Making Up Our Minds:
Imaginative Deconstruction in MathArt, 1920 – Present

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Abstract

The cognitive sciences tell us that the self is a construct. The visual arts illustrate this fact. Mathematics give it full expression, abstracting the self to a Grothendieck site. This self is a haecceity, an ephemeral this-ness and now-ness. We make up our minds and our histories. That our acts are public, that they communicate effectively, becomes a dialetheic paradox, a deep paradox for our times.

Keywords: Condensed object, Creativity, Deleuze, Dialetheia, Erdős, Free Energy principle, Grothendieck, Haecceity, Self, Time

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"I is someone else"
– Rimbaud (1871)

1 Introduction

Mathematics and art have been intertwined throughout their entire history in the West, flourishing together in the Hellenic world before the Roman invasions, in the Arabic world before the Mongol invasions, and in Europe since the Renaissance. Prior to the disciplinization imposed by the modern university, many people – with Leonardo d'Vinci the prime example – engaged openly in both pursuits. Both mathematics and art took dramatic turns toward abstraction in the early 20th century. This history prompts two questions:

Q1: What are we doing when we do mathematics or art – or indeed science or philosophy – that explains their correlation? What are we doing when we do creative activities in general?

Q2: How do we account for the trajectory, through the 20th century and into the present, from some sense of solidity into extreme abstraction, fragmentation, and depersonalization? Why have all creative disciplines followed this trajectory?

We will advance in this Chapter a particular answer to **Q2:**

Claim: What has been deconstructed across all creative disciplines is the self, in particular, the confident Cartesian self as an independent, objective observer and directed, voluntary actor.

We will elucidate and defend this claim by first reviewing, in §2, contemporary answers to **Q1** offered by neuroscience and the physics of information. These disciplines both characterize, in originally different but now rapidly converging languages, all actions as creative, risky, and provocative, and both episodic (i.e. event) memories and the self as imaginative constructs. Indeed, all imaginative activity is both constructive and *active*, in the literal sense of action by an agent on that agent's world. In the context of the variational Free Energy Principle (Friston, 2010, 2013, 2019; Fields et al., 2022), we can consider all creative activity, including all imaginative activity, as problem solving, in particular as the creation of novelty – and hence of uncertainty – in the service of testing and improving a model of the experienced world. Acting creatively is acting on the world with the intention of provoking an informative response from the world. To make art is to perform an experiment. One is reminded of Picabia (1922): "We are not responsible for what we do, for we do not see our deed until it is accomplished." Neither does the scientist: "Unperformed experiments have no results" (Perez, 1978).

An immediate consequence of this is that *all* art is provocative and experimental, and indeed that art is informative – just as science is informative – to the extent that it is provocative and experimental. The idea of art that does not provoke is an oxymoron. From this bio-cybernetic perspective, it is no surprise that Dada (Tzara, 1918), Gödel's incompleteness theorem (Gödel, 1931), and the development of quantum theory (Bohr, 1928) were all nearly contemporaneous. It is no surprise that the computer, which has been a virtual-reality technology from the beginning – indeed virtuality is the core meaning of the Church-Turing thesis (Church, 1936; Turing, 1936) – is the iconic technology of the late 20th century.

It is, however, mathematics that both exemplifies and expresses the dissolution of the self with the greatest clarity and poetry. Grothendiek's notion of a "site" (Grothendieck, 1960-1967) encapsulates this: it removes the solidity and locatedness of the traditional idea of a point, replacing them with a notion that is structureless and ephemeral. Category theory makes this same move, allowing any structure to be transformed, and indeed to be replaced altogether by an operator. The idea of a preferred, foundational object – e.g. the set – thus disappears.

As with the set, so too with the self: all foundational objects stand or fall together. The 20th century deconstruction of the self has been documented, and also furthered, within the philosophical tradition exemplified by Deleuze & Guattari (1987). We introduce in §3 their notion of *haecceity* as an informal analog of Grothendieck's structureless site, and consider in §4 how this notion reconceptualizes the phenomenology of the self. Following Delezue (2002), we focus first on Francis Bacon as artistic provocateur, then turn to Erdős and Grothendieck as mathematical provocateurs, both in their work and in their lives.

We turn in §5 to formalism, constructing the remembered past of a "self" as the action of a functor implementing experienced time on a site representing an interaction defined at a topological boundary. This construction employs sheaf theory and the notion of a "condensed" structure (Clausen and Scholze, 2021), and produces an imagined record of both a "past" experienced world and an agent – a narrative character (Blackmore, 2002; Metzinger, 2004; Graziano & Webb, 2014) – that experiences it. Some initial steps toward this model have been outlined previously (Dobson and Fields, 2021); its full elaboration is left to future work. From this perspective, imagination is inherently mathematical: it is a mathematical act.

We are left in §6 with the question of how we, as artists, as scientists, or as Bayesian uncertainty minimizers, can see each other as fellow agents, communicate, and share ideas. This in turn raises the question: what is language? What does it mean to say that art, math, etc. are languages? What does it mean to be speaker or writer, or to be hearer or reader? The deconstructionist tradition challenges the "objectivity" of any language producer's intended meaning: "a text's unity lies not in its origin but in its destination – we will no longer be fooled by the arrogant antiphrastical recriminations of good society in favour of the very thing it sets aside, ignores, smothers, or destroys" (Barthes, 1977). Indeed we can ask, following Quine (1960), whether a shared semantics can even be assumed.

We conclude that our greatest imaginary construct is our own minds, with their individual and collective histories. To call these constructs "ours," however, is to abuse notation.

2 All Action is Creative

Creativity poses a fundamental paradox to the Western philosophical tradition, which has always valued reflective, deliberate, rules-based thinking – what is now called "type 2" or "process 2" cognition (Evans, 2008; Kahneman, 2011) – over immediate, intuitive, unreflective "type 1" or "process 1" judgement[1]. Creative insight has the phenomenology of type 1 cognition, but a greater value, both aesthetically and socioeconomically, than most type 2 cognition. Hence it is an outlier that demands special explanation. While anecdotal reports throughout history attest to insight's suddenness, sense of certainty, and often ecstatic feeling, integrative theories of insight have lagged far behind; Koestler (1964) being one of the first. This Western discomfort with insight is understandable: one of insight's key attributes is unpredictability. Indeed, the Academy from Plato onward is designed to render the insights of others formulaic and reproducible.

Creativity shares its paradoxical nature with "flow" (Csikzentmihalyi, 2014) states of cognition in which activities are performed with the automaticity that comes with deep experience and expertise (Bargh and Ferguson, 2000). Here again the phenomenology of type 1 cognition – often together with feelings of ease, bliss, or ecstasy – combine with the value of "the best" performances. Hence it is not surprising that flow states have been studied primarily in creatives, including top-level athletes, leaving the commonplace flow-like performance of first-language use and social interaction relatively neglected (but see Bargh et al., 2012, for a counterexample).

With the development of neuroimaging and the emergence of social, cognitive, and affective neuroscience as a subdiscipline in the late 20th century, the mechanisms underlying problem-solving in general, and creative problem-solving in particular, began to be elucidated (Kounios and Beeman, 2015, provides an excellent informal review). The long-standing idea that creativity involves the essentially random – and fully unconscious –

[1]Not being experts in the Eastern tradition, we offer in the below only a few examples, and note here in general that meditative contemplation in search of insight or "apprehension" is much more explicitly valued.

generation of many ideas, following by selection of one or possibly a few alternatives to "present to consciousness" (Campbell, 1960) has been empirically refined to disinhibited idea generation by components of the Default Mode Network (DMN) (Andrews-Hanna et al., 2010) followed by idea selection by the Executive System (ES) (Jung et al., 2013; Beaty, Seli and Schacter, 2018), with the most creative people showing the greatest DMN disinhibition in the generation stage (Chrysikou et al., 2020). The long-hypothesized role of the subcortical reward system – the same system involved in driving addictive behaviors – in producing the feeling of pleasure or even ecstasy associated with "Aha!" breakthroughs (Gopnik, 2000; Fields, 2011) has also been confirmed (Oh et al., 2020).

Such studies have also begun to dispel the paradox posed by creativity, replacing it with another, perhaps deeper paradox. The distinction between unreflective type 1 cognition and deliberate type 2 cognition has never been completely clear even phenomenologically (Melnikoff and Bargh, 2018) and empirical evidence increasingly points toward there being no distinction mechanistically (Chater, 2018; Fields and Glazebrook, 2020). "Deliberate" thought appears to be a sequence of ideas "presented to consciousness" by unconscious processes. Even adding a column of numbers is a sequence of "small" creations that "feel right" and are hence regarded as correct. Checking the sum to make *sure* it is correct is a similar process: the real work is happening below the "threshold of awareness."

What, then, is the threshold of awareness, and why should so much be going below it? A strong hint comes from studies of imagination: the ability to have sensory and emotional experiences without engaging sensory input (Slotnick, Thompson and Kosslyn, 2012; Andrews-Hanna and Grilli, 2021). Imagination is *sensory* because it engages (roughly) the same large-scale functional neuronal networks that process sensory inputs. A consequence of this resource-sharing is that sensation and imagination compete, rendering inner speech difficult in noisy environments and visualization difficult during visually-demanding tasks like driving. Sensory deprivation, on the other hand, enhances imagination; C.F. recalls a professor who wandered the halls with closed eyes, avoiding collisions by gently tracking the wall with one hand. The notorious inattentional blindness of abstract thinkers can similarly be chalked up to competition from imagination. Sensory networks taken over by imagination engage emotion networks in turn, often strongly enough to provoke bodily responses. Such imaginative experiences, including episodic memories (Schacter and Addis, 2007; Simons, Ritchey and Fernyhough, 2022) as well as "thinking," are present to awareness because, being both *sensory* and *emotional*, they are treated by the brain

as important. Indeed, a specialized "reality monitoring" network, involving the most phylogenetically-recent areas of prefrontal cortex, has the task of distinguishing sensory inputs from imaginations, and "labeling" them as such in awareness (Simons, Garrison and Johnson, 2017; Dijkstra, Kok and Fleming, 2022). Deficit functioning of this system manifests as an inability to distinguish sensation from imagination or memory, or an inability to recognize thoughts, memories, or sensations as one's own; "hearing voices" in one's head is a common outcome. Severe chronic cases of deficit reality monitoring can result in dissociative symptoms or in conditions such as schizophrenia. Such "schizotype" symptoms have a nontrivial association, at the population level, with artistic creativity (Nettle, 2001).

Neuroscience, to summarize, presents us with a picture of a brain focused on immediate sensory input, but able to employ "as if" sensory input – including the "as if" auditory input of inner speech (Fields, 2002) – to construct an imagined past, an imagined future, and a counterfactual imagined present. We experience these imaginings as memories, plans, thoughts, pictures, music, etc., complete with their emotional overlays. We use them to understand the world and hence to act appropriately within it.

This picture of a brain as employing all of its available resources to model its world in order to act appropriately within it has been given an elegant and general mathematical formulation (Friston, 2010, 2013, 2019; Fields et al., 2022):

Free Energy Principle (FEP): Any system will behave so as to minimize, to the extent possible, its uncertaintly about its environment's future actions on it.

This uncertainty can be formalized as a variational free energy (VFE); hence the FEP is a *thermodynamic* principle that applies to all classical (Friston, 2019) and quantum (Fields et al., 2022) physical systems, not just to brains. It drives a process of "active inference" that incorporates learning about the world as it presents itself and actively probing the world to provoke informative responses. Within this framework, all living systems – indeed all systems with sufficient information-processing resources – are constantly performing experiments on their worlds.

A "system" in this context comprises a collection of degrees of freedom (or random variables) that, without loss of generality, can be regarded as binary-valued (and hence as classical bits or quantum qubits). *Interesting* systems are ones that have sufficient degrees of

freedom that some remain unexposed to the environment. This implies a "weak" interaction with the environment, and allows the system to have a well-defined internal state that is conditionally independent of the state of its environment. Making the same assumption about the environment allows us to define a Markov blanket (MB) between system and environment as a set of states b such that internal (system) states μ depend on external (environment) states η only as conditioned on the MB states b, and vice-versa. Letting $\pi = (b, \mu)$ be a joint MB plus internal state, we can define a functional (Friston, 2019, Eq. 8.4):

$$F(\pi) \triangleq \underbrace{\mathfrak{I}(\pi)}_{Surprisal} + \underbrace{D_{KL}[Q_\mu(\eta) \parallel P(\eta|b)]}_{Divergence} \geq \mathfrak{I}(\pi). \tag{1}$$

This functional is an upper bound on the Bayesian surprisal $\mathfrak{I}(\pi) = -\log P(\pi)$ because the Kullback-Leibler divergence (D_{KL}) term is always non-negative. It measures the system's uncertainty about its environment's state, as conditioned on the MB. Minimizing this uncertainty is, reading uncertainty as entropy, minimizing VFE; hence it is following a path of least action on a (statistical) manifold representing the "available information landscape" that the system inhabits (Friston, 2019). This framework has been applied extensively in psychology (Adams et al., 2013; Lawson, Rees and Friston, 2014), developmental and evolutionary biology (Campbell, 2016; Fields and Levin, 2020; Kuchling et al., 2020), and even to the Earth's climate system (Rubin et al., 2020).

In the classical FEP, the MB states b encode the system's perceptions of and actions on its environment. The MB is, therefore, the "site" of interaction – hence information exchange – between system and environment in a very natural sense. When the FEP is reformulated in quantum-theoretic language, the MB is abstracted to a holographic screen (Fields et al., 2022); this screen is likewise the site of system–environment information exchange. It is, however, in this latter formulation no longer a *physical* site – a collection of degrees of freedom that take on possible values – but a pure abstraction: a decompositional boundary in the joint system–environment Hilbert space. Hence the FEP in its quantum formulation prefigures the "virtualization" process on which we will focus in §5 below.

The FEP provides a simple explanation for creativity: agents act creatively because creative actions are effective in generating new information. An action is "creative" when it combines existing capabilities in new ways, consistent with the recombinational picture of Campbell (1960) or the role of recombination in "genetic" search algorithms (Holland,

1975). Novel action on the environment can provoke novel responses from the environment, providing new information that can enable the development of new capabilities, as the histories of art, mathematics, and science all demonstrate. The conceptual framework of the FEP also clearly recognizes the risk of creativity: creative actions are valuable precisely because they provoke unpredictable responses, responses that sometimes are distinctly unfavorable to the actor. Minimizing future uncertainty requires maximizing potential information gain, and this inherently involves risk. The political and physical risks of scientific creativity – exemplified by Galileo's persecution and Marie Curies' radiation poisoning, and mythologized by Prometheus – are notorious and lend science a heroic aspect in popular culture; the risks of art are no less significant, as the narrow escapes of some and deaths of many under the Nazis attest. What we wish to explore in what follows is a more subtle existential risk of the creative: a risk of loss of self. We see this risk as, paradoxically, also a goal, one that we will argue art, science, and even mathematics increasingly explicitly pursue.

3 The phenomenology of haecceity

While the phenomenology of scientific creativity has focused on the "Aha!" moment and its sense of sudden mastery, the phenomenology of artistic creativity has given more attention to the sublime, or as Clewis (2021) argues, to the experience of (positive, aesthetic) awe. This introduces a new and much less studied complex of affective responses into the neuroscience of creativity (Allen, 2018; Guan et al., 2018; Takano and Nomura, 2022). Central to this complex is decreased activation of the sense of self.

What is this "sense of self" and how does it relate to the self? Should we say that the self observes the sense of self? Or perhaps that the sense of self observes itself? Such reflexive plays on words are formalized by the idea of an eigenform, a fixed point of a recursive process (Spencer Brown, 1969; von Foerster, 1997; Kauffman, 2017), i.e. an x such that, for some nontrivial operator f:

$$x = f^n(x) \tag{2}$$

for arbitrary n, and hence as $n \to \infty$. This f clearly generalizes the idea of an "Identity" operator. It implements the Eternal Return, mythologized by the sacrificed god and the Ouroboros.

The very nature of f, however, challenges the idea of the "return" as an oversimplification. Delezue (2002) expresses it by:

> It is because nothing is equal, or the same, that 'it' comes back. In other words, the eternal return is predicated only of becoming and the multiple. It is the law of a world without being being, without unity, without identity. Far from presupposing the One or the Same, the eternal return constitutes the only unity of the multiple as such, the only identity of what differs: coming back is the only "being" of becoming...The function of the *eternal return* as Being is never to identify, but to authenticate.

Here Deleuze is fully consonant with von Foerster: *we* authenticate the object by observing it again, and concluding – an inferential *action* and hence a creation – that it has returned.

Following this Deleuzian and yet cybernetic line of thinking, we can conceptualize math and art – and also science, poetry, indeed all creative activity – as ways of distilling experience into meaningful and memorable summaries or abstracts. Hence they are technologies for preserving useful (in a very broad sense) and therefore significant aspects of experience over time. But just as memory is constructive (Schacter and Addis, 2007; Simons, Ritchey and Fernyhough, 2022) as discussed above, "preservation" requires an act of authentication. The Dark Ages lost not so much texts, but the ability to read them.

If authentication is by definition post-hoc, what is being authenticated? Deleuze & Guattari (1987) define *haecceity* as:

> There is a mode of individuation very different from that of a person, subject, thing, or substance. We reserve the name *haecceity* for it. A season, a winter, a summer, an hour, a date have a perfect individuality lacking nothing, even though this individuality is different from that of a thing or a subject....They are haecceities in the sense that they consist entirely of relations of movement and rest between molecules or particles, capacities to affect and be affected.....That awful five in the evening! We say, "What a story!" "What heat!" "What a life!" to designate a very singular individuation....A degree of heat, an intensity of white, are perfect individualities.

The time of the "Aha!" moment does not obey the ordinary time of *doing* and *happening*. If, as we contend, there is no self present during the "Aha!," then who is experiencing

anything?

> Time without present, I without I: this is not anything of which one could say that experience – a form of knowledge – would either reveal or conceal it...Commensurate with it there is no faith (Blanchot, 1995).

We contend that the "Aha!" time of bliss obeys the time of haecceity, which is sharply contrasted with the situational time of doing, since "the individuation of a life is not the same as the individuation of the subject that leads it or serves as its support" (Deleuze & Guattari, 1987).

In particular, haecceity time is not "in the same time, the same temporality." Haecceity time obeys what Deleuze & Guattari (1987) call *Aeon* time.

> *Aeon* is the indefinite time of the event, the floating line that knows only speeds and contiunually divides that which transpires into an already-there that is at the same time not-yet-here, a simultaneous too-late and too-early, a something that is both going to happen and has just happened.

Haecceity time is sharply contrasted with *Chronos* or measurable time, which is:

> the time of measure that situates things and persons, develops a form, and determines a subject...the "pulsed time" of a formal and functional music based on values versus the "nonpulsed time" of a floating music, both floating *and* machinic, which has nothing but speeds or differences in dynamic. IN short, the difference is not at all between the ephemeral and the durable, nor even between the regular and the irregular, but between two modes of individuation, two modes of temporality (Deleuze & Guattari, 1987).

We further invoke Blanchot's concept of *passivity* to categorize as *other* the depersonalized time of bliss as abdication. In this state there is no self to grant the authorization to say anything happens. The statement:

> "I would prefer not to..." is an abstention which has never had to be decided upon, which precedes all decisions and which is not so much a denial as, more than that, an abdication. It belongs to the infinitiveness of patience; no dialectical intervention can take hold of such passivity. We have fallen out of being,

> outside where, immobile, proceeding with a slow and even step, destroyed men come and go (Blanchot, 1995).

This sense of an uncapturable time in which unreportable events occur and possibly intended, but nonetheless uncontrollable actions – Picabia's deeds not seen until they are accomplished – happen is forced onto us also by quantum theory, with its notion of "nonfungible" or unsharable information (Bartlett, Rudolph and Spekkens, 2007). This is precisely the quantum phase information that distinguishes and individuates quantum states. It enforces a radical uniqueness onto quantum systems, including observers, and challenges the very idea of fully-transparent information sharing (Fields and Marcianò, 2019).

The time of haecceity puts the process of reality monitoring in a new light – it is our own reality we are monitoring, including our own ability to remember and authenticate. We are, moreover, only monitoring a simulacrum of the event itself, which is nonfungible and unrecordable. Reality monitoring becomes a rationalizing reconstruction, a form of motivated reasoning (Patterson, Operskalski and Barbey, 2015) that serves the need for a stability that does not actually exist. The continuity of time, and hence of the monitored self, is illusory.

4 The phenomenology of the self

The reportable self, the self that did things in the past and has plans for the future, the self that has beliefs, desires, and other attitudes, the self with a history of relationships, and indeed any history at all, is the constructed self of memory (Seth, 2013; Seth and Tsakiris, 2018). This reportable self is the experienced self, the "I" that speaks and thinks, that directs the body, that is conscious (Blackmore, 2002; Metzinger, 2004; Graziano & Webb, 2014). Sufficiently severe anterograde amnesia can almost surgically remove the experienced self, leaving uses of "I" such as Clive Wearing's (Wilson and Wearing, 1995) that have no enduring 1st person, subjective referent. We will formally distinguish this constructed, Cartesian self from the ungraspable self of haecceity – what Jung called "a virtual point (as it were) between the conscious and the unconscious" (Jung (1935), translated by Sheppard (2004), p. 188) in §5 below. Our interest here is in what this experienced self is like, and in how if differs between individuals.

A striking image of the self is described in Schaller (1991): a deaf-mute from a deaf,

non-signing family who acquired language – and the *concept* of language – only as an adult. Here is a functioning, though languageless and in a deep sense communication-deprived, individual who could, once language was available, reconstruct a past. From what? From memories in modalities other than language, tied together by something other than a descriptive sense of time.

We now know, from experimental techniques that randomly sample ongoing phenomenal experience, that many people live without significant internally-verbalized thoughts (Heavey and Hurlburt, 2008). To writers of philosophy and psychology texts, people for whom verbal introspection is second nature – or maybe even first nature – such a phenomenon is bizarre. To the idea of the WEIRD people (Western, Educated, Industrialized, Rich, Democratic; Henrich, Heine and Norenzayan, 2010) that psychology mainly studies must be added those that regularly experience verbal introspection, and write about it.

Zeman et al. (2020) sharpen this distinction by introducing the phantasia spectrum, with people capable of experiencing rich, multimodal imaginings on one end and people severely deficit in imagination in at least one modality on the other. We ourselves, the present authors, are on opposite ends of this spectrum for the modality of vision. Aphantasia applies to all uses of the imagination, including the reconstruction of episodic memories. Hence aphantasics also experience deficits in episodic memory (Fan et al., 2022), e.g. the purely verbal, textbook-like episodic memories of C.F. Curiously, dream imagery is often preserved in aphantasics (Zeman et al., 2020), suggesting that it is *conscious attention* to imagination that is deficit. This is not surprising: conscious attention is a competitive process (Dehaene and Naccache, 2001; Baars and Franklin, 2003), and who or what wins the competition is not voluntarily chosen.

If the self is a construction, what are its identity conditions? What guarantees, or even suggests, that the self I construct now is the same as the self I constructed as a child, as a student, or even yesterday? In particular, how can we make sense of any proposed continuity of self over time? The standard idea of a "closest causal continuer" (Scholl, 2007) does not straightforwardly apply, as claiming that *I* am still the constructor of my self merely begs the question.

It is, moreover, entirely plausible to see learning – or experience generally – as modifying the self, either gradually or by saltations. Indeed, "Aha!" moments would seem to be precisely such saltations. Meditative, hallucinogenic-induced, and other transformative experiences are regularly reported as involving a "death of the self" (Campbell, 1949;

Masters and Houston, 1966). To learn, and especially to discover or create, is then to "kill the old self" and construct a new one. This new self may have a radically different character. There is, in this case, no "causal continuer" in the usual, smooth sense.

Deleuze & Guattari (1987) go further by introducing their concept of a *body without organs* (Bwo). They claim that a body is "not defined by the form that determines it nor as a determinate substance or subject nor by the organs it posses or the functions it fulfills." They propose instead a *body without organs* as an immanent plane that inhabits its heterogenous populations of frequencies and proceeds via Brownian motion, in sharp protestation of any organized self dominated by subjectivity.

> The Bwo is always swinging between the surfaces that stratify it and the plane that sets it free...For the Bwo is...necessarily a Collectivity (assembling elements, things, plants, animals, tools, people, powers, and fragments of all of these; for it is not "my" body without organs, instead the "me" is on it, or what remains of me, unalterable and changing in form, crossing thresholds) ...

The body without organs proceeds by *contagion* rather than via categorizable identities that "stay stratified — organized, signified, subjected" (Deleuze & Guattari, 1987). After all, "the organism is not at all the body; rather it is a stratum on the body without organs" (Deleuze & Guattari, 1987). In contrast to the pressure of continually having the same self over time, whatever that actually means, Deleuze & Guattari (1987) offer quite the opposite:

> Where psychoanalysis says, "Stop, find your self again," we should say instead, "Let's go further still, we haven't found our Bwo yet, we haven't sufficiently dismantled our self. Substitute forgetting for anamnesis, experimentation for interpretation. Find your Bwo. Find out how to make it...It has nothing to do with fantasy. There is nothing to interpret.

This deconstructed Bwo is reminiscent, from a biological perspective, precisely of the body *with* organs, the living, animal body that is so neglected – or in Freudian circles, so repressed – in Western civilization that many people have great difficulty rediscovering it. It is the body of the protoself (Damasio, 1999), the generally non-conscious homeostatic regulator of heartbeat, blood pressure, immune function, etc., the regulator without which life stops.

Indeed here "there is nothing to interpret" and indeed "this has nothing to do with fantasy" for these functions lie below, and carry on in the absence of, the constructed self.

Delezue (2002) addresses artistic creation in this radical way in his seminal work, *Francis Bacon: the Logic of Sensation*. Here Deleuze views the *becoming* of painting in the form of a *logic of sensation* that functions through hysteria by way of a primordial pre-representative sensation that is foundational to and over any accessory form.

> Painting is hysteria, or converts hysteria, because it makes presence immediately visible. Painting directly attempts to release the presences beneath representation, beyond representation. The color system itself is a system of direct action on the nervous system. This is not a hysteria of the painter, but a hysteria of the painting. With painting, hysteria becomes art. Or rather, with the painter, hysteria becomes painting.

The Body Without Organs is the mediator of painting and hysteria.

> Painting liberates lines and colors from their representative function, but at the same time also liberates the eye from its adherence to the organism. The eye becomes the virtually the polyvalent indeterminate organ that sees the body without organs (the Figure) as a pure presence. Painting gives us eyes all over: in the ear, in the stomach, in the lungs (the painting breathes) ...

In particular, Deleuze posits a symptomologic notion of sensation in the form of replacing subject-object classicism with a color-hysteria taking the form of *autoscopia*:

> Finally, there is a very peculiar feeling that arises from within the body, precisely because the body is felt under the body, the transitory organs are felt under the organization of the fixed organs...This body without organs and these transitory organs are themselves seen, in phenomena known as internal or external autoscopia: it is no longer my head, but I feel myself inside a head. I see and I see myself inside a head. Is there a psychosis in the world that might include this hysterical conditions?

This creative hysteria of Delezue (2002) indeed recalls the "automatic" methods of Dada and Surrealism (Sheppard, 2004). Here, crucially, action and conceptualization are

decoupled. One is reminded of the phenomenon of blindsight, in which objects can be correctly manipulated without been "seen," a consequence of the functional dissociation between dorsal (motion and action processing) and ventral (object identifying) components of the visual system (Goodale, 2014). Indeed most of the world is not seen, heard, or otherwise sensed – at least not consciously – either because it presents itself to the senses only fleetingly (Merikle and Daneman, 2000) or attention is occupied elsewhere (Simons and Rensink, 2005). Hence creative hysteria may involve "seeing" more than can normally be seen, not consciously, but rather *actively*, with the body, not the identifying and categorizing mind. The reduction of art to sense data – seen most clearly perhaps in the work of Mondrian – attempts to capture this world that is seen, but not seen.

Delezue (2002) colors hysteria even further by geometrizing autoscopia as a means for the body to "pass through itself" to escape itself:

> ...by escaping from the organism...It escapes from itself through the open mouth...or the stomach, or through the throat, or through the point of the umbrella. The presence of a body without organs under the organism, the presence of transitory organs under organic representation.

Any presence of autoscopia is an indicator that the Bwo is at play. Further, the hysteria at play in the Bwo is multiple. The eye is polyvalent. The figure is polyvalent. Color is polyvalent, and the very autoscopia itself is polyvalent. We claim that a body passing through itself takes the form of a particular Bwo. We recall that vitrified bodies are not the goal of the Bwo. Care and caution must be exercised to produce such a solarium-spatium of intensities. What is sought is not forlorn bodies that have:

> ... emptied themselves of their organs instead of looking for the point at which they could patiently and momentarily dismantle the organization of the organs we call the organism (Delezue, 2002).

We claim that, in the name of this hysteria symptomology, a body passing through itself is a Bwo that has become a haecceity. For to pass through itself, a body must reach the state where organs are indistinguishable in spatium in *becoming-animal*; these are the very conditions of a haecceity.

> Spatiotemporal relations, determinations, are not predicates of the thing but dimensions of multiplicities....A haecceity has neither a beginning nor end, origin nor destination; it is always in the middle. It is not made of points, only of lines. It is a rhizome (Deleuze & Guattari, 1987).

Doubly so, we can see that the entire process of a body passing through itself is a haecceity, since the body passes through itself in the ways of "spastics and hyperesthetics are indicated by wiped or scrubbed zones, and the anesthetics and paralytics by missing zones." These zones are "rhizomes." When the body is indistinguishable from the paint colors that produced it, the body passes through the paint at the same *time* as it passes through itself. In this radical sense, Bacon's form of painting as hysteresis is a haecceity, recalling that:

> Climate, wind, season, hour are not of another nature than the things, animals, or people that populate them, ..., sleep and awaken with them...This should be read without a pause: the animal-stalks-at-five-o'clock... (Deleuze & Guattari, 1987)

Likewise, this should be read without a pause: *hysteria-painting-body-without-organs-becoming-painting-becoming-animal.*

One is reminded by these "bodies passing through" of Picabia's celebrated Transparencies, in which bodies overlap, merge, passing through each other in a dissolution of individual identities. Picabia's multiple personal re-inventions Camfield (1979) suggest, indeed, the same picture – of an identity that dissolves and is reconstructed so many times that, in the end, it has become *merely* a transparency. As Paul Kantner put it (*Blows Against the Empire*, 1970):

> At first, I was iridescent,
> then, I became transparent,
> finally, I was absent.

This sense of transparency, approaching in the limit complete absence, reveals itself also in the progressive abstractions of 20th century mathematics. Indeed category theory is a tower of abstraction, each layer removing more concrete detail to produce a yet thinner, airier distillation. This process of abstraction appears even in the lives of its practitioners.

If "mathematics is a monster slang" (Deleuze & Guattari, 1987), one must ask: slang for what? Perhaps, we suggest, for an altered state of consciousness in which dissolution is not just an outcome, but a goal.

Consider, as exemplars, Paul Erdős and Alexander Grothendiek. Erdős is arguably most renowned for being a robust problem-solver (Gowers, 2000) and conjecturer, publishing approximately 1500 papers and working with over 500 collaborators as a perpetually peripatetic scholar. Erdős never won the Fields Medal, but he did win the Wolf Prize. On the other hand, Grothendieck is most renowned for his pioneering role in theory-building in the vast areas of algebraic geometry, K-Theory, topoi, schemes, étale cohomology of schemes, crystalline cohomology, motives, the six operations, and derived categories. Grothendieck received the Fields Medal in 1966 and the Crafoord Prize in 1988, but declined the later.

It could be stated that Erdős pursued mathematics as a social activity. But what was the role of his self in his many publications and conjectures? Could we conjecture that he conjectured from and for his self, his reflexive ego, so excited by the prospect of problem-solving that he offered numerous rewards for solving his conjectured problems? Or were his prolific conjectures and proofs products of something more elemental and uncontrollable? Erdős is famous for telling Ron Graham, after winning a bet that he could stop amphetamine use for a month, that:

> You've showed me I'm not an addict. But I didn't get any work done. I'd get up in the morning and stare at a blank piece of paper. I'd have no ideas, just like an ordinary person. You've set mathematics back a month (quoted in Hoffman, 1998).

Just like an ordinary person? From this it appears that Erdős sees the self purely in terms of mathematical value, as a locus from which the activities of publishing and conjecturing could spring. To live out of a suitcase and have no need for possessions, only producing mathematics, and to expect his host to prepare everything for him, including his travel arrangements, is this not a life dedicated to serving mathematics to the exclusion of a personal self? What can we make of this self-as-mathematical-value through his belief in a book of divination containing the most elegant proofs and theorems, which he famously called *The Book*, often stating that "You don't have to believe in God, but you should believe in The Book" (Hoffman, 1998)? In the same passage in which he characterizes the

self as a "virtual point," Jung (1935) quotes St. Paul: "Yet not I live, but Christ liveth in me." Did Erdős not live, but The Book liveth in Erdős?

In stark contrast, it could be stated that Grothendieck's pursuit of mathematics took an activism form and was best accomplished in solitude. His refusal to attend the ICM where he would be presented with the Fields Medal and declining the Crafoord Prize attest to this, as does his statelessless and eventual retreat into the wilderness of Lasserre in the Ariège. What can we make of this literal disappearance, of this not wanting to be found, with he himself his only collaborator, writing his prolific volumes of unpublished work? What can we make of his leaving his mathematical positions and living alone in a mountain cabin on dandelion soup? Is this just for the betterment of prolific writing? Or does disappearance here have a deeper meaning: is Grothendieck exemplifying in his own life the abstraction – the stripping away of extraneous specifics – that so characterizes his work? Can we, in particular, see in Grothendieck's concept of a *site* as an abstraction of a point an urge to grasp the virtuality and dislocation that Jung (1935) ascribes to the self? Grothendieck speaks of "une symbiose profonde entre la main, et l'esprit ou la pensée" (a profound symbiosis between the hand [that writes] and the spirit or thought) and of the creative act as "l'acte archétype de l'esprit humain" (the archetypical act of the human spirit) (Herreman, 2000, translation by CF). We know from these same texts that he was familiar with Jung's thoughts on the matter.

5 The self as a continuous creation

If the self is a momentary construct, an haecceity only experienced as having continuity or identity in the imagination, then "I think therefore I am" gets it radically wrong. There is no thinker, and "I am" is simply false. This comes close, perhaps, to the Buddhist idea of "no self."

How then do we describe this creative process formally? If the "self" is just a site, we must construct its temporal elaboration – the "self" as it appears in memories or plans – by acting on a site. We have outlined such a construction in Dobson and Fields (2021); we summarize it here. We view this construction as implementing, in a rigorous way, the informal ideas of Blackmore (2002); Metzinger (2004); Graziano & Webb (2014) among others. It gives form, we hope, to the "virtual point" of Jung (1935).

We invoke condensed sets in the sense of Clausen and Scholze (2021) to formalize our

notion of *self as a site* from which we create an event-dependent, constructed time of retrospective and prospective memories as an alternative to the objective time of Temporal Logic (Prior, 1957) and its descendants. In essence, our condensed formalism gives representation to how a *self as a site* experiences memory.

A constructive view of memory suggests that what is remembered is not a set of explicitly-represented events, but rather a set of operators with which to construct such events. The previously-firm distinction between "observed" and "constructed" events thus drops away; all (episodic) memory becomes more or less constrained imaginative confabulation. Our condensed formalism re-expresses a sheaf-theoretic "picture" of memory (see Dobson and Fields, 2021, for details) in terms of a "condensed" object located at a single notional "point" that we interpret informally as "the present" without committing ourselves to any particular ontology. This condensed object can, in turn, be considered a representable functor, and so effectively a family of operators invokable at the present. These operators construct an extended, multi-event representation (i.e. a memory) of the (retrospective) past or the (prospective) future.

In particular, we construct an entropic categorization as a sheaf of categorical assignments, which are profinite sets. We then consider the "current" event, $\mathbb{V}c$, located at "the present" regarded as a point. We can, therefore, regard $\mathbb{V}c$ as a pro-étale site on the present. These sites inherit a discrete (indeed Grothendieck) topology of events. We work with the pro-étale topology with its finiteness condition relaxed to a profiniteness condition on the pro-étale morphisms Clausen and Scholze (2021). We take finite jointly surjective families of assignment maps as covers. We recall the following:

Definition 1. *(Clausen and Scholze (2021) Definition 1.2). The pro-étale site $*_{pro\text{-}ét}$ of a point is the category of profinite sets S, with finite jointly surjective families of maps as covers. A condensed set is a sheaf of sets on $*_{pro\text{-}ét}$. Similarly, a condensed ring/group/... is a sheaf of rings/groups/... on $*_{pro\text{-}ét}$.*

With the identification of $\mathbb{V}c$ with $*_{\text{pro-ét}}$ on the present, we state our main theorem:

Theorem 1. *An entropic categorization is a condensed set.*

Proof. With the above identification, the local functor $\mathbf{Fc} : \mathbb{V}c \to \mathcal{C}c$ is a presheaf on $*_{\text{pro-ét}}$, where a categorization $\mathcal{C}c$ of $\mathbb{V}c$ is just an assignment of subgraphs of the category type hierarchy *Type* to the objects in $\mathbb{V}c$. Hence all that is required is to extend $\mathbf{Fc} \to \mathbf{F}$

$\to \tilde{\mathbf{F}}$, and we have a sheaf over $*_{\text{pro-ét}}$ by the Lemma stated below. The required extension is, clearly, the adjoint pair $\mathbf{G}$, $\mathbf{G^{sh}}$ satisfying $\widetilde{\mathbf{F}} \xrightarrow{\mathbf{G}_k} \widetilde{\mathbf{F}_k} \xrightarrow{\mathbf{G}'_k} \mathbf{F}$ and $\widetilde{\mathbf{F}} \xleftarrow{\mathbf{G}^{sh}_k} \widetilde{\mathbf{F}_k} \xleftarrow{\mathbf{G}^{sh\prime}_k} \mathbf{F}$, as explained below. These exist by construction for any entropic categorization. □

To achieve this result, we define an entropic categorization as:

Definition 2. *An* entropic categorization *is a categorization of objects shared by events $\mathbb{V}_i$ and $\mathbb{V}_j$ for which the adjoint time operator $\overleftarrow{T_{ij}}$ is well-defined.*

We note that a categorization may fail to be entropic if it joins a current event involving few relations to a past event involving many relations. We use the language of presheaves to propose that

Lemma 1. *Entropic categorizations are presheaves on events.*

We then consider a set $\mathbb{V} = \{\mathbb{V}_i\}$ of observed events over which some entropic categorization C defines a presheaf $\mathbf{F}$ via Lemma 1 above. We define a *memory* associated with an "observed" entropic categorization $\mathbf{F}$ is a presheaf $\widetilde{\mathbf{F}_k}$ satisfying a particular commutativity constraint. We then consider a particular duality, for each memory k, as the specific left adjoint $\mathbf{G}^{sh}_k$ defined as the sheafification. Whereas $\mathbf{G}_k$ expresses a consistency condition on objects that is imposed by an entropic typing, the adjoint $\mathbf{G}^{sh}_k$ expresses a consistency condition on entropic typings that is imposed by (the assumption of) object identity. The duality becomes

$$\widetilde{\mathbf{F}} \xleftarrow{\mathbf{G}^{sh}_k} \widetilde{\mathbf{F}_k} \xleftarrow{\mathbf{G}^{sh\prime}_k} \mathbf{F} \tag{3}$$

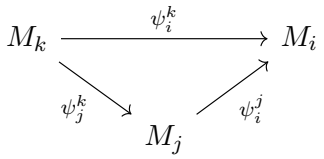
.

The $\mathbf{G}^{sh}_k, \mathbf{G}^{sh\prime}_k$ are effectively embeddings of memories within more-inclusive, but fully consistent, memories involving the same objects and typings, up to the limit specified by $\widetilde{\mathbf{F}}$. We then formalize our claim that consistency of categorization across a collection of memories is effectively a gluing condition in the following theorem.

Theorem 2. *The sheaf over entropic categorizations of objects/events is a projective limit of a sheaf over objects/events. Therefore, the sheaf over categorizations is a pro-object in* **Shv***, the category of sheaves.*

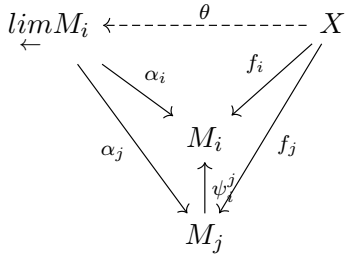
Proof. Let I be a partially ordered set. Recall the following:

Definition 3. Rotman (2000) *Given a partially ordered set I and a category* $\mathbf{C}$*, an inverse system in* $\mathbf{C}$ *is an ordered pair* $((M_i)_{i\in I}, (\psi_i^j)_{j\geq i})$ *abbreviated* $\{M_i, \psi_i^j\}$*, where* $(M_i)_{i\in I}$ *is an indexed family of objects in* $\mathbf{C}$ *and* $(\psi_i^j : M_j \to M_i)_{j\geq i})$ *is an indexed family of morphisms for which* $\psi_i^j = 1_{M_i}$ *for all* i*, and such that the following diagram commutes whenever* $k \geq j \geq i$*.*



Now let M_i be the graded sheaf over i-objects/events, **Shv** the category of sheaves, and $\{M_i, \psi_i^j\}$ an inverse system in **Shv** over I. Take the sheaf over typings of objects/events as an object $\underset{\leftarrow}{lim} M_i$. By definition of entropic typing we have a family of projections $(\alpha_i : \underset{\leftarrow}{lim} M_i \to M_i)_{i\in I}$. For our inverse system to be a projective limit we need:

- i) $\psi_i^j \alpha_j = \alpha_i$ for $i \leq j$,
- ii) for every $X \in obj(\mathbf{Shv})$ and all morphisms $f_i : X \to M_i$ satisfying $\psi_i^j f_j = f_i$ for all $i \leq j$, there exists a unique morphism $\theta : X \to \underset{\leftarrow}{lim} M_i$ making the diagram commute.



Conditions i) and ii) are just the consistency conditions for sequentially embedding the entropic typings $\widetilde{C_k}$; they are met whenever (3) is satisfied as discussed above. □

6 Shareable Information

We are left by the above not with an entity wrapped in a Markov blanket (Friston, 2019), but with an imagined entity wrapped in an imagined blanket. How can such merely

imagined entities communicate? How can they produce works? What is a work of art, or of mathematics, and how is it contemplated by others? What is this text?

Here quantum theory intrudes with a theorem: in the limit in which two agents fully share quantum reference frames, they are entangled (Fields et al., 2022). A quantum reference frame specifies a semantics. Hence in the limit in which two agents fully share a common semantics, and hence a common language, they no longer have separate identities. Learning a common language, a language shared with others, is quite literally dissolving the self. Grinbaum (2017) claims that "physics is about language"; here we see that language is about physics, or rather, that language *is* physics. It is a mechanism of entanglement. Language itself becomes paradoxical. Again Barthes (1977):

> Writing ceaselessly posits meaning ceaselessly to evaporate it, carrying out a systematic exemption of meaning.

Indeed Barthes (1977) gets new teeth: the reader must be allowed her own meanings, not dictated by the author, on pain of both losing their identities.

Information is, nonetheless, sharable. Communications exist. This text exists. You, reading it, exist. The meaning you ascribe to it exists.

Contradiction. Indeed, a true contradiction, a dialetheia, a limit paradox (Priest, 2002, 2006). Such paradoxes arise wherever there is self-reference. Dietrich and Fields (2015) shows how science, applied to itself, generates limit paradoxes. Here we see that creative acts generate limit paradoxes. Indeed creative acts *are* limit paradoxes.

7 Conclusion

Life = art. Not the finished work, but the act. The act, the life as lived now, in the present, is a haecceity. Its history is an imagination.

Our greatest imaginary construct is our own minds. We construct our minds to communicate. Like all of our communicative constructs, they are dialetheic and therefore paradoxical.

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