**The commonly ignored aspects of the history of symmetries.**

**Their link with intuitionist logic**

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… how to understand the status and significance of physical symmetries clearly presents a challenge to both physicists and philosophers. (Brading, Castellani, Teh 2017, last proposition)

*Abstract*:The obscure and punctuated history of symmetry is compared with the history of the celebrated and exalting notion of infinitesimal; some considerations about them are derived. A long list of odd and hidden events concerning symmetry in theoretical physics is offered. The last event is the discovery of the nature of the same word “symmetry” which pertains to non-classical logic, and it is linked to the principle of sufficient reason. A comparison of the roles played by the two mathematical techniques within a classical physical theory is performed. It leads to recognize the different roles played by the two mathematical tools as a manifestation of an incommensurability phenomenon caused by two foundational dichotomies.

*Keywords*: Symmetries, Infinitesimal analysis, Hidden historical events of symmetries, Odd historical events of symmetries, Modal logic, Intuitionist logic, Principle of sufficient reason, Markov constraints, Two dichotomies, Incommensurability.

**1. Introduction**

In theoretical physics there exist two mathematical techniques: the infinitesimal calculus which leads to differential equations and the symmetry, represented by groups of transformations, through which the invariants are found out.

The latter technique was born much later than the former one and with great difficulty. On the contrary, in present theoretical physics symmetries play a great role, maybe the greatest one. But this radical change in the relative importance of the two mathematical techniques did not lead scholars to re-construct their entire historical developments, nor a thorough reflection on the two fundamental questions: what bases and what meanings the two techniques have?

**2. A quick history of of infinitesimal analysis**

Let's quickly retrace their history, as first the history of calculus. Since the year 1635, Bonaventura Cavalieri and Evangelista Torricelli invented this kind of calculation by following a then lost method: they performed a calculation on a unlimited number of "indivisible" geometric elements of a figure, by conceiving them through the “geometric intuition”; and by justifying the use in mathematics of these indivisibles as "a way of speaking" (Drago 1997)[[1]](#footnote-1).

The current historical narrative presents the infinitesimal analysis as fell from the top of two geniuses, Gottfried Leibniz and Isaac Newton. This technique was strongly connected to metaphysics (e.g. an infinitesimal as the inverse of infinity). Unfortunately, a controversy over the priority between the two scientists has obscured the reflections on its fundamental aspects, although thi calculus was strongly linked to metaphysics (e.g. of the actual infinity). At present time one has to consider Leibniz as the only father of infinitesimal analysis.

After the birth of infintesimal analysis no other mathematical theory seemed at this high level, not even geometry (which Lagrange planned to reduce to calculus).

In order to find out an interpretation of his invention we go back to a meritorious work of one century ago (Brunschvigc 1912, pp. 208 ff.). Being impressed by the algebrization of arithmetic – where a number is replaced by a letter: e.g. *a, b, c, m, n*, etc. - Leibniz generalized this symbolization to an infinity of mathematical numbers according to his principle of continuity: he conceived an infinitesimal as the symbol of an infinity of acts of thought leading to conceive a “minimum” mathematical element. Of course, this notion of infinity is not arbitrary since it is constrained by some calculation rules.

However, this is a philosophical interpretation that does not explain why this conception of numbers as abstract ideas then suggests operative calculations. An operational interpretation of infinitesimal analysis is found out in Lazare Carnot writings on the subject (the unpublished L. Carnot 1786, pp. 201-262; L. Carnot 1813, “Appendix”). His main idea is "To generalize is to simplify"; therefore the search for solving inside a given mathematical system a problem is easier when an auxiliary variable generalizing the system is added. Once the solution has been obtained within the new system, the question is how to come back to the original system; as it is commonly said in infinitesimal analysis, one “suppresses” the auxiliary variable by means of a suitable mathematical operation.

About this “suppression” L. Carnot explained that there exist two methods in mathematics and therefore also in differential calculus: the analytic one, which idealistically considers the auxiliary variable as an infinitesimal quantity, regardless the metaphysical content this notion introduces into mathematics; but L. Carnot stressed that the same results can be obtained through the other method, the synthetic one, within which the auxiliary variable is exactly like the others, as long as it ranges with continuity from - ꝏ to + ꝏ; the very difference is rather in how the mathematical suppression operation is then chosen: either tu cut the calculation at the first approximation, or to apply a limit operation. By means of his book on the subject L. Carnot rightly believed that he had clarified the metaphysics that surrounded infinitesimal analysis up to then.

At his time Lazare Carnot's interpretation was very famous, but few decades later it was almost ignored. Rather, common textbooks of the history of mathematics give great relevance to the by Augustin-Louis Cauchy’s “reform of rigour”. Actually, the latter mathematician intensively applied the notion of limit previously introduced by L. Carnot. Moreover, he did not abandon the infinitesimals. In addition, the “rigour” of purging calculus of infinitesimals was actually acquired half a century later, when Richard Dedekind and Karl Weierstrass made clear the basic notion of a real number. Last but not least, this “rigour” of calculus was illusory. As Paul du Bois Raymond promptly remarked that approximations intervals, each defined by their two extreme points, cannot operatively produce, even if they are composed in a convergent series, to only one point, apart the optical illusion of the convergence of ever decreasing intervals; each interval, however little it is, is composed by an infinite number of points.(Kogbetlianz 1978, app. 2) Hence, even present undergraduate teaching presntes a “rigorous” calculus which instead includes the metaphyics of actua infinity. Thus, even present undergraduate teaching present as “rigorous” a mathematical technique including the metaphysics of the actual infinity.

At present time L. Carnot’s interpretation is confirmed by the fact that there exist two kinds of foundations of mathematics: the one, making use of the actual infinity (AI) and therefore has no scruples in idealizing the notions and principles of mathematics (e.g. the infinitesimals, Zermelo's axiom); and the other, making use of only the potential infinity (PI) and hence without metaphysics, requires that every mathematical object is the result of a construction of other mathematical objects (therefore it is called constructive mathematics). The operations of the two methods described by L. Carnot, the analytical one and the synthetic one, correspond to the two kinds of mathematics.

**3. Odd and hidden events occurred in the history of mathematical symmetries**

Let us consider the history and the foundations of symmetries.

The wide literature of reflections on them has not yet clarified their foundational basis (Brading, Castellani, Teh 2017, first propositions of chap. 5) Whereas the infinitesimals represent a mathematical search on *infinity* within continuous variables, symmetries represent a study of a *globality*, as it is appreciated when we see the symmetry of a mosaic or a church façade. To explain this globality in mathematical terms, it is necessary to take into account that an object may be seen from various viewpoints, also by adding some adjunctions to the object; that means that one has to introduce the transformations on the entire object (active transformations) or the transformations of the whole situation (passive transformations, as in special relativity).

These transformations, expressed in a mathematical way, introduce mathematical groups; which pertain to the algebraic attitude of mathematics, in contrast to the analytical attitude to which infinitesimal analysis pertains. In the history of mathematics these two attitudes have alternated and intertwined with each other.

Therefore the foundations of Mathematics elicit a dialectical relationship between the two terms of each couple of the following ones: notion of infinity/notion of globality, analytical/synthetic method, analytical/algebraic mathematics.

Western science has been until recent time mainly an analytical science; a global scientific attitude was introduced with difficulty a century after Newton, at the end of 18th century, by Lavoisier’s chemical theory, Lazare Carnot’s mechanics and Haüy’s crystallography; later, in 1824 by Sadi Carnot’s thermodynamics[[2]](#footnote-2) and at the end of 19th century Haeckel's ecology. All these theories had to wait decades (at least fifty years in the case of ecology) before they received some academic recognition; anyway they remained marginalized with respect to the most important scientific theories.

As a result, symmetries and even group theory gained an important role within science not before two century after calculus’ birth. Let us quickly review its development.

In the history of mathematics the birth of group theory is commonly recognized in the celebrated writings of Evariste Galois (1832). It is well-known the ostracism suffered by his papers (by Cauchy and Jean Baptiste J. Fourier), owing to his at all new mathematical (“ici on fait l’analyse de l’analyse”; Galois 2011, p. 252) and at the same time his revolutionary political ideas. These papers, concerning the use of the simplest group, the group of substitutions, introduced a sophisticated method of application. They waited twenty years to be re-discovered[[3]](#footnote-3). After that, mysteriously his papers had to wait some more years before their publication; which occurred after Cauchy tried again, but again unsuccessfully, to promote his own solution to the same problem.

In 1871 Felix Klein’s Erlangen program introduced group theory in order to classifying all geometries. But after this inaugural *lectio* of his academic settlement as full professor in Erlangen, he did not deal with the subject before Corrado Segre of Turin University translated Klein’s paper in Italian language (1891); in this occasion the essential, if not principal property of a group, never remarked in previous writings of all authors, was eventually declared; i.e. in a group each element enjoys the inverse. However, Klein never more pursued his program.

In meantime, his friend, the Swedish Sophus Lie, investigated the application of group theory to differential equations. He performed his research in a so rushing way and was so sad because the careleness by the mathematicians that in last years of his life he was hospitalized in a sanatorium.

**4.** **Odd and hidden events occurred in the history of physical symmetries**

Histroical textbooks credit the first introduction of symmetries in science to Haüy’s crystallography (Haüy 1784). However, few decades later his theory was attributed to mineralogy, as if its discrete symmetries were exotic mathematical techniques.

A century later, as a side effect of the current use of symmetries in crystallography, in 1894 Pierre Curie wrote a paper for introducing into physics this technique. But his illustration linked the topic to metaphysics. In the style of the metaphysics of Newton’s mechanics, based on (absolute space, absolute time and) force-cause, he finalized his paper to some propositions relating effects to causes, although a cause is not a physical notion. In addition, the logical structure of these propositions were the same of Gottfried Leibniz’s principle of sufficient reason, which at that time represented the metaphysics *par excellence*. No surprise if the paper was controversial. It originated a debate on its physical contents which after 120 years is still alive.

At last, in 1905 an introduction of a transformations group (Lorentz’ one) into theoretical physics was performed by Albert Einstein through his celebrated paper on special relativity. Yet, his paper never nominated the word “group”, apart one time for indicating a “collection”. In addition, in next time he neglected group theory; his subsequent research made use of differential operations (and the traditional notions of analytical mechanics: space, time, matter).

After Einstein’s introduction of Lorentz’ group into mechanics, it was necessary to recognize the group of constant velocity transformations; which was called “Galileian”, although Galilei had merely reiterated without any originality what had been suggested since a century and half before him by Nicholas Cusanus, Giordano Bruno and others; and although the merit of stressing the crucial role played by the relativity of motion within theoretical physics pertains mainly to Leibniz, who instead is rarely mentioned by textbooks on symmetries.

In 1924 Niels Bohr suggested to Hermann Weyl the study of the new theoretical aspects of quanta because Weyl was studying integro-differential equations. Instead, Weyl wrote a book which for a first time applied group theory to the theory of quanta; it was the first textbook of the new theory (Weyl 1928). Yet, a theorem of von Neumann stopped Weyl’s program because it showed that Weyl’s mathematics, essentially based on finite groups, is inappropriate to quantum mechanics. (Drago 2001). As a consequence of this theorem, Weyl abandoned this field of study.

It was the champion of the traditional, formalist Mathematics, Janos von Neumann, who suggested an instrumental approach to Eugene Wigner: to solve differential equations of quantum mechanics through group techniques. The latter one started this field of research and so much expanded it to suggest innumerable theoretical results of symmetries in general. Also Paul A.M. Dirac applied this technique for obtaining very important results; however he did not called it “group theory”, but “transformation theory”, since he only referred to the canonical transformations of Hamiltonian equations.

These unusual attempts to exploit a new mathematical technique in theoretical physics have been charged by some theoretical physicists (e.g. Schroedinger) to be a “pest”[[4]](#footnote-4).

In 1950 Weyl edited a fascinating and celebrated book on symmetries, ranging from ancient to modern symmetries. Surprisingly, he ignored Curie’s paper together with his “laws” and argumentations.

All in all, in the three decades 1930-1960 group theory did not enjoyed a great relevance in theoretical physics. Also because it was a common opinion that there exist “examples of abrupt symmetry changes” denying Curie’s propositions:

To quote just a few: the buckling of symmetrically loaded rods (Euler's *elastica*); the triaxial ellipsoidal shapes of fast rotating fluid masses in self gravitating equilibrium (Jacobi's ellipsoids) and the even more asymmetrical shapes discovered by Curie's colleague, Henri Poincaré (Poincaré's pears); the onset of turbulence in infinitely long circular pipes, etc. In all these cases symmetry breaks down when a scalar parameter reaches a critical value without the intervention of any visible asymmetrical cause, so that the isotropy group of the external action (represented by the equations and the boundary conditions) is larger than that of the resulting state, in contradiction to Curie's principle.(Radicati 1987, p. 202).

Yet, in the late 50’s, two Chinese physicists, [Tsung-Dao Lee](https://en.wikipedia.org/wiki/Tsung-Dao_Lee" \o "Tsung-Dao Lee) and [Chen-Ning Yang](https://en.wikipedia.org/wiki/Yang_Chen-Ning" \o "Yang Chen-Ning) dared to think that asymmetry may play a crucial role in elementary particles physics: their result was the violation of parity symmetry. This novelty originated a theoretical disrupt within physicists community: all scholars of the leading field of physical research of that time, elementary particles, rushed to study symmetries and group theory.

Pressed by the rush for new surprising results on modern physics, all they however did not recovered the entire past history. In fact, it is richer than the common view supposes. The first introduction of symmetries in theoretical physics was not performed by Curie, but, two centuries before him, Leibniz, who attempted an alternative mechanics to Newton’s one by basing it on the idea that “our minds expect conservations” (Leibniz 1698, par. 2). Moroever, one century before Curie, in 1783Lazare Carnot, completed this alternative mechanics; it obtained the invariants of the geometric groups of translations and rotations (L. Carnot 1783, sect. 22, p. 44)[[5]](#footnote-5) by means of the groups of the adjunctions called by him “geometrical motions” (i.e. motions not interacting with other bodies). In addition, the first historian seriously studying Lazare Carnot, Charles C. Gillispie, remarked (in his book about this scientist (Gillispie 1971 a) and also in the specific issue of his Dictionary: Gillispie 1971 b) that Carnot was pride to have generated “a new field of scientific research, intermediate between mechanics and geometry”. Yet, Gillispie did not qualify this field in the modern terms of group theory; that occurred twenty years later (Drago 1989).

As a fact, from L. Carnot’s mechanics started a new kind of theoretical physics, which through the work of his son, Sadi originated thermodynamics. In 1974 Herbert Callen recognized in modern thermodynamics “the Science of symmetries”, because he showed that *via* statistical mechanics each its variable represents a specific kind of symmetry (Callen 1974), a fact ignored along a century and half.[[6]](#footnote-6)

**5. Symmetry and intuitionist logic**

After the year 1960 the field of symmetries in physics proliferated into a great family: geometrical symmetries, (spontaneously or not) broken symmetries, gauge symmetries, static and dynamic symmetries, hidden geometries, etc. However, none investigated in logical terms on the word “symmetry”. Owing to Kantian prejudice for the indispensability of classical logic, previous scholars all ignored that the same word “sym-metry”, coming from the Greek “” = “measuring together”, means measuring (something) according to a modality (say, a proportion); hence, this word belongs to modal logic. In its turn, modal logic is equivalent *via* its S4 model to intuitionist logic (Hughes and Cresswell 1996, pp. 224ff.), where the double negation law fails. Ex. g. of this failure: a Court’s sentence: “Acquitted for insufficient evidence of guilty” **≠** “Acquitted for a loyally correct behavior”.

As a fact, the subject of symmetries is treated through many words each including two negations, whose corresponding affirmative ones lack of evidence: e.g. the words “in-variants“[[7]](#footnote-7) (stressed by Leibniz (1692) and L. Carnot since the first times of the symmetries in physics), “equivalence” (= not-difference), “in-distinguishability”, “im-munity“[[8]](#footnote-8), “in-difference between equivalent alternatives”, “ambiguity” = “not preferable to the other”, etc. An author defines it as a “due proportion”. (MacKay 1986, p. 19); another: “… the situation possesses the possibility of a change that leaves some aspect of the situation un-changed.”(Rosen 2008, p. 1) See also the definition of a conservation as an impossibility to measure an absolute (= not relative) magnitude.

Not only some words, but also the principles of symmetries are doubly negated.

Curie stated his results in the following way (Italic by Curie):

*Lorsque certaines causes produisent certains effets, les éléments de symétrie des causes doivent se retrouver dans les effets produits*.(Curie 1894, 401*)*

*Il n’est pas d’effet* [de symétrie] *sans causes* [de symétrie].

*Il n’est pas de cause* [de symétrie] *sans effets* [de symétrie].(ibidem, 414)

… the properties cannot be less symmetrical than the structure. (McKay 1986, p. 19).

More in general, Rosen established six principles of symmetry through many propositions. Apart three propositions, all his prinicples are either doubly negated or modal propositions[[9]](#footnote-9). (in the following propositions the Italic is by Rosen):

The Equivalence Principle. Roughly: *Equivalent causes — equivalent effects.* Precisely: *Equivalent states of a cause → equivalent states of its effect.* […]

The Symmetry Principle. Roughly: *The effect is at least as symmetric as the cause.* Precisely: *The symmetry group of the cause is a subgroup of the symmetry group of the effect*. […]

The Equivalence Principle for Processes. *Equivalent states, as initial states,* must *evolve into equivalent states, as final states, while inequivalent states* may *evolve into equivalent states*. […]

The Symmetry Principle for Processes. *The "initial" symmetry group (that of the cause) is a subgroup of the "final" symmetry group (that of the effect).* […]

The General Symmetry Evolution Principle. *For a quasi-isolated physical system the degree of symmetry cannot decrease as the system evolves, but either remains constant or increases.* […]

The Special Symmetry Evolution Principle. Usually: *The degree of symmetry of the state of a quasi-isolated system cannot decrease during evolution, but either remains constant or increases.*

Equivalently: *As a quasi-isolated system evolves, the populations of the equivalence subspaces (equivalence classes) of the sequence of states through which it passes cannot decrease, but either remain constant or increase.* […]

*The Symmetry Principle for Processes*. The "initial" symmetry group (that of the cause) is a subgroup of the "final" symmetry group (that of the effect). […] (Rosen 1995, pp. 191-192)

An Italian book lists some other principles:

… the properties cannot be less symmetrical than the structure.

Without a reason a symmetry cannot be created from nothing. (Castellani 2000, p. 67)

Similar problems have similar solutions (Castellani 2000, p. 71)[[10]](#footnote-10)

**6. The complementary role played by Infinitesimal analysis and symmetries within theoretical physics**

Why the birth of symmetries within science occurred so late and in a so difficult way? Surely, the notion of infinity (included in an essential way by the notion of infinitesimal) was more attractive than the idea of a pleasant, yet static figure, as before the 19th Century it was considered a symmetry. Moreover, the notion of infinity played a decisive role in eliciting the birth of modern science, as Akexander Koyré masterly stressed (Koyré 1957). For instance, Galilei discussed for a long time the question of the infinity in physics (Galilei 1638, Day I), whereas he devoted few remarks about the relativity of a motion occurring on a ship with respect to the shore.

Let us consider the relationship mathematics-theoretical physics. The dialectical role played by symmetries with respect to calculus could not be revealed before to have cumulate a great number of physical theories formalizing symmetries, so to easily compare them with the theories based on calculus. Asim O. Barut has the merit to have performed this comparison through a long list of physical theories, even the contemporary ones (Barut 1986). His conclusion is that the roles played by the two techniques are different; he defined their roles as complementary, alternative, etc..

However, he m,ade use of loose terms: he referred to “dynamics” instead of past dominant mathematical technique, “calculus”. Moreover, he ignored the symmetries within L. Carnot’s mechanics, Hauy’s crystallography and S. Carnot’ thermodynamics. By adding them to his list it is manifest that the role played by symmetries within the main theories of classical physics is alternative to the role played by calculus (e.g. within Newton’s mechanics). Which alternative?

First of all, since in L. Carnot’s and Haüy’s theories symmetries born as the result of an algebraic approach to theoretical physics, i.e. as an alternative mathematical technique to the already developed and then thiumphant calculus. More in general, the theories based on symmetries present an alternative in the kind of mathematics: i.e. an elementary mathematics instead of a mathematics relying on infinitesimals or other sophisticated notions of higher mathematics. At present time, the former kind of mathematics is included by the recent constructive mathematics, which makes use of no more than the potential infinity. This kind of mathematics is alternative to the classical mathematics, since the former one presents undecidabilities, which instead are ignored by the latter one, owing to its idealized notions having the power of overcoming a undecidable problem by means of an idealistic notion (as long as it does not lead to contradictions).

This alternative in the kind of mathematics of symmetries may be also specified in formal terms. It is manifested in S. Carnot’s thermodynamics, where the problem is the maximum in the efficiency of heat/work conversions. Calculus allows us to easily solve this problem of the search of a maximum of a function (in our case an input/output function) by means of the derivative of this function equated to 0. But in constructive mathematics the exact equality to 0 is undecidable. This fact gives reason why for stating his result Sadi Carnot rather argued in only a logical way, i.e. through an *ad absurdum* theorem (AAA). In addition, notice that often (also in L. Carnot’ mechanics) symmetries are based on the following argument: *ab* = 0 and *a* ≠ 0, both imply *b* = 0. This implication is contested by constructive mathematics, because there exist constructive algebras in which this implication fails. It is this constructive undecidability that obliges an author to argue in logical terms, through at last one AAA, before to pass to argue in the kind of mathematics which dismisses idealistic notions.

In addition, in the above we already considered L. Carnot’s mechanics; its texts present several doubly negated propositions (his definition of “geometrical motion” constituting a group; his results, i.e. the in-variants; the principle on which his mechanics relies, i.e. the impossibility of a motion without an end). Also S. Carnot’s thermodynamics is based on doubly negated propositions and moreover presents seven AAAs; at last, it applies the principle of sufficient reason (Drago, Pisano 2004). All that manifests that the two different roles played by symmetries and calculus according to Barut rely on the above illustrated different features of the two kinds of organization, respectively either the problem-based one or the deductive-axiomatic one.

The use of a different kind of logic of course implies an entirely new way of thinking. In particular, it introduces a theoretical organization which is different from the deductive-axiomatic one, managed by classical logic; it is aimed at discovering a new scientific method capable of solving a given problem; I call it a problem-based organization. Its general model has been recently discovered by a comparative analysis of those theories which in past times have been suggested by the respective authors according to a non deductive-axiomatic way (Drago 2012).

In this theoretical organization the arguing is based on AAAs; the final doubly negated proposition of each of them is not equivalent to the corresponding affirmative proposition, owing to the failure of the double negation law in intuitionist logic. Moreover, the principle of sufficient reason plays an essential role for translating the last doubly negated predicate into the corresponding affirmative predicate, to be then assumed as a new hypothesis from which one deduces all consequences to be tested with reality. In the past times the application of this principle produced metaphysical and also paradoxical consequences. For this reason it was disputed and most scholars considered it inappropriate to a scientific theory[[11]](#footnote-11).

Its role is more manifest when it (“Nothing is without reason”) is translated into a logical formula: *⌐ x ⌐R (x)*. Leibniz thought that it leads to state the affirmative corresponding proposition: *∀ x R(x).* Dummett’s table about the logical relationships of implication between each couple of intuitionist predicates (Dummett 1977, p. 29) shows that the change suggested by PSR implies the change of all intuitionist predicates into the corresponding affirmative predicates of classical logic. Hence, the resulting change represents the inverse one of the well-known (doubly) negative translation from the classical logic to the intuitionist logic (GKG translation; Troelstra, van Dalen 1988, pp. 56 ff.). Noitice that whereas the translation of real world into a probable or hypothetical world is not problematic, there is of course arbitrariness in translating, as the principle of sufficient reason does, a world of either guesses or probable hypotheses into affirmative propositions about real world. This arbitrariness was considered as unavoidable by past scholars. Yet, Andrey Markov suggested two constraints assuring a correct application of this principle to a predicate: the predicate has to 1) result from an AAA and 2) be decidable (Markov 1962, p. 5). This novelty attributes to the PSR a well-defined role within theoretical physics and in particular the theorization of symmetries[[12]](#footnote-12).

All that proves that symmetries pertain to a heuristic search of an inductive nature, to be formalized within the new theoretical organization which is alternative to the AO.

**7. Conclusion**

After more a century of cultural fight by the supporters of symmetries for attributing to them a role at the par of calculus within theoretical physics, the time is come to start a new history which no longer ignore the hidden events of the two mathematical tools of theoretical physics[[13]](#footnote-13).

By taking into account all the above results I conclude that the alternative between symmetries and calculus concerns *two dichotomies*, i.e. the kind of theoretical organization and the kind of mathematics. It is remarkable that Lazare Carnot introduced his mechanics by illustrating exactly these two dichotomies, although in intuitive terms (L. Carnot 1803, pp. xvii, 3; see also 1783, pp. 101-103).

Now we are in the position of defining in appropriate terms for the foundations of physics the notion of an incommensurability phenomenon of two physical theories insisting on the same field of phenomena; it is caused by a difference in the basic choices of the two theories on the above two dichotomies. It is then clear which is the origin of the particular alternative between two physical theories concerning the same field of phenomena but each relying on a different mathematical techniques, i.e. either symmetries or calculus; it is a manifestation of an incommensurability phenomenon, caused by their different basic choices. This fact gives reason of the hard resistance of past theoretical physicists, who get used of calculus since three centuries; along some decades almost all refused to receive a new mathematical technique which at glance appeared totally weird owing to a philosophical phenomenon of incommensurability.

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1. A (unfortunately posthumous) Torricelli’s paper proved the decisive result for obtaining a calculus from their infinitesimal technique: the inverse theorem, i.e. the derivative of an integral gives the original function (Torricelli 1644, p. 10). [↑](#footnote-ref-1)
2. The original thermodynamic theory of S. Carnot has been misrepresented both in their principles (the current first principle was born 25 years after the second one!), and in the way of approaching reality (its cycle and its *ad absurdum* theorem have always remained indigestible to didactics), so it has been considered as a theory that still had to earn its own specific technique of infinitesimal analysis (Born 1921, Sommerfeld 1962, "Preface"). [↑](#footnote-ref-2)
3. His turbulent life ended in a mysterious way: his death was caused by either a duel for defending the honorability of a not honorable woman, or his political will to sacrifice himself in order to give to his revolutionary friends the opportunity of exploiting the crowding of his funeral for promoting an insurrection against the oppressive Bourbons’ regime (Toti Rigatelli 1996). [↑](#footnote-ref-3)
4. (Pauli 1932). Both Schroedinger and Slater were two physicists that rejected Weyl’s mathematical approach. Weyl himself echoed a widespread criticisms received by his book, when in the “Preface” of the second German edition (1931) wrote: "It has been rumoured that the "group pest" is gradually being cut out of quantum physics"(p. x). See also Wigner, that denounces a past, “great reluctance among physicists toward accepting group theoretical arguments and the group theoretical point of view” (Wigner 1959, p. v). [↑](#footnote-ref-4)
5. Hon and Goldstein 2008 wrote a book attributing to a Legendre’s book of 1794 (concerning geometrical symmetries) the brth of symmetries in modern science. The authors ignored Haüy and L. Carnot. [↑](#footnote-ref-5)
6. Since some centuries the scientific enterprise is subject to the requirements of reproducibility and predictability. Joe Rosen stressed that these requirements essentially are invariants. However in these cases the transformations are not so clear. [↑](#footnote-ref-6)
7. I underline the negative words of a double negation in order to facilitate the reader in recognizing the inequivalence of the suggested words or propositions to their corresponding affirmative ones. Modal words are dotted underlined. [↑](#footnote-ref-7)
8. This word is only allusive to “invariance”, because its etymological meaning is rather “without obligation, without charge”. [↑](#footnote-ref-8)
9. Let us remark that the mathematical proposition are actually translations into classical logic of the most adequate propositions: “… it is not a greater group than …” . Moreover, notice that the author feels necessary to give two versions of almost all his principles; I interpret this fact as caused by his incertitude on the logical formulas of the principles. When instead makes use of mathematics (which is based on equalities instead of equivalences), he acquires certainty and makes use of affirmative propositions. [↑](#footnote-ref-9)
10. Each theory concerning symmetries includes many DNPs. For ex. Leibniz’ mechanics includes the following ones. All physical propositions are not necessary propositions but “contingent propositions” (= “whose contrary propositions do not imply contradiction.“ “Our mind looks for in-variants.” Inertia principle: “In-difference of a body to rest and motion.” The principle of the “Impossibility of a motion without an end.” The principle of sufficient reason: “Nothing is without reason.” L. Carnot’s mechanics includes the following DNPs: Principle of sufficient reason: “Nothing is without reason.”; « Il y a donc… dans chaque percussion ou communication du mouvement… une quantité qui n’est pas altérée par le choc ».(L. Carnot 1783, p. 34) Definition of geometric motion: “Tout mouvement, qui imprimé à un système de corps ne change rien à l’intensité de l’action qu’il exercent ou pourroint exercer les uns sur les autres si on leur imprimoit d’autres mouvements quelconque, seront nommés mouvement géométriques.” (L. Carnot 1803, sect. 136, p. 108) Also Haüy’s crystallography starts from the definition of a crystal as: “une simple structure sans organes et sans functions (particulières) [≠ unitaire], en un mot, un assemblage purement symétrique des molécules réunies successivement les unes aux autres par une force attractive,» (Haüy 1784*,* p. 48) [↑](#footnote-ref-10)
11. I know only Federigo Enriques who in past times attributed to this principle a specific role within scientific theories. (Drago 1988) [↑](#footnote-ref-11)
12. By incidence, let us remark that the translation of Curie’s propositions into logical formulas gives the same logical formula of PSR: *⌐ e ⌐c(e)* and *⌐ c ⌐e(c ).* (where *c* = cause and *e* = effects). Some authors think that they are equivalent. Yet, the functions *c(e)* and *e(c )* should be uniquely invertible functions (Katzir 2004, p. 36, fn. 2). [↑](#footnote-ref-12)
13. And thus the time is also come to reiterate as alternatives the two philosophical notions supported by the two main ancient philosophers, respectively invariants (Plato) and time evolution (Aristotle). [↑](#footnote-ref-13)