Good Guesses

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Abstract

This paper is about *guessing*: how people respond to a question when they aren't certain of the answer. Guesses show surprising and systematic patterns that the most obvious theories don't explain. We argue that these patterns reveal that people aim to optimize a tradeoff between accuracy and informativity when forming their guess. After spelling out our theory, we use it to argue that guessing plays a central role in our cognitive lives. In particular, our account of guessing yields new theories of belief, assertion, and the conjunction fallacy—the psychological finding that people sometimes rank a conjunction as more probable than one of its conjuncts. More generally, we suggest that guessing helps explain how boundedly rational agents like us navigate a complex, uncertain world.

1 Take a Guess

Where do you think Latif will go to law school? He's been accepted at four schools: Yale, Harvard, Stanford, and NYU; now he just has to choose. We don't know his preferences, but here's the data on where applicants who've had the same choice have gone in recent years:

Yale	Harvard	Stanford	NYU
38%	30%	20%	12%

So take a guess: Where do you think he'll go?

Here are some observations. One natural guess is 'Yale'. Another is 'Either Yale or Harvard'. Meanwhile, it's decidedly *un*natural to guess 'not Yale', or 'Yale, Stanford, or NYU'.

Though robust, these judgments are immediately puzzling. First, 'Yale' is a fine guess, but its probability is below 50%, meaning that its negation is strictly more probable (38% vs. 62%); nevertheless, 'not Yale' is a weird guess. Moreover, 'Yale or Harvard' is a fine guess—meaning that it's okay to guess something other than the single most likely school— yet 'Yale, Stanford, or NYU' is a weird guess (why leave out 'Harvard'?). This is so despite the fact that 'Yale or Harvard' is less probable than 'Yale, Stanford, or NYU' (68% vs. 70%).

We'll generalize these patterns (§2, following Kahneman and Tversky 1982; Holguín 2020), then develop an account that explains them (§3). The idea is that guessers aim to optimize a tradeoff between *accuracy* and *informativity*—between, on the one hand, saying something that's as likely as possible to be true; and, on the other, saying something which is as informative or specific as possible. These goals directly compete: the more specific an answer is, the less probable it will typically be. Different guessers, in different contexts, will treat this broadly Jamesian tradeoff in different ways. Some will guess 'Yale'; others will guess 'Yale or Harvard'; still others will guess something else. But we'll show that, however they do so, optimizing this tradeoff is guaranteed to satisfy the structural constraints on guesses that we'll bring out in §2.

After developing this account, we'll use it to argue that guessing— along with its accuracy-informativity tradeoff—plays a central role in our cognitive lives.[1] First (§4.1), we'll argue that our account of guessing underpins a promising theory of *belief*, namely that of Holguín (2020), who argues that your beliefs are your best guesses. Holguín shows how this account unifies recent observations about both the *weakness* and *question-sensitivity* of beliefs. We think our account of guessing helps explain *why* guesses—and hence beliefs—have these features.

Second (§4.2), we'll show that our theory of guessing helps to both explain and generalize the standard pragmatic story about how conversations proceed, suggesting that guessing plays a central role in ordinary exchanges of information.

Finally (§4.3), we'll argue that our theory helps explain the *conjunction fallacy*—the psychological finding that people sometimes rank a conjunction as more probable than one of its conjuncts, contrary to the laws of probability (Tversky and Kahneman, 1983). Our explanation is built on the observation that it is perfectly permissible to *guess* an answer that is less likely—but more informative—than some other guess. We thus propose that the conjunction fallacy arises when subjects rate answers for their *quality as guesses* rather than for their *probability of being true*.

Often (depending on the precise framing), this is no mistake at all. Even when it is, it's a mistake that is easy to make sense of, given the central role that guessing plays in our cognitive lives. Thus our account puts pressure on thoroughly irrationalist interpretations of the conjunction fallacy, and instead helps situate it within a boundedly-rational picture of

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[1]Our interest in guessing was sparked by recent work on its role in epistemology by Horowitz 2017, Builes et al. 2020, and Holguín 2020.

the human mind (§5).

That is part of a broader narrative. We focus on these three applications—belief, assertion, and the conjunction fallacy—not only because they are intrinsically interesting, but also because they help to paint the bigger picture of this paper. On that picture, guessing is a cognitively basic activity—one that we constantly engage in as we think, talk, and reason. Moreover, it's an activity that *makes sense* to engage in, for it's part of how computationally limited creatures like us cope with an intractably complex and uncertain world.

2   What We Guess

We start with a simple question: what sorts of guesses do we tend to make? The answer is both surprising, and surprisingly systematic. In this section we bring out some of these patterns, drawing on observations from Kahneman and Tversky 1982 and Holguín 2020. Along the way, we explain why the most obvious theories of guessing won't predict them—setting the stage for our own theory, which we develop in the next section.

Recall the case of Latif, who's been accepted to four law schools. Assume that your credences—that is, your degrees of confidence—in where Latif will go track the past frequencies, so that they are as follows:

Yale	Harvard	Stanford	NYU
38%	30%	20%	12%

Now suppose you are asked to guess where Latif will go. It seems like there is a range of answers that could reasonably be your guess, given your credences:

(1)   a.   Yale. ✓
       b.   Yale or Harvard. ✓
       c.   Yale or Harvard or Stanford. ✓
       d.   Yale, Harvard, Stanford, or NYU. ✓

A few notes: first, these are all meant to be elliptical for 'He will go to Yale', 'He will go to Yale or he will go to Harvard', etc. We'll use shorthands like this throughout. Second, there may be some subtle effects concerning the order of disjuncts, but we'll ignore them, assuming throughout that a disjunction $p \vee q$ is permissible iff $q \vee p$ is. Third, we'll move freely between speaking of answers as propositions and as the corresponding sentences. Fourth, consider the answer 'Yale, Harvard, Stanford, or NYU' (= 'One of those four'). Although *saying* this in response to a request to guess is weird, we think this is for independent pragmatic reasons (see §4.2). Moreover, there is a sense in which it is fine for this to *be* your guess: that is just to decline to take a stance beyond what you are sure of. We are primarily interested in the *cognitive state* of guessing—of something being your answer to a question—rather than the speech act; and in the former sense, it is clearly a permissible guess. Finally, there are a variety of ways of eliciting guesses: besides explicitly asking for a guess, we could ask questions like

'Where do you think Latif will go?, 'What do you think's likely to happen?', and so on. We'll shift freely between formulations like this, and come back to this point in due course.

There is also a range of answers that are intuitively unacceptable; for example:

(2)   a.   Harvard. ✗
        b.   Stanford. ✗
        c.   NYU. ✗
        d.   Yale or Stanford. ✗
        e.   Yale or NYU. ✗
        f.   Harvard or Stanford. ✗
        g.   Not Yale. ✗
        h.   Harvard, Stanford, or NYU. ✗
        i.   Yale, and it's cold in London today. ✗
        j.   Yale, or he has a birthmark on his left toe. ✗

To be clear, we are not claiming that people never have guesses like these. Our claim is normative: there is something peculiar—something irrational—about guesses like this. Our ultimate aim will be to give a theory which predicts that it's epistemically irrational for any of these answers to be your guess about the question 'Where will Latif go?', given your credences. Insofar as people's practices of guessing (and saying their guess) track this normative structure, our theory will thus predict that they will tend to favor answers like those in (1) over those in (2).

On to generalizing the patterns from (1) and (2). It will be helpful to lay out a formal model of what a *question* is and what its *answers* are, drawing on standard formulations from the semantics and pragmatics literature (Hamblin, 1973; Karttunen, 1977; Groenendijk and Stokhof, 1984). Start with a set of possibilities (possible worlds) that comprise all and only the worlds compatible with the assumptions in a given context—i.e. the *context set* (Stalnaker, 1974, 1978). In the case of a single person thinking to themselves, these will be the set of worlds compatible with what the guesser is certain of. In the case of a conversation, it will be the set of worlds compatible with what the interlocutors are (in some sense) commonly certain of. To keep things simple, we'll focus on the case in which the context set and the guesser's certainties coincide.[2]

A question is a partition of the context set: that is, a set of mutually exclusive and jointly exhaustive subsets of the context set. The cells of the partition are the *complete answers* to the question. So, in our example, we can model our question 'Where will Latif go to law school?' as a partition of the options we leave open in this context, namely the set of propositions {*Latif will go to Yale, Latif will go to Harvard, Latif will go to Stanford, Latif will go to NYU*}. We'll assume that at any given point in a conversation, there's a *question under discussion* (QUD) that guessers aim to address (Roberts, 2012). Sometimes this QUD will be set by explicit questions like, 'Where will Latif go?'; but other times it will be gleaned

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[2]The constraints that we discuss here can all be generalized, with minor tweaks, to the case where the context set and the guesser's certainties don't coincide.

from other parts of the context: in general, by the structure and goals of the relevant inquiry.[3]

We'll assume that the guesser has credences which can be modeled with a probability function P that is regular over the context set (that is, for any proposition p that has a nonempty intersection with the context set, $P(p) > 0$). We will, likewise, assume that questions are always finite partitions.[4]

With this formalism in hand, let's draw some generalizations from the observations above. We'll do this by way of exploring and rejecting some obvious theories of guessing.

The obvious proposal: you should guess p only if you think that p is more likely than not to obtain. As we've seen, this is wrong: while some of the acceptable guesses above have a greater than 50% chance ('Yale or Harvard', 'Yale, Harvard, or Stanford, ...), others, like 'Yale' (38%), do not. So *you don't always have to pick an answer that is more likely than not to obtain* (Kahneman and Tversky, 1982):

> **Improbable Guessing**: It's sometimes permissible to answer p even when $P(p) < 0.5$.[5]

Note, second, that judgments about the reasonableness of answers depend substantially on what question is being answered. Suppose that your credences are as above, but instead of being asked where you think Latif will go, you're asked: 'Will Latif go to Yale?', i.e. the question {*Yale, not Yale*}. Recall that you think there's a 38% chance he'll go to Yale, and thus a 62% chance that he won't. Given that, when addressing *this* question it seems like 'Yale' is not a very natural guess, since it is the substantially less likely of the two complete answers. Thus, again following Kahneman and Tversky 1982:

> **Question Sensitivity**: Whether p is a permissible answer depends not just on the guesser's credence in p but also in what question is being answered.

This means that—holding your credences fixed—p can be permissible relative to one question, but impermissible relative to another. Because of this observation, following Holguín 2020, we'll understand guessing to be a three-place relation between a person x, a question Q, and a proposition p. Thus it's possible that your guess about 'Where will Latif go?' may be 'Yale', while your guess about 'Will Latif go to Yale or not?' may be 'not Yale'. For ease of exposition we'll use 'guess' sometimes to refer to the *state* of guessing that p about a particular question Q, and other times to refer to the proposition p that you guess about Q; we'll use *answer*

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[3]In general, there can be small mismatches between questions explicitly asked and the QUD. For instance, when you're asked 'Do you think p?' the QUD (the one that is relevant for us, anyway) is usually p?, not a partition over possible mental states.

[4]Infinite questions would require some infinity-friendly measure on the QUD. Notably, all of our constraints on guessing could be stated in terms of a comparative confidence ordering, though the model (in §3) that justifies them requires more structure. It's an open question whether some refinement of our model could explain these constraints appealing only to a comparative confidence ordering.

[5]A similar claim is sometimes made about other attitudes, like *belief* or *acceptance* (e.g. Levi, 1967; Hawthorne et al., 2016). While we are sympathetic to this proposal (§4.1), two points: (1) the extension to such further attitudes is separable from our core theory, which is about guesses; and (2) while improbable believing (accepting) is controversial, it should be uncontroversial that you can *guess* something with probability below 0.5, as our cases illustrate.

univocally for the latter (the proposition that you guess).[6]

Next proposal: a natural way to account for Improbable Guessing and Question Sensitivity says that, given a question Q, your answer should be the complete answer you have highest credence in. This theory predicts that the only acceptable answer is 'Yale', so it rules out all the bad responses in (2). But this overgeneralizes: it predicts that only complete answers are permissible guesses; yet a range of partial answers (that is, unions of complete answers) are permissible. In fact, it looks like for any number of complete answers, there is a permissible answer which comprises the union of that number of answers: you can give a one-cell answer ('Yale'), a two-cell answer ('Yale or Harvard'), a three-cell answer ('Yale, Harvard, or Stanford'), or a four-cell answer ('One of those four'). This leads to our next generalization, from Holguín 2020:

> **Optionality**: Given any question Q, for any $k : 1 \leq k \leq |Q|$, it's permissible for your guess about Q to be the union of exactly k cells of Q.

How might we capture Optionality? A natural proposal is that guessers may guess any answer that is *likely enough*, i.e. that is more likely than some threshold set by the context. This would generalize the Lockean Thesis, which says that you should believe any proposition that you have high enough credence in (Foley, 1992; Sturgeon, 2008; Leitgeb, 2014; Dorst, 2019). This theory accounts for Optionality—provided the relevant threshold can be sufficiently low—but it cannot explain why all the answers in (2) are *im*permissible. Since 'Yale' is permissible, the threshold must be below 38%, but that would incorrectly predict that answers like 'Yale or Stanford', or 'Harvard or Stanford', which are both more than 38% likely, are permissible.[7]

What answers *are* permissible, then? The following constraint—also from Holguín (2020)—accounts for the patterns above:

> **Filtering**: A guess about Q is permissible only if it is *filtered*: if it includes a complete answer q, it must include all complete answers that are more probable than q.
> Precisely: p is filtered iff for any $q, q' \in Q$: if $P(q') > P(q)$ and $q \subseteq p$, then $q' \subseteq p$.

In other words, your guess can't include a complete answer q while excluding a strictly[8] more likely complete answer q'.

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[6]If you are first asked 'Where do you think will Latif go?', and your guess is 'Yale', and then you are immediately asked, 'So, do you think he'll go to Yale?' it is very natural to say 'yes'. This is presumably explained by some 'stickiness' in the contextual resolution of the question under discussion: the first QUD 'Where will Latif go?' may remain salient even if the new question, 'Will Latif go to Yale?', is explicitly asked.

[7]*Sophisticated* Lockeans might make the thresholds proposition-sensitive (Easwaran, 2016; Dorst, 2019)—but then they'd need a story for what controls these thresholds.

[8]An alternative formulation of Filtering would say that so long as q' is *at least* as probable a q, it must also be included. Which version you'll like depends on what you think about Optionality in the case of ties. We think that if we're about to toss a fair coin, it's permissible to guess that it'll land heads—despite the fact that tails is equally likely—so we endorse Optionality even in this case, and thus use the strict-inequality version of Filtering. The weak-inequality version could be derived from our theory below if we imposed the constraint that your guess must *uniquely* maximize expected answer-value, in which case we would validate Optionality only when there are no ties. These choices don't matter for our central points.

Why 'Filtering'? Imagine a filter through which the complete answers are strained. The 'size' of a complete answer corresponds to its probability. Whether the mesh of a filter lets such an answer through depends only its size. A guess about Q is *filtered* iff, for some such mesh (some threshold of probability), the answer is the union of the complete answers that survive this filtering process (that are more likely than that threshold).

Filtering rules out the answers in (2-a)–(2-h) above: each of those answers is non-filtered. For instance, *Harvard or NYU* is non-filtered because it includes *NYU* as a subset, but does not include every complete answer which is more likely than *NYU*—it is missing both *Yale* and *Stanford*. Likewise, *Harvard* is non-filtered because it includes *Harvard* as a subset, but does not include the more likely complete answer *Yale*.

Optionality and Filtering together predict the admissibility of the answers in (1), together with the inadmissibility of the answers (2-a)–(2-h). The latter are all non-filtered; the former are all filtered. In fact, for each k between 1 and 4 (the size of the QUD, $|Q|$), there is exactly one filtered answer which is the union of k complete answers, and these are the answers in (1). In general—apart from cases of ties in probability among complete answers—for any k between 1 and $|Q|$, there will be exactly one filtered answer to a question Q which is the union of k complete answers.

What about answers like (2-i)–(2-j), e.g. 'Yale, and it's cold in London today' or 'Yale, or he has a birthmark on his left toe'? Intuitively, such responses include *irrelevant* material. In particular, they crosscut complete answers: (2-i) and (2-j) cannot be derived as a union of complete answers to the QUD. In general:

> **Fit**: If a guess crosscuts a complete answer, it's impermissible.
> Precisely: p is a permissible guess only there are $q_1, ..., q_k \in Q$ such that $p = q_1 \cup ... \cup q_k$.

Though Fit is familiar from the literature on pragmatics (§4.2), this constraint applies just as much to the cognitive act of guessing: if you formulate a guess, to yourself, about where Latif will go, it's bizarre for your guess to be (2-i) or (2-j).

An important complication: some apparent violations of Fit can be felicitous enough—'Latif will go to Yale, and I'm sure he'll love it!'; 'Latif will go to Yale or Harvard, and if he goes to Yale, he'll learn a lot'.[9] Nonetheless, other violations seem robustly bad, like those in (2-i)–(2-j). The standard explanation of the felicity of the former answers is that it is easy to accommodate more fine-grained questions that are in a similar vein to the QUD (e.g. 'Where will Latif go, and will he like it?'); relative to the finer-grained question, the answer satisfies Fit. In contrast, (2-i)–(2-j) are infelicitous because the finer-grained question which would need to be accommodated to satisfy Fit (e.g. 'Where will Latif go, and what is the weather in London?') seem too irrelevant to the original QUD. Given the robustness of many intuitions about Fit, we're inclined to think this is the best way to make sense of the overall picture here. [10]

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[9]Another class of responses that violate Fit are ones like 'I don't want to guess'. As Diego Feinmann has pointed out to us, these feel like ways *opting out* of answering the question (cf. Dorst, 2014); from the point of view of the cognitive attitude, rather than speech act, these do not seem like guesses at all.

[10]But see Feinmann 2020 for a different, probabilistic take.

We'll draw out a couple more constraints on guesses in §3 below, but for now we'll focus on Improbable Guessing, Question-Sensitivity, Optionality, Filtering, and Fit. To strengthen the case for these constraints, let's briefly consider another example, drawn from recent experience. Consider a moment in the 2020 Democratic presidential primary when the only remaining plausible candidates were Biden, Sanders, Warren, Bloomberg, and Buttigieg. Following FiveThirtyEight's model, your credences in who'll win a plurality of delegates are as follows:

Biden	Sanders	Warren	Bloomberg	Buttigieg
35%	28%	18%	13%	6%

What's your guess about who will win? It seems that all and only the following guesses are permissible:

(3)   a.   Biden. ✓
        b.   Biden or Sanders. ✓
        c.   Biden, Sanders, or Warren. ✓
        d.   Biden, Sanders, Warren, or Bloomberg ✓
        e.   Biden, Sanders, Warren, Bloomberg, or Buttigieg ✓

Other guesses, like the following, are not:

(4)   a.   Sanders. ✗
        b.   Warren. ✗
        c.   Biden or Warren. ✗
        d.   Biden or Bloomberg. ✗
        e.   Biden or Sanders or Bloomberg. ✗
        f.   Sanders or Warren or Bloomberg. ✗
        g.   Biden, and it will rain tomorrow. ✗

Once again, we see evidence of Improbable Guessing and Optionality in the range of permissible answers in (3). Filtering accounts for the infelicity of the answers in (4-a)–(4-f) (since each of these contains some complete answer as a part while leaving out some strictly more likely complete answer), and Fit accounts for the weirdness of (4-g).

Based on cases like this, we think the principles above hold robustly (for more examples, see Holguín 2020).

These observations—Improbable Guessing, Question Sensitivity, Optionality, Filtering, and Fit—bring out what guesses people tend to make, revealing surprising yet systematic patterns. The most obvious accounts fail to predict these patterns—and it's by no means obvious how to explain them.

3 How We Guess

We propose to explain these patterns by giving a model of *how* we guess. The basic idea behind our approach is a familiar thought from James 1897. A good guess about a question *Q* is a good picture of how things stand, *Q*-wise. In trying to form such a picture, there's an inevitable tradeoff between two goals. On the one hand, we want our picture to be *accurate*—we want our guess to be true. But being true often doesn't cut it. After all, one way to guarantee that your guess is true is to say very little: when asked 'Where do you think Latif will go?', 'Somewhere' is sure to be true, but is unhelpful. We also want to take a stand on things—to have an *informative* guess, one that helpfully narrows down the space of alternatives we're considering. These two goals compete. Typically, the more informative your guess is ('He'll go to Yale'), the less likely it is to be true; the more likely it is to be true ('He'll go somewhere'), the less informative it is. On this Jamesian approach, trying to form a picture of the world involves trading off informativity (believing substantive truths) and accuracy (avoiding error). In this section, we'll develop this idea of an *accuracy-informativity tradeoff* to give an account of guessing—one which we think is intuitively plausible, and which accounts for all the patterns brought out in the last section.

Our model of guessing is intended to be a computational level explanation in the sense of Marr 1982. Our question is, *What problem is a (rational) mind solving when it forms a guess?* and our answer is, *How to optimally trade off accuracy and informativity.* As always, this style of explanation is neutral on the precise algorithms through which the mind solves this problem, as well as on the question of whether the processes involved will be consciously accessible or not.

3.1 Jamesian guessing

Our approach will be to view guessing as a kind of epistemic decision problem. First, we'll try to say what makes a guess *objectively valuable* (foreshadowing: true guesses are better than false ones; and among true guesses, the more informative the better). Then we'll propose that people aim to maximize this objective value by choosing a guess with the highest *expected* value (given their credences).

We build on a substantial literature in epistemic utility theory,[11] but in particular on Levi 1967, which is the closest precedent for our approach. In particular, Levi's is the only approach we know of which, like ours, measures informativity in terms of ruling out cells in

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[11]The epistemic utility theory literature uses decision-theoretic tools to explain the constraints of epistemic rationality (e.g., Joyce 1998; Pettigrew 2016a; Horowitz 2018; Schoenfield 2019b). Though most of this literature focuses on degrees of belief, some of it focuses on qualitative states of 'outright belief' that are similar to guesses—see Hempel 1962; Levi 1967; Maher 1993; Easwaran 2016; Pettigrew 2016b; Dorst 2019. As mentioned in footnote 1, the work in Sliwa and Horowitz 2015; Horowitz 2017; Builes et al. 2020 is closely related, but puts guessing to a rather different use—arguing that the relationship between credences and guessing can help obviate the need for the tools of epistemic utility theory. By contrast, we'll argue that it is precisely these tools that are needed to explain the relationship between credences and guessing. A referee points out that parallels to Filtering may arise in other domains; the extent to which that's true, and to which we can account for it in a way continuous with our approach here, is an interesting question that deserves further exploration.

a relevant partition (we compare his view to ours in footnote 13, after we lay out our view).

So suppose you are trying to guess the answer to a question Q—say, 'Where do you think Latif will go?'. Your guess could be any proposition: 'Yale'; 'Stanford'; 'I like lentils'; etc. How good your guess is depends on how well it answers the question. That in turn depends on whether your guess is true or false, and how valuable that kind of guess would be if it were true and if it were false. Schematically, let $V_Q(p)$ be a function which yields the *answer-value* of choosing p as your guess about Q, depending on whether p is true or false. Whenever you're unsure whether p is true, you'll be unsure how much answer-value it has. Nevertheless, you can use your credences in the various possibilities to form an *estimate* about how much answer-value it has—p's *expected answer-value*, written $E_Q(p)$. Precisely, we'll assume we can model any permissible measure of answer-value with a real-valued function $V_Q(p)$, such that if p is true, guessing it yields answer-value $V_Q(p) = V_Q^+(p)$, and if it's false, guessing it yields answer-value $V_Q(p) = V_Q^-(p)$. Using our guesser's (probabilistic) credence function, we'll assume that expectations are defined in the standard way (assuming act-state independence, for simplicity). Thus the expected answer-value of p is a weighted average of the various possible values $V_Q(p)$ might take, with weights determined to how likely they are to obtain (where $\bar{p} = \neg p$):

$$E_Q(p) \; := \; P(p) \cdot V_Q^+(p) \; + \; P(\bar{p}) \cdot V_Q^-(p)$$

The core of our theory says that you must make a guess that maximizes this quantity, relative to some epistemically permissible measure of answer-value:

> **Guessing as Maximizing:** A guess is epistemically permissible given a question iff it has maximal expected answer-value relative to that question, for some permissible measure of answer-value.

The crucial question: Which measures of answer-value are epistemically permissible?

True guesses are better than false ones, so any permissible V_Q must be *truth-directed*:

> V_Q is **truth-directed** iff any true guess has higher answer-value than any false guess. Precisely: for all p, r: $V_Q^+(p) > V_Q^-(r)$.

Truth matters. But—on our Jamesian picture—truth isn't *all* that matters. Answer-value also depends on informativity. The informativity of a guess depends on what question it's answering: 'Latif will go to Yale' is an informative answer to 'Where will Latif go?'—but an uninformative answer to 'What are we having for dinner tonight?'. So V_Q^+ and V_Q^- should be sensitive to the informativity of the answer, which, in turn, depends on Q.

In fact, holding fixed a guess's truth-value, Q is arguably the *only* thing V_Q should be sensitive to. Suppose you ask who'll win the election. In *some* sense, 'Latif will go to Yale and my grandpa was bald' is more informative than 'Latif will go to Yale'—but this doesn't seem to be the sense of informativity that governs guesses. After all, if you wanted to know about my grandpa, you would've asked! Similarly, suppose that you guess that Latif will go to NYU. In *some* sense, this is more informative than 'Latif will go to Yale', since learning that it's true would lead to a bigger change in your probabilities than would learning that he'll go

to Yale. But there is another, equally intuitive sense in which this is not a more informative guess than 'Yale': both guesses are maximally informative about the question asked.[12]

Thus a natural idea is that how informative a guess is with respect to Q depends on only the number of answers to Q it rules out. Precisely, given a question Q and an answer p, let the **informativity** of p relative to Q be the proportion of complete answers to Q that p rules out: $Q_p := \frac{|\{q \in Q: \ p \cap q = \emptyset\}|}{|Q|}$. For example, if Q is 'Where will Latif go to law school?', then $Q_{Yale} = Q_{Stanford} = Q_{Harvard} = Q_{NYU} = \frac{3}{4}, Q_{Yale \ or \ Harvard} = \frac{1}{2}$, etc. If Q is 'Who will win a plurality of delegates in the Democratic primary?', then $Q_{Biden} = Q_{Sanders} = Q_{Warren} = Q_{Bloomberg} = \frac{4}{5}$, $Q_{Biden \ or \ Sanders} = Q_{Sanders \ or \ Warren} = \frac{3}{5}$, and so on. Given this, our second constraint is that, given the truth-value of p, $V_Q(p)$ should then be fully determined by p's informativity:

> V_Q is **question-based** iff for all p: $V_Q(p)$ is fully determined by p's informativity together with its truth-value.
> Precisely: for all p, r, if $Q_p = Q_r$, then $V_Q^+(p) = V_Q^+(r)$ and $V_Q^-(p) = V_Q^-(r)$.

Our first main addition to the Guessing as Maximizing account is this: *a measure of answer-value is (epistemically) permissible only if it is truth-directed and question-based.* Why? Truth-directedness is straightforward, but the requirement that V_Q be question-based is somewhat more surprising. Apart from the intuitions just elicited, our primary argument for it is an inference to the best explanation: as we'll see in §3.2, any question-based measure will offer a simple explanation of two of our most distinctive constraints—Fit and Filtering—which is not available to non-question-based measures.

So suppose V_Q is truth-directed and question-based. Although this will establish Fit and Filtering, it doesn't yet say anything about our observed *permissions*—that sometimes permissible guesses can be less likely than not (Improbable Guessing), and that a variety of guesses are always permissible (Optionality). How can we account for these observations? A very permissive approach would be to say that *any* truth-directed, question-based measure of answer-value is permissible. It follows from our results below that this theory would yield all the observations about guessing mentioned in §2, so it is worth flagging this position as a natural, minimal version of our approach.

But we'll do more: we'll motivate a particular subclass of truth-directed, question-based measures as the epistemically permissible ones, which we call **Jamesian measures**. We do

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[12]More generally, the natural alternative to question-based measures are *credal*-based ones, which measure accuracy by how well your guess promotes some desirable quality in your interlocutor's credence function. There are many such measures—(Shannon) information (Shannon, 1948; van Rooy, 2004), probability gain (Baron, 1985), accuracy (Oddie, 1997; Pérez Carballo, 2018), etc.—but they are not well-suited for our purposes. They all involve quantifying how much *learning* (i.e. conditioning on) the answer to a question would improve a credence function. Yet our context involves *guessing* the answer—not learning it—and in general people shouldn't update their credence function by conditioning on guesses. In fact, in cases where the probabilities are common knowledge, often you shouldn't change your credences *at all* in response to a guess. If we all know this coin is 60% biased toward heads, then we know that you won't change your credences when we guess how it'll land. That means that if what we care about is the impact of our guess on your *credences*, then *any* guess is permissible (since none will have any effect) in this case. But that's wrong—'heads' is a permissible guess; 'tails' is not. This, in short, is why standard credal-based measures won't do for our purposes.

this for four reasons. First, giving a more specific class of measures helps bring out the basic idea behind these constraints in a more concrete way. Second, it is important to see that Improbable Guessing and Optionality will fall out of our approach even if we are not nearly so permissive about what measures are allowed (since these encode *permissions* rather than obligations, restricting the set of potential measures has the potential to invalidate them; we'll show that the class of Jamesian measures still validates them). Third, our preferred class of measures will yield an explanation of a further generalization about guessing that we'll bring out in §3.3. Finally, our account of the conjunction fallacy in §4.3 will rely on a systematic theory of how comparisons of expected answer-value will lead people to *rank* non-optimal guesses. We need a more specific account of answer-value in order to make concrete, empirically testable predictions about such rankings.[13]

However, the details of the model we will give are separable from many of the broader proposals of this paper. There are a variety of other sub-classes of the truth-directed, question-based measures that would predict (most of) our constraints on guessing—though none, we think, work quite as elegantly as the Jamesian ones we will explore presently. [14] There are obviously many different ways to work out the details of a model like this, and we hope that our discussion will invite the development and comparison of alternatives.

We also want to emphasize that the goal of these models is not to predict what particular people will guess in particular situations. Rather, the goal is to elucidate the structural

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[13] This puts us in a position to sketch the two core differences between our account of guessing and Levi's (1967) theory of belief—which also analyzes answer-value in terms of (truly) ruling out cells of a salient partition. The first difference is that Levi does not situate his account within the linguistic practices of guessing and asserting. As a result, he does not make use of the notion of a contextually flexible QUD to generate the relevant partition, nor does he motivate his approach as an account of the constraints on guessing we've highlighted. (While his account in fact derives many of those constraints, it is inconsistent with Optionality whenever there are ties in probability: if we're about to toss a fair coin, his approach would disallow you from guessing that it'll land heads.) The second difference is that Levi's approach focuses exclusively on the question of which answers *maximize* expected answer-value, and as a result gives implausible verdicts about *rankings* of expected answer-value. His formula for the expected answer-value of p reduces to $P(p) - \frac{[p] \cdot q}{n}$, where q is a 'boldness' parameter that can take any value between 0 and 1, n is the size of the relevant partition, and $[p]$ is the number of cells of the partition consistent with p. On this measure, the expected value of contradictions will always be 0, while that of contingent claims will often be *negative*. Example: if the question is 'Will Latif go to Yale?', the expected answer-value of 'Yes' is $0.38 - \frac{1 \cdot q}{2}$, which is negative whenever $q > 0.76$. Thus Levi's approach predicts that when people rank guesses in terms of expected answer-value, they will sometimes rank 'He'll go to Yale, and he won't' as a better guess than 'He'll go to Yale'. As we'll argue in §4.3, such rankings of non-optimal guesses are crucial to offering a plausible account of the conjunction fallacy. Our account will generate rankings that predict a variety of empirical findings (§4.3), but examples like this show that Levi's account will not. We take this to show that Levi's (1985, 2004) suggestion that his specific account can explain the conjunction fallacy is incorrect—though we are obviously sympathetic to the more general idea.

[14] An interesting alternative is to shift the location of the optimizing parameter by moving to Rank Dependent Utility (RDU) theory (Quiggin, 1982; Buchak, 2013). RDU introduces a risk function r that is used to modify the weight of a given level of probability. Assuming that $V_Q^+(p) > V_Q^-(p)$, it sets your estimate for the value of guessing p to be $r(P(p)) \cdot V_Q^+(p) + (1 - r(P(p))) \cdot V_Q^-(p)$. We can let variations of r play roughly the role that J plays in our model: when r is convex, you're risk-seeking and care more about informative answers regardless of their low probability; when r is concave, you're risk-averse and care more about making sure your guess is true. If we treat any r function as epistemically permissible, we can use RDU to validate all our constraints and permissions on guessing with a simple question-based measure like $V_Q^+(p) = c + Q_p$, and $V_Q^-(p) = -b$ for positive constants c, b.

features that the practice of guessing is sensitive to, and thus explain why guesses rationally should—and thus in fact tend to—meet constraints like Filtering, Fit, Optionality, and so on. Viewing guesses as attempts to optimize a tradeoff between accuracy and informativity (in the sense above) sheds light on what might otherwise look like arbitrary patterns.

Now to Jamesian measures. We'll give away the ending first, and then explain the reasoning behind it for those interested. Jamesian measures are those for which there is some $J \geq 1$ such that, for all p, $V_Q^+(p) = J^{Q_p}$; and $V_Q^-(p) = 0$. The parameter J represents the guesser's measure of the value of informativity, while Q_p is, again, the proportion of the QUD ruled out by p. This yields the following simple formula for expected answer-value:

$$\textbf{Jamesian Expected Answer-Value:} \quad \begin{aligned} E_Q^J(p) &= P(p){\cdot}J^{Q_p} + P(\bar{p}){\cdot}0 \\ &= P(p){\cdot}J^{Q_p} \end{aligned}$$

Thus the expected answer-value of a guess is determined by two terms: its probability of being true—$P(p)$—and its answer-value-if-true—J^{Q_p}. When J is small, changing the informativity Q_p of your guess will only change J^{Q_p} a small amount—in the limiting case where $J = 1$, it won't change it at all, and the way to maximize expected answer-value is to pick a maximally probable (and so minimally informative) answer. Conversely, as J gets large, changing the informativity of your guess will change J^{Q_p} a large amount, and therefore the way to maximize expected answer-value is to pick an informative guess—in the limit, as $J \to \infty$, the way to do so is to pick a *maximally* informative (filtered) guess, regardless of how low its probability is.

Our formula $P(p) \cdot J^{Q_p}$ thus captures the Jamesian tradeoff between accuracy and informativity. Picking an uninformative but very probable answer ('He'll go somewhere') makes the right term (J^{Q_p}) small but the left term ($P(p)$) large; picking an informative but improbable answer ('He'll go to Yale') makes the right term large but the left term small. Making a good guess requires optimizing the tradeoff between these terms, in light of the probabilities P and your value of informativity J.

We can see this tradeoff graphically in the Latif case by plotting the expected answer-value of 'Yale' ('Y'), 'Yale or Harvard' ('Y or H'), etc. for various values of J (Figure 1). As J increases, the optimal tradeoff between accuracy and informativity shifts towards informativity: when $1.67 > J \geq 1$, the best guess is 'Yale, Harvard, Stanford, or NYU'; when $2.80 > J > 1.67$, it's 'Yale, Harvard, or Stanford'; when $10.25 > J > 2.80$, it's 'Yale or Harvard'; and when $J > 10.25$, it's 'Yale'.

This sums up our preferred way of measuring answer value. The rest of this subsection will explain the reasoning behind Jamesian measures; readers who are eager to see the applications of our model may wish to skip to the next subsection.

Begin with false guesses. How valuable is a false guess? Suppose we're asked where we think Latif will go; one of us says, 'Yale'; the other says 'Yale or Harvard'. Turns out, Latif goes to Stanford. Which of us was a more valuable guess, objectively speaking? Intuitively: neither. Both were maximally far from the truth—since both ruled out the true complete
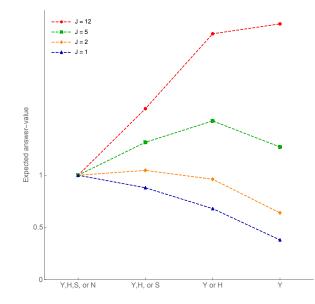
Figure 1: Expected answer-value of law-school guesses, varying the value of informativity (J)

answer to the question. This intuition motivates setting $V_Q^-(p)$ equal to some constant low value, regardless of the informativity of p; we set this value to $V_Q^-(p) = 0$.[15] (This is not to say that all false guesses are the same: some false guesses will have higher *expected* answer-value than others, and thus will be better from that, subjective, perspective.)

What about true guesses? Since V_Q is question-based, $V_Q^+(p)$ will be a determined by the proportion of cells that p rules out, i.e. Q_p. How it's determined should depend on how much you value informativity. If you really value having a complete answer to Q, $V_Q^+(p)$ will increase very quickly with p's informativity; but if you don't particularly value such a precise answer, $V_Q^+(p)$ will increase much more slowly. By truth-directedness, any true answer—even an uninformative one—must have some minimal positive answer-value $t > 0$. This is the value of 'mere truth'. By what factor does guessing a *maximally* informative (true) answer improve on the value t of mere truth? That depends on how you value informativity. Let J be a real-valued parameter that measures this *J*amesian value of (maximal) informativity, so that a maximally informative (true) answer yields value $J \cdot t$. Since a maximally informative (true) answer is at least as valuable as an uninformative one, $J \geq 1$.

How, exactly, should $V_Q^+(p)$ vary as both informativity (Q_p) and the value of informativity (J) change? Note that, ranging over questions Q, informativity has a minimum possible value of 0 (ruling out no complete answers), and has a least upper bound of 1, since a complete answer to Q has informativity $\frac{|Q|-1}{|Q|}$, which goes to 1 as $|Q| \to \infty$. If you don't care at all about informativity (if $J = 1$), then, no matter what Q_p is, $V_Q^+(p)$ should be the value of mere truth, i.e. t. Similarly, no matter how much you value informativity, if p is completely uninformative, then $V_Q^+(p)$ should again be t. Finally, as p becomes *maximally* informative, the value of truly

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[15]In some instances intuitions about verisimilitude may make some false guesses seem closer to the truth than others (Popper, 1963; Oddie, 2019; Schoenfield, 2019a); likewise, intuitions about partial truth (Yablo, 2014); but we'll set such issues aside for our purposes.

guessing p should tend towards multiplying the value of mere truth by a factor of J: as $Q_p \to 1$, we have $V_Q^+(p) \to J \cdot t$. A natural way to capture all these constraints is to raise J to the power of the (true) guess's informativity, and use the resulting value to scale the value of mere truth: $V_Q^+(p) = (J^{Q_p}) \cdot t$. When p has minimal informativity ($Q_p = 0$), $V_Q^+(p) = J^0 \cdot t = t$; likewise, when you don't care about informativity ($J = 1$), then $V_Q^+(p) = 1^{Q_p} \cdot t = t$. And when informativity approaches its maximal value ($Q_p \to 1$), answer-value scales the value of mere truth by a factor of J: $V_Q^+(p) \to J^1 \cdot t = J \cdot t$. Generally: when J is small, answer-value rises slowly with increases in informativity; when J is large, it rises steeply; and as J gets arbitrarily large, increasing informativity dominates all other considerations (Figure 2).[16]
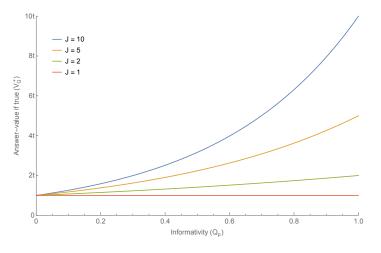

Figure 2: How answer-value varies with J and Q_p

Summing up this discussion, we define our class of Jamesian measures of answer-value:

V_Q is **Jamesian** iff, for some $t > 0$ and $J \geq 1$:

$$
V_Q = \begin{cases} V_Q^+(p) = J^{Q_p} \cdot t & \text{if } p \text{ is true} \\ V_Q^-(p) = 0 & \text{if } p \text{ is false} \end{cases}
$$

Given a Jamesian measure, the expected answer-value of a guess is:

$$
\begin{aligned}
E_Q^J(p) &= P(p) \cdot V_Q^+(p) \; + \; P(\bar{p}) \cdot V_Q^-(p) \\
&= P(p) \cdot (J^{Q_p} \cdot t) \; + \; P(\bar{p}) \cdot 0 \\
&= P(p) \cdot (J^{Q_p} \cdot t)
\end{aligned}
$$

Notably, comparisons of expected answer-value are insensitive to the value t of 'mere truth': for any p, r, $E_Q(p) > E_Q(r)$ iff $P(p) \cdot J^{Q_p} \cdot t > P(r) \cdot J^{Q_r} \cdot t$ iff $P(p) \cdot J^{Q_p} > P(r) \cdot J^{Q_r}$. So without loss of generality we can assume that $t = 1$, and simply say $V_Q^+(p) = J^{Q_p}$. Thus we arrive at the formula above for Jamesian expected answer value: $E_Q^J(p) = P(p) \cdot J^{Q_p}$

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[16]This, intuitively, is why we raise J to the power of Q_p rather than (say) multiplying them: we want high values of J to allow for small increases in informativity to matter more and more.

3.2 Deriving our constraints

We will thus adopt a theory on which p is epistemically permissible iff it has maximal expected answer-value relative to your credences and some Jamesian measure of answer-value. In this section, we will show that the constraints and permissions observed in §2 all follow from this theory.[17]

Start with Fit—the claim that a guess is impermissible if it crosscuts a complete answer, such as: 'I think Latif will go to Yale, and it's cold in London today'. This follows easily. If p violates Fit, then there is some complete answer that it overlaps but does not fully include: there is a $q \in Q$ such that $q \cap p \neq \emptyset$ but $q \not\subseteq p$. Compare p to $p \cup q$. The two rule out exactly the same number of complete answers, and thus by question-basing, have the same answer-value if true and if false. But $p \cup q$ is more likely to be true than p (since we are assuming our probability measure is regular over the context set). Therefore the expected answer-value of $p \cup q$ is strictly higher than that of p. Any non-Fit answer will thus have lower expected answer-value than some alternative Fit answer, and thus will never be a permissible guess. (So, e.g., the expected answer-value of 'Yale' will always be higher than 'Yale, and it's cold in London today'.)[18]

Next, Filtering: a guess is permissible only if it is filtered—if it includes a complete answer q, it must include all complete answers that are more probable than q. This follows from any truth-directed, question-based measure because swapping out the less-probable complete answer for the more-probable one maintains the same level of informativity, but increases the probability of your guess being true. (So a non-Filtered answer like 'Yale or Stanford' will always have lower expected answer-value than a filtered answer with the same size—in this case, 'Yale or Harvard'.)

In fact, Filtering is a special case of a more general constraint which is worth bringing out. If you are asked to *rank* the complete answers to a question, your ranking should follow the probabilities: 'Yale' is a better guess than 'Harvard', which is better than 'Stanford', etc. Likewise if you are asked to rank the two-cell answers: 'Yale or Harvard' is better than 'Yale or Stanford', and so on. Generally:

> **Filtered Rankings**: Equally informative answers should be ranked by probability. Precisely: if $Q_p = Q_r$, then $E_Q(p) > E_Q(r)$ iff $P(p) > P(r)$.

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[17]To be clear, Filtering and Fit will follow for any truth-directed and question-based measures. We'll show that Improbable Guessing, Question Sensitivity, and Optionality also hold provided that all and only Jamesian measures are epistemically permissible—from which it follows that, if we treat a strictly larger class of measures as epistemically permissible, Optionality will still hold.

[18]A referee asks whether we *should* be trying to give a normative explanation of Fit—why not instead say that, since guesses are always relativized to a question Q, the *only options* for a guess about Q are the propositions that are unions of complete answers to Q? We don't have a strong opinion on this question, though we do think the metaphysical constraint is not *obviously* correct. Just as we all know people who are over-specific in their verbal answers ('Yes I'm having a good day; I had cereal for breakfast with plenty of milk and...'), or who obsessively return to a fixed topic ('Yes I'm good, but don't forget to buy my new book!'), it seems we can imagine people who similarly give unfit answers inside their own heads (think Cato the Elder: 'Latif will go to Yale, and Carthage must be destroyed!'). Since this explanation immediately falls out of the same story that accounts for Filtering, it seems to a virtue of our theory that it also explains Fit without further assumptions.

This follows from our model by the same reasoning.[19]

Consider next Improbable Guessing: sometimes it's permissible to guess p even if it's less likely than not to obtain. We've already seen that in our original Latif case, when $J \geq 10.25$, 'Yale' is the optimal guess—despite the fact that $P(\textit{Yale}) = 0.38$. The intuition is that high informativity can outweigh low probability, especially as J grows large.

The next observation was Question Sensitivity: whether a guess is permissible depends on what question it's addressing. This follows because Jamesian measures are based on a guess p's informativity Q_p, which in turn is determined by the question Q. For example, relative to the four-cell question, 'Where will Latif go?' (complete answers: {*Yale, Harvard, Stanford, NYU*}), 'Yale' has informativity $3/4$. But relative to the two-cell question 'Will Latif go to Yale?' (complete answers: {*Yale*, $\overline{\textit{Yale}}$}), it has informativity $1/2$. Thus 'Yale' is a permissible answer to the former (where it's filtered), but not the latter (because it's not filtered with respect to {*Yale*, $\overline{\textit{Yale}}$}). Thus we capture Question Sensitivity in an intuitive way (since 'No' is a good answer to 'Do you think Latif will go to Yale?', but 'Not Yale' is a weird answer to 'Where do you think Latif go?').

Our final observation was Optionality: given a question Q with $|Q|$ possible complete answers, for any $1 \leq k \leq |Q|$, it's permissible to give an answer that is a union of k-cells. In particular, the filtered answer that's a union of the k most-probable cells is always permissible. Thus 'Yale', 'Yale or Harvard', 'Yale or Harvard or Stanford', and 'One of those four' are all permissible guesses. As we've seen above (Figure 1), each of these maximizes expected answer-value relative to certain values of J. The proof of Optionality requires some footwork, so we leave it to an appendix, but the basic idea is straightforward: When J is low, being informative provides little additional value, so the best guess is an uninformative (but definitely true) guess (that is, $\bigcup Q$). As J grows, being more informative gradually matters more and more such that—no matter your credences—you eventually start preferring an answer comprising the union of $(|Q|-1)$-cells, then $(|Q|-2)$-cells, and so on until you prefer a 1-cell answer. We can thus rationalize guesses of different levels of informativity by ascribing to guessers different J-values: that is, different weights on informativity.

3.3 Setting J-values

This discussion raises a natural question: how are J-values set? And how do we know what subjects' J-values are?

We won't offer a definitive answer to either question, but we'll try to illustrate how the structure of our model allows us to say interesting things about this issue, and in so doing explain a further generalization about guessing.

First, it's worth re-emphasizing that we're focusing on the *cognitive attitude* of having something as your best guess about a question. On our account, this attitude is determined entirely by your J-values and credences, given a question. There can of course be all sorts

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[19]This shows that our approach yields Horowitz's (2017) Lockean-like relationship between credences and guessing in a special case: *when p and r are equally informative*, then you should guess p over r if $P(p) > P(r)$. Thanks to Brian Hedden for pointing out this generalization of Filtering.

of other (e.g. practical) values that affect the *speech act* of guessing. If we threaten to punch you unless you correctly guess the exact number of jellybeans in a jar, the rational response is obviously to pick some exact number—say, '457'—even if you have very little idea. Likewise: if we threaten to punch you if you make a mistake, then it makes sense to refrain from guessing—even if in fact you do have an idea. But this doesn't imply that your J-value is high (low), for in these cases the *speech act* of guessing plausibly comes apart from the corresponding cognitive attitude. So J-values aren't straightforwardly sensitive to practical stakes.

Instead, we take your J-value to be determined by your mental state in broadly the way credences, utilities, and (on some views) risk profiles are. But unlike standard interpretations of these states, we take J-values to be very flexible—able to adjust as you switch amongst questions, or address the same question in different contexts. Our discussions later in the paper about the role of guessing (§4.1, §4.2, and §5) will give more of a picture for *why* J-values would need to be flexible like this. But we take it to be clear from our examples that they are: given the credences above, it's perfectly coherent to reply to 'Where do you think Latif will go?' with 'Yale, I think', and yet (perhaps in a different context) reply to 'Who do you think will win the primary?' with 'Biden, Sanders, or Warren'—despite the fact that these two guesses will require different J-values. In fact, this flexibility in J-values helps explain why, when people make guesses, their statements are often peppered with markers that flag various degrees of strength: 'He'll (definitely) go to Yale' vs. 'Yale, surely'; vs. 'I think Yale' vs. 'I'd guess Yale—but it's hard to say'. We think these markers are ways of flagging what J-value you're using, and therefore to what degree your guess is based on confidence in its truth vs. a desire to be informative.

There are many questions about J-values that we don't want to take a stand on: for instance, whether they are more like a subject's priors or more like a subject's preferences—and, if the latter, whether J-values are under the subject's voluntary control in some sense. We suspect that you should answer these questions in the same way for guessing and for risk aversion in general. Nonetheless, we have a bit more to say about how J-values are set.

Consider the following. Although we think Optionality is true—any filtered guess is permissible—there are certain circumstances in which certain filtered guesses seem odd. In particular, as the probabilities of the various complete answers 'cluster' together more tightly, it becomes increasingly strange for your guess to crosscut these clusters—to include some but not all of the cells in a cluster. To see this, consider some variations on our law-school case where your credences about where Latif will go differ:

Scenario:	Yale	Harvard	Stanford	NYU
Original:	38%	30%	20%	12%
Close:	40%	35%	15%	10%
Near-Tie:	40%	39%	11%	10%
Tie:	40%	40%	10%	10%

Consider the guess 'He'll go to Yale, Harvard, or Stanford'. This guess seems fine in the original case, a bit odd in the Close case, quite odd in the Near-Tie case, and pretty bizarre

in the Tie case. We summarize this trend with the following generalization:

> **Clustering:** People tend to avoid making guesses that crosscut clusters of complete answers with similar probabilities.

We state Clustering as a *tendency* because we don't think it imposes a hard constraint on permissible guesses. Two reasons. First, some guesses seem permissible even when they crosscut cells with the same probabilities: in response to 'How do you think this fair coin will land?', it seems perfectly permissible to guess 'Heads'. Even in this Tie case, it doesn't seem outright impermissible to guess 'Yale, Harvard, or Stanford'. Second, Clustering needs to be a 'soft' constraint because its effects are *graded*: 'Yale, Harvard, or Stanford' gets progressively stranger as we move from the Original to the Close to the Near-Tie to the Tie case.

Our proposal is that Clustering reveals how people tend to select J-values—in particular, they tend to select a J-value that makes their guess *distinctive*: one that makes its expected answer-value not only maximal, but distinctively higher than that of alternative guesses. Note, for instance, that in the Tie case, 'Yale, Harvard, or Stanford' will always have the exact same expected value as 'Yale, Harvard, or NYU'—thus even if we pick a J-value that leads both of these to have maximal expected answer-value, neither can ever be *uniquely* maximal. Thus we think that guess is odd because there can be nothing to uniquely recommend it. (Similarly in the Near-Tie case, except that instead of the expected answer-values of 'Yale, Harvard, or Stanford' and 'Yale, Harvard, or NYU' being the *exact same*, they are merely very close— meaning neither can be very distinctive.) In contrast, for many values of J, in the Tie and Near-Tie cases, 'Yale or Harvard' has an expected answer-value that is substantially higher than any other potential answers. Thus our hypothesis is that Clustered guesses are natural because there's a way of valuing informativity that makes them distinctively best.

This notion of distinctiveness can be made precise as follows. Given credences P and a question Q, let the *J-distinctiveness* of a guess p, D_J^p, be the ratio of its expected answer-value to the highest expected answer-value of any other Fit guess (holding fixed J). That is, where F_p is the set of Fit answers to Q other than p, we have:

$$D_J^p := \frac{E_Q^J(p)}{\max\limits_{r \in F_p} \left(E_Q^J(r) \right)}.$$

And define the *distinctiveness* (period) of p, D^p, to be the maximal J-distinctiveness it can receive, for any value of J: $D^p := \sup\{D_J^p : J \geq 1\}$. So defined, we take the distinctiveness D^p of a guess to be a natural measure of its salience. Our proposal is that there is a tendency (but not an obligation) to make guesses that are salient; and thus, *inter alia*, to make guesses with high distinctiveness; and thus to use J-values that allow guesses to have high J-distinctiveness.

This explains Clustering. To see why, note the following. $D^p \geq 1$ iff p is filtered.[20] If p includes complete answer q_1, excludes q_2, and $P(q_1) = P(q_2) + \epsilon$, then p's distinctiveness is no

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[20]If p is not filtered, it can never be maximal in expectation, so $D^p < 1$. And if p is filtered, then by Optionality, there is a setting of J on which it has maximal expectation, so $D^p \geq 1$.

greater than $\frac{P(p)}{P(p)-\epsilon}$.[21] Thus if p is a filtered guess that crosscuts a cluster of equally-probable complete answers, its distinctiveness is the minimal value of 1. And when it crosscuts a cluster of *almost*-equally-probable answers, its distinctiveness will be only marginally greater than 1: ϵ will be small, so $D^p \leq \frac{P(p)}{P(p)-\epsilon} \approx 1$. Meanwhile, when the least-probable cell included in p is significantly more probable than the most-probable cell outside it (i.e. p doesn't crosscut any clusters), then D^p is substantially larger than 1. For example, in the Tie case, 'Yale or Harvard' has the highest distinctiveness of any guess—at 1.33—while that of 'Yale, Harvard, or Stanford' has the minimal distinctiveness of 1; in the Near-Tie case, the distinctiveness of the former is 1.32 while that of the latter is 1.01; in the Close case, the distinctiveness of the former is 1.25 while that of the latter is 1.04; and so on.[22] Thus a preference for distinctive guesses explains Clustering.

Upshot: although J-values are flexible, the structure of Jamesian measures offers the resources to help explain how people select them, and in so doing explains the Clustering generalization. We take this result to both clarify and bolster the case for the Jamesian Guessing-as-Maximizing account.[23]

4   When We Guess

So far we've offered an account of *what* we guess, and *how* we do so. We now turn to *when* we guess. We'll make the case that much of our cognitive lives involves trading off accuracy and

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[21]A relevant alternative to p is a guess p^* that swaps our q_2 for q_1. Since $Q_p = Q_{p^*}$, that means for any value of J, $D_J^p \leq \frac{E_Q^J(p)}{E_Q^J(p^*)} = \frac{P(p) \cdot J^{Q_p}}{P(p^*) \cdot J^{Q_p}} = \frac{P(p)}{P(p^*)} = \frac{P(p)}{P(p-q_1)+P(q_2)} = \frac{P(p)}{P(p-q_1)+P(q_1)-\epsilon} = \frac{P(p)}{P(p)-\epsilon}$.

[22]Supposing p is filtered, a rather complicated proof and calculation (which we omit) offers the following formula for D^p and for the (often unique) J such that $D_J^p = D^p$. Number the cells $q_1, ..., q_n$ of Q such that $P(q_1) \geq ... \geq P(q_n)$ (it doesn't matter how ties are ordered), and define $Q^k := q_1 \cup ... \cup q_k$. If p includes k cells, then for all J, $E_Q^J(p) = E_Q^J(Q^k)$. Let $p' := (Q^k - q_k) \cup q_{k+1}$ (where $q_{k+1} = \emptyset$ if $k+1 > n$). Then if $k = 1$ or $k = n$, $D^p = \frac{P(p)}{P(p')}$; and if $1 < k < n$, then $D^p = \min\left\{\frac{P(p)}{P(p')}, \frac{P(p)}{\sqrt{P(Q^{k-1})P(Q^{k+1})}}\right\}$. The relevant J-value(s) that maximizes D_J^p will be 1 if $k = n$, any high-enough number if $k = 1$, and $\left(\frac{P(Q^{k+1})}{P(Q^{k-1})}\right)^{n/2}$ if $1 < k < n$.

[23] A referee helpfully points out that, combined with a story of how J-values are set, the Jamesian account makes some substantive predictions about how the QUD will affect people's tendencies to guess. For instance, consider two versions of the Latif case. Version 1 is the original. Version 2 replaces both Stanford and NYU with 32 options, each with approximately 1% chance. In Version 1, the option with maximal distinctiveness (1.27) is 'Yale', while in Version 2, the distinctiveness of 'Yale' remains the same, but that of 'Yale or Harvard' becomes maximal (1.33); thus the proposal in this section would predict that people are more likely to guess the latter in Version 2 than Version 1. This seems to us to be a good prediction, insofar as the Clustering intuition is correct. Moreover, it's worth noting that the formula in footnote 22 is *local*, in the sense that the distinctiveness for a size-n filtered answer depends only on the probabilities of the size $n-1$ and $n+1$ filtered answers—thus adding many small-probability outcomes will not affect the distinctiveness of most answers. Finally, this formula predicts that is it the linear distance in probabilities that governs the Clustering. Though perhaps surprising, this seems right to us: if A has a 90% chance to win, B has a 5% chance, and 500 other people each have a 0.01% chance to win, then the natural guess about who will win is that A will (rather than that A or B will)—despite the fact that the ratio between 5% and 0.01% is much larger than that between 90% and 5%. Given that constraints like Question-Sensitivity and Clustering hold, we should expect differences like these in the QUD to have effects on people's guesses. We think it's a virtue of our proposal that it makes concrete predictions about such cases, but we are open to the idea that some other (question-based, truth-directed) proposal will do even better.

informativity—i.e. making good guesses based on limited information—and that, therefore, our theories of cognition (individual and joint) should give a privileged role to guesses. In particular, we'll argue that guessing plays a central role in *believing* (§4.1), *communicating* (§4.2), and *reasoning* (§4.3).

Although we're going to make the strongest case we can for each of our applications, we want to emphasize their modularity. You might be convinced by our theory of guessing and be unconvinced by some or all of our applications. Moreover, we want to flag that our applications come in reverse order of originality: §4.1 primarily refines existing ideas in the literature on belief; §4.2 proposes a revision to—and a new explanation of—the standard pragmatics of assertion; §4.3 develops a new theory of the conjunction fallacy. Regardless of which of these applications you find plausible or exciting, we hope to convince you that understanding guessing in terms of expected answer-value helps us pose and address a variety of fruitful questions in epistemology, philosophy of language, and cognitive science.

4.1   Guess when you believe?

Start with belief.[24] We want to call attention to two relevant threads in the recent literature: the *weakness* and *question-sensitivity* of belief.

Start with weakness. At least in the sense of belief referred to by the ordinary word 'belief', believing p doesn't require having a particularly strong attitude toward p. It doesn't require being sure or taking yourself to know, for it is perfectly sensible to say, 'I $\left\{ {\text{don't know} \atop \text{am not sure}} \right\}$ if it'll rain, but I $\left\{ {\text{think} \atop \text{believe}} \right\}$ it will' (Hawthorne et al., 2016). Nor does it require having non-statistical evidence, since it's perfectly sensible to say 'I $\left\{ {\text{think} \atop \text{believe}} \right\}$ your lottery ticket will lose'. In fact, believing that p doesn't even seem to require believing that p is more likely than not! For in response to the question, 'Where do you think Latif will go?', it's reasonable to reply, 'I $\left\{ {\text{think} \atop \text{believe}} \right\}$ he'll go to Yale' (Kahneman and Tversky, 1982; Hawthorne et al., 2016; Dorst, 2019; Rothschild, 2019). In fact, as Holguín (2020) brings out, this seems true *no matter how unlikely Latif is to go to Yale*, so long it's the most likely complete answer (cf. Windschitl and Wells, 1998).[25]

However, the permissibility of thinking Latif will go to Yale depends on the question you're answering.[26] Although, given the credences at the outset, it's fine to say that you think that Latif will go to Yale in response to the question 'Where do you think he'll go?', if

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[24]Following the recent literature, we'll assume that believing that p is thinking that p, and move freely between the two. See Hawthorne et al. (2016); Dorst (2019); Rothschild (2019); Holguín (2020) for extensive discussion.

[25]Some have replied to this 'belief is weak' picture by arguing that the intended reading of the philosophical term 'belief' is some notion of strong or firm belief—something like *being sure* (Clarke, 2013; Greco, 2015; Friedman, 2019; Moss, 2019; Williamson, 2020). We'll say more about this below (§4.2 and §5), but our proposal is perfectly compatible with the idea that there is such a stronger notion that is philosophically important and plays a different functional role than the weak readings of 'belief' and 'think' that are favored in natural language and are our target here (cf. Buchak, 2014; Staffel, 2016).

[26]See e.g. Levi 1967; Kahneman and Tversky 1982; Thomason 1986; Yalcin 2011, 2018; Drucker 2020; Hoek 2020a,b; Holguín 2020; cf. also Schaffer 2005, 2007; Schaffer and Knobe 2012; Schaffer and Szabó 2014; Gerken and Beebe 2016 for closely related discussion. The theory of belief from Leitgeb 2017 is also partition-sensitive, but in very different ways (for example, it requires beliefs to be more probable than not).

you're instead asked 'Do you think Latif will go to Yale, or not?' it's much more natural to that say that you do not. A simple explanation of this contrast is that belief is not simply a relation to a proposition, but rather a relation to a proposition, relative to a question. Thus *beliefs are answers to questions.*

Drawing these two threads together, Holguín 2020 proposes that to believe p relative to question Q is for p to be entailed by your best guess about Q. The key motivation is that rational best guesses, Holguín observes, obey the distinctive features of both Filtering and Optionality—*and so do beliefs.* For example, with respect to the question 'Where do you think Latif will go?', it's permissible for the strongest thing you believe to be that he'll go to Yale, or that he'll go to Yale or Harvard—but it's not permissible for it to be that he'll go to Harvard, or to be that he'll go to Harvard, Stanford, or NYU. Thus both guessing and believing are weak and question-sensitive, and in the same ways. It's natural to hypothesize that what you believe *just is* whatever's entailed by your best guess.

Our account of guessing nicely complements Holguín's account of belief. His account reduces the weakness and question-sensitivity of belief to that of guessing; our account of guessing, in turn, explains *why* good guesses (and hence rational beliefs) are weak and question sensitive (and Filtered, Fit, Optional, and Clustered): namely, because in forming your best guess, and thus your belief, about Q, you must maximize expected answer-value relative to some Jamesian measure.[27]

4.2   Guess when you talk?

Turn now from thinking to talking. We suspect that guessing plays a key role in ordinary communication. *Explicit* requests for guesses are not all that common in ordinary conversation—but it is very common to ask or report what someone *thinks* or *believes* or *thinks is likely* about some question. A natural thing to ask about Latif is *where you think* he'll go to law school; and a natural reply is that *you think* he'll go to Yale, even if you're not sure. Regardless of whether Holguín is right that you think p iff it's entailed by your guess (§4.1), we take our examples to have shown that these are natural ways to get your interlocutor to take a guess —after all, if we ask you 'Where do you think Latif will go?', we'll be unphased if you give an improbable but filtered answer ('Yale'), yet puzzled if you give a probable but unfiltered one ('Harvard, Stanford, or NYU').

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[27]Two questions. First, where do J-values comes into the semantic calculations of attitude ascriptions: are they supplied by the subject, or by the context of assertion or evaluation? The former answer fits naturally with the picture here, and we think standard arguments against subject-sensitive invariantism are not compelling in the case of 'believe' or 'think': 'If more was riding on it, I wouldn't think that the bank's open—I'd suspend judgment' seems coherent. But our view is also consistent with contextualist or relativist treatments.

Second, given Optionality, does it follow that people who form different guesses—as a result of having different (permissible) J-values—are not genuinely disagreeing? We don't think so. Disagreement is a notoriously thorny phenomenon, showing up in many domains where Optionality is plausibly in play (see Khoo 2015; Khoo and Knobe 2018 for helpful recent discussion). Hence you may think that there is optionality about how risk averse to be (with respect to utility in general), while acknowledging that actual decision makers will disagree vehemently with other decision maker's different levels of risk aversion. So there is still room for disagreement about which guess is best, even if all are permissible in some sense.

But we want to propose that guessing plays an even deeper role in communication. We think that there is a basic norm along the following lines:

> **Say Your Guess:** When the QUD is Q, you should communicate your best guess about Q.

To bring out the motivation for this picture, compare it to the standard pragmatic story about assertion, which has two components:

> **Standard Pragmatics:** When the QUD is Q, you should give an answer that (1) you are certain of, and (2) is a *partial answer* to Q—a union of cells of Q that rules out at least one cell of Q (Grice, 1975; Stalnaker, 1978; Roberts, 2012).

In a slogan: *assert the strongest partial answer you're sure of.*

We have two arguments that Say Your Guess is a better picture about the fundamental rule of conversation than Standard Pragmatics. First, as a matter of fact people often seem to permissibly say things that they are substantially less than sure of—but they never permissibly say things that are not Filtered or not Fit. If so, Say Your Guess simply fits the facts better than Standard Pragmatics. Second, Say Your Guess also *explains* these facts better: it explains both the novel observation that assertions need to be Filtered, as well as the standard observation that they need to be partial answers.

Take these points in turn. First, it seems to us that people often say things when they are clearly not certain of them. It's not unusual to overhear exchanges like, 'What's going to happen in the primary?', 'It'll be Biden or Bernie'; 'Where's Latif going to go?', 'He'll end up at Yale'; 'Is teaching going to be in person in the fall?', 'No way—it's going to be online'; 'It looks like it's going to rain; will the concert be cancelled?', 'Nah, it'll happen'; 'Will Bernie win in South Carolina?', 'No way—that's Biden's state'; etc. These assertions are clearly not certain, or even plausibly known, yet they seem perfectly ordinary. Nevertheless, it's felicitous to report the speaker has having *said p*, rather than having said that they think or guess that p ('Jim said it'll be Biden or Bernie.') Thus it looks like, at least in some contexts, assertions can be *Improbable*, like guesses.

But—again, like guesses—not just anything goes. While 'Latif will end up at Yale' is an acceptable response to 'Where do you think Latif will go?', 'Latif will go to Harvard' is a very weird thing to say, given your credences; so is 'He'll go to Harvard or Stanford'. Thus it seems that assertions must be Filtered. And they must also be Fit: that is, again, a standard pragmatic assumption, which follows from the requirement that your assertion should be a partial answer to the QUD. Hence saying 'Latif will go to Yale, and it is cold in London' is unacceptable in reply to the question 'Where will Latif go to law school?'

These patterns are immediately explained if the basic rule of assertion is Say Your Guess. And while a proponent of Standard Pragmatics might maintain that these cases of improbable assertions are simply unremarkable violations of the standard rules of assertion, she would not thereby have a good explanation of why these assertions still must be Filtered and Fit, even though they can be Improbable.

Now to our second point: the second part of Standard Pragmatics, that your assertion should be a partial answer, is usually built in as a pragmatic primitive. By contrast, this is *explained* by Say Your Guess. For, as we have seen, any permissible guess in our framework is Fit. Moreover, the Gricean Maxim of Quantity plausibly entails that you shouldn't assert a contextual tautology. But a guess that is Fit but that also rules out some worlds from the context set is just a partial answer! So Say Your Guess (together with the Maxim of Quantity) explains a fundamental feature of Standard Pragmatics.

Thus it's plausible that the fundamental norm of conversation is to Say Your Guess. This explains patterns that are inconsistent with Standard Pragmatics—namely, that assertions can be Improbable. It explains patterns that are consistent with but not entailed by Standard Pragmatics—namely, that assertions must be Filtered. And it explains patterns that are typically built in but not explained by Standard Pragmatics—namely, that assertions must be partial answers.

Clearly there is much more to do to defend a picture like this—a project we take up in further work (Mandelkern and Dorst, 2021). There we adduce more arguments that guessing is the fundamental norm of assertion, and confront the best arguments *for* Standard Pragmatics: the infelicity of Moorean sentences.

For the present, we want to just briefly answer two objections. First: just as some want to distinguish an attitude of full belief from the target of our analysis above (the weak, ordinary denotation of 'think'), some might want to distinguish something like *fully asserting* from ordinary run-of-the-mill speech acts that can indeed be relatively Improbable—call them *sayings*. The idea would be that sayings is a broad category, which includes a special subkind—assertions. On this approach, all sayings are governed by the basic rule Say Your Guess, but assertions are governed by a further rule, 'Say only what you're certain of'. We are open to an ecumenical approach like this. Even on that way of going, we'd still take Say Your Guess to play an important explanatory role, since (i) much of what happens in ordinary conversations are sayings and not assertions in this special sense, and (ii) our account would still play a key role in explaining why sayings in general—including assertions—must be Fit and Filtered.

Second: you might wonder how much we are really doing to *explain* why assertions/sayings must be partial answers to the QUD since it is simply stipulated in Say Your Guess that you should say your guess *about the QUD*. But there are many different rules of assertion that are QUD sensitive. For instance, you might have a theory that says: *in response to a QUD Q, say the thing you know that will most change your interlocutors' credences in some cell of Q.* Indeed, theories like this have been defended (e.g. Feinmann 2020). And theories like that— although they also take on board QUD sensitivity—do not require assertions to be partial answers. So, while QUD sensitivity is indeed built into Say Your Guess from the start, our theory still does important work in explaining why we assert partial answers to the QUD (in addition, of course, to explaining why assertions must be Filtered, and can be Improbable).

4.3 Guess when you reason?

We turn finally to the role of guesses in reasoning. We'll argue that our theory helps explain the *conjunction fallacy*—the well-known observation that people sometimes judge a conjunction to be more probable than one of its conjuncts.

We think that this is a particularly interesting application for two reasons. First, the patterns we've tried to explain above involve intuitive patterns in guessing, belief, and assertion. All of those patterns could be tested experimentally, but—to our knowledge—have not yet been. We think it is significant, then, that our account can also explain surprising and intricate patterns of judgments that *have* been explored experimentally. Second, the conjunction fallacy is a cornerstone of a fairly standard case in psychology that humans are fundamentally not very good at reasoning under uncertainty.[28] Our account gives a different diagnosis of what is going on here, consistent with a different picture of human reasoning. This is philosophically interesting in its own right; and, we think, provides further support for the thesis that guessing in general—and something along the lines of our theory in particular—has an important role to play in the theory of human cognition.

To get a feel for the conjunction fallacy, begin with the most famous case, from Tversky and Kahneman 1983. Subjects were first given the following vignette:

> Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Subjects were then asked which of two alternatives was more probable:

> · Linda is a bank teller.
> · Linda is a bank teller and is active in the feminist movement.

85% of the subjects chose the second option over the first, contrary to the laws of probability.

To see how our account of guessing might help explain this result, recall a central lesson of our discussion of guessing above: it is acceptable to guess a less likely answer when it is correspondingly more informative—if asked where Latif is likely to go, it can be acceptable to guess 'Yale' over 'Yale or Harvard'. In particular, when we measure informativity in terms of the proportion of cells of the QUD that are ruled out, a conjunction will be more informative than its conjuncts when both conjuncts address the QUD. So, although the conjunction will never be more probable than one of its conjuncts, it may nevertheless be a better guess. Our hypothesis is that the accuracy-informativity tradeoff of guessing can explain the conjunction fallacy:

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[28]For classic and modern statements of this kind of picture, see Tversky and Kahneman 1974; Kahneman and Tversky 1982, 1996; Fine 2005; Ariely 2008; Kahneman 2011; Lewis 2016. For resistance to it, including the rise of the 'rational analysis' program in cognitive science, see Anderson 1990; Gigerenzer 1991; Gigerenzer and Goldstein 1996; Oaksford and Chater 1994, 1998; Kelly 2004, 2008; Oaksford and Chater 2007; Tenenbaum and Griffiths 2006; Hahn and Oaksford 2007; Tenenbaum et al. 2011; Griffiths et al. 2012; Hahn and Harris 2014; Hedden 2018; Miller and Sanjurjo 2018.

The Answer-Value Account: People commit the conjunction fallacy because they rank outcomes according to their expected answer-value, rather than their probability.

A key motivation for this account is that evaluating expected answer-value—that is, comparing potential guesses—is something we all do all the time (if our arguments so far are right: whenever we think or talk).

Let's work through the idea in more detail. Take the Linda case, where Tversky and Kahneman found that a large proportion of subjects rated 'Linda is a feminist bank teller' (FT) as more likely than 'Linda is a bank teller' (T). First, we assume that the vignette and the alternatives offered (T vs. FT) generate a QUD based on the relevant characteristics in play, namely, the four-cell QUD obtained from crossing the question of whether Linda is a feminist or not ($\{F, \overline{F}\}$), and whether she is a bank teller or not ($\{T, \overline{T}\}$): $Q = \{FT, F\overline{T}, \overline{F}T, \overline{F}\overline{T}\}$.[29] Then, given credences P and any J, the expected answer-values of FT and T are as follows:

$$E_Q^J(FT) = P(FT) \cdot (J^{3/4})$$
$$E_Q^J(T) = P(T) \cdot (J^{1/2})$$

Thus the expected answer-value of FT is greater than that of T iff:

$$P(FT) \cdot (J^{3/4}) > P(T) \cdot (J^{1/2})$$
$$\Leftrightarrow \quad \frac{P(FT)}{P(T)} > \frac{J^{1/2}}{J^{3/4}}$$
$$\Leftrightarrow \quad P(F|T) > \frac{1}{J^{1/4}}$$

When the value of informativity is minimal ($J = 1$), the right-hand side equals $\frac{1}{1^{1/4}} = 1$, and the expected answer-value of FT is never higher than that of T (since $P(F|T) \leq 1$). But as J grows, the right-hand side shrinks. And, when $J > 1$, the expected answer-value of 'feminist bank teller' is higher than that of 'bank teller' iff the conditional probability of Linda being a feminist, given that she's a bank teller, is sufficiently high—where what counts as 'sufficient' is determined by the value of informativity J.

Why does the conditional probability $P(F|T)$ matter, on our account? Because although the conjunction FT always has a lower probability than the conjunct T, the *degree* to which it's lower is determined by how likely F is given T, since $P(FT) = P(T) \cdot P(F|T)$. Thus when the conditional probability $P(F|T)$ is high, FT will be only slightly less likely than T—which speaks in favor of trading the (slightly) more probable but less informative guess T for the (slightly) less probable but more informative guess FT.

Concretely, suppose that you judge the probability that Linda is a feminist to be 0.8,

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[29]Importantly, our theory's predictions don't depend on the details of the QUD selected: one that draws more distinctions about what Linda is like would yield the same (expected) answer-value scores for T, FT, etc., provided those distinctions are all (contextually) orthogonal to F and T. So the same story extends to cases where, say, more options about Linda's livelihood are given (provided they are mutually consistent), as in one version of the Linda case given in Tversky and Kahneman 1983.

and to be independent of whether she's a bank teller, so that $P(F|T) = P(F) = 0.8$. Then you should guess 'feminist bank teller' over 'bank teller' iff $P(F|T) = 0.8 > \frac{1}{J^{1/4}}$, which in turn holds iff $J > 2.44$, i.e. iff the value of a (true) maximally informative answer is a bit more than twice that of a (true) completely uninformative answer. While we don't expect to have direct intuitions about J-values, recall that in our original law-school case, 'Yale' was the best guess iff $J > 10.25$—so the value of informativity required to yield the conjunction fallacy is quite modest.[30]

To be clear: we're not claiming that subjects have the judgments they do in conjunction fallacy cases because they are actually *guessing*. In fact, while the guess FT often has higher expected answer-value than T, neither guess is filtered given intuitive probability assignments—'feminist and *not* a bank teller' is more probable, but equally informative as, 'feminist bank teller'; and 'feminist' is more probable, but as informative as, 'teller'. So our claim is not that FT is a *good guess*, but rather that it is a *better* guess than T, given many natural credence functions and J-values.

This brings out something interesting about this application. We motivated our theory mainly via binary observations about which guesses (and later, beliefs and assertions) are *permissible*. To account for these judgments, we gave a theory which actually gives us something more general than a division among permissible and impermissible guesses: namely, *rankings* of all possible answers according to their expected answer-value. While this may have at first seemed over-committal, we hope to have shown how these rankings naturally arise out of an intuitive picture of guessing—and now we are claiming that they in fact do important theoretical work.

With the basic idea in hand, we want to briefly highlight some key predictions of our account. The conjunction fallacy is an intricate phenomenon which has generated an enormous amount of literature; for reasons of space, our discussion here must be limited. In future work, we plan to offer a more detailed examination of both the empirical literature and the ways our theory stacks up against other approaches. Here, we'll simply highlight what we take to be the key selling points of our account.

Start with an observation that follows from the discussion above:

Prediction 1: Ranking AB over B will be more common as $P(A|B)$ goes up.

We have already seen why this follows: when the conditional probability, say, $P(F|T)$ is high, then FT will be only slightly less likely than T, which means subjects will (for many J-value)

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[30]One strategy for making concrete predictions here is to use the distinctiveness measure from §3.3 to predict how people will set J-values. We have some hesitancy about this, since that measure is best motivated as a way of seeing what the salient answers are, yet a conjunction fallacy case is one in which, by design, the options for answers are artificially restricted (e.g. 'feminist non-bank-teller' is not an option). But setting this hesitancy aside, here's how the approach would go. Suppose for illustration the probability of Linda being a bank teller is 0.1. Then the probabilities of the various complete answers are: $F\overline{T}$: 0.72; $\overline{F}\,\overline{T}$: 0.18; FT: 0.08; $\overline{F}T$: 0.02. This makes the distinctiveness of 'feminist non-teller' $\frac{0.72}{0.18} = 4.00$, while the next highest value is 1.07; thus we predict that there'll be a strong preference for setting J to a value that makes $F\overline{T}$ substantially higher than its alternatives. $F\overline{T}$ becomes more distinctive as J grows, thus predicting that people will have a high J-value and thus will likely commit the conjunction fallacy.

be inclined to select the slightly less probable but much more informative guess FT over the slightly more probable but much less informative one T.

This prediction is confirmed by a variety of empirical studies (e.g. Gavanski and Roskos-Ewoldsen 1991; Fantino et al. 1997; Costello 2009a,b; Tentori and Crupi 2012; though see Tentori et al. 2013 for a challenge). For instance, Tentori and Crupi 2012 asked subjects about two claims about a character Mark and a 100-ticket lottery, giving two stimuli: 'Mark is a scientist' (S) and 'Mark is a scientist and will win the lottery' (SW). Subjects were given different information about how many lottery tickets Mark has (either none, 1, 20, 50, 80, or all). The rates of conjunction fallacy (i.e. the rate of ranking SW as more probable than S) increased strictly with the number of lottery tickets Mark had—see Figure 3. This means, in turn, that they increased strictly with the conditional probability of 'Mark will win the lottery' on 'Mark is a scientist' (since these are probabilistically independent, $P(W|S) = P(W)$).
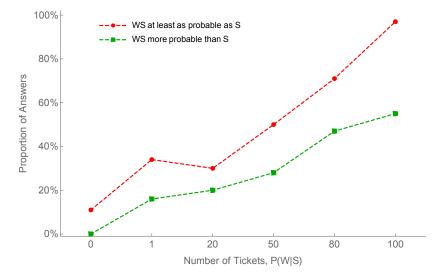

Figure 3: Conjunction-fallacy rates, varying $P(W|S)$, recreated with permission (Tentori and Crupi, 2012).

> **Prediction 2:** Ranking AB over B will not generally depend on the content of A and B, but instead on their (conditional) probabilities.

This prediction follows because the Answer-Value Account of the conjunction fallacy is only sensitive to (i) how much more informative AB is than B, and (ii) how much less probable it is. Thus it is not generally sensitive to what A and B are about.[31] Perhaps surprisingly, this is empirically confirmed: the conjunction fallacy is found even for unrelated conjunctions of events. For example, Yates and Carlson (1986) found that with two probable but completely unrelated events—namely, 'Governor Blanchard will succeed in raising

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[31]We include the 'generally' rider because this holds in general only if A and B are equally informative; but sometimes that is not plausible. In comparing 'feminist bank teller' to 'bank teller', a sensible overall question is 'What are Linda's social and political positions?' But if we ask you to compare 'Linda is a bank teller' to 'Linda is a bank teller and has at least seven eyelashes', you'll be hard-pressed to come up with a sensible overall question in which the second conjunct could play a part. We suspect this observation may help account for the data found in Tentori et al. 2013.

the Michigan state income tax' and 'Bo Derek will win an Academy Award for the movie she is currently making'—people committed the conjunction fallacy 56% of the time. Similarly for Costello (2009a) with unrelated weather events. (This is particularly challenging for representativeness-based accounts, like that of Tversky and Kahneman 1983, to make sense of.)

> **Prediction 3:** When $P(A|B)$ and $P(B|A)$ are *both* high, 'double'-conjunction fallacies will be common: people will rank $AB \succ A, B$. Meanwhile, when $P(A|B)$ is high but $P(B|A)$ is low, 'single'-conjunction fallacies will be common: people will rank $A \succ AB \succ B$.

These predictions follow because our account is symmetric: provided A and B are both relevant to the QUD and are contextually orthogonal, the expected answer-value of AB is higher than that of B iff $P(A|B) > \frac{1}{J^{1/4}}$; and it is higher than that of A iff $P(B|A) > \frac{1}{J^{1/4}}$.[32] And indeed, in cases where both conditional probabilities are high, people standardly rank the conjunction as more probable than both conjuncts as in this famous case from Tversky and Kahneman (1983):

> A young college runner, Peter, has already run the mile in 4:06. Please rank the following for probability:
>
> (a) Peter will complete the mile under 4 min.
>
> (b) Peter will run the second half-mile under 1:55 and will complete the mile under 4 min.
>
> (c) Peter will run the second half-mile under 1:55

48% of subjects ranked (b) as the most probable of (a)–(c). (See Crupi et al. 2018 for more cases.) This is expected from our view, since $P(a|c)$ and $P(c|a)$ are both relatively high.

> **Prediction 4:** Ranking AB over B will be equally common regardless of how exactly the conjunction AB and conjunct B are phrased.

In particular, nothing about our account requires that the relevant expressions are literally conjunctions and conjuncts—what matters is simply their informativity and probability. As a result, different ways of expressing pairs of claims, one of which is more informative (but less probable than) the other, will result in the same effect. Thus the account predicts that the effect can occur when one of the claims is a disjunct and the other is a disjunction, or when one is a broad category and the other is a narrow one ('humanities' vs. 'literature', etc.; see Bar-Hillel and Neter 1993; Costello 2009a). Moreover, the account predicts the effect will

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[32]In the Linda case people standardly rank rank $F \succ FT \succ T$ (Tversky and Kahneman, 1983). This is in line with our predictions, for in this case $P(F|T)$ is high (so likely above the threshold $\frac{1}{J^{1/4}}$), while $P(T|F)$ is low (so likely below it).

still occur even when the claims are carefully phrased to avoid various implicatures, (e.g., 'Linda is a bank teller, whether or not she is a feminist'). This is empirically confirmed.[33]

Prediction 5: AB will often be ranked over B regardless of whether any evidence relevant to A or B is provided.

That is, we predict that whether AB will be ranked over B depends on how high $P(A|B)$ is, which in turn may be so regardless of whether subjects have received any confirming evidence for A (or B) as part of the experimental setup. Such evidence *is* part of the experimental setup in the Linda case—the vignette provides evidence that she is a feminist—and that has motivated 'confirmation-theoretic' accounts of the conjunction fallacy, based on the idea that people may say 'feminist bank teller' is more likely because it's *more confirmed* than 'bank teller' by the relevant evidence (Sides et al., 2002; Crupi et al., 2008; Tentori and Crupi, 2012; Tentori et al., 2013; Crupi et al., 2018). But the conjunction fallacy is also observed in scenarios wherein subjects are provided with no relevant evidence by the experimenters, meaning neither answer is confirmed (Tversky and Kahneman, 1983; Yates and Carlson, 1986; Costello, 2009a). For example, Tversky and Kahneman (1983) asked some subjects to evaluate the probability of (a), and others to evaluate the probability of (b):

(a) There will be a massive flood somewhere in North America in 1983, in which more than 1000 people drown.

(b) There will be an earthquake in California sometime in 1983, causing a flood in which more than 1000 people drown.

The average estimates for (b) were higher than for (a). Our account (unlike confirmation-theoretic accounts) generalizes immediately to this instance of the conjunction fallacy, since (b) is more informative than (a) relative to a salient QUD (*will there be an earthquake, and will there be a flood?*); and (b) is plausibly only somewhat less probable than (a).[34]

Prediction 6: Since informativity relative to the QUD drives the effect, we expect that corresponding effects will diminish in cases involving estimation of frequencies.

For example, tell subjects that 100 individuals fit Linda's description, and ask them to estimate the proportion of them that are ___s, where the blank is filled in either by 'bank teller' or 'feminist bank teller.' Here the QUD is 'What number of people have property ___?', so

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[33]E.g. Tversky and Kahneman 1983; Adler 1984; Agnoli and Krantz 1989; Macdonald and Gilhooly 1990; Politzer and Noveck 1991; Dulany and Hilton 1991; Gigerenzer 1991; Messer and Griggs 1993; Hertwig and Gigerenzer 1999; Mosconi and Macchi 2001; Tentori et al. 2004; Hertwig et al. 2008. Moro (2009) gives a helpful overview.

[34] Another objection to confirmation-based accounts come from cases like this. Consider Mark, who buys one ticket to a five-million ticket lottery. Then his friend gives him nine more tickets, so he now has ten tickets total. What do you think is more likely, that Mark is right-handed, or that Mark is right-handed and will win the lottery? We suspect subjects will judge the former to be more likely, even though it is not confirmed at all by the vignette, while the latter is massively confirmed. This needs to be tested experimentally, but if it is confirmed, it would show that confirmation alone does not drive the conjunction fallacy (which, of course, is not to say confirmation plays no role).

regardless of what fills in the blank, each answer is equally informative in our sense (saying that 20 people are *FT* is no more informative than saying that 10 are *T*; both are complete answers to their respective questions). This prediction—that conjunction-fallacy rates diminish or disappear in estimate settings like these—is empirically confirmed (Tversky and Kahneman, 1983; Gigerenzer, 1991; Costello, 2009a; Moro, 2009).

In sum: our account makes a variety of subtle predictions which match a variety of robust empirical trends in the literature.

Having explained the contours of the Answer-Value Account of the conjunction fallacy, we want to briefly reflect on it more broadly. The general idea that an accuracy-informative tradeoff is behind the conjunction fallacy has been discussed before, most prominently by Tversky and Kahneman (1983).[35] Their discussion is brief, concluding that 'it is unlikely that our respondents interpret the request to rank statements by their probability as a request to rank them by their expected (informational) value' (p. 312). The worry seems to be that an account like ours is simply undermotivated (compare Moro, 2009, 18–19): *why* would people who are asked about probabilities respond using an accuracy-informativity tradeoff?

We think that the work we've done in this paper helps put this worry to rest. We've argued that assessing expected answer-value is a cognitively basic practice that plays a central role in guessing, believing, and talking. If this is right, assessing expected answer-value is a natural *default* mode of evaluating potential answers to questions. In other words, we think our discussion changes the dialectic: rather than introducing a new apparatus to explain the conjunction fallacy, we are showing how it arises naturally from a mechanism that arguably plays a central role in our cognitive lives.

Of course, there are undoubtedly many dynamics behind the conjunction fallacy; we don't claim that the Answer-Value Account is the whole story. In particular, we leave it open that other proposed factors—like confirmation, similarity, implicature, and noise—also influence the effect; but we maintain that assessments of expected answer-value play a central role in explaining the core phenomenon.

Zooming out a bit more: we are talking about the conjunction fallacy not just because it's a promising application of our theory, but because our account of it opens up philosophically interesting avenues. The conjunction fallacy is often held up as part of the core evidence that humans are fundamentally quite bad at dealing with uncertainty (Kahneman and Tversky, 1996; Kahneman, 2011). While our account is of course consistent with this picture, it's also consistent with a very different one.

Here's what we mean. Everyone should agree that people are bad at *conscious* probabilis-

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[35]Two other key precedents: first, again, a framework similar to ours was developed by Levi 1967, who suggested it could be applied to the conjunction fallacy—but we don't think the details work (see footnote 13). Second, Cross (2010) also proposes that a Jamesian tradeoff of some kind is behind the conjunction fallacy, though he suggests spelling this out in terms of explanatory power instead of informativity in our sense. More generally, the idea that we trade off accuracy and informativity is also present in Yaniv and Foster (1995), but they do not give a general framework for evaluating informativity. Similar ideas are taken by Adler (1984); Moro (2009) gives a helpful discussion of the idea. For the general idea that question sensitivity plays a central role in human reasoning, spelled out in a different framework, see Koralus and Mascarenhas 2013, 2018.

tic reasoning. This is demonstrated by the conjunction fallacy (among others), and is made especially clear by the fact that people sometimes choose to *bet* on conjunctions rather than their conjuncts (Tversky and Kahneman, 1983; Bar-Hillel and Neter, 1993; Sides et al., 2002; Bonini et al., 2004). What should we conclude from this fact? In particular, does it show that the way people form judgments under uncertainty is fundamentally non-probabilistic—that they make do with other (worse) ways of managing uncertainty? Though popular,[36] that conclusion sits uneasily with the burgeoning literature on Bayesian cognitive science, which uses probabilistic models to help explain the remarkable feats of human learning and inference (e.g. Anderson, 1990; Gopnik, 1996; Tenenbaum et al., 2011; Lake et al., 2016). That literature suggests that people are very *good* are (implicit) probabilistic reasoning.

Our account of the conjunction fallacy may help to reconcile these pictures. On our view, conscious 'probabilistic' reasoning is influenced by the calculation of expected answer-value, which in turn is the product, in part, of (implicit) probabilistic calculations. Even if those implicit calculations are probabilistic—as they are on our account—people's conscious probabilistic reasoning will be poor because expected answer-value doesn't conform to the rules of probability, and it is the outputs of expected answer-value assessments that are most prominently consciously accessible. In other words: people may be bad at conscious probabilistic reasoning not because they are bad probabilistic reasoners full stop, but because they are bad at *pulling apart* judgments about probability from judgments about expected answer-value.

Of course, probability is a *component* of expected answer-value. So should it be surprising, on our view, that people are good at assessing the latter and bad at assessing the former?[37] We don't think so—for two reasons.

First, the *language* we use to talk about probabilities is very close (in fact, often identical) to the language we use to elicit guesses. Questions like 'What's most likely?', or 'What would you bet will happen?', are naturally used to elicit guesses, not probability judgments. Because of this, we think most people simply don't have much practice distinguishing these two types of judgments.[38] This may help make sense of why they sometimes have stubborn responses to

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[36]See, for example, Kahneman et al. 1982; Tversky and Kahneman 1983; Gigerenzer and Goldstein 1996; Kahneman and Frederick 2002; Hastie and Dawes 2009; Kahneman 2011; Thaler 2015; Tetlock and Gardner 2016.

[37]Thanks to Josh Knobe for helpful discussion on this point.

[38]A question we want to remain neutral on here: do words like 'likely' and 'probably' have a *meaning* according to which they mean 'has high expected answer value'? This is not by any means outlandish; for instance Yalcin 2010 argues on the basis of cases like those we have focused on (following Windschitl and Wells 1998) that 'probably' is assessed relative to a salient QUD. He leaves open the exact form of QUD-sensitivity; one could naturally incorporate our account of guessing into a story about the meaning of 'probably'. This would lay the foundation for a very hard line on the rationality of the conjunction fallacy: if 'probably'/'likely' literally have a *meaning* on which they are measures of expected answer value rather than probability, then there is no mistake at all in conjunction fallacy judgments in response to questions about what is probable/likely. A softer line would say that the literal meaning of these words is about probability, but for reasons of pragmatics, questions about probability are naturally (mis/over)interpreted as questions about expected answer value. This softer line is still consistent with a broadly rationalist line on the conjunction fallacy (compare implicatures: one might deny that 'John had some cookies' literally *means* that he didn't have all of them, without thus thinking that subjects make an *error* if they conclude that he didn't). The choice between these

criticisms of their answers when they commit the conjunction fallacy. For example, Michael Lewis recounts the following illustrative anecdote. Kahneman tried to convince a group of his students that the conjunction fallacy was an error, asking: 'Do you realize you have violated a fundamental rule of logic?' Lewis recounts: "So what!' a young woman shouted from the back of the room. 'You just asked for my opinion!" (Lewis, 2016, 325). If opinions are beliefs, and hence guesses, then this woman is protesting in exactly the way we would expect—she thought she was being asked for her guess, not for a probability judgment.

Second, and more generally, it's normal for components of core cognitive competences to be required for—but hard to separate from—those competences. You can effortlessly recognize a face, but would struggle to articulate any of its distinctive features. You can easily press the brake pedal just hard enough to avoid a collision, but would be at a loss to articulate the underlying principles. Likewise: you can effortlessly respond to 'Where do you think Latif will go?' with 'Yale or Harvard', without having any conscious access to the calculations that went into this. In short: assessing probabilities is a crucial step in doing something we do all the time (the quality of a guess), but one that can be consciously separated from it only with care and practice.

5   Why We Guess

We've covered a lot of ground. What guesses do people make? The answer is subtle but surprisingly systematic (§2; §3.3). How do people make guesses? By optimizing a tradeoff between accuracy and informativity (§3). When do people make guesses? All the time: they make guesses whenever they form beliefs, (§4.1), communicate (§4.2), or reason (§4.3) under uncertainty.

But *why*? We've argued that guessing aims at both accuracy and informativity. We've given this hypothesis a simple exposition, and argued that it helps to explain a variety of patterns. We think the abductive case for it is strong.

Yet it may retain an air of mystery. When asked about guessing, the first things that come to mind are quiz shows and country fairs: 'Guess the exact number of jellybeans in this jar'; 'Try to guess which cup I hid the prize under—you get two tries'; etc. These guessing games have their own idiosyncratic rules: often only maximally informative guesses are allowed; sometimes multiple guesses are permitted; etc. Considering examples like this, it may seem that the practice of guessing itself will have no intrinsic rules or standards. Yet we've argued at length that it *does*: that the cognitive attitude of guessing the answer to a question—of figuring out what you think—always aims at accuracy and informativity. If we are correct that the practice of guessing plays a central role in our cognitive lives, there must be some explanation of why it involves *these* rules. What is that explanation?

Here we can only speculate. Start with a more general question: why would you want to form a guess at all? If you already have credences—which, after all, are an *input* to our theory of guessing—why not just use them in making your way through the world? A natural

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approaches involves interesting methodological issues and deserves careful exploration.

answer, in the spirit of theories of bounded rationality, is that your credences can at best be extremely partial given your limited computational powers and the intractability of general probabilistic inference.[39] Thus you can only form credences over a relatively small set of propositions—for instance, those generated by the QUD. Those credences can be used to form a guess, which highlights a particular region of possibilities—the region that you take most seriously, given the question at issue and your contextual priorities.

What, then, do you do with this region? Our proposal is that you use it to guide further investments of cognitive resources. You *reason within your guess*: supposing it to be true, you can form (and discuss) plans, preferences, or opinions about what to do, want, or think, if so. Having a small region like this can greatly simplify thought and talk about these activities. If we ask, 'Where do you think Latif will go?' and you reply, 'I think he'll go to Yale or Harvard', then it's natural to follow up with plans ('So we'll be able to visit him on the weekends'), preferences ('So I'll want to put him in touch with Jane'), or more fine-grained credences ('So it's likely he'll need a car'). You can do all of this without losing sight of the fact that the region is just your guess; possibilities inconsistent with your guess remain in your peripheral vision, so to speak—preventing you from betting the farm on your guess the moment you make it. But by being able to highlight certain regions of possibilities for further thought and talk, you can reach a more fine-grained assessment of the various routes to action *in the scenario you guess will occur*.[40]

If something like this were right, it would make sense of why guessing has the profile we've argued it does. Guesses should be accurate, since if your guess turns out to be false, any contingency-planning you've done within it will be wasted. But guesses should also be informative: in choosing a region of possibilities to highlight for further investigation, it pays to have a specific answer to the live question because this cuts down on the number of distinctions you need to track—an informative guess allows you to make fine-tuned plans even when you don't have the resources to plan for every contingency. For example: if you guess that Latif will go to Yale, you can focus on apartment listings in New Haven; if you guess that he'll go to Yale or Harvard, you can at least look at flight prices to the Northeast; but if you guess that he'll go to Yale, Harvard, or Stanford, your plans within this guess can't be nearly as specific. Thus the 'reason within your guess' picture may have the resources to explain why guesses are subject to an accuracy-informativity tradeoff of the kind we've spelled out.

Moreover, there's empirical evidence that people *do* tend to reason within their guesses. A common claim in the literature on confirmation bias in psychology—and on theory-choice in philosophy of science—is that people have a tendency latch onto a specific, favored hypothesis, and expend most of their cognitive effort using its predictions to guide their investigations.[41] A

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[39]E.g. Simon 1957; Cherniak 1986; Dagum and Luby 1993; Weirich 2004; Bradley 2017.

[40]This picture is related to (but interestingly distinct from) views on which subjects choose a set of possibilities to treat *as certain* in a given context—for discussion, see Harsanyi 1985; Lance 1995; Lin and Kelly 2012; Lin 2013; Clarke 2013; Tang 2015; Greco 2015; Leitgeb 2017, and Staffel 2019.

[41]See, for example, Wason 1960; Kuhn 1962; Koriat et al. 1980; Klayman and Ha 1987; Kuhn 1989; Maher 1993; Gopnik 1996; Nickerson 1998; Whittlestone 2017.

common mistake in poker is to 'put someone on a hand'—guess what they have, and use that guess to guide your betting. The mistake is not in guessing per se, but in having a guess that's overly specific (and hence improbable); *good* poker players put their opponents on a *range* of hands.[42] Studies of doctors' reasoning shows that they tend to commit the conjunction fallacy—that is (we think) to *guess*—when proposing diagnoses, which presumably has a direct impact on which procedures they go on to perform (Tversky and Kahneman, 1983; Rao, 2009; Crupi et al., 2018). And, as mentioned above, people have some tendency to commit the conjunction fallacy when selecting bets—indicating that they're using their guess to frame and guide their actions.

As some of these examples illustrate, reasoning within guesses can lead to mistakes. But, again, we think situating them within a broader, bounded-rationality theory of guessing suggests that they are very different *kinds* of mistakes than is standardly thought. For the same mechanism that leads the human mind into conjunction-fallacy betting also—perhaps—helps it consistently outperform computers in novel situations of intractable complexity (Tenenbaum et al., 2011; Huang and Luo, 2015; Lake et al., 2016). There are over 10,000 known human diseases, over 2.5 million different poker hands, and always infinitely many empirically adequate scientific theories. Yet doctors, poker players, and scientists don't simply freeze up; instead, they make good guesses that allow them to reason within such complexity.

We can illustrate this point close to home, for we've reached the stage where we speculate about future directions for our theory—that is, the stage where we guess. Obviously we can't formulate detailed opinions or plans about all the directions a theory like this could go. What we *can* do is highlight a region of possibilities for future investigation. That region should be small enough that we can see—albeit dimly—how such investigations might proceed; but it should be large enough that it is likely to contain promising directions. That is: it should be a good guess.

Our guess, then, is that this approach could be usefully applied to a range of topics in both cognitive science and philosophy.

In cognitive science: expected answer-value may help to explain other peculiar patterns in human judgments, like sub-additivity effects.[43] The way people generate and then reason within guesses may help to explain or refine the data surrounding confirmation bias—such as the well-known Wason selection task[44] And, of course, our predictions about the conjunction fallacy may lead to new discoveries about it and related phenomena.

In philosophy: the accuracy-informativity tradeoff of guessing may help refine our theories of both conversational implicature and prediction[45] The connection between guessing, (weak) belief, and action suggests that previous authors may have been too quick to treat weak

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[42]Thanks to Ben Holguín for the example.

[43]Given a proposition q and partition Q of q, people often report probability judgments $P(\cdot)$ such that $\sum_{x \in Q} P(x) > P(q)$ (see Tversky and Koehler 1994; Redelmeier et al. 1995; Rottenstreich and Tversky 1997). Notably, (Jamesian) expected answer-value is subadditive: for any partition Q of q, $\sum_{x \in Q} E_Q^J(x) \geq E_Q^J(q)$, with equality only if $Q = \{q\}$ or $J = 1$.

[44]E.g. Wason 1966; Klayman and Ha 1987; Nickerson 1998.

[45]E.g. Benton 2012; Ninan 2019; Cariani 2020.

belief as a linguistic phenomenon that plays no role in decision theory.[46] And our account of epistemic value and its relation to new constraints on guesses, beliefs, and questions may open up new territory for the tools of epistemic utility theory.[47]

In sum: the accuracy-informativity tradeoff of guessing may contribute to our understanding of both human rationality and the role of question-sensitivity in our cognitive lives.

Appendix: Proof of Optionality

Theorem (Optionality). If P is regular over a question Q with $|Q| = n$, then for any $1 \leq k \leq n$, there is some $J \geq 1$ such that any filtered k-cell answer maximizes $E_Q^J(\cdot)$.

Proof. Order the cells q_i of the QUD by probability, so that $P(q_1) \geq P(q_2) \geq \ldots \geq P(q_n)$, and let $Q^i := q_1 \cup \ldots \cup q_i$ be the union of the first i cells. The only guesses that can maximize expected answer-value, for any value of J, are the filtered ones. And if p is filtered, then there is a Q^i that it is equivalent to in expectations—in particular, if p is a union of k cells and p is filtered, then $P(p) = P(Q^k)$ and $Q_p = Q_{Q^k}$, so—recalling that $E_Q^J(r) := P(r) \cdot J^{Q_r}$—we have that for all J, $E_Q^J(p) = E_Q^J(Q^k)$. Thus it suffices to show that for any $1 \leq k \leq n$, there is a $J \geq 1$ such that $E_Q^J(Q^k) > E_Q^J(Q^i)$ for all $i \neq k$.

We begin with several observations about the probabilities and expected values of the Q^i.

Lemma 1. The pairwise conditional probabilities are ordered: $P(Q^1|Q^2) < P(Q^2|Q^3) < \cdots < P(Q^{n-1}|Q^n)$.

Proof. Take an arbitrary $1 \leq i \leq n-2$ and consider $P(Q^i)$, $P(Q^{i+1})$, and $P(Q^{i+2})$, relabelling them p_0, p_1, p_2 respectively. Note that $P(Q^i|Q^{i+1}) = \frac{P(Q^i)}{P(Q^{i+1})} = \frac{p_0}{p_1}$, and similarly $P(Q^{i+1}|Q^{i+2}) = \frac{p_1}{p_2}$. Thus to establish Lemma 1 it suffices to show that $\frac{p_0}{p_1} < \frac{p_1}{p_2}$.

Note that by construction, $0 < p_0 < p_1 < p_2$, and moreover that $p_1 - p_0 = P(Q^{i+1}) - P(Q^i) = P(q_{i+1})$ and similarly $p_2 - p_1 = P(q_{i+2})$. Since we know $P(q_{i+1}) \geq P(q_{i+2})$, it follows that $p_1 - p_0 \geq p_2 - p_1$. Thus we have that $2p_1 \geq p_0 + p_2$ and so $\frac{p_0 + p_2}{2} \leq p_1$. Note that what we want to show is that $\frac{p_0}{p_1} < \frac{p_1}{p_2}$, which holds iff $p_0 p_2 < (p_1)^2$; by the above inequality it suffices to show that $p_0 p_2 < (\frac{p_0 + p_2}{2})^2$. This holds iff

$$
\begin{aligned}
p_0 p_2 \;&<\; \frac{p_0^2 + 2p_0 p_2 + p_2^2}{4} \\
\Leftrightarrow\; 4p_0 p_2 \;&<\; p_0^2 + 2p_0 p_2 + p_2^2 \\
\Leftrightarrow\; 0 \;&<\; p_0^2 - 2p_0 p_2 + p_2^2 \\
\Leftrightarrow\; 0 \;&<\; (p_0 - p_2)^2
\end{aligned}
$$

which of course is true. It follows that $P(Q^i|Q^{i+1}) = \frac{p_0}{p_1} < \frac{p_1}{p_2} = P(Q^{i+1}|Q^{i+2})$, and since i was arbitrary, Lemma 1 follows in turn. □

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[46]E.g. Christensen 2004; Križ 2015; Dorst 2019; Friedman 2019; Moss 2019; Williamson 2020.
[47]Cf. Horowitz 2018; Pérez Carballo 2018; Schoenfield 2019b.

Lemma 2. For any $1 \le i \le n-1$: $E_Q^J(Q^i) \gtrless E_Q^J(Q^{i+1})$ iff $P(Q^i|Q^{i+1}) \gtrless \frac{1}{J^{1/n}}$.

Proof. Noting that the proportion of Q ruled out by Q^i is $\frac{n-i}{n}$, we have that $E_Q^J(Q^i) \gtrless E_Q^J(Q^{i+1})$ iff

$$P(Q^i) \cdot J^{\frac{n-i}{n}} \; \gtrless \; P(Q^{i+1}) \cdot J^{\frac{n-(i+1)}{n}}$$

$$\Leftrightarrow \frac{P(Q^i)}{P(Q^{i+1})} \; \gtrless \; \frac{J^{\frac{n-i-1}{n}}}{J^{\frac{n-i}{n}}}$$

$$\Leftrightarrow P(Q^i|Q^{i+1}) \; \gtrless \; J^{\frac{n-i-1}{n} - \frac{n-i}{n}} \; = \; J^{\frac{n-i-1-n+i}{n}} \; = \; J^{\frac{-1}{n}} \; = \; \frac{1}{J^{1/n}}$$

as desired. $\square$

From here we establish that for any J, the expectations of the Q^i are 'single-peaked':

Lemma 3 (Single-Peaked Expectations). For any $1 < i < n$: if $E_Q^J(Q^{i-1}) > E_Q^J(Q^i)$, then $E_Q^J(Q^i) > E_Q^J(Q^{i+1})$; and if $E_Q^J(Q^i) < E_Q^J(Q^{i+1})$, then $E_Q^J(Q^{i-1}) < E_Q^J(Q^i)$.

Proof. Suppose $E_Q^J(Q^{i-1}) > E_Q^J(Q^i)$. By Lemma 2 this implies that $P(Q^{i-1}|Q^i) > \frac{1}{J^{1/n}}$. By Lemma 1, we know that $P(Q^i|Q^{i+1}) > P(Q^{i-1}|Q^i)$. Stringing these inequalities together yields:

$$P(Q^i|Q^{i+1}) \; > \; P(Q^{i-1}|Q^i) \; > \; \frac{1}{J^{1/n}}$$

But Lemma 2 again tells us that since $P(Q^i|Q^{i+1}) > \frac{1}{J^{1/n}}$, we have $E_Q^J(Q^i) > E_Q^J(Q^{i+1})$, as desired.

If $E_Q^J(Q^i) < E_Q^J(Q^{i+1})$, parallel reasoning establishes that $E_Q^J(Q^{i-1}) < E_Q^J(Q^i)$.
$\square$

We're now in a position to complete the proof of Optionality. Given an arbitrary k such that $1 \le k \le n$, we find a $J \ge 1$ for which $E_Q^J(Q^k)$ is maximal amongst the Q^i.

If $k = n$, then, by setting $J = 1$, Lemma 2 implies that $E_Q^J(Q^{n-1}) < E_Q^J(Q^n)$ iff $P(Q^{n-1}|Q^n) < \frac{1}{1^{1/n}} = 1$, which (by regularity) holds. Lemma 3 then implies that $E_Q^J(Q^1) < \cdots < E_Q^J(Q^n)$, as desired. Meanwhile, if $k = 1$, then sending $J \to \infty$ suffices, since this sends $\frac{1}{J^{1/n}} \to 0$, and once $P(Q^1|Q^2) > \frac{1}{J^{1/n}}$, Lemma 2 implies the $E_Q^J(Q^1) > E_Q^J(Q^2)$, and then Lemma 3 implies that $E_Q^J(Q^1) > \cdots > E_Q^J(Q^n)$, as desired.

Now consider the case when $1 < k < n$. Given the single-peaked expectations from Lemma 3, it suffices to show that there is a $J \ge 1$ such that $E_Q^J(Q^{k-1}) < E_Q^J(Q^k) > E_Q^J(Q^{k+1})$. By Lemma 2, this holds iff both $P(Q^{k-1}|Q^k) < \frac{1}{J^{1/n}}$ and $P(Q^k|Q^{k+1}) > \frac{1}{J^{1/n}}$. By Lemma 1 we know that there are $t, \epsilon > 0$ such that $P(Q^{k-1}|Q^k) = t < t + \epsilon = P(Q^k|Q^{k+1})$. Thus it suffices to show that there is a $J \ge 1$ such that $\frac{1}{J^{1/n}}$ is strictly between these thresholds—say, $\frac{1}{J^{1/n}} = t + \frac{\epsilon}{2}$. This holds iff $J = \frac{1}{(t+\epsilon/2)^n}$. Since $t + \epsilon/2$ is strictly between 0 and 1, likewise $(t+\epsilon/2)^n$ is as well, and hence $\frac{1}{(t+\epsilon/2)^n} \ge 1$ meaning that J can indeed match that value. At this value of J, $E_Q^J(Q^{k-1}) < E_Q^J(Q^k) > E_Q^J(Q^{k+1})$, completing the proof. $\square$

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