# IN SEARCH OF AESTHETICS, ALHAZEN'S OPTICA 

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# IN SEARCH OF AESTHETICS, ALHAZEN'S OPTICA 

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## Praise be to God, The Cherisher and Sustainer of the Universes:

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# ABSTRACT <br> IN SEARCH OF AESTHETICS, ALHAZEN'S OPTICA 

Ahmed Elkady

## Marco Frascari

The aesthetic theory of AlHasan Ibn AlHaytham, known in Latin as Alhazen (965-1041 A.D.), has hardly received any attention in the realm of aesthetic investigations. The aesthetic theory is based on twenty-two criteria, which he describes in his book, Kitab Almanazir, better known in Latin as Optica. To illustrate the notion of beauty presented in Optica, one has to understand Alhazen's cultural encyclopedia based on classical Greek knowledge and Islamic sciences. The Greeks and the Egyptians before them, relied on the concepts of measure and geometry as the foundation of aesthetic criteria. By examining the third chapter of Optica, one can see that Alhazen does not look upon these traditional concepts as the fundamental principles of beauty, but he approaches the problem differently. He addresses it as a continuation of his research theme in Optica-to present a proper theory of vision. He dilates upon his twenty-two characteristics as they can be perceived by sight. Alhazen calls these different characteristics the "visible properties"
(light, color, distance, position, solidity, shape, size, separation, continuity, number, motion, stillness, roughness, smoothness, transparency, opacity, shadow, darkness, beauty, ugliness, similarity, and dissimilarity). Explaining each of these properties and the way they are perceived, Alhazen comes up with a hypothesis of "how to entitle an object to be considered beautiful" and a new interpretation of beauty, which is based upon the visual qualities of visible objects. This search of Alhazen hypothesis is essential for drawing architecture out of the present condition of impasse.
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The idea behind this dissertation inaugurated from examining the second chapter written by Erwin Panofsky on the history of the theory of human proportions as a reflection of the history of styles. ${ }^{1}$ Panofsky mentions that Lorenzo Ghiberti (1378-1455) was one of the earliest post classical champions to set principles of aesthetic perfection. Moreover, Ghiberti derived his principles from an Arabic source, Kitab Almanazir (كتاب المناظر) of Alhazen (965-1041), which is known in Latin as Optica. Even more interesting is the fact that Ghiberti, while drawing from Alhazen, promoted the idea of proportionality to an entirely different status. Alhazen does not look upon proportionality as "the" fundamental principle of beauty; rather, he mentions it, as one might say en passant. ${ }^{2}$

Alhazen's reputation is well known as a polymath. Although his fame as a physicist and his theory of vision ${ }^{3}$ are noticeable in the history and evolution of science and scientific investigation, the core of my interest in his work deliberates around his writings on what we would call aesthetics.

[^0]In Optica, Alhazen enumerates twenty-two principles or criteria of beauty as an essential part of his theory of vision. According to him, there is no category of optical perception (such as light, color, size, distance, position, continuity, roughness, transparency, shadow, similarity, etc.) which cannot operate as an aesthetic criterion under certain conditions, either individually or combined with other categories. Alhazen's theory is primarily a theory of how we come to qualify an object as beautiful.

The structure of this dissertation attempts to illustrate the apprehension of beauty in Optica, through the foundation of the classical Greek knowledge which played an important role in Alhazen's early learnings.

The first chapter investigates the transformation of classical knowledge through the course of time, the influence of Islamic science, Alhazen's biography and his conjunction to Greek philosophy. The second chapter explains the concepts of measure, which formed the base of the science of mathematics and the concepts of geometry as the basics of an aesthetic criterion. The third chapter discusses the different ways of perception that are mentioned in Optica. The fourth chapter explains each of what Alhazen has called the visible properties. Then it leads to his interpretation of the perception of beauty and how an object is entitled to be considered beautiful.

This dissertation is a philosophical investigation of Alhazen's hypothesis of aesthetics through his writings in the third chapter of Optica. My study primarily concerns shedding the light on the aspects of Alhazen's theory, which has hardly been researched thoroughly. Although this research does not investigate the influence and inspiration of Alhazen's theory of aesthetics on the Renaissance period, it seems obvious through the extraction in the Commentari or Memoirs written in 1450 by Lorenzo Ghiberti, the famous sculptor and a friend of Alberti ${ }^{4}$, that indeed it did have an impact.

[^1]
# Transformation of Classical knowledge 

Prologue:
The notion of Islamic Science:
Alhazen's concepts:
Biography:
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## Prologue

With the demise of Classical civilization, the scholarship of Greek scientists was absorbed into the language and learning of the Arab and thereby begun its circuitous route back to the West. Arabic science began to flourish about 800 AD . In the process of translation and transmission of older scientific texts, the Arabic language became the vehicle par excellence for scientific and philosophical thought. Thus began what is called the Arabic scientific tradition - the activity of scholars of different nationalities, who came from various parts of the Muslim world (Persia, Arabia, Syria, Egypt, Spain, India, etc.), who had different religions (mainly Muslims, but also Christian, Jewish and others) and who mainly wrote in Arabic. Arabic science flourished with few interruptions until ca. 1450. The transmission of Arabic texts or Arabic translations of Greek texts to (Western) Europe was a decisive factor in the development of science and philosophy in the twelfth and thirteenth centuries, which in turn paved the way for the Renaissance in Europe.

The Arabs were the greatest contributors to the science of optics. Their work, which often combined highly original experimentation with the groundwork of classical theories, had a profound effect with its revival in Europe. One of the most influential Arabic treatises on optics was AlHasan ibn AlHaytham's (known as Alhazen in Latin) "Kitab Almanazir" (كتاب المناظر) or simply Perspectiva, as it was entitled in Latin (also known as Optica). Written in Cairo during the early eleventh
century, the work was introduced to the West around 1200 and became the primary source for thirteenth century optical investigation by Vitellio, ${ }^{5}$ Roger Bacon and John Pechman and other notable European scientists such as Leonardo da Vinci, Kepler and Descartes. ${ }^{6}$ It was translated into Italian during the fourteenth century and a version of it was abridged and inserted by Lorenzo Ghiberti (1378-1455), ${ }^{\text { a a famous }}$ sculptor and a friend of Alberti, in his "Commentari" or Memoirs. ${ }^{8}$ One of Alhazen's great contributions was that he effectively integrated the old Greek mathematical attitude toward the visual cone with the physicist's appraisal of how sight works. The result was a model explaining how the "figure" (sura) of things passes into the eyes and thence, to the brain. His mechanistic theory of vision is as simple as it is ingenious and it prevailed in the West at the time of Kepler.

5 In, Erwin Panofsky, "The History of the theory of human proportions as a reflection of the history of styles" Meaning in the visual arts. Chicago: The University of Chicago Press, 1982, p. 90, he wrote "On the whole, the pertinent passage of the Optica was taken over word for word and not selectively, by a medieval writer like Vitellio".

6 J. P. Hogendijk, Ibn al-Haytham's Completion of the Conics. New York, Berlin, Heidelberg, Tokyo: Springer-Verlag, 1985. p. 57.

7 Samuel Edgerton, The Renaissance rediscovery of linear perspective. New York: Harper \& Row, Publishers, 1976. p. 73.

8 The Commentari was written around 1450, the third of this three-book memoir is filled with glosses lifted verbatim from the writings of Alhazen, Bacon, Vitellio, Pecham, Avicenna and Vitruvius.

The objective of this study endeavors to illustrate the concept of beauty in Optica, through the foundation of the classical Greek knowledge which was integrated into the scholarship of the Arab and elaborated by the ethics and principles of Islam.

## The notion of Islamic Science

The rise of the Muslims to the climax of civilization in a period of less than one hundred years and the evolution of Islamic Science were based on Islam's over insistence on learning. The Quran and the heritage of the Prophet Mohammed are saturated with references to learning, education, observation and the use of reason. The very essence of Islam is summed up in the first verse of the Quran, revealed to the Prophet Mohammed in 611 A.D.:

> Read: In the Name of thy Lord who created, Created Man of a blood-clot. Read: And thy Lord is the most Generous Who taught by the Pen, Taught Man, that he knew not. (96: 1-5)

The Prophetic traditions supplement the emphasis on learning and investigation as Prophet Mohammed says "An hour's study of nature is better than a year's adoration" and "Seek knowledge from the cradle to the grave".

The main task facing Islamic thinkers since the birth of Islam is the basic religious phenomenon of the Sacred Book, the law of life within this world and the
guide beyond it. The first and last task is to understand the true meaning of the Quran. Another primary fact in Islam is the absence of the phenomenon of the Church. Just as Islam has no clergy which possess of the "means of grace", so it has no dogmatic magisterium, no pontifical authority, no Council which is responsible for defining dogma. ${ }^{9}$

It is this open, non-authoritative environment plus the insistence on learning that motivated many Muslim philosophers and scientists to achieve their accomplishments ${ }^{10}$ and one of them was Alhazen. Alhazen did all of his studies on optics within the pattern of the Islamic philosophical and sociological universe. While performing experiments with light, Alhazen never failed to remind himself and the readers of his textbooks that 'God is the light of the heavens and the earth'. For many Muslim scientists, science was integral to Islam and equivalent to piety.

The understanding of light in Islam stems from the belief of absolute oneness of God and from verses of the Quran, like the one in which God describes himself as follows:

God is the light of the heavens and the earth. The parable of his light is as if there were a

[^2]niche and within it a lamp, the lamp enclosed in glass, the glass as it were a brilliant star, lit from a blessed tree, an olive, neither of the east nor of the west, whose oil is well-nigh luminous, though fire scarce touched it, light upon light, God doth guide whom he will to his light, God doth set forth parables for people and God doth know all things. (24: $35)^{11}$

It can also be found in the verse that describes the Quran as light:

O mankind, verily there have come to you a convincing proof from your Lord, for we have sent unto you a light that is manifest. (4:174)

Rationalism, as a philosophical movement, made an appearance in Islamic history almost immediately after the death of prophet Mohammed. Citizens of the newly established state and expanding empire would question and argue with the officials about their unreasonable behavior. Rationalism in Islam, thus began both as
${ }_{11}$ AlGhazali ( $1059-1111$ ) explains this verse by stating that the niche, the glass, the lamp, the tree and the oil symbolize the five spirits of the human soul. (1) The sensory spirit (which takes in the information brought by the senses) is symbolized by the niche. (2) The imaginative spirit (which records the information conveyed by the senses and presents it to the intelligential spirit, when required), is symbolized by the glass which is clear and transparent. (3) The intelligential spirit (which apprehends ideas beyond the spheres of sense and imagination) is symbolized by the lamp. (4) The discursive or ratiocinative spirit (which takes the data of pure reason, combines them and deduces from them abstract knowledge) is symbolized by the tree. (5) The transcendental prophetic spirit (by which the unseen tables of the law are revealed together with the sciences of realms and deity) is symbolized by the oil. All of these are lights, for it is through them that every kind of living is manifested.
a weapon against injustice and a method for elaborating religious beliefs. During the first two hundred years of Islam, philosophy stemmed from the reading of the Quran and the Sunnah ${ }^{12}$ and it was used to provide rational grounds for the beliefs that the world and matter are created and not eternal. Also, it was used to demonstrate the existence of a creative God, unique and incorporeal. Intellectual discourse and debates were a common occurrence of this period. There was not much of a difference in the positions of those who labeled themselves theologians and those who labeled themselves Mutazilites.

The Mutazilites were totally monopolized by Greek philosophy. ${ }^{13}$ While the Mutazilites were highly critical of Islamic sources, they uncritically accepted all the basic assumptions of Greek philosophy, even when they were contradictory to the basic teachings of Islam.

Muslim rulers, many of whom saw themselves as Plato's philosopher kings, were quick to realize that Greek philosophy provided certain legitimacy for their authoritarianism and the perpetuation of their dynasties, positions which could not be justified within Islam. The Abbasids, a dynasty of caliphs who ruled the Islamic

12 Sunnah is the Prophet's tradition.
${ }^{13}$ Such as Ibn Sina (980-1037), known in Latin as Avicenna who wrote, Book of Healing, a collection of treatises on Aristotelian logic, the nature of sciences and other subjects. Also AlFarabi (870-950) who built the philosophical system of Brethren of Purity "Ikbwan AlSaffa", which sought to purify their souls by philosophy.

Empire from 750-1258 in Baghdad, were very impressed with Greek philosophy. ${ }^{14}$ Caliph AlMamun, who funded the translation of a vast corpus of Greek thought into Arabic, made Mutazilism the official doctrine of the state between 833 and $848 .{ }^{15}$ Thus rationalism, which originally emerged as a tool to fight injustice, changed sides and was allied with the state.

The majority of Muslim scholars rejected Greek philosophy because it was clearly an alien way of thinking. The conclusions that Greek philosophy offered the only demonstratively respectable conclusions, often ran against the principles of Islamic theology. But in rejecting Greek philosophy, Muslim scholars did not reject reason; ${ }^{16}$ they were aware of and positively conscious of the importance of reason in general and in particular, the exposition of religious arguments. What they were objecting to was the dogmatic belief that reason alone can lead to absolute truth and that reason can become the basis of an ethic. ${ }^{17}$

14 The study of Greek science was much encouraged by Caliph Harun AlRashid (reigned 786-809) and even more by his son Caliph AlMamum (reigned 813-833). In this period, a large number of Greek manuscripts were collected in Baghdad.
${ }^{15} \quad$ Ziauddin Sardar, Islamic Futures and Policy Studies, Explorations in Islamic Science. London: Mansell Publishing Limited, 1989. p. 16.
${ }^{16}$ The opposition to the Mutazilites led to the production of many schools of thought. These schools boasted such brilliant thinkers among their ranks as AlRazi (d. 924), AlGhazali (d. 1111) and ibn Khaldun (d. 1406).
${ }^{17}$ From the point of view of some scholars, there was nothing new about the methodology of

## Alhazen's concepts

Alhazen has been described by many Western historians of science as the most secular of Muslim scientists because of his unquestioned commitment to science for science's sake. This is because he placed a high level of confidence in observation, experimentation and empirical analysis. However, he did not lose sight of philosophical and metaphysical methods.

A person who studies scientific books with a view to knowing the truth, ought to turn himself into a hostile critic of everything that he studies. ... He should criticize it from every point of view and in all its aspects. And while thus engaged in criticism he should also be suspicious of himself and not allow himself to be easy going and indulgent with regard to (the object of his criticism). If he takes this course, the truth will be revealed to him and the flaws . . . in the writings of his predecessors will stand out clearly. ${ }^{18}$

Alhazen came to glorify, above everything, the commitment to truth for its own sake, independent of school traditions and sectarian dogmatism Although he had the highest regard for Ptolemy and Aristotle, he advocated the most vigorous

Greek philosophy. AlGhazali went so far as to demonstrate that logic and Aristotelian syllogisms were already recommended in the Quran and even illustrated Aristotelian logic with examples from Islamic law.

[^3]criticism of scientific writngs to the point that his program of methodological criticism has been compared to that of Descartes. ${ }^{19}$

Alhazen, in his scientific practice, combined experience and reason. Such expressions, such as "Let us make it clear by reasoning and experience", frequently occurring in his works, show that he believed in combining rational and empirical elements for studying Nature. ${ }^{20}$ Although Alhazen emphasized the pursuit of science for its own sake, he also emphasized the fact that it should be pursued within a framework of philosophy and theology. In a letter dated some thirteen years before his death, Alhazen wrote:

There are three disciplines which go to make philosophy: mathematics, physical sciences and theology. I have discovered that duality and controversy are natural to human beings and man is mortal; so that while in his youth man can ponder over these three disciplines which govern his existence on earth, he cannot do so when he grows old. So I thought over these three philosophical disciplines so far as my ratiocinative and intellectual faculties could allow me and summarized and explained them and their branches. ... I have three objects in adopting this view: first, to be of service to those who are in search of truth; second, that the disciplines which have been able to
19 Anton Heinen, "Al-Biruni and al-Haytham: a comparative study of scientific method", in Hakim Said, Al-Biruni Commemorative Volume, Karachi: The Times Press, 1979. pp. 501-513.
${ }^{20}$ Muhammad Saud, The Scientific Method of Ibn al-Haytham. Islamabad: Islamic Research Institute, 1990. p. 18.
understand to some extent should be extended and studied; and, third, the knowledge that I possess may turn out to be the wherewithal of my old age. ${ }^{21}$

The ethical framework of Alhazen is based on two fundamental principles: sacrifice for truth and search for knowledge. For him, ethical perfection requires that truth should be looked for and desired both in the worldly and religious aspects of human life, truth should take priority over falsehood and honesty and justice should be the mode of his life.

For Alhazen theology was just as real as science. He believed that knowledge and wisdom went hand in hand: "I have always been haunted by the desire to seek knowledge and wisdom and it has also dawned on me that there is nothing better than these two things to bring man closer to God," he wrote. ${ }^{22}$

Alhazen considers the pursuit of science, without an ethical framework, to be inconceivable. Consequently, ethics for Alhazen is a pragmatic concern, not some abstract philosophical notion. He equates every action with accountability on the Day of Judgment. His ethical system is based on three main points:
${ }^{21}$ Quoted by Ziauddin Sardar, Islamic Futures and Policy Studies, Explorations in Islamic Science. London: Mansell Publishing Limited, 1989. p. 91.
${ }^{22}$ Quoted by Naseer Ahmed Nasir, "Ibn al-Haytham and his philosophy", in Said, $I b n$ Al-Haytham. Karachi: The Times Press, 1969. pp. 80-93, p.80.

1. Beautification and perfection of morality are not possible without the quest for knowledge;
2. The acquisition of truth, knowledge and realization of self depends upon (a) a clean and thorough understanding of theology, (b) the achievement of good through noble deeds and (c) the avoidance of evil;
3. The main object of beautification and perfection of morals is to enjoy a happy, eternal life in Paradise in the hereafter.

It is this ethical edifice that forms the base of Alhazen's works. ${ }^{23}$
Alhazen, a scientist from the classical period of Islam, introduced the inductive method and was an arch believer in rationality, a belief that has led many orientalists and western historians to dub him a secularist, an Aristotelian, even a scientist in the tradition of the Enlightenment, whose rationality was subservient to his ethical system. His firmness of faith was very obvious in his opposition to the Mutazilites, who were regarded as the most free thinking and heterodox group of Muslims. Since the Mutazilites dallied with all sorts of metaphysical and ethical questions, Alhazen should normally have been sympathetic to their cause. What was repugnant to him, though, was their recourse of naturalism into matters of faith; he wrote several treatises against them. ${ }^{24}$ It is, in fact, an irony of fate that Basra, where
$23 \quad$ Ibid., p. 84.
24 Abdul Ghafur Chaudhri, "Ibn al-Haitham: The educational and scientific importance of his writings", in Said, Ibn Al-Haytham. Karachi: The Times Press, 1969. pp.109-123, p. 122.
the Mutazilite movement had its origins, was also the birthplace of one of the greatest physicists of Islam and mankind, whose other field of interest was the refutation of the rationalist doctrine of the Mutazilites.

Western historians accepted the contributions of Alhazen and placed them in the linear progress of science from the days of the city-states of Greece. Yet, even a casual examination of Alhazen's methodology reveals a different system of science: a system which believes not in a single, all-pervasive method but in a multiplicity of methods, giving due importance to all; a system that believes in rationality, but in a rationality that is subservient to an ethical code; and a system that draws its strength from a matrix of Quranic concepts and values which it seeks to promote. For Alhazen, science is a means of seeking the pleasure of Allah; it is considered to be a form of worship which has a spiritual and a social function. He also believed, as a Muslim scientist, in revelation and regarded reason as one instrument for moving toward God. This system of science is very substantial in grasping the essential of any true Islamic quest as it was guided by Alhazen and through his accomplishments, contributing to the formation of the notion of Islamic science.

## Biography

Alhazen is mentioned in general biographies of scholars, such as the Sources of Information on the Generations of Physicians "Uyun Alanba fi Tabaqat Alatibba"
by ibn Abi Usaybia (1270); the History of the Scholars "Tarikh Albukama" by ibn AlQifti (1248) and the History of the Scholars of Islam "Tarikh Hukama Alislam" by Ali AlBayhaqi (1169/70). The accounts are sometimes contradictory and are less specific than one would expect for a scholar as famous as Alhazen.

Another source is of a more exceptional nature. It is a short autobiography which Alhazen wrote in a letter in 1027 , informing us about his own intellectual development. The letter is lost, but the autobiography is quoted by ibn Abi Usaybia in the Sources of Information on the Generations of Physicians mentioned above.

Alhazen's full name was Abu Ali ibn AlHasan ibn AlHusayn ibn AlHaytham and he was nicknamed Ptolemaeus secundus ${ }^{25}$. He was a physicist, an astronomer, a mathematician, a philosopher, an engineer and a theologian. He had a good knowledge of medicine but did not practice it. According to ibn Abi Usaybia, none of Alhazen's contemporaries was equal to him (Alhazen) in mathematics. ${ }^{26}$

All authorities agree that Alhazen was born in Basra, a city in southeastern Iraq, in 355 H./965 A.D. ${ }^{27}$ and he spent the first part of his life there. He was the product of an age which, in many ways, was characterized by intense intellectual

25 Henry Corbin, History of Islamic Pbilosophy. Translated by Liadain Sherrard. London: Kegan Paul International, 1993. p. 149.

26 Ahmad ibn AlQasim ibn Abi Usaybia, Uyun Alanba fi Tabaqat Alatibba, Volume III, 1270, Beirut: Dar Elfikr, 1957. p. 149.
$27 \quad$ Alhazen wrote his autobiography at the end of 417 H./ February 1027 A.D. in his sixty third (lunar) year.
activities and the climax of medieval thought. ${ }^{28}$
In his autobiography, Alhazen states that in his youth, he investigated the various doctrines of the religious sects. Then he realized that there is only one truth and that the differences in the doctrines must be differences in approach. When he had completed his intellectual education, he started searching for a criterion through which truth could be discerned.

Alhazen concluded that the only foundation of truth was in rational opinions on perceptible things and these he found in the Writings of Aristotle. ${ }^{29}$ He also read and studied Euclid, Plato and Archimedes, summarizing and commenting on everything he read. Then he devoted all of his energy to the study of the three philosophical sciences: the triad of mathematics, physical sciences and theology (metaphysics). ${ }^{30}$

The middle part of Alhazen's life has been told differently. ibn AlQifti says that Alhazen had composed a treatise, Hilat-ul-Bur, on the construction of a dam across the Nile that would regulate its annual flow by achieving the dual purpose of controlling the floods on the Nile and storing water for irrigation purposes for the

28 George Sarton, Introduction to the History of Science. Volume I. Baltimore: The Williams \& Wilkins Company, 1962. p. 693.

29 J. P. Hogendijk, Ibn al-Haytham's Completion of the Conics. New York, Berlin, Heidelberg, Tokyo: Springer-Verlag, 1985. p. 54.
${ }^{30}$ Ahmad ibn AlQasim ibn Abi Usaybia, "Uyun al-Anba fi Tabaqat al-Atibba", Volume III, 1270, Beirut: Dar Elfikr, 1957. p. 153.
whole year. Alhazen's fame ${ }^{31}$ reached the ruler of Egypt, the Fatimid Caliph AlHakim, who ruled Egypt between 996-1021 and who was a patron of scholars and had attracted philosophers, scientists and theologians of repute to his court. ${ }^{32}$ AlHakim assigned him to lead an expedition group of professional builders and architects. Alhazen traveled down the Nile with his group to achieve a method of regulating the annual Nile inundation. He reached a place south of Aswan called Elganadel, where water falls from a higher level to the stream of the Nile. He realized then that his ideas would not work out as he had planned it. This was due to a shortage of labor and a lack of the necessary earth-excavating and earth-moving equipment. ${ }^{33}$ He concluded that his project was not feasible. After his failure in this task, he went back to Cairo and unwillingly accepted to be in charge of an administrative office due to his fear of AlHakim. After a while, realizing the harshness of the caliph, he simulated madness; The caliph and his representatives confiscated his belongings and confined him to his house. Subsequent to the death of AlHakim, Alhazen revealed his sanity and his belongings were then returned to him.

[^4]32 Allama Alauddin Siddiqui, "The legacy of Ibn al-Haitham" in Said, Ibn al-Haytham, Karachi: The Times Press, 1970. pp. 69-70.
${ }^{33}$ Muhammad Saud, The Scientific Method of Ibn al-Haytham. Islamabad: Islamic Research Institute, 1990. p. 2.

Alhazen spent the last part of his life in Cairo under a dome ${ }^{34}$ nearby the gate of ElAzhar Mosque earning his living by copying the manuscripts of the Uqlidas (Euclid's work). He died in 432 H./ 1041 A.D. at the age of around 75 years. This last period of his life seems to have been a time of enormous scientific activity, as he produced 21 treatises on geometry, physics, astronomy, mathematics and philosophy between February 1027 and July 1028. After July 1028, he had written several writings on mathematics ${ }^{35}$ and physics.

## Optica

Alhazen's main work, ${ }^{36}$ Kitab Almanazir or Optica, deals with the theory of vision, the theory of perception, visual deception, the laws of reflection, mathematical
${ }^{34} \quad$ Alhazen's life is mentioned in general biographies of scholars, such as Ibn Abi Usaybia's "Uyun al-Anba fi Tabaqat al-Atibba" (1270), Ibn AlQifti's "Tarikb al-Hukama" (1248) and Ali AlBayhaqi's "Tarikb Hukama al-Islam" (1169/70). These biographies give short accounts of the circumstances of his life. During that part of his life, living under a dome is mentioned by Ibn AlQifti; the other sources mention that Alhazen addressed himself to a simple life and lived in a small room in the ElAzhar area. This accounts for the relation of simplicity and modesty to the expression and the act of "living under a dome".
${ }^{35}$ One of the most important of Alhazen's writings on mathematics is "On the Resolution of Doubts in Euclid's Elements and Interpretation of its Special Meanings ". He does not only confine himself to explaining the difficult passages of the Euclidean proofs, but he also finds alternatives to the given proofs; for instance, instead of indirect proof (reductio ad absurdum), he used direct demonstration.
${ }^{36}$ Alhazen left several minor writings on physical optics, among them On Light, On Twilight
problems concerning reflection in mirrors and errors of vision due to reflection and refraction. The Arabic text survived in five copies, only one of which is complete. ${ }^{37}$ The oldest and best copy is referred to as "ElAskary copy" named after its copyist. It


Figure 1: Albazen's theory of vision.
is dated forty-three years after the death of Alhazen and is preserved in Istanbul.
Optica stands high above the optical works of Alhazen's predecessors. The work contains many new results and many assertions are proved by systematic experiments. The detailed technical descriptions in Optica as well as in other works show that Alhazen really performed his experiments. ${ }^{38}$

Phenomena, On the Buming Glass and other treatises that deal with the rainbow, the halo and with spherical and parabolic mirrors.
${ }^{37}$ Optica consists of seven maqalas or books, which are divided into two main parts. The first, consisting of the first three books, expounds a theory of direct radiation and direct vision. The second, consisting of the last four books, deals with optical reflection and refraction.
${ }^{38}$ J. P. Hogendijk, Ibn al-Haytham's Completion of the Conics. New York, Berlin, Heidelberg,


Figure 2: Diagram of the eye from Kamal Aldin's copy of Optica in Istanbul.

Alhazen's objective in Optica was to achieve a synthesis between the geometrical optics of Ptolemy and Euclid and the traditions of natural philosophy, including the Aristotelian tradition. Alhazen disapproved the Euclidean and Ptolemaic doctrine of visual rays emerging from the eye and proved that "forms" of


Figure 3: Diagram of the eye from the Royal Observatory copy in Edinburgh, 1269.

Tokyo: Springer-Verlag, 1985. p. 57.
light propagate from any point on the luminous object in all directions. He states that visibility is the result of the transmission of the image of the object first [see (Figure number 1)] to the retina, then to the optic nerve and then through the latter to the brain. Alhazen not only presented a correct representation of the mechanism of vision but also discussed the anatomy and physiology of the eye. [see (Figure number 2) and (Figure number 3)].

The Latin translation of Optica was first mentioned in a book, named "In trigonometry", by Gordanous De Nimory who was famous between 1220 and 1230. ${ }^{39}$ There are 20 Latin translations of Optica, ${ }^{40}$ of which at least seven were copied in the thirteenth century; Among them is a copy dated 1269, which is at the Royal Observatory library in Edinburgh. There is also an Italian copy of the fourteenth century translated from Latin at the Vatican Library. ${ }^{41}$

## Aesthetics

Erwin Panofsky cites in his study, "The history of the theory of human proportions as a reflection of styles", that Ghiberti (1378-1455) was one of the

39 Marshall Clagett, Archimedes in the Middle Ages, Madison, 1964. pp. 668-669.
${ }^{40}$ David Lindberg, A Catalogue of Medieval and Renaissance Optical Manuscripts, Toronto: Pontifical Institute of Mediaeval Studies, 1975.
${ }^{41}$ Abdelhamid Sabra, Kitab Al-Manazir, the National Council for Culture, Arts and letters, Kuwait, 1983. pp. 47.
earliest post classical champions to set principles of aesthetic perfection. Ghiberti derived his principles from an Arabic source, the Optica of Alhazen, "Kitab Almanazir". ${ }^{42}$ Even more interesting is the fact that Ghiberti, while drawing from Alhazen, promoted the idea of proportionality to an entirely different status. Alhazen does not look upon proportionality as "the" fundamental principle of beauty; rather, he mentions it, as one might say en passant.

In his phenomenal excursus on what we would call aesthetics, Alhazen enumerates no fewer than twenty-two principles or criteria of beauty because according to him, each category of optical perception (such as light, color, size, position, continuity, etc.) operates as an aesthetic criterion under certain conditions; In the context of this long list there appears, quite inorganically connected with the other "categories," the paean to the "relationship of the parts." Ghiberti ignored all the other categories and, with a remarkable instinct for that which is classical, appropriated only the passage in which the catchword "proportionality" occurred.

The title of the third chapter in the second book in Optica is "In the distinction of each partial criterion perceived by the visual sense". Alhazen devoted this chapter to detail the perception of the characteristics of objects. Although he says it is difficult to enumerate these characteristics, he suggested twenty-two criteria of aesthetics. They are either simple or compounded of two or more varieties. These
${ }^{42}$ Erwin Panofsky, " The history of the theory of human proportions as a reflection of the history of styles " Meaning in the visual arts. Chicago: The University of Chicago Press, 1982. p. 89.
crıterıa are: Light, Color, Distance, Position, Solidity, Shape, Size, Separation, Continuity, Number, Motion, Stillness, Roughness, Smoothness, Transparency, Opacity, Shadow, Darkness, Beauty, Ugliness, Similarity and Dissimilarity.

Alhazen's aesthetics is remarkable, not only for the division of the beautiful into as many criteria as there are categories of visual experience, but also for its pervasive relativism. Distance can be conducive to beauty in that it subdues imperfections and irregularities, but the same is true of proximity in that it renders effectively the refinements of the design, etc. ${ }^{43}$ On the whole, considering the fact that Alhazen's main sources were the classical writings of Plato and Euclid and that he was commenting on and adding to their treatises - the relevant chapter of Optica (which was taken over word for word and not selectively by medieval writers, like Vitellio) deserves attention, due to the fact that it had its influence on Lorenzo Ghiberti and others. ${ }^{44}$

## Conclusion

The absorption of classical knowledge and the scholarship of Greek civilization into the learnings of the Arab through translations in the eighth and
${ }^{4}$ Ibid., p. 90.
44 Alberti (1404-1472), who dedicated his treatise on painting "Della pittura" to Ghiberti and four other artists, was one of the attendants of Lorenzo Ghiberti's (1378-1455) workshop, where artists from all over the city of Florence came to meet and discuss their daily activities.
ninth century was one of the influential determinants that led to the emergence of many Islamic thinkers and philosophers. Through this atmosphere, Alhazen emerged with his philosophy and had a remarkable impact in defining the perception of Islamic science through an ethical framework. Alhazen's main work, Optica, formed the groundwork of education in the field of optics. Judging from the fact that textbooks on optics through the seventeenth century were essentially summaries and extractions of Optica, it was translated much earlier into the Latin language.

Understanding the transformation of classical knowledge inheres in grasping the idea of origins of measure and origins of geometry. The apprehension of measure which formed the base of the science of mathematics, along with the science of geometry, were essential in devising Greek philosophy. By its turn, Greek philosophy was the domain of the studies and the comments of physicists and mathematicians, such as Alhazen, which led to the production of an enormous body of Arabic thought. Subsequently, the transformation took its way back again to Europe through the spread of Arabic scientific texts and its Latin translations.

## Origins of Alhazen's conceptions

A: The perception of measure and mathematics

B: Inception of geometry and its aesthetics, the golden section

## (A)

Introduction: The philosophy of measure: Symbolic concepts: Beliefs associated with measure: Anthropometric measures:

Egyptian perception:
Greek concept:
Conclusion:

## Introduction

> If language is to be a means of communication there must be agreement not only in definitions but also (queer as this may sound) in judgments. This seems to abolish logic, but does not do so. It is one thing to describe methods of measurement. But what we call "measuring" is partly determined by a certain constancy in results of measurement. ${ }^{45}$

In Alhazen's early stages of life, he studied philosophical sciences which forms the body of classical knowledge. Philosophical sciences are thought to have originated from Geometry and in turn, Geometry is considered to result from the concept of Measure. The sources of classical knowledge originate from the establishments of the Greeks and their main source, the Egyptians. Greek philosophy was influenced and dominated by a mathematical, or geometrical, concept of the universe. ${ }^{46}$ The earliest notion of dividing the universe is found in Homer, who divided it into five equal parts. Later, the Pythagoreans claimed the ruling principle of numbers, believing in a geometrically-ordered cosmos. The dogma regarding the geometrical form of the universe also appears throughout the works of Plato and Aristotle and is reaffirmed later by Plutarch in his Moralia. ${ }^{47}$

[^5]Muslim philosophers and thinkers in the tenth century were influenced by the luminaries of Pythagoras, Plato and Aristotle, which were devised and revived in a new Islamic firmament. Concurrent to Alhazen, we find the clear influence in the writings of many thinkers, such as the Brethren of Purity. In their Rasail or epistles ${ }^{48}$, we find clear and unmistakable traces of the Pythagorean doctrine, which held that numbers and their properties could explain the whole structure of the universe and that mathematical principles were the root of everything.

This phenomenon points out the importance of understanding the original essence of Classical knowledge, which lies in the perception of Measure, Mathematics, Geometry and the development of aesthetics, based upon this knowledge in order to comprehend the aesthetic hypothesis of Alhazen.

## The philosophy of Measure

Ludwig Wittgenstein presents us with an interpretation of the meaning of "measuring". The idea of measuring is partly determined by certain a constancy in the
${ }^{47}$ They maintained the idea that the universe was based on the five regular polyhedra: earth was based on the cube, fire on the pyramid, air on the octahedron, cosmos on the dodecahedron and water on the icosahedron. Plato. Timaeus, translated by Francis M. Cornford, 1959. pp. 60-61.
${ }^{48}$ These are fifty-two epistles written by a group of philosophers, who lived in Basra during the tenth century. Their subject is vast and ranges from mathematics, music and logic through mineralogy, botany and embryology to philosophy and theology and finally a treatise on magic. The epistles are divided into four main sections: fourteen on Mathematical sciences, seventeen on Natural sciences, ten on Psychological and Rational sciences and eleven on Theological sciences.
results of measurement, but as the expression itself reveals, there must have already been some disciplined observations and comparisons (i.e., measurements) for the constancy to have been made visible. Which came first, the activity or the goal? For example, consider a very young child being struck, in a vague way, by similarities in things. This sharpens his attention to relative sizes, so he comes to notice more and to finer regularities. The mutually-supporting process continues, guided in part by his growing interest in particular sorts of comparisons (for example, between two portions of food). At some point he will learn (probably from adults) that one size can be taken as a standard by which others may be counted. ${ }^{49}$ This understanding of counting the units in something's size (or weight, etc.) comes with a realization of what we call "measuring" and for the first time, makes it possible to view an object's particular "size" in such a way that it can be authoritatively compared with other sizes not present. Now, as adults, we tend to forget the original process and think that "measuring" came about because we needed a name for the natural procedure of counting units of size. Actually, there were no "units of size" visible until we went through an elaborate stage-setting, which culminated in the significant use of the word "measure".

Man needed a unit of measure to allow himself to size up his own creatıve work and nature's creation around him, in comparison with his own size and in
$49 \quad$ Jeffrey Price, Language and Being in Wittgenstein's Pbilosophical Investigations. The Hague: Mouton, 1973. p. 50.
relation with his own strength. He needed a unit that was stable, invariable, already established by his constant vision and absolutely clear in meaning in his mind. Naturally, he should find the answer in the dimensions of his own body as this was for him a constant and immediate system of reference.

It is a matter of common knowledge that the old measures bearing the same names can signify vastly different magnitudes, depending on the time, the place and the substance measured (ratione loci, ratione temporis and ratione materiae). It is not enough merely to be aware of this, nor to be able to translate the old measures into their metric equivalent. It is necessary to understand their varied but hidden, social content. The core of this understanding is to be found in their derivation from solid phenomena in daily life, in contrast to the meter, which has been agreed to by convention.

Today's standard measures signify nothing more than a common denominator for all the dimensions measured, e.g., length, area, mass, time and exchange value. The size of the unit is a matter of indifference; what matters is that the unit should be invariable. The fact that the kilogram stands for the weight of ten cubic centimeters of water at the temperature of zero degree centigrade, or that the meter stands for $1 / 40,000,000$ part of the meridian of the earth ${ }^{50}$ (or, strictly speaking,
${ }^{50}$ That particular meridian was chosen to be the meridian of Paris. Although such a dimension cannot be an absolute invariant, it was as near an approximation as possible. A rod was cast and has been kept ever since at a constant temperature after two notches were engraved on it. It was called
initially stood), has no inherent social significance whatsoever. The majority of people who employ measures are ignorant of these facts and none are mindful of them when actually using such measures. By contrast, the measures of ancient civilizations which have a certain definite social significance explain the size of units and their variety across space. After all, the metric system, whose acceptance meant that the unit of measurement was based on an astronomical phenomenon independent of man, has been with us only for two centuries.

Old measures, when we stop to consider them, may appear to us to be very inexact and to offer much scope for misunderstanding. But let us not look at them just through late twentieth century spectacles, for under different circumstances, different degrees of exactitude are socially requisite. The exactitude of the metric system was more than sufficient for the construction of ferro-concrete buildings and airplanes, but it proved to be far from adequate in the planning of lunar landings by interplanetary rockets.

Naturally, the conditions of life and work dictated the lines along which the metrological system, or its component parts, would develop. In societies where land was relatively abundant, the system of area measures tended to be poorly developed. The Ashanti of Ghana, in whose economy the extraction of gold dust played a major part, had a very advanced system of weights. ${ }^{51}$ On the other hand, the Saharan the meter.
nomads, for whom the exact distance from one water hole to another may be a matter of life or death, have a rich vocabulary of measures for long distances. Thus, they calculate in terms of a stick's throw or a bow shot; or the carrying distance of the voice; or the distance seen with the naked eye from ground level, or from a camel's back; or walking distances from sunrise to sunset, or from early morning, midmorning, or late morning; or a man's walking distance with no load to carry, or with a laden ass, or with an ox; or a walk across an easy or a difficult terrain. Such measures are still in use today and we find reference to them in historical sources going back a thousand years or more. ${ }^{52}$

The diversity of representational measures in different periods and countries is astonishing. In old Ethiopian recipes, we find the following description of the measure of salt, "enough to cook a chicken." The bow shot has been found almost everywhere as a measure of distance, but the measure disappears with the demise of the bow. A hatchet's throw as a measure will not surprise us, but a hatchet's throw backward from a sitting posture is a more unusual unit.

In stratified societies, even in very early stages of evolution, integrity in the application of weights and measures is extremely regarded and given all manner of
${ }^{51} \quad$ Niangoran-Bouah, "Weights for the weighing of gold. One of the aspects of African Philosophical and Scientific Thought before Colonization," First International Congress of Africanists (mimeo), Accra, 1962.

52 Witold Kula, Measures and men. Princeton: Princeton University Press, 1986. p. 5.
warranties. Therefore, in addition to a guarantee from the secular authority, one of a sanctified nature emerges as well. Very early, we find that "the just measure" becomes symbolic of justice in general.

## Symbolic concepts

Practices bound up with man's attitude to measurement constitute a character of a symbolic expression that can be found in the Bible. In the book of Moses from the Bible, which constitutes a code of social guide enclosed with sacred sanctions, the norms relating to measures are still put literally. By the time of Solomon and the Prophets, the phraseology becomes symbolic. For example, Solomon writes, "A just weight and balance are the Lord's; all the weights of the bag are His work. ${ }^{153}$ In the New Testament, with the metaphorical words of Christ, "With what measure ye mete, it shall be measured to you: and unto you that hear shall more be given. ${ }^{154}$

The Quran also condemns metrological offenses in a totally realistic fashion. In the eighty third Sura, dating from the Medina period, Allah says:

> Woe to those that deal in fraud. Those who, when they have to receive by measure from men, exact full measure. But when they have to give by measure or weight to men, give less than due. Do they not think that they
${ }^{53}$ Proverbs 16.11. The weights and measures serve here as symbols: "the weights in the bag" stand for men's deeds, just or unjust.

Mark 6.11.
will be called to account. On a mighty day. A day when (all) mankind will stand before The Lord of the Worlds? (83:1-6)

The acme of man's symbolic conception of measurements is found in many references, in various civilizations, to the "great day," as it is called in Islam, or the "awful day," the day of the last judgment.

All three monotheistic religions of Near East origin, Judaism, Christianity and Islam, share this belief. It is not at all surprising that in those civilizations, the knowledge of measure symbolizes - is even synonymous with - civilization itself. This was even true in older civilizations as in ancient Egypt.

For the ancient Egyptians, exactitude was symbolized by a feather that served as a weight on scales used for the weighing of souls. This light feather, called Maat, ${ }^{55}$ also stood for a unit of length -the 33 centimeters of the standard brick- and for the fundamental note of the flute. ${ }^{56}$ Figure number 4, showing a papyrus from the Book of the Dead of Qenna, XIXth Dynasty (Thebes), between 1405 and 1367 B.C., is an exceptional representation of the standard Egyptian judgment scene of weighing the heart of the deceased against the feather, where the pole is marked by seven distinct

55 Maat is the daughter of Re and the Egyptian goddess of truth, who wore a single ostrich feather that has the same name. As a personification of truth and justice, her feather was placed in one pan of the balance used for weighing the soul of a dead man in the judgment before Osiris, king of the 'other land'.
${ }^{56}$ Italo Calvino, Six memos for the next millennium. Cambridge, Massachusetts: Harvard University Press, 1988. p. 55.
nodules below the balance beam and an eighth above it. The nose of Osiris' monstrous watchdog, the Swallower (a mixture of crocodile, hippopotamus and lion who is to swallow the soul if the heart is heavier than the feather), cuts directly across the pole between its third and fourth nodules (read third and fourth Chakras, which furthermore, is exactly the level of a platform, across the way, supporting a seated baboon (the animal symbolic of Thoth, Egyptian counterpart of the Greek Hermes, guide of souls to the knowledge of eternal life). If the aims of the deceased in life were no higher than those of Chakra 3, the swallower claims the soul; whereas, if the "sound not made by any two things striking together" had been heard (at Chakra 4) and heeded in the lifetime, Thoth will conduct the blessed soul (light as a feather) to Osiris's throne by the Waters of the Eternal Life. ${ }^{57}$ [see also (Figure number 5)]

## Beliefs associated with measure

In Athens, the standards of weights and measures were in safekeeping on the Acropolis, additionally secure in their dedication to the gods (in Rome they were kept in custody on the Capitoline Hill) and specialists were employed to authenticate them. Newly emergent city-states created their own standards as symbols of their supremacy, while those that had the adversity to be defeated had the measures of the conqueror imposed upon them as symbols of the new domination.

[^6]For a long time in medieval Europe, there were some dogmas affiliated with measure and measuring. As it was Cain who devised measures, ${ }^{58}$ it was considered sinful to count or to measure. Since it is a well known fact that the devil himself gave David the idea of counting God's people, ${ }^{59}$ it is clear that to count and especially to count people, is sinful. Similarly, it is sinful to measure a human being.


Figure 4:
The Weighing of the Heart. (from the Papyrus of Qenna)

Taking a man's measure, or the measure of some part of his body, one invests him with symbolic and ambivalent significance. A thread or a ribbon that has the length of the circumference of the head may be offered as a votive offering for the
${ }^{58}$ According to Witold Kula , Measures and men. Translated by R. Szreter, Princeton: Princeton University Press. p. 3, Cain, the son of Adam and Eve who killed his brother and committed other sins, was the author of weights and measures. It was he who invented measures, but many could thought it was an anonymous invention like the wheel or fire.
$59 \quad$ Witold Kula, Measures and men. Translated by R. Szreter, Princeton: Princeton University Press. p. 13.
recovery of the person who was measured. It may also be used in the practice of black magic, to harm him and if the subject is already deceased, it may be hung on the altar to prevent his undesired return.


Figure 5:
The Weighing of the Heart. (from the Papyrus of Hu-nefer)

The right to determine measures is an attribute of authority in all advanced societies. It is the prerogative of the ruler to make measures mandatory and to retain the custody of the standards, which are invested here and there with sacred character. The controlling authority, moreover, seeks to unify all measures within its territory and claims the right to punish metrological transgressions.

## Anthropometric measures

"Man is the measure of all things". This sentence of Protagoras had a dual significance. On one hand, it was the synthesis of the anthropometric philosophical
position as well as a declaration of a cognitive faith. At the same time, it was a simple statement of the existing state of affairs, a generalized view of a system in which man used himself, the parts of his body, to measure all other objects.

The emergence of man's metrological concepts and habits is a very important aspect of his apprehension of the world and of the formulation of taxonomic systems and abstract concepts. Primitive man measured the world by himself; to measure objects independent of him, he employed his own parts: his foot, arm, finger, palm of his hand, outstretched arms, pace, etc. There were a great many potential units, since there are a great number of measurable parts of the human body. However, the intellectual turning point came with the transition from concrete to abstract concepts, from the particular my finger, your finger, to the general the finger. Measures such as the ell (elbow), the span and the foot enjoyed currency in our civilization until quite recently - pending the complete dominance of the metric system - but they had become abstractions. The unit was the foot in general; at any one time, its length was fixed (though it was variable over time) and it would be somewhat longer than $m y$ foot or somewhat shorter than your foot. On the other hand, Ethiopian medical prescriptions that refer to your finger as a measure as late as the sixteenth or seventeenth century are singularly significant. ${ }^{60}$ What is important to us is the

60
Naturally, the author of the prescription would take into consideration the fact that one patient's finger might be longer than another's. We may safely assume that the range of differences would not materially affect the dose, reducing it to ineffectiveness or increasing it to the point of
phrasing itself as evidence that the stage of employing the finger as an abstract measure at large had yet to be reached.

The major inconvenience of the anthropometric measures was the lack of simple multiples. The pace could not be divided into the whole number of the ells, nor the ell into so many spans, etc.

Once firmly established, the system of anthropometric metrology was an all-embracing one. It reduced to common measures nature and culture the world around and man's artifacts as well. It did not only enable man to measure fields, trees and roads, but also imposed its proportions on the dimensions of the weaver's loom and on bricks and church belfries, the dimensions of the bricks being part of the same system as the proportions of the church architecture. Reconstruction of medieval buildings in France by Viollet-le-Duc were criticized, inter alia, for using the meter in the rebuilding of an architecture that had been governed by the span and the foot and thus distorting fundamental proportions. ${ }^{61}$

## Egyptian perception

In ancient Egypt, measure and proportion was adapted to the purpose and the symbolic meaning of the idea to be expressed. The cubit would not necessarily be the same from one temple to another, since these temples are in different places and their danger. 61
M. Bataille, "Viollet-le-Duc, jardinier des pierres," Archeologie, number 6, 1965. pp. 50-56.
purposes are different. Although, the cubit will not be the same for measuring one Neter ${ }^{62}$ or another, according to whether the Neter is Horian or Osirian, primordial created or natural, but each cubit corresponds in itself to a definite value, which means that it can be used under specific conditions. ${ }^{63}$

To illustrate the "vital" nature of measure, let us imagine the following example: It is a question of measuring a tree X and a tree Y . Today a yardstick of some kind would be used. The measurement would be a relative determination, calculated by units and fractions and we should obtain only a comparison of sizes between the circumferences of the trunks, the heights, etc., of these two trees.

With the system of measurement based on a philosophic, "vital" principle, two different cubits would be employed, one for the tree X and the other for tree Y , but these cubits would not be chosen arbitrarily. In fact, each of these individual members of the vegetable kingdom belong to a genus, this genus to a family and these families belong to an original "lineage". At the head of this lineage is a Neter, a "principle" synthesizing all the characteristics of this lineage: its number, its rhythm, its classification in the general harmony.
$62 \quad$ Neter signifies the idea and the principle of life and the temple is its house.
63 The author of "The Nilometric Cubit" in the Bulletin de la société Royale de Geographie, vol. 21, (1943), Kamel Ghalet Pasha, reports the existence of a cubit, called the "black cubit," of unknown origin. This cubit is carved on the sole of the black granite colossus to the east of the entrance leading from the court of Ramses to the great colonnade. This black cubit is found only on black stones or on what corresponds to their symbol. It is measured length is 54.02376 cm .

Let us suppose tree X is attributed to the Neter X and tree Y to the Neter Y . Each of these Principles has a measure that corresponds to its rhythm and this measure-applied to any one of the individuals that, in each kingdom, belongs to this lineage-will therefore determine all its particular proportions and characteristic qualities, in its growth as well as in its appearance, its behavior and its affinities.

Thus the "cubit," whose strange divisions are geometric, astronomical and geodetic coordinates, has a vital meaning. On the contrary, the comparison of relative sizes is a quantitative conclusion, which in this case has no meaning.

The principle of the Neter is associated with the cubit and found in the hieroglyphic system. The sign of the cubit is represented by a section of the forearm assuming the outline of the recumbent sign of the Neter. Also, the importance of the color of the sign has to be mentioned. Often the "stalk" of the hieroglyph Neter is colored green, a symbol for "vegetable." The Neter is considered as the seed that summarizes all the possibilities of a particular rhythm. [see (Figures number 6 and number 7)].

Some may object that it is not very likely that a particular cubit can give universal indications. This has no significance when it is solely a matter of knowing a quantitative size; an ordinary measure will suffice. But when it is a matter of speaking about this tree as a sacred nature, the use of the particular cubit-or inversely, the attribution of this tree to a definite Neter-takes on an extremely important
significance. This simple comparison, sums up an entire philosophy, placing this object in relation with everything that concerns the lineage of this tree in the vital harmony of the world.
R. Lepsius ${ }^{64}$ was the first to formulate the theory of a connection between a metrological system and the authentic grids on many reliefs and sculptural works


Figure 6: The sign "Neter". The sign of the arm.
from almost all periods of Egyptian art up to the end of the Ptolemaic period 400 A.C.

The Egyptian grids can be divided into two groups, an older and younger. In the older group, which is often called the first canon, the total height of the human figure is divided into 19 parts [see (Figure number 8)] and the younger group, the second group, into 22.5 parts [see (Figure number 9)]. The alteration of the grids has been found to have taken place at the time of the XXVIth Dynasty (663-525 B.C.).

The most important Egyptological observations regarding the length-units in the above mentioned periods can be summed up in general as follows:

1. The small cubit represented the original standard, which was used for all everyday purposes until the XXVIth Dynasty. It was further used as a measurement for building. Nowadays, it is generally considered that the small cubit consisted of six units, each containing four fingers (or inches) and representing the length of the forearm from the tip of the elbow to the first joint of the middle finger. From the XXVIth Dynasty the small cubit fell into disuse and was replaced for all purposes by the royal cubit.


Figure 8:
The first canon grid as published by Iversen.


Figure 9:
The second canon grid
2. In addition to the small cubit, one of the most important units is said to be the Ser. It represents the length of the arm from elbow to wrist and is expressed as 4 units each having 4 fingers. It seems to be the general opinion
that this unit is identical with the unit which in Greek metrology is called a foot $=16$ inches. The foot-unit is said to be unknown in Egyptian metrology. ${ }^{65}$
3. Beside the small cubit there was the royal cubit ${ }^{66}$, which was supposed to have been introduced for fiscal purposes. It was also used for building purposes and after the XXVIth Dynasty, it seems to have replaced the small cubit. The royal cubit is said to be a "natural" extension of the small cubit, where one unit was added to the former, since it appears certain that the royal cubit can only have consisted of 7 units.
4. Another major unit in Egyptian metrology is the fathom [see (Figure number $10)$ ] which is the distance between the tips of the middle fingers of the outstretched arms and is equal to 3 royal cubits. ${ }^{67}$

## Greek concept

One of Vitruvius' main objectives in his De Architectura was to tell something about how the Greek architects planned their temples. Initially, he started to tell us a

65 Eivind Lorenzen, Technological studies in ancient metrology. Copenhagen: NYT nordisk forlag, 1966. p. 61.
${ }^{66}$ There was a distinction between the royal cubit, used only for measuring things pertaining to pharaoh and the small cubit, common unit for everyone else. Pharaoh and princes of royal blood always were pictured as larger figures than the surrounding people.
${ }^{67} \quad$ Eivind Lorenzen, Technological studies in ancient metrology. Copenhagen: NYT nordisk forlag, 1966. p. 109.
little about the units of measurement that were used for this purpose. As the units of length used by the ancients were derived from the lengths of parts of the human body (the feet, arms, etc.), each of the units being given a name corresponding to its human counterpart. Vitruvius tells us in two lines that sculptors used these units also, ${ }^{68}$ but he might just as well have said that they were used by shoemakers or tailors. Vitruvius mentions a field of employment of the length measure system, namely that of the sculptor, but he doesn't tell us how the sculptors went to work.


Figure 10:
Greek representation of the fathom, the Metrological Relief (the Ashmolean Museum, Oxford).

The description he gives is intended to serve as a principal for establishing a norm for units of length and especially with regard to its use in the planning of temple. From the Vitruvian point of view all other possible fields of employment are beside the purpose. The preference for statues as objects of study, lies in the fact that several of the so called "human measures" in the metrological system can be found reflected as

[^7]lengths of limbs of statues. Polyclitus of Argos ${ }^{69}$ was the formulator of classical Greek anthropometry. He was obsessed with rules of mathematical proportion in composing his figures. He sculpted the famous bronze statue of an athlete carrying a lance, the Doryphoros, about 440 B.C. [see (Figure number 11)] which was considered to be the classical Canon. ${ }^{70}$ Polyclitus also wrote a book explaining the principles of symmetry and proportion upon which he based the construction of the Doryphoros. ${ }^{71}$

Diodorus of Sicily tells, in the ninety-eighth chapter of his first Book, the following story: In ancient times (that is to say, the sixth century B.C.) two sculptors, Tekeles and Theodoros, made a cult statue in two separate parts. While the former prepared his portion in Samos, the latter made his in Ephesus and on being brought together, each half matched the other perfectly. This method of working was not customary among the Greeks but among the Egyptians. For with them the proportions of the statue were not determined according to visual experience as with the Greeks. ${ }^{72}$
${ }^{69} \quad$ His name was mentioned twice in Vitruvius' De Architectura among the theoreticians of proportions.

70 Erwin Panofsky, " The History of the theory of human proportions as a reflection of the history of styles." Meaning in the visual arts. Chicago: The University of Chicago Press, 1982. p.64.

71 Bernard S. Myers and Trewin Copplestone, The History of Art. Dorset Press: New York, 1985. p. 81.

For the Greeks, the original concept of measure in architecture along with the units of measure is considered to be a pure aesthetic value linked tightly to the idea of proportion. According to the classical theorist Vitruvius, "proportion is a correspondence among the measures of the members of an entire work and of the


Figure 11:
A Roman copy of
Polycleitos' Doryphoros. (the National Museum, Naples.)
whole to a certain part selected as standard . . . . as in the case of those of a well shaped man ${ }^{173}$ [see (Figure number 12)]. This description and the fact that units of

72 Erwin Panofsky, "The History of the theory of human proportion as a reflection of the history of styles." Meaning in the visual arts. Chicago: The University of Chicago Press, 1982. p.69.

73 Marcus Pollio Vitruvius, De Architectura, 27 b.c. Translated by Frank Granger. Cambridge Massachusetts: Harvard University Press. 1985. p. 159.
measurements themselves are derived from members of the human body - the palm, the foot, - are not unique to Greek architecture. The phrase "as in the case of those of a well-shaped man" implies a physical analogy between measure and aesthetics.

In book IV of De Architectura, chapter one, Vitruvius addresses the establishment of the sanctuaries of the immortal gods and the beginning of temple construction in the large cities of Ephesus, Miletus and Myus. They called it Doric temple because they had first seen it built in the Dorian cities. When they wished to place the columns in that temple, not having their proportions and seeking a method


Figure 12:
Vitruvian man by Leonardo da Vinci.
that could make them appear to have an approved grace, they measured a man's foot step and applied it to his height. Finding that the foot was a sixth of a man's height, they applied this proportion to the column.

Down to the tenth century in the epistles of the Brethren of Purity ${ }^{74}$ the scholarly brotherhood, we find a system of proportions ${ }^{75}$ that expresses the dimensions of the body by one fairly large unit or a module. They used a unit of measurement, the "span" of the hand, as a module for proportioning the whole body. This is different from the use of the face-length as the basic unit in the Vitruvian system of human proportion. Furthermore, their system was not ${ }^{76}$ applied in the case
${ }^{74}$ Greek philosophy and thoughts are very obvious and can be traced in their first epistles. Also see footnote number 4.
${ }^{75}$ This system of proportion is mentioned in their fifth epistle that discusses music. It is as follows: For a properly built new-born, regarding his span as a unit of measurement, his length is eight spans- two spans from the top of his knees to the bottom of his feet, two spans from the top of his knees to his loins, two spans from his loins to the top of his heart and two spans from the top of his heart to the top of his head. The length between the tips of his fingers when his right and left arms are extended is eight and a quarter spans. By extending his arms above his head, a circle can be drawn centered in his navel and touching the tips of his fingers and the bottom of his feet; and the length between them is ten spans. The length of his face from his chin to the beginning of his hair above his forehead is one and one eighth spans. The length between his ears is one and a quarter spans. The length of his nose is a quarter span. The length of his eye opening is a quarter of an eighth of a span. His forehead is one third the length of his head. The length of his mouth opening and both of his lips, each is equal to the length of his nose. The length of his feet is one and a quarter spans. The length of his hand from his wrist to the tip of the middle finger is one span. The lengths of his thumb and his pinkie are equal. The tip of his ring finger exceeds his pinkie with one eighth of a span, the same as the difference between his middle finger over his pinkie and his index finger. The width of his chest is one and a half spans. The distance between his nipples is one span. From his navel to his crotch is one span. From the top of his heart to his collarbone is one span. The distance between his shoulders is two spans.
${ }^{76}$ This system of the Brethren of Purity, was presented to give insight into a vast harmony
of "a well-shaped man" but was applied to the new born-child, who plays a fundamental role in cosmological and astrological thinking.

## Conclusion

Generally, we may say that the earliest stage in the development of man's metrological concepts is the anthropomorphic, in which the most important measures correspond to parts of the human body. It is in a later stage that reference is made to units of measure derived from conditions, objectives and outcomes of human labor.

The ancients used the proportions of the human body for measuring short distances. For instance, the length of a man's forearm with the hand outstretched was called a cubit, which had several variants. The Egyptians had a smaller cubit, which consisted of six "handbreadth" and a larger one, the Royal Cubit, which was seven handbreadth. The Egyptian "hand" was made up of the dimensions of four "fingers" or "digits". A further measure, the "fist", equaling one hand and a third handbreadth, was used by the Egyptian master builders and craftsmen to establish the square grids used for the proportioning of their royal statuary. Many unfinished Egyptian sculptures have been found which have such grids, used by the artists and workmen. For the Greeks, their philosophy was influenced and controlled by a mathematical concept of the universe. Their initial perception of measure in architecture, along that unifies all parts of cosmos by numerical and musical correspondences.
with the units of measure, accounts for a pure aesthetic value that is closely connected to the idea of proportion.

# Origins of Alhazen's conceptions 

## A: The perception of measure and mathematics

B: Inception of geometry and its aesthetics, the golden section

(B)

Philology and extraction:
Egyptian mathematics:
Greek geometry:
The Golden section:
Conclusion:

## Philology and extraction

"Let no one destituted of geometry enter my doors" 77
The word "Geometry" means "measure of the earth". In ancient Egypt, from which Greece inherited this study, the Nile would flood its banks each year, covering the land and obliterating the orderly marking of plot and farm areas. This yearly flood symbolized to the Egyptian the cyclic return of the primal watery chaos and when the water receded, the work of redefining and reestablishing the boundaries began. This work was called geometry and was seen as a re-establishment of the principle of law and order on earth. Similarly the same account has been given by many Greek writers on the origin of geometry. Herodotus says that Sesostris (Ramses II, circa 1300 B.C.) distributed the land among all the Egyptians in equal rectangular plots, on which he levied an annual tax. When the river swept away a portion of a plot and the owner applied for a corresponding reduction in the tax, surveyors had to be sent down to certify what the reduction in the area had been. "This, in my opinion," Herodotus continues, "was the origin of geometry, which then passed into Greece". ${ }^{78}$

The previous approach considers the question of the origin of geometry from a philological-historical point of view, i.e., as the search for the first geometers who
$77 \quad$ Inscription over the door of Plato's school.
78 Sir Thomas Heath, A bistory of Greek Mathematics, Volume I from Thales to Euclid. Oxford: The Clarendon Press, 1929. p.121.
actually uttered pure geometrical propositions, proofs, theories, or propositions they discovered, or the like. Although the previous tradition has been applied widely in many treatises that deal with the history of geometry, the way to address the question should be an inquiry back into geometry's most original sense, which was present as the heritage of millennia, is still present for us and is still being worked on in an active onward progression. ${ }^{79}$ Edmund Husserl criticizes the notion of "universal historical a priori"; he poses the question of "original historical meaning . . . which was required to give the whole development of geometry a persistent sense of truth" and insists on the importance of an "inner history", aligned with a "universal teleology of reason" for all "common factual history".

The Egyptians believed that a cosmic order had once and for all been established when the world was created by $\mathrm{Ptah}^{80}$ [see (Figure number 13)]. The

79 Edmund Husserl writes: "The geometry which is ready-made, so to speak, from which the regressive inquiry begins, is a tradition. Our human existence moves within innumerable traditions. The whole cultural world, in all its forms, exists through tradition. These forms have arisen as such not merely causally; we also know already that tradition is precisely tradition, having arisen within our human space through human activity, i.e., spiritually, even though we know nothing, or as good as nothing of the particular provenance and of the spiritual source that brought it about. And yet there lies in this lack of knowledge, everywhere and essentially, an implicit knowledge, which can thus also be made explicit, a knowledge of unassailable self-evidence." Quoted from the appendix of Jacques Derrida. Edmund Husserl's Origin of Geometry: an introduction. Translated by John P. Leavy, Jr. New York: Nicolas Hays, Ltd,. 1978. p. 158.

80 The leading member of the Egyptian pantheon. His cult center was at Memphis, where he had the lion goddess, Sakhmet, as his wife and Nefertem, as his son. At the beginning of things Ptah
other gods were various manifestations of Ptah's being and the Pharaoh was the king-god. Everything which had significance, originated in the life of the cosmos and the task of the Pharaoh was to maintain harmony in this scheme of things.


Figure 13: Ptah of Memphis, standing by a djed pillar and leaning on bis scepter-measuring rod, after Budge.

In the famous Memphis papyri, which are believed to go back to the time of the first dynasties, there is a parallel to the account in the Gospel according to St. John of the creation of the world: "In the beginning was the word and the word was with god and the word was god." The next phrase is formulated (according to different translations) as follows: "And so Ptah rested (or was satisfied) after he had
existed as Nun, the primeval waters. By speech or by kneading mud, Ptah-Nun created the world. He is even credited with the birth of Atum, the form Re assumed in the Heliopolitan creation myth.
made all things and all divine words." According to another translation, "divine words" has been substituted for "divine order", which implies that the gods have a system into which all of the created elements should fit as soon as they are created.

In the fifth section of the Memphis papyrus in which the gods, i.e., the various manifestations of Ptah, "enter into their bodies (statues) of all kinds of material, stone, metal or wood, which had grown out of earth that is out of Ptah". The gods, then enter the statues and pictures of them, which the artists made. The art of creating the pictures in which the gods may dwell must be subject to strict rules. The artist becomes the medium, which submits the work of art he is making to the law of eternity, the law which Ptah laid down when the world was created.

An inscription of a chief craftsman, painter and sculptor says, "I know how to work up clay, how to proportion ${ }^{81}$ (it) according to rule, how to mould or introduce (it) by taking away or adding to it so that each member comes to its (proper) place". Wilson ${ }^{82}$ sums up the inscription by concluding that it was in the realm of proportion, balance and weight that the craftsmen claimed to be proficient.
${ }^{81}$ In the question of recognizing what is the real value, proportion or measurement? R. A. de Lubicz, in his book The temple in Man, The secrets of Ancient Egypt, says "Proportion belongs to geometry and harmony, measurement to the object and to arithmetic; and one necessitates the other. Proportion is the comparison of sizes; harmony is the relationship of measures; geometry is the function of numbers." p. 61.

82 John Albert Wilson. The present state of Egyptian studies, Chicago: The Oriental institute of the University of Chicago, 1938.

Kielland, ${ }^{83}$ through her studies and analyses of Egyptian art, concludes that geometry was above all the means that the Egyptians had used in order to achieve balance and order. A work of art is a cosmos, in which the laws of geometry dominate.

In explaining the Egyptians' use of geometry in art, we must consequently look for the ritualistic reason, not for the aesthetic one. Their work of art can be regarded as embodiments of sacred geometry and by virtue thereof, they become sacred. A main point to be remembered is that, practically speaking, all of the art we know from Egypt comes from temples and tombs. It is an expression of the Egyptian's ideas of Existence and Man, of Life and Death. The objective of the Egyptian sculptor at work was not primarily to make a piece of sculpture which would satisfy the aesthetic demands of himself and his fellows; the whole process was infinitely more serious and belonged to a constant and eternal cult. It is not until recently by the discoveries that all of this was regarded and admired exclusively as art. We note that its qualities as art have survived the changing ages and proved that they constitute a reality. It is another question to what extent this quality is conditioned by the Egyptians' attitude of life. The created work is for them a unity, in which every detail and all measurements are determined in relation to the whole, based on

83 Else Christie Kielland. Geometry in Egyptian Art. London: Alec Tiranti ltd, 1955.
sacred geometry. It is the consequence of this attitude, this view, which has preserved it and actually helped it to survive the passing of time.

## Egyptian mathematics

Kielland, after examining six papyri dealing with mathematics and establishing their terminology, states that the Egyptians used quite different methods in making their mathematical calculations, as a natural result of their system of numbers. Their arithmetic tended to reduce multiplication and division, respectively to addition and substraction.

The Rhind Papyrus that is preserved in the British Museum ${ }^{84}$ begins as follows: "Accurate computation. The gateway to knowledge of all things and of dark mysteries." The papyrus was written by a scribe named Ahmes during the 17th century B.C. It contains many problems of practical value for the farmer in everyday life. It seems more like a modern manual of elementary arithmetic "ad usum populi" than the total knowledge of Egyptian higher mathematics, as many eminent scholars have seen in it. ${ }^{85}$ It seems quite believable that the Egyptians kept their more

[^8]important findings secret and only gave the people some practical and even crude ways of measuring and reckoning to help them solve their everyday living problems. One of the problems concerns the comparison between the area of the circle and the square ascribed to it. It shows that Ahmes gave the value of $\pi$ as $3.1605,{ }^{86}$ while it is certain that the Egyptian mathematicians of his time had already found a very simple construction to figure $\pi$ geometrically with an exact value down to the fourth decimal place, 3.1416.

The Egyptian numbers and system of counting were directly governed by geometric factors. Their ideas and theories were bound up in geometric rules. ${ }^{87}$ To them a square was always two or more triangles; these were their primary subdivisions and it was from these that they drew their knowledge of numbers.

The Egyptians made use only of fractions with the numerator 1 , with the sole exception of $2 / 3$, for which they had a special symbol. When they divide a line into 7 segments, the result is one seventh which is to be regarded rather as a quality than a quantity. The remainder is not $6 / 7$ but a complementary fraction (i.e., 1-1/7th). In this way an intimate relationship arose between the original whole and the part, between the part and its complementary fraction. These are the very qualities which are characteristic of the golden section division, where the relationship between the
${ }^{86}$ This value, given by Ahmes' Papyrus, seems rather to be a sort of decoy to seduce the people away from the great secrets of science.

87 This is a reversed order of the approach in our modern times, where we use numbers as the primary factor and geometry as the subsidiary.
whole and the larger part is the same as that between the larger part and its complementary fraction.

## Greek geometry

Geometry is considered to be the study of spatial order through the measure and relationships of forms. Geometry and arithmetic, together with astronomy, constituted the major intellectual disciplines of classical education. The fourth element of this great fourfold syllabus, the Quadrivium, was the study of harmony and music. The laws of simple harmonics were considered to be universals, which defined the relationship and interchange between the temporal movements and events of the heavens and the spatial order and development on earth.

Egypt, for thousands of years, remained the inspirational center for the whole of Africa and Europe and travelers came from every compass point to seek the knowledge and wisdom of the Egyptian Temple.

Thales (624-548 B.C.), after his visit to Egypt, spending time with the priests, was the first to bring the science of geometry back to Greece. ${ }^{88}$ He made many discoveries and indicated many principles to those who came after him. ${ }^{89}$

88 Ettore Carruccio, Mathematics and logic in History and in Contemporary thought. Chicago: Aldine publishing company, 1964. pp. 20-23. He is quoting Proclus (A.D. 412-485).

89 Thales figured out a way of determining the height of a pyramid, by measuring the shadow at the moment in which the body and its shadow are equal.

Figure 14 demonstrates Pythagoras (582-507 B.C.), who is credited with first establishing the relationship between number ratios and sound frequencies. He is


Figure 14: Pythagoras at his musical experiments, after Franchino Gaforio, Theorica musice, Milan 1492, fol. $b 6$.
shown experimenting with bells, water glasses, stretched cords and various sized pipes. After his visit to Egypt, Pythagoras systematized geometry and transported it into Greece, where he set up his school of speculative geometry and initiated a number of young-thinking Greeks in its secrets. The learning was passed directly from teacher to pupil without the aid of books or documents. Education at Pythagoras' school was not free in the sense that his pupils were banned from telling friends and acquaintances of their new-found knowledge. Their learning was something religious, something sacred, something to be defended, protected. It was
not the personal property of the individual. Study lasted for several years, of course and it became evident early if any of the pupils broke his oath of silence. Students in the upper and senior classes kept an eye on new pupils and any irregularities observed were immediately reported to the school's master, who took appropriate action.

Quite apart from the Pythagoras school of learning there were the usual, older established Greek Temple schools, at which initiation was considerably more involved and difficult and the punishment for breaking the oath of silence much harder, often leading to the risk of one's life. One of the Greek Temple secrets was geometry. It was learned from the Egyptians as Pythagoras visited Egypt, but the Greek Temple had obtained the learning infinitely earlier than Pythagoras' era. The information was held within a tightly-knit, religious brotherhood, were not even outside Temple brethren were initiated in the occult mysteries of numbers.

The priests applied their geometric knowledge in planning and building temples or as part of their religious ritual, but few could hope to gain entry into the inner circle of mathematical priesthood. The few selected had illustrated, over a long period of years, their reliability to keep lesser secrets. Their silence and oath were unbreakable.

Whereas the ancient Greek Temple admitted to its ranks primarily men of mature years, Pythagoras opened the doors of his school to large groups of young
students. One of his principal subjects was arithmetic combined with geometry. He considered this to be the best form of training in logical thought.

According to ancient tradition, initiation in the higher degrees of learning was obtained - on recommendation - from the temples of Egypt. Anyone who had been admitted to and trained by the Egyptian Temple was supreme anywhere else in the world. He was part of the very Temple superstructure. Intrigue, on the part of the Temple and back-stabbing by certain citizens who saw their powerful positions threatened by the Pythagorean school, had its desired effect. Pythagoras was forced to shut down his influential school and flee the country at the age of 80 years. Although his teachings were forbidden, they were extensively practiced. Soon Pythagorean schools and societies were sprouting up in countries and states bordering Greece and within an amazingly short time, they had crept back over the border and became underground movements all over Greece.

Plato considers geometry and number as the most reduced and essential and therefore the ideal, philosophical language. In the Republic, he says of geometry that it is "pursued for the sake of the knowledge of what eternally exists and not of what comes for a moment into existence and then perishes," and that it "must tend to draw the soul towards truth and to give the finishing touch to the philosophic spirit. ${ }^{190}$

[^9]Around the year 300 B.C. (roughly 250 years after the death of Pythagoras), the situation regarding mathematics, geometry and its teaching was as follows:

1. The old Greek Temple Mystery societies still existed, with tradition and ritual as strong as ever. They were headed by priests, who virtually lived and died by secrecy. Within the priesthood were an even closer brotherhood of mathematical experts, with a full knowledge of and familiarity with ancient geometry's origin and application.
2. After the Pythagorean school had been totally wiped out of Greece, its followers had started similar schools outside Greece and had worked their way back over the border again under different names. A well-known example of these schools was Plato's Academy.

Those two groups, the Greek Temple and the various Pythagoreans, both stemmed from the same source, i.e. Egypt, where they had collected their knowledge of ancient geometry. Both had shared the principle of secrecy, the former as an age old tradition, the latter as a way to fight for survival.

The ruler of Greece at that time was Ptolemy Philadelphus who well-appreciated that education was essential to the nation's advancement at home and abroad. He set up a place of learning across the Mediterranean at Alexandria. It was a center where wise men and students could discuss scientific and other matters
for the benefit of public listeners. This place was called the Museum ${ }^{91}$; it represented the third school of geometric thought in Greece.

Euclid, who belonged to the third group of geometers, the Museum of Alexandria, included the fundamental propositions of Pythagorean and Platonian mathematicians in his famous treatise The Elements. He was also familiar with the existence of the five-pointed star or the pentacle [see (Figure number 15)], drawn as a signature and representing a symbolic figure used as a sign of recognition by all initiated Pythagoreans. Furthermore, he dealt with geometry and the construction of


Figure 15: The pentacle after Brunes.
polyhedra and used the mathematical basis of optics, astronomy and music for educational purposes.

The importance of geometry as an intellectual medium ${ }^{92}$ can be encountered furthermore in Vitruvius' De Architectura. The Vitruvian definition "architectura est
$91 \quad$ The building itself was impressive, with dozens of shady courtyards and cloisters where visitors, students and professors could gather in large or small groups to muse upon problems. The library alone is reported to have contained 400,000 papyri obtained by Ptolemy's researchers from every corner of the learned world.
scientia" was fulfilled by linking architecture with mathematics in order to have it participate in the universal principles and "truth" itself by basing it on geometry and arithmetic. ${ }^{93}$ In the introduction to his sixth book, Vitruvius brings out the story of


Figure 16: Aristippus after a happy landing, from Jacqes Ozanan, Recreation mathematique et physique, Paris, 1778.
the Greek philosopher Aristippus, who was shipwrecked off the coast of Rhodos and

2 Kant reasoned in relation to the "metaphysical origin of the natural sciences" that any natural science contained only so much of science "proper" as it "contained mathematics"; a statement reformulated by neo-Kantians to "so much science, so much mathematics".

93 According to Daniele Barbaro's explanation, the "dignity" of architecture rested on its position among the sciences.
washed ashore [see (Figure number 16)]. Without orientation, Aristippus soon finds geometrica schemata ${ }^{94}$ in the sand, exclaiming: "bene speremus! bominum enim vestigia video." (Let us be hopeful for I see traces ${ }^{95}$ of men.) August Rode had translated this as. "Confidence, my friends, I see tracks of men" after explicitly describing the geometrical figures as a schemata. Daniele Barbaro, in his edition of De Architectura, was more concerned with Aristippus' inference from geometrical figures to human beings; he specifies Vitruvius' text with an additional remark, "not the tracks of wild animals for they do not think." Barbaro, in his commentary, goes to great lengths to make his point that Aristippus related the geometric figures not to the human body, but to his "men", his "ratio", i.e. his intellect. He adds that the mathematical figures had previously been developed by human intellect, "prius mente . . . conceptae". Although there are many points of view that explained Aristippus' words and the importance of the story in Vitruvius' text, the word "vestigia" has variant meanings. The meaning that should be considered for the passage is "the trace or indication of the presence at any time of a person or a thing ${ }^{196}$ considering that "a thing" could refer to a building or a structure. ${ }^{97}$

94 The word schema, single of scbemata, means: a diagrammatic representation, a hypothetical outline or plan; a theoretical construction; a draft, design.

95 Granger translated it as "There are good hopes for us; for I see human footsteps!"
\% P. G. W. Glare, ed., Oxford Latin Dictionary. Oxford: The Clarendon Press, 1986. p. 2049.
97 As in the meaning of the word vestige in J. A. Simpson and E. S. C. Weiner, The Oxford

## The Golden section

The proportion of the Golden Section is mathematically expressed as $\mathrm{A}: \mathrm{B}=\mathrm{B}:(\mathrm{A}+\mathrm{B})$. Geometrically speaking, this means that when a line is divided according to the proportion of the Golden Section $\varnothing$ the smaller portion, A, also called the minor, is in relation to the larger section, B , the major, as B is to the whole line $(A+B)$. As the Golden Section proportion expresses the relationship between three parts by means of two of them, it has been called the most "economic" relationship existing.

Figure 17 shows how to divide a line $A B$ in $\emptyset$. The angle at $A$ is equal to 90 degrees and $A C$ is $1 / 2 \mathrm{AB}$. An arc is drawn from C through A and an arc is drawn from $B$ through the point of intersection of $C B$ and the first arc. The intersection point D then divides AB in $\varnothing$. The different dimensions in the $\varnothing$ proportion can be found by means of addition and subtraction of lines and we can perceive the unity existing between arithmetic and geometrical relations, which is also typical of the Egyptians, who made no distinction between them.

Brunes believes that Euclid and his followers arrived to the golden section as a geometric ratio and it bears no relation to the geometric system that emerged and operated in earliest history, nor does it have any connection with the practical

English Dictionary. Oxford: The Clarendon Press, 1989. Volume XI p. 578, "A mark, trace, or visible sign of something esp. a building or other material structure, which no longer exists or is present; a piece of material evidence of this nature; something which remains after the destruction or disappearance of the main portion."
application of geometry, i.e. building and planning. ${ }^{98}$
In Ad Quadratum, Macody Lund tried to track down the origin of the Golden


Figure 17: Golden section construction
Section. The actual name Golden Section was first mentioned in literature towards the end of the Middle Ages by Luca Paccioli, in his De Divina Proportione, Venice, 1509. Lund was unable to discover the term, Golden Section, in print before 1509.99 It was an impossible task to trace the stage at which literary references to a harmonious factor in building were first linked with Euclid's speculation, but Lund at any rate, identifies these references, without question, with Euclid's line-division.

Leonardo Fibonacci, ${ }^{100}$ alias Leonardo of Pisa, was born in 1175. His early years were passed in a Christian community, but he received his academic education among the Arabs. There he learned the Arabic, or decimal, system of numbering as
${ }^{98}$ Tons Brunes, The secrets of ancient geometry and its use. volume II. Rhodes, Copenhagen, 1967. p. 84.
$9 \quad$ Ibid. p. 85.
100 Filius Bonacci, son of Bonacci, shortened to Fibonacci.
well as AlKhawarismi's (founder of Algorithm) teaching of algebra. At the age of twenty-seven, he turned to his native land and there he published Liber Abaci (the book of the Abacus), in which he demonstrated the great advantages of the Arabic system of numbering over the Roman.

The summation series, in which each number is the sum of the two previous


Figure 18: The five Platonic solids, after Leonardo da Vinci. The tetrabedron, the octahedron, the icosabedron, the dodecabedron and the cube.
ones, was called the Fibonacci series in 1877 by Edward Lucas:

$$
0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987,1597,2584, \ldots
$$

Any number in this series divided by the following one approximates $0.618 \ldots$ and any number divided by the previous one approximates $1.618 \ldots$, these being the characteristic proportional rates between minor and major parts of the golden section, which is referred to as $\varnothing$ or $\frac{1+\sqrt{5}}{2}$.

The concept of the "regular solids" in geometry has a close connection to the Golden Section. A regular solid is a three-dimensional form with all its faces equal and all its angles equal. As the ancients knew, they are only five: the cube, the
tetrahedron, the octahedron, the icosahedron and the dodecahedron [see (Figure number 18)]. Pythagoras was credited with discovering the five regular solids. ${ }^{101}$ For the Pythagoreans, the five solids were the playthings or "dice" of baby Bacchus and


Figure 19: The five regular solids identified with an appropriate element, after Kepler's Harmonices mundi libri, Linz, 1619.
the archetypal patterns from which all things in the universe were made. ${ }^{102}$
In the Pythagorean tradition, each solid was associated with one of the four elements, while the dodecahedron was assigned to the heavens in their entirety ${ }^{103}$ [see

101 Proclus, Commentary on Euclid, Book 1 in Ivor Thomas, Selections Illustrating the History of Greek Mathematics: From Thales to Euclid. Cambridge, Massachusetts: Harvard University Press, 1939. p. 149.

102 Anne Griswold Tyng, "Resonance between eye and archetype". VIA 6, The jourral of the Graduate School of Fine Arts, University of Pennsylvania. 1983. p. 50.
${ }^{103}$ The reasoning behind these assignments was not arbitrary. Kepler explains: The uprightness of the cube conveys a certain impression of stability, it was assigned to earth; the octahedron can be suspended by two opposite corners and spun as in a lathe, "a certain image of mobility," it was assigned to air; the sharpness of the tetrahedron suggests the complexion of fire; while the globular form of the icosahedron, the figure with the largest number of faces, suggests "a water drop";
(Figure number 19)]. This lore, a strange mixture of mysticism and science typical of


Figure 20: Spheres within spheres, after Kepler's Harmonices mundi libri, Linz 1619.

Pythagorean thought, was prominently displayed by Plato in his Timaeus. So the regular solids also became known as the Platonic solids.

The concept of fitting the five platonic solids into spheres within spheres was the archetypal image of the cosmos for Kepler. It was his proposal for the planetary orbits of the five planets that were then known. ${ }^{104}$ [see (Figure number 20)] Kepler found out that by successively placing each solid within the sphere inscribed within

Finally, the dodecahedron is left for the celestial form, having the same number of faces as the celestial zodiac has signs.

104
Ann Griswold Tyng, "Resonance between eye and archetype". VIA 6, The journal of the Graduate School of Fine Arts, University of Pennsylvania. 1983. p. 57.
another of the five solids placed in a certain order, the diameter of the spheres would be related to one another in proportion to the distance of the five major planets. Each sphere was given a thickness corresponding to the extreme distances of a given planet to the sun, at its closest position (peribelion) and its farthest (aphelion) on its orbit. ${ }^{105}$

## Conclusion

Geometry in ancient times was simply the culture-creating tool with which most things were formulated. The system has its roots in the very beginning of time, further back in Man's history through the ancient Egyptian dynasties and the Greek civilization.

The philosophy of Plato regarding numbers as the sole establishment of things has influenced Western thought, art and architecture. Plato's aesthetics, his concept of beauty, evolved from the role of numbers and geometry and was taken from the Pythagorean doctrine and then developed by Plato and his school. A great factor in Plato's mathematical philosophy and, in a subsidiary manner, in his system of aesthetics was the importance given to the five regular bodies and the interplay of proportions they reveal. This point of view transmitted throughout the Middle Ages to the Renaissance and beyond, with the study and application to artistic composition of the same proportions.

105 Later on Kepler realized that this concept was not accurate.


In the second book of Optica, which is entitled "On the visible properties, their causes and the manner of their perception", Alhazen uses the term "alma-ani almubsira" (المعانىىالمصره) [visible properties] for the characteristics of objects which man can see and comprehend. He has specified the perception of the characteristics of objects as follows:

1. Brightness or dimness of the objects: their colors and the distance between the objects and the eye, exclusive of the distance between two or more than two objects.
2. The shape, volume, magnitude and movement, or inertia of the objects: perception of the characteristics that establishes the similarities or distinctions between such objects.

Alhazen devoted the third chapter of the second book of Optica, which is entitled "In the distinction of each partial criterion perceived by the visual sense", to detail the perception of the characteristics of objects. Although he says it is difficult to enumerate these characteristics, nevertheless, they can be divided into several kinds, for which Alhazen has devised the term "alma-ani almubsira" which is translated as visible properties. These visible properties are twenty-two factors for perception. They are either simple or made up of two to three or more varieties. These twenty-two criteria are: Light, Color, Distance, Position, Solidity, Shape, Size, Separation, Continuity, Number, Motion, Stillness, Roughness, Smoothness,

Transparency, Opacity, Shadow, Darkness, Beauty, Ugliness, Similarity and Dissimilarity.

## Manners of perception

Alhazen begins to describe the visible properties with the following statement: "The sense of sight perceives no visible properties that are not in a body"106

Bodies combine many visible characteristics and there occur on them many others. The sense of sight perceives many of the properties that inhere in or occur on them. As an example, color is one of the characteristics that inhere in bodies and light is another characteristic that either inheres in bodies or occurs on them. These two properties in bodies along with other ones can be perceived by the sense of sight.

Moreover the sense of sight does not perceive all properties in the same manner, nor does it perceive every property by absolute sensation. As when the sense of sight perceives two individual objects of similar figure at the same time, it is a perception of their agreement or disagreement with respect to a certain property and the existence of that property in each of them. Therefore, the sense of sight's perception of similarity and dissimilarity of figures is not by pure sensation, but rather by comparing the figures it perceives by pure sensation.

106 Al-Hasan Ibn Al-Haytham, Kitab Al-Manazir, Books 1-11-111 (On Direct Vision), Edited by Sabra. Kuwait: The National Council for Culture, Arts and Letters, 1983. p. 216.

Sight also perceives colors of the same kind, of which one is stronger than the other. When one of them is rust-green and the other is pistachio-green, the sense will perceive that they are both green, but one is greener than the other, thus differentiating between the two greens. That is the perception of similarity with respect to greenness and the dissimilarity with respect to strength and weakness.

It is similarly the case of light with regard to the sense of sight and perceiving strength and weakness in them. Therefore, the sense of sight's perception of the similarity and dissimilarity of color and light as well as the outline of visible objects, which is not due to mere sensation but to being distinguished and compared with one another.

Furthermore the sense of sight perceives transparency by judgment and inference. Sight does not perceive transparency unless it perceives what lies behind the transparent body, or perceives the penetration of light through it. The power of judgment perceives that which appears behind the transparent body as something other than the body itself.

Alhazen now comes to a statement that not everything perceived by the sense of sight is perceived by pure sensation; rather, many visible properties are perceived by judgment and inference in addition to sensing the visible object's image. Sight does not possess the power to judge; rather, it is the power of judgment that
distinguishes between those properties. This ability to distinguish performed by the power of judgment cannot take place without the mediation of the sense of sight.

Recognition is the only way for sight to perceive what a visible object is. When sight perceives an individual object which is taken away for a while and then sees it again without remembering having seen it before, sight does not recognize it, but only recognizes that which it remembers having seen before. Recognition is perception through a kind of inference; it is a perception of the similarity of the two figures, one when the sight perceives the visible object at the time of Recognition and another time in an earlier instance. Recognition ${ }^{107}$ cannot, therefore take place without remembering. Sensing similarity is a kind of perception by inference to achieve recognition. This does not occur as a result of inspecting all properties in the figure, but rather through perception of signs.

Similarly, a large number of visible things that are perceptible by means of inference are perceived only after inspection of all their features. As for a literate person when he glances at the written name of Allah (اللّ) ), be He exalted-he perceives by recognition, at the moment of glancing at it, that it is Allah's name, be He exalted. This is also true with all well-known written words. The case is different for the same person when he notices a strange word, which he has not come upon before or he has not already read. He will perceive such a word after inspecting its

[^10]letters one by one; then he will be perceiving the meaning of the word. This is very similar with everything perceptible by the sense of sight if the latter has not come upon it. Sight will perceive the identity of this thing only after inspecting all or many of the features of that thing and discerning them.

That which is perceived by recognition is perceived by sıgns, but not everything perceived by inference is perceived by signs. Recognition is distinct from inference, as perception by recognition is characterized by quickness because it is perception by signs.

Many of the visual properties perceived by judgment and inference occur in an extremely short interval of time and in many cases, their perception occurs by means of judgment and inference because of the quickness of process. Consequently, the speed with which these properties are perceived by inference is due only to the clarity of their premises and to the fact that the power of judgment has been much accustomed to differentiate between those properties. Thus, it happens that the power of judgment may perceive all properties in an image as soon as that image presents itself to it. When it perceives all of them, they become distinct to it at the very moment they are perceived, when the power of judgment comes to the perception of the image. For example, if a person of a sound judgment hears someone say "This (thing) ${ }^{108}$ is able to write", he will perceive at the moment he understands

108 In the Arabic text Alhazen meant by the word "thing" a person, an animal or anything.
these words that the (thing) he has heard describes a man. His perception that the (thing) that can write is a man must be due to the universal assumption "Everything that can write is a man". Similarly, if someone says "How effective this sword is!", a listener in possession of judgment will immediately understand that the sword referred to is sharp. The perception that the sword is sharp must be due to the universal assumption "Every effective sword is sharp".

Similarly, the power of judgment perceives the conclusions of all syllogisms whose premises are manifest and established in the soul and present to the memory. At the moment it hears the particular premises and in an extremely short interval of time, without there being an appreciable time between the moment of understanding the particular premise, the power of judgment perceives the conclusion. The reason for this is that the power of judgment does not syllogize by ordering, composing and repeating the premises as in the verbal ordering of a syllogism.

Most of the visible properties that are perceptible by inference are perceived extremely quickly and because of the speed of this perception, it does not become apparent in most cases that they are perceived by inference and discernment. The process is accomplished very fast because the premises of the visible properties are manifest and the power of judgment has become well-accustomed to discerning them. When sight has repeatedly perceived the visible properties of some objects, its
perception of them turns into a perception by recognition that has no need to resume the inference through which it has perceived their exactitude.

Many of the notions perceptible by inference are wrongly thought to be primary notions, which the mind perceives by its own nature and not achieved by means of inference. For example, "the whole is greater than the part" is called a primary notion. It is thought that the assertion of its truth is due to the intellect of its own nature and not to inference, because it is quickly accepted by the understanding and because the power of judgment does not doubt it at any time. In fact, these primary notions are perceived by recognition immediately because its truth has been established in the soul and because the soul remembers it and its truth, recognizing the proposition at the moment of its appearance.

It is not apparent how visible properties are perceived by inference and recognition at the moment of perceiving them. As for differing perceptions, some are achieved by the mind's own nature, others by recognition and still others by discernment and contemplation. There is a need for a second inference to understand the manner of their perception. The power of judgment does not employ this second inference at the moment of perceiving a visible property. This second inference, through which the power of judgment perceives the manner of perceiving what it perceives, is not an inference that can be performed extremely quickly; rather, it requires further contemplation.

It is the nature of man to judge and to make inferences without effort or the exercise of thought and also without being aware of performing familiar inferences. Thus, familiar inferences, of which the premises are evident and which do not require undertaking the process of inferring, are natural to man; for at the moment of perceiving their conclusions, one is not aware of having perceived them through inference. A clear evidence of this phenomena is furnished by what can be observed in the behavior of children in their early development and at the beginning of their awareness. They perceive many of those things that a man of perfectly developed judgment perceives and many of his acts are due to discerning and comparing one another. For example, if a child is shown two things of the same kind and is made to choose between them, then assuming that one of them is beautiful in appearance and the other ugly, he will choose the beautiful and refuse the ugly one, provided that he has reached awareness and is not extremely young.

## Conclusion

Some objects which are perceptible to the sense of sight are perceived by:

- pure sensation,
- others by recognition and
- still others by judgment and an inference that goes beyond recognition.

That, which is perceived by judgment and inference that exceeds recognition, will be subject to perception by recognition after it has been repeatedly perceived by sight and understanding that has been established in the soul. The manner of perceiving the visible properties does not, in most cases, become apparent because of the speed with which they are perceived. The speed of the inference by means of visible properties, in most cases, are perceived not by thinking and exertion but by nature and habit. This is due to the fact that the power of judgment performs these inferences naturally.

Furthermore, as time passes, man repeatedly perceives visible objects from childhood and early development until there is no particular visible property which sight has not repeatedly perceived. Consequently, all particular properties that are perceptible by inference will have become understood to the power of judgment and will be established in the soul. In this manner, the faculty perceives all particular properties (that are repeatedly presented in visible objects) by recognition and habit without needing to resume inference for the sake of perceiving any of those repeated properties.
light and color: distance: position: solidity: shape: size: separation: continuity:
number:
motion:
stillness: roughness:
smoothness:
transparency: opacity: shadow:
darkness:
beauty:
ugliness:
similarity: dissimilarity:

As discussed previously, Alhazen lists the particular properties that can be perceived by the sense of sight in general into twenty-two characteristics, namely: Light, Color, Distance, Position, Solidity, Shape, Size, Separation, Continuity, Number, Motion, Stillness, Roughness, Smoothness, Transparency, Opacity, Shadow, Darkness, Beauty, Ugliness, Similarity and Dissimilarity. All other visible properties fall under one of these, such as order which falls under position; or writings, scripts and drawings which fall under shape and order; or straightness, curvature, convexity or concavity which are modes of configuration and therefore fall under shape; or quantity, which fall under number; or equality or inequality which fall under similarity and dissimilarity; or laughter, joy or cheer, which sight perceives from the configuration of the form of the face and therefore fall under shape; or weeping which is perceived from the configuration of the face together with the movement of tears and therefore falls under motion and rest, since wetness is only perceived by the sense of sight from the fluidity of the wet body and the motion of its parts with respect to one another and dryness is only visible from the coherence of the dry body and the absence of fluidity from it. Similarly, when all the particular visible properties are discerned concerning the manner of their perception by sight, they will be seen to fall under some of the divisions that have been mentioned or its detailed ones.

## Light and Color

The first property to be perceived by the power of judgment in a colored figure is color. The power of judgment can only perceive that by recognition, provided that the color existıng in the object is a familiar one. Therefore the perception of the quiddity of the color by recognition is due to the comparison it makes between the figure of the color and the figures it previously perceived of the forms of similar colors and to its remembering of those forms. For when sight perceives a red color, it will perceive that it is red only because it recognizes it and this recognition must be to assimilating the color's form to those it previously perceived of similar colors. If sight had not perceived a red color before finally perceiving a red color, it would not know the final red color to be red upon perceiving it. If the color is a familiar one, sight will percenve what it is by recognition, but its quiddity will not be perceived if it is a rare color, which sight has not previously perceived. If sight does not perceive the quiddity of the color or recognize it, it will assimilate it to the nearest color it knows. Therefore, color is originally perceived by pure sensation; when it has been seen repeatedly, sight will then perceive what color it is by recognition.

Sight also perceives the quiddity of light by recognition. Thus, it recognizes the light of the sun and differentiates it among the light of the moon or of fire.

Sight's perception of the quiddity of each one of these lights as achieved by recognition.

Therefore, that which light perceives by pure sensation is light as light and color as color. But nothing that is visible, apart from light and color, can be perceived by pure sensation, but by discernment, inference and recognition, in addition to sensation; all visible properties that are perceptible by discernment and inference can be perceived by discerning the properties in the sensed figure. Similarly, all perceptions by recognition can be achieved only by perceiving the signs in the figure that is sensed. Thus, sight perceives the light that exists in the self-luminous body as it is and by itself through the sensation itself.

Therefore, sight perceives color and senses that it is color and the beholder who looks at it knows that it is color before realizing what color it is. For the eye is colored at the moment when the figure appears in it and when it is colored, it senses that it is colored and when it senses that it is colored, it senses the color. Then, by discerning the color and comparing it with colors known to it, sight perceives the quiddity of the color. Proof, that sight perceives color as color before perceiving what color it is, is furnished by visible objects of strong colors, such as dark blue or wine when they exist in a somewhat obscure place. When sight perceives one of these colors in a dim place, it perceives it only as a dark color, realizing that it is color without at first discerning what color it is. If the place is not very dim, sight will
perceive what the color is after contemplating it further. It is, therefore, clear from this experiment that sight perceives color as color before perceiving what color it is.

Unusual colors also furnish proof that sight perceives color as color prior to perceiving what color it is. When sight perceives an unusual color, which it has not previously seen, it perceives it to be color without knowing what color it is, but upon contemplating it further, sight assimilate it to the closest color known to it.

A clear and visible proof that perception of the quiddity of color must take place in time is furnished by what can be observed in a revolving top. If the top is painted in different colors forming lines that extend from the middle of its visible surface close to its neck to the limit of its circumference, then forcefully made to revolve, it will turn round with great speed. Looking at it, the observer will now see one color that differs from all the colors in it, as if this color was composed of all the colors of those lines; he will neither perceive the lines nor their colors. Thus, no point on the surface will remain fixed in any one place for a sensible interval of time , but rather travel in the smallest amount of time. All points at equal distances from the center will move with the top's rotation on the circumference of a single circle. Therefore, the colors of all those points will appear in the whole circumference of that circle as mixed and undistinguished by sight; and thus, sight will perceive the color of the top's surface as one color that is a mixture of all the colors in its surface. Therefore, sight does not perceive the quiddities of the colors in the top's surface
when the top moves quickly, but perceives them when the top is at rest or moving slowly.

That is to say sight does not perceive the quiddity of color unless the color is fixed in one place for a sensible interval of time, or moves in a sensible interval of time through a distance whose magnitude does not greatly affect the position of that color in regard to the eye.

Perception of light and color as such, requires the lapse of a certain interval of time, however small; that is to say, that the instant at which such perception occurs is later than that at which the surface of the eye first comes into contact with the illuminated air. For this purpose, Alhazen employs an argument from an analogy with the physical situation, in which external light passes through apertures to objects placed behind them. The passage of light from the eye to the common nerve, he says, is analogous to its passage from the aperture to the opposite body; and he expressly states that the light's arrival from the aperture at the body facing the aperture must take place in time, though this time is imperceptible. If, along the path from the aperture to the illuminated body, the light arrives at one point after another, then, he asserts, we have to deal with movement which must take place in time. Alhazen replies to the other hypothesis that if the intervening air receives the light all at once, it must also take place in time. ${ }^{109}$

109 Alhazen had no experimental proof of the finite speed of light. This is not surprising as such proof was not forthcoming for centuries after his time. However, unlike Aristotle (De anima, II, 7;

## Distance

Sight does not perceive distance ${ }^{110}$ by pure sensation. Nor is perception of the distance of an object the same as perception of the place where the object is. The place of a visible object is determined by three things: distance [remoteness], direction and the magnitude [or the measure] of the distance between the eye and the object.

The magnitude of distance is not the same as distance as such. Distance between two bodies is non-contiguity; it is the existence of a certain interval between the two separate bodies. The magnitude of the distance is the magnitude of that interval. Therefore, perception of distance, i.e. non-contiguity, is different from perception of the magnitude of distance; these two properties are not perceived in the same manner.

Perception of the magnitude of distance is due to perception of size. Perception of both the visible object's distance and direction depends on perception of position. Thus, the perception of where the visible object is, is different from the perception of the object's distance.

De sensu, VI) and AlKindi (De aspectibus, Proposition 15) before him and many others after him down to the early seventeenth century, Alhazen had theoretical reasons for believing that light took time to travel from one place to another. In Book VII, he asserts that light moves faster in a rare medium like air, than a denser medium like water or glass.

110 The Distance or remoteness of visible objects from the eye.

The perception of a visible object in its own place consists of perceiving five things:

1. Perception of the light that is in the object.
2. Perception of the object's color.
3. Perception of the object's distance [remoteness].
4. Perception of the object's direction.
5. Perception of the magnitude of its distance.

None of these properties is perceived separately, or one after another, but all perceived at once because they are perceived by recognition without the discernment and inference being resumed. Therefore, there exists no perception of distance by itself at the time of sensation.

At this point, Alhazen establishes his theory of vision over the adherents who held the doctrine of the visual ray ${ }^{111}$ theory. Their proposition was "if vision takes place by means of a figure which passes from the visible object to the eye and if the figure occurs within the eye, then why does sight perceive the object in its own place outside the eye while its figure exists inside the eye?" He answered that these people have ignored the fact that vision is not achieved by pure sensation alone and that it is accomplished only by means of discernment and prior knowledge. Without them,
${ }^{11}$ They believed that the vision takes place by means of a ray, which goes out of the eye and reaches the object and that vision occurs through the ray's extremities (the points on the base of the cone of vision).
sight would achieve no vision, nor would there be perception of what the visible object is at the moment of seeing it.

Alhazen returns to describe the manner of perceiving distance, by saying that distance of a visible object can be perceived separately only by discernment. Yet distance is one of the notions that have settled in the mind during the course of time, as much as the soul has not been aware of its settlement, due to the continual existence of this notion and its repeated presence before the power of judgment. To perceive it, therefore, there is no need to reuse the discernment and inference at the time of perceiving each visible object. Rather, it perceives distance by prior knowledge along with the other properties contained in a visible object.

The manner, which the power of judgment perceives distance by means of discernment, is as follows: when the eye turns towards a visible object which it has not been facing, it perceives the object; and when it turns away from it, the perception ceases. Similarly, when the eyelids are opened and facing an object, sight will perceive that object. When the eyelids are closed after perceiving the object, that perception comes to an end. It is natural for the mind to judge that which is produced in the eye while in a certain position and cease when the eye turns away; is not something fixed inside the eye nor does its assignee reside within the eye. When this happens, the power of judgment perceives that what is produced in the eye is something that comes from the outside and that its assignee resides outside the eye
and there is a distance between the object and the eye. It is extremely manifest to the power of judgment, that that which is not inside a body or in contact with it must be at a distance from it.

The power of judgment does not require the above detailed account in order to perceive distance. It perceives the conclusion at the moment of vision without the need to go into details. Because of the continuity and repetition of this phenomena, the realization that all visible objects lie outside the eye and have a distance from the eye is established in the soul, without the awareness of how this is established or of the manner in which it has been established.

Nevertheless, distance is not perceived separately, since it is perceived only in conjunction with other properties. Perceiving distance along with position as well as how the object is perceived in its own place, will be discussed in the manner of perceiving position.

Sight's perception of the magnitude of distance is either certain or uncertain. The amount of distance from any visible object to the eye is perceived and realized in the case of every object. Some visible objects are such that ordered and connected bodies exist between them and the eye, while others are not of this description, as there are no connected bodies ordered along their distances. Thus, when sight perceives the magnitude of these intervals, it will perceive the magnitude of the object's distance. Sight, therefore, perceives the magnitude of distances of visible
objects, whose distances stretch along a series of continuous bodies, from perceiving the magnitudes of the ordered and continuous bodies lying along their distances.

Some of the visible objects are at moderate ${ }^{112}$ distances, while others are not. As for those at moderate distances, sight perceives, correctly and with certainty, the magnitudes of their distances. Thus, if the distances of visible objects extend along ordered and continuous bodies and they are moderate distances, then sight will perceive their magnitudes correctly and with certainty. ${ }^{113}$ But sight does not perceive correctly and with certainty the magnitudes of distances of those immoderately distant objects whose distances extend along a continuous series of bodies which are themselves perceived. That is because sight does not distinctly perceive visible objects whose distances are immoderate.

However, sight does not perceive the magnitude of distances of objects when these distances do not extend along ordered and continuous bodies. For this reason, when sight perceives clouds in the plains or in regions that has no mountains, it will take them to be excessively far, by analogy with celestial bodies. If, however, a discontinuous cloud forms between mountains, the mountain tops will appear above it and sight will perceive pieces of the cloud attached to the mountain's side. Therefore, it will appear from this consideration that the distances of clouds are not

A statement of "the size-distance constancy principle", as modern psychologists have called 1 t.
${ }_{113}$ By certainty, Alhazen adds that he means the utmost of what the sense can perceive.
excessively great and that many of the clouds are closer to the earth than the mountain tops and that what is thought about their excessive distance is erroneous and untrue.

The following is another example to show that sight does not perceive the magnitude of an object's distance unless the latter lies along a series of continuous bodies and unless sight perceives those bodies and ascertains their magnitude. Let the experimenter go to a house which he has not entered before and let there be a hole in one of the walls of this house. Behind that hole, let there be an open space which the experimenter has not previously observed. Let two walls stand in that space so that one of them will be closer to the hole than the other. Let there be a sizable distance between the two walls and let the nearer wall hide part of the farther one and let the other part of the farther wall be visible. Let the hole be above the ground so that upon looking through it the observer will not see the ground surface behind the wall that has the aperture. By looking through the aperture, the experimenter will see the two walls together without perceiving the distance between them. If the distance of the first wall from the aperture is excessively large, he will perceive the two walls as contiguous and may take them to be a single continuous wall if their color is the same. If the first wall is moderately distant from the aperture and the observer senses the two walls as two, he will take them to be close together and fail to ascertain the distance between them. Thus, the distance between the two bodies cannot be
ascertained by the sense of sight if it had not previously observed that place and those two walls, or if it had had no previous knowledge of the distance between them.

As sight cannot perceive the distance between two such bodies, then it cannot perceive the magnitude of the distance of the far body, even though it perceives the body's figure. That is to say that there is nothing, by means of which, sight may estimate the object's distance and with which it may compare the distance, other than the ordered bodies extending along the object's distance. For an estimation by anything other than those bodies would be conjectural and not certain.

Perception of the distances between separated objects is due to perception of the objects' separation. And perception of the magnitudes of these distances is like the perception of the magnitudes of distances of visible objects from the eye.

When sight perceives objects with magnitudes of distances that cannot be determined, the power of judgment immediately conjectures their magnitudes by comparing their distances with those of objects which sight has previously perceived and whose magnitudes it has ascertained. This comparison will rely on the figure of the object and the similar figures, which it has previously perceived and the magnitudes of those distances, the power of judgment has ascertained. If the discerning power cannot determine the outline of the object's figure, it will compare the magnitude of the figure as a whole with the magnitudes that have been
ascertained before, thus assimilating the object's distance whose magnitude it cannot determine, to the already ascertained magnitudes of objects equal in size to the present object.

That is the limit of what the discerning faculty is capable of in the process of attaining perception of the magnitudes of the distances of visible objects. Sometimes it happens that it correctly perceives by means of this comparison the distances of such objects and sometimes it errs. Also, when it achieves correct perception, it cannot be sure that it has done so. This conjecture is made extremely quickly on account of the many times in which the discerning faculty has been accustomed to perceiving the distances of visible objects by conjecture or with certainty.

As for familiar objects at familiar distances which have been repeatedly and frequently perceived, sight will perceive the bodies along their distances and ascertain the magnitudes of those distances because of being presented repeatedly and frequently to the eye; because their distances have appeared many times before it, sight will perceive the magnitudes of those distances by recognition based on its conjecture of the magnitudes of the distances. There being no great discrepancy between such conjecture and their true magnitudes, it is in this manner that most of the distances of visible objects are perceived.

## Position

Alhazen divides positions of any visible object that sight can perceive into three types:

1. The position of the object as a whole, or of one part of it, relative to the eye. This type is opposition.
2. The position of the object's surface relative to the eye, or of the object's surfaces opposite of the eye (if the object has a number of surfaces, some of which are visible) and the positions, relative to the eye, between any two points or any two objects which sight simultaneously perceives and visualizes a distance between them.
3. The positions of the object's surfaces and the extremities of these surfaces in relation to one another. This type is arrangement or order.

The opposition between a visible object and the eye consists of the object's distance from the eye and its location relative to the eye. As for the perception of the object's distance, this has been demonstrated previously to be something which has been established in the soul. But regarding the location of the object, it is perceived by the sentient on account of the eye's position at the moment of vision. Sight can only perceive an object placed opposite from it and only when the eye faces the object's direction. Locations of objects relative to the eye are perceived by sense and judgment. The power of judgment differentiates between a location facing the eye
and others close to that location and it perceives all directions by imagination and discernment. Therefore, when sight perceives a visible object upon facing a certain direction, then turns away from that direction, the perception of that object ceases. Upon the sight's facing again in the previous direction, vision of that object will return. Thus, from the eye's facing in the direction of the visible object at the moment of vision, the direction of the object becomes determined for the sentient and for the power of judgment.

One of the characteristics of sight is perceiving the object's figure by the straight lines of ray coming out of the object ${ }^{114}$. These figures extend in the body of the eye along the lines of the ray. Thus, when the figure of a visible object occurs in the eye, the sentient senses the figure, senses the part of the eye in which the figure occurs and senses the direction of the radial lines which extend between the eye and the object. When sight perceives the location of the figure in the eye and the line of direction in which the figure has extended, the power of judgment will perceive the direction in which that line extends towards the object. In this manner, the visible objects are distinguished with regard to their direction, as sight distinguishes dispersed visible objects by distinguishing the separate locations on the surface of the sentient organ in which the figures of the dispersed objects occur.

[^11]Alhazen has an analogy for the perception of the direction/location of visible objects and the subject of hearing. The sentient perceives sounds by the sense of hearing and it perceives the direction from which the sound has come. It also differentiates between sound coming from the right and from the left, or coming from the front and from behind. The sense of hearing can also distinguish between close locations from which the sound has come, as between a sound coming directly from an opposite direction and one from a direction inclined to it. Thus, just as the directions of sounds are perceived by the sense of hearing and the power of judgment perceives them by means of that sense, so the directions of visible objects are perceived by the power of judgment by means of the sense of sight.

A rationalization of how a sentient perceives figures by lines of ray is clearly shown by what is perceived in mirrors by reflection. Visible objects seen in a mirror are perceived as being opposite to the eye. Therefore, when sight senses the figure through the lines of the ray, it assumes the object to be at the extremities of those lines and that the figure must have passed along those lines because it lies at their extremities.

As the perception of distance being established in the soul, therefore, at the moment when the figure occurs in the eye, the power of judgment perceives the direction of the object, in addition to the notion of distance which has been
established in itself and the conjunction of both distance and direction is opposition. It is in this manner that perception of opposition takes place.

Sight perceives the figure of a visible object by pure sensation. At the moment when the object's figure occurs in the eye, the sentient perceives the object's color, light, and the place in the eye that has been colored and illuminated by that figure; the power of judgment perceives the object's direction and distance at the moment when the sentient perceives its light and color. As direction and distance constitute opposition and light and color constitute the objects figure, so that perception of an object opposite from the eye consists of perception of the figure and opposition. Because of the continuity and frequent repetition of this procedure, the figure becomes a sign for the sentient and the power of judgment. At the moment of the figure's occurrence in the eye, the sentient perceives the figure and the power of judgment perceives the opposition and thus, perception of the object in its own place is constituted. It is in this manner, that perception of any object occurs and of any part of it, in its own place.

Opposition consists of direction and distance as such. Perception of any object, as to be opposed to the eye, is always certain, even if the magnitude of its distance is ascertained.

Positions of the surfaces of visible objects fall into two categories: frontality and inclination. A surface is frontal to the eye, being perceived in this position by the
eye, the axis of the ray perpendicularly meets a point in it. A surface is inclined when, being perceived in this position by the eye, the ray's axis, meeting a point in it will be inclined to the surface and will not be perpendicular to it.

Also, edges of any objects' surfaces, lines in objects and intervals between objects or between their parts, divide into two classes:

1. lines and intervals that intersect the radial lines. These also divide into frontal (the ray's axis perpendicularly meets at a point in it) and inclined (inclined to the ray's axis when it meets it in a point) positions.
2. lines and intervals that are parallel to, or collinear with the radial lines.

Sight perceives the inclination and frontality of surfaces and lines from perceiving the difference or equality between the distances of the extremities of those surfaces and lines from the eye. The same applies to perception of positions of frontal or inclined lines and intervals; sight will distinguish it by perceiving the distances of their extremities, or when it perceives the equality or inequality of the distances of any two points on the line or interval from the point at which it is gazing. Such equality or inequality are perceived by the sentient by means of conjecture and signs. It is in this way that sight perceives inclination and frontality.

As for the positions of lines and intervals that are parallel to the lines of the ray, sight perceives them from its perception of opposition.

Therefore, it is in these ways sight perceives the positions of surfaces, lines and intervals in relation to the eye.

For the surfaces, lines and intervals that intersect the lines of the ray, some are excessively inclined to the latter lines, others are only slightly inclined to them and others are perpendicular to one of the radial lines, these being the surfaces, lines and intervals that lie frontal to the eye. Sight distinguishes the inclination of surfaces, lines and intervals that are inclined to the lines of ray, as a result of perceiving the locations of their extremities. Also, perception of inclination of surfaces, lines and intervals takes place if the distances of their extremities from the eye are moderately large in comparison with their magnitudes.

When an observer wants to ascertain the positions of a surface or a line in a visible object, or the position of an interval in the surface of a visible object, he contemplates the figure of the object and the manner in which that surface, line or interval extends. If the figure of the object is clear and distinct and the inclination of the surface or line or interval is excessive, then sight will perceive its true inclination from its perception of how they extend and of the locations of their opposed extremities. However, if the figure of the object is clear, the inclination of the object is not excessive and its distance extends along ordered bodies, then he will notice bodies along the distances of its extremities from the eye and estimate their magnitudes, thus, he will perceive the inclination or the frontality of that surface or
line or interval from his perception of the magnitudes of the distances of its extremittes.

For indistinct or clear figures with moderate inclination, but the distance does not lie along ordered bodies, sight will not perceive the true position of such a surface or line or interval. Nevertheless, sight, at once, will become aware that the position of that surface or line or interval is uncertain, if it is aiming to estimate their position.

Therefore, it is in these ways that sight perceives the positions of the surfaces of visible objects and the positions of lines and intervals in the surfaces of objects when they all intersect the radial lines.

Positions of visible objects are mostly perceived by conjecture. If the objects are moderately far, there will be no great difference between their conjectural and their true positions. If they are excessively far and sight does not perceive a difference between the distances of their extremities from the eye, then sight will perceive them as frontal-oriented in relation to it, even if they are oblique, because if it fails to perceive the difference between the distances of two ends of the object, then it will perceive these two distances as equal and consequently, it will judge the object to be frontal-oriented.

The positions of the parts and the edges of visible objects or of its surfaces and the positions of separate objects relative to one another (all of which fall under order), sight will perceive them from perception of those places in the eye where the
figures of the parts occur and from the power of judgment's perception of the parts of the figure produced in the eye for the whole object. If the object's surface has different colors or if gaps exist between its parts, separating them from one another, the figure produced in the eye will be of different colors or its parts will be separate as those of the object's surface. The power of judgment will perceive the order and color of those parts from the total figure produced in the eye for the whole object; it will perceive the order of those parts and their position in relation to each other. As for the limits of the surface or surfaces of an object, sight will perceive them and their order by perceiving the part in the surface of the eye where the color and the light of that surface occur and by the power of judgment's perception of the limits of that part and of the order of its periphery. It is in these ways that sight perceives the positions of the parts of objects, of their surfaces and their limits and the positions of the distinct parts of visible objects and those of separate objects.

## Solidity

Solidity is the extension of a body in the three dimensions. Sight perceives solidity in some bodies but not in others. It has been established by knowledge and experiment that the sense of sight perceives only bodies; thus, upon looking at a visible object, a person endowed with judgment will know it to be a body and on the basis of immediate vision, will judge it to be a body, even without perceiving its
extension in the three dimensions. Sight perceives the extension of all bodies in length and width from its perception of the surfaces of bodies in front of it, along with the established knowledge that the visible object is a body, only the third dimension will remain to be perceived.

Some bodies are surrounded by plane and intersecting surfaces that fold into one another. Other bodies are surrounded by convex or concave surfaces. Other bodies are surrounded by surfaces of different shapes that intersect and fold into one another. Other bodies are contained by a single, round surface.

When sight perceives a body surrounded by intersecting surfaces of which one 1s plane, assuming the plane surface to be frontal and facing the eye and the remaining surfaces that intersect the frontal surface to be either perpendicular or inclined to it in such a way as to cover behind it, so that only the frontal surface is visible, then sight will only sense their extension in length and breadth; it will not sense the solidity of bodies of this description. However, if the surface facing the eye is inclined to the frontal direction in such a way that the eye perceives another intersecting surface to the frontal one, then sight will perceive the bending of that body's surface and its extension in depth. Therefore, sight will perceive the solidity of bodies situated in this manner with respect to the eye. In general, sight will perceive the solidity of every body, of which it perceives two intersecting surfaces.

Bodies that have convex or concave surfaces that face the eye, the distances of its parts from the eye will differ. If sight perceives the convexity or the concavity of the surface, it will perceive the surface's middle to be closer or farther than its borders, so it will sense the body's extension in depth, thereupon, it will perceive the body's solidity. Similarly, if a surface other than the one facing the eye is convex or concave and sight distinguishes its convexity or concavity, then it will perceive the body's extension in the three dimensions.

Therefore, sight perceives the solidity of bodies by perceiving the bending of their surfaces only in the case of bodies, which are moderately distant from the eye and at distances which are ascertained by the sight. Of excessively distant bodies or those at a distance whose magnitude is not ascertained, sight will perceive neither the bending of the surfaces nor the relative positions of the parts of these bodies and it will fail to distinguish the solidity of these objects. Alternatively, sight will perceive these bodies only as flat.

The solidity of other visible objects cannot be perceived by the sense of sight, but can be perceived only by precedent knowledge.

## Shape

Alhazen categorizes shapes ${ }^{115}$ of visible objects into two kinds:

115 Alhazen uses the word shape or "shakl" for both two dimensional and three dimensional bodies, while reserving "bay'a" for three dimensional ones only. "Hay'a" corresponds to form. The

1. The shape of the object's circumference or of the circumference of a part of the object's surface.
2. The bodily shape of the object, or the bodily shape of a part of it. This being the form of the surface or part of the surface of the object whose solidity sight perceives.

The sentient perceives the shape of an object's circumference or every part of the object's surface by sensing the order of segments of the boundaries of every part in the figure. If the sentient wants to ascertain the shape of the circumference of an object's surface or of a part of the surface, it moves the radial axis of vision on the object's periphery, thus determining by means of this motion the positions of parts of the limits of the figure and through this procedure, sight will perceive the shape of the object's circumference.

Sight can perceive the form of the object's surface only by perceiving the positions of the parts of the surface and the similarity or dissimilarity of these positions. Sight ascertains the form of the surface by perceiving the inequality or equality of the distances from the eye to the parts of the object's surface.

Therefore, sight's perception of whether the surface of a visible object is convex, concave or flat by perceiving the inequality or equality of the distances, heights or breadths of the parts and the amount of difference between these

Latin translation has figura for shakl and renders hay'a by forma.
distances, heights or breadths. It is for this reason that sight perceives convexity and concavity only in the case of moderately distant objects when it can ascertain the magnitudes of their distances, the amount of difference between their distances from the eye or their heights or breadths. If some of the parts of a surface are prominent or depressed, sight will infer this status from the bending, intersection or curving of those parts at those places and from the relative positions of the surfaces of those parts. That is if sight has not previously perceived that surface or anything like it. But if the object is a familiar one, sight will perceive its form and the form of its surface by prior knowledge. Sight frequently errs in its perception of the forms of objects and their surfaces without being aware of its error. In cases of slight convexity, concavity, corrugation or protrusion, when the differences between the distances of the surface's parts are small, sight often fails to perceive these differences, even though the distances may be moderate, provided that these objects are not very close to the eye.

## Size

The manner of perceiving the size or magnitude of a visible object is an uncertain subject. Most mathematicians assume that the size or magnitude of a visible object is perceived from the size of the angle produced at the center of the eye by the rays surrounding the visible objects ${ }^{116}$. Thus they base perception of size on the

[^12]angles alone and give no consideration to anything else in this mode of perception. Some of them, however, hold such perception and also take account of the object's distance and position.

The truth of the matter is that sight cannot perceive the magnitudes of visible objects by an estimation based upon only on the angles which the object subtend at the center of the eye. The same object does not look different in magnitude when its distance is moderately varied ${ }^{117}$. Thus, when a near object, whose magnitude is perceived by sight, moves through a moderately large distance away from the eye, it does not look smaller but is rather perceived to be of the same magnitudes it looked from the first distance, provided the second is a moderate one. All familiar objects appear to be of constant magnitudes when their distances vary within moderate limits.

Alhazen demonstrates some experiments to prove his point as follows:

- Equal objects at different distances are always perceived to be equal, provided that the farthest among them is moderately distant. The angles subtended by the same
seen through smaller angles appear smaller; and those seen through equal angles appear to be equal" Ver Eecke, Paul (trans.), L'Optique [d'Euclide], p. I. On perception of size in Euclid and Ptolemy, see Lejeune, Albert. Euclide et Ptolémée, deux stades de l'optique géométrique grecque, Louvain 1948. pp. 95-103.
${ }^{117}$ Sabra comments that the principle of moderate distances, derived from Ptolemy and is mentioned in this paragraph and the following ones. Excessively remote objects (as the stars) appear to be smaller than they are.
object from different distances vary appreciably in size. If an object at a cubit's distance from the eye moves farther away to a distance of two cubits, the difference between the two angles produced in the eye by that object will be of a noticeable amount. But sight will not perceive the object to be smaller at two cubits than it was at one cubit.
- If a square with equal sides and right angles is drawn on the surface of a body, which is then raised close to eye level so that the figure on its surface can still be seen, sight will perceive the square figure to be of equal sides; although the angles subtended by the sides at the center of the eye will differ greatly if the eye lies close to the plane of the square, sight will not perceive the sides of the square as unequal.

After this introduction, Alhazen shows exactly how perception of size takes place. As it has been shown that perception of most sensible properties depends on inference and judgment. Perception of size is one of the properties perceived by inference and judgment. The criterion on which the power of judgment depends in discerning the size of a visible object is the magnitude of the portion of the eye's surface, in which the figure of the object occurs. That portion is limited and measured by the angle which exists at the center of the eye and which is contained by the radial cone surrounding both the object and the portion of the eye containing the object's figure. Therefore, the basis on which the power of judgment relies for
discerning the size of a visible object's is the part of the eye, in which the object's figure occurs and the angle subtended by this part at the eye's center.

In order to perceive size, the power of judgment cannot be satisfied merely with considering the angle or the magnitude of the portion of the eye that subtends it. Therefore, if the sentient perceives the place in which the object's figure occurs together with its magnitude, then it will perceive the decrease in magnitude of that place as the object recedes from the eye.

The movement of objects farther off and closer to the eye is repeatedly experienced by sight at all times. Sight perceives these movements and relates their farther distances along with the decrease in magnitude of the areas where the figures occur in the eye and it perceives the increase in magnitude of those areas as the objects draw nearer to it. When that is established in the soul, the power of judgment, when discerning the object's magnitude, will not take into account the angle alone, but will consider both the angle and the distance. The magnitude of objects is, therefore, perceived only by judgment and inference. And the inference through which the object's magnitude is perceived consists in estimating the base of the radial cone, i.e. the object's surface, by the angle of the cone and by its length, namely the distance of the object from the eye.

It has been shown in the manner of perceiving distance that the perception of the magnitude of the distance of any object from the eye is either ascertained or
conjectured. By perceiving the positions of the radial lines surrounding the object's limits and the magnitude of the part of the eye's surface contained by them, i.e. the size of the angle and imagining the magnitude of the object's distance, hence it will imagine the object's magnitude.

The following is clear evidence to show that perception of size of a visible object is the result of comparing the size with the object's distance:

- Let someone face a wide wall at such a moderate distance from his eyes that his sight will ascertain the magnitude of the distance and the width of that wall; then, having raised one hand before one eye so as to lie between that eye and the wall, let him look at the wall with the other eye closed; he will find that his hand has screened a large portion of the wall and will perceive the magnitude of his hand in this situation and also perceive that the part of the wall screened by his hand is much larger than his hand. Now, sight will perceive that the angle subtend by the hand and by the screened part of the wall is one and the same and also perceive that the screened part of the wall is much greater than the hand. That being so, the power of judgment will, in this situation, perceive that of two unequally distant visible objects that subtend the same angle, the farther will be larger in size.
- When the observer turns his eye to look at another wall farther off, placing his hand in front of his eye, he will find the magnitude of the screened part to be greater than that of the first wall.
- While in the same situation, if he looks at the sky, he will find that his hand has screened a large portion of the visible sky.

Therefore, it is clear from this experiment that sight perceives the size of an object by means of the magnitude of its distance as well as by estimation by means of the angle and not by estimation by angle alone. In other words, the power of judgment can only perceive the size of a visible object as a result of imagining the cone surrounding the object and imagining the magnitude of the cone's angle and length and estimating the cone's base by the magnitude of both the angle and the length. That then, is the manner of perceiving size.

Sight is accustomed to judge the distances of visible objects. Distinguishing the magnitudes of distances is a substantial component in perceiving objects' sizes. Magnitudes of objects' distances are perceived, as mentioned in perception of position, with either certainty or conjecturally. Those, which are conjectured are perceived only by assimilating the object's distance to determined distances of equivalent objects and distances, whose magnitudes are certain, are those that extend along a series of continuous bodies. The following is an attempt to clarify how sight perceives the magnitudes of objects' distances that extend along ordered and continuous bodies:

The ordered and contınuous bodies that lie along the distances of visible objects are, in most cases, those parts of the ground close to the feet. Familiar
objects, which are seen continuously and at all times, are those standing on the surface of the ground with the body of the ground lying between them and the body of the spectator. Now, sight always perceives and measures the magnitudes of those parts of the ground that intermediate between the beholder and the visible objects that stand on the surface of the ground and lie along the distances of those objects. Sight perceives the magnitudes of those intermediate parts of the ground as a result of measuring them by one another, measuring the farther parts by those closer to it, whose magnitudes it has determined. Then, as a result of continually perceiving and measuring these parts of the ground repeatedly, sight come, to perceive the magnitudes of parts of the ground close to the feet by recognition and assimilating them to analogous magnitudes it has already perceived. Thus, when sight glances at a part of the ground between it and a visible object, it will recognize the magnitude of that part as a result of having repeatedly perceived similar intermediate parts. That is one of the notions, which the sentient has acquired from the beginning of growth and childhood and in the course of time, as a result of which the magnitudes of distances of familiar objects are formed in the imagination and established in the soul without our being aware of how they have become established.

As for how the sentient begins to perceive the magnitudes of the parts of the ground extending between it and the visible object, the first part whose magnitude it ascertains is that lying close to the feet. Sight will perceive the magnitude of that part
and the power of judgment will perceive that part and its magnitude and it will ascertain its extent as a result of measuring and comparing it to our human body. We always measure such parts unintentionally by our feet whenever we step upon them or by our arms whenever we stretch our hands to them. Thus, all parts of the ground next to us are always measured unintentionally by our body. In the course of time, the angles subtended by the parts of the ground close to us come to be comprehended by the sentient and their figure imagined in the soul. The magnitudes of the radial lines, which extend from the center of the eye to the limits of the parts are always ascertained by the power of judgment, since the lengths of these lines are always measured unintentionally by our body. Hence, when someone standing looks at the ground close to his feet, the length of the radial lines will be measured by his height and the power of judgment will comprehend with certainty the distance between his eyes and the part of the ground close to his feet, which is his height.

Also, when someone looks at the ground close to where he is sitting, the power of judgment will perceive that the distance, from the eyes of the ground at the place where he is sitting is the same as his height in this posture.

The parts nearest those surrounding our feet are also measured by our body. For when we walk by our feet and our steps for a distance, the power of judgment perceives that distance's magnitude. The sentient always perceives and performs this measurement and judgment unintentionally. This perception is achieved as a result of
the fact that the eye always looks at those parts of the ground on which we walk. As a result of the continuation of this state of affairs and its frequent repetition and from the sight's repeated perception of the magnitudes of the ground's parts, the magnitudes of parts lying nearest the feet and those adjoining them will be determined. This acquisition takes place at the beginning of childhood, after which the magnitudes of the distances of familiar objects existing on the earth's surface are established for the sentient and for the power of judgment. Perception of the distances of these familiar objects come to be performed by recognition and assimilation of their distances to one another, at the moment of glancing at the intermediate bodies between those objects and the eye and without recommending the process of judgment and inference, rather than through recognition and assimilation alone.

Perception of the magnitudes of distances of visible objects on the surface of the ground by acquisition does not mean that it is the perception of how many arm-lengths each one of these distances is; rather, there occurs for each distance and part of the ground a determinate, imagined magnitude to which they compare and liken the magnitudes of distances of the objects they subsequently perceive. Each one of the magnitudes used for measurements, such as the cubit or the hand, has a determinate magnitude in the sentient. Thus, when the onlooker perceives a certain distance or interval and wishes to know how many cubits it is, it compares the figure
produced in the imagination for that distance or interval with the figure it has previously perceived for the cubit. Thus, when people say "there was between me and such and such person ten or five steps, or so many cubits, or the range of a spear, or course of a horse, or an arrow shot", comparing the distance between themselves and that person by reference to the step or the cubit or the arm's reach or some other magnitude whose figure exists in their soul.

When an observer wishes to ascertain the distance of a visible object on the surface of the ground, he contemplates the continuous intermediate part of the ground and moves his sight over its length, thereby surveying and perceiving it one part after another and sensing its smaller parts, provided that the end-point of the distance is moderately far.

As for immoderate distances, sight cannot ascertain their magnitudes.
The sentient is aware of whether the magnitude of a distance is certain or uncertain. Close or moderately distant objects are seen more correctly, that is, sight perceives their figures and their distances more clearly than excessively remote objects; sight cannot ascertain the form of a visible object at a very great distance from it.

The sentient may conjecture the magnitude of an object's distance from the size of the angle subtended by the object. This happens when it perceives the size of familiar objects by recognition. Perception of the object's size by recognition
together with the angle subtended at that time by the object, it immediately perceives the magnitude of the object's distance since the angle subtended by the object, at the eye's center, must depend on the distance's magnitude. That means the sentient can infer the magnitude of the distance from the size of a previously-recognized object, together with the subtended angle at the center of the eye. By the repetition of perceiving familiar object's distance in this manner, the size of the angle subtended by the familiar object will be a sign, which indicates the magnitude of that object's distance. It is in this manner that most of the distances of familiar objects are perceived. ${ }^{118}$ This perception is not perfectly accurate, but it does not differ greatly from the accurate distance.

Sight may infer the magnitude of distances of unfamiliar objects by assimilating the magnitudes of unfamiliar objects to those of familiar ones that have been previously perceived. It is in this manner that magnitudes of distances of visible objects are perceived by the sense of sight.

Sizes perceived by sight when facing visible objects are the magnitudes of their surfaces, the extents of parts of their surfaces, boundaries of the objects, intervals between the boundaries of parts of the objects' surfaces and intervals between separate objects. These are all the kinds of magnitude that are perceptible to sight
${ }^{118}$ It was from this kind of perception that mathematicians derived that an object's size is perceived by means of the angle; Although this perception occurs only in the case of familiar objects and is based on conjecture, not ascertainment.
upon facing a visible object. However, sight does not perceive the magnitude of the object's body upon facing it because it cannot, in this situation, perceive the whole surface of the object but only that surface or surfaces facing it, even though the object may be small. Thus, when sight perceives the solidity of a body, it does not perceive the magnitudes of the body but only that it is solid. Only when the body moves, or when the eye moves around the object so as to perceive its whole surface by sensation or inference, will the power of judgment perceive the magnitudes of the body's solidity by a second inference other than that used at the moment of vision. The magnitudes perceived by sight when it faces them are, therefore, only those of the surfaces and lines we have specified.

The sentient ascertains an object's size and distance by moving the radial axis over their parts. When the ray's axis moves over such an object, it will survey it, perceiving its parts one by one and determining their magnitudes and by means of this motion, ${ }^{119}$ it will ascertain the magnitude of the part of the sentient organ in which the object's figure occurs and the size of the angle of the surrounding cone which this part subtends. By this procedure, a grasp of the object's magnitude will occur.

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The movement of the axis over the object's parts does not leave its central position and move separately over those parts. The ray's axis always extends rectilinearly to the bend in the nerve where the eye is set and it maintains its position relative to the eye. Rather the eye moves as a whole before the object so that the middle of the sensitive area in it may face each one of the object's parts.

When sight perceives the magnitudes of the radial lines extending between the eye and the limits of the object or those of the parts of the object's surface, it will sense their equality or inequality. If the perceived surface or interval is inclined, sight will sense their inclination by sensing the inequality of the distances of their extremities. On the contrary, if the surface or interval is frontal, sight will sense its frontality by sensing the equality of the distances of their extremities. When sight senses the inclination or frontality of an interval, the power of judgment will not mistake its size because it will perceive the inclination of the cone containing the interval by perceiving the unequal distances of the interval's extremities and consequently, will sense the difference in the size of the cone's base owing to its inclination. The size of an inclined magnitude will be confused with that of a frontal one only if estimation is made by means of the angle alone. No confusion in regard to size will occur if estimation is based upon both the angle and the lengths of the radial lines between the eye and the object extremities.

## Separation

Sight perceives separation from the segregation of the two figures produced in the eye for the two separate bodies that are seen. For any two separate bodies, there either appears in the gap between them a light or a shining colored body, or the gap is dark and does not show what lies behind it. When sight perceives two separate
bodies, the figure of the light, or the colored body or the darkness which exists in the gap between them, will occur in that part of the eye which lies between the two figures produced in the eye for the two separate bodies. But light, color or darkness may exist in a body that may lie between the two bodies so as to be continuous with them. Consequently, if sight does not sense that the light, color or darkness in the place of separation does not exist in a body that is continuous with the two bodies on either side of it, then sight will not sense the separation of the two bodies. The surface of one or each of the two separate bodies will bend back at the place of separation and this bending may or may not be visible to the eye. If the bending is visible, sight will, in consequence, sense the separation of the two bodies. Thus, all that sight perceives of the separation of bodies is perceived by inference from one or more of the above-mentioned arguments.

Separation may exist between two detached bodies or between two bodies that join in some of their parts but not in others, such as fingers, the limb of animals, many walls and the branches of trees. In either case, sight will perceive separation only in the ways mentioned above, whether the separate bodies are entirely disconnected or joined in some parts and detached in others. Sight may perceive the separation of bodies by recognition and by prior knowledge, but this perception is not due to the sensation in the eye.

The separation of bodies may be wide and large or it may be narrow and small. Wide separation, in most cases, is apparent to the eye because of the visibility of the body that is in line with the gap and because of sight's sensing of that body. However, sight can perceive small degrees of separation, such as narrow wrinkles, from a distance at which a body equal in magnitude to the grade of separation does not disappear or is not distinguishable.

## Continuity

Sight perceives continuity from the absence of separation. If a hidden separation exists in a body and sight does not perceive it, then sight will perceive that body to be continuous. Sight also perceives contiguity and differentiates between contiguity and continuity, by perceiving the junction of the edges of two bodies while knowing that each of the them is detached from one another. Sight cannot judge contiguity except with the knowledge that the two contiguous bodies are not one but are disjunct from each other, for something that looks like disjunction of adjacent bodies may exist in continuous ones.

## Number

Sight perceives number by inference from things that can be counted. Whereas sight may perceive several separate objects at the same time, it perceives the separate objects and perceives their separation and will perceive that each of them is
not the same as the other. By distinguishing that, sight will perceive multiplicity. By perceiving multiplicity, the power of judgment will perceive number. Number is, therefore, perceived by the sense of sight when sight distinguishes several separate objects, provided that it perceives them simultaneously, perceives their separation and that each is different from the other.

## Motion

Sight perceives motion by inference from comparing the varying position of the moving object with other visible objects. Sight perceives a moving object together with other visible objects, it perceives the position of the object in relation to the others and its alignment with them.

Sight perceives motion in three different ways:

- By comparing the moving object with other objects. Sight perceives the moving object and perceives it first in line with some object and then in line with another object, while the eye maintains the same position.
- By comparing the moving object with a single object. This happens by perceiving the change in the first object's position to that other object either by receding farther from or drawing closer to it, by changing sides, or by changing the position of some parts of it. This happens while the eye maintains the same position.
- By comparing the moving object with the eye itself. When sight perceives the moving object, it perceives its direction and distance.
- If the eye is stationary while the object moves, then the object's position will move relative to the eye.
- If the motion of the object takes place on a frontal oriented interval, then sight will sense the change in the body's direction.
- If the motion of the object takes place on the line extending between the object and the eye, away from or towards the eye, then the object will recede from or approach the eye.
- If the object's motion is rotary, then the part facing the eye will not stay the same.

It is in these ways that sight perceives motion when it maintains the same position.
Sight may also perceive motion in any one of these ways, even if the eye moves. Sight has become accustomed to the motion of the objects' figures on its surface when the objects are stationary and, therefore, does not judge the object to be in motion on account of the motion of its figure, unless the figure of another object occurs in the eye and sight perceives the varying position of the moving object's figure relative to the figure of the other object or unless a succession of forms takes place in the eye as a result of the rotary motion.

An object's position can vary only in time; it can be in two different places and in two different positions only at two different moments. A certain duration must exist between any two different moments. Therefore, sight can perceive motion only in time.

Sight perceives the inequality or equality of motions in quickness or slowness by perceiving the intervals, on which the moving objects move. If sight perceives two moving objects together with the intervals on which they move and if it senses that one of the intervals covered by the two moving objects in the same time is larger than the other, then it will sense the quickness of the object that has covered the larger interval.

Alhazen sums up the way in which sight perceives motion by saying that sight perceives motion by perceiving the moving visible object in two different positions at two different moments separated by a sensible time.

## Stillness

Sight perceives stillness by perceiving the visible object in the same place and position at two different moments separated by a sensible time. It will perceive the object as stationary during that amount of time. Sight perceives the position of a stationary object in relation to other objects and in relation to the eye itself.

## Roughness

Sight perceives roughness in most cases from the light that appears on the surface of the rough body. Roughness is a difference in position of the parts of the object's surface, some parts of the surface are protruding and the others are depressed. When light shines upon that surface, the bulging parts will in most cases cast shadows on the depressed ones. When light reaches the sunken parts, it will be accompanied by shadows cast by some of the lights; the amount of the light will vary over the surface of that body. On the other hand, the parts of a smooth surface are similarly situated, so that when light shines upon it the form of the light will be similar over the whole surface. Sight recognizes the figure of the light on rough and smooth surfaces as a result of having frequently looked at rough and smooth surfaces. Therefore, if sight senses the light on the surface of a body to be of the quality it has been accustomed to seeing in rough bodies, it will judge the surface of that body to be rough. Sight, therefore, perceives roughness in most cases from the figure of the light, which it perceives in the surface of the rough body.

Roughness can also be excessive, in which the protruding parts will be fairly large. In the case of such surfaces, sight will perceive the protruding parts together with their protrusion and the difference in position of the parts of that body's surface, by perceiving the separation between the parts. Consequently, sight will perceive this kind of roughness without the need to examine the light.

If a strong light radiates opposite from a rough surfaced body, sight will not perceive the roughness of such body unless it perceives its parts distinctly and perceives the protrusion of some of them and the depression of others. If the body is slightly rough, the depressed parts and the pores in it being extremely small, then its roughness will not, in most cases, be visible to the eye if the light radiating upon the body is strong and no variation of its figure appears in the surface of the body. Sight will not perceive the roughness of such body except when it is very close and the parts of the body's surface are contemplated.

## Smoothness

Smoothness is the evenness of the surface of the body. Sight perceives it in most cases from the figure of the light, which appears in the surface of the smooth body and which sight has been accustomed to seeing in smooth surfaces. If the light on the surface of the body is of similar figure and even, sight will infer the smoothness of that surface. Smoothness may also be perceived by contemplation, from the perception of the flatness and evenness of the surface of a body. The state of being very smooth, being polished, sight perceives it in no other way than from the glitter and shine of the light in the body's surface.

## Transparency

Sight distinguishes transparency by perceiving what lies behind the transparent body. In order for sight to perceive transparency, the opacity of a transparent body should be denser than the transparency of the air mediating between that body and the eye. If the body is perfectly transparent, sight will neither perceive nor sense its transparency but will only perceive what lies behind it. Sight does not sense the transparency of a body, unless it senses that the light and the color it perceives through the transparent body is a light and color that exist behind that body and is not a property of the body itself.

## Opacity

Opacity is perceived from the absence of transparency. When sight perceives a body without sensing any transparency in it, it will judge it to be opaque.

## Shadow

Sight perceives shadow by comparison with neighboring lights or with lights of which it previously experienced. Shadow is the absence of some light, while the shadowed place is being illuminated with a light other than the absent light. If sight perceives a dimly lit place and perceives the light on neighboring bodies to be stronger than that dim light, it will perceive shadow in that place.

## Darkness

Darkness is perceived by inference from the lack of sensation of light. Darkness is the total absence of light. Therefore, if sight perceives a certain place without distinguishing any light in it, it will sense darkness.

## Beauty

For the beauty ${ }^{120}$ that is perceptible to the sense of sight, sight perceives it by perceiving each one of the visible properties, of which the manner of perception by sight has been shown. Each of these properties separately produces one of the kinds of beauty and they produce other kinds of beauty in conjunction with one another.

The types of beauty that sight perceives from visible objects are many:

- An object perceived to be beautiful because it possesses a single beautiful looking property.
- An object is judged to be beautiful because it has two or more properties each of which is beautiful independently of the others. This is an additive effect of several properties acting individually on the perceiver.
- The conjunction or combination "iqtiran" (إقتزان) of the properties one with another and not by the properties themselves.
- The harmony "ta'aluf" (تآلف) that may be produced by the composed "murakkaba" (مر كبه) properties.

120 Alhazen uses the Arabic word "al-busn", for the word beauty. In the Arabic language besides its meaning of conceptually pleasing forms, it refers also to ethical, practical and spiritual virtues.

Sight perceives each one of the properties in each one of the figures individually, it perceives them in composition and it perceives their conjunction and harmony. ${ }^{121}$ Therefore, sight perceives beauty in various ways, all of which reduce to perception of the particular properties.

These visible properties produce beauty separately, ${ }^{122}$ which will be evident from a brief contemplation. Light produces beauty and, therefore, the sun, the moon and the stars are found to be beautiful. ${ }^{123}$ There is no other cause in their figure to appear beautiful and appealing other than their radiant light. Therefore, light by itself produces beauty.
${ }^{121}$ In his book The Optics of Ibn Al-Haytham, II. London: The Warburg Institute, 1989. pp. 97-101, Sabra uses in his commentary the word "conjunction" to translate the Arabic word "Igtiran", which means a kind of combination endowed with harmony or proportionality and in general, the co-existence in an object of a number of properties or features. The Latin translation rendered the word "Iqtiran" with coniugatio and coniunctio. Alhazen uses the three cognate terms: " ta'lif; ta'aluf and $t^{\prime} t i l a f "$ ", which derive from the same root "lf" and have the general sense of union, combination, composition, concord, harmony. From the ninth century, "ta'lif and talifi " had become the chosen equivalents of the Greek harmonia and barmonikos, respectively; and by Alhazen's time, the passive "mu'allaf" and the verbal noun " ta'lif" were in current scientific use in expressions like "alnisba almu'allafa", harmonic ratio and "ilm ta'lif alalban ", the science of the composition of melodies. From the use of the above three terms, "ta'aluf and i'tilaf" is rendered as harmony, while "ta'lif" is rendered as composition.
${ }_{122}$ Alhazen means by producing beauty that it is the production of an effect, in the soul, that compliments figures that are considered beautiful.
${ }^{123}$ Alhazen uses the word " mustabsan " which means "to be found beautiful", "considered to be beautiful" or "felt to be beautiful" and does not use the word " basan " which means "beautiful".

Color also produces beauty. Every bright color-such as purple, vegetable-green, rose, reddish-yellowish and others that look like them-appeals to the beholder and pleases the eye. Similarly, dyed clothes, covers and utensils, as well as flowers, blossoms and meadows are felt to be beautiful. Therefore color by itself produces beauty.

Distance, may produce beauty by accident. Some apparently beautiful figures may have marks, wrinkles or pores that spoil them and agitate their beauty. But when moved farther from the eye, these miniature marring features disappear and the beauty of the figure stands out. Similarly, many beautiful looking figures possess certain refinements, such as minute designs or outlines or order which account for the beauty of the figure. Many of these features may not appear to the eye from moderate distances, but when brought closer to it they become visible and the beauty of the figure becomes manifest. Thus, increasing or diminishing the distance from the eye may cause beauty to appear and therefore, distance by itself produces beauty.

Position may produce beauty and many things that look beautiful do so only because of order and position. Beautiful writing ${ }^{124}$ also is regarded as such because of
${ }^{124}$ In the middle of the tenth century, the concept of proportion, expressed in the term "tanasub", consciously entered the art of calligraphy, or at least discussions of it. Ibn Muqla in 940 A.D., a famous calligrapher, is attributed the invention of a special kind of writing, the proportioned script, "alkbatt almansub". Later at the end of the tenth century, the Brethren of Purity devoted one of their anonymous Letters, " Rasa'il ", to "the most valued science" of ratio and proportion "for the use of the philosophically minded and not the accountants in the administrative offices of the
order alone. The beauty of writing is only due to the soundness of the shapes of letters and their composition among themselves, so that when the composition and order of the letters is not regular and proportionate, the writing will not be beautiful, even though the shapes of individual letters may be correct and refined. Similarly, many figures of visible objects are felt to be beautiful and appealing only because of the composition and order of their parts among themselves.

Solidity produces beauty and thus, the full grown bodies of individual human beings and of many animals are considered beautiful.

Shape produces beauty and thus, a crescent moon looks beautiful. Figures of individual human beings and many individual animals, trees and plants that are considered to be beautiful, are this way only because of their shapes and the shapes of the parts of their figures.

Size produces beauty and that is why the moon is more beautiful than any one of the stars and the larger stars are more beautiful than the smaller ones.

Separation produces beauty. Thus, separated stars are more beautiful than smudges and the Milky Way. That is also why separated lamps or candles are more beautiful than a continuously clustered fire. For this reason, too, blossoms and
government". They distinguished the various kinds of ratio, arithmetical, geometrical and harmonic. Moreover, they argued that geometry (reviving the Greek slogan) was the basis of every art and that no art could achieve the perfection of which it was capable without drawing upon the science of ratios. They also included a lengthy discussion of proportionality in writing that became one of the main sources on the subject.
flowers dispersed in meadows look more beautiful than when they are gathered and crowded together.

Continuity produces beauty. Thus, meadows with continuous and dense vegetation are more beautiful than those in which the vegetation is interrupted and discontinuous. For the meadows that look beautiful because of their colors, those which are continuous are more beautiful than the others. The additional beauty in these is produced by continuity alone.

Number produces beauty; portions of the sky with many stars are more beautiful than those with few stars. For this reason, lamps and candles look beautiful when many of them are gathered in one place.

Motion produces beauty; beauty is found in dancing, the motion of the dancer and the many gestures and movements of man in speech and in action.

Stillness produces beauty and therefore being dignified, bearing and steady appear beautiful.

Roughness produces beauty. Thus, many rough clothes and covers are found to be beautiful and for this reason, many of the goldsmith's artifacts become beautiful by having their surfaces roughened and textured.

Smoothness produces beauty and, therefore, it is considered beautiful in cloth and utensils.

Transparency produces beauty, therefore, transparent precious stones and transparent utensils are felt to be beautiful.

Opacity produces beauty, for colors, lights, shapes, outlines and all beautiful-looking features that are seen in the forms of visible objects are perceptible to sight only on account of opacity.

Shadow may cause the appearance of beauty, for many of the figures of visible objects have in them minute marks, wrinkles or pores which vitiate them and eclipse their beauty. When these objects are placed in the sun's light or in some other strong light, their marks and pores will be visible, thus, causing their beautiful features to disappear. But when placed in the shadow or in faint light their beautiful features become manifest as a result of the disappearance of those marring marks, wrinkles and pores. Rainbow colors that appear in birds feathers and in Peacock species become only visible in shadow or in subdued lights. When placed in sunlight or in other strong light those rainbow colors and beautiful features, which were visible in shadow and in subdued light, become invisible.

Darkness produces beauty, for the stars are visible only in darkness. Similarly, the beauty of lamps, candles and fires appears only in the darkness of night or in darkened places, but not in daylight or in strong light. Moreover, stars are more beautiful on dark nights than on moonlit nights.

Similarity produces beauty for paired organs of an animal are beautiful only when they are similar. Thus, if the eyes are of different shapes, as when one is round and the other is elongated, they will be extremely ugly. They will also be found ugly if one is black and the other is blue and likewise, if one is larger than the other. Also, if one cheek is sunken and the other is bulging, both will look extremely ugly. In the same way eyebrows are extremely ugly if one is thick and the other is narrow; they will also look ugly if one is long and the other is short. Thus, all paired organs of animals are beautiful only when they are similar. Also, scripts and letters are beautiful only when identical letters or parts are similar.

Dissimilarity produces beauty, for the shapes of animal organs are of dissimilar parts and without this dissimilarity, they would cease to be beautiful. A nose would look very ugly if it were of equal thickness from beginning to end; its beauty is only due to the difference between its ends and to its conical shape. Also, eyebrows are beautiful only when they are narrower at the ends than anywhere else. When all the organs of animals are examined, the beauty will be found to be due to the difference in the shapes of their parts. Correspondingly, scripts and letters will not look beautiful if their parts are of equal thickness. The extremities of letters and the ends of their deep curves are beautiful only when they are narrow and narrower than the remaining parts of the letters. A script would be very ugly if its letters were of equal thickness and of the same shape at their ends, middles, beginnings, junctions
and joints. Therefore, dissimilarity produces beauty in many of the figures of visible objects.

It is now clear that each of the particular visible properties produces beauty by itself in many situations. These properties do not produce beauty in all situations nor do any of them produce it in every figure in which they occur, but in some figures rather than others. For example, magnitude does not produce beauty in every body of a sizable magnitude. Nor does the same color produce beauty in every body in which this color exists. Similarly, not every shape produces beauty.

These properties also produce beauty by being joined with one another. Whereas beautiful script, whose letters have beautiful looking shapes and are in beautiful composition with one another, is considered perfect beauty in a script, a script which combines these two properties is more beautiful than the one which has one of them without the other. Perfect beauty in a script comes only from the conjunction of shape and position.

Correspondingly, bright and pure colors and scripts are more beautiful when regularly and uniformly ordered than when they have no regular order ${ }^{125}$. Also, beauty may appear in figures of individual human beings and animals on account of the combination of particular properties in them. Eyes of moderate size and almond shape are more beautiful than eyes having only one or the other of these properties.

[^13]Similarly, cheeks which are both flat and of delicate color are more beautiful than cheeks that are flat but of dull color or those that are pouching and of delicate color. Also, the roundness of the face with a delicate color is together more beautiful than one of them without the other. Similarly, a small mouth with thin and moderately-sized lips is more beautiful than one that is small with thick lips or one that is wide with thin lips.

The conjunction of the particular properties for visible objects is found to produce kinds of beauty which is not brought up by any of these properties alone. Most of the beauty perceived by the sense of sight consists in the combination of these properties with one another. Accordingly, the particular properties that we have mentioned originate beauty independently and in combination with one another.

Proportionality ${ }^{126}$ and harmony may establish beauty. Composed figures that consist of various organs and parts, these parts and organs may differ in respect of
${ }^{126}$ The idea of proportion as the most responsible cause for beauty is Greek and was transmitted and developed in the Islamic science up to and through Alhazen's time. In the ninth century the famous writer AlJahiz gives an early expression to this idea, using a different vocabulary from what we find in Alhazen. AlJahiz, in his Risalat al-Qiyan, gives an explanation of beauty as a quality of the human body, in terms of what he calls "tamam " fullness, " i'tidal " moderateness and "wazn" which means measure, balance, rhythm. AlJahiz also explained that he used "wazn" to mean the right shape and composition in a thing. He adds that "wazn" is a quality that can be found in vessels, furnishings, embroidered fabric and clothes.
their contiguity and separateness and thus, a number of particular properties may occur in each figure without these properties being all proportionate and harmonious.

- For not every shape is beautiful with every shape, nor is every size beautiful with every size, nor every position with every position.
- Moreover, not every shape is beautiful in every size nor every size is beautiful in every position, but rather each one of these particular properties is proportionate to some magnitudes and proportionless to others.

For example:

- An aquiline nose does not look beautiful together with sunken eyes, nor do large eyes look beautiful with an extremely large nose.
- Likewise, a bulging forehead with sunken eyes or a low forehead with prominent eyes are not considered to be beautiful.

Thus, every organ has a shape that makes its figure beautiful and yet every shape of any one of the organs agrees only with some shapes of the other organs to the exclusion of others. The beauty of the figure results from the combination of shapes that are proportionate to the organs in it.

The same thing applies to the size, position and order of composed figures:

- Large eyes look beautiful when their beautiful shape combines with an aquiline nose whose moderate size is proportionate to that of the eyes.
- Also, the almond shape of the eyes and the sweetness of that shape, even when the eyes are small, looks beautiful when combined in a face with a narrow nose of the right shape and size.
- Likewise, minute lips are beautiful in a small mouth, provided that the smallness of the mouth is proportionate to the thinness of the lips. They should not be extremely thin when the mouth is not very small, but rather the mouth should be moderately small and the lips thin in addition to being proportionate to the size of the mouth.
- Similarly, a face looks beautiful when it is proportionate in width to the size of its organs. That means the face should not be very wide while its organs are small, i.e. unproportionate to the size of the face as a whole.
- Considering the face is excessively large while its organs are small and unproportionate to its size, it does not look beautiful even if the organs are proportionate in magnitude to one another and their shapes are beautiful.
- Also, a face will be ugly looking when it is small and narrow, while its organs are large and unproportionate to its size.
- But the face will be considered beautiful when the organs are proportionate to each other and to the width of the face although each of the organs may not by itself be beautiful in shape or size.

Proportionality alone may produce beauty, provided that the organs are not in themselves ugly, nevertheless not perfect in their beauty. Thus, when a figure combines the beauty of the shapes of all of its parts, the beauty of their magnitudes, the beauty of their composition, the proportionality of its parts in regard to shape, size, position and all other properties required by proportionality and moreover, when the organs are proportionate to the shape and size of the face as a whole-that is perfect beauty. A figure that has some of these properties to the exclusion of others will be believed beautiful in accordance with what it has of the beautiful properties.

Furthermore, writing in calligraphy is not considered to be beautiful unless its letters are proportionate in consideration of their shapes, magnitudes, positions and order. The same is also legitimate of all visible objects which are combined of various parts.

Therefore, in all kinds of visible objects that have beautiful figures, proportionality will be found to produce in them a different beauty other than that produced by any one of the particular properties by itself and other than that made by the conjunction of the particular properties existing together in the visible object. When the beautiful effects produced by the conjunction of particular properties are examined, the beauty due to that conjunction will be found to be only the result of the proportionality and harmony gained between those combined properties. This is because beauty does not come about whenever these two or more particular
properties come together, but only in some figures rather than others, owing it to proportionality that composes harmony to the two or more properties combined in the figure. Therefore, beauty is a product of the particular properties, but its completion and perfection is only due to the proportionality and harmony that may transpire between the particular properties.

It is clear now that beauty of figures perceived by the sense of sight is only due to the visible particular properties, to their conjunction or to their proportionality. Sight perceives these properties either separately, or in conjunction and it perceives the figures composed out of them. When sight perceives a visible object in which there exists one of the particular properties that independently produce beauty, sight contemplates that property by itself. The figure of that property will present itself to the sentient, therefore, the power of judgment will perceive the beauty of the object in possession of this property. When sight perceives an object whose beauty consists in the conjunction of the properties and in their proportionality, it contemplates the object, thus, distinguishing the properties that produce beauty by being conjoined or by being proportionate to one another. This perception also occurs in the sentient and the power of judgment compares those properties with one another. It will then perceive the beauty of the object that consists in the conjunction of the harmoniously combined properties in it. Sight,
therefore, perceives the beauty of visible objects by relating those properties to one another in the manner that have been illustrated in detail.

## Ugliness

Ugliness is the figure that lacks the existence of all properties that are considered beautiful. It has been mentioned that the particular properties produce beauty, but not in every situation nor in every figure, but in some figures rather than others. Also, proportionality does not exist in all figures but in some rather than others. Therefore, beauty will be lacking from figures in which no particular property produces beauty either independently or in conjunction and in which no proportionality exists among the parts. Thus ugliness in a figure is the absence of beauty from it. There may exist in the same figure both beautiful and ugly properties and in this case, sight will perceive their respective beauty and ugliness once it has distinguished and contemplated the properties in the figure.

## Similarity

Similarity is the identicalness of two figures or properties with regard to the factor in which they resemble one another. Sight perceives figures and their properties the way they are. Thus when it perceives at the same time two similar figures or properties:

1. Sight will observe their similarity from ts perception of each one of the figures or properties.
2. And from comparing each of the figures or properties with the other.
3. Then from the perception of their identity with regard to the factor in which they resemble one another.

## Dissimilarity

The sense of sight perceives dissimilarity from its perception of each one of the figures or properties alone, from comparing them with one another and from the sentient's sensing of the lack of identicalness between them.

## Conclusion

Alhazen divides perception of the visible properties into two main groups: brightness, that is reflective and refractive properties and inertia, that is the visual potentiality of dynamic perception of objects. Then he specifies twenty-two properties for perception: Light, Color, Distance, Position, Solidity, Shape, Size, Separation, Continuity, Number, Motion, Stillness, Roughness, Smoothness, Transparency, Opacity, Shadow, Darkness, Beauty, Ugliness, Similarity and Dissimilarity. According to Alhazen, all other visible properties fall under one or more of these categories, such as order, which falls under position; or straightness,
curvature and convexity which all fall under shape; and fluidity that belongs to motion.


Figure 21: Analysis of Albazen's visible properties which induce the perception of beauty.

Alhazen's main concern is explaining the perception of each visible property with examples and experiments as a continuation of his research theme in Optica to accomplish an appropriate theory of vision. To illustrate how these properties are
related and grouped in such a matter that they can generate the perception of beauty or ugliness, I attempt, through the diagram of figure 21, to analyze the relationship between Alhazen's visible properties, which induce the perception of beauty, as divided into the groups of brightness and inertia.

There are two properties and two pairs of opposite properties that branches out of the brightness category. They are:

- Light/Darkness. Light is the essential common factor in defining the brightness category. In visual perception, it is the core property. Without it, the eye can observe neither the reflective/refractive properties nor the properties that define the potentiality of dynamic perception of objects. Light is the physical cause of what we see. As for darkness, it is the lack of sensation of light.
- Color, as a quiddity, is perceived by sensation, then by recognition. The relation between light and color in Alhazen's interpretation of their perception is inseparable as he expresses it in his text.

Shadow is the absence of some lights, while the shadowed place is being illuminated with a light other than the absent light.
. Transparency/Opacity, is the ability for distinction of what lies behind the visible object.

Properties that fall under the visual inertia can be divided into three subcategories: magnitude, composition and time.

- There are five properties that fall under magnitude. They are:
- Solidity, which is the extension of a body in the three dimensions.
- Shape, which is the two dimensional representation of the object's circumference or part of the object's surface. Also, shape can be a three dimensional representation, that is the bodily shape of the object or part of it.
- Size, which depends on the angle of vision and the object's distance.
- Roughness/Smoothness, a pair of opposite properties, that are dealing with texture. Lights and shadows play an important role in the definition and perception of this pair of properties.
- There are five properties that can fall under the composition subcategory, of which one is a property and two are pairs of opposites. These are: number, separation/continuity and similarity/dissimilarity.
- There are two single properties and one pair of opposites that fall under time. They are:
- Distance, which is the perception of a visible object in its own place. This includes the perception of the object's remoteness, its direction and the magnitude of the object's distance.
- Position, which is three types: The relativeness of the object to the eye, the orientation of the object's surface to the eye and the arrangement/order of the object's surface to one another.
- Motion/Stillness, which is the relative position and orientation of objects in relation to the object itself, or to the eye or to other objects.

The absorption of classical knowledge and scholarship into the learning of the Arabs and Muslim scholars through translations in the eighth and ninth century was one of the inspirational reasons that led to the emergence of many Islamic thinkers and philosophers. AlHasan ibn AlHaytham (965-1041 A.D.), known as Alhazen in Latin, accepted the scientific knowledge of his predecessors after its experimental verification and with the help of his scientific method, he himself made a number of remarkable discoveries. In this environment, the emergence of Alhazen's philosophy had a remarkable influence in defining the perception of Islamic science through an ethical framework. He proved in his main work, Optica, that objects were seen by means of the light rays emanating from them. Before him, through classical knowledge, it was believed that rays from the eye impinged on visible objects and thus, caused them to be sighted. Alhazen's major work, Optica, formed the groundwork of education in the field of optics, judging from the fact that textbooks on optics throughout the seventeenth century were essentially summaries and extractions of Optica, as it was translated much earlier into the Latin language.

The transformation of classical knowledge should be understood through apprehending the idea of origins of measure and origins of geometry. The concept of measure that formed the base of the science of mathematics, along with the science of geometry, were the essentials in devising Greek philosophy. By its turn, Greek philosophy was the domain of the studies and the comments of Alhazen that led to
the production of an enormous body of Arabic thought. Afterward, transformation took its way back again to Europe through the spread of Arabic scientific texts and their Latin translations.

Alhazen's concepts of beauty established a noticeable exception to a tendency that preserved itself in the philosophic literature of the Islamic world. It also had an impact on European readers of Optica, one of whom was the Renaissance sculptor Lorenzo Ghiberti (1378-1455), who copied parts of Alhazen's paragraphs in his Commentari from a fourteenth century Italian translation of Optica.

In the second book of Optica, Alhazen applied the term "alma-ani almubsira" [visible properties] for the characteristics of objects that can be seen and comprehended. Alhazen started by categorizing how the sense of sight perceives these characteristics and divided this process into three different methods:

1. Pure sensation,
2. Recognition and
3. Judgment or inference that goes beyond recognition.

Although, he says it is difficult to enumerate these characteristics, nevertheless he partitioned them into twenty-two properties and discussed the methods of their perception. These properties are either simple or associated with two to three or more varieties and they are: Light, Color, Distance, Position, Solidity, Shape, Size, Separation, Continuity, Number, Motion, Stillness, Roughness, Smoothness,

Transparency, Opacity, Shadow, Darkness, Beauty, Ugliness, Similarity and Dissimilarity.

In his discussion on the perception of beauty, Alhazen prefers to use the word "mustabsan" (مستحسن) which means "to be found beautiful", "considered to be beautiful" or "felt to be beautiful" and he does not use the word " hasan " (حَسَن") which means "beautiful", to emphasize the subjective aspect of the aesthetic judgment. He distinguishes four causes of visual beauty:

1. A single visible property.
2. The additive effect of several visible properties actung individually on the perceiver.
3. The conjunction or combination of a number of visible properties with one another.
4. The harmony that may be generated by the composed visible properties.

Alhazen's theory of beauty is primarily a theory of how we come to qualify an object as beautiful. The Arabic word "al-busn" (الـدّنْ )- like the Greek "to kalon" for the word beauty- refers in the Arabic language (besides meaning perceptually pleasing forms), to ethical, moral, practical and spiritual virtues. Alhazen's whole discussion about beauty is conducted in purely aesthetic terms as a part of a theory of visual perception and is phenomenal for its consistent approach.

Alhazen's theory on aesthetics demonstrates the shift from a theory that is based on tangible geometric criteria to one that is based on an usual approach. In order to cope with a system of criteria for qualifying an object to be beautiful, he presents us with all the visible properties that any visual object may possess.

This search of aesthetics is an accomplishment in defining and understanding what we see and why we see. Although Alhazen's theory was formulated almost ten centuries ago, it is still very helpful and valid in the present time in shedding some new perspectives on developing our current aesthetic analyses in architecture. This will take place by perceiving the visual properties and making them explicit, by extracting foundational principles and by showing structural relations at work, in order to sharpen instinctive intuition shore it up and make its elements communicable.

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[^0]:    1 Erwin Panofsky, " The history of the theory of human proportions as a reflection of the history of styles " Meaning in the visual arts. Chicago: The University of Chicago Press, 1982. pp. 55-107.
    $2 \quad$ Ibid., p. 89.
    ${ }^{3}$ He opposes the theory of Euclid and Ptolemy that the eye sends out visual rays to the object of vision.

[^1]:    $4 \quad$ Alberti (1404-1472), who dedicated his treatise on painting "Della pittura" to Ghiberti and four other artists, was one of the attendants of Lorenzo Ghiberti's (1378-1455) workshop, where artists from all over the city of Florence came to meet and discuss their daily activities.

[^2]:    9 Henry Corbin, History of Islamic Pbilosophy. Translated by Liadain Sherrard. London: Kegan Paul International, 1993. pp. 1-4.

    10 The same motives led to the establishment of ElAzhar (c. 800) in Cairo, known to be the first university in the world and other institutions, such as the celebrated House of Wisdom (c. 815) of Baghdad.

[^3]:    18 S. Pines, "Ibn al-Haytham's Critique of Ptolemy". Actes $d u$ Xe Congres internationale d'bistoire des sciences, Paris, 1964. I., p. 574.

[^4]:    ${ }^{31}$ Alhazen had said, "If I were in Egypt, I would have done a project that brings benefit from the Nile in its both cases of increasing or decreasing. As I have heard that the Nile rolls down from a higher level in Upper Egypt".

[^5]:    ${ }^{45} \quad$ Ludwig Wittgenstein, Pbilosophical investigations. Oxford: Basil Blackwell, 1974. p. 88.
    46 C. A. Doxiadis, Arcbitectural Space in Ancient Greece. Translated and edited by Jaqueine Tyrwhitt. Cambridge, Massachusetts: Massachusetts Institute of Technology Press, 1985. p. 16.

[^6]:    ${ }^{57}$ Joseph Campbell, The inner reaches of outer space. New York: Harper \& Row Publishers, 1988. p. 83.

[^7]:    ${ }^{68} \quad$ Marcus Pollio Vitruvius, De Architectura, 27 b.c. Translated by Frank Granger. Cambridge, Massachusetts: Harvard University Press, 1985. pp. 159 \& 161.

[^8]:    ${ }^{34}$ The definitive work on the Rhind Mathematical Papyrus was written (complete with translation and comment) by T. Eric Peet in 1923 and was published in Liverpool by the University Press.
    ${ }^{85}$ Paul Jacques Grillo, Form Function \& Design. New York: Dover Publication, Inc., 1960. p. 159 .

[^9]:    90
    Quoted from: Tons Brunes, The secrets of ancient geometry and its use. volume I. Rhodes, Copenhagen, 1967. p. 235.

[^10]:    107 Alhazen divided Recognition into two categories: recognttion of individual objects and recognition of a species.

[^11]:    114 Alhazen devoted a whole chapter entitled "On distinguishing the lines of the ray" where he discussed these lines. These are the radial lines coming out of visible objects and meet at the center of the eye through which sight perceives any visible object.

[^12]:    116
    Euclid's Optics, Definition IV: "Magnitudes seen through larger angles appear larger; those

[^13]:    125 "Tartib muntazim" is translated as regular order. It may also be rhythmic order, which is very close to the Greek eurbythmia referred by Vitruvius in his De architectura.

