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Knowledge of Validity

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Abstract

What accounts for how we know that certain rules of reasoning, such as reasoning by Modus Ponens, are valid? If our knowledge of validity must be based on some reasoning, then we seem to be committed to the legitimacy of rule-circular arguments for validity. This paper raises a new difficulty for the rule-circular account of our knowledge of validity. The source of the problem is that, contrary to traditional wisdom, a universal generalization cannot be inferred just on the basis of reasoning about an arbitrary object. I argue in favor of a more sophisticated constraint on reasoning by universal generalization, one which undermines a rule-circular account of our knowledge of validity.

1 Introduction

1.1 The Issue

We have knowledge of the validity of certain belief-forming patterns of ours. We know, for example, that reasoning by Modus Ponens is valid: anytime that we draw a conclusion of the form **Q** on the basis of beliefs of the forms **P** and **if P then Q**, there is no possibility that makes both the premises true and the conclusion false. What accounts for how we are able to know this? What is the source of our justification here?

If we are required to *use* valid reasoning in order to acquire knowledge *about* validity, then inevitably our reasoning will follow the very same patterns or *rules*[1] whose validity we purport to know about. Such reasoning would be circular in some way. It need not be *premise-circular*: the rules whose validity we're trying to defend need not appear as premises in our argument. But it will at least be what's commonly called *rule-circular*.

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It's widely accepted that premise-circular arguments are epistemically ineffective.[2] You can't *acquire* justification to believe that reasoning by Modus Ponens is valid by reasoning:

Premise: Reasoning by Modus Ponens is valid.
∴ Conclusion: Reasoning by Modus Ponens is valid.

With rule-circular arguments, on the other hand, things are less clear.

A rule-circular argument concludes that a particular rule of reasoning has some positive epistemic quality, where that very same rule of reasoning is applied in the course of the argument. For example,

Premise: Mom says that reasoning by Modus Ponens is valid.
Premise: If Mom says that reasoning by Modus Ponens is valid, then reasoning by Modus Ponens is valid.
∴ Conclusion: Reasoning by Modus Ponens is valid.

This argument doesn't have very plausible premises, but ignoring that, is it vicious? If some reasoner were justified in believing its premises, would its rule-circularity prevent her from acquiring justification for its conclusion? Many philosophers have argued that rule-circularity need not be vicious. They suggest that by going through certain rule-circular reasoning, you can acquire justification to believe that reasoning by Modus Ponens is valid.

This assumes that you can rationally *reason by Modus Ponens* before rationally *believing that such reasoning is valid*.[3] But it seems fair to grant that assumption: in order to rationally infer a conclusion of the form **Q** on the basis of beliefs of the forms **P** and **if P then Q**, you don't need the further explicit premise that the latter beliefs validly entail the former.[4] Indeed, you don't need any premise about the quality of reasoning by Modus Ponens, not even the premise that if the beliefs of the forms **P** and **if P then Q** are true, then the conclusion of the form **Q** is true. Requiring a further premise about the quality of the reasoning would trigger the regress that the Tortoise sets Achilles on in Carroll (1895).

A rule-circular justification is, of course, unavailable to a skeptic who refuses or is unable to rationally reason by Modus Ponens. But the advocates of rule-circular reasoning do not recommend using it against such a skeptic. They claim that rule-circular arguments are important only because by using them we can provide *ourselves* with the justification to believe that our own rules are valid.

The list of contemporary philosophers who have endorsed a rule-circular justification for our knowledge of validity is impressive. The classic defense is Michael Dummett's; see Dummett (1973) and Dummett (1991). Michael Friendman echoed Dummett's endorsement in Friedman (1979).

More recently, Paul Boghossian has defended rule-circularity in a series of articles consisting of Boghossian (2000), Boghossian (2001), and Boghossian (2003). Crispin Wright has also recently endorsed the rule-circular argument in Wright (2004), though in a slightly reserved tone.[5] And Alvin Goldman has said that he at least sees nothing wrong with a rule-circular justification of our knowledge of validity; see Goldman (1986).

In this paper, I will argue that rule-circular reasoning cannot account for our knowledge of validity. I will make my argument while granting to the defenders of rule-circularity that it may generally be rationally permissible to reason according to rules before rationally believing that those rules are valid. (Throughout the paper, I focus on reasoning by Modus Ponens for clarity, but what I will argue applies equally well to all of our canonical deductive rules of inference).

The problem with the rule-circular route to our knowledge of validity is not the use of the rule itself. The problem, rather, concerns the argument's introduction of a universal generalization. We're traditionally taught in logic and math classes that if we have shown some result for an arbitrary object, that suffices to allow us to draw a general conclusion about all objects. As I'll show, the rational rules for introducing universal generalizations cannot be that simple. The true restrictions on introducing a universal generalization turn out to prevent rule-circular arguments from accounting for our knowledge of the validity.

After I establish this conclusion, I will argue that it has a significant upshot. The conclusion's upshot is that it seals off the only plausible route on which all our knowledge of validity is *inferential*. This provides new and powerful motivation for views according to which our knowledge of validity is, at its foundation, *non-inferential*. In particular, this may serve as further fuel for an age-old view that has gained some new popularity in contemporary epistemology, namely the view that *intuitions* play a fundamental justificatory role. However, in this paper, my goal is only to motivate the exploration of an alternative view by closing the door on the inferential route.

1.2 Some Quick Preliminaries

1.2.1 The Rule of Reasoning by Modus Ponens

My use of 'rules of reasoning' and 'reasoning by Modus Ponens' is intended to avoid controversial commitments. I take no stand as to whether, when we reason, we *follow* or are *guided by* rules in any substantive sense. I do not assume, for example, that reasoners must have any representations of any rules of any kind. Indeed, it would be in many ways preferable to use the less loaded description 'pattern of belief revision' in place of 'rule of reasoning', but I have used 'rule' to emphasize connections with other literature on so-called rule-circularity. I use 'reasoning by Modus Ponens' only to describe a rule (pattern) of rational change in beliefs. I call it Modus Ponens because this rule concerns the forms **P**, **if P then Q** and **Q**.

Reasoning by Modus Ponens. If you believe propositions of the forms **P** and **if P then Q**, and if you have no reasons to disbelieve the proposition of the form **Q**, then you may, on the basis of those two beliefs, infer the proposition of the form **Q**.

This rule makes no mention of validity as such. I do not assume that the validity of this rule,[6] or our knowledge thereof, plays an important role in explaining how we can rationally reason by Modus Ponens. The datum that this paper rests on is just that we *do* have knowledge of the validity of this rule as well as of others. I intend to show something about rational reasoning by exploring what accounts for how we arrive at that knowledge.

1.3 Validity

What do we mean when we say someone's reasoning is valid? We mean that necessarily, if all her premise beliefs are true, then her conclusion is true. But this will mean different things depending on what sort of modality is expressed here by 'necessarily'. There are two philosophically interesting notions of validity.

Let's say that, if the conclusion of some reasoning is true in every possible world in which the premises are true, then the reasoning is 'metaphysically valid'. All reasoning by Modus Ponens is metaphysically valid because there is no possible world in which the propositions of the forms **P** and **if P then Q** are true but the proposition of the form **Q** is false.

Textbooks in logic are usually concerned with a different notion, a semantic notion, which I'll call 'logical validity'. According to this, reasoning is valid if and only if no interpretation assigns semantic values to the non-logical terms occurring in the reasoning so as to make the premises true and the conclusion false. The canonical demarcation of the logical terms includes 'if' and 'then'.[7] Given that, all reasoning by Modus Ponens comes out logically valid because no interpretation can make its premises true and its conclusion false.

It's debatable to what extent this semantic notion of logical validity is present in the mind of the ordinary person on the street. So, with respect to just logical validity, it might be safest to restrict our question: how do logic students come to know that certain patterns of reasoning are logically valid? Metaphysical validity is likelier to be a more widely shared concept, though still perhaps not well articulated by the person on the street.

So, we have at least two interesting notions of validity: preservation of truth in all possible worlds, and preservation of truth in all interpretations. Any piece of reasoning by Modus Ponens is both metaphysically valid and (given the usual choice of logical terms) logically valid. Just for convenience, I will only present my main argument in the terms of logical validity, but my argument can easily be extended to the case of metaphysical validity simply by replacing 'interpretation' with 'possible world' throughout.

1.3.1 Enumerative Induction

Many philosophers have endorsed a rule-circular justification for the belief that our inductive methods are reliable.[8] I am not going to criticize these inductive arguments for the reliability of induction. For all I will say in this paper, such a rule-circular justification of induction is perfectly good.

We have inductive evidence, i.e. evidence that allows us to reason by enumerative induction, that reasoning by Modus Ponens is reliable. Each of the many times we have observed reasoning by Modus Ponens, the reasoning preserved truth, so this is good inductive evidence for concluding, or at least raising our confidence, that the *next* observed instance of reasoning by Modus Ponens will also preserve truth. It's even a small amount of inductive evidence for raising our confidence that reasoning by Modus Ponens *always* preserves truth, i.e. that Modus Ponens is valid.

But our inductive evidence for the validity claim hardly accounts for the extremely high level of confidence that we take ourselves to have justification for. For a long time, we had only inductive evidence that Fermat's Last Theorem is true, but it was not until Wiles proved the theorem that we were justified in taking our current high level of confidence, the sort of level of confidence we think is justifiable for the claim that Modus Ponens is valid. As long as a conclusion is known to have only inductive support, there will be doubt about the conclusion. We doubt it because we know that there is some positive chance that the conclusion is false, even if we relied on no false premises in arriving at it. The appropriate level of doubt may even be very substantial, in light of the fact that some mathematical conjectures had tremendous inductive support of this sort before eventually being disproved.[9] We seem to have confidence in the validity of certain rules of reasoning without any doubt. (If you don't feel absolutely certain that reasoning by Modus Ponens is valid, consider the law of non-contradiction.) Moreover, our high confidence is also extremely *resilient*. It is resilient in the sense that we view any purported counterexample to validity with extreme suspicion, even before we can articulate any reason for why the purported counterexample fails. In light of how high and resilient it is, then, it's implausible that our confidence in the validity of certain rules could have been arrived at inductively.

This only shows that our knowledge of validity must be arrived at non-inductively, that is, using only valid rules.[10] It does not follow that the only remaining route to the conclusion that some rule is valid is to reason through an argument using that very same (valid) rule. There are alternative valid routes, which I shall discuss in my conclusion, once I have motivated them by closing off the rule-circular route. Let's now turn to my main argument, which will show that rule-circular reasoning about validity presupposes antecedent knowledge of validity.

2 The Problem with the Standard Rule-Circular Argument for Validity

Let's take a close look at how reasoning through the standard rule-circular argument is supposed to lead someone to rationally believe that reasoning by Modus Ponens is valid. In the next section, I will consider other ways of constructing rule-circular arguments, arguing that they do not avoid the problem I will raise for the standard argument.

Suppose that Sasha is able to rationally reason by Modus Ponens: when she has high confidence in both of two propositions of the logical forms **P** and **if P then Q**, and she has no reason to doubt the corresponding proposition of the form **Q**, she is disposed to transmit her high confidence to that proposition of the form **Q**. Let's even be generous and suppose that Sasha knows the meanings of logical terms like 'valid' and 'interpretation', and she knows the truth-condition for a conditional: if its antecedent is true, then its consequent is true. But Sasha is wondering whether reasoning by Modus Ponens is valid. In an attempt to settle her question, she reasons as follows:

Argument MP

(Step 1) She starts by introducing an assumption. She assumes that *a*, *b* and *c* are three arbitrary propositions with the logical forms **P**, **if P then Q**, and **Q** respectively, and *I* is an arbitrary interpretation that makes *a* and *b* true.

(Step 2) Next, Sasha appeals to her general knowledge of the truth-conditions of conditionals. She infers that if *I* makes *b*'s antecedent, *a*, true, then *I* makes *b*'s consequent, *c*, true.

(Step 3) Observing that *I* indeed makes *b*'s antecedent, *a*, true, Sasha reasons by Modus Ponens to infer that *I* makes *b*'s consequent, *c*, true.

(Step 4) Now Sasha reasons by Conditional Proof to discharge the assumption she started with. She infers that IF: *a*, *b* and *c* are three arbitrary propositions with the logical forms **P**, **if P then Q**, and **Q** respectively, and *I* is an arbitrary interpretation that makes *a* and *b* true, THEN: *I* makes *c* true.

(Step 5) Finally, recalling that her assumption was arbitrary, Sasha generalizes the conclusion of step 4. She infers that for any two propositions with the logical forms **P** and **if P then Q**, and for any interpretation that makes those two propositions true, that interpretation also makes true the corresponding proposition of the form **Q**. She concludes that reasoning by Modus Ponens is valid.

I am going to argue that the above reasoning does not allow Sasha to rationally conclude that Modus Ponens is valid, at least not unless she has an antecedent justification to believe that Modus Ponens is valid. The above reasoning is, at best, redundant.

To show this, let me introduce another argument that is structurally similar in its reasoning to Argument MP, but obviously fails to justify its conclusion. We will shortly compare the two arguments.

In Argument MP, the interpretations Sasha is considering leave the semantic value of the logical terms 'if' and 'then' fixed. Now, there may be principled reasons for privileging the canonical demarcation of the logical terms, or it may just be a purely conventional demarcation. But whatever the outcome of that debate may be, nothing prevents us from investigating whether truth is preserved in all the interpretations that fix the semantic values of certain canonically *non*-logical terms. We can perfectly well be curious about the properties of all the interpretations that leave the expression '99.999% of . . . s are . . . s' fixed with its ordinary meaning.

Let's allow Sasha to indulge in this. Sasha treats '99.999% of . . . s are . . . s' *as if* it were logical vocabulary, and now she wants to investigate the associated interpretations. So, for example, in the different interpretations Sasha is now considering, the expression '99.999% of animals are insects' only has different extensions[11] assigned to the predicates 'animal' and 'insect', leaving the ordinary meaning of the rest fixed. An interpretation makes the resulting sentence true just in case 99.999% of the objects in the extension assigned to the predicate 'animal' are also assigned to the predicate 'insect', and suppose Sasha knows this as well.

Suppose that, like any normal person, Sasha rationally reasons by a rule called statistical inference: *if you believe propositions of the forms* ***Fm*** *and* ***99.999% of F's are G's****, then you may believe the corresponding proposition of the form* ***Gm****, as long as you have no independent reason to disbelieve it.* Suppose Sasha now wonders whether, given her choice of what to treat as logical vocabulary, this rule of reasoning that she rationally reasons with will also turn out to be valid. Of course, we know better: no matter what we treat as logical vocabulary, statistical inference is, though rational to reason by, invalid. We don't even need to consider interpretations that alter the meanings of our ordinary terms to come up with a counterexample; e.g. '99.999% of people are not Ringo Starr' and 'Ringo Starr is a person' are both true, but 'Ringo Starr is not Ringo Starr' is false even on the ordinary meanings of these sentences. But suppose that Sasha is considering the question of whether statistical inference is valid for the first time, and she has no outside considerations bearing on the matter. She has not thought of or even searched for any counterexamples to statistical inference. Would it now do her any good to reason as follows?

Argument SI

(Step 1*) She starts by introducing an assumption. She assumes that *a*, *b* and *c* are three arbitrary propositions with the logical forms **Fm**, **99.999% of F's are G's**, and **Gm** respectively, and *I* is an arbitrary interpretation that makes *a* and *b* true.

(Step 2*) Next, Sasha appeals to her general knowledge of the truth-conditions of these claims. She infers that *I* includes the referent of **m** in the extension it assigns to **F**, and that the extension *I* assigns to **G** contains 99.999% of the objects in the extension it assigns to **F**.[12]

(Step 3*) By statistical inference, Sasha infers that *I* includes the referent of **m** in the extension it assigns to **G**, i.e. *I* makes *c* true.

(Step 4*) Now Sasha reasons by Conditional Proof to discharge the assumption she started with. She infers that IF: *a*, *b* and *c* are three arbitrary propositions with the logical forms **Fm**, **99.999% of F's are G's**, and **Gm** respectively, and *I* is an arbitrary interpretation that makes *a* and *b* true, THEN: *I* includes the referent of **m** in the extension it assigns to **G**, i.e. *I* makes *c* true.

(Step 5*) Finally, recalling that her assumption was arbitrary, Sasha generalizes the conclusion of step 4. She infers that for any two propositions with the logical forms **Fm** and **99.999% of F's are G's**, and for any interpretation that makes those two propositions true, that interpretation also makes true the corresponding proposition of the form **Gm**. She concludes that reasoning by statistical inference is valid.

Clearly, Sasha cannot reason as in Argument SI to justify much confidence in the validity of statistical inference, certainly not the extremely high and extremely resilient level of confidence we have in the validity of reasoning by Modus Ponens. Argument SI shows that being able to rationally reason by some rule doesn't by itself guarantee that you can successfully make a rule-circular argument for its validity. It also raises the question of whether and how Argument MP could succeed even though Argument SI fails. To answer this question, we need to understand why Argument SI fails.

2.1 Where Does Argument SI Fail?

Which step, or steps, of Sasha's reasoning in Argument SI are irrational?

Step 1* is just an assumption, so there's nothing to criticize there. Step 2* invokes some of Sasha's knowledge of semantics. We stipulated she had this knowledge, and it doesn't seem like an incoherent stipulation, so step 2* looks fine as well. Step 3* is the rule-circular step, an application of statistical inference. Now, an assumption throughout this paper is that there's no general obstacle to rationally reasoning by a rule before you have a rational belief that the rule is valid. Without that assumption, rule-circular arguments are non-starters, so let's continue to allow it.

Someone might raise the separate complaint about step 3* that is wrongly treats an arbitrary case as though it were a random case. This complaint derives from the idea that an arbitrary case is not really a case; the idea is that 'arbitrary case' is actually a code word to indicate that the relevant steps of the true proof take place within the scope of universal generalizations. According to this complaint, Argument MP does not represent how we rationally come to know that reasoning by Modus Ponens is valid. Instead, in a more perspicuous presentation, reasoning with arbitrary cases would be omitted in favor of reasoning that uses universal quantifiers. This complaint is mistaken: 'arbitrary case' is not a code word for 'any case'. 'Arbitrary case' refers to an arbitrary case, and there is no general problem with reasoning using arbitrary cases. But this is not the place to defend views about the meaning of 'arbitrary case' and the legitimacy of reasoning with arbitrary cases.[13] Instead, my tactic will be to argue that even if the complaint were not mistaken, it would not help. I will do this later by arguing in sections 3.2 and 3.3 that Argument MP cannot be rewritten in a way that avoids generalizing on an arbitrary case. But for now, as long as we understand step 3* of Argument SI with its face-value meaning, I see no reason to criticize it.

Next, if the reasoning from steps 1* though 3* is acceptable, then it is hard to doubt that step 4* must also be acceptable. After all, if the conditional in step 4* is false, then its consequent, which is the conclusion drawn in step 3*, is false. So if step 3* is rational then step 4* should be rational.[14]

So, Sasha's reasoning and her conclusions in all of steps 1* through 4* of Argument SI seem unproblematic. The step that lands her in trouble, and I think this was fairly evident from the start, is when she proceeds to draw the very *un*intuitive conclusion stated in step 5*. Let's call the rule of reasoning (mis)applied in step 5* 'reasoning by universal generalization', or more briefly 'reasoning by UG', (not to be confused with the standard rule of derivation, which I'll call 'derivation by UG').

You can't rationally reason by UG to draw a generalization just on the basis of an instance of that generalization, of course. Rational reasoning by UG requires further constraints to be in place. Standards textbooks offer a constraint on the rule of derivation: the universalized term must be arbitrary. This means that the term in the instance which becomes a universally bound variable must not appear either in the target generalization or in any undischarged assumptions of the reasoning that led to the instance.[15] But Argument SI shows that this constraint doesn't suffice for rational reasoning by UG, since this constraint is actually met in step 5*. If it is rational to reason by UG, the proper formulation of the rule must be more complicated.

2.2 Constraints on Rational Reasoning by UG

Our concern now is why Sasha's reasoning by UG in step 5* is irrational. What constraint on rational reasoning by UG does her reasoning here fail to meet? We want to know this because we want to know if step 5 of Argument MP does any better than step 5*.

2.2.1 UG Involves Recognizing the Quality of the Earlier Reasoning

Let's call the sequence of steps leading up to an application of UG in a given context the 'earlier reasoning'. In Argument MP, the earlier reasoning is steps 1 - 4, and in Argument SI it's steps 1* - 4*. Clearly the earlier reasoning has an important effect on—likely determines—whether the context allows for rational reasoning by UG. Something about the earlier reasoning in Argument SI (something missing from it, perhaps) explains why step 5* is irrational. What sort of something should we be looking for then?

I claim that rational reasoning by UG requires Sasha to *base* her inference on a rational attitude that her earlier reasoning has certain features. To justify this claim, let me say why we should reject the *prima facie* tempting alternative view.

The alternative view is that it is enough that Sasha's earlier reasoning *have* certain features, whether or not Sasha herself has access to them or has any attitudes about them. This alternative view may initially tempt those with externalist sympathies, for example reliabilists. But in fact, even Goldman, the arch externalist, would have to reject this view, for the following reason. When you reason by UG, you infer a generalization on the basis (in part) of an instance. This type of inference will be reliable only under certain conditions. In order for your inferences to be reliable when you reason by UG, you must perform those inferences only in the context of certain (as yet unspecified) kinds of earlier reasoning. But now, unless your cognitive processes are somehow sensitive to the features of the earlier reasoning, you will not be able to consistently perform only the reliable type of inference. In order to avoid systematically making unreliable inferences like the one in Argument SI, Sasha must only reason by UG when she, somehow or other, *tracks* the appropriate conditions in her earlier reasoning. Since Goldman requires rational inferences to issue from a reliable *process*[16], he must require the process that implements rational reasoning by UG to track the features of the earlier reasoning.

Barring implausibly extreme positions that carry externalism even farther than Goldman does, we can conclude that in order for Sasha's reasoning by UG to be rational she must exhibit sensitivity to certain (as yet unspecified) features of her earlier reasoning. We have yet to specify exactly *how* she must manifest this sensitivity. In order for reasoning by UG to license a rational conclusion, does Sasha have to *know* that her earlier reasoning possesses the relevant features? Is it enough if she has a *rational belief* about her earlier reasoning? Perhaps she doesn't even need to use a propositional attitude to manifest the sensitivity: perhaps it's enough if some non-intentional psychological mechanism tracks the relevant feature. I am going to defer settling how she must manifest her sensitivity to her earlier reasoning; we'll be in a better position to answer this after we've agreed on exactly what features of her earlier reasoning Sasha must be sensitive to. In the meanwhile, I will say that Sasha must *recognize* the relevant features of her earlier reasoning, stipulating that 'recognize' expresses however it may be that Sasha must be sensitive to her earlier reasoning.

The question now is: *what* features of Sasha's earlier reasoning must she recognize in order to rationally reason by UG? Let's consider the possibilities. We'll see that there is no proposal that is both plausible and able to vindicate Argument MP as a rule-circular route to our knowledge of validity.

2.2.2 Recognize that Earlier Reasoning Involves No Loss of Confidence?

A first proposal is that for Sasha to rationally reason by UG, she must recognize that she arrived at the instance she plans to generalize only using rules of reasoning that transmit 100% of her confidence in (the conjunction of) the premises to the conclusion. On this account, step 5* would be irrational because Sasha recognizes that the instance in step 4* was inferred in part using a rule, statistical inference, which transmitted less than 100% of the confidence the agent had at step 2* to the conclusion drawn at step 3*. Unfortunately, this proposal is unlikely to save Argument MP. It is unrealistic to suppose that anyone like Sasha, anyone who is *wondering* whether Modus Ponens is valid, would transmit exactly 100% of their confidence from premises to conclusion when reasoning by Modus Ponens. (It's doubtful that even we who know that Modus Ponens is valid transmit 100% of our confidence.) And it's even less realistic to expect the agent to also *recognize* this 0.001% difference in how much more confidence she transmits using Modus Ponens instead of statistical inference.

A revision of the above proposal may be offered: perhaps for Sasha to rationally reason by UG, she must recognize that, in her earlier reasoning, she only used rules of reasoning by which an *ideally rational reasoner* would transmit 100% of her confidence. Unfortunately, this is still unlikely to save Argument MP. In the first place, it is far from clear that an ideally rational reasoner would indeed transmit 100% of her confidence in reasoning by Modus Ponens. Even in our most idealized theories of rational reasoning, for example Bayesianism or AGM-style dynamic reasoning theories[17], it is not asserted that an ideally rational reasoner would transmit 100% of her confidence in reasoning by Modus Ponens. In these theories, it is simply assumed that at any time, the agent's confidence in the conclusion is at least 100% of her confidence in the premises. These theories simply make no comment on the *dynamics* of reasoning by Modus Ponens. And in the second place, it is mysterious how Sasha, who is not yet aware that reasoning by Modus Ponens is valid, could be in a position to recognize how much confidence an ideally rational reasoner would transmit when reasoning by Modus Ponens.

2.2.3 Recognize that Earlier Reasoning Uses Only Indefeasible Rules?

Let's turn to a new proposal for what feature of her earlier reasoning Sasha must recognize. Perhaps for Sasha to rationally reason by UG, she must recognize that her earlier reasoning used only indefeasible rules. This is really two different proposals, depending on whether we give a strong or a weak interpretation of 'indefeasible'. On the strong interpretation, when an indefeasible rule allows you to infer some conclusion from certain premises, then you can still rationally infer that same conclusion from any superset of those premises. But this proposal won't rescue Argument MP. On this interpretation, all reasoning, including reasoning by Modus Ponens, is defeasible.[18] For example, suppose that initially, I rationally believe that George is crying and that if George is crying then he is unhappy. It may be rational, at that point, for me to infer that George is unhappy. But if I later acquire the belief that George is happily practicing his audition for a play, then it is not necessarily still rational for me to infer that George is unhappy. Instead, it might be better for me to rather infer that it is *not* the case both that George is crying and that if George is crying then he is unhappy. And, to be consistent, I'd give up at least one of my initial beliefs.

To distinguish reasoning by Modus Ponens from statistical reasoning, we need a weaker interpretation of an indefeasible rule. A natural proposal is that a weakly indefeasible rule allows the original conclusion to be drawn from any logically consistent superset of the original premises. But even this is unlikely to help. It's not clear that the class of weakly indefeasible rules we reason by is any bigger than the (empty) class of strongly indefeasible rules we reason by. Suppose I initially believe that George is crying and that if George is crying then he is unhappy. Then, I'm completely taken in by some skeptical thesis about whether these premises give any epistemic support to the conclusion that George is unhappy. Or, another example, suppose I'm taken in by the skeptical thesis that the form of inference in question is not at all reliable. Yet another example, suppose I'm fooled into believing that I've been given a drug that makes unreliable forms of inference appear to be valid. Such skeptical theses are logically consistent with my initial beliefs, but any of them would appear to still interfere with my ability to rationally infer that George is unhappy. So, it's doubtful that constraints concerning indefeasibility vindicate Argument MP as the fundamental source of our knowledge of validity.[19]

2.2.4 Recognize that Earlier Reasoning is Generally Truth-Preserving

Neither of the previous two proposed constraints on what Sasha must recognize in her earlier reasoning succeeds in vindicating Argument MP. Neither proposal is, in any case, intrinsically very plausible. The proper basis for reasoning by UG does not concern transmission of confidence or indefeasibility. I claim the proper basis is rather the following: In order to rationally reason by UG, Sasha's inference must be based on her recognition *that all of her earlier reasoning was generally truth-preserving*.

I should immediately clarify what I mean by 'generally truth-preserving'. I do not mean 'generally' in the loose sense of 'mostly' or 'largely'. I use it more strictly than that. However, I also do not mean that Sasha can never rationally reason by UG unless she recognizes her earlier reasoning was *logically valid*. A young student who hasn't yet learned the concept of validity can come to know some generalization about all natural numbers by following a proof which invites him to reason by UG. In his case, the proof introduces an arbitrary object to stand in for any natural number. When the arbitrary object is stipulated just to be a natural number, the student need only recognize that his earlier reasoning is truth-preserving when the arbitrary object is replaced by *any natural number*. *The proper basis of rational reasoning by UG is recognition that the earlier reasoning is truth-preserving when the arbitrary object is replaced by any of the individual objects that it is standing in for. In other words, Sasha must recognize that her earlier reasoning* *is truth-preserving throughout the class of cases which the application of UG aims to generalize to*.

Now, in Arguments MP and SI, the arbitrary object is an arbitrary *interpretation*. So, the relevant general class of cases just is all interpretations, and preservation of truth in all interpretations just is logical validity. So, for these particular arguments, reasoning by UG does in fact require recognizing that the earlier reasoning was logically valid.

Recall that 'recognize' here is just a place-holder for however it is that Sasha must be sensitive to the claim that her earlier reasoning is generally truth-preserving. Now that we've fixed *what* we want Sasha to be sensitive to, let's say more about *how* she must be sensitive to it. First, notice that the feature of her earlier reasoning that we want Sasha to be sensitive to, i.e. truth-preservation, is a *semantic* property. This supports the view that recognition must itself be an intentional state; it's certainly hard to see what non-intentional mechanism could track whether Sasha's earlier reasoning was truth-preserving.

We can narrow things down even more if we pick up on the paradigmatically *inferential* nature of reasoning by UG. The *basis* of an inferential process (its input condition) is, intuitively, comprised of cognitive propositional attitudes of the reasoner's. Externalists may be inclined to construe these as factive attitudes, such as knowledge, awareness, or some other truth-tracking attitude which can ensure that the resultant reasoning by UG will in fact be reliable. Internalists may only require the subject to rationally believe that her earlier reasoning exhibits the appropriate features, allowing that rational belief and reasoning can be unreliable. Both camps can agree though, when one rationally reasons by UG, one's inference is partly based on a propositional attitude toward the presence of certain features in one's earlier reasoning. I happen to be a thorough-going internalist, but what I say in this paper is neutral over the internalist-externalist debate.

There is one more thing to add about recognition. Externalists and internalists can agree that, at a minimum, rational reasoning by UG requires Sasha to have a *rational* attitude that her earlier reasoning was valid. If the conclusion of an inference is to be categorically rational, it cannot have an irrational basis. Indeed, the inferred conclusion acquires *no more* rational support than its basis has. In adding this requirement, it may sound like I am blindly favoring a foundationalist general epistemological theory over a coherentist one. The coherentist might ask: why do I dismiss out of hand the possibility that an inference's conclusion may be rational even while its basis is not? In fact, I do not mean to take sides on any debates about the structure of the general theory of rationality. In the present, very localized context, however, it would be disastrous to allow Sasha to use a rule-circular argument to convert an irrational belief in the validity of a rule into a rational belief. It would, for instance, allow someone with the irrational belief that statistical inference is valid to reason along the lines of Argument SI and arrive at the rational belief that statistical inference is valid.

So, to sum up, my proposal is that for Sasha to rationally reason by UG, she must partly base her conclusion on a rational cognitive propositional attitude that her earlier reasoning was generally truth-preserving.

We are finally ready to apply the proposal to Arguments SI and MP. Presumably Sasha has no (misleading) evidence supporting a rational belief that reasoning by statistical inference is valid, so, step 5* of Argument SI is not a rational application of reasoning by UG. What about step 5 of Argument MP? Since reasoning by Modus Ponens was part of her earlier reasoning, Sasha requires some justification to think that Modus Ponens is valid, and she must have this *antecedently* to drawing the conclusion of Argument MP. Thus, Sasha's knowledge of validity is not accounted for by Argument MP.

However, the proposed restriction on reasoning by UG does not imply that step 5 is strictly irrational. It implies that reasoning by UG in step 5 cannot account for how Sasha could *acquire* her justification to believe that reasoning by Modus Ponens is valid. If Sasha must already recognize that reasoning by Modus Ponens is valid before she can rationally reason through all the steps of Argument MP, then her reasoning in Argument MP is completely redundant at best.

As I mentioned at the outset, I am not a skeptic about our justification to believe that reasoning by Modus Ponens and many other rules is valid. I only believe that our justification has its source elsewhere. If we do have some kind of non-inferential justification, that could explain why Argument MP does not seem nearly as absurd as Argument SI. Argument MP is less absurd because it does not, as Argument SI does, describe an irrational sequence of transitions of reasoning. It is only a completely redundant sequence. Sasha may well settle her doubts about the validity of reasoning by Modus Ponens while she is running through the steps of Argument MP. But, when she reaches the conclusion stated in step 5, she will need to draw on, and will add nothing to, an independent justification that she had all along.

At this point, let me observe that it was a convenient but dispensable choice to frame the discussion in terms of the *logical* validity (truth-preservation in all interpretations) of transitions in *reasoning* by *Modus Ponens*. We could equally well have talked about how we recognize *metaphysical* validity, or validity of transitions in *derivation*. We could have talked about any other rule we take to be valid, not just *Modus Ponens*. This is because the heart of the problem concerns the use of reasoning by UG in a rule-circular argument. The solution, then, must be that we can recognize validity without relying on a rule-cicular argument that includes reasoning by UG. My proposal is that our recognition, and ultimately our knowledge, of validity is non-inferential. For certain forms of argument, we can recognize non-inferentially that every interpretation or every possible world that makes the premises true also makes the conclusion true. This proposal will receive further motivation after section 3, where I reject a number of other strategies for showing that reasoning by Modus Ponens is valid. But before coming to that, there are other important issues to delve into.

2.2.5 Why Is UG Constrained in This Way?

There are several questions to be answered about my proposed constraint on reasoning by UG.

First: Arguments by elimination aside, what is it about this proposal itself that makes it attractive? This proposal may be found attractive for different reasons. From this basis, the transition to the UG conclusion will be both highly reliable[20] and highly intuitive to a normal reasoner like Sasha[21]. Some may think it is by virtue of an external connection, such as reliability, while others may think it is rather by virtue of an internal connection, such as what the reasoner finds intuitive. Either way, it's plausible that reasoning by UG is only *supported* when the reasoner recognizes that the earlier reasoning was truth-preserving in the relevant range of cases.

Second: Why might this construal of UG have eluded us so long? Part of the difficulty is that it is only easy to see why rational reasoning by UG needs this constraint after we disassociate UG from the contexts in which we received our first lessons in it. In the context of a logic classroom, we are taught or at least encouraged to make the (false!) assumption that rational rules are valid rules. In the context of a math classroom, we only deal with valid rules, and so we won't need to exercise much care to ensure that our reasoning by UG is reliable. This makes it very easy for a student to unthinkingly take it for granted that his earlier reasoning (which he would surely believe was rational) was valid, and hence generally truth-preserving. Once we introduce UG as a rule of *reasoning*, a rule for use in contexts like Argument SI where invalid rules can be rational rules, we must require Sasha to explicitly believe that her earlier reasoning was generally truth-preserving (and not merely rational) before she can be allowed to reason by UG.

Third: What *explains why* reasoning by UG has this unusual constraint? This is a deep and difficult question, and one which we can only begin to tackle here. We can gain at least some perspective on this question, though, by understanding how knowledge of truth-preservation is essential for rational high confidence in certain inferential conclusions. Let me explain.

Consider a case where Sasha wants to draw some general conclusion $\forall x Fx$ about a domain of individuals $a_1 \ldots a_n$. Suppose that in this example Sasha can name every individual and that she can reason separately about each individual to infer, following the same form of reasoning each time, that it has the property F. These inferences will provide her with a large heap of conclusions, $Fa_1 \ldots Fa_n$. Now Sasha can perform inferences according to *conjunction introduction* a heap of times to eventually arrive, after a tediously long sequence of iterations, at a long conjunction Fa_1 & ... & Fa_n. Supposing Sasha knows when she's considered every last individual in the class, from this long conjunction she can finally infer the desired universal generalization $\forall x Fx$.

But, even though this procedure might allow her to, in a sense, reach the desired generalization, it involves the following serious drawback: by the time her long sequence of inferences brings her to her final conclusion, Sasha's levels of confidence may have eroded to almost nothing. Unless Sasha is completely certain in her beliefs, each time she reasons by conjunction introduction, her confidence in each successive conjunction must be lower than her confidence in the previous conjuncts. Over a sufficiently long sequence of inferences, the attrition that would accrue by the time she reaches her final conclusion would leave her with hardly any confidence.

Now, there is a way to avoid this attrition of confidence. The crucial tool that Sasha needs is just what reasoning by UG called for: Sasha must recognize that all her steps in the above long sequence of inferences are truth-preserving. Attrition occurs when Sasha proceeds to *reason her way* through the long sequence of steps, assigning confidence to each of the new sub-conclusions only as she infers them one after another. If she instead laid all of the steps before her *in the form a proof*, and recognized that every step was truth-preserving, then, appealing to the transitivity of truth-preservation, she could infer in one single step that if her original premises are true then the final conclusion is true. The difference here is between performing a long sequence of inferences, and representing that sequence of steps in the form of a proof. Once you know that you've given a proof of (or at least a truth-preserving argument for) some conclusion from some premises, you can the infer that conclusion on the basis of the premises plus the knowledge that if those premises are true then that conclusion is true. Thus, by collapsing the entire sequence of steps into a single Modus Ponens inference, Sasha circumvents the attrition problem.[22]

Notice that the crucial claim for Sasha to recognize here is that *all* the steps of her proof are truth-preserving, and thus that the transition from the initial premise(s) to the final conclusion is itself truth-preserving. If Sasha only recognizes *of each step* that *it* is truth-preserving, then the attrition problem would remain. She would still need to use conjunction introduction a heap to times to infer that the *entire* sequence is truth-preserving. (Her original sequence of reasoning already applied conjunction introduction a heap of times to arrive at the claim Fa_1 & ... & Fa_n. What I'm talking about now is using conjunction introduction to collect together the individual claims about the truth-preservation of each step of the original sequence of reasoning.) Even if she is highly confident of each step that it is truth-preserving, her aggregate doubt that she is mistaken about at least one step will prevent her from confidently recognizing that the entire sequence is truth-preserving.

Sasha's recognition that her reasoning is truth-preserving throughout the entire range of cases in question cannot be based on a collection of individual judgements about each case. To avoid the attrition problem, Sasha must have some independent basis that directly licenses rational high confidence that all her transitions are truth-preserving.

How can Sasha have direct license for high confidence that an entire class of transitions is truth-preserving? The solution should sound familiar. First, she must represent all the inferences about each individual, $a_1 \ldots a_n$, in some common form, namely as reasoning about an appropriately arbitrary object.[23] And now, the final key to solving to the attrition problem is just to grant that Sasha can directly recognize that this reasoning with the arbitrary object is generally truth-preserving, specifically, truth-preserving at least throughout the domain $a_1 \ldots a_n$.

So, we wanted to know what explains why reasoning by UG requires Sasha to recognize that her earlier reasoning, her reasoning about the arbitrary object, was generally truth-preserving. My suggestion has been that this recognition allows her to avoid a severe attrition in her confidence, even though her reasoning implicitly covers a large, usually infinite, number of inferential transitions. Even if it is right, though, this suggestion only begins to explain why reasoning by UG requires recognition that the earlier reasoning generally preserves truth. A fully adequate explanation can only be given within a general theory of epistemic rationality, one that uncovers the most fundamental explanation of why any reasoning is rational. But explanations, in any case, can only be found after we've agreed on what it is we're trying to explain, and my primary purpose here is to argue that reasoning by UG *is* constrained in a way that makes traditional rule-circular arguments for validity problematic.

3 Can Universal Generalization Be Avoided?

So far, I've only argued that the *standard* rule-circular argument cannot account for how we know that reasoning by Modus Ponens is valid. Since the problem I raised concerned the use of reasoning by UG, it is natural to wonder whether Argument MP could be reconstructed in a way that doesn't apply a rule of universal generalization. In this section, I consider and reject several such strategies for reconstructing Argument MP.

3.1 Reductio ad Absurdum

It might claimed that the rule-circular argument for the validity of Modus Ponens could be recast as a *reductio*, as follows:

> Suppose *a*, *b*, and *c* are arbitrary propositions of the forms **P**, **if P then Q**, and **Q** respectively, and *I* is an arbitrary interpretation that makes *a* and *b* true, and *c* false. Since *I* makes *b* true, it follows from the truth-conditions of conditionals

> that if I makes a true then it makes c true, and I in fact makes a true, so (by Modus Ponens) I makes c true. But that contradicts our assumption, so there exists no such interpretation that makes a and b both true while making c false.

But, the claim goes, the proof does not make use of universal generalization, so there is no problem.

We can see that the claim must be wrong even before we understand why it is wrong because an analogous *reductio* would prove the validity of statistical inference, as follows:

> Suppose a, b, and c are arbitrary propositions of the forms **Fm**, **99.999% of F's are G's**, and **Gm** respectively, and I is an arbitrary interpretation that makes a and b true, and c false. By the semantics of '99.999%' and the truth of b, I assigns 99.999% of the objects in the extension of '**F**' to the extension of '**G**'. Since I makes a true, I assigns the referent of '**m**' to the extension of '**F**'. So (by statistical inference), I assigns the referent of '**m**' to in the extension of '**G**', which means I makes c true. But that contradicts our assumption, so there exists no such interpretation that makes a and b both true while making c false.

Now, it initially appears that since there's no obvious use of universal generalization here, our previous complaints with rule-circular arguments don't apply. Well, there actually *is* an application of universal generalization in the proof by *reductio*, but it is hidden in the above proof sketches. The proof would not do its job (i.e. it would not be a complete proof of the validity of Modus Ponens) if it only refuted the claim that particular sentences a, b and c are thus and so. It needs to refute the claim that there *exist* sentences x, y and z that are thus and so, i.e. it needs to refute an existential generalization.[24] The arbitrary instances a, b and c can be used in the course of that refutation; they can be used to show the negation of the relevant existential generalization, i.e. to show that there do not exist sentences x, y and z that are thus and so. But that proof of the negation of an existential generalization will involve applying the rule of universal generalization.[25]

The main point can be summarized thus: you don't just want to refute the claim that the particular trio of sentences a, b, and c fails to constitute a counterexample to the validity of Modus Ponens; you want to show that *no* sentences constitute a counterexample.

3.2 Schematic Reasoning

What if we considered a very different sort of argument for the validity of Modus Ponens? Consider the following argument that makes use of *schematic* letters **P** and **Q** and applies *schematic reasoning* to them:

(1)	**P** and (if **P** then **Q**)	Assumption	
(2)	**P**	(and-elimination, 1)	[assuming 1]
(3)	if **P** then **Q**	(and-elimination, 1)	[assuming 1]
(4)	**Q**	(by MP, 2,3)	[assuming 1]
(5)	if (**P** and (if **P** then **Q**)) then **Q**	(by Conditional Proof, 1,4)	

The last line is a schematic statement of the validity of Modus Ponens and rests on no assumptions.

In some passages, it appears that this sort of schematic reasoning may be at least part of what some defenders of a rule-circular argument for the validity of Modus Ponens had in mind.[26] Unfortunately, as I will now argue, this sort of reasoning does not help to provide us with an argument for the validity of Modus Ponens.

First of all, once we let schematic reasoning in the door, an equally good argument for a schematic statement of the validity of statistical inference is possible:

(1)	**Fm** and (**99.999% of F's are G's**)	Assumption	
(2)	**Fm**	(and-elimination, 1)	[assm. 1]
(3)	**99.999% of F's are G's**	(and-elimination, 1)	[assm. 1]
(4)	**Gm**	(by statistical reasoning, 2,3)	[assm. 1]
(5)	if (**Fm** and (**99.999% of F's are G's**)) then **Gm**	(by Conditional Proof, 1,4)	

It would do no good to reject the above argument by claiming that, when it comes to schematic reasoning, only Modus Ponens is justified, not statistical reasoning. What could be the defense of that claim other than the question-begging point that Modus Ponens is valid while statistical reasoning is not? It is plausible that schemas and schematic reasoning are *invented technical notions*. If that's so, we cannot rationally use schematic reasoning unless we *first* provide a justification for it, and this justification will have to proceed on the basis of premises about validity. So, while I don't deny that in one's schematic reasoning it might be reasonable to use only Modus Ponens and not statistical reasoning, I am sceptical that *any* schematic reasoning can be accepted *prior* to having justified beliefs about validity.

Independently of the above, a fatal second problem with the schematic route to the validity of Modus Ponens is that the schematic statement of the validity of Modus Ponens was never our target. What we wanted to conclude is the statement that Modus Ponens is valid, and that is a not a schema. It is, in its definition, a generalization.

I should mention that we could use schematic reasoning to derive the validity of Modus Ponens (in full generality) if we appealed to a theory of schematic reasoning that Hartry Field developed for completely unrelated purposes in a recent article.[27] Unfortunately, this would not help the advocate of schematic reasoning in the present context.[28] Field's view includes a special rule of universal generalization, one that takes us from statements about arbitrary individual schematic letters to generalizations about all sentences. Such a rule raises all the now familiar problems; for one, it would prove the validity of statistical inference.[29]

3.3 Axiomatic Proof Theories

Argument MP represents a sequence of reasoning, not a derivation in some formal proof theory. Still, it's clear that it could easily be modeled in a natural deduction proof theory. All of the rules it uses, including universal generalization, are informal analogs of derivation rules in a natural deduction proof theory.

That observation may inspire the following thought. Classical predicate logic need not be formalized using a natural deduction proof theory. We could equally well give a sound and complete formalization using an axiomatic proof theory. In particular, some axiomatic proof theories have Modus Ponens as their only rule of derivation.[30] The thought, now, is this: if Sasha reasoned her way to the validity of Modus Ponens using such an axiomatic proof theory, then there will be no application of universal generalization to complain about, and there also will be no analogous statistical proof to complain about either.

My reply begins by noting that it is of no significance for a reasoner whether this or that conclusion can be derived using this or that proof theory *until she has established to her own satisfaction that the proof theory is sound*. For example, consider a proof theory whose only rule of derivation is Modus Ponens, and whose axioms are (1) $1 + 1 = 3$, and (2) If $1 + 1 = 3$, then reasoning by Modus Ponens is valid. The validity of reasoning by Modus Ponens can easily be derived in this proof theory, yet this shows nothing about how anyone might come to know that reasoning by Modus Ponens is valid. For an axiomatic proof theory to be of any use to us, we must first be able to satisfy ourselves that it is sound. Then we could argue that, since the proof theory is sound, anything derivable in it must be true.

Furthermore, in order for an axiomatic proof theory to help account for how we know that reasoning by Modus Ponens is valid, we must be able to establish its soundness without relying on antecedent knowledge that reasoning by Modus Ponens is valid. Consider, for example, a proof theory whose only rule of derivation is Modus Ponens, and whose axioms are (1) All bachelors are unmarried, and (2) If all bachelors are unmarried, then reasoning by Modus Ponens is valid. We can establish that this system is sound, and in it we can derive that reasoning by Modus Ponens is valid. But how could the second axiom be established except by appealing in part to our knowledge that reasoning by Modus Ponens is valid? This proof theory will not help account for how we know that reasoning by Modus Ponens is valid.

The standard axiomatic proof theories found in textbooks are more sophisticated. These proof theories, which are not only sound but complete, include universal generalizations among their axioms. For example, such systems require the following axiom (or something very similar):

$$\text{(A1)}\quad \forall \boldsymbol{x}, \boldsymbol{y} \ldots \{\, \forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{A} \supset \mathbf{B}) \supset [\forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{A}) \supset \forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{B})]\,\}.$$

'**A**' and '**B**' are schematic letters; plug any grammatical formula in their places and you have an axiom. Also, you may add as many universal quantifiers as you want to the head of the formula and it's still an axiom. Axiom A1, or something very similar, is essential to any reasonable proof theory that uses Modus Ponens as its only rule of derivation.[31] If you're going to make Modus Ponens your only rule of derivation, you'll need an axiom like this, or else you'll be unable to apply Modus Ponens 'inside the scope of quantifiers'.

In an appendix, I show (just for illustration) how to derive the conclusion that Modus Ponens is valid in a sophisticated (sound and complete) axiomatic system that has Modus Ponens as its only rule of derivation. But, as I've been urging, a derivation is just a piece of formal symbol-manipulation; until more is said, it won't convince a rational reasoner to accept any derived results. In particular, we wouldn't accept reasoning that started from axioms as sophisticated as (instances of) A1 with no further defense of those axioms. Just because a proposition is an axiom in a formal proof theory does not mean that its truth can be taken for granted. We do *not* believe axioms like (instances of) A1 with no argument.

What reasoning is required, then, to establish the truth of a (closed) sentence that is an instance of A1 that is headed by a single universal quantifier, '$\forall x$'? First you do some reasoning to show that the formula occurring inside the quantifier '$\forall x$', i.e. the (possibly open) formula of the form $\forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{A} \supset \mathbf{B}) \supset [\forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{A}) \supset \forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{B})]$, is true in an arbitrary assignment (where an 'assignment' fixes the denotation of any occurrences of the unbound variable 'x'). Doing this will require reasoning by Modus Ponens and many other rules.[32] Next, you point out that, since that formula was shown to be true in an arbitrary assignment, the quantified sentence of the form $\forall \boldsymbol{x}\{\forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{A} \supset \mathbf{B}) \supset [\forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{A}) \supset \forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{B})]\}$ is true (i.e. true *simpliciter*). And this, of course, is an application of reasoning by universal generalization; in particular, it's an application to a claim that was established using Modus Ponens in the earlier reasoning. We thus see that taking a detour through an axiomatic proof theory doesn't help to avoid the problems created by universal generalization.

4 What Has Been Shown? Where Do We Go Now?

I have argued that the standard rule-circular argument cannot account for our justified beliefs in the validity of reasoning by Modus Ponens. The problem lies with the fact that rational reasoning by universal generalization presupposes justification for believing that one's earlier reasoning was generally truth-preserving. In a rule-circular argument for validity, this has the effect of requiring antecedent justification to believe the very conclusion drawn by the argument. And universal generalization cannot be avoided, I argued, even if we turn to proof by *reductio ad absurdum*, schematic reasoning, or axiomatic formal proof theories. I conclude, therefore, that our knowledge of validity cannot be accounted for by a rule-circular argument.

Supposing that I have in fact succeeded in closing off the rule-circular account of our knowledge of validity, what is the significance of this? I claim that the upshot of this result is to give a tremendous amount of new motivation for the view that we have knowledge of validity *non-inferentially*. I do not claim that the demise of the rule-circular route *entails* that a non-inferential account is right. But, it is not hard to see why the natural choice is between these two options. A non-inferential account of our knowledge of validity is the only other *prima facie* plausible account. To see this, take a look at the following exhaustive list of possibilities:

(1) *We do **not** have knowledge of validity.* This is a highly skeptical view, not at all *prima facie* plausible.
(2) *We have knowledge of validity, but we have this knowledge via **invalid** (inductive) reasoning.* It is not *prima facie* plausible that the extremely high and extremely resilient degree of confidence we seem to rationally give to claims about validity is supported by inductive reasoning.
(3) *We have knowledge of validity, and we have this knowledge via valid reasoning, but there are **blind spots**: we know of rule R_1's validity via the use of valid rule R_2, but we are in principle unable to know of R_2 that it is valid.* This view is also excessively skeptical. It is much bolder, for instance, than the reasonable claim that most people who know that reasoning by Modus Ponens is valid do not know exactly how they have that knowledge.
(4) *We have knowledge of validity, via valid reasoning, with no blind spots, and this knowledge is accounted for by **rule-circular reasoning**.* This is the account we've attacked in this paper.

So, we find ourselves in the remaining territory of possibilities. I divide this space into two, as follows:

(5) *We have knowledge of validity, via valid reasoning, with no blind spots, and without relying on rule-circular reasoning, but we nonetheless still somehow acquire all this knowledge **inferentially**.*

(6) *We have knowledge of validity, via valid reasoning, with no blind spots, and without relying on rule-circular reasoning, but, crucially, we acquire some of this knowledge* ***non-inferentially***.

This list, (1)–(6), represents a partition of all possibilities. I've claimed that rejection of (4) leads us to (6). Let me say now, then, why (5) should be skipped over.

According to the rule-circular account, we learn that rule R is valid via an argument that uses rule R. But even if (5) and (6) avoid such rule-circular arguments, they must both still involve circularity in a broader sense. In order not to leave any blind spots, they must both include 'loops' of valid reasoning: *We can first learn that rule* R_1 *is valid by using rule* R_2*, we can next learn that rule* R_2 *is valid by using rule* $R_3, \ldots$ *we can finally learn that* R_N *is valid by using* R_1.[33] The difference between (5) and (6) is that, on (5), all these rules are inferential, and this turns out to be problematic. To see how, let's look at a more concrete example of a view that falls under option (5).

In this example, we first learn that reasoning by Modus Ponens is valid by the following argument:

P1 Either reasoning by Modus Ponens is valid or else reasoning by Disjunctive Syllogism[34] is invalid.

P2 Reasoning by Disjunctive Syllogism is not invalid, but valid.

C Therefore, using reasoning by Disjunctive Syllogism, reasoning by Modus Ponens is valid.

Next, we use the following argument to learn that reasoning by Disjunctive Syllogism is valid:

P1′ If reasoning by Modus Ponens is valid, then reasoning by Disjunctive Syllogism is valid.

P2′ Reasoning by Modus Ponens is valid.

C′ Therefore, using reasoning by Modus Ponens, reasoning by Disjunctive Syllogism is valid.

This example indicates why views in option (5) are not *prima facie* plausible. Such views invite the charge of *premise-circularity*. In the example given, what could account for how the premises, in particular premises P2 and P2′, are justified? P2 seems like it can only be justified after one has justification for C′, while P2′ seems like it is only justified after C is. The problem is not that each argument is justified if and only if the other is. The problem is that each argument contains premises which can be rationally accepted only *after* the other argument has already been rationally followed.[35] The problem would not go away if we moved to a larger circle with more than just two arguments.

To eliminate premise-circularity, we must get rid of the problematic premises, namely premises stating that some rule of ours is valid. But now (5) seems forced to say we infer that reasoning by Modus Ponens is valid on the basis of premises which say nothing about the validity of anything. What could make such an account *prima facie* plausible? What could the appropriate inference look like? Should we infer that reasoning by Modus Ponens is valid from the premise that ducks quack? Such an inference is presumably valid, but only because the conclusion validly follows from the empty set of premises. The *prima facie* plausible option is to leave the premises out of the account altogether. Our knowledge that reasoning by Modus Ponens is valid is simply non-inferential.

My dismissal of option (5)'s *prima facie* plausibility might seem rash if one reads this as a dismissal of the view held by Quine[36], which of course should be taken seriously, more seriously than I take option (5). Is (5) Quine's view? Perhaps the example about reasoning by Disjunctive Syllogism evokes the rough idea of 'global coherence' in a way reminiscent of certain alleged aspects of Quine's view. But if we pay attention to how intellectually demanding of reasoners that example is, it becomes clear that (5) is not the Quinean view. (5) says that having a rational belief that reasoning by Modus Ponens is valid requires having performed an inference. There is much about Quine's view that may be open to interpretation and debate, but it is very clear that on the Quinean picture, our metalogical knowledge does *not* need to be propped up by an inference.

Whatever the details of Quine's view may be, and however implausible some of those details may be, there is promise in the view that we have non-inferential knowledge of validity. This paper's attack on rule-circularity can be taken to provide new motivation for the following sort of view.

Our knowledge of validity is non-inferential in roughly the way our perceptual knowledge is non-inferential, or in the way some philosophers have thought our knowledge of metaphysical essences is non-inferential. Even before ever learning of the rule-circular argument, ordinary reasoners find it intuitive or obvious that reasoning by Modus Ponens is valid. Many epistemologists are currently developing and advocating theories that give a justificatory role to such intuitions.[37] What we have here, then, is new motivation to explore a notion of intuition that will provide a non-inferential basis for rational belief in the validity of reasoning by Modus Ponens.

Now, there is even room in such an intuition-based view for rule-circular arguments, as long as these play a non-foundational role. After we non-inferentially acquire a foundational layer of knowledge of validity, a rule-circular inference can provide us with additional knowledge of validity. Let *R* be the (very local) rule that instructs us to non-inferentially adopt the belief that reasoning by Modus Ponens is valid. One way we might come to know that *R* is valid is via the following rule-circular inference: (1) *R* is valid if reasoning by Modus Ponens is valid; (2) reasoning by Modus Ponens is valid; therefore, (3) R is valid. Step two is justified by R itself. So, we may still end up including a rule-circular inference in our account of how we know about the validity of rules in our general belief-forming method. It's no problem, of course, since what I have objected to is the use of reasoning by UG in rule-circular arguments for validity. But if we have antecedent non-inferential knowledge of validity, there is nothing to worry about. A strategy like this, one that provides a non-inferential foundation for our knowledge of validity, seems to be the sort of idea that we are pushed to explore in light of the problems uncovered here for the rule-circular strategy.[38]

Appendix

Here is a proof of the validity of Modus Ponens in an axiomatic formal proof theory. Suppose our proof theory has at least the following two axiom schemas:

(A1) $\forall \boldsymbol{x}, \boldsymbol{y} \ldots \{ \forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{A} \supset \mathbf{B}) \supset [\forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{A}) \supset \forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}(\mathbf{B})] \}$,

(A2) $\forall \boldsymbol{x}, \boldsymbol{y} \ldots \{ [\mathbf{A} \supset (\mathbf{B} \supset \mathbf{C})] \supset [(\mathbf{A}\&\mathbf{B}) \supset \mathbf{C})] \}$.

Again, '**A**','**B**' and '**C**' are schematic letters; plug any grammatical formula in their places and you have an axiom. And you may add as many universal quantifiers as you want to the head of the formula and it's still an axiom. I already introduced A1. Axiom A2 closely resembles (and is equivalent to) an axiom that you see in logic textbooks[39], except that I've made use of an '&' whereas most textbooks use only '⊃' and '∼' in their formal vocabularies. (I'm using A2 instead of the more usual textbook axiom only because it makes the derivation that follows far more readable; no philosophical issues turn on this, since exactly the same results are derivable, only less conveniently, using just the textbook axioms.)

Using only these two axiom schemas, plus the compositional truth rule for conditionals (as we had in our proof in section 2), and using Modus Ponens as our only rule of derivation, we can derive the validity of Modus Ponens as follows:

Let '$Cxyz$' mean 'y is a conditional with antecedent x and consequent z'. Let 'Tix' mean 'Interpretation i makes x true'.

(1) $\forall x, y, z, i \{ Cxyz \supset [Tiy \supset (Tix \supset Tiz)] \}$[40]

(compositional truth rule for '⊃')

(2) $\forall x,y,z,i \{ (Cxyz \supset [Tiy \supset (Tix \supset Tiz)]) \supset ([Cxyz\&Tiy] \supset [Tix \supset Tiz]) \}$ (A2)
(3) Line2 ⊃ (Line1 ⊃ $\forall x,y,z,i([Cxyz\&Tiy] \supset [Tix \supset Tiz])$ (A1)
(4) Line1 ⊃ $\forall x,y,z,i([Cxyz\&Tiy] \supset [Tix \supset Tiz])$ (MP 2,3)
(5) $\forall x,y,z,i([Cxyz\&Tiy] \supset [Tix \supset Tiz])$ (MP 1,4)

(6) $\forall x,y,z,i$ { ([Cxyz&Tiy] ⊃ [Tix⊃Tiz]) ⊃ ([Cxyz&Tiy&Tix] ⊃ Tiz) } (A2)
(7) Line6 ⊃ {Line5 ⊃ $\forall x,y,z,i$([Cxyz&Tiy&Tix] ⊃ Tiz) } (A1)
(8) Line5 ⊃ $\forall x,y,z,i$([Cxyz&Tiy&Tix] ⊃ Tiz) (MP 6,7)
(9) $\forall x,y,z,i$([Cxyz&Tiy&Tix] ⊃ Tiz) (MP 5,8)

The last line states that Modus Ponens is valid.

Notes

*Pronunciation: ***sin***·on ***dor***·uh·*mudge*·uh. Email: sinan.dogramaci@gmail.com

[1] I will very shortly describe the innocuous meanings I give to 'rule' and 'reasoning by Modus Ponens'.

[2] Widely, but not universally accepted. Sorensen (1991) offers a number of potentially compelling exceptions. I won't engage with Sorensen's examples; this paper is not about premise-circularity. But even granting his examples, the general deficiency of premise-circularity is clear.

[3] I mean the same thing by the adjectives 'rational' and 'justified'. For stylistic reasons only, it's convenient to be able to use inflections of both words, for example the noun 'justification' and the adverb 'rationally'.

[4] This is not to say that the inference would be permissible if you believe it to be invalid and irrational. What seems to be permissible is performing the inference while not yet having any attitude about its validity.

[5] Wright (2004) does favor the view that a reasoner may acquire knowledge of validity by making a rule-circular argument. However, Wright adds, a rule-circular argument will be no help in earning us whatever warrant we might have for the higher-order claim that we have knowledge of validity. If a reasoner makes the higher-order claim that she has knowledge of validity, then the epistemic status of that claim is at best what Wright calls 'entitlement'. Entitlement is a positive epistemic status, but one that is not based on the thinker's making some argument or indeed on her having evidence of any kind; entitlement falls short of the achievement expressed by our ordinary notions of knowledge or even justification. These reservations of Wright's are clearly significant, since no anti-skeptical philosopher would be fully satisfied by the view that she can have knowledge in some area while her claim to that knowledge must be of an inferior epistemic status.

[6] More fully: 'the validity of the reasoning described in this rule'.

[7] I want to apply the above definition of logical validity to reasoning, for example I want to say that any piece of reasoning by Modus Ponens is logically valid. But this faces two wrinkles. Some philosophers doubt whether the objects of beliefs, such as the proposition that if cats meow then dogs bark, contain terms that allow it to have a logical form. (For example, Stalnaker (1984).) And many philosophers doubt that ordinary indicative conditionals can be understood as truth-valued propositions. (For background on this, see Bennett (2003).) I intend to sidestep these issues. I will talk as if it is okay to describe a proposition as having a logical structure, including the structure of a conditional. But if it is not okay, then we can safely understand the discussion to follow as if, in any given context, we have (tacitly) agreed on a linguistic representation of the belief in question (e.g. the sentence occurring in the that-clause we're using to attribute and refer to the belief), and are debating the logical form of that representation.

[8] Friedman (1979) and Goldman (1986) endorse rule-circularity for both projects. For defenses exclusively of the inductive case, see, Black (1958), Braithewaite (1953) and Van Cleve (1984).

[9] For example, the Pólya conjecture states that more than 50% of the natural numbers under any given number have an odd number of prime factors. The smallest counterexample, found many decades after the conjecture was first made, turned out to be 906,150,257!

[10] I'm counting non-inferential beliefs as formed by rules. Remember, I use 'rule of reasoning' to just mean a pattern of belief formation. When a belief is non-inferential, i.e. based on no premise beliefs, then I count the associated rule as valid just in case the belief validly follows from the empty set of premises.

[11] For convenience, let's ignore intensional predicates, like 'is sought by Lois Lane'. This won't affect our purposes.

[12] For clearer exposition, I sloppily use '**F**', '**m**' and '**G**' to refer to the terms in *a*, *b*, and *c* that have those logical forms.

[13] For a defense, see Fine (1985).

[14] Seeing that step 4* is rational might depend on first appreciating the point of the previous paragraph that an arbitrary case really is an individual case, not a covert generalization. Knowing that 99% of little boys like chocolate, it is perfectly rational to infer with high confidence that *if* Joey is a little boy *then* Joey likes chocolate. I have no complaint with these conditionals concerning individuals. In fact, nothing in this paper raises a complaint with arguments that show of any *specific instance* of reasoning by Modus Ponens that *it* preserves truth. In this paper, I am targeting general theses, claims of *validity*.

[15] If we represented step 5* in a formal natural deduction system, it would actually involve several applications of derivation by UG, since each application only generalizes a single term. But I'm passing over this detail since it makes no difference to my arguments.

[16] See Goldman (1986).

[17] For an overview of Bayesianism, see Howson and Urbach (2005). For an overview of AGM theory, see Hansson (1999).

[18] See Harman (1995).

[19] There's an additional problem that can be pressed against the weak indefeasibility proposal: consistency and validity are interdefinable. If Sasha must recognize that Modus Ponens is weakly indefeasible before she can reason by UG in step 5, then she must be able to recognize whether certain sets of propositions are logically consistent. But if she can already recognize consistency, then she can already recognize validity. This puts the weak indefeasibility proposal in the same boat as my favored proposal for what Sasha must recognize, which I discuss next.

[20] Reliability requires the accuracy of the attitude that the earlier reasoning is generally truth-preserving. Choosing a factive attitude like knowledge to play the role of recognition will guarantee this.

[21] Whether a reasoner will find the transition intuitive obviously depends on her cognitive make-up. Philosophers who place heavy explanatory weight on the thinker's intuitions may take the rationality of these inferences to be relative to a thinker's cognitive make-up. For philosophers with such sympathies, the present proposal is an explication of the conditions on rational inference for a normal human like Sasha. I myself think intuitions are fundamental to explaining rationality, while reliability is irrelevant.

[22] Sasha circumvents the danger of a severe loss of confidence. This doesn't mean she can draw her conclusion with no less confidence than she had in her initial premises. If Sasha's proof begins with multiple (independent) premises, then her confidence in her conclusion will be capped by her confidence in the conjunction of the premises, which will be lower than the individual premises. Likewise, to the extent that Sasha's confidence that the proof is truth-preserving is less than 100%, this will diminish her transmission of confidence to the conclusion. These one-time only losses are, however, manageable losses; not like the severe attrition that would accrue over a very long sequence of reasoning.

[23] Of course, the idea of Sasha herself reasoning one by one about each individual in some large domain, $a_1 \ldots a_n$ is just a illustrative fiction. If attrition of confidence is a concern, then

this domain will be too large for anyone to realistically be able to reason about every individual. So the use of an arbitrary object serves two purposes: one, to allow Sasha to represent a large, possibly infinite, collection of inferences, and two, to represent specifically the fact that those inferences share a common form.

[24] I use '*a*' as a constant, a name, while I use '*x*' as a variable.

[25] It might not strictly invoke universal generalization by name; it might use some logically equivalent rule(s), for example some sequence of the usual introduction and elimination rules for the existential quantifier and negation. But the same old problems will arise for any such rule(s) that are logically, if not nominally, equivalent to universal generalization.

[26] See p. 68 of Wright (2001), p. 13 of Boghossian (2001), and p. 19 of Wright (2004). But also see p. 12 of the last article for a presentation that is more distinctly entirely in natural deduction.

[27] Field (2006).

[28] In a personal correspondence, Field agreed that his system won't help the advocate of schematic reasoning respond to my objection.

[29] To be precise, Field's rule says that from any claim in which arbitrary schematic letters occur in single quotes, one may draw the corresponding generalization about all sentences. To prove the validity of Modus Ponens, we'd begin by proving the schema, as we did in the text above, that if [**P** and [if **P** then **Q**]] then **Q**. Using that and also using instances of the disquotational truth schema, we could then show (the derivation gets a little monotonous here; see Field's article for the basic strategies) that if ['**P**' is true and [if the conditional with antecedent '**P**' and consequent '**Q**' is true]] then '**Q**' is true. Then, using Field's proposed rule, we could generalize that last claim to infer the validity of Modus Ponens, i.e. for all sentences p and q, if [p is true and [if the conditional with antecedent p and consequent q is true]] then q is true. A corresponding procedure would work for statistical inference.

[30] Two standard textbooks that include such systems are Hunter (1971) and Enderton (1972). Note that the rule of derivation by Modus Ponens is a different thing than the rule of reasoning by Modus Ponens. The former describes how derivations may be constructed in a proof theory. The latter describes a rational pattern of reasoning. Harman (1986) urges us not to confuse reasoning and derivation.

[31] See Hunter (1971) p. 167, and Enderton (1972) p. 112.

[32] The reasoning is roughly as follows. First assume the two antecedents. Then take arbitrary instances of them. Apply Modus Ponens to derive an instance of the final consequent. Next, you can apply universal generalization to that instance of the consequent. And finally apply Conditional Proof twice. (It is clear that the point of the present paragraph in the body text applies to the reasoning in this footnote as well.)

[33] A rule-circular argument is just a loop of size 1.

[34] Reasoning by Disjunctive Syllogism follows a rule the takes one from premises of the forms $\mathbf{A} \vee \mathbf{B}$ and $\sim\mathbf{A}$ to the conclusion of the form **B**.

[35] These arguments are thus similar to the argument using the unsophisticated axiomatic proof-theory which includes the axiom *if all bachelors are unmarried, then reasoning by Modus Ponens is valid*. Where is the justification for believing this axiom to come from, if we are not antecedently justified in believing that reasoning by Modus Ponens is valid? The appeal to the axiomatic proof-theory offers no explanation, while the above argument using reasoning by Disjunctive Syllogism offers a premise-circular justification.

[36] See Quine (1951). For slightly more elaboration, see Harman (2003).

[37] See, for example, BonJour (1998), Bealer (2000), Huemer (2007), and Sosa (2007).

[38] For invaluable help in preparing this paper, I would like to thank an anonymous referee for *Noûs*, Paul Boghossian, Shamik Dasgupta, Hartry Field, Kit Fine, Matt Kotzen, Jim Pryor, Karl Schafer, and Crispin Wright.

[39] Namely, $\forall \boldsymbol{x}, \boldsymbol{y} \ldots \{\ [\mathbf{A} \supset (\mathbf{B} \supset \mathbf{C})] \supset [(\mathbf{A} \supset \mathbf{B}) \supset (\mathbf{A} \supset \mathbf{C})]\ \}$. See, e.g., axiom QS2 on p. 167 of Hunter (1971).

[40] This line is my translation of 'For any conditional, and for any interpretation, the interpretation makes the conditional true only on the condition that if the interpretation makes the conditional's antecedent true, then the interpretation makes the conditional's consequent true.' I've represented the truth-condition of the conditional in a way that allows the axiomatic derivation of Modus Ponens's validity in the easiest way I've been able to come up with. We could have defined it using ≡, ∨ and ∼, but that would require an even messier proof than this one and would make no difference to the epistemological issues.

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