Normative Decision Theory

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1. Introduction

Decision theory is the interdisciplinary study of choice. Given two or more incompatible options—say, between finishing off a pile of marking or trying a new karaoke place with colleagues—how does one choose between them? Much of the work done within decision theory concerns descriptive questions. For example: what patterns exist in our decision-making behaviour, what are the psychological mechanisms behind those patterns, and how might our choices be most accurately predicted? Most of the research done by philosophers in decision theory, however, concern normative questions—essentially, how *should* we make our choices?

This review will briefly introduce some of the major debates within normative decision theory over the past decade or so. I'll stay focused on topics that are directly concerned with how we ought to make decisions, where the 'ought' in question is subjective and pragmatic in nature. As such, I won't discuss recent applications of decision-theoretic ideas to epistemic or moral issues. Despite this, the amount of new philosophical work on decision theory is vast and highly varied, so the reader should not assume that the review is complete in all respects.[1]

There are four main sections. After providing some background in §2, in §3 I will discuss the ongoing debate between causalist and evidentialist versions of expected utility theory. In §4 I'll discuss an orthogonal debate regarding attitudes towards risk. And finally, in §5 I'll discuss a number of issues concerning the need to relax the idealising assumptions standardly made in decision theory.

2. Background

Normative decision theory primarily concerns how an agent ought to choose when faced with some *decision problem*. We can think of a decision problem as consisting in a set of *acts*, each within the agent's power to choose. These acts are mutually exclusive and jointly exhaustive, and each one might have a range of different *outcomes* depending on which *state of the world* is actual.

According to orthodoxy—that is, expected utility theory (EUT)—the decision-making agent will assign to each outcome a subjective value (or *utility*) that reflects the strength of her preference for that outcome obtaining. Fur-

[1] Noteworthy omissions include decision theory and transformative experience (e.g., Paul 2014), expectation gaps and the Pasadena game (e.g., Hájek 2014), and a variety of developments on the foundations of decision theory (e.g., Easwaran 2014). These topics could not be given their due here without detracting substantially from the issues already discussed.

thermore, she will usually be uncertain as to which state of the world is actual. The agent wants to attain the best outcome in any decision problem she might happen to find herself in. However, because she's uncertain about the actual state of the world, she will likewise be uncertain about which of the available acts will in fact maximise utility. The best she can do is maximise *expected utility*.

Causal decision theory (CDT) and evidential decision theory (EDT) are two different ways of precisifying this basic idea—they correspond to two ways of defining the 'expected' in 'expected utility theory'. But before we can discuss these two theories (and their alternatives) in more detail, I'll need to introduce some formalities.

From here on, I'll use 'α' to refer to the decision-making agent, and I'll say:

> $P \succ Q$ iff α *prefers* P to Q
> $P \sim Q$ iff α is *indifferent* between P and Q
> $P \succcurlyeq Q$ iff $P \succ Q$ or $P \sim Q$

Preference should here be understood as a kind of comparative conative propositional attitude—essentially, *wants more* or *desires more strongly*. As an imperfect heuristic, you might read '$P \succ Q$' as saying that α would be happier to learn that P than she would be to learn that Q.

Next, the acts. It is common to describe acts as things an agent might *do* under some intentional description. However, most philosophers today follow Jeffrey (1983) in treating acts as *propositions*—usually, those about what the agent does. I'll use these two ways of talking about acts interchangeably. We will let $A_1, A_2, \ldots, A_n$ designate the available acts; $O_1, O_2, \ldots, O_n$ their possible outcomes; and $S_1, S_2, \ldots, S_n$ the relevant states of the world. To simplify, I'll assume throughout that there are only finitely many acts, states, and outcomes.

We'll assume that the outcomes are maximally specific with respect to what α cares about: for each outcome O and any doxastically possible P entailing O, $O \sim P$. And we'll let the states be what Lewis (1981) calls *dependency hypotheses*: conjunctions of counterfactual conditionals that specify, for each act, the outcome that would result were that act chosen. For example,

$$S_1 = (A_1 \mathbin{\Box\!\!\rightarrow} O_1) \& (A_2 \mathbin{\Box\!\!\rightarrow} O_2) \& \ldots \& (A_n \mathbin{\Box\!\!\rightarrow} O_n)$$

Consequently, every doxastically possible act-state conjunction $A \& S$ determines a specific outcome O, where $(A \& S) \sim O$. We can represent the relationship between acts, states and outcomes using a *decision matrix*, like so:

	S_1	S_2	…	S_n
A_1	O_1	O_3	…	$O_{n'}$
A_2	O_2	O_4	…	$O_{m'}$
…	…	…	…	…
A_n	O_n	O_m	…	O_k

We'll let u be α's *utility function*; this assigns real numbers to propositions so as to represent α's preferences over them. This implies (amongst other things) that $P \succcurlyeq Q$ iff $u(P) \geq u(Q)$; hence $u(A\&S) = u(O)$ whenever $(A\&S)\sim O$. And finally, we let c be α's *credence function*, representing her credences. We assume that c satisfies the usual axioms of probability.

Given all that, EDT can be understood as saying that an act A is permissible for α just in case, of all the acts available to her, A has maximal *evidentially expected utility*, which we can define as:

$$\mathcal{V}_{\text{EDT}}(A) = \sum_{S} c(S_i|A)u(A\&S_i)$$

So, α ought to choose the act (or one of the acts) which would be most likely to have better results, were she to conditionalise on having chosen that act. By contrast, CDT will say that an act A is permissible for α just in case A has maximal *causally expected utility*:

$$\mathcal{V}_{\text{CDT}}(A) = \sum_{S} c(S_i)u(A\&S_i)$$

So, α ought to choose the act (or one of the acts) which, by her current *unconditional* credences over the various dependency hypotheses, is most likely to have better results.

3. Causal versus evidential decision theory

In most cases, there will be no difference between $\mathcal{V}_{\text{EDT}}$ and $\mathcal{V}_{\text{CDT}}$. This is because in most decision problems the choice of act will be evidentially independent of which state is actual, so $c(S_i) = c(S_i|A)$. But it is possible to devise situations where the two theories come apart. Historically, most of the debate between EDT and CDT has centred on one case like this: *Newcomb's Problem* (see Nozick 1969). Over the past decade, however, attention has shifted somewhat to another kind of case where CDT and EDT appear to generate conflicting recommendations, and which seem to tell against CDT in particular.[2]

For instance, consider the following case, which was made especially prominent by Egan (2007):

> *Psychopath Button*
> α can choose to press or not press the "kill all psychopaths" button. She's very confident she's not a psychopath, and it would, she thinks, be better to live in a world with no psychopaths. On the other hand, α is also confident that only a psychopath would press the button, and she strongly prefers living in a world *with* psychopaths to dying. Should α press the button?

Many commentators report the intuition that α should not press the button, and that seems to be the more common response. (It is not the universal response; see, e.g., Ahmed 2012: 387.)

[2] *Newcomb's Problem* is still frequently discussed, of course. See, *inter alia*, (Price 2012), (Ahmed 2014a), (Hare and Hedden 2016), (Spencer and Wells forthcoming), (Wells forthcoming).

According to EDT, α should not press the button. Pressing would constitute strong evidence that she is a psychopath, and hence in a state where pressing leads to the worst outcome (her death); whereas not pressing in all states merely leads to living in a world with psychopaths. According to CDT—at least on the way I've framed it—α should press the button. She's confident she's not a psychopath, so by that measure pushing is most likely to lead to the best results. Given the (apparently) widespread anti-pushing intuition, the *Psychopath Button* is often touted as a counterexample to CDT.

A range of responses on behalf of CDT have been offered. Cantwell (2010) argues that the informal description of the decision problem on which the anti-pushing intuition is grounded is inconsistent with the description under which CDT recommends pushing the button. Bales (forthcoming) argues that if we think of α's acts as the intentions she might form, and combine that with a Bratmanian view of intentions, then CDT will recommend not pushing. Ahmed (2012, 2014a: 61-65) argues that the anti-pushing intuition is incompatible with CDT's preferred response to *Newcomb's Problem*, unless preferences can be intransitive or there are some (relatively simple and straightforward) decision problems in which there are *no* rationally permissible acts. And Joyce (2012) argues that CDT gives the correct recommendations for the case, for reasons that I'll describe in a moment.

Several philosophers have taken the *Psychopath Button* (and similar) to show that CDT is unsatisfactory, and have offered modified—or even wholly new—theories to replace it. Arntzenius (2008) suggests we use what he calls deliberational causal decision theory (DCDT), a modified version of CDT based ultimately on a 'No Regrets' principle: agents should not be able to foresee that they'll regret their decisions.[4] CDT seems to violates this principle in the *Psychopath Button*: upon choosing to push the button, and updating her beliefs on having made that choice, α should come to believe that not pushing would have been more likely to lead to better results. If she changes her mind, and decides not to push after all, *then* she'll come to believe that pushing would have been the better option. We might imagine the committed causal decision theorist flip-flopping between pushing and not pushing indefinitely, or until a choice is forced at random.

Borrowing an account of deliberation from Skyrms (1990), Arntzenius proposes to modify CDT by invoking *mixed decisions*. A mixed decision can be modelled as a probability distribution p over the space of acts—say, $p(\text{Push}) = 0.6$, $p(\neg\text{Push}) = 0.4$—where this might at first pass be taken to represent a decision to act 'randomly', with a probability $p(A)$ of performing A (but cf. Arntzenius 2008: 292, for discussion on the intended interpretation). What's key is that upon having 'chosen' the mixed decision associated with the distribution p, α's credence that she'll perform A should equal $p(A)$. With this as background, we're to imagine that α begins her deliberative process with cre-

[4] See (Wallace 2010) for a view very similar to Arntzenius', and (Gustafsson 2011) for an alternative take on the *Psychopath Button* also centred around a 'No Regrets' principle.

dences not only about whether she is a psychopath, but also about whether she will perform this or that act. She determines the expected utility of the acts available to her on this basis, but *does not yet* make a choice. Instead, when she calculates that $\mathcal{V}_{\text{CDT}}(\text{Push}) > \mathcal{V}_{\text{CDT}}(\neg\text{Push})$, she merely *raises her credence* that she'll push. This provides evidence that she's a psychopath, so she updates her credences and performs the expected utility calculations anew—perhaps this time finding that $\mathcal{V}_{\text{CDT}}(\text{Push}) < \mathcal{V}_{\text{CDT}}(\neg\text{Push})$. This process is repeated, raising and lowering $c(\text{Push})$ and $c(\neg\text{Push})$, until finally an equilibrium is reached—a stable point at which further expected utility calculations no longer shift her credences regarding the available acts in either direction. The equilibrium corresponds to the mixed decision that α should 'choose'.

Arntzenius' proposal has met with some resistance. Plommer (2016) argues that it requires us to calculate expected utilities in a subtly inappropriate way. Ahmed (2014b) suggests a variation on Gibbard and Harper's (1978) *Death in Damascus* case that he argues DCDT gets wrong. (See also Ahmed 2014a: 69-73.) And Joyce (2012) argues that Arntzenius' 'No Regrets' principle is acceptable only to the extent that it's already implied by CDT. With that said, Joyce's response to the *Psychopath Button* is at a glance very close to Arntzenius' own. According to Joyce, CDT properly characterised should include the constraint that α is only to make a choice using her assessment of the expected utilities *once all relevant information is in*. Since introspective evidence about one's own expected utilities counts as relevant information, Joyce argues that CDT already forces a deliberational process much like the one Arntzenius describes, and ultimately prescribes being *indifferent* between pushing and not pushing.

A wholly distinct response to the *Psychopath Button* is Wedgwood's (2013) benchmark theory (BT). Like CDT, BT is designed to be sensitive especially to the counterfactual consequences of the available acts, but unlike CDT (and like EDT), it's also designed to be sensitive to the evidence that those acts provide about the causal structure of the world. The foundational idea is that the merits of an act in a state should be evaluated relative only to how well other acts do *at that state*, not how well they might have done in other states. Thus, we define for each state S_i a 'benchmark' value, b_i, and define the comparative utility of an action's outcome at a state relative to that benchmark:

$$cu(A,S_i) \;=\; u(A\&S_i) - b_i.$$

According to BT, α should choose the act with maximal expected *comparative* utility, with the expectations being determined by her credences for each state conditional on the act being chosen:

$$\mathcal{V}_{\text{BT}}(A) \;=\; \textstyle\sum_S \; c(S_i|A)cu(A,S_i)$$

Wedgwood argues that BT gives the intuitively correct results for the *Psychopath Button* case. For criticisms, see (Briggs 2010: 15-17) and (Bassett 2015).

One can see BT as a hybridisation of CDT and EDT, intended to accommodate intuitions that in some hypothetical cases apparently tell in favour of CDT, while in other cases in favour of EDT. Alternative approaches to accommodating these mixed intuitions include MacAskill's (2016) meta-decision theory, which builds uncertainty about the correct theory of decision-making into the decision rule itself; and Bales' (2018) decision-theoretic pluralism, according to which the concept of *permissible choice* admits of indeterminacy, with EDT and CDT corresponding to distinct precisifications thereof. There is also what could be called the 'no theory' theory—for instance, Briggs (2010) applies Arrow's (1950) classic impossibility theorem for preference aggregation to argue that no single decision rule can adequately accommodate all the intuitive data.

Beyond the *Psychopath Button*, still more counterexamples to CDT have been raised in recent years. Hare and Hedden (2016: 615ff) put forward an enhanced *Newcomb*-like problem in which, they argue, CDT (and DCDT) will lead self-aware decision-makers to choose in a clearly self-destructive manner. And Ahmed (2013a, 2013b, 2014a) has suggested a variety of cases aimed at taking down CDT. To take just a single example, consider:

> *Betting on the Past*
> α places high confidence in a deterministic system of laws *L*, and must choose between two bets. The first is such that she'll win \$10 if *P*, lose \$1 otherwise. The second is such that she'll win \$1 if *P*; lose \$10 otherwise. *P* is the proposition that at some point in the past the world was thus-and-so, where *P*&*L* entails α will take the second bet.

Ahmed argues that CDT recommends taking the first bet, since that option dominates and the choice has no way of influencing the causal structure of the situation. On the other hand, Ahmed argues, α should take the second bet, which is what EDT advises.

4. Attitudes towards risk

Orthogonal to the debates between CDT and EDT (and DCDT and BT and…) is another debate concerning the appropriate way to incorporate considerations of *risk* into normative decision theory. Going at least as far back as Allais (1953), we have known that there are decision problems where ordinary agents seem to have preferences that conflict with EUT in general, regardless of whether that theory is cashed out in causalist or evidentialist terms. What's more, the preferences in question don't seem obviously *irrational*.

For example, imagine that α is ⅓ confident that *S*, ⅔ confident that ¬*S*, and she faces a choice between two options. On the one hand (A_1), she might take a ticket in a lottery that pays out either \$100 or \$1, depending on whether *S* or ¬*S* respectively. On the other hand (A_2), she might take \$34 unconditionally:

	S	$\neg S$
A_1	\$100	\$1
A_2	\$34	\$34

If we assume that $u(\$x) = x$, then EUT implies that α ought to be indifferent between A_1 and A_2:

$$\begin{aligned}\mathcal{V}_{\text{EUT}}(A_1) &= c(S)u(\$100) + c(\neg S)u(\$1)\\ &= ⅓(100) + ⅔(1) = 34\end{aligned}$$

$$\begin{aligned}\mathcal{V}_{\text{EUT}}(A_2) &= c(S)u(\$34) + c(\neg S)u(\$34)\\ &= ⅓(34) + ⅔(34) = 34\end{aligned}$$

But it's not at all hard to imagine α strictly preferring A_2. Given A_1, she'd have a reasonable shot at the higher payout, but that option also has a significantly lower *minimum* payout. By contrast, there's no risk of being left with just \$1 if α takes A_2—it's a *sure thing*.

Proponents of EUT have responded to this kind of example in one of two ways. On the one hand, many have argued that since EUT sets the correct standards of rational decision making, any preferences other than indifference here must be irrational. On the other hand, some proponents of EUT have argued that some relevant aspect of the decision problem might have been mischaracterised, and that a preference for A_2 need not be irrational after all. For example, we might need to redescribe the outcomes to better reflect how α perceives her situation, or at least we might want to check our assumptions about the utilities she assigns to those outcomes. By assuming that $u(\$x) = x$, we're saying that the utility α assigns to \$34 is situated one third of the way between the utilities she assigns to \$1 and \$100. This isn't required by EUT, which is consistent with preferring A_2 whenever $u(\$34) > ⅓(u(\$100) – u(\$1))$.

Buchak (2013, 2014) takes a different approach. Building on earlier theoretical work by Quiggin (1982) and Machina and Schmeidler (1992), her new risk-weighted expected utility theory (REU) permits *rational* sensitivity to risk in cases like these, with or without the assumption that $u(\$x) = x$. To see the difference between REU and EUT, it helps to first reformulate EUT somewhat. Assuming $u(\$x) = x$, consider again the expected utility of A_1:

$$\mathcal{V}_{\text{EUT}}(A_1) = c(S)u(\$100) + c(\neg S)u(\$1)$$

We can read this as saying that the *value of* A_1 is the value of a ⅓ chance at \$100, plus a ⅔ chance at \$1. But we can also think of it like this: if α choses A_1, then she's guaranteed to get *at least* \$1 regardless of what happens, and if S is true she'll get \$99 *more*. Say that \$1 is the *guaranteed minimum*, and \$99 is the *conditional bonus*; the utility of the latter is equal to the utility of the better outcome minus the utility of the guaranteed minimum. Then, EUT says that value of A_1 for α is equal to:

(i) her utility for the guaranteed minimum, *plus*
(ii) her utility for the conditional bonus, *weighted by* her credence in the relevant condition obtaining.

Hence, we can rewrite the formula above as follows:

$$\begin{aligned}\mathcal{V}_{\mathrm{EUT}}(A_1) &= u(\$1) + c(S)(u(\$100) - u(\$1)) \\ &= 1 + ⅓(100 - 1) = 34\end{aligned}$$

To put that more generally, EUT dictates that rational agents will always weight conditional bonuses by their credences towards the conditions in question. Buchak's REU denies exactly this: decision-makers' credences *matter* according to REU, but they're not the whole story. We also need to consider *attitudes towards risk*.

Formally, we model α's risk-attitudes using a function r, which transforms α's credences before they interact with her utilities to determine the overall value of the act. So, where r is a (strictly increasing and continuous) function from [0,1] to [0,1], with $r(0) = 0$ and $r(1) = 1$, we define the *risk-weighted expected utility* of A_1 as:

$$\mathcal{V}_{\mathrm{REU}}(A_1) = u(\$1) + r(c(S))(u(\$100) - u(\$1))$$

Thus, the utility of the conditional bonus ($u(\$100) - u(\$1)$) is weighted *not* by $c(S)$, but by $r(c(S))$. In the special case where $r(c(S)) = c(S)$, then REU and EUT will amount to precisely the same thing and we say that α is *risk-neutral*. But if α is *risk-averse*, then $r(c(S)) < c(S)$, and she will end up assigning *less* value to A_1 than would be allowed under EUT.

It will come as no surprise that numerous objections to REU have already been put forward in the literature.[6] Thoma and Weisberg (2017) attempt to undermine the support for REU. They argue that once all the relevant details of the agents' decision problems have been spelled out *in full*, REU fails to recapture the intuitively permissible preferences of apparently risk-averse agents that Buchak uses to motivate her theory. Thoma (2019) argues that REU and EUT amount to (at least approximately) the same thing: for an agent who sees any small-stakes decision problem she's presently faced with as just one in a long series of similar choices she'll need to face over the course of her life, REU will (under reasonable assumptions) recommend acting as if one is risk-neutral. And Briggs (2015) and Joyce (2017) both argue that REU permits irrational decisions in cases of sequential choice, leading agents to accept dominated strategies or leaving them susceptible to Dutch Books.

Pettigrew (2015) demonstrates that preferences for A_2 can be explained within an EUT framework—indeed, that *any* of the preferences permitted by REU can be so explained—if we're allowed to redescribe the outcomes to make them more fine-grained. In particular, we're to suppose that one and the same coarse-grained outcome (e.g., \$1, or \$34) might have different utilities contingent on whether it was brought about by this or that act. Agents' attitudes towards risky options can then be encoded in their utilities towards *act-outcome pairs*, while everything else about the EUT decision rule is left the same. Thus, we have a range of 'risk-averse' preferences over acts, which might be rationalised by assuming *either* (i) that the decision-maker is follow-

[6] For responses to some of these objections, see (Buchak 2015, 2017).

ing the REU decision rule with the outcomes as originally described, or (ii) the she is following the EUT rule with the more fine-grained redescription of those outcomes.

Stefansson and Bradley (forthcoming) also adopt a version of the redescription strategy in order to make sense of risk-averse preferences. Adapting Jeffrey's (1983) axiomatic framework, they enrich the underlying space of propositions to include propositions about objective chance distributions over outcomes, and represent agents' risk-attitudes *via* their utilities regarding these propositions. To motivate their way of accommodating risk-aversion, Stefansson and Bradley argue that (i) unlike their own account, REU is unable to accommodate the seemingly rational preference patterns that ordinary agents tend to display in the *Ellsberg Paradox* (Ellsberg 1961), and (ii) the REU model misrepresents the psychology of risk-attitudes. Buchak (2013: 80-81) raises the former issue as a potential worry for REU as well, though it's set aside to be dealt with under future developments of the theory—specifically, those which might allow for 'imprecise' credences (see §5.2 below).

5. Deidealising decision theory

It's possible to view REU as one way of *deidealising* expected utility theory. That is—and, setting aside the redescription strategy—we *could* interpret the situation described in §4 as one in which EUT implicitly presupposes that rational agents are risk-neutral, whereas we might want our theory of good decision-making to incorporate a wider range of risk-attitudes.

Put in these terms, REU becomes one part in a much larger project to broaden the scope of normative decision theory by relaxing some of EUT's many idealising assumptions. For example, on the standard way of setting things up, we typically assume that α is *aware* of all the acts available to her, that she has *precise utilities* towards all relevant outcomes (represented by the real-valued function u), and that she has *precise* and *probabilistically coherent credences* towards all relevant states (represented by the probability function c). These are strong assumptions by any measure, and most theorists today think that at least some of them are too strong.

There are in fact two projects here, and philosophers have made contributions to both. On the one hand, you might think that the some of the idealisations built into EUT are too demanding for agents *like us*. Our rational capacities are bounded in a variety of ways, and so we need a theory of rational decision making that we mere human beings can actually live up to. This idea forms the basis of the *bounded rationality* project, which I will not focus on here. (But see Weirich 2015, Elliott 2017, and Bradley 2018, for recent work connected to this project.) On the other hand, you might think that some of these idealisations ask too much even of *ideally* rational beings. In the remainder, I will consider two important strands of this latter deidealisation project.

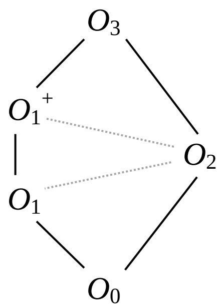
5.1 Incomplete preferences

Hare (2010) considers a case in which one has the option to preserve from destruction at most one of either:

> O_1: An item of significant historical value, such as the Fabergé egg
> O_2: An item of significant personal value, such as a wedding album

Even if she were ideally rational, it would not be unreasonable for α to lack any all-things-considered preference between O_1 and O_2. Moreover, α's lack of preferences might display *insensitivity to sweetening*. Let O_1^+ be just like O_1, but for the addition of a mild benefit—e.g., the Fabergé egg plus $10. If α were merely *indifferent* between O_1 and O_2, then since we can assume that $O_1^+ \succ O_1$, we'd also expect that $O_1^+ \succ O_2$. Yet this need not be the case: α might just as reasonably lack any all-things-considered preference between O_1^+ and O_2.

Overall, then, it seems that α's preferences might permissibly take the following kind of structure (where the solid lines represent asymmetric preference relations, and the dotted lines represent a symmetric *lack-of-preference* relation that *isn't* indifference):



In this case, α's preferences are *incomplete*. It is a trivial matter to show that an incomplete preference ranking cannot be faithfully represented by a real-valued function u in the sense that was characterised in §2: since the $\geq$-ordering over the real numbers is not itself incomplete, it cannot be ordinally equivalent to any incomplete $\succcurlyeq$-ordering over propositions.[7]

There has been some debate about whether the incompleteness here should be analysed as α's having *vague* preferences, or whether perhaps it highlights the need for a new kind of symmetrical preference relation that usually goes by the name 'parity' (cf. Rabinowicz 2009, Gustafsson and Espinoza 2010, and especially Chang 2014). But independent of the *source* and *nature* of the incompleteness, if incomplete preference rankings are rationally permissible then they should be incorporated somehow into our best theories of decision making.

Hare's own (weakly) preferred account of decision-making with incomplete preferences he calls *prospectism*. Let an *admissible completion* of α's

[7] Rabinowicz (2009) also shows that only slightly more complicated incomplete preference structures cannot be adequately represented by *interval-valued* functions.

preferences refer to any way of rendering her overall preference structure complete without altering any pre-existing preferences, while remaining consistent with basic requirements of coherence (e.g., transitivity). For instance, in the case above there are five admissible completions corresponding to where we might place O_2 in relation to O_1 and O_1^+:

1. $O_3 \succ O_2 \succ O_1^+ \succ O_1 \succ O_0$
2. $O_3 \succ O_2 \sim O_1^+ \succ O_1 \succ O_0$
3. $O_3 \succ O_1^+ \succ O_2 \succ O_1 \succ O_0$
4. $O_3 \succ O_1^+ \succ O_2 \sim O_1 \succ O_0$
5. $O_3 \succ O_1^+ \succ O_1 \succ O_2 \succ O_0$

Each will correspond to different utility functions (perhaps more than one). Prospectism then says that the choice of an act is *permissible,* in general, just in case it would be permissible according to EUT under some admissible extension of α's preferences.

Bales, Cohen and Handfield (2014) have objected to Hare's proposal by giving a case where it conflicts with the following dominance-like principle that they call *competitiveness*: An action *A* is permissible if, for every state, its consequences are *not worse* than the consequences of any alternative actions. Doody (forthcoming) discusses the competitiveness principle in some depth and ultimately finds it wanting, and suggests a weaker principle in its place—one that prospectism also happens to violate

Finally, Peterson (2015) has argued that if α chooses in accord with prospectism, then she might be subjected to a 'weak money pump'—i.e., a sequence of bets, all of which she is *permitted* to choose, but which in combination she knows in advance are guaranteed to lose her money. In response, Kaivanto (2017) has proposed a variation on prospectism that avoids Peterson's money pumps.

5.2 Imprecise credences

A related strand of the deidealisation project relates to the possibility of *imprecise credences*. Imagine, for example, that before you is an old pack of cards. You know that some cards are missing, but you have no idea about how many nor which ones. Now compare:

P = The global population in 2100 will be greater than 12 billion
Q = The next card drawn from this old deck will be a heart

If you're like most people, you won't think that *P* is exactly as probable as *Q*. But is *P* more, or less, probable than *Q*—and if so, by how much exactly? You should find this hard to answer. (I certainly do.) Moreover, the difficulty doesn't seem to be a mere lack of introspective transparency regarding one's own credences. The problem (at least arguably) goes deeper than that: it's not especially plausible that there must be some precise value *n* such that *P* is exactly *n* times more (or less) probable than *Q*.

Joyce (2010) forcefully argues that credal imprecision need not be limited to non-ideal agents like us, but in fact represents the appropriate rational response to evidence which is itself often imprecise and fragmented. Along slightly different lines, Williams (2014) argues that imprecise credences are an appropriate epistemic response to indeterminate subject-matters. Probability functions are not well-suited for representing credal imprecision; hence, we need to generalise our representation of credences.

There's a wide range of models for imprecise credences which have been proposed—see (Augustin *et al* 2014) for a recent review—but the one that philosophers typically prefer is the *credal sets* model. Rather than modelling α's credences using a single probability function *c*, on the credal sets model we instead use a non-empty *set* of probability functions, $\boldsymbol{C}$. Exactly how we're supposed to interpret $\boldsymbol{C}$ as a model of α's credences usually differs somewhat from person to person. Nevertheless, almost everyone agrees on at least the following two interpretive points:

1. If $c(P) = c(Q)$ for all c in $\boldsymbol{C}$, then α takes P to be as probable as Q
2. If $c(P) > c(Q)$ for all c in $\boldsymbol{C}$, then α takes P to be more probable than Q

Under certain conditions, a credal set $\boldsymbol{C}$ induces an interval-valued function which summarises the range of values towards a given proposition that the different functions in $\boldsymbol{C}$ might take.[8] In practice, philosophers tend to discuss imprecise credences in terms of these intervals, though there will be cases where this leaves out some of the information contained in $\boldsymbol{C}$. (See Joyce 2010 for examples.)

Most philosophical work on decision-making with imprecise credences has been framed in response Elga's (2010) *Two Bets* argument, which is aimed at showing that credences should always be precise. Imagine that α's imprecise credence for *P* falls within the range [0.1,0.8], and she knows she'll be offered the following two choices (in short sequence):

> *Choice 1*: Accept or reject bet B1: lose \$10 if *P*; win \$15 otherwise
> *Choice 2*: Accept or reject bet B2: win \$15 if *P*; lose \$10 otherwise

If α *accepts* both bets, she's guaranteed a net profit of \$5; Elga considers this reason enough to conclude that *rejecting* both is impermissible. And yet, he argues, no *plausible* decision rule for imprecise credences gets us this result.

In saying this, Elga considers a wide range of possible decision rules—far too many to consider here. But one obvious example is worth noting: the 'permissive' rule, which is analogous to prospectism. According to this rule, an act *A* is permissible just in case it would be permissible according to standard EUT under *any* of the probability functions in $\boldsymbol{C}$.[9] If we consider each of the two choices in isolation from one another, then EUT permits rejecting B1

[8] That is, let $C(P) = \{c(P) \mid c \in \boldsymbol{C}\}$; where $\boldsymbol{C}$ is convex, $C(P)$ will be an interval.

[9] See (Moss 2015a) for detailed discussion on this view, and (Moss 2015b) for a closely related alternative.

whenever $c(P) \geq 0.8$, and permits rejecting B2 whenever $c(P) \leq 0.4$. Consequently, Elga argues, the permissive rule permits rejecting both bets.

The most common strategy of response to Elga's argument has been to highlight a number of possible decision rules for imprecise credences which, in combination with a sophisticated approach to sequential decision making, rule out rejecting both bets. Versions of this response can be found in (Sahlin and Weirich 2014), (Chandler 2014), (Bradley and Steele 2014), and (Sud 2014). A *sophisticated* decision maker knows how she is liable to choose in future decision problems, and recognises that the choices she makes now can affect which problems she's faced with in the future. Thus, when deciding on B1, the sophisticated chooser will incorporate into her evaluation of the outcomes how her choice *now* will affect her *later* choice regarding B2.

Now consider this sophisticated approach to sequential decision-making in combination with the decision rule usually known as Γ-maximin, which says that an act A is permissible just in case its *minimal expected utility* (i.e., the lowest expected utility relative to any c in $\boldsymbol{C}$) is *maximal* (i.e., no less than the minimal expectations of any of A's alternatives). According to Sahlin and Weirich (2014), the sophisticated follower of Γ-maximin knows, while deciding on B1, that if she takes B1 she'll also end up also taking B2, since in the later choice she'll be effectively choosing between (i) having taken B1 only, versus (ii) having taken both B1 and B2, and the minimal expected utility of (ii)_is greater than that of (i). For similar reasons, if she rejects B1 then she knows she'll end up rejecting B2. These predictions need to be factored into her initial choice—in a rough sense, she's choosing from the start whether to accept or reject both bets. And relative to any probability function in $\boldsymbol{C}$, the minimal expectation of accepting both will be greater than the minimal expectation of rejecting both; hence, α should take B1 and B2.

In an erratum to his paper, Elga (2012) has agreed that sophisticated choice will help to save some decision rules for imprecise credences from his *Two Bets* argument—thought it does not save all (including the permissive rule). Mahtani (2018), however, has more recently argued that sophisticated choice will *not* help in general: the key assumption of the strategy is that α will be able to predict her later choices under different suppositions about what she chooses now. But, Mahtani argues, if α has imprecise credences then she will display 'unstable' betting behaviour when faced with B2, regardless of her choice regarding B1, rendering her future choices unpredictable.

Rinard (2015) takes a different line in response to Elga, and in the process advocates for a new 'supervaluationist' decision theory. According to Rinard, we can interpret *imprecise* credences as *indeterminate* credences, with the various probability functions in $\boldsymbol{C}$ seen as admissible *precisifications* of α's indeterminate credal state. Following the usual supervaluationists' line, we then say that an action is determinately permissible (or impermissible) just in case it's permissible (impermissible) relative to every admissible precisification. For each bet B1 and B2, it will be indeterminate whether it's permissible to reject that bet, but rejecting *both* bets will be determinately impermissible.

In a recent paper, Bradley (forthcoming) has objected to Rinard's proposed decision theory, along with two other positive proposals put forward in Sud (2014) and Moss (2015b) both aimed at dealing with Elga's *Two Bets* argument. The essence of Bradley's objection is that these decision theories are unable to adequately explain the (apparently) rational phenomenon of *ambiguity aversion*, as exemplified in the classic *Ellsberg paradox*. Although other decision rules have been suggested which do manage to adequately deal with the *Two Bets* and can also capture the Ellsberg preferences—such as sophisticated Γ-maximin—these suffer from distinctive problems of their own. So, if we take the Ellsberg preferences to be rationally permissible, the upshot is that as of today, we still lack an adequate decision theory for imprecise credences.

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