On the self-predicative universals of category theory

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Abstract

This paper shows how the universals of category theory in mathematics provide a model (in the Platonic Heaven of mathematics) for the self-predicative strand of Plato's Theory of Forms as well as for the idea of a "concrete universal" in Hegel and similar ideas of paradigmatic exemplars in ordinary thought.

The paper also shows how the always-self-predicative universals of category theory provide the "opposite bookend" to the never-self-predicative universals of iterative set theory and thus that the paradoxes arose from having one theory (e.g., Frege's Paradise) where universals could be either self-predicative or non-self-predicative (instead of being always one or always the other).

Keywords: universals, category theory, Plato's Theory of Forms, set theoretic antinomies

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1 Introduction: "Bad Platonic Metaphysics"

Consider the following example of "bad metaphysics."

> Given all the entities that have a certain property, there is one entity among them that exemplifies the property in a universal, paradigmatic, archetypical, ideal, essential, or canonical way. It is called the "self-predicative universal." There is a relationship of "participation" or "resemblance" so that all and only the entities that have the property (including itself) "participate in" or "resemble" that perfect example, the self-predicative universal.

To the modern ear, all this sounds like the worst sort of "bad Platonic metaphysics." Yet there is a mathematical theory developed within the last seventy years, category theory (MacLane [13]; Awodey [3]), that provides precisely that treatment of self-predicative universals within mathematics.

A simple example using sets will illustrate the points. Given two sets a and b, consider the property of sets: $F(x) \equiv x \subseteq a \;\&\; x \subseteq b$. In other words, the property is the property of being both a subset of a and a subset of b. In this example, the participation relation is the subset inclusion relation. There is a set, namely the intersection or meet of a and b, denoted $a \cap b$, that has the property (so it is a "concrete" instance of the property), and it is universal in the sense that any other set has the property if and only if it participates in (i.e., is included in) the universal example:

self-predication: $F(a \cap b)$, i.e., $a \cap b \subseteq a$ and $a \cap b \subseteq b$, and

universality: x participates in $a \cap b$ if and only if $F(x)$, i.e., $x \subseteq a \cap b$ if and only if $x \subseteq a$ and $x \subseteq b$.

This example of a self-predicative universal is quite simple, but all this "bad metaphysical talk" has highly developed and precise models in category theory.

- This interpretation of the universals of category theory as self-predicative universals[1] is the first point of this paper (written for a non-mathematical philosophical audience).
- In terms of the old theme of universals in philosophy, we show how the self-predicative universals of category theory provide a rigorous model (in the "Platonic Heaven" of mathematics) for the self-predicative strand in Plato's thought and the "concrete universal" [11, Entry at "Concrete Universal"] synthesis in Hegel's thought as well as for the common "Form" of thought that considers a paradigmatic [22], canonical, iconic, archetypical, or quintessential exemplar of some property.
- Moreover, this interpretation of category-theoretic universals gives a new perspective on the set-theoretic antinomies. We show that the always-self-predicative universals of category theory are the "opposite bookend" to the never-self-predicative sets of iterative set theory. This shows that the problem in the paradoxes was not self-predication *per se* but negated self-predication.

2 Criteria for a Theory of Universals

In Plato's Theory of Ideas or Forms ($\varepsilon\iota\delta\eta$), a property F has an entity associated with it, the universal u_F, which uniquely represents the property. An object x has the property F, i.e., $F(x)$, if and only if (iff) the object x participates in the universal u_F.

Let μ (from $\mu\varepsilon\theta\varepsilon\xi\iota\varsigma$ or *methexis*) represent the participation relation so

$$\text{"}x \;\mu\; u_F\text{" reads as "}x \text{ participates in } u_F\text{".}$$

Given a relation μ, an entity u_F is said to be a *universal* for the property F (with respect to μ) if it satisfies the following universality condition:

$$\text{for any } x,\; x \;\mu\; u_F \text{ if and only if } F(x).$$

A universal representing a property should be in some sense unique. Hence there should be an equivalence relation ($\approx$) so that universals satisfy a uniqueness condition:

$$\text{if } u_F \text{ and } u'_F \text{ are universals for the same } F, \text{ then } u_F \approx u'_F.$$

A mathematical theory is said to be a *theory of universals* if it contains a binary relation μ and an equivalence relation $\approx$ so that with certain properties F there are associated entities u_F satisfying the following conditions:

(I) Universality: for any x, $x \;\mu\; u_F$ iff $F(x)$, and

(II) Uniqueness: if u_F and u'_F are universals for the same F [i.e., satisfy (I) above], then $u_F \approx u'_F$.

[1] Or "concrete universals" in Ellerman [6] although all the entities are abstract mathematical entities.

A universal u_F is said to be *abstract* or *non-self-predicative* if it does not participate in itself, i.e., $\neg(u_F\ \mu\ u_F)$. A universal u_F is *self-predicative* or *concrete* if it participates in itself, i.e., $u_F\ \mu\ u_F$.

3 Set Theory as The Theory of Abstract Universals

There is a modern mathematical theory that readily qualifies as a theory of universals, namely set theory. In the naïve form of set theory ("Frege's Paradise"), the universal representing a property F is the set of all elements with the property:

$$u_F = \{x|F(x)\}.$$

The participation relation is the set membership relation usually represented by $\in$. The universality condition in (naïve) set theory is the (naïve) comprehension axiom: there is a set y such that for any x, $x \in y$ iff $F(x)$. Set theory also has an extensionality axiom, which states that two sets with the same members are identical:

$$\text{for all } x,\ (x \in y \text{ iff } x \in y') \text{ implies } y = y'.$$

Thus if y and y' both satisfy the comprehension axiom scheme for the same F then y and y' have the same members so $y = y'$. Hence in set theory, the uniqueness condition on universals is satisfied with the equivalence relation ($\approx$) as equality ($=$) between sets. Thus naïve set theory qualifies as a theory of universals.

The hope that naïve set theory would provide a general theory of universals proved to be unfounded. The naïve comprehension axiom lead to inconsistency for such properties as

$$F(x) \equiv \text{"}x \text{ is not a member of } x\text{"} \equiv x \notin x$$

If R is the universal for that property, i.e., R is the set of all sets which are not members of themselves, the naïve comprehension axiom yields a contradiction.

$$R \in R \text{ iff } R \notin R.$$

Russell's Paradox

The characteristic feature of Russell's Paradox and the other set theoretical paradoxes is the negated self-reference wherein the universal is allowed to qualify for the negated property represented by the universal, e.g., the Russell set R is allowed to be one of the x's in the universality relation: $x \in R$ iff $x \notin x$. The set-theoretic formulation of the paradox was not essential. Russell himself expounded the paradox in term of the property of predicates that they are not self-predicative, i.e., the "predicates which are not predicable of themselves" (Russell [21, p. 80]).

There are several ways to restrict the naïve comprehension axiom to defeat the set theoretical paradoxes, e.g., as in Russell's type theory, Zermelo-Fraenkel set theory, or von-Neumann-Bernays set theory. The various restrictions are based on an iterative concept of set (Boolos [4]) which forces a set y to be more "abstract", e.g., of higher type or rank, than the elements $x \in y$. As Russell himself put it:

> It will now be necessary to distinguish (1) terms, (2) classes, (3) classes of classes, and so on ad infinitum; we shall have to hold that no member of one set is a member of any other set, and that $x \in u$ requires that x should be of a set of a degree lower by one than the set to which u belongs. Thus $x \in x$ will become a meaningless proposition; and in this way the contradiction is avoided. (Russell [21, p. 527])

Thus the universals provided by the various set theories are "abstract" universals in the technical sense that they are relatively more abstract (i.e., of higher type or rank) than the objects having the property represented by the universal. Sets may not be members of themselves.[2]

With the modifications to avoid the paradoxes, a set theory still qualifies as a theory of universals. The membership relation is the participation relation so that for suitably restricted predicates, there exists a set satisfying the universality condition. Set equality serves as the equivalence relation in the uniqueness conditions. But set theory cannot qualify as a general theory of universals. The paradox-induced modifications turn the various set theories into theories of abstract (never-self-predicative) universals since they prohibit the self-membership of sets. That clears the ground for another theory of always-self-predicative universals.

4 Self-Predicative or Concrete Universals

Philosophy has long contemplated another type of universal, variously called a *self-predicative*, *self-participating*, or *concrete universal*. Indeed, it is a common Form of thought. The intuitive idea of a self-participating universal for a property is that it is an object that has the property and has it in such a universal sense that all other objects with the property resemble or participate in that paradigmatic, archetypal, canonical, iconic, ideal, essential, or quintessential exemplar. Such a universal u_F for a property F is self-predicative in the sense that it has the property itself, i.e., $F(u_F)$. It is universal in the intuitive sense that it represents F-ness is such a perfect and exemplary manner that any object resembles or participates in the universal u_F if and only if it has the property F.

> If anything else is beautiful besides Beauty itself, it is beautiful for no other reason than because it participates in that Beauty. (*Phaedo* 100, translation from [7, p. 35])

The intuitive notion of a concrete universal or paradigmatic instance occurs in ordinary thought as in the "all-American boy" or any case of a quintessential iconic example (e.g., Sophia Loren as "the" Italian women or Michelangelo's David or da Vinci's Mona Lisa as "the" exemplars for certain artistic categories), or when resemblance to an "defining" example becomes an adjective like "Lincolnesqe." In Greek-inspired Christian theology, there is the "Word made Flesh" (Miles [17]) together with *imitatio Christi* as the participation or resemblance relation to that concrete universal. The idea of the concrete universal is often associated with Hegel (Stern [23]) where it was one way to think about the synthesis between an abstract universal thesis and the antithesis of diverse particulars. One sensible application by Hegel was in the arts and literature (Wimsatt [28]) to explicate the old idea that great art uses a concrete instance to universally exemplify certain human conditions, e.g., Shakespeare's Romeo and Juliet as "the" romantic tragedy (Desmond [5]).

The notion of a self-predicative universal goes back to Plato's Theory of Forms (Vlastos [24], [25], [26]; Malcolm [16]). Plato's forms are often considered to be abstract or non-self-predicative universals quite distinct from and "above" the instances. In the words of one Plato scholar, "not even God can scratch Doghood behind the Ears" (Allen [1]). But Plato did give examples of self-participation or self-predication, e.g., that Justice is just [Protagoras 330] or that Beauty is beautiful. Moreover, Plato used expressions that indicated self-predication of universals.

> But Plato also used language which suggests not only that the Forms exist separately ($\chi\omega\rho\iota\sigma\tau\alpha$) from all the particulars, but also that each Form is a peculiarly accurate or good particular of its own kind, i.e., the standard particular of the kind in question or the model ($\pi\alpha\rho\alpha\delta\varepsilon\iota\gamma\mu\alpha$) to which other particulars approximate. (Kneale and Kneale [12, p. 19])

[2] Quine's system ML [20] allows "$V \in V$" for the universal class V, but no standard model of ML has ever been found where "$\in$" is interpreted as set membership (Hatcher [10, Chapter 7]).

But many scholars regard the notion of a Form as *paradeigma* or self-predicative universal as an error.

> For general characters are not characterized by themselves: humanity is not human. The mistake is encouraged by the fact that in Greek the same phrase may signify both the concrete and the abstract, e.g. $\lambda\varepsilon\upsilon\kappa o\nu$ (literally "the white") both "the white thing" and "whiteness", so that it is doubtful whether $\alpha\upsilon\tau o\ \tau o\ \lambda\varepsilon\upsilon\kappa o\nu$ (literally "the white itself") means "the superlatively white thing" or "whiteness in abstraction". (Kneale and Kneale [12, pp. 19-20])

Thus some Platonic language is ambivalent between interpreting a form as a paradigmatic exemplar ("the superlatively white thing") and an abstract universal ("whiteness in abstraction").

The literature on Plato has reached no resolution on the question of self-predication. Scholarship has left Plato on both sides of the fence; many universals are not self-predicative but some are. It is fitting that Plato should exhibit this ambivalence since the self-predication issue has only come to a head in the 20^{th} century with the set theoretical antinomies. Set theory had to be reconstructed as a theory of universals that were never-self-predicative.

The reconstruction of set theory as the theory of never-self-predicative universals cleared the ground for a separate theory of universals that are always-self-predicative. Such a theory of self-predicative universals would realize the self-predicative strand of Plato's Theory of Forms. The antinomies show that there cannot be one theory of universals (e.g., Frege's Paradise) that could be self-predicative or non-self-predicative because one could then consider the universal for all those universals that are non-self-predicative.

A theory of (always) self-predicative universals would have an appropriate participation relation μ so that for certain properties F, there are entities u_F satisfying the universality condition:

$$\text{for any } x,\ x\ \mu\ u_F \text{ iff } F(x).$$

The universality condition and $F(u_F)$ imply that u_F is a concrete universal in the previously defined sense of being self-predicative, $u_F\ \mu\ u_F$. A theory of self-predicative universals would also have to have an equivalence relation so the self-predicative universals for the same property would be the universal up to that equivalence relation.

Is there a precise mathematical theory of the common Form of thought, the self-predicative universal? Our claim is that category theory is precisely that theory where the self-predicative universals are the universal constructions, usually as universal morphisms or universal arrows.

> Universal constructions appear throughout mathematics in various guises - as universal arrows to a given functor, as universal arrows from a given functor, or as universal elements of a set-valued functor. (MacLane [13, p. 55])

The simplest examples of categories are partially ordered sets.[3] Consider the universe of subsets or power set $\wp(U)$ of a set U with the inclusion relation $\subseteq$ as the partial ordering relation. Given sets a and b, consider the property

$$G(x) \equiv a \subseteq x \ \&\ b \subseteq x.$$

The participation relation is set inclusion $\subseteq$ and the union $a \cup b$ is the universal u_F for this property $G(x)$. The universality relation states that the union is the least upper bound of a and b in the inclusion ordering:

$$\text{for any } x,\ a \cup b \subseteq x \text{ iff } a \subseteq x \ \&\ b \subseteq x.$$

[3] A binary relation $\leq$ on U is a partial order if for all $u, u', u'' \in U$, it is reflexive ($u \leq u$), transitive ($u \leq u'$ and $u' \leq u''$ imply $u \leq u''$), and anti-symmetric ($u \leq u'$ and $u' \leq u$ imply $u = u'$).

The universal has the property it represents, i.e., $a \subseteq a \cup b$ and $b \subseteq a \cup b$, so it is a self-predicative or concrete universal.[4] Two self-predicative universals for the same property must participate in each other. In partially ordered sets, the antisymmetry condition, $y \subseteq y'$ & $y' \subseteq y$ implies $y = y'$, means that equality can serve as the equivalence relation in the uniqueness condition for universals in a partial order.

There is much controversy in Platonic scholarship about self-predication (see Malcolm [16] for a summary and more references). Our purpose is *not* Plato exegesis. But the development of a mathematical theory of self-predicative universals in category theory does help by showing one way to sort out Plato's Ideas. One simple point is that there can be both non-self-predicative and self-predicative universals for the same property $F(x)$, and both are distinct from that property. Given subsets $a, b \subseteq U$, consider the property: $F(x) \equiv x \subseteq a$ & $x \subseteq b$. The self-predicative universal (or "paradigmatic instance" in Platonic language) for that property is the intersection $a \cap b$ where the participation relation is inclusion: $\forall x \subseteq U$, $x \subseteq a \cap b$ iff $F(x)$. The power set $\wp(a \cap b)$ is the non-self-predicative universal for that property where the participation relation is set membership: $\forall x \subseteq U$, $x \in \wp(a \cap b)$ iff $F(x)$. The self-predicative universal $a \cap b$ has the property, i.e., $a \cap b \subseteq a$ & $a \cap b \subseteq b$, while the non-self-predicative universal $\wp(a \cap b)$ does not have the property, and neither universal is to be confused with the property $F(x) \equiv x \subseteq a$ & $x \subseteq b$ itself.

5 Self-predicative Universals in General Categories

For the self-predicative universals of category theory,[5] the participation relation is the *uniquely-factors-through* relation. It can always be formulated in a suitable category as:

$$\text{"}x \ \mu \ u_F\text{" means "there exists a unique arrow } x \to u_F\text{".}$$

Then x is said to *uniquely factor through* u_F, and the arrow $x \to u_F$ is the unique factor or participation morphism. In the universality condition,

$$\text{for any } x,\ x \ \mu \ u_F \text{ if and only if } F(x),$$

the existence of the identity arrow $1_{u_F} : u_F \to u_F$ is the self-participation of the self-predicative universal that corresponds with $F(u_F)$, the self-predication of the property to u_F. In category theory, the equivalence relation used in the uniqueness condition is the isomorphism ($\cong$).[6]

It is sometimes convenient to "turn the arrows around" and use the dual definition where "$x \ \mu \ u_G$" means "there exists a unique arrow $u_G \to x$ that can also be viewed as the original

[4] These universals are "concrete" in the technical sense of being "One among the many." They are not relatively abstract or "One over the many" like the universals of iterative set theory. Of course, the concrete universals of category theory are abstract (in the usual sense) mathematical entities in "Plato's Heaven" and are not ordinary concrete objects.

[5] In the general case, a category may be defined as follows (e.g., MacLane and Birkhoff [14] or MacLane [13]):

A category C consists of

(a) a set of objects $a, b, c, \ldots$,

(b) for each pair of objects a, b, a set $hom_C(a, b) = C(a, b)$ whose elements are represented as arrows or morphisms $f : a \to b$,

(c) for any $f \in hom_C(a, b)$ and $g \in hom_C(b, c)$, there is the composition $gf : a \to b \to c$ in $hom_C(a, c)$,

(d) composition of arrows is an associative operation, and

(e) for each object a, there is an arrow $1_a \in hom_C(a, a)$, called the *identity* of a, such that for any $f : a \to b$ and $g : c \to a$, $f1_a = f$ and $1_a g = g$.

An arrow $f : a \to b$ is an *isomorphism*, $a \cong b$, if there is an arrow $g : b \to a$ such that $fg = 1_b$ and $gf = 1_a$. A *functor* is a map from one category to another that preserves composition and identities.

[6] Thus it must be verified that two concrete universals for the same property are isomorphic. By the universality condition, two concrete universals u and u' for the same property must participate in each other. Let $f : u' \to u$ and $g : u \to u'$ be the unique arrows given by the mutual participation. Then by composition $gf : u' \to u'$ is the unique arrow $u' \to u'$ but $1_{u'}$ is another such arrow so by uniqueness,$gf = 1'_u$. Similarly, $fg : u \to u$ is the unique self-participation arrow for u so $fg = 1_u$. Thus mutual participation of u and u' implies the isomorphism $u \cong u'$.

definition stated in the dual or opposite category. The above treatment of the intersection $a \cap b$ and the union $a \cup b$ are dual to one another. If we think of the "uniquely-factoring-through" arrows as *transferring* the property from the universal to the instances, then the transferring may go along the direction of the arrow—so the property may be said to be *transmitted* to the instance—or against the direction of the arrow—so the property may be said to be *reflected* to the instance.

Category theory as the theory of self-predicative universals has quite a different flavor from set theory, the theory of abstract non-self-predicative universals. Given an appropriately delimited collection of all the elements with a property, set theory can postulate a more abstract entity, the set of those elements, to be the universal. But category theory cannot postulate its universals because those universals are self-predicative, i.e., are the "One among the many"–if any. Category theory must find its universals, if at all, among the entities with the property.

6 Self-Predicative Universals and the Antinomies

The flaw or "mistake" in the set-theoretic paradoxes and similar self-referential antimonies is often taken to be the self-reference.

> In all the above contradictions (which are merely selections from an indefinite number) there is a common characteristic, which we may describe as self-reference or reflexiveness. (Whitehead and Russell [27, p. 61])

The iterative notion of a set requires the universal for a property to be of higher type or rank than the instances so that "$x \in x$ will become a meaningless proposition; and in this way the contradiction is avoided." To avoid the paradoxes, Whitehead and Russell postulated the *vicious circle principle*: "Whatever involves all of a collection must not be one of the collection." (Whitehead and Russell [27, p. 37]) But a self-predicative universal of category theory is the "One among the many" that transfers the property to all the instances so it is "impredicative" or self-predicative[7] in violation of the vicious circle principle. Indeed, the universals of category theory are *always* self-predicative via the identity morphisms so the question arises of how category theory avoids similar paradoxes.

All morphisms can be seen as "uniquely factoring through" themselves by the identity morphism (at either the head or tail of the arrow)—so the construction of something like "a universal morphism for all those morphisms that don't factor through themselves" would always come up empty. Abstractly put, there can be no self-predicative universal for the property of not being self-predicative—since the universal needs to have the property that is "transferred" to the instances by their "participation" in the universal and that particular negative property would always be defeated by the universal's identity morphism.[8] Thus the problem with the paradoxes was *not* the self-predication *per se* but the *negated* self-predication, and that is defeated in category theory by the universals being *always* self-predicative (by the identity morphisms). The "circle" or self-reference is not the problem if all the circles are required to be "virtuous" so that a "vicious" circle cannot arise.

[7] The connection between the self-predicative or "impredicative" definitions (which caused the problems in naïve set theory) and the self-predicative universals of category theory has not escaped the attention of category theorists. For instance, Michael Makkai notes that the "Peano system" of natural numbers is the self-predicative universal for the property of being a "pre-Peano system": "we can say that a Peano system is distinguished among pre-Peano systems by the fact that it has exactly one morphism to any pre-Peano system. (An 'impredicative' definition if there ever was one!)" (Makkai [15, p. 52]) See MacLane and Birkhoff for a full explanation of that "Peano-Lawvere Axiom" [14, p. 67] characterizing the Natural Numbers as the self-predicative universal for counting systems.

[8] From the purely syntactic viewpoint, it is "as if" all sets always had self-membership $x \in x$ so the paradox-causing negation $x \notin x$ could never apply.

7 Category Theory and the Third Man Argument

Much of the modern Platonic literature on self-participation and self-predication (Nehamas [18]; Allen [2]; Malcolm [16]) stems from the work of Geach [8] and Vlastos [24] on the Third Man Argument. The name derives from Aristotle (who denied self-predication), but the argument occurs in the dialogues.

> But now take largeness itself and the other things which are large. Suppose you look at all these in the same way in your mind's eye, will not yet another unity make its appearance—a largeness by virtue of which they all appear large?
>
> So it would seem.
>
> If so, a second form of largeness will present itself, over and above largeness itself and the things that share in it, and again, covering all these, yet another, which will make all of them large. So each of your forms will no longer be one, but an indefinite number. [Parmenides, 132]

If a form is self-predicative, the participation relation can be interpreted as "resemblance." An instance has the property F because it resembles the paradigmatic example of F-ness. But then, the Third Man Argument contends, the common property shared by Largeness and other large things gives rise to a "One over the many", a form Largeness* such that Largeness and the large things share the common property by virtue of resembling Largeness*. And the argument repeats itself giving rise to an infinite regress of forms. A key part of the Third Man Argument is what Vlastos calls the Non-Identity thesis:

> NI If anything has a given character by participating in a Form, it is not identical with that Form. [24, p. 351]

It implies that Largeness* is not identical with Largeness.

P. T. Geach [8] has developed a self-predicative interpretation of Forms as standards or norms, an idea he attributes to Wittgenstein. A stick is a meter long because it resembles, lengthwise, the standard meter measure. Geach avoids the Third Man regress with the exceptionalist device of holding the Form "separate" from the many so they could not be grouped together to give rise to a new "One over the many." Geach aptly notes the analogy with Frege's *ad hoc* and unsuccessful attempt to avoid the Russell-type paradoxes by allowing a set of all and only the sets which are not members of themselves—except for that set itself (Quine [19]; Geach [9]).

Category theory provides a mathematical model for the Third Man Argument, and it shows how to avoid the regress. The category-theoretic model shows that the flaw in the Third Man argument lies not in self-predication but in the Non-Identity thesis. "The One" is not necessarily "over the many"; it can be (isomorphic to) one among the many. In the special case of sets ordered by inclusion, the union or intersection of a collection of sets is not necessarily distinct from the sets in the collection; the "One" could be one among the many.

For example, let $A = \cup\{A_\beta\}$ be the One formed as the union of a collection of many sets $\{A_\beta\}$. Then add A to the collection and form the new One* as

$$A^* = \cup\{A_\beta\} \cup A.$$

This operation leads to no Third Man regress since $A^* = A$.

8 Conclusion

Whitehead described European philosophy as a series of footnotes to Plato, and the Theory of Forms was central to Plato's thought. We have seen that the self-predicative universals of category theory provide a rigorous mathematical model for the self-predicative strain in Plato's Theory of Forms and for the intuitive notion of a concrete universal or paradigmatic exemplar elsewhere in philosophy, literature, and ordinary thought.

Moreover, the always-self-predicative universals of category "complete the picture" of the antinomies in Frege's Paradise; one cannot have one theory of universals that could be either non-self-predicative or self-predicative. The "problem" in the antinomies was not the self-predication *per se* but allowing *negated* self-predication. That leaves two ways to avoid the antinomies in a theory of universals, have never-self-predicative universals or always-self-predicative universals. The always-self-predicative universals of category theory form the "opposite bookend" to the never-self-predicative universals of iterative set theory.

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