A valid conjunction principle for fallible knowledge

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The *multi premise closure principle* states that the logical conjunction of known facts yields again a known fact. For absolute knowledge this principle holds. We show that for *fallible* knowledge, assuming knowing requires a minimum level of statistical certainty (whatever else it requires), and that there is a sufficient number of known facts above a given level of ***un***certainty, it does *not* hold, for simple statistical reasons. We present a modified version, the *dependent conjunctive closure principle*, that *does* hold.

1 Introduction

The multi premise closure principle states that the (finite) conjunction of known facts yields again a known fact. For absolute knowledge this is true; for fallible knowledge the principle has been attacked through a series of paradoxes. These paradoxes assume that knowledge requires a minimum level of statistical certainty.

- The **lottery paradox** assumes a large fair lottery with a single prize – large enough to make it a virtual certainty, and thus known, that a given ticket will not win. Yet, it is also known that *either ticket 1 wins, or ticket 2 wins, or ticket 3 wins, or .. or ticket n wins* – where n is the number of tickets in the lottery.
- The **preface paradox** envisions someone finishing a huge, carefully-researched, work of factual information, and writing in the preface: I know there are errors in this book, and I apologise for them.
  A recent variant of the preface paradox strengthens it by having the book be *about* errors in scholarly works, and concluding that *any* such book above a certain size is bound to contain errors.
- The **pill paradox** (Backes 2019) tells the story of a researcher inventing and ingesting a pill that is guaranteed to alter some – very few – beliefs randomly into false ones. The researcher now *knows* that the conjunction of his beliefs is false.

Solutions to these paradoxes have generally attempted to put additional conditions on knowledge. Merely inferring statistically that the probability of winning is low would not amount to *knowing* the ticket is a loser, or merely knowing in general that all facts in the book have some chance to be false does not amount to *knowing* that their conjunction is false. Obviously, proposing additional conditions on knowledge is a rich field, with endless opportunities. Those are all bound to fail in the end, however, unless they restrict the field of “knowledge” to an impractical extent. Section 2 will show this, and section 3 will present another, valid, principle to take the place of the multi premise closure principle. Section 4 will discuss the utility of stricter valid closure principles.

2 The closure principle breaks down.

Be c^- the minimum certainty required for something to be called *knowledge*. Whatever other conditions may be required for knowledge, *at least* a known fact should have a probability no less than .c^-

Let $F_1..F_n$ be a number of known facts, each of which has a certainty of at most c^+, with those probabilities independent, i.e. uncorrelated. This means:

1. $c^- \le c^+$, as otherwise the facts would not be known.
2. $P\left(\bigcap_{i=1}^{n} F_i\right) = \prod_{i=1}^{n} P(F_i) \le \prod_{i=1}^{n} c^+$, because of the independence of the probabilities.
3. $n > (c^+ \log c^-) \Rightarrow P\left(\bigcap_{i=1}^{n} F_i\right) < c^-$, i.e. $P\left(\bigcap_{i=1}^{n} F_i\right)$ is not known, even though each F_i is known.

So for the closure principle to hold, for any c^+, the number of facts with $P(F_i) \le c^+$ must be smaller than $c^+ \log c^-$, which means that with a good number of facts, almost all of them must have an extremely high probability – a criterion that is not in general met, which means that the multi premise closure principle is false[2].

Does this mean that there is no conjunctive closure principle for fallible knowledge? No – it turns out that there is another closure principle, that will be introduced in the next section.

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2 This is of course not the strictest requirement for the principle to hold. For that, see section 4 below.

3 A new closure principle

Elsewhere[3] I present the *can/ken foundation* for knowledge – a source of often fallible yet reliable knowledge. Does this presentation require revision in the light of the above? It turns out it does not, for the simple reason that where it presents a conjunction of facts as known, the uncertainties connected to those facts are not independent. In fact they correlate fully.

Imagine I have a copy of the Encyclopedia Britannica and am asked questions of the form: *What word is on page x, line y, position z in the Encyclopedia Britannica?* I can give the answer, because I have the source and can simply look it up. Now someone asks whether the conjunction of all those facts is still known. The answer depends on two things: my fallibility in looking up and producing the answer, and the fallibility of the source – maybe I have a fake, a mock-up of the real encyclopedia.

Obviously, my fallibility makes the answers prone to the argument against the multi premise closure principle. But what if I were infallible? Then there is the issue of the encyclopedia – is it the real one? But since I have been using the same copy all the time, those uncertainties in the individual answers don't grow by conjoining them – the certainty that the conjunction is correct is the same as the certainty that any of the answers is correct.

So we have the **dependent conjunctive closure principle**: *if the uncertainties of the individual bits of knowledge correlate fully and positively, then the conjunction of known facts yields a known fact.*

4 Stricter closure principles.

The dependent conjunctive closure principle is not the strictest closure principle that can be formulated. In section 2 above we saw that there was some leeway even for fully uncorrelated conjuncts, if their individual probability was sufficiently higher than the minimum probability required for knowing. It would be possible to work out a formula that gave the precise conditions under which conjunctions could still be considered known.

However, this would be of little use. Suppose we found a limit p for the number of conjuncts in a given situation. Then any conjunction with fewer than p conjuncts would be known – but what about a conjunction of such conjuncts? The resulting principle would not be transitive, and depend on probability distributions that are seldom met, and even more seldom *known* to be met.

In the end the requirement is simply that the probability of the conjunction of *all* known facts is higher than the threshold c^-. In the case of independent facts, this means that the product of the probabilities of *all* known facts exceeds c^-. In cases of partial dependency between the probabilities there is more leeway, and maybe there are a few cases worth being formulated on their own. But I am fairly convinced that the most useful and most easily applicable of those will be the principle for fully dependent facts, the *dependent conjunctive closure principle*, as it is the only one that does not involve limiting the number of known facts, or the probabilities of those facts beyond what the criterion for knowing already does[4].

5 References

Backes, Marvin (2019). *A Bitter Pill for Closure*. Synthese 196, pp. 3773-3787.

3 In my *Knowing in the Teeth of the Diallelus*.

4 One slight extension that might be useful is allowing a finite collection of independent or partly dependent sets of fully interdependent known facts, such that the probability for a conjunction containing precisely one member of each set does not fall below the threshold. For fully independent sets this would mean that the product of the probabilities for each set exceeds c^-.