

# Carnap's Principle of Tolerance and logical pluralism

#### RESUMO

Pluralismo lógico é a tese de que há mais de uma lógica adequada. Diversos autores apontam Carnap como um dos precursores do pluralismo lógico. Mais que isso, afirmam que o Princípio de Tolerância consiste em uma das primeiras formulações explícitas de um pluralismo lógico. Não obstante, há poucas e esparsas investigações detalhadas para avaliar se o Princípio de Tolerância implica necessariamente em um pluralismo lógico e, caso implique, de qual tipo. O objetivo deste artigo é analisar o Princípio de Tolerância, bem como o contexto no qual tal princípio está inserido e, por fim, investigar qual a relação entre esse princípio e o pluralismo lógico.

Palavras-chave: Carnap; princípio de tolerância; logical syntax; pluralismo lógico.

#### **ABSTRACT**

Logical pluralism is the claim that there is more than one adequate logic. Many authors consider Carnap as one of the forerunners of logical pluralism. More than that, they claim that Carnap's Principle of Tolerance consists in one of the first explicit formulations a logical pluralism. Nonetheless, there is little detailed investigation to evaluate if the Principle of Tolerance necessarily implies a logical pluralism, and if so, of which kind. The aim of this paper is to analyze the Principle of Tolerance, as well as its context, and to investigate the relation between such principle and logical pluralism.

**Keywords**: Carnap; Principle of Tolerance; logical syntax; logical pluralism.

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## Introduction

This paper has a double aim. On the one hand, it intends to analyze Carnap's Principle of Tolerance presented on his book *Logical Syntax* of *Language*. For this, we will investigate not only this work, but also the influence of other thinkers for Carnap's thought. On the other hand, this paper intends to answer the following question, namely: what is the relation between Carnap's Principle of Tolerance and a possible logical pluralism? This is an important task since, although many scholars consider Carnap as one of the forerunners of logical pluralism, there is little detailed investigation to evaluate if the Principle of Tolerance necessarily implies a logical pluralism, and if so, of which kind.

## Logical syntax of language and the Principle of Tolerance

Let us begin with the Principle of Tolerance. This is formulated for the first time in *Logical Syntax* of *Language*. In general terms, the main goal of the book is

to provide a system of concepts, a language, by the help of which the results of logical analysis will be exactly formulable. Philosophy is to be replaced by the logic of science – that is to say, by the logical analysis of the concepts and sentences of the sciences, for the logic of science is nothing other than the logical syntax of the language of science. (CARNAP, 1937, p. viii).

With the formulation of a general syntax, applicable to any language, Carnap intends to present a solution to many philosophical problems. In fact, the idea of a general syntax was meant to replace philosophy itself<sup>1</sup>.

Nonetheless, there is a particular problem that occupies a central position in this book, namely: the discussion between formalism, intuitionism and logicism regarding the foundation of mathematics.

It is precisely in this context that the Principle of Tolerance emerges<sup>2</sup>. Once a syntactical metalanguage is formulated, it is possible to see that the three proposed solutions consist merely in formulations of different languages. In other words, from this perspective, logicism, formalism and intuitionism consist of three different ways of formulating a language, i.e., of stipulating a set of symbols together with some rules for their manipulation. In this level

<sup>&</sup>lt;sup>1</sup> We will limit ourselves to evidence the aspects of *Logical Syntax* that allows the formulation of the Principle of Tolerance. We will not investigate any specific problems of *Logical Syntax*, nor its application to other philosophical problems.

<sup>&</sup>lt;sup>2</sup> Carnap claims, years later, that "it might perhaps be called more exactly the 'principle of the conventionality of language forms'". (CARNAP, 1963, p. 5).

there is no need for an external justification for such formulation. Thus, the Principle of Tolerance consists in affirming that

In logic, there are no morals. Everyone is at liberty to build up his own logic, i.e. his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments. (CARNAP, 1937, p. 52).

Therefore, this principle does not offer a solution to the problem regarding the foundation of mathematics, but represents a dissolution of the problem. One must understand the proposed solutions as suggestions to build a language. After that, what remains is to investigate and evaluate the consequences that follow from each language.

In a schematic way, Carnap's aim with his general syntax and his Principle of Tolerance is to put an end to any discussion regarding the justification, or 'truth' of a logic. According to the German philosopher, it is precisely this pursuit for justification that, on the one hand, precludes investigations on countless languages different than classical logic and, on the other, creates several pseudo-problems on the subject. In this – and only in this – sense it is possible to understand Carnap's project as a solution for the foundations of mathematics.

The solution, therefore, does not consist of a synthesis between formalism, logicism and intuitionism, but of the possibility to establish a common framework on which each proposal can be formulated and its advantages – as well as its disadvantages – can be explored.

From this change of perspective, several questions about the foundations of mathematics must be reformulated. Take, for instance, the debate on the possibility of using impredicative definitions. Carnap asserts that

The proper way of framing the question is not 'Are [...] impredicative symbols admissible?', for, since there are no morals in logic [...] what meaning can 'admissible' have here? The problem can only be expressed in this way: 'How shall we construct a particular language? Shall we admit symbols of this kind or not? And what are the consequences of either procedure?' It is therefore a question of choosing a form of language – that is, of the establishment of rules of syntax and of the investigation of the consequences of these. (CARNAP, 1937, p. 164).

It's important to highlight that the notion of logic in the *Logical Syntax* is much more comprehensive than today's concept of logic. For Carnap, tolerance towards logic means tolerance with respect to the language adopted. The languages may contain different inferential apparatus, such as arithmetic and type theory. All different kinds of formulations are allowed, since the symbols introduced and the syntactical rules for their manipulation are explicitly presented. Put in other words, a language or a linguistic framework

is syntactically specified by its formation and transformation rules. So far, this is the common procedure nowadays. Nonetheless, for Carnap, a language can possess formal and empirical components. Thus, both logical and physical rules are needed. In Carnap's terminology, we have the L-rules – the logical rules –, and the P-rules – the physical rules –; the L-consequences, and the P-consequences.

Although there is a precise distinction between L-sentences and P-sentences once a language is formulated, both kinds of sentences are open to revision. Nevertheless, when a P-rule is reformulated, this is done within the same language. Thus, this is only a reformulation of the empirical sentences formulated in the language. Reformulating an L-rule, on the other hand, represents changing the language, since we are changing the behavior of its symbols. Moreover, there is no absolute extra linguistic division between these rules. Such division is only possible after the establishment of a language. Thus it is possible in principle that a rule is logical in a language, and empirical in another one<sup>3</sup>.

The very notion of syntax in this context is different from its present meaning. Carnap "developed the idea of the logical syntax of a language as the pure analytical theory of the structure of its expression." (CARNAP, 1963, p. 53). Some concepts present on his general syntax, such as the concept of analyticity, would be considered as semantic concepts today. In this sense, it is curious to note that Carnap has named this project, on moments prior to its publication, as metalogic and even as semantics.

After presenting such rules,

The investigation will not be limited to the mathematico-logical mart of the language [...] but will be essentially concerned also with synthetic, empirical sentences. The latter, so-called "real" sentences, constitute the core of science; the mathematico-logical sentences are analytic, with no real content, and are merely formal auxiliaries. (CARNAP, 1937, p. xiv).

In this quotation one can see an essential point. The distinction between the empirical and conventional sentences is established by the distinction between analytical and synthetic sentences. Once again, this distinction is no longer absolute. To formalize a language is, at the same time, to distinguish analytical from synthetic sentences. And this distinction is fundamental, for "all of logic including mathematics, considered from the point of view of the total language, is [...] no more than an auxiliary calculus for dealing with synthetic statements". (CARNAP, 1935/1953, p. 127).

<sup>&</sup>lt;sup>3</sup> For an example, cf. FRIEDMAN, 1999, p. 85.

Therefore it is the concept of analyticity that assures that both logic and mathematics do not say anything about the word, although they may be used as auxiliaries to analyze empirical sentences<sup>4</sup>.

As seen, according to the Principle of Tolerance one can develop different kinds of languages. The next question is: How to formulate different linguistic frameworks and why? The answer is found once it is acknowledge the auxiliary role of logic and mathematics inside a linguistic framework. According to Carnap:

if we regard interpreted mathematics as an instrument of deduction within the field of empirical knowledge rather than as a system of information, then many of the controversial problems are recognized as being questions not of truth but of technical expedience. The question is: Which form of the mathematical system is technically more suitable for the purpose mentioned? Which one provides the greatest safety? If we compare, e.g., the systems of classical mathematics and of intuistionistic mathematics, we find that the first is much more simples and technically more efficient, while the second is more safe from surprising occurrences, e.g., contradictions. (CARNAP, 1939, p. 50).

So, this choice is determined by a purely pragmatic question, namely: which is the purpose of this formulation? In this way, questions regarding the validity of a given argument can be understood in two senses. First, we can look at it as an internal question. This means that we choose a logic, formalize the argument, and then we evaluate its validity. Another option is to understand this question as an external one. We may ask: Should this argument be interpreted in Language L or L\*? Note that, in both cases, there's no absolute answer regarding the validity of a given argument.

# Influences on Carnap

Carnap states in his autobiography that the ideas presented in *Logical* Syntax occurred to him during a sleepless night:

the whole theory of language structure and its possible applications in philosophy came to me like a vision during a sleepless night in January 1931, when I was ill. On the following day, still in bed with a fever, I wrote down my ideas on forty-four pages under the title "Attempt at a Metalogic". These shorthand notes were the first version of my book *Logical Syntax* of *Language*. (AWODEY, CARUS, 2007, p. 24).

This does not mean, however, that Carnap developed these ideas in isolation. It is well known that Carnap was influenced not only by the other

<sup>&</sup>lt;sup>4</sup> There is a long dispute regarding whether Carnap's project and specially his concept of analyticity is subject to Gödel's incompleteness theorems. Since this problem is tangential to the subject of this paper, we will not discuss about it. For an inquiry on this debate, (FRIEDMAN, 1999 and RICKETTS, 1994).

members of the Vienna Circle, but also by other authors working on logic and foundations of mathematics. Thus, this section aims to analyze these authors' influence on Carnap's thought.

As said before, Carnap proposes not a synthesis, but an overcoming of the debate between formalism, logicism and intuitionism. Nonetheless, Carnap's proposal is very different from these three doctrines. The introduction of the numbers and the induction principle as primitive symbols and axiom seems to indicate that logicism was rejected. Similarly, the use of non-constructive methods suggests that intuitionism was abandoned. Furthermore, the presence of infinitary reasoning indicates that formalism was also ruled out. This allows us to question which aspects are preserved from these three schools in *Logical Syntax*.

Starting with formalism, it is clear that, despite the divergences noted above, Carnap retain Frege's idea that there is no need for a foundation of logic, for every rational discourse presupposes logic. In addition to that he also incorporates Fregean anti-empirism regarding mathematics. And this leads him to accept the logicist's thesis that mathematics can be reduced to logic. Thus, mathematics is also analytic. The main differences are in the rejection of a universal conception of logic, as well as in the acceptance of the axiomatic method and the distinction between language and metalanguage.

It is precisely this acceptance above that Carnap retains from formalism. Languages I and II formulated on *Logical Syntax* are presented in an axiomatic way. Besides that, Tarski's and Hilbert's investigations on metalanguage allow Carnap to formulate a general syntax, i.e., the possibility of discussing on the metalanguage the general features of any language. Tarski himself, while visiting the Vienna Circle in January of 1930, was responsible to personally stress to Carnap that

concepts used in logical investigations, e.g., the consistency of axioms, the probability of theorems in a deductive system, and the like, are to be expressed not in the language of axioms (later to be called the object-language), but in the meta-mathematical language (later called the metalanguage). (CARNAP, 1963, p. 30).

On the other hand, after Gödel's incompleteness theorems, Carnap abandons any formalist pretention to present a finitary consistent proof for classical mathematics. Thus, he rejects the thesis that consistency proofs of axiomatic theories assure the existence of this theory's objects. Formalism also holds the universality of logic, something which Carnap rejects, as seen. Furthermore, the languages investigated in *Logical Syntax* differ from the formalist project in the sense that they are not limited in formalizing mathematics, but are also used as formalization of science and allow the use of non-decidable concepts.

With respect to intuitionism, Carnap acknowledges that the lecture given by Brower to the Vienna Circle in 1928 had a strong influence on him. Carnap himself recognizes that influenced by Brower he had "a strong inclination toward a constructivist conception." (CARNAP, 1963, p. 49). Thus, language I is formulated based only on primitive recursive arithmetic which in principle only uses constructive reasoning and concepts. Furthermore, Carnap asserts several times that constructive methods are safer than others.

It is curious to note however that it seems that from the three doctrines, intuitionism is the least influent for Logical Syntax's project. Although language I is constructive, the fact that it is formulated axiomatically and that the intuitionist concept of continuum cannot be formalize on it suggests that Brower himself would reject it. The very use of a classical metalanguage to formalize language I would also be rejected by intuitionism. In addition to that, one of the main criticisms from intuitionism towards classical logic – that classical treatment of logical connectives is incoherent – becomes unfounded in Carnap's project.

Other authors were also fundamental to the development of Carnap's thinking. The first version of Logical Syntax - called Attempt at a Metalogic did not contain the Principle of Tolerance. This appears, partly, as an answer to the criticism made by Gödel to the first drafts of this work. In this first version, Carnap tried to define a general concept of analyticity. In personal communication, Gödel showed him that such definition was flawed<sup>5</sup>. Carnap acknowledges, after the works of Tarski and Gödel, that such concept was to be defined in a metalanguage. But at that time, he thought that by using "Godel's method of arithmetizing the metalanguage in the object language, [...] one could now get by with only a single language after all." (AWODEY, CARUS, 2009, p. 93). Gödel was responsible for showing Carnap that this method was subject to his incompleteness results. So it was necessary not only a metalogic with a greater power of expression than the object language, but also a hierarchy of such metalanguages. Thus, given a language L, the concept of analytic-in-L must be formulated in the metalanguage of L. Therefore, analyticity is always defined relatively to a language. From these considerations, Carnap extends this notion of linguistic relativity and starts to claim that many linguistic frameworks are legitimate.

Once the legitimacy of several languages is recognized, the choice among them becomes a merely pragmatic issue. Here we can clearly see the influence of Poincaré: "What, then, are we to think of the question: Is Euclidian geometry true? It has no meaning [...]. One geometry cannot be more true than another; it can only be more convenient." (POINCARÉ, 1905, p. 50). Thus, Carnap extends Poincaré conventionalism to logic itself.

<sup>&</sup>lt;sup>5</sup> For a detailed exposition of this discussion cf. AWODEY, CARUS, 2007 and AWODEY, CARUS, 2009.

Lastly, we have to acknowledge Wittgenstein's importance. *Tractatus* exerted a strong influence on the Vienna Circle. In particular it is fundamental to highlight that Carnap accepts Wittgenstein's position that logical truth are tautologies. Thus, logic does not talk at all about the world. Now, since mathematics can be reduced to logic – something that Carnap incorporated from logicism –, it results that mathematical truths are also tautological. But in order to achieve *Logical Syntax*'s project, it was necessary to depart from some fundamental ideas on *Tractatus*:

the members of the Circle, in contrast with Wittgenstein, came to the conclusion that it is possible to speak about language and, in particular, about the structure of linguistic expressions. On the basis of this conception, I developed the idea of the logical syntax of a language as the purely analytic theory of the structure of its expression. My way of thinking was influenced chiefly by the investigations of Hilbert and Tarski in metamathematics. (CARNAP, 1963, p. 53).

In addition to that, the *Tractatus* also holds, in certain sense, the absolute character of logic. Therefore, not a general theory of language, nor the principle of tolerance could be formulated in the *Tractatus* terms. Here it is important to clarify some points regarding the posterior development of Wittgenstein's thought. A few years after the publication of *Tractatus*, and before the publication of *Logical Syntax*, Wittgenstein wrote about the possibility of formalizing freely different languages. Nonetheless, this change did not exert any influence on the formulation of the Principle of Tolerance. Firstly, Carnap did not read those texts. When Schlick was reading one of the drafts of *Logical Syntax*, he wrote a letter to Carnap warning him that Wittgenstein was also developing something along the same line. Thus, in the first version of *Logical Syntax*'s Foreword, Carnap affirms that:

A propos of the remarks made – especially in §17 and §67 – in opposition to Wittgenstein's former dogmatic standpoint, Professor Schlick now informs me that for some years, in writings as yet unpublished, Wittgenstein has taken the view that the rules of language can be chosen freely. Perhaps his view too is developing in the direction of the Principle of Tolerance. (UEBEL, 2009, p. 59, quoting from unpublished letters from Carnap to Schlick).

After reading this, Schlick wrote another letter to Carnap reinforcing the similarities between Wittgenstein's ideas and the Principle of Tolerance. It is clear from Carnap's response that his ideas are different from Wittgeinstein's:

I myself do not have the impression that Wittgenstein adopts the conception which I designate as the Principle of Tolerance. To be sure, it seems as if he now adopts a more tolerant conception than he (and we all) adopted earlier on. But according to what I have learnt from you (especially from the last paper) and from Waismann, his views do not coincide wholly with

mine on this point. (E.g., he rejects, if I am informed correctly, sentences that cannot be conclusively verified; moreover, you, and so I suspect he as well, allow as analytic sentences (tautologies) only those for which we possess a decision procedure.) We can talk about these questions later on at our leisure. Here what matters is only that I do not believe that we are in agreement. (UEBEL, 2009, p. 60, quoting from unpublished letters from Carnap to Schlick).

Therefore, as important as Wittgenstein has been to the Vienna Circle, this relevance is limited, in general terms, to the aspects from the *Tractatus* that could be incorporated "as far as we could assimilate them to our basic conceptions" (CARNAP, 1963, p. 24–5). Specially, the acceptance of freedom of choice regarding languages did not have any impact on the development of the Principle of Tolerance.

## Logical pluralism

We reach now the final section of this paper. We will investigate whether the Principle of Tolerance implies a logical pluralism and, if so, of which kind.

In general terms, logical pluralism is the claim that there is more than one adequate, coherent, or even, true logic. Note that, from a pure abstract point of view, this thesis may seem trivial today. It's obvious that there are different pure logics. But logical pluralism is not exactly, or not only, about this. It amounts to acknowledge, for instance, that given a certain domain, there are at least two logics that formalize it in a fundamentally different way. And that, nonetheless, both are equally adequate for this task. There are many kinds of logical pluralism (Cf. BEALL, RESTAL, 2006 and SHAPIRO, 2014), but that's not really relevant for the moment. In a schematic way, a pluralist logician claims that exists situations such that

- i)  $\beta$  is a logical consequence-in-L from  $\alpha$  and  $\sim \alpha$ ; and
- ii)  $\beta$  is not a logical consequence-in-L\* from  $\alpha$  and  $\sim \alpha$ .

So, there are at least two distinct logics that evaluate differently the validity of the same argument (BEALL, RESTALL, 2006; RESTALL, 2001). From what was discussed in the previous sections we can affirm that the Principle of Tolerance does not imply necessarily a logical pluralism. For, in the first place, it does not claim that every linguistic framework is legitimate. Analyzing the following passage, it is clear the Principle of Tolerance has its limits:

According to my principle of tolerance, I emphasized that, whereas it is important to make distinctions between constructivist and nonconstructivist definitions and proofs, it seems advisable not to prohibit certain forms of procedure but to investigate all practically useful forms. It is true that

 $<sup>^6</sup>$  Where  $\alpha$  and  $\beta$  are metavariables for formulas,  $\sim$  is the symbol for negation, and L and L\* denotes different logics.

certain procedures, e.g., those admitted by constructivism or intuitionism, are safer than others. Therefore it is advisable to apply these procedures as far as possible. However, there are other forms and methods which, though less safe because we do not have a proof of their consistency, appear to be practically indispensable for physics. In such a case there seems to be no good reason for prohibiting these procedures so long as no contradictions have been found. (CARNAP, 1963, p. 49).

Note that we don't have so much freedom to formulate a language. The Principle of Tolerance does not embrace contradictory languages. And the reason is that, despite his tolerance regarding different languages, Carnap still endorses the now called principle of explosion that states that from contradictories premises it is possible to deduce any conclusion. This is clear from the following quote, on which Carnap presents in an informal way the distinction between analytical and synthetic sentences:

an analytic sentence is absolutely true whatever the empirical facts may be. Hence, it does not state anything about facts. On the other hand, a contradictory sentence states too much to be capable of being true; for from a contradictory sentence each fact as well as its opposite can be deduced. A synthetic sentence is sometimes true – namely, when certain facts exist – and sometimes false; hence it says something as to what facts exists. Synthetic sentences are the genuine statements about reality. (CARNAP, 1937, p. 41).

That is why, even allowing the formulation of different languages, a proof of the consistency of a given language is still used as a security parameter. Besides that, Carnap never claims that two distinct languages are equally legitimate, which is a core statement of logical pluralism.

The Principle of Tolerance limits itself to allow for different languages to be presented, and for its consequences to be evaluated according to purely pragmatic criteria. Yet, such tolerance opens the possibility for a logical pluralism. But what kind of pluralism is that?

To answer this question, it is enlightening to recall a commentary by Quine – one of Carnap's students - which is well known for those who study non-classical logics:

whoever denies the law of excluded middle changes de subject. This is not to say that he is wrong in so doing. In repudiating "p or  $\neg p$ " he is indeed giving up classical negation [...]; and he may have his reasons." (QUINE, 1960, p. 100).

To put in other words, when a classical logician claims that a given proposition is a logical law, and another logician claims the opposite, they are talking past each other, that is, they are talking about different things.

This reasoning is already present in Carnap's thought. By claiming that each one is free to choose a logic, Carnap gives a huge step towards a logical

conventionalism and pluralism. Nonetheless, since accepting a logic implies in accepting a language that formalizes it, we have at the end a pluralism of languages. This means that, in particular, a classical and an intuitionist logician are not discussing the same subject; they are talking in different languages and, therefore, are even talking about different mathematics. In the final analysis, there's no disagreement between a classical and an intuitionist logician, just a difference in their languages. There's no difference in the way they evaluate a single argument, but rather in the very form of the argument.

Hence, if we compare to the original scheme presented earlier, Carnap's logical pluralism asserts that

- i)  $\beta$  is a logical consequence from  $\alpha$  and the L-negation of  $\alpha$ ; and
- ii)  $\beta$  is not a logical consequence from  $\alpha$  and L\*-negation of  $\alpha$ .

But that's not all. From Carnap's point of view, the previous formulation of logical pluralism is just incoherent. Each logic comes with a subjacent language. Therefore, once this language is set forward, there's no internal dispute regarding the validity of a given argument. This means, using today's terminology, that Carnap admits the thesis that changes in the syntactical rules of a logical connective implies changes in its meaning.

Hence, the analytical character of each logic is preserved, as well as their universality. Nonetheless, each logic is universal only in its own domain; there is no possibility of interaction between them.

## Final Remarks

In sum, even though it is necessary to recognize the historical value of the principle of tolerance and the logical pluralism presented by Carnap, today, this form of pluralism is too restrictive for someone trying to defend the idea that there are more than one adequate logic. For, in the final analysis, Carnap proposes that we develop different logics, and decided about its usefulness, in a sense, according to the purpose of this logic. At no time there's an explicit defense of the possibility of two logics being equally adequate. And, even though it's not possible to talk about adequacy outside a logical system, Carnap still believes there are criteria to determine the usefulness of a logic, such as safety, or simplicity. In the same way, Carnap rejects the possibility of a language containing contradictory sentences, which indicates that his principle of tolerance is not that tolerant after all.

Hence, if someone wants to defend a logical pluralism, Carnapian pluralism is not a good choice. Firstly, because it does not explicitly argue for the adequacy of rival logics, but merely states the possibility of formulate them and investigate their consequences. Secondly, it amounts to a form of pluralism with respect to languages, that is, different languages disagree because they are speaking about different things. And thirdly, for a pluralism

within the same language is not only rejected, but prohibited as a starting point: its formulation is nonsense, and this is based on the assumption that the meaning of a logical constant is given by its syntactical rules.

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