Why Delayed Choice Experiments do NOT imply Retrocausality

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Abstract

There is a fallacy that is often involved in the interpretation of quantum experiments involving a certain type of separation such as the: double-slit experiments, which-way interferometer experiments, polarization analyzer experiments, Stern-Gerlach experiments, and quantum eraser experiments. The fallacy leads not only to flawed textbook accounts of these experiments but to flawed inferences about retrocausality in the context of delayed choice versions of separation experiments.

Contents

1 Introduction

Following John Archibald Wheeler's paper on delayed choice experiments [15], a number of quantum physicists, philosophers, and popular science writers seem to have just accepted the implication of retrocausality as part of quantum "weirdness." For instance, concerning the quasar-galaxy version of Wheeler's delayed choice experiment, Anton Zeilinger remarks:

We decide, by choosing the measuring device, which phenomenon can become reality and which one cannot. Wheeler explicates this by example of the well-known case of a quasar, of which we can see two pictures through the gravity lens action of a galaxy that lies between the quasar and ourselves. By choosing which instrument to use for observing the light coming from that quasar, we can decide here and now whether the quantum phenomenon in which the photons take part is interference of amplitudes passing on both side of the galaxy or whether we determine the path the photon took on one or the other side of the galaxy. [Zeilinger 2008, pp. 191-192]

Occasionally instead of stating that future actions can determine whether the particle passes "on both sides of the galaxy" or only "on one or the other side," the euphemism is used of saying the photon acts like a wave or particle depending on the future actions.

The important conclusion is that, while individual events just happen, their physical interpretation in terms of wave or particle might depend on the future; it might particularly depend on decisions we might make in the future concerning the measurement performed at some distant spacetime location in the future. [Zeilinger 2004, p. 207]

In the Mach-Zehnder interferometer version of the delayed-choice experiment, the delayed insertion or not of a second beam-splitter is pictured as determining whether or not a photon goes through both arms of the interferometer.

[W]hen does the system (the light pulse) "make the choice" to behave like a wave or like a particle? Is it on the first beam splitter? If it is so, this choice must depend on the kind of arrangement that will be encountered later on the path, and that might be changed afterwards ("delayed choice" experiment [16]). [1, p. 58]

First, note that we can make the two paths through the beam-splitter as long as we please. Second, note that we can choose to insert or not to insert the second half-silvered mirror after the photon has begun to traverse the beam-splitter. We can delay making our choice as to whether the photon must take just one path through the beam splitter or both paths, which is why the delayed-choice experiment is so called. [6, p. 41]

There is a certain (fallacious) pattern of interpreting delayed-choice experiments that seems to imply retrocausality. The purpose of this paper is to point out the fallacy in non-technical terms for the usual delayed-choice experiments:

- double-slit experiments,

- which-way interferometer experiments,

- polarization analyzer experiments, and

- Stern-Gerlach experiments,

and then in slightly more technical terms for

- quantum eraser experiments.

The fallacy can be first described in general terms. In each case, given an incoming quantum particle, the "separation" apparatus creates a *superposition* of certain eigenstates; it is not a measurement. Detectors can be placed in certain positions so that when the evolving superposition state is finally projected or collapsed by the detectors, then only one of the eigenstates can register at each detector. It is then fallaciously assumed that the particle had already been projected or collapsed to an eigenstate at the entrance to the separation apparatus–as if it had been a measurement.

And if after the particle had entered the apparatus, the delayed-choice is made to suddenly remove the detectors (prior to arrival of the particle), then the superposition would continue to evolve and have distinctive effects (e.g., interference patterns in the two-slit experiment). Hence the fallacy makes it *seem* that by the delayed choice to insert or remove the appropriately positioned detectors or measurement devices, one can *retrocause* either a collapse to an eigenstate or not at the particle's entrance into the apparatus.

2   Double-slit experiments

In the well-known setup for the double-slit experiment, if detectors D_1 and D_2 are placed a small finite distance after the slits so a particle "going through the other slit" cannot reach the detector, then this is seen as "measuring which slit the particle went through" and a hit at a detector is usually interpreted as "the particle went through that slit."

> The natural image for this behaviour is that of a particle that passes either through hole [slit 1] or through [slit 2], but not through both holes. [1, p. 45]
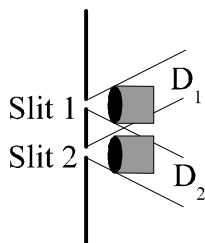

Figure 1

But this is incorrect. The particle is in a superposition state, which we might represent schematically as $|S1\rangle + |S2\rangle$, that evolves until it hits the detector which projects (or collapses) the superposition to one of (the evolved versions of) the slit-eigenstates. The particle's state was not collapsed earlier so it was not previously in the $|S1\rangle$ or $|S2\rangle$ eigenstate, i.e., it did not "go through slit 1" or "go through slit 2."

Thus what is called "detecting which slit the particle went through" is a misinterpretation. It is only placing a detector in such a position so that when the superposition projects to an eigenstate, only one of the eigenstates can register in that detector. It is about the spatial *detector placement*; it is not about which-slit information.

By erroneously talking about the detector "showing the particle went through slit 1," we imply a type of retrocausality. If the detector is suddenly removed after the particle has passed the slits, then the superposition state continues to evolve and shows interference on the far wall (not shown)—in which case people say "the particle went through both slits." Thus the "bad talk" makes it seem

that by removing or inserting the detector after the particle is beyond the slits, one can retrocause the particle to go through "both slits" or one slit only.

This sudden removal or insertion of detectors that can only detect one of the slit-eigenstates is a version of Wheeler's delayed choice thought-experiment [15].
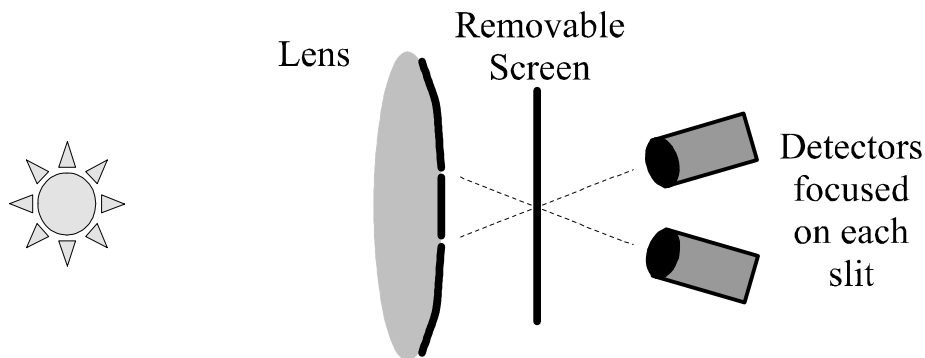

Figure 2: Wheeler's Delayed Choice 2-Slit Experiment

In Wheeler's version of the experiment, there are two detectors which are positioned behind the removable screen so they can only detect one of the projected (evolved) slit eigenstates when the screen is removed. The choice to remove the screen or not is delayed until after a photon has traversed the two slits.

> In the one case [screen in place] the quantum will transform a grain of silver bromide and contribute to the record of a two-slit interference fringe. In the other case [screen removed] one of the two counters will go off and signal in which beam–and therefore from which slit–the photon has arrived. [15, p. 13]

The fallacy is involved when Wheeler incorrectly infers from the fact that one of the specially-placed detectors went off that the photon had come from one of the slits–as if there had been a projection or collapse to one of the slit eigenstates at the slits rather than later at the detectors. Wheeler makes a similar mistake when he infers from a photon now registering a certain polarization upon measurement–that the photon always had that polarization. Hence by changing the angle of the polarization detector we could change the polarization in the past.

> There is an inescapable sense in which we, in the here and now, by a delayed setting of our analyzer of polarization to one or other angle, have an inescapable, an irretrievable, an unavoidable influence on *what we have the right to say about what we call the past.* [17, p. 486]

3   Which-way interferometer experiments

Consider a Mach-Zehnder-style interferometer with only one beam-splitter (e.g., half-silvered mirror), the "separation," which creates the photon superposition: $|T1\rangle + |R1\rangle$ (which stand for "Transmit" to the upper arm or "Reflect" into the lower arm at the first beam-splitter).
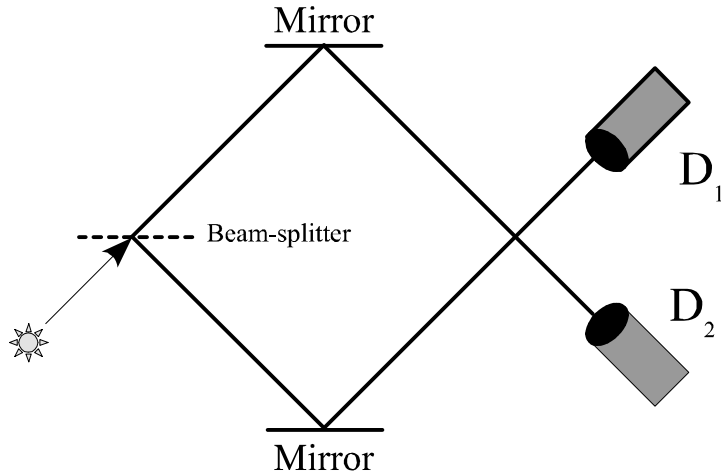
Figure 3: Interferometer with one beam splitter

When detector D_1 registers a hit, it is said that "the photon was reflected and thus took the lower arm" of the interferometer and similarly for D_2 and passing through into the upper arm.

> Indeed, if we want to visualize what happens in this experiment, the only possible image is that "something" is either reflected, or transmitted, on the beam-splitter, but it is not split: this corresponds to the behavior of a classical particle. [1, p. 58]
>
> So we can say in this case, without fear of paradox, that each photon went through just one path through the beam-splitter. In fact, if the photon were to take both paths, it would be hard to understand why it should appear to have taken just one or the other paths, why, that is, it is detected at [D1] (say) rather than at both [D1 and D2]. [6, p. 40]

This is the interferometer analogue of putting two up-close detectors after the two slits in the two-slit experiment.

And this standard description is incorrect for the same reasons. The photon stays in the superposition state until the detectors force a projection to one of the (evolved) eigenstates. If the projection is to the evolved $|R1\rangle$ eigenstate then only D_1 will get a hit, and similarly for D_2 and the evolved version of $|T1\rangle$. The point is that the placement of the detectors (like in the double-slit experiment) only captures one or the other of the projected eigenstates–but that does *not* mean the photon was in that eigenstate prior to the detection/measurement.

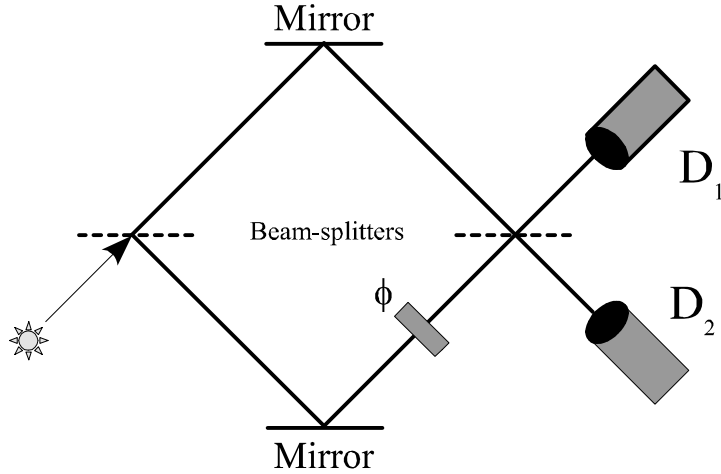Now insert a second beam-splitter as in the following diagram.

Figure 4: Interferometer with two beam splitters

It is said that the second beam-splitter "erases" the "which-way information" so that a hit at either detector could have come from either arm, and thus an interference pattern emerges by varying the phase ϕ.

But this is also incorrect. There is no "which-way information" to be erased in the superposition state $|T1\rangle + |R1\rangle$ which is further transformed at the second beam-splitter (where $|T2\rangle, |R2\rangle$ refer to transmit or reflect at the second beam-splitter) to the superposition $|T1, T2\rangle + |T1, R2\rangle + |R1, T2\rangle + |R1, R2\rangle$ that can be regrouped according to what can register at each detector:

$$[|T1, R2\rangle + |R1, T2\rangle]_{D_1} + [|T1, T2\rangle + |R1, R2\rangle]_{D_2}.$$

The so-called "which-way information" was not there to be "erased" since the particle did not take one way or the other in the first place. The second beam-splitter only allows the superposition state $[|T1, R2\rangle + |R1, T2\rangle]_{D_1}$ to be registered at D_1 or the superposition state $[|T1, T2\rangle + |R1, R2\rangle]_{D_2}$ to be registered at D_2. By using a phase shifter ϕ, an interference pattern can be recorded at each detector since each one is now detecting a superposition that will involve interference.

By inserting or removing the second beam-splitter after the particle has traversed the first beam-splitter (as in [15] or [16]), the fallacy makes it *seem* that we can retrocause the particle to go through both arms or only one arm.

4 Polarization analyzers and loops

Another common textbook example of the fallacy is the treatment of polarization analyzers such as calcite crystals that are *incorrectly said* to create two orthogonally polarized beams in the upper and lower channels, say $|v\rangle$ and $|h\rangle$ from an arbitrary incident beam.
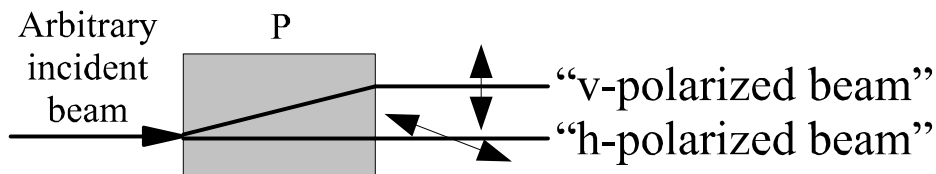
Figure 5: *vh*-analyzer (incorrectly described)

The output from the analyzer P is routinely described as a "vertically polarized" beam and "horizontally polarized" beam as if the analyzer was itself a measurement that collapsed or projected the incident beam to either of those polarization eigenstates. This *seems* to follow because if one positions a detector in the upper beam then only vertically polarized photons are observed and similarly for the lower beam and horizontally polarized photons. A blocking mask in one of the beams has the same effect as a detector to project the photons to eigenstates. If a blocking mask in inserted in the lower beam, then only vertically polarized photons will be found in the upper beam, and vice-versa.

But here again, the story is about the spatial placement of the detector (or blocking mask); it is not about the analyzer supposedly projecting a photon into one or the other of the eigenstates. The analyzer puts the incident photons into a superposition state, a superposition state that associates the compatible polarization and spatial modes for a particle. This could be schematically represented as:

$$|\text{vertical polarization; upper channel}\rangle + |\text{horizontal polarization; lower channel}\rangle.$$

There is a certain ambiguity in the practice of representing two eigenstates or eigenvalues in a ket: $|\text{state 1; state 2}\rangle$. This could be interpreted as shorthand for the tensor product of two states $|\text{state 1}\rangle \otimes |\text{state 2}\rangle$ of two particles (and the same observable) so a superposition would be an entangled state–or it could be interpreted as giving two eigenstates of one particle for two compatible observables (e.g., as in Dirac's complete set of commuting observables) and then we could consider a superposition of those single-particle states. We are using here the latter single-particle version of the superposition: $|\text{vertical; upper}\rangle + |\text{horizontal; lower}\rangle$. This sort of a superposition state is thus similar to an entangled state but only involves a single particle.

If a polarization detector is spatially placed in, say, the upper channel and it registers a hit, then *that* is the measurement that collapses the evolved superposition state to $|\text{vertical; upper}\rangle$ so only a vertically polarized photon will register in the detector, and similarly for the lower channel. Thus it is misleadingly said that the "upper beam" was already vertically polarized and the "lower beam" was already horizontally polarized as if the analyzer had already done the projection to one of the eigenstates.

If the analyzer had in fact induced a collapse to the eigenstates, then any prior polarization of the incident beam would be lost. Hence assume that the incident beam was prepared in a specific polarization of, say, $|45°\rangle$ half-way between the states of vertical and horizontal polarization. Then follow the *vh*-analyzer P with its inverse P^{-1} to form an *analyzer loop* [5].
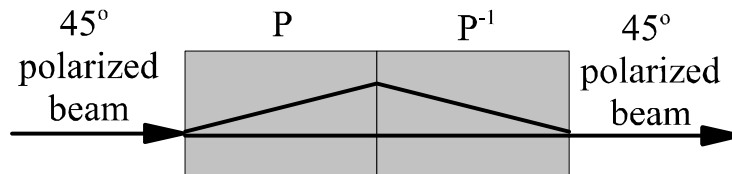

Figure 6: *vh*-analyzer loop

The characteristic feature of an analyzer loop is that it outputs the same polarization, in this case $|45°\rangle$, as the incident beam. This would be impossible if the P analyzer had in fact rendered

all the photons into a vertical or horizontal eigenstate thereby destroying the information about the polarization of the incident beam. But since no collapsing measurement was in fact made in P or its inverse, the original beam can be the output of an analyzer loop.

Some texts do not realize there is a problem with presenting a polarization analyzer such as a calcite crystal as creating two beams with orthogonal eigenstate polarizations—rather than creating a superposition state so that appropriately positioned detectors can detect only one eigenstate when the detectors cause the projections to eigenstates.

One (partial) exception is Dicke and Wittke's text [3]. At first they present polarization analyzers as if they measured polarization and thus "destroyed completely any information that we had about the polarization" [3, p. 118] of the incident beam. But then they note a problem:

> The equipment [polarization analyzers] has been described in terms of devices which measure the polarization of a photon. Strictly speaking, this is not quite accurate. [3, p. 118]

They then go on to consider the inverse analyzer P^{-1} which combined with P will form an analyzer loop that just transmits the incident photon unchanged.

They have some trouble squaring this with their prior statement about the P analyzer destroying the polarization of the incident beam but they struggle with getting it right.

> Stating it another way, although [when considered by itself] the polarization P completely destroyed the previous polarization Q [of the incident beam], making it impossible to predict the result of the outcome of a subsequent measurement of Q, in [the analyzer loop] the disturbance of the polarization which was effected by the box P is seen to be revocable: if the box P is combined with another box of the right type, the combination can be such as to leave the polarization Q unaffected. [3, p. 119]

They then go on to correctly note that the polarization analyzer P did not in fact project the incident photons into polarization eigenstates.

> However, it should be noted that in this particular case [sic!], the first box P in [the first half of the analyzer loop] did not really measure the polarization of the photon: no determination was made of the channel ... which the photon followed in leaving the box P. [3, p. 119]

Their phrase "in this particular case" makes it seem that the delayed choice to not add or add the second half P^{-1} of the analyzer loop will retrocause a measurement to (respectively) be made or not made in the first box P.

There is some classical imagery (like Schrödinger's cat running around one side or the other side of a tree) that is sometimes used to illustrate quantum separation experiments when in fact it only illustrates how classical imagery can be misleading. Suppose an interstate highway separates at a city into both northern and southern bypass routes–like the two channels in a polarization analyzer loop. One can observe the bypass routes while a car is in transit and find that it is in one bypass route or another. But after the car transits whichever bypass it took without being observed and rejoins the undivided interstate, then it is said that the which-way information is erased so an observation cannot elicit that information.

This is not a correct description of the corresponding quantum separation experiment since the classical imagery *does not contemplate superposition states*. The particle-as-car is in a superposition of the two routes until an observation (e.g., a detector or "road block") collapses the superposition to one eigenstate or the other. Thus when the undetected particle rejoins the undivided "interstate," there was no which-way information to be erased. Correct descriptions of quantum separation experiments require *taking superposition seriously*–so classical imagery should only be used *cum grano salis*.

It is fallacious to reason that "we know the photons are in one polarization state in one channel and in the orthogonal polarization state in the other channel *because* that is what we find when we measure the channels," just as it is fallacious to reason "the particle has to go through one slit or another (or one arm or another in the interferometer experiment) *because* that is what we find when we measure it." This purely *operational* (or "Copenhagen") description does not take superposition seriously since a superposition state is never "what we find when we measure."

5 Stern-Gerlach experiments

We have seen the fallacy in the standard treatments of the double-slit experiment, which-way interferometer experiments, and in polarization analyzers. In spite of the differences between those separation experiments, there was that common (mis)interpretative theme. Since the "logic" of the polarization analyzers is followed in the Stern-Gerlach experiment (with spin playing the role of polarization), it is not surprising that the same fallacy occurs there. Many texts represent the Stern-Gerlach apparatus as separating particles into spin eigenstates denoted by, say, $+S, 0S, -S$.
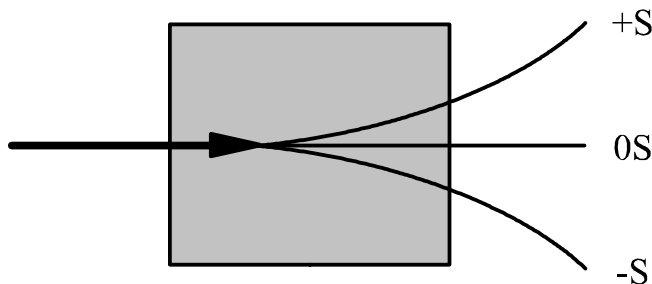

Figure 7: S-G Apparatus (incorrectly described)

But a careful "expert" analysis of the experiment (e.g., [7, p. 171]) shows the apparatus does not project the particles to eigenstates. Instead it creates a single particle superposition state that combines or "entangles" spin with spatial location so that with a detector in a certain position, it will only see particles of one spin state (analogous to the previous polarization example). If the collapse is caused by placing blocking masks over two of the beams, then the particles in the third beam will all be those that have collapsed to the same eigenstate. It is the detectors or blockers that cause the collapse or projection to eigenstates, not the prior separation apparatus.

And again, the fallacy is revealed by considering the Stern-Gerlach analogue of an analyzer loop. The idea of a Stern-Gerlach loop seems to have been first broached by David Bohm [2, 22.11] and was later used by Eugene Wigner [18]. One of the few texts to consider such a Stern-Gerlach analyzer loop is *The Feynman Lectures on Physics: Quantum Mechanics (Vol. III)* where it is called a "modified Stern-Gerlach apparatus" [4, p. 5-2].
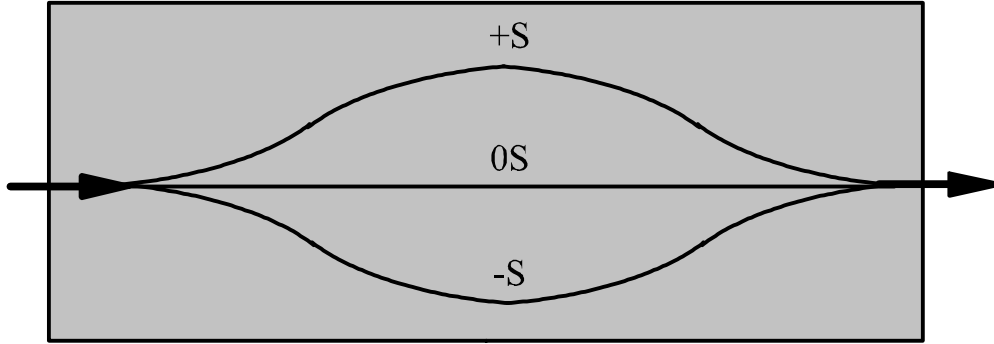
Figure 8: Stern-Gerlach Loop

We previously saw how a polarization analyzer, contrary to the statement in many texts, does not lose the polarization information of the incident beam when it "separates" the beam (into a positionally-entangled superposition state). In the context of the Stern-Gerlach apparatus, Feynman similarly remarks:

"Some people would say that in the filtering by T we have 'lost the information' about the previous state $(+S)$ because we have 'disturbed' the atoms when we separated them into three beams in the apparatus T. But that is not true. The past information is not lost by the *separation* into three beams, but by the *blocking masks* that are put in...." [4, p. 5-9 (italics in original)]

6   The Separation Fallacy

We have seen the same fallacy of interpretation in two-slit experiments, which-way interferometer experiments, polarization analyzers, and Stern-Gerlach experiments. The common element in all the cases is that there is some separation apparatus that puts a particle into a certain superposition of spatially "entangled" eigenstates in such a manner that when an *appropriately spatially-positioned* detector induces a collapse to an eigenstate, then the detector will only register one of the eigenstates. The separation fallacy is that this is misinterpreted as showing that the particle was already in that eigenstate in that position as a result of the previous "separation." In fact the superposition evolves until some distinction is made that constitutes a measurement, and only then is the state reduced to an eigenstate. The quantum erasers are more elaborate versions of these simpler experiments, and a similar separation fallacy arises in that context.

7   One photon quantum eraser experiment

A simple quantum eraser can be devised using a single beam of photons as in [9]. We start with the two-slit setup where a $+45°$ polarizer in front of the slits to control the incoming polarization. Then we can represent the system after the polarizer as a tensor product with the second component giving the polarization state. The evolving state after the two slits is the superposition:

$$\tfrac{1}{\sqrt{2}} \left(|S1\rangle \otimes |45°\rangle + |S2\rangle \otimes |45°\rangle \right).$$
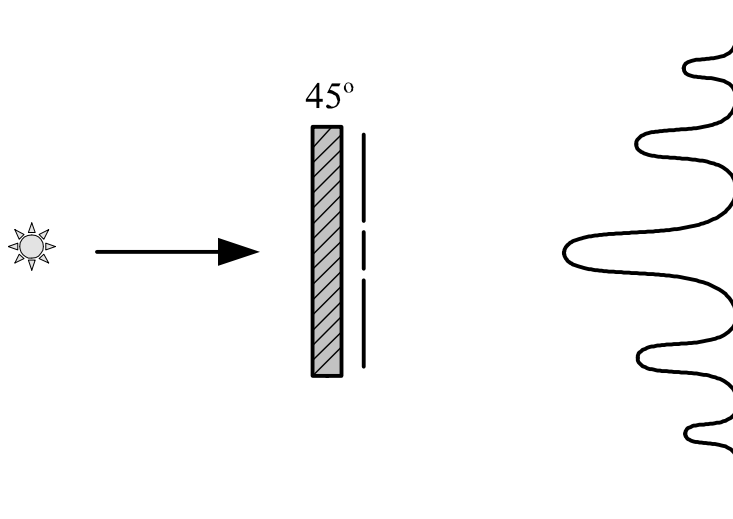
Figure 9: Interference in two-slit setup

Then a horizontal polarizer is inserted after slit 1 and a vertical polarizer after slit 2. This will change the evolving state to: $\frac{1}{\sqrt{2}}\left(|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right)$ but since these new polarizers involve some measurements, not just unitary evolution, it may be helpful to go through the calculation in some detail. The state that "hits" the H, V polarizers is:

$$\frac{1}{\sqrt{2}}\left(|S1\rangle \otimes |45°\rangle + |S2\rangle \otimes |45°\rangle\right).$$

The $45°$ polarization state can be resolved by inserting the identity operator $I = |H\rangle\langle H| + |V\rangle\langle V|$ to get:

$$|45°\rangle = [|H\rangle\langle H| + |V\rangle\langle V|]\,|45°\rangle = \langle H|45°\rangle\,|H\rangle + \langle V|45°\rangle\,|V\rangle = \frac{1}{\sqrt{2}}\left[|H\rangle + |V\rangle\right].$$

Substituting this for $|45°\rangle$, we have the state that hits the H, V polarizers as:

$$\frac{1}{\sqrt{2}}\left(|S1\rangle \otimes |45°\rangle + |S2\rangle \otimes |45°\rangle\right)$$
$$= \frac{1}{\sqrt{2}}\left(|S1\rangle \otimes \frac{1}{\sqrt{2}}[|H\rangle + |V\rangle] + |S2\rangle \otimes \frac{1}{\sqrt{2}}[|H\rangle + |V\rangle]\right)$$
$$= \frac{1}{2}\left[|S1\rangle \otimes |H\rangle + |S1\rangle \otimes |V\rangle + |S2\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right]$$

which can be regrouped in two parts as:

$$= \frac{1}{2}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right] + \frac{1}{2}\left[|S1\rangle \otimes |V\rangle + |S2\rangle \otimes |H\rangle\right].$$

Then the H, V polarizers are making a degenerate measurement that give the first state $|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle$ with probability $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$. The other state $|S1\rangle \otimes |V\rangle + |S2\rangle \otimes |H\rangle$ is obtained with the same probability, and it is blocked by the polarizers. Thus the state that evolves is the state (after being normalized):

$$\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right].$$

11

After the two slits, a photon is in a state that entangles the spatial slit states and the polarization states (for a discussion of this type of entanglement, see [11]). But as this superposition evolves, it cannot be separated into a superposition of the slit-states as before, so the interference disappears.
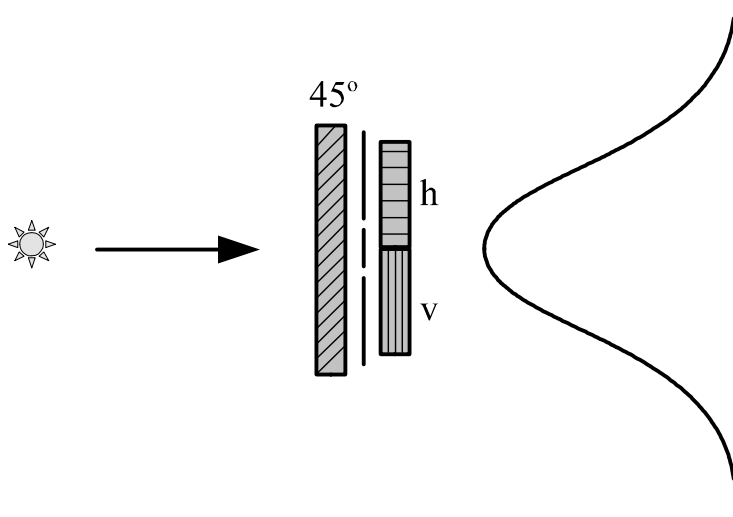

Figure 10: "Mush" pattern with interference eliminated by "which-way" markings

Technically, if $P_{\Delta y}$ is the projection operator representing finding a particle in the region Δy along the wall, then that probability in the state $\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right]$ is:

$\frac{1}{2}\langle S1 \otimes H + S2 \otimes V|P_{\Delta y} \otimes I|S1 \otimes H + S2 \otimes V\rangle$
$= \frac{1}{2}\langle S1 \otimes H + S2 \otimes V|P_{\Delta y}S1 \otimes H + P_{\Delta y}S2 \otimes V\rangle$
$= \frac{1}{2}[\langle S1 \otimes H|P_{\Delta y}S1 \otimes H\rangle + \langle S1 \otimes H|P_{\Delta y}S2 \otimes V\rangle$
$+ \langle S2 \otimes V|P_{\Delta y}S1 \otimes H\rangle + \langle S2 \otimes V|P_{\Delta y}S2 \otimes V\rangle]$
$= \frac{1}{2}[\langle S1|P_{\Delta y}S1\rangle \langle H|H\rangle + \langle S1|P_{\Delta y}S2\rangle \langle H|V\rangle$
$+ \langle S2|P_{\Delta y}S1\rangle \langle V|H\rangle + \langle S2|P_{\Delta y}S2\rangle \langle V|V\rangle]$
$= \frac{1}{2}[\langle S1|P_{\Delta y}S1\rangle + \langle S2|P_{\Delta y}S2\rangle]$

which is the average of separate slit probabilities that shows no interference. The key step is how the orthogonal polarization markings decohered the state since $\langle H|V\rangle = 0 = \langle V|H\rangle$ and thus eliminated the interference between the $S1$ and $S2$ terms. The state-reduction occurs only when the evolved superposition state hits the far wall which measures the positional component (i.e., $P_{\Delta y}$) of the entangled state and shows the non-interference pattern.

The key point is that in spite of the bad terminology of "which-way" or "which-slit" information, the polarization markings do NOT create a half-half mixture of horizontally polarized photons going through slit 1 and vertically polarized photons going through slit 2. It creates the *superposition* (pure) state $\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right]$ which evolves until measured at the wall.

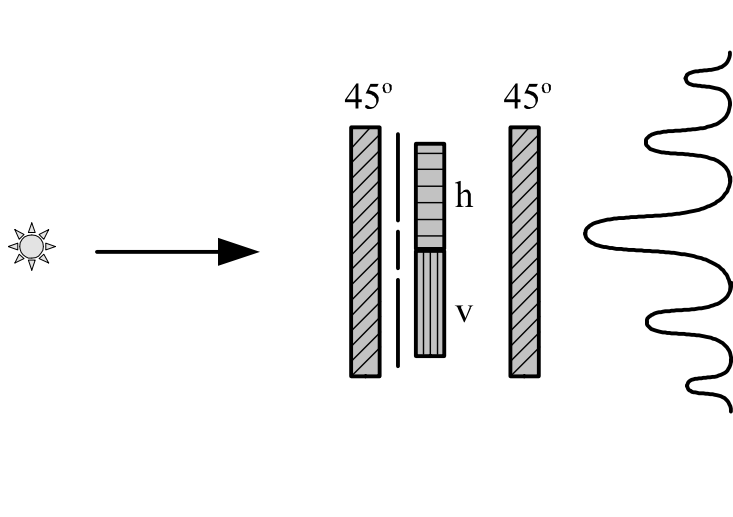This can be seen by inserting a $+45°$ polarizer between the two-slit screen and the far wall.

Figure 11: Fringe interference pattern produce by $45°$ polarizer between slits and wall

Each of the horizontal and vertical polarization states can be represented as a superposition of $+45°$ and $-45°$ polarization states. Just as the horizontal polarizer in front of slit 1 threw out the vertical component so we have no $|S1\rangle \otimes |V\rangle$ term in the superposition, so now the $+45°$ polarizer throws out the $-45°$ component of each of the $|H\rangle$ and $|V\rangle$ terms so the state transformation is:

$$\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right]$$
$$\rightarrow \frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |+45°\rangle + |S2\rangle \otimes |+45°\rangle\right] = \frac{1}{\sqrt{2}}\left(|S1\rangle + |S2\rangle\right) \otimes |+45°\rangle.$$

It might be useful to again go through the calculation in some detail.

1. $|H\rangle = \left(|+45°\rangle \langle+45°| + |-45°\rangle \langle-45°|\right)|H\rangle = \langle+45°|H\rangle |+45°\rangle + \langle-45°|H\rangle |-45°\rangle$ and since a horizontal vector at $0°$ is the sum of the $+45°$ vector and the $-45°$ vector, $\langle+45°|H\rangle = \langle-45°|H\rangle = \frac{1}{\sqrt{2}}$ so that: $|H\rangle = \frac{1}{\sqrt{2}}\left[|+45°\rangle + |-45°\rangle\right]$.

2. $|V\rangle = \left(|+45°\rangle \langle+45°| + |-45°\rangle \langle-45°|\right)|V\rangle = \langle+45°|V\rangle |+45°\rangle + \langle-45°|V\rangle |-45°\rangle$ and since a vertical vector at $90°$ is the sum of the $+45°$ vector and the negative of the $-45°$ vector, $\langle+45°|V\rangle = \frac{1}{\sqrt{2}}$ and $\langle-45°|V\rangle = -\frac{1}{\sqrt{2}}$ so that: $|V\rangle = \frac{1}{\sqrt{2}}\left[|+45°\rangle - |-45°\rangle\right]$.

Hence making the substitutions gives:

$$\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right]$$
$$= \frac{1}{\sqrt{2}}\left[|S1\rangle \otimes \frac{1}{\sqrt{2}}\left[|+45°\rangle + |-45°\rangle\right] + |S2\rangle \otimes \frac{1}{\sqrt{2}}\left[|+45°\rangle - |-45°\rangle\right]\right].$$

We then regroup the terms according to the measurement being made by the $45°$ polarizer:

$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |+45°\rangle + |S2\rangle \otimes |+45°\rangle\right] + \frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |-45°\rangle - |S2\rangle \otimes |-45°\rangle\right]\right]$$
$$= \frac{1}{2}\left(|S1\rangle + |S2\rangle\right) \otimes |+45°\rangle + \frac{1}{2}\left(|S1\rangle - |S2\rangle\right) \otimes |-45°\rangle.$$

13

Then with probability $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$, the $+45°$ polarization measurement passes the state $(|S1\rangle + |S2\rangle) \otimes |+45°\rangle$ and blocks the state $(|S1\rangle - |S2\rangle) \otimes |-45°\rangle$. Hence the normalized state that evolves is: $\frac{1}{\sqrt{2}} (|S1\rangle + |S2\rangle) \otimes |+45°\rangle$, as indicated above.

Then at the wall, the positional measurement $P_{\Delta y}$ of the first component is the evolved superposition $|S1\rangle + |S2\rangle$ which again shows an interference pattern. But it is not the same as the original interference pattern before H, V or $+45°$ polarizers were inserted. This "shifted" interference pattern is called the *fringe* pattern of figure 11.

Alternatively we could insert a $-45°$ polarizer which would transform the state $\frac{1}{\sqrt{2}} [|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle]$ into $\frac{1}{\sqrt{2}} (|S1\rangle - |S2\rangle) \otimes |-45°\rangle$ which produces the interference pattern from the "other half" of the photons and which is called the *anti-fringe* pattern.
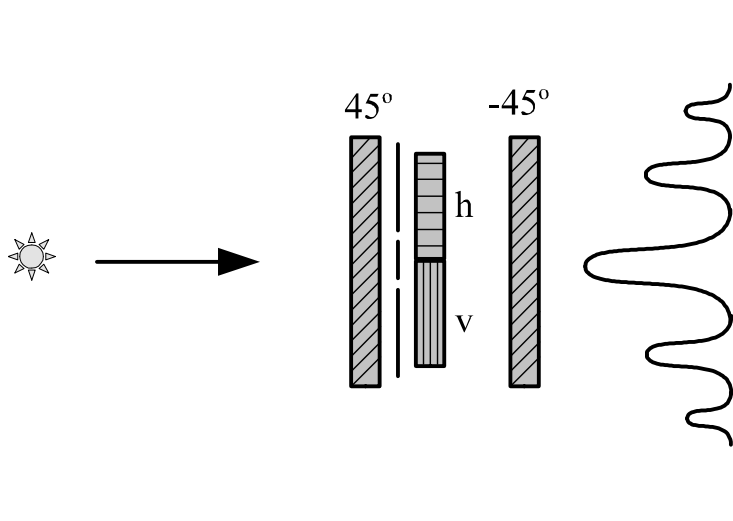

Figure 12: Anti-fringe interference pattern

The all-the-photons sum of the fringe and anti-fringe patterns reproduces the "mush" non-interference pattern of Figure 10.

This is one of the simplest examples of a quantum eraser experiment.

1. The insertion of the horizontal and vertical polarizers marks the photons with "which-slit" information that eliminates the interference pattern.

2. The insertion of a $+45°$ or $-45°$ polarizer "erases" the which-slit information so an interference pattern reappears.

But there is a **mistaken** interpretation of the quantum eraser experiment that leads one to infer that there is *retrocausality*.

[Start of erroneous example]

1. The markings by insertion of the horizontal and vertical polarizers creates the half-half *mixture* where each photon is reduced to either a horizontally polarized photon that went through slit 1 or a vertically polarized photon that went through slit 2. Hence the photon "goes through one slit or the other."

2. The insertion of the $+45°$ polarizer erases that which-slit information so interference reappears which means that the photon had to "go through both slits."

Hence the delayed choice to insert or not insert the $+45°$ polarizer–after the photons have traversed the screen and H, V polarizers–**retrocauses** the photons to either:

- go through both slits, or

- to only go through one slit or the other.

[End of erroneous example]

Now we can see the importance of realizing that prior to inserting the $+45°$ polarizer, the photons were in the *superposition* (pure) state $\frac{1}{\sqrt{2}}[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle]$, not a half-half mixture of the reduced states $|S1\rangle \otimes |H\rangle$ or $|S2\rangle \otimes |V\rangle$. The **proof** that the system was not in that mixture is obtained by inserting the $+45°$ polarizer which yields the (fringe) interference pattern.

1. If a photon had been, say, in the state $|S1\rangle \otimes |H\rangle$ then, with 50% probability, the photon would have passed through the filter in the state $|S1\rangle \otimes |+45°\rangle$, but that would not yield any interference pattern at the wall since their was no contribution from slit 2.

2. And similarly if a photon in the state $|S2\rangle \otimes |V\rangle$ hits the $+45°$ polarizer.

The fact that the insertion of the $+45°$ polarizer yielded interference *proved* that the incident photons were in a superposition (pure) state $\frac{1}{\sqrt{2}}[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle]$ which, in turn, means there was no mixture of "going through one slit or the other" in case the $+45°$ polarizer had not been inserted.

Thus a correct interpretation of the quantum eraser experiment removes any inference of *retro-causality* and fully accounts for the experimentally verified facts given in the figures.

8 Two photon quantum eraser experiment

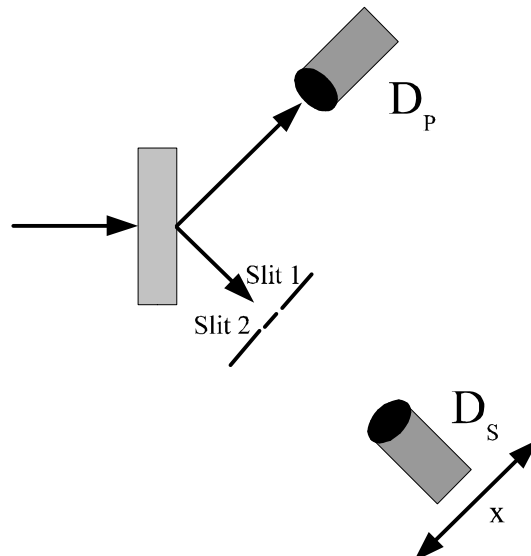We now turn to one of the more elaborate quantum eraser experiments [14].

Figure 13: Setup with two slits

A photon hits a down-converter which emits a "signal" p-photon entangled with an "idler" s-photon with a superposition of orthogonal $|x\rangle$ and $|y\rangle$ polarizations so the overall state is:

$$|\Psi\rangle = \tfrac{1}{\sqrt{2}} \left[|x\rangle_s \otimes |y\rangle_p + |y\rangle_s \otimes |x\rangle_p \right].$$

The lower s-photon hits a double-slit screen, and will show an interference pattern on the D_s detector as the detector is moved along the x-axis.

Next two quarter-wave plates are inserted before the two-slit screen with the fast axis of the one over slit 1 oriented at $|+45°\rangle$ to the x-axis and the one over the slit 2 with its fast axis oriented at $|-45°\rangle$ to the x-axis.
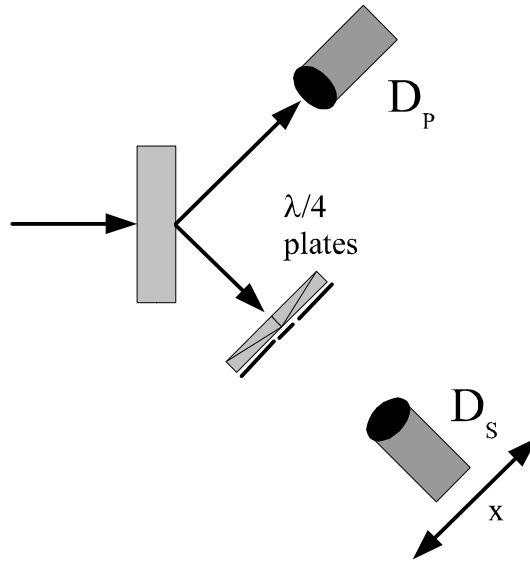

Figure 14: Setup with $\lambda/4$ plates

Then Walborn et al. [14] give the overall state of the system as (where $S1$ and $S2$ refer to the two slits):

$$|\Psi\rangle = \tfrac{1}{2} \left[\left(|L, S1\rangle \otimes |y\rangle_p + i\,|R, S1\rangle \otimes |x\rangle_p \right) + \left(i\,|R, S2\rangle \otimes |y\rangle_p - i\,|L, S2\rangle \otimes |x\rangle_p \right) \right].$$

Then by measuring the linear polarization of the p-photon at D_p and the circular polarization at D_s, "which-slit information" is said to be obtained and no interference pattern recorded at D_s.

For instance measuring $|x\rangle$ at D_p and $|L\rangle$ at D_s imply $S2$, i.e., slit 2. But as previously explained, this does *not* mean that the s-photon went through slit 2. It means we have positioned the two detectors *in polarization space*, say to measure $|x\rangle$ polarization at D_p and $|L\rangle$ polarization at D_s, so only when the superposition state collapses to $|x\rangle$ for the p-photon and $|L\rangle$ for the s-photon do we get a hit at both detectors.

This is the analogue of the one-beam-splitter interferometer where the positioning of the detectors would only record one collapsed state which did not imply the system was all along in that particular arm-eigenstate. The phrase "which-slit" or "which-arm information" is a misnomer in

that it implies the system was already in a slit- or way-eigenstate and the so-called measurement only revealed the information. Instead, it is only at the measurement that there is a collapse or projection to an evolved slit-eigenstate (not at the previous separation due to the two slits).

Walborn et al. indulge in the separation fallacy when they discuss what the so-called "which-path information" reveals.

> Let us consider the first possibility [detecting p before s]. If photon p is detected with polarization x (say), then we know that photon s has polarization y before hitting the $\lambda/4$ plates and the double slit. By looking at [the above formula for $|\Psi\rangle$], it is clear that detection of photon s (after the double slit) with polarization R is compatible only with the passage of s through slit 1 and polarization L is compatible only with the passage of s through slit 2. This can be verified experimentally. In the usual quantum mechanics language, detection of photon p before photon s has prepared photon s in a certain state. [14, p. 4]

Firstly, the measurement that p has polarization x after the s photon has traversed the $\lambda/4$ plates and two slits [see their Figure 1] does not retrocause the s photon to already have "polarization y before hitting the $\lambda/4$ plates." When photon p is measured with polarization x, then the two particle system is in the superposition state:

$$i \left|R, S1\right\rangle \otimes \left|x\right\rangle_p - i \left|L, S2\right\rangle \otimes \left|x\right\rangle_p = \left[i \left|R, S1\right\rangle - i \left|L, S2\right\rangle\right] \otimes \left|x\right\rangle_p$$

which means that the s photon is *still* in the slit-superposition state: $i \left|R, S1\right\rangle - i \left|L, S2\right\rangle$. Then only with the measurement of the circular polarization states L or R at D_s do we have the collapse to (the evolved version of) one of the slit eigenstates $S1$ or $S2$. It is an instance of the separation fallacy to infer "the passage of s through slit 1" or "slit 2", i.e., $S1$ or $S2$, instead of the photon s being in the entangled superposition state $|\Psi\rangle$ after traversing the slits.

Let us take a new polarization space basis of $\left|+\right\rangle = +45°$ to the x-axis and $\left|-\right\rangle = -45°$ to the x-axis. Then the overall state can be rewritten in terms of this basis as (see original paper for the details):

$$|\Psi\rangle = \tfrac{1}{2} \left[\left(\left|+, S1\right\rangle - i \left|+, S2\right\rangle \right) \otimes \left|+\right\rangle_p + i \left(\left|-, S1\right\rangle + i \left|-, S2\right\rangle \right) \otimes \left|-\right\rangle_p \right].$$
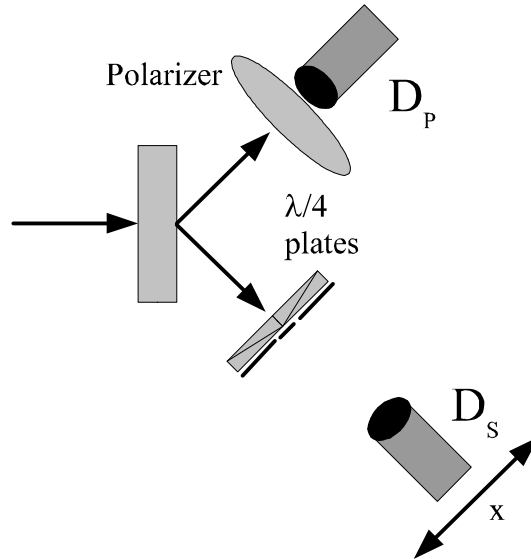
Figure 15: Setup with D_p polarizer

Then a $|+\rangle$ polarizer or a $|-\rangle$ polarizer is inserted in front of D_p to select $|+\rangle_p$ or $|-\rangle_p$ respectively. In the first case, this reduces the overall state $|\Psi\rangle$ to $|+, S1\rangle - i\,|+, S2\rangle$ which exhibits an interference pattern, and similarly for the $|-\rangle_p$ selection. This is misleadingly said to "erase" the so-called "which-slit information" so that the interference pattern is restored.

The first thing to notice is that two complementary interferences patterns, called "fringes" and "antifringes," are being selected. Their sum is the no-interference pattern obtained before inserting the polarizer. The polarizer simply selects one of the interference patterns out of the mush of their merged non-interference pattern. Thus instead of "erasing which-slit information," it selects one of two interference patterns out of the both-patterns mush.

Even though the polarizer may be inserted after the s-photon has traversed the two slits, there is no retrocausation of the photon going though both slits or only one slit as previously explained.

One might also notice that the entangled p-photon plays little real role in this setup (as opposed to the "delayed erasure" setup considered next). Instead of inserting the $|+\rangle$ or $|-\rangle$ polarizer in front of D_p, insert it in front of D_s and it would have the same effect of selecting $|+, S1\rangle - i\,|+, S2\rangle$ or $|-, S1\rangle + i\,|-, S2\rangle$ each of which exhibits interference. Then it is very close to the one-photon eraser experiment of the last section.

9   Delayed quantum eraser

If the upper arm is extended so the D_p detector is triggered last ("delayed erasure"), the same results are obtained. The entangled state is then collapsed at D_s. A coincidence counter (not pictured) is used to correlate the hits at D_s with the hits at D_p for each fixed polarizer setting, and the same interference pattern is obtained.
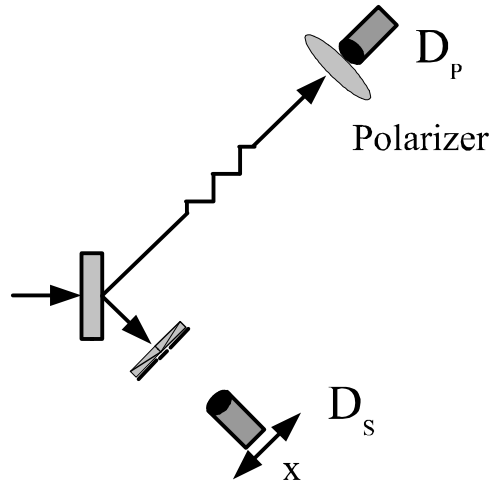
Figure 16: Setup for "delayed erasure"

The interesting point is that the D_p detections could be years after the D_s hits in this delayed erasure setup. If the D_p polarizer is set at $|+\rangle_p$, then out of the mush of hits at D_s obtained years before, the coincidence counter will pick out the ones from $|+, S1\rangle - i\,|+, S2\rangle$ which will show interference.

Again, the years-later D_p detections do not retrocause anything at D_s, e.g., do not "erase which-way information" years after the D_s hits are recorded (in spite of the "delayed erasure" talk). They only pick (via the coincidence counter) one or the other interference pattern out of the years-earlier mush of hits at D_s.

> "We must conclude, therefore, that the loss of distinguishability is but a side effect, and that the essential feature of quantum erasure is the post-selection of subensembles with maximal fringe visibility." [12, p. 79]

The same sort of analysis could be made of the delayed choice quantum eraser experiment described in the papers by Scully et al. [13] or Kim et al. & Scully [10]. Brian Greene [8, pp. 194-199] gives a good popular analysis of the Kim et al. & Scully experiment which avoids the separation fallacy and thus avoids any implication of retrocausality.

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19

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