The Metaphysics in Counterfactual Logic

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Abstract

This paper investigates the metaphysics contained within higher-order counterfactual logic. I prove the necessity of identity and distinctness as well as vacuism, from which it follows that all counteridenticals are true. I then prove the Barcan, its converse, Necessitism and the Being Constraint. I show how to derive the Identity of Indiscernibles, before discussing maximalist ontology—a plenitude so expansive that some have claimed it is inconsistent. I establish that maximalism is equivalent to the collapse of the counterfactual conditional into the material conditional. This, in turn, is provably equivalent to the modal logic TRIV. Because TRIV is consistent, maximalism is, surprisingly, consistent. I close by arguing that stating the Limit Assumption requires shifting from a first-order to a higher-order counterfactual logic.

Introduction

Over the past few decades, philosophers have systematically investigated counterfactual and higher-order logic. Given the theoretical applications of these systems, this focus is unsurprising. Counterfactuals figure in debates ranging from the necessity of mathematics to decision theory—and form the basis for prominent analyses of causation.[2] Higher-order logic, for its part, has shed light on debates ranging from Leibniz's Law to propositional granularity to grounding.[3] The uses for these systems are undoubtedly broad.

What is surprising is how little has been written on their interaction. Currently, there is no literature on systems that describe what would have been the case and quantify over terms in any syntactic category.[4] To date, higher-order counterfactual logic does not exist.

I aim to remedy this oversight. The ensuing system—which I dub 'HOCL'—governs the logic of higher-order counterfactuals: those that embed a higher-order claim in either

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[2]For their use in the necessity of mathematics, see Yli-Vakkuri and Hawthorne (2020). For their use in decision theory, see Bradley and Steffánson (2017). For their use in theories of causation, see Lewis (1973*a*).

[3]For discussions of its implications for Leibniz's Law, see Bacon and Russell (2019); Caie, Goodman and Lederman (2020). For discussions of its implications for grounding, see Fritz (2021, 2022); Elgin (2024). For discussions of its implications for propositional identity, see Dorr (2016); Bacon and Dorr (2024).

[4]However, two papers that use some form of higher-order inferences in counterfactual logic are Goodman and Fritz (2017) and Kocurek (2022*b*). To the best of my knowledge, this list is exhaustive.

their antecedent or consequent. Natural language has plausible examples of higher-order counterfactuals. For example, 'If Sarah and Jane had nothing in common, they would not both be Norwegian' appears to assert that if there were not to exist a property borne by both Sarah and Jane, then they would not both bear the property *is Norwegian*. One reason to study this system is to understand the logic governing these sentences. However, it is worth noting that HOCL does not *merely* govern the logic of higher-order counterfactuals. It not only regiments counterfactuals that embed higher-order claims, but also counterfactuals that are themselves embedded in higher-order inferences. For example, from 'If it were raining, the street would be wet' it follows that there exists a relation between 'It is raining' and 'The street is wet.'

My primary focus is metaphysics, rather than the logic of natural language. Originally, I focused solely on the higher-order aspect of this system. However, it quickly became clear that there are significant—yet underdeveloped—first-order implications as well. So, after discussing the counterfactual definition of necessity—and after axiomatizing HOCL—I discuss its implications for the necessity of identity and distinctness, counteridenticals, the Barcan and Converse Barcan, and Necessitism. In these debates, the higher-order aspect of this system is largely (though not entirely) auxiliary. While there are higher-order instances of these theorems, versions of many could be stated in a language with quantifiers that only range over objects. I then turn to debates where higher-order quantification is indispensable: the Identity of Indiscernibles, Maximalism and the Limit Assumption.

A quick note on this project's aims: nearly every assumption I make about counterfactual logic is controversial. Moreover, what these assumptions entail is equally controversial. There is thus ample room to reject HOCL. But there is a sense in which dissidents need not disagree with anything that I say. I do not claim that HOCL settles debates correctly—nor that its axioms are true. Rather, I take it to be a natural starting point (perhaps even *the* natural starting point) for reasoning about higher-order counterfactuals. Those who would rule differently ought to employ a different logic—and even those who ultimately reject this system may find it illuminating to understand how it works.

Modality and Counterfactuality

I make a number of logical assumptions throughout this paper. I assume that classical logic is true. I assume that sentences certified by truth-tables to be true (in the standard way) are indeed true—and that sentences so-certified to be false are indeed false. I further assume that the results of valid arguments carried out in first-order logic are true if their premises are true; that is, I assume that the conclusions of sound arguments are true.

I do not mean to suggest that this assumption is incontrovertible. Every assumption I make is open to dispute—and my commitment to classicality is no exception. Nevertheless, I will provide no defense of it here. Those who deny classical logic need read no further.

I also assume the counterfactual definition of necessity, which I dub 'Definition$_1$':

$$\Box p := \neg p \,\Box\!\!\rightarrow \bot$$

For p to be necessary is for it to be the case that, if p were false, the absurd would be true.[5] Definition$_1$ is an immediate consequence of the Lewis (1973*b*)/Stalnaker (1968) semantics for counterfactual conditionals—which holds that sentences of the form 'If p were true then q would be true' hold just in case the closest possible world(s) in which p are true are world(s) in which q is true. After all, if p is true in every possible world, then the closest possible world in which p is false is an absurdity.

I do not assume that the Lewis/Stalnaker semantics is correct—nor do I assume that it is incorrect. While the widespread appeal of their accounts offers some support for Definition$_1$, an arguably stronger motivation occurs in Williamson (2007*b*)—who notes that it follows from the weakest standard modal logic—K—and the following two principles:

Necessity: $\Box(p \rightarrow q) \rightarrow (p \,\Box\!\!\rightarrow q)$
Possibility: $(p \,\Box\!\!\rightarrow q) \rightarrow (\Diamond p \rightarrow \Diamond q)$

Necessity asserts that strict implication entails counterfactual implication; if it is necessary that if p is true then q is true, then if p were true then q would be true. **Possibility**, for its part, asserts that anything counterfactually implied by a possible proposition is itself possible; if it is possible for p to be true, and if p were true then q would be true, then it is possible for q to be true.

When I first encountered **Necessity**, it struck me as overwhelmingly plausible.[6] Appealing to it requires more than conditional truth at the closest possible worlds—truth at *all* worlds is needed. Suppose we were to canvas the entirety of modal space, and found not even a single world in which p is true and q is false. In this case, every possible p situation is a q situation—so, if it were to be the case that p, then, surely, it would be the case that q.

I now recognize that matters are not so simple. **Necessity** takes a stand on a contentious debate. In particular, it presupposes *vacuism*—the claim that all counterpossibles (counterfactuals with impossible antecedents) are true. Take an arbitrary proposition p that is necessarily false. Because p is false in every possible world, the conditional $p \rightarrow q$ is true in every world (for every q). So, $p \rightarrow q$ holds necessarily. **Necessity**, then, entails $p \,\Box\!\!\rightarrow q$.

[5]Two other potential counterfactual definitions of necessity are $\Box p := \neg p \,\Box\!\!\rightarrow p$ and $\Box p := \forall q(\neg p \,\Box\!\!\rightarrow q)$. As Williamson (2007*b*) established, minimal assumptions entail that the three are equivalent. There is thus no need to choose between competitors. Opting for the formulation I have is merely a stylistic preference.

[6]**Possibility** also strikes me as overwhelmingly plausible. Even Lange (2009), who is loath to commit to any generalizable principles of counterfactual logic, repeatedly endorses **Possibility**. If we determine that a proposition p could be the case, then the counterfactual 'If p were true then q would be true' takes us from one possibility to another. I note, however, that Williamson (2020) denies **Possibility** on the grounds that there could be contextual shifts between 'If p were true then q would be true' and 'It is possible that p.'

Since this holds for every impossible p and for every q, **Necessity** validates vacuism.

Vacuism is controversial—and rightly so. Nonvacuists (who hold that at least some counterpossibles are substantive) maintain that their view better accords with ordinary judgments.[7] Intuitively, the sentence 'If paraconsistent logic were true, then Graham Priest would be incorrect' is false (since Priest has offered an impassioned defense of paraconsistent logic), but vacuists must maintain that it is true.[8] After all, its antecedent could not possibly obtain.[9]

Despite the existence of challenging cases, many paths lead to vacuism. Dominant accounts of counterfactuals—like the Lewis/Stalnaker—hold that it is true. If there are no worlds in which p is true then, trivially, all of the closest p worlds are q worlds. As Lewis said, "Confronted by an antecedent that is not really an entertainable supposition, one may react by saying, with a shrug: If that were so, anything you like would be true!" (Lewis, 1973*b*, pg. 24). Beyond the appeal of particular semantic accounts, vacuists typically highlight the theoretical virtues of their position.[10] Quite generally, vacuist systems are simpler and more elegant than their nonvacuist counterparts.

HOCL is deeply committed to vacuism, but this will only become clear after the system is formalized. For the moment, suffice it to say that **Necessity** offers some support for Definition$_1$, but there remains room for disagreement—as nonvacuists ought to reject it.

We can then define possibility in terms of necessity—a connection I dub 'Definition$_2$':

$$\Diamond p := \neg\Box\neg p$$

For p to be possible is for it to be the case that it is not necessary that p is false. Definition$_2$ is standard in the literature.[11] If we interpret 'Necessarily p' as the claim that p is true

[7]Examples of nonvacuists include Zagzebski (1990); Nolan (1997); Brogaard and Salerno (2007); Kment (2014).

[8]At least, on the assumption that paraconsistent logic is not merely actually false, but necessarily false.

[9]A stronger motivation for nonvacuism is given by Jenny (2018)—who argues that mathematics employs substantive counterpossibles. There are pairs of problems p and q, such that neither p nor q are computable (in that no algorithm given a finite time could solve them), but that q is computable relative to p (in that any solution to p generates a solution to q). For example, although neither the validity problem for First-Order Logic nor the halting problem are computable, the validity problem is computable relative to the halting problem. For this reason, 'If the halting problem were computable, then the validity problem for First-Order Logic would be computable' is true. These sentences are nontrivial; it takes mathematical work to establish that one problem is computable relative to another. However, given that uncomputable problems are necessarily uncomputable, they are also counterpossibles. To account for the substance of relative computability theory, perhaps we ought to endorse nonvacuism.

[10]Examples of vacuists (beyond Lewis and Stalnaker) include Kratzer (1979); Bennett (2003); Williamson (2007*b*, 2010, 2015); Emery and Hill (2017).

[11]To the best of my knowledge, the only philosophers who reject Definition$_2$ are intuitionists—such as Bobzien and Rumfitt (2020). I myself find the consequences of intuitionism untenable. While intuitionists claim that not all propositions are either true or false ($\neg\forall p(p \vee \neg p)$), they cannot claim that some propositions are neither true nor false ($\exists p(\neg p \wedge \neg\neg p)$) on pain of contradiction. Intuitionists thus lose the inference from $\neg\forall x\phi$ to $\exists x\neg\phi$—an unacceptable loss in my view. Suffice it to say that my first assumption—that classical

in every possible world, then $\neg\Box\neg p$ asserts that it is false that p is false in every possible world. This naturally seems to require p to be true in at least one possible world—and so it is possible for p to be true.

The upshot is this: in addition to accepting classical logic, I endorse both the claim that $\Box p = \neg p \mathrel{\Box\!\!\rightarrow} \bot$ and that $\Diamond p = \neg\Box\neg p$. While these assumptions are not uncontroversial, they have enough support to make this discussion worthwhile. I also note that those who reject the definition of either necessity or possibility may have a use for the system that follows. These definitions serve one purpose: to translate claims involving counterfactuals into claims involving modals. Without these definitions, such translations are impossible—but the remainder of the theorems still hold, and claims that purely involve counterfactuals may be of interest in their own right.

Higher-Order Counterfactual Logic

The system I employ is not the propositional logic considered so far—or even a first-order extension of that system. Rather, I operate with a *higher-order* counterfactual logic: one that allows for quantification over terms in any syntactic category. At the beginning of the analytic tradition, higher-order systems played a pivotal role in philosophical inquiry. However, following Quine (1970)'s impassioned insistence on the primacy of first-order logic, these systems largely fell out of favor. A few years ago, it would have been incumbent to provide a general introduction to higher-order logic before this project could commence. Fortunately, matters have much improved; there are now excellent overviews of higher-order logic—and the system is widespread enough that little introduction is needed.[12] My discussion of the non-counterfactual fragment will be brief; I dedicate the bulk of my attention to counterfactual logic.

The Syntax of HOCL

I operate with a simply-typed language with λ-abstraction.[13] There are two basic types: a type e for entities and a type t for sentences. 'Socrates' and 'The Mona Lisa' are of type e, while 'Roses are red' and 'Violets are blue' are of type t. Additionally, there are complex types that consist of functional relations between the basic ones; for every types τ_1 and $\tau_2 \neq e$, $(\tau_1 \rightarrow \tau_2)$ is a type. Nothing else is a type.

Terms of diverse syntactic categories are regimented in the standard way. Monadic first-order predicates are identified with terms of type $(e \rightarrow t)$; they are functions with entities as inputs and sentences as outputs. Diadic first-order predicates are terms of

logic is true—rules out this strategy.

[12] See Bacon (2023); Fritz and Jones (2024) for introductory texts.

[13] The simply-typed λ calculus differs from the pure-type theory of Berardi (1989); Terlouw (1989), which has the power to perform operations on the types themselves.

type $(e \to (e \to t))$, monadic second-order predicates are of type $((e \to t) \to t)$, etc. The negation operator $\neg$ is of type $(t \to t)$, and the binary connectives $\wedge, \vee, \to$ and $\leftrightarrow$ are all of type $(t \to (t \to t))$.

There are also terms for identity and the usual quantifiers. For every type τ there is a term $=$ of type $(\tau \to (\tau \to t))$. We allow there to be infinitely many variables of every type, as well as λ-abstracts that serve to bind these variables. We also introduce the quantifiers $\forall$ and $\exists$ of type $((\tau \to t) \to t)$ for every type τ. Effectively, first-order quantifiers are second-order properties: the property of *having every object in its extension* and of *having an object in its extension* respectively. There are the modal operators $\Box$ and $\Diamond$ of type $(t \to t)$ and, lastly, the counterfactual conditional $\Box\!\!\rightarrow$ of type $(t \to (t \to t))$. It represents sentences like 'If the shampoo were cheaper, I would have bought it' and 'If kangaroos had no tails, they would topple over.'[14]

The Axioms and Rules of HOCL

The nonmodal axioms and inferential rules I employ are the following:

Axiom Schemes:

PC:	$\vdash p$ if p is a theorem of classical propositional logic
UI:	$\vdash \forall F \to Fa$
EG:	$\vdash Fa \to \exists F$
UD:	$\vdash \forall x(P \to Q) \to (P \to \forall x Q)$ if x is not free in P
ED:	$\vdash \forall x(P \to Q) \to (\exists x(P) \to Q)$ if x is not free in Q
Ref:	$\vdash a = a$
LL:	$\vdash a = b \to (\phi \leftrightarrow \phi^{[a/b]})$
Eβ:	$\vdash \lambda x.F(a) \leftrightarrow F^{[a/x]}$

Rules:

MP:	If $\vdash p \to q$ and $\vdash p$ then $\vdash q$
Gen:	If $\vdash p$ then $\vdash \forall x p$

Many of these axioms either are included in—or are natural extensions of—First-Order Logic. **PC** and **MP** jointly ensure that classical propositional logic holds within HOCL. **UI**, **EG**, **UD** and **ED** likewise stipulate that quantifiers act classically (though here the

[14]In what follows, I occasionally omit various symbols when ambiguity does not result. I also omit the types of terms where the type is either contextually evident, or the term is taken as a schema with applications in every type. I also suppress the λ terms that immediately follow the quantifiers $\forall$ and $\exists$.

axioms should be interpreted as schemata; quantifiers of arbitrary type obey analogues of first-order inferences). Likewise, **Ref** and **LL** govern the logic of identity. Everything is identical to itself—and terms that co-denote can be substituted for one another in any formula.[15] The most novel axiom is **E**β—the principle of β-reduction. This permits the inference from $\lambda x.Fx(a)$ to Fa.

A few points about this system. The axioms and inferences have instances involving free variables, as well as constants. Thus, $x = x$ is a theorem of this system. However, we will only speak of formula as being 'true' when they contain no free variables. To that end, **Gen** can be applied to *formula* with free variables to arrive at *sentences* that lack them. Additionally, this system is extremely weak in some respects. In particular, it sidesteps many controversial debates over propositional granularity. For example, **E**β merely stipulates that $\lambda x.Fx(a)$ and Fa have the same truth-value; it does not take a stand on whether the two are identical.[16] It is thus available to many metaphysicians.

The counterfactual axioms and rules I employ are the following:

Counterfactual Axiom Schemes:

ID: $\vdash p \mathrel{\Box\!\!\rightarrow} p$
Vac: $\vdash (\neg p \mathrel{\Box\!\!\rightarrow} p) \rightarrow (q \mathrel{\Box\!\!\rightarrow} p)$
B$\Box\!\!\rightarrow$: $\vdash p \rightarrow ((p \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot)$

Counterfactual Rules:

Closure: If $\vdash p \rightarrow q$ then $\vdash (r \mathrel{\Box\!\!\rightarrow} p) \rightarrow (r \mathrel{\Box\!\!\rightarrow} q)$
REA: If $\vdash p \equiv q$ then $\vdash (p \mathrel{\Box\!\!\rightarrow} r) \equiv (q \mathrel{\Box\!\!\rightarrow} r)$

ID is the principle of reflexivity for counterfactuals.[17] It reflects the thought that when we construct a counterfactual situation from a given supposition, we start with the supposition itself.[18] **Vac** (or Vacuity) generates vacuous counterfactuals. The underlying thought is that a situation in which p is false is the 'worst' situation from the perspective of p. If p would be true *even in a $\neg p$ situation*, then p would be true in any situation whatsoever. So, in any situation in which q were true, p would be true (for an arbitrary q).

[15]Some may be more accustomed to a version of Leibniz's Law according to which identicals bear all of the same properties. That formulation is provably equivalent to the version given here.

[16]For an argument that they are identical, see Dorr (2016). For an argument that they are not, see Rosen (2010); Fine (2012*b*).

[17]Williamson (2007*b*) dubs this principle '**Reflexivity**.' I depart from his terminology in order to avoid ambiguity with my axiom of **Ref**—according to which terms are self identical.

[18]While I take **ID** to be extraordinarily intuitive, I note that some have argued that it fails in at least some cases. See, for example, Lowe (1995); Nolan (1997); Kocurek (2022*a*).

There are two rules within this system. **Closure** allows us to generate counterfactual conditionals from (provable) material conditionals, while **REA** (or Replacement of Equivalent Antecedents) allows for the substitution of logically equivalent expressions in the antecedents of counterfactuals.[19] I also assume polyadic extensions of these axioms and rules hold. **Closure**, in particular, licenses the inference from 'If $\vdash (p_1 \wedge p_2 \wedge ... \wedge p_n) \rightarrow q$,' to '$\vdash ((r \mathrel{\Box\!\!\rightarrow} p_1) \wedge (r \mathrel{\Box\!\!\rightarrow} p_2) \wedge ... \wedge (r \mathrel{\Box\!\!\rightarrow} p_n)) \rightarrow (r \mathrel{\Box\!\!\rightarrow} q)$.'[20]

REA is particularly controversial. Given other plausible assumptions about counterfactual logic, it fails at least some of the time.[21] However, I do not appeal to the controversial instances of **REA** within this paper. I only replace antecedents with their double negations or double negatums; i.e., I assume $(p \mathrel{\Box\!\!\rightarrow} q) \leftrightarrow (\neg\neg p \mathrel{\Box\!\!\rightarrow} q)$.[22] Prominent accounts of counterfactuals that deny **REA** in its full generality still license this particular instance.[23] So, despite the controversy surrounding **REA**, I doubt that rejecting it would significantly impact the results that follow.

B$\Box\!\!\rightarrow$ is so-named because it is the counterfactual analog of the **B** modal axiom $p \rightarrow \Box\Diamond p$.[24] We can establish that **B**$\Box\!\!\rightarrow$ entails **B** as follows:

i.	$p \rightarrow ((p \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot)$	**B**$\Box\!\!\rightarrow$
ii.	$p \equiv \neg\neg p$	**PC**
iii.	$(p \mathrel{\Box\!\!\rightarrow} \bot) \equiv (\neg\neg p \mathrel{\Box\!\!\rightarrow} \bot)$	*ii*, **REA**
iv.	$(\neg\neg p \mathrel{\Box\!\!\rightarrow} \bot) \equiv \neg\neg(\neg\neg p \mathrel{\Box\!\!\rightarrow} \bot)$	**PC**
v.	$(p \mathrel{\Box\!\!\rightarrow} \bot) \equiv \neg\neg(\neg\neg p \mathrel{\Box\!\!\rightarrow} \bot)$	*iii*, *iv*, **PC** and **MP**
vi.	$((p \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot) \equiv ((\neg\neg(\neg\neg p \mathrel{\Box\!\!\rightarrow} \bot)) \mathrel{\Box\!\!\rightarrow} \bot)$	*v*, **REA**
vii.	$p \rightarrow ((\neg\neg(\neg\neg p \mathrel{\Box\!\!\rightarrow} \bot)) \mathrel{\Box\!\!\rightarrow} \bot)$	*i*, *vi*, **PC** and **MP**
viii.	$p \rightarrow (\neg\neg(\Box\neg p) \mathrel{\Box\!\!\rightarrow} \bot)$	*vii*, Definition$_1$
ix.	$p \rightarrow (\Box\neg(\Box\neg p))$	*viii*, Definition$_1$
x.	$p \rightarrow \Box\Diamond p$	*ix*, Definition$_2$

This proof can be more-or-less reversed to demonstrate that **B** entails **B**$\Box\!\!\rightarrow$:

[19] Williamson (2007*b*) refers to this as 'Equivalence.' I note that (Pollock, 1976, pg. 11) states that **Closure** is "So obvious as to need no defense."

[20] In the final section, I consider infinite extensions of **Closure**.

[21] The principles I allude to are **Simplification**: $\vdash ((p \vee q) \mathrel{\Box\!\!\rightarrow} r) \rightarrow (p \mathrel{\Box\!\!\rightarrow} r)$ and the **Failure of Antecedent Strengthening**: $\nvdash (p \mathrel{\Box\!\!\rightarrow} r) \rightarrow ((p \wedge q) \mathrel{\Box\!\!\rightarrow} r)$. This conflict was noted independently by Fine (1975) and Nute (1975). I do not want to cast too much doubt on **REA**; plausible principles also entail that it holds in full generality. If necessarily equivalent propositions are identical, and if Leibniz's Law is true, then **REA** universally succeeds.

[22] There is one exception. In the discussion of Maximalism, I replace an antecedent with its vacuous β-conversion. I take this also to be uncontroversial.

[23] See, e.g., Fine (2012*a*).

[24] Williamson (2007*a*) operates with the provably equivalent axiom **BS**: $\vdash (p \mathrel{\Box\!\!\rightarrow} (q \mathrel{\Box\!\!\rightarrow} \bot)) \rightarrow (p \rightarrow (q \mathrel{\Box\!\!\rightarrow} \bot))$. I opt for my axiom due to its comparative simplicity.

i.	$p \rightarrow \Box\Diamond p$	**B**
ii.	$p \rightarrow (\Box\neg(\Box\neg p))$	*i,* Definition$_2$
iii.	$p \rightarrow (\neg\neg(\Box\neg p) \Box\!\!\rightarrow \bot)$	*ii,* Definition$_1$
iv.	$p \rightarrow ((\neg\neg(\neg\neg p \Box\!\!\rightarrow \bot)) \Box\!\!\rightarrow \bot)$	*iii* and Definition$_1$
v.	$p \equiv \neg\neg p$	**PC**
vi.	$(p \Box\!\!\rightarrow \bot) \equiv (\neg\neg p \Box\!\!\rightarrow \bot)$	*v,* **REA**
vii.	$(\neg\neg p \Box\!\!\rightarrow \bot) \equiv \neg\neg(\neg\neg p \Box\!\!\rightarrow \bot)$	**PC**
viii.	$(p \Box\!\!\rightarrow \bot) \equiv \neg\neg(\neg\neg p \Box\!\!\rightarrow \bot)$	*vi, vii,* **PC** and **MP**
ix.	$((p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \equiv ((\neg\neg(\neg\neg p \Box\!\!\rightarrow \bot)) \Box\!\!\rightarrow \bot)$	*viii*, **REA**
x.	$p \rightarrow ((p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$	*iv, ix*, **PC**, **MP**

These axioms and rules are too weak to constitute 'counterfactual logic' in any comprehensive sense of the term. HOCL includes neither the weak nor strong centering axioms—and lacks the ability to prove the counterfactual excluded middle. I do not omit these because I have any principled objection to them, but rather because they play no role in the theorems that follow. For our purposes, weak axioms are enough.

The Necessity of Identity and Distinctness

A natural starting point is the necessity of identity—due to both the simplicity of the proof and the significance of the result. We can establish the necessity of identity using **Closure** and **ID** as follows:

i.	$x = x$	**Ref**
ii.	$x \neq x \rightarrow \bot$	*i,***PC** and **MP**
iii.	$(x \neq x \Box\!\!\rightarrow x \neq x) \rightarrow (x \neq x \Box\!\!\rightarrow \bot)$	*ii,* **Closure**
iv.	$x \neq x \Box\!\!\rightarrow x \neq x$	**ID**
v.	$x \neq x \Box\!\!\rightarrow \bot$	*iii, iv*, **MP**
vi.	$x = y \rightarrow ((x \neq x \Box\!\!\rightarrow \bot) \leftrightarrow (x \neq y \Box\!\!\rightarrow \bot))$	**LL**
vii.	$x = y \rightarrow (x \neq y \Box\!\!\rightarrow \bot)$	*v, vi,* **PC** and **MP**
viii.	$\forall x, y(x = y \rightarrow (x \neq y \Box\!\!\rightarrow \bot))$	*vii*, **Gen** (x2)
ix.	$\forall x, y(x = y \rightarrow \Box(x = y))$	*viii,* Definition$_1$

Counterfactual systems that are committed both to reflexivity and to the substitution of entailments in consequent position (like HOCL) are thus committed to the necessity of identity. While higher-order resources are unneeded in this proof, the result holds for terms of arbitrary type; identical properties are necessarily identical, identical sentential operators are necessarily identical, and identical connectives are necessarily identical.

The necessity of distinctness follows from **ID**, **Closure**, **B**$\Box\!\!\rightarrow$, **REA**, and **Vac**:

i.	$x = y \rightarrow (x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	Previous Theorem
ii.	$\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y$	*i*, **PC** and **MP**
iii.	$\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y) \rightarrow \bot$	*ii*, **PC** and **MP**
iv.	$((\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y)) \mathrel{\Box\!\!\rightarrow} (\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y))) \rightarrow ((\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y)) \mathrel{\Box\!\!\rightarrow} \bot)$	
v.	$(\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y)) \mathrel{\Box\!\!\rightarrow} (\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y))$	**ID**
vi.	$\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y) \mathrel{\Box\!\!\rightarrow} \bot$	*iv*, *v*, **MP**
vii.	$((\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y) \wedge (\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot))) \rightarrow x \neq y$	**PC**
viii.	$((\neg x \neq y \mathrel{\Box\!\!\rightarrow} (\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y)) \wedge (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot))) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} x \neq y)$	*vii*, **Closure**
ix.	$(x \neq y \wedge \neg x \neq y) \rightarrow \bot$	**PC**
x.	$((\neg x \neq y \mathrel{\Box\!\!\rightarrow} x \neq y) \wedge (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \neg x \neq y)) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*ix*, **Closure**
xi.	$\neg x \neq y \mathrel{\Box\!\!\rightarrow} \neg x \neq y$	**ID**
xii.	$(\neg x \neq y \mathrel{\Box\!\!\rightarrow} x \neq y) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*x*, *xi*, **PC** and **MP**
xiii.	$\bot \rightarrow p$	**PC**
xiv.	$(\neg p \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (\neg p \mathrel{\Box\!\!\rightarrow} p)$	*xiii*, **Closure**
xv.	$(\neg p \mathrel{\Box\!\!\rightarrow} p) \rightarrow (q \mathrel{\Box\!\!\rightarrow} p)$	**Vac**
xvi.	$(\neg p \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (q \mathrel{\Box\!\!\rightarrow} p)$	*xiv*, *xv*, **PC** and **MP**
xvii.	$(\neg\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot))$	Instance of *xvi*
xviii.	$(\neg(\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y) \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} (\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow x \neq y))$	Instance of *xvi*
xix.	$(\neg\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*vi*, *viii*, *xii*, *xvii*, *xviii*, **PC** and **MP**
xx.	$(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \equiv \neg\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	**PC**
xxi.	$(\neg\neg(x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot) \equiv (((x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot))$	*xx*, **REA**
xxii.	$((x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*xix*, *xxi*, **PC** and **MP**
xxiii.	$x \neq y \rightarrow ((x \neq y \mathrel{\Box\!\!\rightarrow} \bot) \mathrel{\Box\!\!\rightarrow} \bot)$	**B**$\mathrel{\Box\!\!\rightarrow}$
xxiv.	$x \neq y \rightarrow (\neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*xxxxii*, *xxiii*, **PC** and **MP**
xxv.	$\forall x, y(x \neq y \rightarrow \neg x \neq y \mathrel{\Box\!\!\rightarrow} \bot)$	*xxiv*, **Gen**(x2)
xxvi.	$\forall x, y(x \neq y \rightarrow \Box x \neq y)$	*xxv*, Definition$_1$

I was unable to prove the necessity of distinctness without each of these axioms—by comparison, far more than those needed to prove the necessity of identity. Complexity corresponds to controversy. A philosopher who rejected **B**$\mathrel{\Box\!\!\rightarrow}$, for example, could deny the necessity of distinctness but remains committed to the necessity of identity (assuming, of course, that they accept **ID** and **Closure**).

The necessity of distinctness impacts the debate over counteridenticals (counterfactuals with a false identity claim in their antecedent).[25] Potential examples of counteridenticals include 'If I were you, I would leave by 4:00 pm' and 'If John were Einstein, he would pass his physics exam.' By itself, the necessity of distinctness does not force a position on whether counteridenticals are substantive. But—in combination with vacuism—it does.[26] **Closure** and **REA** jointly entail vacuism—which can be shown as follows:

i.	$\bot \rightarrow q$	**PC**
ii.	$(p \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (p \mathrel{\Box\!\!\rightarrow} q)$	*i*, **Closure**
iii.	$\forall q((p \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (p \mathrel{\Box\!\!\rightarrow} q))$	*ii*, **Gen**
iv.	$\forall q((p \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow (p \mathrel{\Box\!\!\rightarrow} q)) \rightarrow ((p \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow \forall q(p \mathrel{\Box\!\!\rightarrow} q))$	**UD**
v.	$(p \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow \forall q(p \mathrel{\Box\!\!\rightarrow} q)$	*iii*, *iv*, and **MP**
vi.	$p \equiv \neg\neg p$	**PC**
vii.	$p \mathrel{\Box\!\!\rightarrow} \bot \equiv \neg\neg p \mathrel{\Box\!\!\rightarrow} \bot$	*vi* and **REA**
viii	$(\neg\neg p \mathrel{\Box\!\!\rightarrow} \bot) \rightarrow \forall q(p \mathrel{\Box\!\!\rightarrow} q)$	*v*, *vii*, **PC** and **MP**
ix.	$\Box\neg p \rightarrow \forall q(p \mathrel{\Box\!\!\rightarrow} q)$	*viii* and Definition$_1$

While this derivation relies upon **Closure** and **REA**, I suspect that the real culprit is **Closure**. **Closure** allows for the substitution of entailments in counterfactuals' consequents (i.e., it licenses the inference from $p \mathrel{\Box\!\!\rightarrow} q$ and $q \vdash r$ to $p \mathrel{\Box\!\!\rightarrow} r$)—and this comes very close to vacuism itself.[27] Given an impossible proposition p, **ID** entails that $p \mathrel{\Box\!\!\rightarrow} p$. Because p is impossible, it entails absolutely everything—including an arbitrary q, so **Closure** permits the inference to $p \mathrel{\Box\!\!\rightarrow} q$. Those who reject vacuism ought to reject **Closure**.

HOCL is committed both to the necessity of distinctness and to the vacuity of counterpossibles. Therefore, it is committed to the claim that all counteridenticals are true; those who maintain that counteridenticals are substantive are wrong. Adherents of HOCL thus owe a response to plausible examples of substantive counteridenticals.

In my view, the best response is that natural language examples are not genuine counteridenticals. Rather, they are paraphrases of sentences that do not involve identity—and whose antecedents are contingent. While they are substantive, they do not violate vacuism. We need not hold that all counteridenticals paraphrase in the same way; they may be paraphrases for different sorts of counterfactuals. The example 'If I were you, I would leave by 4:00 pm' seems to gloss 'If I were in your situation, I would leave by 4:00 pm'—and 'If John were Einstein, he would pass his physics exam' seems to gloss 'If John were as smart as Einstein, he would pass his physics exam.'[28]

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[25] For defenses of substantive counteridenticals, see Kocurek (2018); Wilhelm (2021).

[26] Recall that vacuism is the claim that all counterpossibles hold vacuously—i.e., that $\Box\neg p \rightarrow \forall q(p \mathrel{\Box\!\!\rightarrow} q)$

[27] More precisely, it follows from the **Deduction Theorem**—according to which $p \vdash q$ entails $\vdash p \rightarrow q$. For an arbitrary $p \vdash q$, the **Deduction Theorem** then entails $\vdash p \rightarrow q$—and, given **Closure**, this in turn entails $\vdash (r \mathrel{\Box\!\!\rightarrow} p) \rightarrow (r \mathrel{\Box\!\!\rightarrow} q)$.

[28] Adherents of substantive counterexamples reject the gloss from 'If I were you, then p' as 'If I were in your

There are natural-language sentences that indisputably involve identity—but they do not seem synonymous with standard examples of counteridenticals. Take, for example, 'If I were identical to you, I would leave by 4:00 pm.' This explicitly invokes identity—yet does not appear to mean the same thing as 'If I were you, I would leave by 4:00 pm.' Since the former sentence involves identity and is not synonymous with the latter, there is room to deny that the latter involves identity. Moreover, expressions like 'If I were identical to you' are sufficiently removed from ordinary use that we ought not revise counterfactual logic in light of them.

The upshot is this: HOCL entails both the necessity of identity and the necessity of distinctness. Because it is also committed to vacuism, it entails that all counteridenticals are true. While this is controversial, there is room to resist putative examples of substantive counteridenticals in the literature.

Necessitism and the Barcan Formula

One of the most pivotal choice-points in quantified modal logic is the **Barcan Formula**:

Barcan Formula (BF): $\forall x \Box Fx \rightarrow \Box \forall x Fx$

If all objects are necessarily F, then, necessarily, all objects are F. Because all objects are necessarily self-identical, it is necessary that all objects are self-identical. Quantified modal logic regiments inferences involving both necessity and generality; more importantly, it formalizes the interaction between the two. The **Barcan**—and its converse—describe that interaction.

Some of the most significant implications of the **Barcan** concern **Necessitism**, which are discussed in greater depth below. However, it ought to be controversial irrespective of its implications for what must exist. Suppose the world consisted of nothing but two electrons that repel each other—accelerating in opposite directions for eternity. Quite plausibly, each of these electrons is necessarily negatively charged.[29] Because the world contains nothing but these electrons, everything is necessarily negatively charged. But it would be absurd to claim that it is necessary that everything is negatively charged. After all, there could exist protons, neutrons and the like—particles of positive or neutral charge.

Many paths lead to the **Barcan**—challenging cases notwithstanding.[30] It follows from

situation, then p' due to sentences like 'If I were you, I would not be in your situation.' But there is contextual variation in 'your situation' that allows the gloss to succeed even in this case. Suppose, for example, that you have not begun a consequential assignment until the night before it was due. I might then claim 'If I were in your situation (i.e., the situation of having a consequential assignment due) I would not be in your situation (i.e., the situation of having left it so late).'

[29]That is, it is plausible that it is *essential* to—or *in the nature of*—these electrons that they are negatively charged. Anything that was not negatively charged would not be these electrons.

[30]Marcus (1947) establishes that the **Barcan** holds in S2—a system I do not discuss in depth here—and the

REA, Closure, Vac, and **B**$\Box\!\!\rightarrow$. To the best of my knowledge, this has gone overlooked in the literature; this is the first derivation of the **Barcan** in counterfactual logic.

It is helpful to first prove some derived rules that facilitate the proof of the **Barcan**.

Derived Rule 1 (DR1): If $\vdash p \rightarrow q$ then $\vdash (\neg p \Box\!\!\rightarrow \bot) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$

i.	$p \rightarrow q$	Supposition
ii.	$((p \rightarrow q) \wedge \neg(p \rightarrow q)) \rightarrow \bot$	**PC**
iii.	$\neg(p \rightarrow q) \rightarrow \bot$	*i*, *ii*, **PC** and **MP**
iv.	$(\neg(p \rightarrow q) \Box\!\!\rightarrow \neg(p \rightarrow q)) \rightarrow (\neg(p \rightarrow q) \Box\!\!\rightarrow \bot)$	*iii*, **Closure**
v.	$\neg(p \rightarrow q) \Box\!\!\rightarrow \neg(p \rightarrow q)$	**ID**
vi.	$\neg(p \rightarrow q) \Box\!\!\rightarrow \bot$	*iv*, *v*, **MP**
v.	$\bot \rightarrow p$	**PC**
vi.	$(\neg p \Box\!\!\rightarrow \bot) \rightarrow (\neg p \Box\!\!\rightarrow p)$	*v*, **Closure**
vii.	$(\neg p \Box\!\!\rightarrow p) \rightarrow (\neg q \Box\!\!\rightarrow p)$	**Vac**
viii.	$(\neg p \Box\!\!\rightarrow \bot) \rightarrow (\neg q \Box\!\!\rightarrow p)$	*vi*, *vii*, **PC** and **MP**
ix.	$((p \rightarrow q) \wedge p) \rightarrow q$	**PC**
x.	$((\neg q \Box\!\!\rightarrow (p \rightarrow q)) \wedge (\neg q \Box\!\!\rightarrow p)) \rightarrow (\neg q \Box\!\!\rightarrow q)$	*ix*, **Closure**
xi.	$((\neg(p \rightarrow q) \Box\!\!\rightarrow \bot) \wedge (\neg p \Box\!\!\rightarrow \bot)) \rightarrow (\neg q \Box\!\!\rightarrow q)$	*viii*, *x*, **PC** and **MP**
xii.	$(q \wedge \neg q) \rightarrow \bot$	**PC**
xiii.	$((\neg q \Box\!\!\rightarrow q) \wedge (\neg q \Box\!\!\rightarrow \neg q) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*xii*, **Closure**
xiv.	$\neg q \Box\!\!\rightarrow \neg q$	**ID**
xv.	$(\neg q \Box\!\!\rightarrow q) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*xiii*, *xiv*, **PC** and **MP**
xvi.	$(\neg p \Box\!\!\rightarrow \bot) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*vi*, *xi*, *xv*, **PC** and **MP**

Derived Rule 2 (DR2): If $\vdash p \rightarrow q$ then $\vdash \neg(p \Box\!\!\rightarrow \bot) \rightarrow \neg(q \Box\!\!\rightarrow \bot)$

i.	$p \rightarrow q$	Supposition
ii.	$\neg q \rightarrow \neg p$	*i*, **MP** and **PC**
iii.	$(\neg\neg q \Box\!\!\rightarrow \bot) \rightarrow (\neg\neg p \Box\!\!\rightarrow \bot)$	*ii* **DR1**
iv.	$p \equiv \neg\neg p$	**PC**
v.	$(p \Box\!\!\rightarrow \bot) \equiv (\neg\neg p \Box\!\!\rightarrow \bot)$	*iv* and **REA**
vi.	$(q \Box\!\!\rightarrow \bot) \equiv (\neg\neg q \Box\!\!\rightarrow \bot)$	**PC** and **REA**
vii.	$(q \Box\!\!\rightarrow \bot) \rightarrow (p \Box\!\!\rightarrow \bot)$	*iii*, *v*, *vi*, **PC** and **MP**
viii.	$\neg(p \Box\!\!\rightarrow \bot) \rightarrow \neg(q \Box\!\!\rightarrow \bot)$	*vii*, **PC** and **MP**

strict conditional. That is, the relevant modal operator was $\Box(A \rightarrow B)$, rather than $\Box$. Prior (1956) proves that the **Barcan** holds in a quantified version of S5. The proof of the **Barcan** in the weaker B system is attributed to John Lemmon in Prior (1967). See, also, Cresswell and Hughes (1996).

Derived Rule 3 (DR3): If $\vdash \neg((\neg p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$ then $\vdash p$

i.	$\neg((\neg p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$	Supposition
ii.	$\neg p \rightarrow ((\neg p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$	$\mathbf{B\Box\!\!\rightarrow}$
iii.	$\neg((\neg p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow \neg\neg p$	*ii*, **MP** and **PC**
iv.	$\neg\neg p$	*i*, *iii*, **MP**
v.	p	*iv*, **MP**, **PC**

Derived Rule 4 (DR4): If $\vdash \neg(p \Box\!\!\rightarrow \bot) \rightarrow q$ then $\vdash p \rightarrow (\neg q \Box\!\!\rightarrow \bot)$

i.	$\neg(p \Box\!\!\rightarrow \bot) \rightarrow q$	Supposition
ii.	$(\neg\neg(p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*i* and **DR1**
iii.	$((p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*ii*, **REA**, **MP** and **PC**
iv.	$p \rightarrow ((p \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$	$\mathbf{B\Box\!\!\rightarrow}$
v.	$p \rightarrow (\neg q \Box\!\!\rightarrow \bot)$	*iii*, *iv*, **MP**, **PC**

With these rules in place, the **Barcan Formula** can be derived as follows:

i.	$\forall x(\neg Fx \Box\!\!\rightarrow \bot) \rightarrow \neg Fx \Box\!\!\rightarrow \bot$	**UI**
ii.	$\neg(\forall x(\neg Fx \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow \neg((\neg Fx \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot)$	*i* and **DR2**
iii.	$\neg(\forall x(\neg Fx \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow Fx$	*ii*, **DR3**, **MP** and **PC**
iv.	$\neg(\forall x(\neg Fx \Box\!\!\rightarrow \bot) \Box\!\!\rightarrow \bot) \rightarrow \forall x Fx$	*iii*, **Gen**, **MP**, **UD** and **PC**
v.	$\forall x(\neg Fx \Box\!\!\rightarrow \bot) \rightarrow (\neg\forall x Fx \Box\!\!\rightarrow \bot)$	*iv*, **DR4**
vi.	$\forall x \Box Fx \rightarrow \Box\forall x Fx$	*v* and Definition$_1$

The **Converse Barcan Formula** is nearly as significant to quantified modal reasoning as the **Barcan** itself:

Converse Barcan Formula (CBF): $\Box\forall x Fx \rightarrow \forall x \Box Fx$

This follows from **REA, Closure** and **Vac**:

i.	$\forall x Fx \rightarrow Fx$	**UI**
ii.	$(\neg\forall x Fx \Box\!\!\rightarrow \bot) \rightarrow \neg Fx \Box\!\!\rightarrow \bot$	*i*, **DR1**
iii.	$(\neg\forall x Fx \Box\!\!\rightarrow \bot) \rightarrow \forall x(\neg Fx \Box\!\!\rightarrow \bot)$	*ii*, **Gen, UD, PC** and **MP**
iv.	$\Box\forall x Fx \rightarrow \forall x \Box Fx$	*iii* and Definition$_1$

HOCL is thus committed both to the **Barcan** and its converse. The primary controversy surrounding these principles concerns **Necessitism**—the claim that, necessarily, everything necessarily exists. Necessitists hold that this is true: contingentists that this is false. More formally, we can represent **Necessitism** as:

Necessitism: $\Box\forall x\Box\exists y(x = y)$

It is worth acknowledging the limit to the disagreement between necessitists and contingentists. For any given object, both may hold that the object necessarily exists; they might agree that God necessarily exists—or that there necessarily is a prime between 3 and 7. Contingentists are only committed to the claim that it is possible for *something-or-other* to exist contingently—not to what that something is.

Necessists are sometimes charged with holding that there are the same *number* of objects in every possible world. This is only accurate if they also accept the necessity of identity and distinctness. If distinctness were contingent, then, even given **Necessitism**, there could be fewer objects than there actually are—if objects that are actually distinct were identical. And if identity were contingent, there could be more objects than there actually are—if objects that are actually identical were distinct. However, we have already established the necessity of identity and distinctness in HOCL. So, within this system, we can frame the disagreement between necessitists and contingentists in terms of whether possible worlds contain the same number of objects.

Contingentism aligns with common sense; intuitively, had my parents never met I would not have existed—and had the Chicxulub Impactor not struck the Earth, no humans would have existed at all.[31] Necessists need compelling arguments to justify their position. One argument Williamson (2013) provides relies on the **Being Constraint**: the claim that, necessarily, if an object bears a property (or, in layman's terms, if there is a way that the object is), then the object exists. More formally:

Being Constraint (BC): $\forall x\Box(\exists F(Fx) \rightarrow \exists z(x = z))$

The **Being Constraint** has intuitive appeal.[32] After all, if we were to count the number of objects which are F, we would presumably assume that if something is an F, then it must exist—and so is worthy of being counted. Without the **Being Constraint**, it is difficult to see why the fact that object a is an F would impact the number of objects that are F (since, without the **Being Constraint** the fact that a is F is compatible with a not existing).

The step from the **Being Constraint** to **Necessitism** is straightforward. Necessarily, all objects bear the property *is self-identical*—so, necessarily, every object bears some property other. The **Being Constraint** then allows us to conclude that there exists an object that

[31]Necessitists typically respond to these cases by claiming that the use of 'exists' in these sentence is restricted; only when we speak unrestrictedly is it true that I necessarily exist.

[32]However, for objections to the use of the **Being Constraint** (largely on the grounds that it seems unmotivated for contingentism) see Dorr (2016); Goodman (2016); Litland (2023).

each object is identical to—and that this holds necessarily. Those who endorse the **Being Constraint** accept **Necessitism**.

The **Being Constraint**—and the principles that generate it—crucially rely upon higher-order logic; the ability to express higher-order quantifiers is essential to this sort of principle. We can prove the **Being Constraint** within HOCL as follows:

i.	$x = x$	**Ref**
ii.	$\exists y(x = y)$	*i*, **EG**, **PC** and **MP**
iii.	$\exists F(Fx) \rightarrow \exists y(x = y)$	*ii*, **MP** and **PC**
iv.	$\forall x(\exists F(Fx) \rightarrow \exists y(x = y))$	*iii* and **Gen**
v.	$\neg\forall x(\exists F(Fx) \rightarrow \exists y(x = y)) \rightarrow \bot$	*iv*, **PC** and **MP**
vi.	$(\neg\forall x(\exists F(Fx) \rightarrow \exists y(x = y)) \mathrel{\Box\!\!\rightarrow} \neg\forall x(\exists F(Fx) \rightarrow \exists y(x = y))) \rightarrow (\neg\forall x(\exists F(Fx) \rightarrow \exists y(x = y)) \mathrel{\Box\!\!\rightarrow} \bot)$	*v* and **Closure**
vii.	$\neg\forall x(\exists F(Fx) \rightarrow \exists y(x = y)) \mathrel{\Box\!\!\rightarrow} \neg\forall x(\exists F(Fx) \rightarrow \exists y(x = y))$	**ID**
viii.	$\neg\forall x(\exists F(Fx) \rightarrow \exists y(x = y)) \mathrel{\Box\!\!\rightarrow} \bot$	*vi*, *vii* and **MP**
ix.	$\Box\forall x(\exists F(Fx) \rightarrow \exists z(x = z))$	*viii* and Definition$_1$
x.	$\forall x\Box(\exists F(Fx) \rightarrow \exists z(x = z))$	*ix* and **Converse Barcan**

Thus, the **Being Constraint** holds if the **Converse Barcan** holds. It is also possible to prove **Necessitism** directly in HOCL—without appealing to the **Being Constraint**. This can be shown as follows:

i.	$x = x$	**Ref**
ii.	$\exists y(x = y)$	*i*, **EG** and **MP**
iii.	$\neg\exists y(x = y) \rightarrow \bot$	*ii*, **PC** and **MP**
iv.	$(\neg\exists y(x = y) \mathrel{\Box\!\!\rightarrow} \neg\exists y(x = y)) \rightarrow (\neg\exists y(x = y) \mathrel{\Box\!\!\rightarrow} \bot)$	*iii*, **Closure**
v.	$\neg\exists y(x = y) \mathrel{\Box\!\!\rightarrow} \neg\exists y(x = y)$	**ID**
vi.	$\neg\exists y(x = y) \mathrel{\Box\!\!\rightarrow} \bot$	*iv*, *v*, **MP**
vii.	$\forall x(\neg\exists y(x = y) \mathrel{\Box\!\!\rightarrow} \bot)$	*vi*, **Gen**
viii.	$\neg(\forall x(\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot)) \rightarrow \bot$	*vii*, **PC** and **MP**
ix.	$(\neg(\forall x(\neg\exists y(x = y) \mathrel{\Box\!\!\rightarrow} \bot)) \mathrel{\Box\!\!\rightarrow} \neg(\forall x(\neg\exists y(x = y) \mathrel{\Box\!\!\rightarrow} \bot))) \rightarrow (\neg(\forall x(\neg\exists y(x = y) \mathrel{\Box\!\!\rightarrow} \bot)) \mathrel{\Box\!\!\rightarrow} \bot)$	*viii*, **Closure**
x.	$\neg(\forall x(\neg\exists y(x = y) \mathrel{\Box\!\!\rightarrow} \bot)) \mathrel{\Box\!\!\rightarrow} \neg(\forall x(\neg\exists y.(x = y) \mathrel{\Box\!\!\rightarrow} \bot))$	**ID**
xi.	$\neg(\forall x(\neg\exists y(x = y) \mathrel{\Box\!\!\rightarrow} \bot)) \mathrel{\Box\!\!\rightarrow} \bot$	*ix*, *x*, **MP**
xii.	$\Box\forall x\Box\exists y(x = y)$	*xi*, Definition$_1$

HOCL validates the **Barcan**, **Converse Barcan**, **Being Constraint** and **Necessitism**.

The Identity of Indiscernibles

The Principle of the Identity of Indiscernibles (the **PII**) is the principle that objects cannot differ only numerically. It is metaphysically necessary that distinct objects must differ in some non-numerical respect. Despite the prevalence of counterexamples (most notably, Black (1952)'s pair of indiscernible spheres), many maintain that one interpretation of this principle is trivially true: the claim that objects bearing all of the same properties are identical.[33] This is held to be trivial due to haecceities: properties like *is identical to* a (which we represent with $\lambda x.(x = a)$). Any objects that bear the same properties (in general) bear the same haecceities (in particular). And, clearly, all objects that bear the property *is identical to a* are identical to one another.

Like the **Being Constraint**, the **PII** crucially relies on higher-order modal inferences. The sentence 'Objects *cannot* differ only numerically' has modal force. It is not only a claim about what is actually so, but rather about what must be so.[34] Moreover, 'Objects that bear all of the same properties' overtly quantifies over properties themselves—so we cannot hope to reconstruct this proof in a first-order language. Establishing this principle requires modal and counterfactual resources. We can prove the **PII** in HOCL as follows:

i.	$\forall X \forall x, y(Xx \leftrightarrow Xy) \rightarrow \forall x, y(\lambda z.(x = z)(x) \leftrightarrow \lambda z.(x = z)(y))$	**UI**
ii.	$\forall x, y(\lambda z.(x = z)(x) \leftrightarrow \lambda z.(x = z)(y)) \rightarrow (\lambda z.(x = z)(x) \leftrightarrow \lambda z.(x = z)(y))$	**UI**
iii.	$\lambda z.(x = z)(x) \leftrightarrow x = x$	**E**β
iv.	$x = x$	**Ref**
v.	$\forall X \forall x, y(Xx \leftrightarrow Xy) \rightarrow \lambda z.(x = z)(y)$	*i, ii, iii, iv,* **PC,** and **MP**
vi.	$\lambda z.(x = z)(y) \leftrightarrow x = y$	**E**β
vii.	$\forall X \forall x, y((Xx \leftrightarrow Xy) \rightarrow x = y)$	*v, vi,* **PC** and **MP**
viii.	$\neg(\forall X \forall x, y(Xx \leftrightarrow Xy) \rightarrow x = y) \rightarrow \bot$	*vii,* **PC** and **MP**
ix.	$(\neg(\forall X \forall x, y(Xx \leftrightarrow Xy) \rightarrow x = y) \mathbin{\Box\!\!\rightarrow} \neg(\forall X \forall x, y(Xx \leftrightarrow Xy) \rightarrow x = y)) \rightarrow (\neg(\forall X \forall x, y(Xx \leftrightarrow Xy) \rightarrow x = y) \mathbin{\Box\!\!\rightarrow} \bot)$	*viii,* **Closure**
x.	$\neg(\forall X \forall x, y(Xx \leftrightarrow Xy) \rightarrow x = y) \mathbin{\Box\!\!\rightarrow} \neg(\forall X \forall x, y(Xx \leftrightarrow Xy) \rightarrow x = y)$	**ID**
xi.	$\neg(\forall X \forall x, y(Xx \leftrightarrow Xy) \rightarrow x = y) \mathbin{\Box\!\!\rightarrow} \bot$	*ix, x,* **MP**
xii.	$\Box \forall X \forall x, y((Xx \leftrightarrow Xy) \rightarrow x = y)$	*xi* and Definition$_1$

[33] The first derivation of this triviality occurs in Whitehead and Russell (1952). One philosopher who denies that there is a trivial version of the PII is Rodriguez-Pereyra (2022)—on the grounds that objects could differ 'only numerically' while not bearing all of the same properties.

[34] However, for contingent versions of the PII, see Casullo (1984)—and French (1989) for a response.

Maximalism

Many metaphysicians endorse versions of plenitude: the view that the world is 'full'—in some sense of the word. For example, mereological universalists hold that any collection of objects composes another.[35] In addition to ordinary objects like tables and chairs, there are also strange objects—like the one composed of the galaxy Alpha Centauri and an electron on the tip of my nose. Essential universalists, for their part, hold that if there is an object that bears a collection of properties, then, for each subset of that collection, there is a coincident object that bears that subset essentially and the rest accidentally.[36] While there is an object co-located with me that is contingently seated, there is another which is essentially seated—one that ceases to exist the moment I stand.[37]

Those susceptible to this line of thought might wonder how full the world could be. A particularly extreme form of plenitude is **Maximalism**: "What maximalism says is that for any type of object such that there can be objects of that type...there are such objects" Eklund (2008).[38] We can formally represent this as:

Maximalism: $\forall X(\Diamond(\exists x Xx) \rightarrow \exists x Xx)$

Maximalism largely serves as an object of curiosity—rather than a view that serious philosophers endorse. As stated, it faces a problem.[39] Let the property *is an xheart* be the property *is a heart and is such that there are no livers*—and let the property *is an xliver* be the property *is a liver and is such that there are no hearts*. While it seems possible for there to be xhearts, and seems possible for there to be xlivers, there cannot be both xhearts and xlivers. But if it really *were* possible for there to be xhearts and xlivers, **Maximalism** would entail that they both actually exist. For this reason, some have suggested that **Maximalism** is logically inconsistent.[40]

HOCL does not entail **Maximalism**; it is compatible both with the claim that it is true and that it is false. However, it can be used to establish an important result: **Maximalism**

[35] There are far too many universalists to provide a comprehensive list here. Notable adherents include Lewis (1986); Sider (2001).

[36] Some who subscribe to plenitude include Fine (1999); Johnston (2006); Koslicki (2008).

[37] Different arguments support different versions of plenitude. Many hold that there is no non-arbitrary way to restrict which objects exist—and, in the absence of a non-arbitrary restriction, we ought to accept no restriction at all. There seems no principled reason to claim that body exists, but deny that an object consisting of my body an an electron hovering next to my left thumb does not. And many accept that the statue is distinct from the clay (because the statue is essentially shaped thus-and-so, while the clay is only accidentally shaped thus-and-so)—but find no principled reason to deny that properties other than shape give rise to coincident objects as well.

[38] Eklund also includes the modifier 'given that the empirical facts are exactly what they are'—a modification that he acknowledges requires clarification.

[39] In addition to Eklund (2008), see Thomasson (2015); Fairchild (2019).

[40] Fairchild, for example, claims "Maximalism is *wickedly difficult* to formulate consistently" (Fairchild, 2019, pg. 3).

entails that material implication entails counterfactual implication (the converse of Lewis's Weak Centering axiom). We can establish this as follows:

i.	$\forall X(\Diamond(\exists x Xx) \rightarrow \exists x Xx)$	**Maximalism**
ii.	$\forall X(\neg(\exists x Xx \;\Box\!\!\rightarrow \bot) \rightarrow \exists x Xx)$	*i*, Definition$_1$ and Definition$_2$
iii.	$\neg(\exists x(\lambda y.\neg(p \rightarrow q)(x)) \;\Box\!\!\rightarrow \bot) \rightarrow \exists x(\lambda y.\neg(p \rightarrow q)(x))$	*ii*, **UI**
iv.	$\neg(p \rightarrow q) \equiv \exists x(\lambda y.\neg(p \rightarrow q)(x))$	**E**β, **EG**, **PC** and **MP**
v.	$\neg(\neg(p \rightarrow q) \;\Box\!\!\rightarrow \bot) \rightarrow \neg(p \rightarrow q)$	*iii*, *iv*, **REA**, **PC** and **MP**
vi.	$(p \rightarrow q) \rightarrow (\neg(p \rightarrow q) \;\Box\!\!\rightarrow \bot)$	*v*, **PC** and **MP**
vii.	$(p \wedge (p \rightarrow q)) \rightarrow q$	**PC**
viii.	$((p \;\Box\!\!\rightarrow p) \wedge (p \;\Box\!\!\rightarrow (p \rightarrow q))) \rightarrow (p \;\Box\!\!\rightarrow q)$	*vii*, **Closure**
ix.	$p \;\Box\!\!\rightarrow p$	**ID**
x.	$(p \;\Box\!\!\rightarrow (p \rightarrow q)) \rightarrow (p \;\Box\!\!\rightarrow q)$	*viii*, *ix*, **PC** and **MP**
xi.	$\bot \rightarrow (p \rightarrow q)$	**PC**
xii.	$(\neg(p \rightarrow q) \;\Box\!\!\rightarrow \bot) \rightarrow (\neg(p \rightarrow q) \;\Box\!\!\rightarrow (p \rightarrow q))$	*xi*, **Closure**
xiii.	$(\neg(p \rightarrow q) \;\Box\!\!\rightarrow (p \rightarrow q)) \rightarrow (p \;\Box\!\!\rightarrow (p \rightarrow q))$	**Vac**
xiv.	$(p \rightarrow q) \rightarrow (p \;\Box\!\!\rightarrow (p \rightarrow q))$	*vi*, *xii*, *xiii*, **PC** and **MP**
xv.	$(p \rightarrow q) \rightarrow (p \;\Box\!\!\rightarrow q)$	*x*, *xiv*, **PC** and **MP**

While important in its own right, this result also relates to the strongest consistent modal logic: TRIV.[41] TRIV is characterized by the axiom $p \leftrightarrow \Box p$. If we were to describe modality in terms of world accessibility, it corresponds to the assumption that accessibility is *reflexive* and *unique*; the actual world can access itself, and nothing else.

Converse Weak Centering entails TRIV. Select an arbitrary true proposition p. Because $\neg p$ is false, **PC** entails that $\neg p \rightarrow \bot$ is true—and Converse Weak Centering then entails $\neg p \;\Box\!\!\rightarrow \bot$. Given Definition$_1$, this is equivalent to $\Box p$. So, p entails $\Box p$. Because **Maximalism** entails Converse Weak Centering—and Converse Weak Centering entails $p \rightarrow \Box p$, **Maximalism** entails $p \rightarrow \Box p$.

As it turns out, we can also establish that TRIV entails **Maximalism**. If the only possible world is the actual world, then if it is possible for an object to bear property F, then some object actually does bear property F. This holds for every property; so, **Maximalism** is true. Moreover, because TRIV entails **Maximalism**—and because TRIV is consistent—it follows that **Maximalism** is consistent. In some respects, the connection between TRIV and **Maximalism** ought to be unsurprising. The maximalist is guided by the thought that the world is as full as it could be; it takes only a slight shift in emphasis to arrive at the view that the world could only be as full as it (actually) is.

[41] See (Cresswell and Hughes, 1996, pg. 67) for proof that this is the strongest consistent modal logic.

The Limit Assumption

Thus far, I have focused on the counterfactual logic's proof theory; I have largely omitted its semantics.[42] This restriction makes the discussion more easily intelligible—but limits the topics I can address. I close by broadening my horizons—and addressing a debate that has almost entirely occurred within counterfactual semantics: the **Limit Assumption**. The reason that this debate has occurred within the context of semantics is that it cannot even be *stated* in a first-order counterfactual language—or so I argue. Philosophers who would express the **Limit** in the object language—either to endorse or to reject it—ought to shift from a first-order language to HOCL.

The **Limit Assumption** holds that, for any possible antecedent p, there are most-similar possible worlds in which p is true.[43] The **Limit** allows for ties: cases where two (or more) p worlds are equally—and maximally—similar to the actual world. However, it forbids infinite sequences of p worlds—each of which is more similar to the actual world than the last. At some point in the sequence, we arrive at a 'limit': a p world so similar to actuality that nothing is more similar than it. The **Limit** is thus of particular interest to philosophers who analyze counterfactuals in terms of world similarity.

A classic counterexample was introduced by Lewis (1973*a*). Suppose there was a line of precisely one inch in length—and consider counterfactuals of the form 'If the line were longer, it would be of length $1 + x$.' For every nonzero value of x, a world in which the line has length $1 + x$ is not maximally similar to the actual world. After all, a world in which the line has length $1 + \frac{x}{2}$ is more similar still. There is thus an infinite sequence of worlds, each of which is more similar to the actual world than the last as the value of x diminishes. Because such sequences exist, Lewis holds that the **Limit Assumption** is false.[44]

Intuitive as this case is, there has been pushback; Pollock (1976) and Herzberger (1979) argue that rejecting the **Limit** conflicts with an independently appealing principle of counterfactual logic: the claim that if p were true, then everything counterfactually implied by p would be true simultaneously. More formally:

Pollock/Herzberger: $p \mathrel{\Box\!\!\rightarrow} \forall q((p \mathrel{\Box\!\!\rightarrow} q) \rightarrow q)$

(Note that, if the counterfactual excluded middle—$p \mathrel{\Box\!\!\rightarrow} q \vee p \mathrel{\Box\!\!\rightarrow} \neg q$—holds, then the consequent is a world-proposition: one that determines the truth-value of every proposition whatsoever). For reasons Lewis provides, given each positive value of x, the counterfactual 'If the line were longer, it would be of length $1 + x$' is false. The

[42] My thanks to Jeremy Goodman for suggesting a discussion of the **Limit Assumption** in this paper.

[43] Adherents include Stalnaker (1968); Pollock (1976); Herzberger (1979); Warmbrōd (1982). Dissidents include Lewis (1973*b*); Hàjek (forthcoming). For a discussion of various ways to precisify the assumption, see Kaufman (2017).

[44] To his credit, Lewis does not find this example to be definitive, stating, "This and other examples are not quite decisive; but they should suffice at least to deter us from rashly assuming there *must* be a smallest antecedent-permitting sphere." (Lewis, 1973*b*, pg. 20)

Pollock/Herzberger then implies that 'If the line were longer, it would not be of length $1 + x$ for every x' is true. But, if the line would not be of length $1 + x$ for every x, then the line would not be longer than one inch. So, it seems that denying the **Limit** licenses the counterfactual 'If the line were longer, it would not be longer'—an absurdity.[45]

This reasoning appeals to a version of **Closure**. It follows from the claim that, for every x, the line would not be of length $1 + x$, that the line would not be greater than one inch in length. This entailment is what justified concluding that if the line were longer, it would not be longer.

However, **Closure** does not license this inference as it stands. This is because every instance of **Closure** only takes *finitely-many* premises before it can be applied, and this case involves infinitely many premises. For this reason, Pollock and Herzberger claim that adherents of the **Limit Assumption** grant infinite instances of **Closure**, while dissidents only grant finite instances.[46]

At the outset, we ought to question our ability to distinguish finite from infinite entailment. In first-order languages, every instance of infinite entailment is accompanied by an instance of finite entailment. So, within these languages, those who subscribe to finite **Closure** are committed to infinite **Closure** as well.[47] In order to distinguish finite from infinite cases (which we must in order to distinguish opponents from adherents to the **Limit Assumption**), we require a language where infinite entailment is distinguishable from finite entailment. HOCL fits the bill.

Let us represent single-proposition entailment with $\leqslant$, so that $p \leqslant q$ iff $p \vdash q$. It is straightforward to define finite propositional entailment in terms of single-propositional entailment. We say that a collection Γ entails that p just in case the conjunction of Γ single-proposition entails that p. This definition of entailment cannot be straightforwardly extended to infinite cases if our language lacks conjunctions that are infinitely long.

Higher-order logic provides the resources to define infinite entailment. Effectively, an infinitely large collection of propositions Γ entails p just in case every proposition that entails every element of Γ also entails p. More precisely, we represent the infinite collection of propositions with a propositional operator X of type $(t \to t)$ (which asserts that a proposition is a member of the relevant connection). Infinite entailment can then be represented as:

[45] Note that the counterfactual 'If the line were longer it would not be longer' is equivalent to the claim that it is impossible for the line to be longer given Definition$_1$.

[46] More precisely, Pollock argues that the **Limit Assumption** is equivalent to the claim that, for an infinite $\Gamma \models r$, if $\forall q \in \Gamma, p \,\Box\!\!\rightarrow q$, then $p \,\Box\!\!\rightarrow r$.

[47] Take a satisfiable and infinite Γ such that $\Gamma \models r$. $\Gamma \bigcup \{\neg r\}$ is therefore not satisfiable. The compactness theorem for First-Order Logic states that an infinite collection of sentences is satisfiable just in case every finite subset of sentences is satisfiable. Therefore, there must exist a finite $\Delta \subset \Gamma : \Delta \bigcup \{\neg r\}$ that is not satisfiable. Δ must be satisfiable, so we have that $\Delta \models r$. Given the completeness of first-order logic, we then have $\Delta \vdash r$. But because Δ is finite, a finite instance of **Closure** will allow us to infer that, for every $q \in \Delta$, if $p \,\Box\!\!\rightarrow q$ then $p \,\Box\!\!\rightarrow r$. So, in a first-order language, any infinite instance of **Closure** entails the existence of a finite instance of **Closure**.

$$\leqslant_{\infty} := \lambda X.\lambda p.\forall r(\forall q(Xq \to r \leqslant q) \to r \leqslant p)$$

Armed with this definition of entailment, the infinite extension of **Closure** is:

$$(\Gamma \leqslant_{\infty} r) \to ((\forall q \in \Gamma(p \mathrel{\Box\!\!\to} q)) \to (p \mathrel{\Box\!\!\to} r))$$

Those who endorse the **Limit Assumption** claim that this is true; those who deny it claim that it is false.

Conclusion

Relatively weak assumptions about counterfactual logic have substantial metaphysical implications. Adherents of HOCL must endorse the necessity of identity and distinctness—and the vacuity of counterpossibles (in general) and counteridenticals (in particular). They must also accept the **Barcan**, **Converse Barcan**, **Being Constraint**, **Necessitism** and the **Principle of the Identity of Indiscernibles**. They have the resources to demonstrate that **Maximalism** collapses the counterfactual conditional into the material conditional—and can state the **Limit Assumption** in the object language.

If nothing else, I hoped to have piqued readers' interest in higher-order counterfactual logic. HOCL is, in many respects, an extraordinarily weak system. I have no doubt that a stronger logic will yield yet more metaphysical implications. While this is among the first papers to explore the metaphysics contained within higher-order counterfactual logic, I very much hope that it will not be the last.

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