\alpha(p) \rightarrow \beta(q)) \rightarrow (\mathcal{W}_\theta\alpha(p) \rightarrow \mathcal{W}_\theta\beta(q)))$

(For all situations, if God knows a state of affairs to imply another state of affairs, then God knows the latter, provided that He knows the former.)

Proof.

1. $\mathcal{W}_\theta(\alpha(p) \rightarrow \beta(q)) \leftrightarrow (\alpha(p) \rightarrow \beta(q))$ [T3.4, $\alpha(p)/(\alpha(p) \rightarrow \beta(q))$, Spec]
2. $\mathcal{W}_\theta\alpha(p) \leftrightarrow \alpha(p)$ [T3.4, Spec]
3. $\mathcal{W}_\theta\beta(q) \leftrightarrow \beta(q)$ [T3.4, $\alpha(p)/\beta(q)$, p/q, Spec]
4. $\mathcal{W}_\theta(\alpha(p) \rightarrow \beta(q)) \leftrightarrow (\mathcal{W}_\theta\alpha(p) \rightarrow \mathcal{W}_\theta\beta(q))$ [Eq, 3 and 2 in 1]
5. $\mathcal{W}_\theta(\alpha(p) \rightarrow \beta(q)) \rightarrow (\mathcal{W}_\theta\alpha(p) \rightarrow \mathcal{W}_\theta\beta(q))$ [**PC**, 4]
6. $\forall p \forall q(\mathcal{W}_\theta(\alpha(p) \rightarrow \beta(q)) \rightarrow (\mathcal{W}_\theta\alpha(p) \rightarrow \mathcal{W}_\theta\beta(q)))$ [Gen, 5]

□

T3.6. $\forall p((\alpha(p) \rightarrow \beta(q)) \rightarrow (\mathcal{W}_\theta\alpha(p) \rightarrow \mathcal{W}_\theta\beta(q)))$

(If a state of affairs implies another, then, if God knows the former, He also knows the latter.)

Proof.

1. $(\alpha(p) \rightarrow \beta(q)) \rightarrow \mathcal{W}_\theta(\alpha(p) \rightarrow \beta(q))$ [T3.3, $\alpha(p)/(\alpha(p) \rightarrow \beta(q))$, Spec]
2. $\mathcal{W}_\theta(\alpha(p) \rightarrow \beta(q)) \rightarrow (\mathcal{W}_\theta\alpha(p) \rightarrow \mathcal{W}_\theta\beta(q))$ [T3.5, Spec]
3. $(\alpha(p) \rightarrow \beta(q)) \rightarrow (\mathcal{W}_\theta\alpha(p) \rightarrow \mathcal{W}_\theta\beta(q))$ [**PC**, 1, 2]
4. $\forall p((\alpha(p) \rightarrow \beta(q)) \rightarrow (\mathcal{W}_\theta\alpha(p) \rightarrow \mathcal{W}_\theta\beta(q)))$ [Gen, 3]

□

T3.7. $\forall p \forall q((\alpha(p) \leftrightarrow \beta(q)) \rightarrow (\mathcal{W}_\theta\alpha(p) \leftrightarrow \mathcal{W}_\theta\beta(q)))$

(For all situations, equivalent states of affairs are equivalently known by God.)

Proof.

1. $(\alpha(p) \leftrightarrow \beta(q)) \rightarrow (\alpha(p) \leftrightarrow \beta(q))$ [**PC**-Theorem]
2. $\mathcal{W}_\theta\alpha(p) \rightarrow \alpha(p)$ [T3.4, Spec]
3. $\mathcal{W}_\theta\beta(q) \rightarrow \beta(q)$ [T3.4, $\alpha(p)/\beta(q)$, p/q, Spec]
4. $(\alpha(p) \leftrightarrow \beta(q)) \rightarrow (\mathcal{W}_\theta\alpha(p) \leftrightarrow \mathcal{W}_\theta\beta(q))$ [Eq, 2 and 3 in 1]
5. $\forall p \forall q((\alpha(p) \leftrightarrow \beta(q)) \rightarrow (\mathcal{W}_\theta\alpha(p) \leftrightarrow \mathcal{W}_\theta\beta(q)))$ [Gen, 4]

□

T3.8. $\forall p(\mathcal{W}_\theta \mathcal{W}_\theta \alpha(p) \leftrightarrow \mathcal{W}_\theta \alpha(p))$

(For all situations, God knows that He knows some state of affairs *iff* He knows such state of affairs.)

Proof.

1. $\mathcal{W}_\theta \mathcal{W}_\theta \alpha(p)) \leftrightarrow \mathcal{W}_\theta \alpha(p)$ [T3.4, $\alpha(p)/\mathcal{W}_\theta \alpha(p)$, Spec]
2. $\forall p(\mathcal{W}_\theta \mathcal{W}_\theta \alpha(p) \leftrightarrow \mathcal{W}_\theta \alpha(p))$ [Gen, 1]

□

T3.9. $\forall p(\mathcal{W}_\theta \alpha(p) \leftrightarrow \neg \mathcal{W}_\theta \neg \alpha(p))$

(For all situations, if God knows some state of affairs, then it is not the case that He knows the opposite.)

Proof.

1. $\mathcal{W}_\theta \alpha(p) \leftrightarrow \alpha(p)$ [T3.4, Spec]
2. $\neg \alpha(p) \leftrightarrow \mathcal{W}_\theta \neg \alpha(p)$ [T3.4, $\alpha(p)/\neg \alpha(p)$, Spec]
3. $\neg\neg \alpha(p) \leftrightarrow \neg \mathcal{W}_\theta \neg \alpha(p)$ [**PC**, 2]
4. $\alpha(p) \leftrightarrow \neg \mathcal{W}_\theta \neg \alpha(p)$ [**PC**, 3]
5. $\mathcal{W}_\theta \alpha(p) \leftrightarrow \neg \mathcal{W}_\theta \neg \alpha(p)$ [Eq, 4 in 1]
6. $\forall p(\mathcal{W}_\theta \alpha(p) \leftrightarrow \neg \mathcal{W}_\theta \neg \alpha(p))$ [Gen, 5]

□

The theorem T3.10 expresses the attribute of divine omnipotence; furthermore, together with A3.2 and A3.3, the theorem establishes the basic $\mathsf{S4}$ system for the operator $\mathcal{C}_\theta$:

T3.10. $\forall p(\mathcal{C}_\theta \alpha(p) \to \alpha(p))$

(For all situations, if God wills some state of affairs, then it is the case.)

Proof.

1. $WW \wedge NM \wedge WM$ [T3.1]
2. WW [**PC**, 1]
3. $WW \leftrightarrow \forall p(\mathcal{C}_\theta \alpha(p) \to \alpha(p))$ [Def.3.5, Spec]
4. $\forall p(\mathcal{C}_\theta \alpha(p) \to \alpha(p))$ [MP, 2, 3]

□

As discussed before, in classical theism, God cannot be contradictory. The theorems below makes explicit a similar insight:

T3.11. $\forall p(\neg\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p)))$

(For all situations, it is not the case that God wills contradictory states of affairs.)

Proof.

1. $\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p)) \rightarrow (\alpha(p) \wedge \neg\alpha(p))$ [T3.10, $\alpha(p)/\alpha(p) \wedge \neg\alpha(p)$, Spec]
2. $\neg(\alpha(p) \wedge \neg\alpha(p)) \rightarrow \neg\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [**PC**, 1]
3. $\neg(\alpha(p) \wedge \neg\alpha(p))$ [**PC**-Theorem]
4. $\neg\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [MP, 2, 3]
5. $\forall p(\neg\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p)))$ [Gen, 4]

□

T3.12. $\forall p(\mathcal{C}_\theta\neg(\alpha(p) \wedge \neg\alpha(p)))$

(For all situations, God wills contradictory states of affairs not to be the case.)

Proof.

1. $\neg(\alpha(p) \wedge \neg\alpha(p))$ [**PC**-Theorem]
2. $\mathcal{C}_\theta\neg(\alpha(p) \wedge \neg\alpha(p))$ [Nec, 1]
3. $\forall p(\mathcal{C}_\theta\neg(\alpha(p) \wedge \neg\alpha(p)))$ [Gen, 2]

□

From T3.13 to T3.26, many relations between will, permission and opposition of God concerning states of affairs are provided. These theorems are all obtained within a $\mathsf{S4}$ framework; thus, many of them exhibit the structure of $\mathsf{S4}$ for the operator $\mathcal{C}_\theta$:

T3.13. $\forall p(\mathcal{S}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p))$

(For all situations, God is opposed to some state of affairs *iff* it is not the case that He permits it.)

Proof.

1. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [A3.7, Spec]
2. $\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [A3.6, Spec]
3. $\neg\neg\mathcal{C}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p)$ [**PC**, 2]

4. $\mathcal{C}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p)$ [**PC**, 3]

5. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p)$ [Eq, 1 in 4]

6. $\forall p(\mathcal{S}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p))$ [Gen, 5]

□

T3.14. $\forall p(\mathcal{C}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\neg\alpha(p))$

(For all situations, God wills a state of affairs *iff* He does not permit the opposite.)

Proof.

1. $\mathcal{D}_\theta\neg\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\neg\alpha(p)$ [A3.6, $\alpha(p)/\neg\alpha(p)$, Spec]

2. $\mathcal{D}_\theta\neg\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\alpha(p)$ [**PC**, 1]

3. $\mathcal{C}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\neg\alpha(p)$ [**PC**, 2]

4. $\forall p(\mathcal{C}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\neg\alpha(p))$ [Gen, 3]

□

T3.15. $\forall p(\neg\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p))$ □

(For all situations, it is not the case that God permits a state of affairs *iff* He wills such state of affairs not to be the case.)

T3.16. $\forall p(\mathcal{D}_\theta\neg\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\alpha(p))$ □

(For all situations, God permits a state of affairs not to be the case *iff* it is not the case that He wills such state of affairs.)

T3.17. $\forall p(\alpha(p) \rightarrow \mathcal{D}_\theta\alpha(p))$

(For all situations, if a state of affairs is the case, then it is permitted by God.)

Proof.

1. $\mathcal{C}_\theta\neg\alpha(p) \rightarrow \neg\alpha(p)$ [T3.10, $p/\neg\alpha(p)$]

2. $\neg\neg\alpha(p) \rightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [**PC**, 1]

3. $\neg\neg\alpha(p) \leftrightarrow \alpha(p)$ [**PC**-Theorem]

4. $\alpha(p) \rightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [Eq, 3, 2]

5. $\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [Def. 3.6, Spec]

6. $\alpha(p) \rightarrow \mathcal{D}_\theta\alpha(p)$ [Eq, 4, 5]

7. $\forall p(\alpha(p) \rightarrow \mathcal{D}_\theta\alpha(p))$ [Gen, 6]

□

T2.31 is a first-order version of the formula **D**, that characterizes KD modal system.

T3.18. $\forall p(\mathcal{C}_\theta \alpha(p) \rightarrow \mathcal{D}_\theta \alpha(p))$

(For all situations, if God wills some state of affairs, then He permits such state of affairs.)

Proof.

1. $\mathcal{C}_\theta \alpha(p) \rightarrow \alpha(p)$ [T3.10, Spec]
2. $\alpha(p) \rightarrow \mathcal{D}_\theta \alpha(p)$ [T3.17, Spec]
3. $\mathcal{C}_\theta \alpha(p) \rightarrow \mathcal{D}_\theta \alpha(p)$ [**PC**, 1, 2]
4. $\forall p(\mathcal{C}_\theta \alpha(p) \rightarrow \mathcal{D}_\theta \alpha(p))$ [Gen, 3]

□

T3.19. $\forall p(\mathcal{C}_\theta \alpha(p) \rightarrow \neg \mathcal{S}_\theta \alpha(p))$

(For all situations, God wills some state of affairs *iff* it is not the case He is opposed to such state of affairs.)

Proof.

1. $\mathcal{C}_\theta \alpha(p) \rightarrow \mathcal{D}_\theta \alpha(p)$ [T3.18, Spec]
2. $\mathcal{S}_\theta \alpha(p) \leftrightarrow \neg \mathcal{D}_\theta \alpha(p)$ [T3.13, Spec]
3. $\neg \mathcal{S}_\theta \alpha(p) \leftrightarrow \mathcal{D}_\theta \alpha(p)$ [**PC**, 2]
4. $\mathcal{C}_\theta \alpha(p) \rightarrow \neg \mathcal{S}_\theta \alpha(p)$ [Eq, 3 in 1]
5. $\forall p(\mathcal{C}_\theta \alpha(p) \rightarrow \neg \mathcal{S}_\theta \alpha(p))$ [Gen, 4]

□

T3.20. $\forall p(\mathcal{S}_\theta \alpha(p) \rightarrow \neg \alpha(p))$

(For all situations, if God opposes a state of affairs, then such state of affairs is not the case.)

Proof.

1. $\mathcal{C}_\theta \neg \alpha(p) \rightarrow \neg \alpha(p)$ [T3.10, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\mathcal{S}_\theta \alpha(p) \leftrightarrow \mathcal{C}_\theta \neg \alpha(p)$ [Def. 3.7, Spec]
3. $\mathcal{S}_\theta \alpha(p) \rightarrow \neg \alpha(p)$ [Eq, 2 in 1]
4. $\forall p(\mathcal{S}_\theta \alpha(p) \rightarrow \neg \alpha(p))$ [Gen, 3]

□

T3.21. $\forall p(\mathcal{S}_\theta\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p))$

(For all situations, if God opposes some state of affairs, then He permits the opposite.)

Proof.

1. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [Def. 3.7, Spec]
2. $\mathcal{C}_\theta\neg\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p)$ [T3.18, $\alpha(p)/\neg\alpha(p)$, Spec]
3. $\mathcal{S}_\theta\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p)$ [Eq, 1 in 2]
4. $\forall p(\mathcal{S}_\theta\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p))$ [Gen, 3]

□

T3.22. $\forall p(\neg\mathcal{D}_\theta\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p))$

(For all situations, if it is not the case that God permits a state of affairs, then God permits such state of affairs not to be the case.)

Proof.

1. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\alpha(p)$ [T3.13, Spec]
2. $\mathcal{S}_\theta\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p)$ [T3.21, Spec]
3. $\neg\mathcal{D}_\theta\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p)$ [Eq, 1 in 2]
4. $\forall p(\neg\mathcal{D}_\theta\alpha(p) \to \mathcal{D}_\theta\neg\alpha(p))$ [Gen, 3]

□

T3.23. $\forall p(\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\mathcal{C}_\theta\alpha(p))$

(For all situations, God wills to will a state of affairs *iff* He wills such state of affairs.)

Proof.

1. $\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\alpha(p)$ [T3.10, $\alpha(p)/\mathcal{C}_\theta\alpha(p)$, Spec]
2. $\mathcal{C}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{C}_\theta\alpha(p)$ [A3.3, Spec]
3. $\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\mathcal{C}_\theta\alpha(p)$ [**PC**, 1, 2]
4. $\forall p(\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\mathcal{C}_\theta\alpha(p))$ [Gen, 3]

□

T3.24. $\forall p(\mathcal{C}_\theta\mathcal{D}_\theta\alpha(p) \to \mathcal{D}_\theta\alpha(p))$

(If God wills to permit a state of affairs, then He permits such a state of affairs.)

Proof.

1. $\mathcal{C}_\theta \mathcal{D}_\theta \alpha(p) \to \mathcal{D}_\theta \alpha(p)$ [T3.10, $\alpha(p)/\mathcal{D}_\theta\alpha(p)$, Spec]
2. $\forall p(\mathcal{C}_\theta \mathcal{D}_\theta \alpha(p) \to \mathcal{D}_\theta \alpha(p))$ [Gen, 1]

□

T3.25. $\forall p(\mathcal{D}_\theta \mathcal{D}_\theta \alpha(p) \leftrightarrow \mathcal{D}_\theta \alpha(p))$

(For all situations, God permits to permit a state of affairs *iff* He permits such state of affairs.)

Proof.

1. $\mathcal{C}_\theta \mathcal{C}_\theta \neg\alpha(p) \leftrightarrow \mathcal{C}_\theta \neg\alpha(p)$ [T3.23, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\mathcal{C}_\theta \neg\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta \alpha(p)$ [T3.15, Spec]
3. $\mathcal{C}_\theta \neg\mathcal{D}_\theta \alpha(p) \leftrightarrow \neg\mathcal{D}_\theta \alpha(p)$ [Eq, 2 in 1]
4. $\mathcal{C}_\theta \neg\mathcal{D}_\theta \alpha(p) \leftrightarrow \neg\mathcal{D}_\theta \mathcal{D}_\theta \alpha(p)$ [T3.15, $\alpha(p)/\mathcal{D}_\theta\alpha(p)$, Spec]
5. $\neg\mathcal{D}_\theta \mathcal{D}_\theta \alpha(p) \leftrightarrow \neg\mathcal{D}_\theta \alpha(p)$ [Eq, 4 in 3]
6. $\mathcal{D}_\theta \mathcal{D}_\theta \alpha(p) \leftrightarrow \mathcal{D}_\theta \alpha(p)$ [**PC**, 5]
7. $\forall p(\mathcal{D}_\theta \mathcal{D}_\theta \alpha(p) \leftrightarrow \mathcal{D}_\theta \alpha(p))$ [Gen, 6]

□

T3.26. $\forall p(\mathcal{C}_\theta \mathcal{S}_\theta \alpha(p) \leftrightarrow \mathcal{S}_\theta \alpha(p))$

(For all situations, God wills to oppose a state of affairs *iff* He opposes such state of affairs.)

Proof.

1. $\mathcal{C}_\theta \mathcal{C}_\theta \neg\alpha(p) \leftrightarrow \mathcal{C}_\theta \neg\alpha(p)$ [T3.23, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\mathcal{S}_\theta \alpha(p) \leftrightarrow \mathcal{C}_\theta \neg\alpha(p)$ [Def. 3.7, Spec]
3. $\mathcal{C}_\theta \mathcal{S}_\theta \alpha(p) \leftrightarrow \mathcal{S}_\theta \alpha(p)$ [Eq, 2 in 1]
4. $\forall p(\mathcal{C}_\theta \mathcal{S}_\theta \alpha(p) \leftrightarrow \mathcal{S}_\theta \alpha(p))$ [Gen, 3]

□

T3.27. $\forall p(\mathcal{C}_\theta \mathcal{C}_\theta \alpha(p) \to \neg\mathcal{C}_\theta \mathcal{C}_\theta \neg\alpha(p))$

(For all situations, if God wills some state of affairs, then it is not the case that He wills to will such state of affairs not to be the case.)

Proof.

1. $\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\alpha(p)$ [T3.23, Spec]
2. $\mathcal{D}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{D}_\theta\alpha(p)$ [T3.25, Spec]
3. $\mathcal{C}_\theta\alpha(p) \rightarrow \mathcal{D}_\theta\alpha(p)$ [T3.18, Spec]
4. $\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \rightarrow \mathcal{D}_\theta\mathcal{D}_\theta\alpha(p)$ [Eq, 1 and 2 in 3]
5. $\mathcal{D}_\theta\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\mathcal{D}_\theta\alpha(p)$ [A3.6, $\alpha(p)/\mathcal{D}_\theta\alpha(p)$, Spec]
6. $\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \rightarrow \neg\mathcal{C}_\theta\neg\mathcal{D}_\theta\alpha(p)$ [Eq, 5 in 4]
7. $\neg\mathcal{D}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [T3.15, Spec]
8. $\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \rightarrow \neg\mathcal{C}_\theta\mathcal{C}_\theta\neg\alpha(p)$ [Eq, 7 in 6]
9. $\forall p(\mathcal{C}_\theta\mathcal{C}_\theta\alpha(p) \rightarrow \neg\mathcal{C}_\theta\mathcal{C}_\theta\neg\alpha(p))$ [Gen, 8]

□

The following theorems show the relation between some of the irreducible modalities in **N2**. They can be deduced from A3.2, A3.3, and T3.10:

T3.28. $\forall p(\mathcal{C}_\theta\alpha(p) \rightarrow \mathcal{C}_\theta\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p))$ □

T3.29. $\forall p(\mathcal{C}_\theta\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \rightarrow \mathcal{D}_\theta\mathcal{C}_\theta\alpha(p))$ □

T3.30. $\forall p(\mathcal{C}_\theta\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \rightarrow \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p))$ □

T3.31. $\forall p(\mathcal{D}_\theta\mathcal{C}_\theta\alpha(p) \rightarrow \mathcal{D}_\theta\mathcal{C}_\theta\mathcal{D}_\theta\alpha(p))$ □

T3.32. $\forall p(\mathcal{C}_\theta\mathcal{D}_\theta\alpha(p) \rightarrow \mathcal{D}_\theta\mathcal{C}_\theta\mathcal{D}_\theta\alpha(p))$ □

T3.33. $\forall p(\mathcal{D}_\theta\mathcal{C}_\theta\mathcal{D}_\theta \rightarrow \mathcal{D}_\theta\alpha(p))$ □

Hence, many properties regarding God's attitudes toward states of affairs are established. There are more restrictions to what can be done here, compared to the previous reproposal; as $\mathcal{C}_\theta$ is based on $\mathsf{S4}$, many modalities are irreducible one another, and therefore, many equivalences proved in **N1** cannot be proved in **N2**.

In what follows, the formal axiology is presented, in order to evaluate situations.

3.3 N2: Formal axiology

The first feature of **N2**'s axiology is the definition of the goodness of God. As this feature should be considered an attribute, it serves as a starting point to consider what it means for a situation to be good and what relation there is between God and these situations:

T3.34. $\forall p(\mathcal{C}_\theta P(p) \to \delta(p))$

(For all situations, if God wills that some situation to be the case, then it is good.)

Proof.

1. $\mathcal{C}_\theta P(p) \leftrightarrow \delta(p)$ [A3.9]
2. $\mathcal{C}_\theta P(p) \to \delta(p)$ [**PC**, 1]
3. $\forall p(\mathcal{C}_\theta P(p) \to \delta(p))$ [Gen, 2]

□

Def. 3.9 (Goodness of God)**.** $DB(\theta) :\leftrightarrow \forall p(\mathcal{C}_\theta \alpha(p) \to \delta(p))$

(God is good *iff*, for all situations, if God wills some state of affairs, then it is good.)

T3.35. $DB(\theta)$

(God is good.)

Proof.

1. $\delta(p) \leftrightarrow \mathcal{C}_\theta P(p)$ [A3.9, Spec]
2. $\mathcal{C}_\theta P(p) \to \delta(p)$ [**PC**, 1]
3. $\forall p(\mathcal{C}_\theta P(p) \to \delta(p))$ [**PC**, 2]
4. $DB(\theta) \leftrightarrow \forall p(\mathcal{C}_\theta \alpha(p) \to \delta(p))$ [Def. 3.9]
5. $DB(\theta)$ [**PC**, 3, 4]

□

The following theorems from T3.36 to T3.43 describe some of the relations between situations and their values:

T3.36. $\forall p(\neg\nu(p) \leftrightarrow (\delta(p) \vee \xi(p)))$

(For all situations, a situation is not neutral *iff* it is good or evil.)

Proof.

1. $\nu(p) \leftrightarrow (\neg\delta(p) \wedge \neg\xi(p)$ [A3.11, Spec]
2. $\nu(p) \leftrightarrow \neg(\delta(p) \vee \xi(p))$ [**PC**, 1]

3. $\neg\nu(p) \leftrightarrow (\delta(p) \vee \xi(p))$ [**PC**, 2]

4. $\forall p(\neg\nu(p) \leftrightarrow (\delta(p) \vee \xi(p)))$ [Gen, 3]

□

T3.37. $\forall p(\nu(p) \vee \delta(p) \vee \xi(p))$

(All situations are either neutral, good, or evil.)

Proof.

1. $\neg\nu(p) \leftrightarrow (\delta(p) \vee \xi(p))$ [T3.36, Spec]
2. $\neg\nu(p) \rightarrow (\delta(p) \vee \xi(p))$ [**PC**, 1]
3. $\neg\neg\nu(p) \vee (\delta(p) \vee \xi(p))$ [**PC**, 2]
4. $\nu(p) \vee \delta(p) \vee \xi(p)$ [**PC**, 3]
5. $\forall p(\nu(p) \vee \delta(p) \vee \xi(p))$ [Gen, 4]

□

T3.38. $\forall p\forall q(Op(p, q) \rightarrow (\delta(p) \rightarrow \neg\delta(q)))$

(For all situations, if two situations are opposite, then if one is good, the other is not good.)

Proof.

1. $Op(p, q)$ [Hip.]
2. $\quad Op(p, q) \rightarrow (\delta(p) \leftrightarrow \xi(q))$ [A3.8, Spec]
3. $\quad \delta(p) \leftrightarrow \xi(q)$ [MP, 1, 2]
4. $\quad \delta(q) \rightarrow \neg\xi(q))$ [A3.4, p/q, Spec]
5. $\quad \xi(q) \rightarrow \neg\delta(q)$ [**PC**, 4]
6. $\quad \delta(p) \rightarrow \neg\delta(q)$ [Eq, 3 in 5]
7. $Op(p, q) \rightarrow (\delta(p) \rightarrow \neg\delta(q))$ [DT, 1–6]
8. $\forall p\forall q(Op(p, q) \rightarrow (\delta(p) \rightarrow \neg\delta(q)))$ [Gen, 7]

□

T3.39. $\forall p(\xi(p) \rightarrow \neg\delta(p))$

(For all situations, if a situation is evil, then it is not good.)

Proof.

1. $\delta(p) \rightarrow \neg\xi(p)$ [A3.4, Spec]

2. $\xi(p) \to \neg\delta(p)$ [**PC**, 1]

3. $\forall p(\xi(p) \to \neg\delta(p))$ [Gen, 2]

□

T3.40. $\forall p(\neg\delta(p) \to (\nu(p) \lor \xi(p)))$

(If a situation is not good, then it is neutral or evil.)

Proof.

1. $\nu(p) \lor \delta(p) \lor \xi(p)$ [T3.37, Spec]
2. $\delta(p) \lor \nu(p) \lor \xi(p)$ [**PC**, 1]
3. $\neg\delta(p) \to (\nu(p) \lor \xi(p))$ [**PC**, 2]
4. $\forall p(\neg\delta(p) \to (\nu(p) \lor \xi(p)))$ [Gen, 3]

□

T3.41. $\forall p((\nu(p) \lor \xi(p)) \to \neg\delta(p))$

(For all situations, if a situation is neutral or evil, then it is not good.)

Proof.

1. $\delta(p) \to \neg\xi(p)$ [A3.4, Spec]
2. $\xi(p) \to \neg\delta(p)$ [**PC**, 1]
3. $\nu(p) \leftrightarrow (\neg\delta(p) \land \neg\xi(p))$ [A3.11, Spec]
4. $\nu(p) \to \neg\delta(p)$ [**PC**, 3]
5. $(\nu(p) \lor \xi(p)) \to \neg\delta(p)$ [**PC**, 2, 4]
6. $(\nu(p) \lor \xi(p)) \to \neg\delta(p)$ [Gen, 5]

□

T3.42. $\forall p(\neg\delta(p) \leftrightarrow (\nu(p) \lor \xi(p)))$

(For all situations, a situation is not good *iff* it is neutral or evil.)

Proof.

1. $\neg\delta(p) \to (\nu(p) \lor \xi(p))$ [T3.40, Spec]
2. $(\nu(p) \lor \xi(p)) \to \neg\delta(p)$ [T3.41, Spec]
3. $\neg\delta(p) \leftrightarrow (\nu(p) \lor \xi(p))$ [**PC**, 1, 2]
4. $\forall p(\neg\delta(p) \leftrightarrow (\nu(p) \lor \xi(p)))$ [Gen, 3]

□

T3.43. $\forall p \forall q(Op(p,q) \to (\nu(p) \leftrightarrow \nu(q)))$

(If two situations are opposite, then one of them is neutral *iff* the other is also neutral.)

Proof.

1. $Op(p,q)$ [Hip]
2. $Op(p,q) \to (\delta(p) \leftrightarrow \xi(q))$ [A3.8, Spec]
3. $\delta(p) \leftrightarrow \xi(q)$ [MP, 1, 2]
4. $\nu(p) \leftrightarrow (\neg\delta(p) \wedge \neg\xi(p))$ [A3.11, Spec]
5. $\nu(q) \leftrightarrow (\neg\delta(q) \wedge \neg\xi(q))$ [A3.11, p/q, Spec]
6. $\delta(q) \leftrightarrow \xi(p)$ [US, 3]
7. $\nu(p) \leftrightarrow (\neg\xi(q) \wedge \neg\delta(q))$ [Eq, 3 & 6 in 4]
8. $\nu(p) \leftrightarrow (\neg\delta(q) \wedge \neg\xi(q))$ [**PC**, 7]
9. $\nu(p) \leftrightarrow \nu(q)$ [Eq, 5 in 8]
10. $Op(p,q) \to (\nu(p) \leftrightarrow \nu(q))$ [DT, 1–9]
11. $\forall p \forall q(Op(p,q) \to (\nu(p) \leftrightarrow \nu(q)))$ [Gen, 10]

□

So far, the axiology is the feature of **N2** which has most to do with **N1**. Despite their different symbols for good, evil and neutral situations, all of the results derived here can also be derived in **N1** and vice versa.

3.4 Refutation of determinism

Let us recover the statement **DET1**, one of the main determinist claims addressed in our systems:

$$\textbf{(DET1)}\ \forall p(P(p) \to \mathcal{C}_\theta P(p))$$

T3.44 is an answer to such claim:

T3.44 (¬ DET1)**.** $\neg\forall p(P(p) \to \mathcal{C}_\theta P(p))$

(Not all situations are such that, if some situation is the case, then God wills such situation to be the case.)

Proof.

1. $\forall p(P(p) \to \mathcal{C}_\theta P(p))$ [Hip]
2. $P(p) \to \mathcal{C}_\theta P(p)$ [1, Spec]
3. $\mathcal{C}_\theta P(p) \leftrightarrow \delta(p)$ [Def. 3.9, Spec]
4. $P(p) \to \delta(p)$ [Eq, 3 in 2]
7. $\forall p(P(p) \to \delta(p))$ [Gen, 6]
8. $\neg\forall p(P(p) \to \delta(p))$ [A3.10]
9. $\neg\forall p(P(p) \to \mathcal{C}_\theta P(p))$ [¬ Hip, 7, 8]

□

Thus, in **N2**, **DET1** is also false. The following theorems establish some consequences of T3.44: not all situations are such that, if God permits some state of affairs involving it, then such state of affairs holds; furthermore, not all situations are such that, if God permits a state of affairs, He wills it.

It is noteworthy that refuting religious determinism in Nieznański's corresponding system depends on a axiom analogous to A3.5. In his system, the formula $\sim \forall p(Dbp \to CbDbp)$ was assumed as an axiom. Such formula is analogous to the negation of a first-order version of **5**, the characteristic axiom of S5, and in fact, he calls it "the negation of specific axiom of S5 system" (NIEZNAŃSKI, 2008, p. 259). However, the same could not have been obtained here, for A3.5 employs the predicate P, and not a generic formula that can be substituted through **US**. If A3.5 were defined as $\neg\forall p(\mathcal{D}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p))$, then, it would be possible to deduce that God permits contradictions:

$$\neg\forall p(\mathcal{D}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p)) \vdash \exists p\mathcal{D}_\theta(\alpha(p) \land \neg\alpha(p))$$

Proof.

1. $\neg\forall p(\mathcal{D}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p))$ [Hip.]
2. $\exists p\neg(\mathcal{D}_\theta\alpha(p) \to \mathcal{C}_\theta\mathcal{D}_\theta\alpha(p))$ [**FOL**, 1]
3. $\neg(\mathcal{D}_\theta\alpha(r) \to \mathcal{C}_\theta\mathcal{D}_\theta\alpha(r))$ [Inst, 2]
4. $\mathcal{D}_\theta\alpha(r) \land \neg\mathcal{C}_\theta\mathcal{D}_\theta\alpha(r)$ [**PC**, 3]
5. $\mathcal{D}_\theta\alpha(r)$ [**PC**, 4]
6. $\mathcal{D}_\theta(\alpha(r) \land \neg\alpha(r))$ [5, $\alpha(r)/(\alpha(r) \land \neg\alpha(r))$]

7. $\exists p \mathcal{D}_\theta(\alpha(p) \land \neg\alpha(p))$ [**FOL**, 5]

□

However, this result would be inconsistent with other theorems of **N2**. Take, for instance, T3.12 and T3.18: by *Modus Ponens*, one may deduce from such theorems the negation of what is expressed in line 6 of the deduction above.

Other results are provided in the following, showing how divine permission and will are related to states of affairs and situations.

T3.45. $\neg\forall p(\mathcal{D}_\theta\alpha(p) \to \alpha(p))$

(Not all situations are such that, if God permits a state of affairs, then it is the case.)

Proof.

1. $\forall p(\mathcal{D}_\theta\alpha(p) \to \alpha(p))$ [Hip.]
2. $\mathcal{D}_\theta\alpha(p) \to \alpha(p)$ [Spec., 1]
3. $\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [A3.6, Spec]
4. $\neg\mathcal{C}_\theta\neg\alpha(p) \to \alpha(p)$ [Eq, 3 in 2]
5. $\neg\alpha(p) \to \mathcal{C}_\theta\neg\alpha(p)$ [**PC**, 4]
6. $\alpha(p) \to \mathcal{C}_\theta\alpha(p)$ [5, $\alpha(p)/\neg\alpha(p)$, **PC**]
7. $P(p) \to \mathcal{C}_\theta P(p)$ [6, $\alpha(p)/P(p)$]
8. $\forall p(P(p) \to \mathcal{C}_\theta P(p))$ [Gen, 7]
9. $\neg\forall p(P(p) \to \mathcal{C}_\theta P(p))$ [T3.44]
10. $\neg\forall p(\mathcal{D}_\theta\alpha(p) \to \alpha(p))$ [¬Hip, 1]

□

T3.46. $\neg\forall p(\mathcal{D}_\theta P(p) \to \mathcal{C}_\theta P(p))$

(Not all situations are such that, if God permits a situation to be the case, then God wills such situation to be the case.)

Proof.

1. $\forall p(\mathcal{D}_\theta P(p) \to \mathcal{C}_\theta P(p))$ [Hip]
2. $\mathcal{D}_\theta P(p) \to \mathcal{C}_\theta P(p)$ [1, Spec]
3. $\mathcal{C}_\theta P(p) \to \mathcal{C}_\theta\mathcal{D}_\theta P(p)$ [T3.30, $\alpha(p)/P(p)$, Spec]
4. $\mathcal{D}_\theta P(p) \to \mathcal{C}_\theta\mathcal{D}_\theta P(p)$ [**PC**, 2, 3]
5. $\forall p(\mathcal{D}_\theta P(p) \to \mathcal{C}_\theta\mathcal{D}_\theta P(p))$ [Gen, 5]

6. $\neg\forall p(\mathcal{D}_\theta P(p) \to \mathcal{C}_\theta \mathcal{D}_\theta P(p))$ [A3.5]

7. $\neg\forall p(\mathcal{D}_\theta P(p) \to \mathcal{C}_\theta P(p))$ [¬ Hip, 1]

□

Additionally, there is at least one situation such that God permits both the situation to be the case and not to be the case. The following theorem states more rigorously this statement:

T3.47. $\exists p(\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p))$

(There is at least one situation such that God permits such situation to be the case and permits it not to be the case.)

Proof.

1. $\neg\forall p(\mathcal{D}_\theta P(p) \to \mathcal{C}_\theta \mathcal{D}_\theta P(p))$ [A3.5]
2. $\exists p \neg(\mathcal{D}_\theta P(p) \to \mathcal{C}_\theta \mathcal{D}_\theta P(p))$ [**FOL**, 1]
3. $\neg(\mathcal{D}_\theta P(r) \to \mathcal{C}_\theta \mathcal{D}_\theta P(r))$ [Inst, 2]
4. $\mathcal{D}_\theta P(r) \wedge \neg\mathcal{C}_\theta \mathcal{D}_\theta P(r)$ [**PC**, 3]
5. $\neg\mathcal{C}_\theta \mathcal{D}_\theta P(r) \to \mathcal{D}_\theta \neg\mathcal{D}_\theta P(r)$ [T3.15, $\alpha(p)/\mathcal{D}_\theta P(r)$]
6. $\mathcal{D}_\theta P(r) \wedge \mathcal{D}_\theta \neg\mathcal{D}_\theta P(r)$ [**PC**, 5]
7. $\mathcal{D}_\theta P(r)$ [**PC**, 6]
8. $\mathcal{D}_\theta \neg\mathcal{D}_\theta P(r)$ [**PC**, 6]
9. $\neg\mathcal{D}_\theta P(r) \leftrightarrow \mathcal{C}_\theta \neg P(r)$ [T3.15, $\alpha(p)/P(r)$, Spec]
10. $\mathcal{D}_\theta \mathcal{C}_\theta \neg P(r)$ [Eq, 9 in 8]
11. $\mathcal{D}_\theta \mathcal{C}_\theta \neg P(r) \to \mathcal{D}_\theta \mathcal{C}_\theta \mathcal{D}_\theta \neg P(r)$ [T3.31, $\alpha(p)/\neg P(r)$, Spec]
12. $\mathcal{D}_\theta \mathcal{C}_\theta \mathcal{D}_\theta \neg P(r) \to \mathcal{D}_\theta \neg P(r)$ [T3.33, $\alpha(p)/\neg P(r)$, Spec]
13. $\mathcal{D}_\theta \mathcal{C}_\theta \neg P(r) \to \mathcal{D}_\theta \neg P(r)$ [**PC**, 11, 12]
14. $\mathcal{D}_\theta \neg P(r)$ [MP, 10, 13]
15. $\mathcal{D}_\theta P(r) \wedge \mathcal{D}_\theta \neg P(r)$ [**PC**, 7, 14]
16. $\exists p(\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p))$ [**FOL**, 15]

□

The following definition establishes what it means for God to be a "will-it-all", *i.e.*, to hold the property of willing any situation to be or not to be.

Def. 3.10 (Will-it-all)**.** $CW :\leftrightarrow \forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$

(God is a ‘will-it-all’ *iff*, for all situations, God wills some state of affairs to be the case or not to be the case.)

As a result, God is not a “will-it-all”, as the following theorems show:

T3.48. $\neg CW$

(God is not a ‘will-it-all’.)

Proof.

1. $\exists p(\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p))$ [T3.47]
2. $\neg\forall p\neg(\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p))$ [**FOL**, 1]
3. $\neg(\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p)) \leftrightarrow (\neg\mathcal{D}_\theta P(p) \vee \neg\mathcal{D}_\theta \neg P(p))$ [**PC**-Theorem]
4. $\neg\mathcal{D}_\theta P(p) \leftrightarrow \mathcal{C}_\theta \neg P(p)$ [T3.15, $\alpha(p)/P(p)$, Spec]
5. $\mathcal{C}_\theta P(p) \leftrightarrow \neg\mathcal{D}_\theta \neg P(p)$ [T3.14, $\alpha(p)/P(p)$, Spec]
6. $\neg(\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p)) \leftrightarrow (\mathcal{C}_\theta \neg P(p) \vee \mathcal{C}_\theta P(p))$ [Eq, 5 & 4 in 3]
7. $\neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [Eq, 6 in 2]
8. $CW \leftrightarrow \forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [Def. 3.10]
9. $\neg CW \leftrightarrow \neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [**PC**, 8]
10. $\neg CW$ [**PC**, 8, 9]

□

T3.49 refutes the claim **DET2**, restated in the following:

$$\textbf{(DET2)}\ \forall p(\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p))$$

T3.49. $\neg\forall p(\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p))$

(Not all situations are such that if God knows that a situation is the case, then God wills such situation to be the case.)

Proof.

1. $\forall p(\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p))$ [Hip]
2. $\quad\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p)$ [Spec, 1]
3. $\quad\mathcal{W}_\theta P(p) \leftrightarrow P(p)$ [T3.4, $\alpha(p)/P(p)$, Spec]
4. $\quad P(p) \rightarrow \mathcal{C}_\theta P(p)$ [Eq, 3 in 2]
5. $\quad\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [Gen, 4]
6. $\quad\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [T3.44]

7. $\quad \forall p(P(p) \to \mathcal{C}_\theta P(p)) \land \neg\forall p(P(p) \to \mathcal{C}_\theta P(p))$ [**PC**, 5, 6]

7. $\neg\forall p(\mathcal{W}_\theta P(p) \to \mathcal{C}_\theta P(p))$ [¬ Hip, 1]

□

Finally, the definition of contingency of situations is stated, as well as a definition of the responsibility of God concerning states of affairs. Several theorems (from T3.50 to T3.58) are demonstrated in order to describe how God's will, permission, opposition and responsibility are related to contingent situations.

Def. 3.11 (Contingency)**.** $K(p) :\leftrightarrow (\mathcal{D}_\theta P(p) \land \mathcal{D}_\theta \neg P(p))$

(A situation is contingent *iff* God permits it to be the case and permits it not to be the case.)

T3.50. $\forall p(K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \land \neg\mathcal{C}_\theta \neg P(p)))$

(For all situations, a situation is contingent *iff* God neither wills such situation to be the case, nor wills it not to be the case.)

Proof.

1. $K(p) \leftrightarrow (\mathcal{D}_\theta P(p) \land \mathcal{D}_\theta \neg P(p))$ [Def. 3.11]
2. $\mathcal{D}_\theta P(p) \leftrightarrow \neg\mathcal{C}_\theta \neg P(p)$ [A3.6, Spec]
3. $\mathcal{D}_\theta \neg P(p) \leftrightarrow \neg\mathcal{C}_\theta P(p)$ [T3.15, $\alpha(p)/P(p)$, Spec]
4. $K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \land \neg\mathcal{C}_\theta \neg P(p))$ [Eq, 2 & 3 in 1]
5. $\forall p(K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \land \neg\mathcal{C}_\theta \neg P(p)))$ [Gen, 4]

□

T3.51. $\forall p(K(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \lor \mathcal{C}_\theta \neg P(p)))$

(For all situations, a situation is contingent *iff* it is not the case that God wills such situation to be the case or that He wills it not to be the case.)

Proof.

1. $K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \land \neg\mathcal{C}_\theta \neg P(p))$ [T3.50, Spec]
2. $(\neg\mathcal{C}_\theta P(p) \land \neg\mathcal{C}_\theta \neg P(p)) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \lor \mathcal{C}_\theta \neg P(p))$ [**PC**-Theorem]
3. $K(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \lor \mathcal{C}_\theta \neg P(p))$ [Eq, 2 in 1]
4. $\forall p(K(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \lor \mathcal{C}_\theta \neg P(p)))$ [Gen, 3]

□

T3.52. $\exists pK(p) \leftrightarrow \exists p\neg(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$

(There is at least one contingent situation *iff* there is at least one situation such that it is not the case that God wills it to be the case or that God wills it not to be the case.)

Proof.

1. $K(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [T3.51]
2. $\exists pK(p) \leftrightarrow \exists p\neg(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [**FOL**, 1]

□

T3.53. $\exists pK(p) \leftrightarrow \neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$

(There are contingent situations *iff* it is not the case that, for all situations, God wills a situation to be the case or wills it not to be the case.)

Proof.

1. $\exists pK(p) \leftrightarrow \exists p\neg(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [T3.52]
2. $\exists p\neg(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p)) \leftrightarrow \neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [**FOL**-Theorem]
3. $\exists pK(p) \leftrightarrow \neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [Eq, 2 in 1]

□

T3.54. $\exists pK(p) \leftrightarrow \neg CW$

(There is a contingent situation *iff* God is not a 'will-it-all'.)

Proof.

1. $CW \leftrightarrow \forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [Def. 3.10]
2. $\exists pK(p) \leftrightarrow \neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$ [T3.53]
3. $\exists pK(p) \leftrightarrow \neg CW$ [Eq, 1 in 2]

□

T3.55. $\exists pK(p)$

(There is a situation such that some state of affairs involving it is contingent.)

Proof.

1. $\exists pK(p) \leftrightarrow \neg CW$ [T3.55]
2. $\neg CW$ [T3.48]
3. $\exists pK(p)$ [MP, 1, 2]

□

In what follows, a definition of divine responsibility is provided.

Def. 3.12 (Responsibility of God)**.** $\mathcal{O}_\theta\alpha(p) :\leftrightarrow (\mathcal{C}_\theta\alpha(p) \vee \mathcal{C}_\theta\neg\alpha(p)))$

(God is responsible for a state of affairs *iff* He wills such state of affairs or wills it not to be the case.)

There is a significant difference between this definition and that of **N1** (Def. 2.13). In **N1**, God is responsible for a state of affairs when He *wills* such state of affairs; in **N2**, He is responsible for a state of affairs when He *wills* or *wills not* a state of affairs.

T3.56. $\forall p(\mathcal{O}_\theta\alpha(p) \leftrightarrow (\mathcal{C}_\theta\alpha(p) \vee \mathcal{S}_\theta\alpha(p)))$

(God is responsible for some state of affairs *iff* He wills it or opposes to it.)

Proof.

1. $\mathcal{S}_\theta\alpha(p) \leftrightarrow \mathcal{C}_\theta\neg\alpha(p)$ [A3.7, Spec]
2. $\mathcal{O}_\theta\alpha(p) \leftrightarrow (\mathcal{C}_\theta\alpha(p) \vee \mathcal{C}_\theta\neg\alpha(p))$ [Def. 3.12]
3. $\mathcal{O}_\theta\alpha(p) \leftrightarrow (\mathcal{C}_\theta\alpha(p) \vee \mathcal{S}_\theta\alpha(p))$ [Eq, 1, 2]
4. $\forall p(\mathcal{O}_\theta\alpha(p) \leftrightarrow (\mathcal{C}_\theta\alpha(p) \vee \mathcal{S}_\theta\alpha(p)))$ [Gen, 3]

□

T3.57. $\forall p(\mathcal{O}_\theta P(p)) \leftrightarrow CW$

(For all situations, God is responsible for some state of affairs involving it *iff* He is not a 'want-it-all'.)

Proof.

1. $\forall p(\mathcal{O}_\theta P(p) \leftrightarrow \mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta\neg P(p))$ [Def.3.12, $\alpha(p)/P(p)$]
2. $\forall p(\mathcal{O}_\theta P(p)) \leftrightarrow \forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta\neg P(p))$ [**FOL**, 1]
3. $CW \leftrightarrow \forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta\neg P(p))$ [Df.3.10]
4. $\forall p(\mathcal{O}_\theta P(p)) \leftrightarrow CW$ [Eq, 2 in 3]

□

T3.58. $\neg\forall p(\mathcal{O}_\theta P(p))$

(Not all situations are such that God is responsible for such situations to be the case.)

Proof.

1. $\forall p\mathcal{O}_\theta P(p) \leftrightarrow CW$ [T3.57]

2. $\neg CW \leftrightarrow \neg\forall p \mathcal{O}_\theta P(p)$ [PC, 1]

3. $\neg CW$ [T3.48]

4. $\neg\forall p \mathcal{O}_\theta P(p)$ [**PC**, 2, 3]

□

T3.59. $\forall p(\neg\mathcal{O}_\theta P(p) \leftrightarrow K(p))$

(God is not responsible for a situation to be the case *iff* the situation is contingent.)

Proof.

1. $\mathcal{O}_\theta P(p) \leftrightarrow (\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta\neg P(p))$ [Def. 3.12, $\alpha(p)/P(p)$]
2. $\neg\mathcal{O}_\theta P(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta\neg P(p))$ [**PC**, 1]
3. $K(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta\neg P(p))$ [T3.51, Spec]
4. $\neg\mathcal{O}_\theta P(p) \leftrightarrow K(p)$ [Eq, 2, 3]
5. $\forall p(\neg\mathcal{O}_\theta P(p) \leftrightarrow K(p))$ [Gen, 4]

□

3.5 N2 and N1: some remarks

In this chapter, the system **N2** was described. It consists of a revisiting of Nieznański's system, which is in many senses similar to **N1**: both establish relations between God's will and knowledge to good, evil, and contingent situations, refuting **DET1** and **DET2**. As it is possible to see, the systems have also many differences – for instance, the set of axioms of each system is different; there are significant differences on some definitions (for instance, the definition of divine responsibility); many theorems derived in **N1** cannot be derived in **N2**, as the former includes S5 and the second includes S4; and so forth.

However, concerning these systems, two more issues still remain. The first issue is the following: why should one require such a long list of theorems in order to deal with the problem of evil? Some of the definitions, axioms, and theorems of both systems seem to be superfluous to answer religious determinism and the logical problem of evil. Why argue, for instance, that if God wills some state of affairs, then He wills to permit to will such state of affairs, as T3.28 shows? Why suppose that God's will is regulated by S4 or S5, two of the strongest systems in modal logic? Perhaps it may serve as a technical tool to reduce modalities (in $\mathsf{S5}$, for instance, any combination of modalities can be reduced to just one), or perhaps it may be related to other religious features within classical theism, such as divine simplicity, but it does not mean neither that God's will works as a $\mathsf{S4}$ or $\mathsf{S5}$ system, nor that it provides a clear solution to many of the philosophical questions described in the chapter 1.[8]

Moreover, at least one issue is noteworthy: A2.3, an axiom of **N1**, corresponds to a quantification over the axiom **5** of $\mathsf{S5}$:

$$\textbf{A2.3.}\ \forall p(\mathcal{D}_\theta \alpha(p) \rightarrow \mathcal{C}_\theta \mathcal{D}_\theta \alpha(p))$$

From A2.3, it is easy to obtain, as a theorem, the following formula (by specification, then by substituting $\alpha(p)$ for $P(p)$, and then by generalization):

$$\vdash_{N1} \forall p(\mathcal{D}_\theta P(p) \rightarrow \mathcal{C}_\theta \mathcal{D}_\theta P(p))$$

However, **N2** includes the following axiom:

$$\textbf{A3.5.}\ \neg\forall p(\mathcal{D}_\theta P(p) \rightarrow \mathcal{C}_\theta \mathcal{D}_\theta P(p))$$

As it is possible to see, the formulas are explicitly contradictory. These contradictions lead us to at least two possible conclusions: either one or both systems are trivial and thus all of the results could be trivially obtained, or there must be a set of axioms which is enough to reach some of the most interesting conclusions for both systems, avoiding contradictions.

In the light of these remarks, a new proposal can be developed. Our aim is to answer religious determinism in order to deal with the problem of evil in a clearer way; thus, for that reason, we sought to a system that could address such questions more directly, for while **N1**, **N2**, and Nieznański's systems have the merits of dealing with many issues, a smaller system could contribute to the discussion in a more significant way, and its contribution could be assessed much easier. As the contradiction between **N1** and **N2** is not associated with the axioms and theorems related to **DET1**, **DET2**, and the answer to the problem of evil, and in any case, the answer to this problem is not logically trivial.

[8] Nieznański's intention was probably to deal with many other features of classical theism in his systems. This may justify, for instance, such a long description of God's attributes in both articles (2007; 2008).

Thus, our research found that an underlying minimal set of axioms is enough to settle the questions proposed. This minimal system, called **N3**, is enough to solve the same issues tackled by **N1** and **N2**, but with less assumptions than these systems. Let us now present this axiomatic approach to theodicy via formal applied systems.

Chapter 4

The final proposal

In the previous chapters, the systems **N1** and **N2** were presented. Although different from those systems formulated by Nieznański, they somehow followed his fundamental insights. Both **N1** and **N2** characterize the attributes of God, provide a formal axiology, and give an answer to a form of religious determinism; all of these results are also obtained from their distinct sets of axioms. However, as discussed in the end of the last chapter, one may ask whether all of these axioms and theorems are necessary to solve the problem of evil. Much more seems to be done by them than just establishing an answer to the question.

Besides, as exposed before, when the systems are considered together, one of the outcomes is surprising: they are mutually contradictory. Let us consider the following formulas: the first, a theorem of **N1**, and the second, an axiom of **N2**:

$\vdash_{N1} \forall p(\mathcal{D}_\theta P(p) \rightarrow \mathcal{C}_\theta \mathcal{D}_\theta P(p))$
(For all situations, if God permits a situation to be the case, then He wills to permit such a situation to be the case.)

A3.5 $\neg\forall p(\mathcal{D}_\theta P(p) \rightarrow \mathcal{C}_\theta \mathcal{D}_\theta P(p))$
(Not all situations are such that, if God permits some situation to be the case, then He wills to permit such situation to be the case.)

These issues suggest that a new system, called **N3**, could be sufficient to prove the most relevant results of **N1** and **N2**. In the following, the formal structure of **N3** is presented and discussed. Some of these results are going to be published soon in an article (DA SILVA; BERTATO, 2020).

4.1 The system N3

4.1.1 Language, rules, and axioms of N3

The system **N3** is described in a first-order modal logic. The language $\mathcal{L}_{N3}$ of **N3** is very similar to that of the previous systems. Such language has the following symbols as primitives:

(i) Symbols for unary predicates: B, P, d, z;

(ii) A symbol of constant: θ;

(iii) Variables for situations: p, q, r, possibly with indexes;

(iv) The symbols for connectives: $\neg, \rightarrow$;

(v) The symbol of universal operator: $\forall$;

(vi) Two symbols for modal operators: $\mathcal{C}_\theta, \mathcal{W}_\theta$.

The formation rules are the same as of the previous systems.[1] Some rules of deduction of **N3** are: *Modus Ponens* (MP), *Uniform Substitution* (US), *Rule of Necessitation* (Nec) and *Substitution of Equivalents* (Eq).[2] This characterizes **N3** as a normal modal system. The Theorem of Deduction **(DT)** is valid in the system.[3]

The symbols of the language $(\wedge, \vee, \leftrightarrow, \exists)$ are defined as in **N1** and **N2**,[4] and other symbols are defined as usual. As a convention, $\alpha(p)$ stands for any *wff* that involves *only* the variable p, where p is free.

Let us now establish the formal definition of the attributes of God.

4.1.2 Attributes of God

The divine attributes are defined as follows. The main difference here is that, in **N3**, the goodness of God is considered a divine attribute, and not an isolate feature of the system.

[1]See p. 45 for these formation rules.
[2]See p. 45 for their definitions.
[3]Its definition is found on p. 46. See also note 5, p. 46.
[4]See p. 46.

Def. 4.1. $WW :\leftrightarrow \forall p(\alpha(p) \to \mathcal{W}_\theta \alpha(p))$

(God is omniscient *iff* for all situations, if a state of affairs is the case, then God knows it.)

Def. 4.2. $NM :\leftrightarrow \forall p(\mathcal{W}_\theta \alpha(p) \to \alpha(p))$

(God is infallible *iff*, for all situations, if God knows a state of affairs, then it is the case.)

Def. 4.3. $WM :\leftrightarrow \forall p(\mathcal{C}_\theta \alpha(p) \to \alpha(p))$

(God is omnipotent *iff*, for all situations, if God wills a state of affairs, then it is the case.)

Def. 4.4. $DB :\leftrightarrow \forall p(\mathcal{C}_\theta P(p) \to \delta(p))$

(God is omnibenevolent *iff*, for all situations, if God wills a situation to be the case, then such situation is good.)

The definitions above are in accordance with **N1**, **N2**, and Nieznański's definitions (p. 204 NIEZNAŃSKI, 2007, 2008, p. 255). The description of omnipotence, in particular, is also in accordance with Curt Christian (p. 152-153 NIEZNAŃSKI, 1987; ŚWIĘTORZECKA, 2011, p. 309). In addition, as affirmed in chapter 1, it is important to interpret the material implication not as a cause and effect relation, but much more as a relation of pertinence: if the implication is true, whenever the antecedent holds, the consequent also holds. And, finally, it is possible to affirm that even if the concepts of the attributes of God may be stronger than the formalization presented, they at least include the formalization presented here.[5]

In accordance with such characterizations, God is defined as follows:

Def. 4.5 (God)**.** $B(\theta) :\leftrightarrow (WW \land NM \land WM \land DB)$

There are only three axioms in **N3**. All of them are directly related to the problem of evil. As expressed in the first chapter, Mackie states that there is an inconsistency between the existence of God and the existence of evil. Furthermore, he states that good is opposed to evil (MACKIE, 1955, p. 201). Thus, the following axioms express the formal components of this debate:

A4.1. $B(\theta)$

[5]See. p. 36-40 of this dissertation for an extended discussion of these features.

(God is divine)

A4.2. $\neg\forall p(P(p) \rightarrow \delta(p))$

(Not all situations are such that, if a situation is the case, then such situation is good.)

An equivalent formulation of A4.2 would be $\exists p(P(p) \wedge \neg\delta(p))$.

A4.3. $\forall p(\delta(p) \rightarrow \neg\xi(p))$

(For all situations, if a situation is good, then it is not evil.)

One may find that the axiom A4.2 is not the same as to say that there is evil. But there are two reasons to state the axiom in this form. First, if there are situations that are the case but are not good, among these situations we could count evil ones, if evil exists in our world. There are possible worlds, however, in which there are no evil situations; but it does not mean that all of these situations are good, if there is no evil. Think about a falling leaf; is this situation good? Perhaps yes, but perhaps no. It is not necessary to discuss whether these situations are morally relevant; thus, we can, for the sake of argumentation, assume that they are not good, either because they do not involve a free moral action, or because they are neutral in some other sense.

But the second reason, compatible with the first, is that Mackie argues that God could create a world in which there is no evil (MACKIE, 1955, p. 209.). If this is so, the statement “evil exists” is not necessarily true, and it cannot be assumed as an axiom: it is true in the actual world, but it is not necessarily true. Furthermore, as **N3** is a normal modal system, it includes **Nec**: if γ is an axiom or a theorem, then, $\mathcal{C}_\theta\gamma$ is a theorem. But if we formalize and assume “evil exists” as one of our axioms, we fall in a petition of principle: to prove that God wills evil in the world, we suppose, in principle, that evil exists, which is the same of assuming, implicitly, that God wills evil to exist. Is it necessarily true? Not in classical theism. If one wills to assume this as an axiom, much more is assumed than just holding the logical problem of evil: it is affirming that evil necessarily exists, and that there is a God other than the God of classical theism.

Let us now present the theorems of **N3**. In the following, the attributes of God are introduced:

T4.1. $WW \wedge NM \wedge WM \wedge DB$

(God is omniscient, infallible, omnipotent, and omnibenevolent.)

Proof.

1. $B(\theta)$ [A4.1]
2. $B(\theta) :\leftrightarrow WW \wedge NM \wedge WM \wedge DB$ [Def. 4.5]
3. $WW \wedge NM \wedge WM \wedge DB$ [**PC**, 1, 2]

□

T4.1.1 $\forall p(\alpha(p) \rightarrow \mathcal{W}_\theta \alpha(p))$ □

(For all situations, if a state of affairs is the case, then God knows such a state of affairs.)

T4.1.2 $\forall p(\mathcal{W}_\theta \alpha(p) \rightarrow \alpha(p))$ □

(For all situations, if God knows a state of affairs, then such state of affairs is the case.)

T4.1.3 $\forall p(\mathcal{C}_\theta \alpha(p) \rightarrow \alpha(p))$ □

(For all situations, if God wills a state of affairs, then such state of affairs is the case.)

T4.1.4 $\forall p(\mathcal{C}_\theta P(p) \rightarrow \delta(p))$ □

(For all situations, if God wills some situation to be the case, then such situation is good.)

The axioms and theorems stated above characterize the divine attributes of omnipotence, omniscience, infallibility and omnibenevolence of God. In all situations, and for arbitrary states of affairs, if a state of affairs is the case, then God knows it, for He is omniscient; if God knows a state of affairs, then it is the case, for God is infallible in His (fore)knowledge; if God wills a state of affairs, then it is the case, for he is omnipotent; and finally, as God is omnibenevolent, if he wills a state of affairs, then it is good.

In this regard, David Hume asks: why is there evil in the world? As quoted in the chapter 1, he states the problem of evil in the following terms:

> "*Epicurus*' old questions are yet unanswered. Is he willing to prevent evil, but not able? then is he impotent. Is he able, but not willing? then is he malevolent. Is he both able and willing? Whence then is evil?" (1779/2007, p. 74)

Hume's statement has been discussed in the first chapter of this work. But one may perceive here that he seems to assume that if God does not will a state of affairs, then such a state of affairs is not the case.[6] This claim is ambiguous; in fact, if God is omnipotent (T4.1.3), His willing some state of affairs not to occur implies that such state of affairs does not occur. But if God is omnipotent, it does not mean that if some situation is the case, then God knows it. A similar affirmation can be said about Mackie's claims: if there is evil in the world, God wills such to be the case, for He could have avoided it by creating another world.[7]

And here we arrive to the core of our work: the philosophers seem to assume a determinist account on the divine attribute of omnipotence. Both seem to assume a fallacy: if God is omnipotent, then if some situation is the case implies that God wills it. Stating the first determinist claim once again,

(DET1) $\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$

(For all situations, if a situation is the case, then God wills such situation to be the case.)

4.1.3 Religious determinism defeated

In **N3**, **DET1** is, once again, refuted. We begin by this refutation because of its centrality to the problem we are concerned with:

T4.2 (¬DET1)**.** $\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$

(Not all situations are such that, if a situation is the case, then God wills such situation to be the case.)

Proof.

1.	$\neg\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$	[Hip]
2.	$\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$	[**PC**, 1]
3.	$P(p) \rightarrow \mathcal{C}_\theta P(p)$	[2, Spec]
4.	$\mathcal{C}_\theta P(p) \rightarrow \delta(p)$	[T4.1.3, Spec]
5.	$P(p) \rightarrow \delta(p)$	[**PC**, 3, 4]

[6] I thank Professor Fábio Bertato for this insight on Hume's claim.

[7] In his article, Mackie even casts doubt on free will; as he says, concerning a free will solution to the problem of evil: "I think that this solution is unsatisfactory primarily because of the incoherence of the notion of freedom of the will" (MACKIE, 1955, p. 209).

6. $\forall p(P(p) \rightarrow \delta(p))$ [Gen, 5]

7. $\neg\forall p(P(p) \rightarrow \delta(p))$ [A4.2]

9. $\neg\neg\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [$\neg$ Hip, 7, 8]

10. $\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [**PC**, 9]

□

The claim that infallibility and omniscience are implicit in many allegations related to the problem of evil, labeled as **DET2**, (see p. 36-40), is also refuted here:

T4.3 ($\neg$DET2)**.** $\neg\forall p(\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p))$

(Not all situations are such that if God knows a situation to be the case, then God wills such a situation to be the case.)

Proof.

1. $\forall p(\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p))$ [Hip]

2. $\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p)$ [1, Spec]

3. $P(p) \rightarrow \mathcal{W}_\theta P(p)$ [T4.1.1, $P(p)/\alpha(p)$, Spec]

4. $\mathcal{W}_\theta P(p) \rightarrow P(p)$ [T4.1.2, $P(p)/\alpha(p)$, Spec]

5. $P(p) \leftrightarrow \mathcal{W}_\theta P(p)$ [**PC**, 3, 4]

6. $P(p) \rightarrow \mathcal{C}_\theta P(p)$ [**Eq**, 5, 2]

7. $\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [Gen, 6]

8. $\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [T4.2]

9. $\neg\forall p(\mathcal{W}_\theta P(p) \rightarrow \mathcal{C}_\theta P(p))$ [$\neg$ Hip]

□

Thus, omnipotence, omniscience, infallibility and omnibenevolence of God are consistent with the existence of situations outside God's control. The reader may have realized that, in T4.2, the only attribute necessary to refute **DET1** is the attribute of omnibenevolence.[8] Since God determines only what is good, and there are situations which are not, the natural result is that He should not be blamed for that.

As a last contribution, let us now explore, in some more few theorems, the problem of evil, in order to enrich our explanation.

[8] This led us to develop a first-order system to consider the incompatibility of religious determinism and the divine attribute of omnibenevolence. The system shall be published soon (BERTATO; DA SILVA, forthcoming).

4.1.4 Further consequences: God, values and determinism

The first three theorems of this subsection are devoted to relate evil and the will of God:

T4.4. $\forall p(\xi(p) \to \neg\delta(p))$

(For all situations, if a situation is evil, then such situation is not good.)

Proof. Easily deduced from A4.3, by contraposition. □

T4.5. $\forall p(\neg\delta(p) \to \neg\mathcal{C}_\theta P(p))$

(For all situations, if a situation is not good, then it is not the case that God wills it to be the case.)

Proof. Easily deduced from T4.1.4, by contraposition. □

T4.6. $\forall p(\xi(p) \to \neg\mathcal{C}_\theta P(p))$

(For all situations, if a situation is evil, then it is not the case that God wills it to be the case.)

Proof.

1. $\forall p(\neg\delta(p) \to \neg\mathcal{C}_\theta P(p))$ [T4.4, Spec]
2. $\neg\delta(p) \to \neg\mathcal{C}_\theta P(p)$ [T4.5, Spec]
3. $\xi(p) \to \neg\mathcal{C}_\theta P(p)$ [**PC**, 1, 2]
4. $\forall p(\xi(p) \to \neg\mathcal{C}_\theta P(p))$ [Gen, 3]

□

The next two theorems show the relation between contradictions and the will of God. The novelty is T4.8, which is deduced using the rule of necessitation **Nec**. Both theorems refute, together, the claim that there are no logical limits to what God can do, a claim that classical theists usually reject (see p. 25):

T4.7. $\forall p\neg\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p))$

(For all situations, it is not the case that God wills some contradiction.)

Proof.

1. $\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p)) \rightarrow (\alpha(p) \wedge \neg\alpha(p))$ [T4.1.3, $\alpha(p)/(\alpha(p) \wedge \neg\alpha(p))$, Spec]
2. $\neg(\alpha(p) \wedge \neg\alpha(p)) \rightarrow \neg\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [**PC**, 1]
3. $\neg(\alpha(p) \wedge \neg\alpha(p))$ [**PC**-Theorem]
4. $\neg\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [MP, 2, 3]
5. $\forall p \neg\mathcal{C}_\theta(\alpha(p) \wedge \neg\alpha(p))$ [Gen, 4]

□

T4.8. $\forall p \mathcal{C}_\theta \neg(\alpha(p) \wedge \neg\alpha(p))$

(All situations are such that God wills non-contradictions.)

Proof.

1. $\neg(\alpha(p) \wedge \neg\alpha(p))$ [**PC**-Theorem]
2. $\mathcal{C}_\theta \neg(\alpha(p) \wedge \neg\alpha(p))$ [**Nec**, 1]
3. $\forall p \mathcal{C}_\theta \neg(\alpha(p) \wedge \neg\alpha(p))$ [Gen, 2]

□

Permission is defined in **N3** as the dual operator of $\mathcal{C}_\theta$:

Def. 4.6 (Permission)**.** $\forall p(\mathcal{D}_\theta\alpha(p) :\leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p))$

The following theorem flows naturally from this definition.

T4.9. $\forall p(\mathcal{C}_\theta\alpha(p) \leftrightarrow \neg\mathcal{D}_\theta\neg\alpha(p))$

(For all situations, God wills a state of affairs *iff* He does not permit the opposite.)

Proof. Easily deduced from Def. 4.6. □

Because God is omnipotent, all of the states of affairs that are the case are permitted by Him:

T4.10. $\forall p(\alpha(p) \rightarrow \mathcal{D}_\theta\alpha(p))$

(For all situations, if a state of affairs is the case, then it is permitted by God.)

Proof.

1. $\mathcal{C}_\theta\neg\alpha(p) \rightarrow \neg\alpha(p)$ [T4.1.3, $\alpha(p)/\neg\alpha(p)$, Spec.]
2. $\mathcal{D}_\theta\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$ [Def. 4.6, Spec]

3. $\alpha(p) \to \mathcal{D}_\theta \alpha(p)$ [**PC**, 1, 2]

4. $\forall p(\alpha(p) \to \mathcal{D}_\theta \alpha(p))$ [Gen, 3]

□

But as **DET1** is false, not all situations that God permits are the case. To say that everything God permits (perhaps evil included) is the case is to implicitly accept **DET1**:

T4.11. $\neg\forall p(\mathcal{D}_\theta \alpha(p) \to \alpha(p))$

(Not all situations are such that, if God permits some state of affairs, then it is the case.)

Proof.

1. $\forall p(\mathcal{D}_\theta \alpha(p) \to \alpha(p))$ [Hip.]
2. $\mathcal{D}_\theta \alpha(p) \to \alpha(p)$ [Spec., 1]
3. $\mathcal{D}_\theta \alpha(p) \leftrightarrow \neg\mathcal{C}_\theta \neg\alpha(p)$ [Def. 4.6, Spec]
4. $\neg\mathcal{C}_\theta \neg\alpha(p) \to \alpha(p)$ [Eq, 3 in 2]
5. $\neg\alpha(p) \to \mathcal{C}_\theta \neg\alpha(p)$ [**PC**, 4]
6. $\alpha(p) \to \mathcal{C}_\theta \alpha(p)$ [5, $\neg\alpha(p)/\alpha(p)$]
7. $P(p) \to \mathcal{C}_\theta P(p)$ [6, $\alpha(p)/P(p)$]
8. $\forall p(P(p) \to \mathcal{C}_\theta P(p))$ [Gen, 7]
9. $\neg\forall p(P(p) \to \mathcal{C}_\theta P(p))$ [T4.2]
10. $\neg\forall p(\mathcal{D}_\theta \alpha(p) \to \alpha(p))$ [¬Hip, 1]

□

Finally, what is the nature of evil, when it comes to necessity and contingency? To say this, one more concept is established. It is possible to define that a situation is *contingent* when both such situation and its complementary are possible[9]. Meanwhile there is an analogy between $\Diamond$ and $\mathcal{D}_\theta$; thus, we can define the concept as follows:

Def. 4.7. $K(p) :\leftrightarrow (\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p))$

(A situation is contingent *iff* God permits it to be or not to be the case.)

This is equivalent to our next theorem:

T4.12. $\forall p(K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{C}_\theta \neg P(p)))$

[9]As Carnielli and Pizzi affirm, "it is considered to be contingently true what is neither necessarily true nor necessarily false" (CARNIELLI; PIZZI, 2008, p. 27)

(For all situations, a situation is contingent *iff* neither God wills such situation to be the case, nor wills it not to be the case.)[10]

Proof.

1. $K(p) \leftrightarrow (\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p))$ [Def. 4.7]
2. $\mathcal{D}_\theta P(p) \leftrightarrow \neg\mathcal{C}_\theta \neg P(p)$ [Def. 4.6, Spec]
3. $\mathcal{C}_\theta P(p) \leftrightarrow \neg\mathcal{D}_\theta \neg P(p)$ [T4.9, Spec]
4. $\mathcal{D}_\theta \neg P(p) \leftrightarrow \neg\mathcal{C}_\theta P(p)$ [**PC**, 3]
5. $K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{C}_\theta \neg P(p))$ [Eq, 2 & 4 in 1]
6. $\forall p(K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \neg\mathcal{C}_\theta \neg P(p)))$ [Gen, 5]

□

The following corollaries are easily deduced from T4.12, and serve to deduce some relevant results:

T4.12.1. $\forall p(K(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p)))$

(For all situations, a situation is contingent *iff* either it is not the case that God wills such situation to be the case or He wills such situation not to be the case.)

T4.12.2. $\exists p K(p) \leftrightarrow \neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p))$

(There is a contingent situation *iff* it is not the case that, for all situations, either God wills a situation to be the case or He is opposed to that.)

T4.12.3 $\forall p(K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \mathcal{D}_\theta P(p)))$

(For all situations, a situation is contingent *iff* either it is not the case that God wills such situation to be the case or He permits such situation not to be the case.)

The following theorem is relevant to prove a result on the existence of contingent situations.

T4.13. $\neg\forall p(\mathcal{C}_\theta \alpha(p) \vee \mathcal{C}_\theta \neg\alpha(p))$

(Not all situations are such that God wills either a state of affairs or its opposite.)

[10] Although preserving a similarity with that of Carnielli and Pizzi (2008, p. 27), this definition is formally different. Instead of defining contingency as an operator, which could be applied to generic states of affairs, the purpose of this definition is to provide a treatment for contingent *situations*.

Proof.

1. $\forall p(\mathcal{C}_\theta\alpha(p) \vee \mathcal{C}_\theta\neg\alpha(p))$ [Hip]
2. $\mathcal{C}_\theta\alpha(p) \vee \mathcal{C}_\theta\neg\alpha(p)$ [1, Spec]
3. $\neg\mathcal{C}_\theta\alpha(p) \rightarrow \mathcal{C}_\theta\neg\alpha(p)$ [**PC**, 2]
4. $\mathcal{C}_\theta\neg\alpha(p) \rightarrow \neg\alpha(p)$ [T4.1.3, $\alpha(p)/\neg\alpha(p)$, Spec]
5. $\neg\mathcal{C}_\theta\alpha(p) \rightarrow \neg\alpha(p)$ [**PC**, 4, 5]
6. $\alpha(p) \rightarrow \mathcal{C}_\theta\alpha(p)$ [**PC**, 5]
7. $\forall p(\alpha(p) \rightarrow \mathcal{C}_\theta\alpha(p))$ [Gen, 6]
8. $\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [7, $P(p)/\alpha(p)$]
9. $\neg\forall p(P(p) \rightarrow \mathcal{C}_\theta P(p))$ [T4.2]
10. $\neg\forall p(\mathcal{C}_\theta\alpha(p) \vee \mathcal{C}_\theta\neg\alpha(p))$ [¬ Hip, 1]

□

As an outcome, there are contingent situations. This result is a consequence of refuting **DET1**:

T4.14. $\exists pK(p)$

(There is at least one situation that is contingent.)

Proof.

1. $\exists pK(p) \leftrightarrow \neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta\neg P(p))$ [T4.12.2]
2. $\neg\forall p(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta\neg P(p))$ [T4.13, $P(p)/\alpha(p)$]
3. $\exists pK(p)$ [**PC**, 1, 2]

□

Finally, one of the most striking results. The last theorem of **N3** demonstrates the exact relation between evil, permission of God and contingency:

T4.15. $\forall p((\xi(p) \wedge \mathcal{D}_\theta P(p)) \rightarrow K(p))$

(For all situations, if a situation is evil, and God permits it, then such situation is contingent.)

Proof.

1. $\xi(p) \wedge \mathcal{D}_\theta P(p)$ [Hip.]
2. $\xi(p)$ [**PC**, 1]

3. $\mathcal{D}_\theta P(p)$ [**PC**, 1]

4. $\xi(p) \to \neg\mathcal{C}_\theta P(p)$ [T4.6, Spec]

5. $\neg\mathcal{C}_\theta P(p)$ [MP, 2, 4]

6. $\neg\mathcal{C}_\theta P(p) \wedge \mathcal{D}_\theta P(p)$ [**PC**, 5, 3]

7. $K(p) \leftrightarrow (\neg\mathcal{C}_\theta P(p) \wedge \mathcal{D}_\theta P(p))$ [T4.12.3, Spec]

8. $K(p)$ [Eq, 6 in 7]

9. $(\xi(p) \wedge \mathcal{D}_\theta P(p)) \to K(p)$ [DT, 1–14]

10. $\forall p((\xi(p) \wedge \mathcal{D}_\theta P(p)) \to K(p))$ [Gen, 15]

□

T4.15 is equivalent to $\forall p((\xi(p) \wedge \neg\mathcal{C}_\theta \neg P(p)) \to K(p))$. Such theorem has an explanatory dimension: it means that if there is evil in the world and God does not "determine" the opposite, then this very situation does not depend on his will.

4.2 Some final remarks

As the results proved from T4.9 to T4.15 are theorems, in **N3**, they are valid in any possible world. For any possible world in **N3**, **DET1** is false: there are situations that are not good (but we do not have to assume that these situations are evil because they are not good), and evil situations that God permits do not depend on the will of God. The attributes of God, as far as we can conceive and to the best of this present characterization, are logically compatible with the existence of non-good situations, evil ones included.

I think these results have many consequences to the way we look to the logical problem of evil. Mackie's classical statement is, indeed, a confusion: on affirming that God could create a world in which there would be no evil, he implicitly assumed two more statements: the first, that situations are not good *iff* they are evil; the second, that contingent situations do not exist, for God is omnipotent. Both assumptions are false. Moreover, the results established in **N3** propose a new answer to the question, as **N1** and **N2** also do; but it consists on a simpler answer, which also establishes a new path to consider the question, a path different of what has been done before.

Conclusion

The aim of this research was to establish an axiomatic approach to theodicy via formal applied systems. In the first chapter, an overview on the problem of evil was provided: some historical formulations and contemporary ones. Among such formulations, the logical problem of evil has been presented as a *logical* issue, an allegation that the existence of God and the existence of evil are mutually inconsistent. The free will defense proposed by Alvin Plantinga, one of the main solutions to it, has been presented, as well as the status of contemporary debate on the defense.

As said in the Introduction, among the main tenets of this work is the belief that logical questions deserve logical answers, and as such, this research looked for a different approach: two systems proposed by Edward Nieznański as axiomatic systems to solve the problem of evil. While many merits of these systems can be recognized, other details required a thorough revisiting. Thus, the research began by revisiting two systems originally proposed by Edward Nieznański, providing them with a more appropriate formalization to them. The new resulting systems, called **N1** and **N2**, were reformulated in first-order modal logic; they retain much of the original basic structure, but some different results were obtained.

Furthermore, our research found an underlying minimal set of axioms that suffices to settle the questions proposed. Thus, we developed a minimal system, called **N3**, that solves some of the issues tackled by **N1** and **N2**, but with less assumptions than these systems. **N1**, **N2**, and principally **N3** aim at solving the logical problem of evil through the refutation of two versions of religious determinism, showing that the attributes of God in Classical Theism, namely, those of omniscience, omnipotence, infallibility, and omnibenevolence, when formalized, are consistent with the existence of evil.

Of course, as I affirmed in the introduction, the problem of evil has many dimensions, some that go beyond the logical issue; but, when considered together, both traditional responses (such as the free will defense) and the responses presented here cast serious doubts on the logical problem of evil, understood as a challenge to the rationality of classical theism.

On developing the work, many features of the systems remained untouched, both technical and philosophical. This may lead to other works, mainly in the field of semantics of possible worlds, completeness theorems and other logical properties. A first step to deal with these issues is addressed in the models developed for the systems presented here. One of them is presented in Da Silva and Bertato (2019), and the others shall be published soon (DA SILVA; BERTATO, 2020, forthcoming; BERTATO; DA SILVA, forthcoming). Concerning the philosophical interpretation of the underlying logic, features regarding *de dicto* and *de re* modalities, the Barcan formula and issues on identity are still relevant and could not be addressed as they deserve. Finally, one more question still remains: how could we give a formal treatment to individuals in first-order multimodal logic? In the systems provided, the simplest assumption has been made, and just one individual was associated to the modal operators. But what if we consider, for instance, these operators of knowledge and volition for many agents? How would such a logic look like, both syntactically and semantically? What should be the best strategy to give a solution to the formal issues regarding individuals in first-order modal logic? These questions are certainly relevant and can be addressed by further investigations.

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