Philosophical consequences of the Gödel theorem

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Abstract

In this contribution an attempt is made to analyze an important mathematical discovery, the theorem of Gödel, and to explore the possible impact on the consistency of metaphysical systems. It is shown that mathematics is a pointer to a reality that is not exclusively subjected to physical laws. As the Gödel theorem deals with pure mathematics, the philosopher as such can not decide on the rightness of this theorem. What he, instead can do, is evaluating the general acceptance of this mathematical finding and reflect on the consistency between consequences of the mathematical theorem with consequences of his metaphysical view.

The findings of three mathematicians are involved in the argumentation: first Gödel himself, then the further elaboration by Turing and finally the consequences for the human mind as worked out by Penrose. As a result one is encouraged to distinguish two different types of intellectual activity in mathematics, which both can be carried out by humans. The astonishing thing is not the distinction between a formalized, logic approach on the one side and intuition, mathematical insight and meaning on the other. Philosophically challenging, however, is the claim that principally only one of these intellectual activities can be carried out by objects exclusively bound to the laws of physical reality.

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1. Introduction

Looking at the program of this symposium and reading the title of the talks could easily provoke certain astonishment: why should one bring all these things together? What has physics, mathematics in common with philosophy or even theology? Are we are not mixing disciplines which should not be confused and ending up in a big mess without scientific value. To make this point more clear, why should or even principally could mathematics change a pure philosophical argument?

In order to answer these questions one could start by observing the human desire to bring all knowledge about reality together in a single, consistent view. It is the dream of the philosophers of all times to create such a view, a consistent metaphysics without internal contradiction and in accordance with experience. It is based on the assumption that reality is by definition consistent, complete, without contradiction and accessible to the intellect.

Already in the times of Aristotle the possibility of a sound metaphysics was rejected by the skepticism of sophists, who denied the possibility of finding the truth. Aristotle argued that the fundamental statement of philosophical skepticism, that it is not possible to affirm anything truly, is already an absolute statement not demonstrable by science. By accepting this statement as an absolute truth, one is therefore in contradiction with its meaning[1]. The search for a unifying metaphysical view, therefore, is challenging, but in my opinion the only one in accordance to human intellectual dignity.

Reflecting on the huge amount of information provided by his senses man is confronted with a very fundamental question: is there a reality that can not be reached by senses even not with the aid of sophisticated instruments? Is there a reality, which does not obey the physical laws of nature? This is certainly a philosophical question that is answered differently in different metaphysical systems, but as will be shown below, certain experiences support or make difficult certain metaphysical systems. For example, Aristotle based his metaphysics on the everyday experience of the occurrence of real changes. In the philosophy of Parmenides these changes were considered to be only apparent and not based on reality. His philosophy, therefore should and has in fact be disregarded.

Today the empirical facts have largely been increased, and, what is probably more important, these facts are only accessible to a small number of well-prepared specialists. How can a philosopher understand these new empirical facts – often only expressed in the highly formalized language of physicists, mathematicians or other scientists - and evaluate their relevance with regard to his metaphysics? Not to speak about a scientific discussion on the validity of these facts.

In the following an attempt is made to analyze an important mathematical discovery, the theorem of Gödel and to demonstrate that the acceptance of this theorem has a direct impact on the consistency of metaphysical systems. It will be shown that there are elements of reality, in this case the human intellect, that can perform actions that exceed the potential of operators exclusively subjected to physical laws.

To make it very clear, the Gödel theorem is in the field of pure mathematics. The philosopher as philosopher can not decide on the rightness of this theorem. What he, instead can do, is evaluating the general acceptance of this mathematical finding and reflect on the consistency between consequences of the mathematical theorem with consequences of his metaphysical view.

In the following the contribution of three mathematicians will be presented: first Gödel [2] himself, then the further elaboration by Turing [3] and finally the consequences for the human mind as worked out by Penrose [4]. Schins[5],[6] presented in several contributions the basic ideas of these authors. In this paper often use will be made of his argumentations. Finally we will present a philosophical evaluation

2. The Gödel incompleteness theorem

In order to grasp the impact of the work of Gödel one has to consider the foundation of mathematics at the beginning of the 20th century. There was a strong school considering mathematics just a product of human logic. In this view it is principally possible to arrive at any mathematical truth by applying exclusively a set of axioms and formal deduction. The German mathematician David Hilbert expresses this conviction by: *In Mathematics there is no Ignorabimus* [7]. He formulated in 1900 the 10 last real problems of mathematics to be solved. The last of these, the *Entscheidungsproblem*, concerned the existence of an algorithm able to solve general classes of mathematical problems[8]. The solution of this problem would be the triumph of the formalistic approach: just following an algorithm, i.e. a sequence of deductive steps one would arrive on the solution. The question of meaning would be irrelevant; as long as the formal procedures would be used one would arrive at the mathematical truth.

In his communication of 1931, Gödel[2], an Austrian mathematician, demonstrated the fundamental inadequacy of the pure formal approach. He proved that in every consistent, sufficiently general axiomatic system (i.e., systems which are based on axioms and specific rules of deduction)

- there always exists a true proposition which cannot be deduced from the axioms (Gödel's incompleteness theorem);
- the consistency of the axioms cannot be deduced from the axioms (Gödel's consistency theorem).

In both theorems Gödel states that a certain mathematical truth cannot be obtained exclusively by deduction, i.e. exclusively by a formal approach. Mathematical meaning and mathematical truth are relevant concepts not reducible to formal logic. To prove his incompleteness theorem, Gödel provides a proposition whose correctness and truth everyone with a minimum of mathematical education can confirm, but which is undecidable within the formal system in question. It is here not the place to follow in detail the proof of Gödel; it is, however, worthwhile to express in words the so-called Gödel proposition in the incompleteness theorem: *there exists no proof for the Gödel proposition*.

In order to get a taste of Gödel's argumentation one can start with the observation that the Gödel proposition is legitimate (as is demonstrated formally by Gödel). There are now two alternatives, there is a proof for this proposition or not. If there is a proof, a legitimate proposition that is formulated according the axioms of the formal system is wrong. Consequently there is a contradiction within the axiomatic system which is impossible and the assumption has to be rejected. If there is no proof, the proposition is true and the incompleteness theorem is correct. This is the only alternative without formal contradiction. One is therefore forced to accept the validity of a proposition which, being true, is undecidable and cannot be proven in the formal system.

It is not so easy to realize the impact of the work of Gödel. What directly is shown is that mathematics goes beyond applying deductive or formal steps, also insight, intuition or meaning on a higher level come into play. Further, no formal system is complete, as a mathematician can always provide true statements that are formally improvable.

A first important conclusion can already be drawn now. For some physicist, the reduction of physical reality to a single theory or even a single formula would be the ultimate triumph of science. A physical theory, however, or a formula are part of a formal system, that after Gödel we know to be necessarily incomplete. Reality and also physical reality is by definition complete. The Grand Unified Theory (GUT) and other ultimate approaches will be necessarily incomplete and inadequate to describe the full richness of physical reality.

3. The contribution of Turing

In 1936 Turing, a mathematician inspired by Gödel, conceived the ultimate computer, the Turing machine[3]. We are used to the fact that every three years we have a new computer with largely improved capacity. The Turing machine is the end of any possible evolution with regard to software as well as hardware: with infinite speed and memory, using digital logic, neural networks, quantum logic or any other up to now known or unknown technology for data processing. It is the best what physical laws and the most advanced design can offer with in addition unlimited speed and unlimited memory. With this machine any finite sequence of processing steps could be carried out in an infinitesimally small amount of time.

Turing posed himself the question and gave an answer with regard to the halting-problem. Is there a general algorithm that can predict whether the Turing machine will stop with a given program and given input? If that would be possible, one could look for certain prepositions and demonstrate, whether they are of the Gödel-type or not. Once these have been identified, one could isolate them and could continue with a ‘clean’ formal system. Turing showed that this algorithm does not exist, the algorithm that should mark Gödel-type propositions would be caught in an infinite loop in the Turing machine.

What can one learn from Turing?[9] Scientists and fiction writers are speaking about intelligence in man-made apparatus. There is a clear evolution: increasingly processing speed, memory and code complexity is introduced. But even after evolution and many centuries of future research there will be mathematical problems, these artificial intelligence (AI) devices can not solve. For the Turing machine, which exceeds these AI devices by far, has been shown to never come up with a result on well-defined algorithmic problems.

The question remains, would it not be fantastic if human beings would have the capacity of the Turing machine. Would this be a step forward or something destroying humanity in its very root? To answer this question one should not forget that the

Turing machine is the best matter or beings exclusively bound to physical laws can deliver.

4. Penrose and the human mind

In his book *The Emperor's new mind* Penrose[4] uses the results of Gödel and Turing to analyze the cognitive activity of the human mind. His argumentation is based on meta-mathematics, a level achievable by mathematicians but not exclusively bound to formal argumentation. His argumentation starts with two observations well within the field of mathematics.

1) There are mathematical results on the truth of certain prepositions, which can be recognized by any mathematician, but can not formally be proven (for example the Gödel preposition).

2) In addition, he learns from Turing, that the ideal computing device, the so-called Turing machine, can only be used for solving problems by a formal approach.

Penrose explains:

The point of view that one can dispense with the meaning of mathematical statements, regarding them as nothing but strings of symbols in some formal mathematical system, is the mathematical standpoint of formalism. Some people like this idea, whereby mathematics becomes a kind of 'meaningless game'. It is not an idea that appeals to me, however. It is indeed 'meaning'—not blind algorithmic computation—that gives mathematics its substance[10].

In drawing a conclusion from his meta-mathematical observations, Penrose leaves mathematics and enters the field of anthropology. And his conclusion is that the human mind is able to carry out certain activities that artificial devices, including the Turing machine, can not. It is not the question whether the human mind is superior in all aspects to the Turing machine, for example in computing speed even present-day computers exceed by far human capacity. The remarkable is that man can do something, for example grasping the meaning of a mathematical truth, a machine never can do. Penrose writes:

Mathematical truth is not something that we ascertain merely by use of an algorithm. I believe, also, that our consciousness is a crucial ingredient in our comprehension of mathematical truth. We must 'see' the truth of a mathematical argument to be convinced of its validity. This 'seeing' is the very essence of consciousness. It must be present whenever we directly perceive mathematical truth. When we convince ourselves of the validity of Gödel's theorem we not only 'see' it, but by so doing we reveal the very non-algorithmic nature of the 'seeing' process itself[11].

The significance of this 'seeing' a truth becomes even clearer if one uses the equivalent Latin verb *intueri* or the derivative 'intuition'. The latter points to a special intellectual activity that nearly instantaneous in a creative way is able to understand or conceive a solution to a problem. The foundation of this peculiar intellectual capacity is, according to Penrose, related to consciousness. One should, however have in mind that investigating this foundation one is not working any more in the realm of mathematics or even science in general, but more within anthropology or philosophy of man and metaphysics.

What about artificial intelligence? In the light of Penrose's conclusions one could speak of a misleading term. A person who is able to memorize very well or to solve certain formal problems in a rapid way, but who is missing insight and understanding is not considered to be intelligent. And the artificial devices like the Turing machine exclusively based on physical laws, can do it perhaps more rapidly, but with the same fundamental lack of insight. In stead of AI one should speak of ADP: artificial data processors.

5. Philosophical evaluation

As already stated in the introduction, the ultimate goal of the philosopher is to find an overall consistent view where all empirical knowledge could be integrated. With the work of Gödel, Turing and Penrose one is confronted with two different types of intellectual activity in mathematics, which both can be carried out by humans. The astonishing thing is not the distinction between a formalized, logic approach on the one side and intuition, mathematical insight and meaning on the other. Being surely relevant for the philosophy of mathematics it does not have strong implications for the overall view on reality. Philosophically challenging, however, is the claim that principally only one of these approaches can be carried out by devices bound exclusively to the laws of physical reality. Turing machines, namely, are not able to grasp issues related to mathematical meaning, and these machines are the best physical laws can offer

Accepting the evidence provided by mathematics the philosopher should allow for two aspects of reality in his metaphysics: one exclusively bound to physical laws and the other not. A plain, ordinary materialistic view considering reality restricted to things that are exclusively subjected to physical laws is not able to incorporate the distinction provided by mathematics. The well-known quote by Karl Marx, *Der Mensch ist, was er ißt*, (The human is that what he eats) is an example of a metaphysical statement that refutes this distinction: a human being is like his food, i.e. something exclusively bound to physical laws of nature.

Accepting the distinction suggested by mathematics the philosopher could provide more information about the ontological base of humans that allows them carrying out intellectual activities that Turing machines and other objects subjected exclusively to physical laws are not able to do. The principle of causality expressed in the classical statement *agere sequitur esse* (acting is according to the way of being) indicates that if an activity is observed that exceeds the potential physical laws can offer, there should be a subject exceeding the potential physical laws can offer. Already some of the Greek philosophers like Plato and Aristotle made a distinction between a material and spiritual reality. Especially Aristotle and later Aquinas studied the intellectual activity of human beings and found, interesting enough, a similar distinction in two principal approaches: strict logical argumentation with syllogism, a kind of formalized, deductive or inductive mental activity and intuition or in Greek *theoria* that accounts for insight and creativity.

It is interesting to note that in the metaphysics of Aquinas an intellectual activity is possible in pure spiritual beings and in man, who is composed of a spiritual and material component. According to Aquinas it is the material component in man that is responsible for the formal, logic intellectual activity with the aid of syllogism, whereas pure spiritual intellects know only by intuition. This is a remarkable parallelism with the findings in mathematics. A Turing machine, being a pure material device, can do the formalized processing, whereas only the human mind with his spiritual dimension according to Aquinas is able to carry out the intuitive intellectual activity needed to grasp mathematical meaning.

Aristotle like Aquinas did not know the Gödel theorem but arrived by a different analysis at the same conclusions as Penrose. This is, of course, no proof of the validity of the metaphysics of Aristotle and Aquinas, but an important consistency check with experience. There are, of course many other checkpoints, but after Gödel, Turing and Penrose this specific one provided by mathematics should be taken seriously, leading in some cases to the abandonment of certain metaphysical views, as, for example, provided by plain materialism.

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[1] See Aristotle, Metaphysics, Book XI - Part 5.
[2] K. Gödel, *Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme, I,* Monatshefte für Mathematic und Physik, 38, p. 173, 1931.
[3] A. Turing, *On Computable Numbers with an Application to the Entscheidungsproblem*, Proc. London Mathem. Soc., 42, pp 230-265, 43, pp. 544-546, 1936-1937.
[4] R. Penrose, *The Emperors New Mind*, Oxford University Press, Oxford, UK, 1989.
[5] J.M. Schins, *Mathematics: a Pointer to an Independent Reality,* in A. Driessen and A. Suarez (Eds.) *Mathematical Undecidability, Quantum Nonlocality and the Question of the Existence of God,* Kluwer Academic Publishers, Dordrecht, NL, 1997, pp. 49-56.
[6] J.M. Schins, *Hoeveel geest kan de wetenschap verdragen?* Agora, Kampen, The Netherlands, 2000.
[7] D. Hilbert, *Gesammelte Abhandlungen*, III. Band, pp. 297-298, Springer Verlag, Berlin, 1935.
[8] D. Hilbert, *Mathematical Problems (tenth problem),* 2nd Intern. Congress of Mathematics, Paris, France, 1900.
[9] F. Cacace, *Meaning, Reality and Algoritms:Iimplications of the Turing Theorem*, in A. Driessen and A. Suarez (Eds.) *Mathematical Undecidability, Quantum Nonlocality and the Question of the Existence of God,* Kluwer Academic Publishers, Dordrecht, NL, 1997, pp. 27-40.
[10] ref. 4, p. 105.
[11] ref. 4, p. 418.