Theory of Fuzzy Time Computation (3)

$(\mathbf{TC} + \mathbf{CON}(\mathbf{TC}^*) \vdash \mathbf{P} \neq \mathbf{NP})$

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Abstract. Here, we give the second proof for $\mathrm{TC} + \mathrm{CON}(\mathrm{TC}^*) \vdash \mathrm{P} \neq \mathrm{NP}$. The first proof is in [1]. In the second proof, we do not employ the concept of scope. *(Lost flash)*

Keywords. TC^*, $\mathrm{P} \neq \mathrm{NP}$, $\mathrm{P}^* \neq \mathrm{NP}^*$, Fuzzy time

Introduction

In [1], we gave the first proof by employing the concept of scope and scope* . Here , we give the second way.

1. The spaces

To give the proof more exactly, first define $\mathrm{W_m}$. Let .

$$\mathrm{W_m} = \{C_{i,t} \overset{m}{\Rightarrow} C_{j,t} : C_{i,t} \;\&\; C_{j,t} \text{ are configurattions for } M_t\}$$

Now, we define $\mathrm{C} \subsetneq \mathrm{W_m}$ as follows

C= $\{C_{i,t} \overset{m}{\Rightarrow} C_{j,t}\colon \mathrm{C_{i,t}}$ & $\mathrm{C_{j,t}}$ are configurattions for $\mathrm{M_t}$ and in m steps by transition functtion associatted to $\mathrm{M_t}$, we reach from $\mathrm{C_{i,t}}$ to $\mathrm{C_{j,t}}\}$ (Classical computational world)

We define $\circ$ over $\mathrm{W_m}$

$\circ\colon \mathrm{W_m} \times \mathrm{W_m} \to \mathrm{W_m}$

$$(C_{i,t} \overset{m}{\Rightarrow} C_{j,t}) \circ (C_{j,t} \overset{n}{\Rightarrow} C_{k,t}) = (C_{i,t} \overset{m+n}{\Longrightarrow} C_{k,t})$$

Furthermore, C induces a directed graph on the space of all configurations, like $\vec{\mathrm{G}}$. We call the underlying graph of this undirected graph, G.

(Remark1. In the case of nondeterministic Turing Machines, we define the concepts in the same way.)

By considering, fuzzy time, we have turning back in time. So, any path in the graph G, is a path of possible computation, when in our model we consider instants of time as fuzzy number.

In the definition of path here, the nodes could be repeated but the lengths of paths are finite. $\mathrm{P(G)}$ is the set of all paths of G.

$$W_{\mathrm{FUZZY}} = \left\{\mathrm{C_{i,t}} \overset{\mathrm{m}}{\Rightarrow} \mathrm{C_{j,t}}\colon m \in N, \mathrm{C_{i,t}} \text{ \& } \mathrm{C_{j,t}} \text{ belong to a path in } \mathrm{P(G)}\right\}.$$

We call $\mathrm{W_{FUZZY}}$ the Fuzzy world. In the fuzzy world, all of these paths are possible.

Here, any instant of time is a fuzzy number, which its support is R, the set of real numbers.*

Remark. Here, we are able to define the "fuzzy computational model" more exactly, in the case that the area under the instant of time is finite.

Computational-Model$= \{(C_{i,t} \overset{m}{\Rightarrow} C_{j,t}, \eta(C_{i,t} \overset{m}{\Rightarrow} C_{j,t}): \mathrm{m} \in \mathrm{N}, \mathrm{C_{i,t}}$ & $\mathrm{C_{j,t}}$ blong to a path in $\mathrm{P(G)}, \eta(\mathrm{C_{i,t}} \overset{\mathrm{m}}{\Rightarrow} \mathrm{C_{j,t}})$ is the probbility of reching from $\mathrm{C_{i,t}}$ to $\mathrm{C_{j,t}}$ in m steps$\}$

η could be computed by fuzzy function.

Now we define $\mathrm{R(M_t)}$ as the set of possible computational worlds for $\mathrm{M_t}$.

Let $\mathrm{R(M_t)} = \{\mathrm{W_{i,t}}\}_{\mathrm{i\in I}}$, which the following four conditions hold

1. $\mathrm{W_{i,t}} \subset \mathrm{W_{FUZZY}}$

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2. $\forall \mathrm{m}(\mathrm{C_{l,t}} \overset{\mathrm{m}}{\Rightarrow} \mathrm{C_{j,t}} \in \mathrm{W_{i,t}})$ implies there is a path between $\mathrm{C_{i,t}}$ &$\mathrm{C_{j,t}}$ in $\mathrm{W_{FUZZY}}$

3. $\mathrm{C_{l,t}} \overset{\mathrm{m}}{\Rightarrow} \mathrm{C_{j,t}} \in \mathrm{W_{i,t}}$ &$\mathrm{C_{l,t}} \overset{\mathrm{m}}{\Rightarrow} \mathrm{C_{k,t}} \in \mathrm{W_{i,t}}$ implies $\mathrm{k = j}$

4. $\mathrm{W_{i,t}}$ is closed by $\circ$.

Examples:

1. $\{\mathrm{C_{0,t}} \overset{\mathrm{m}}{\Rightarrow} \mathrm{C_{0,t}}: \mathrm{m} \in \mathrm{N}\} \in \mathrm{R(M_t)}$, void world.

2. $C_{l,t} \overset{m}{\Rightarrow} C_{j,t} \in W_{c,t}$ iff $C_{l,t} \overset{m}{\Rightarrow} C_{j,t}$ in classical time in Turing machine M_t. $W_{c,t}$ is the classical world associated to M_t.

2. In the case of Non determinism, we do not consider the third condition in above.

Now, we define $S = \{(w_{i,t})_{t\in N}: W_{i,t} \in R(M_t)\}$, this is the possible worlds of computation. We give here two members of S as examples.

1. Void world of computation. $V \in S$ is Void world of computation by definition if any component Of V is a void world.
2. Classical world of computation. $W_{classical} \in S$ is classical world by definition, if any component of V is a classical world.

2. Complexity Classes

In this section first we define the related complexity classes, secondly we give the proof.

Definition. The problem X is solved by $W_{k,t}$ in polynomial time means, for some polynomial function p and in less than $p(|a|)$ steps we have either 1 as output if a belongs to X or we have 0 as output if a does not belong to X.

Definition. For $\mathrm{W_k} \in \mathrm{S}$, $\mathrm{X} \in (\mathrm{P}, \mathrm{W_k})$, or is a $(\mathrm{P}, \mathrm{W_k})$ problem if X is solved by $\mathrm{W_{k,t}}$ in polynomial time, which $\mathrm{W_{k,t}}$ is a component of $\mathrm{W_k}$.

Definition. $\mathrm{X} \in (\mathrm{NP}, \mathrm{W_k})$, if for some polynomial function Q there is a set $\mathrm{Y} = \{(x, a): x \in X,$ |a | is less than $\mathrm{Q}($ |x |$)\}$, such that $\mathrm{Y} \in (\mathrm{P}, \mathrm{W_k})$.

Remark. In the case $\mathrm{W_k} = \mathrm{W_{classical}}$, it is easy to see that, the above definition is equivalent to the following definition

Definition′. $\mathrm{X} \in (\mathrm{NP}, \mathrm{W_{classical}})$, if X is solvable by non deterministic Turing machine in polynomial time.

Actually, $\mathrm{X} \in (\mathrm{NP}, \mathrm{W_{classical}})$ if and only if $\mathrm{X} \in \mathrm{NP}$ and $\mathrm{X} \in (\mathrm{P}, \mathrm{W_c})$ iff $\mathrm{X} \in \mathrm{P}$.

Poposition. $\mathrm{X} \in (\mathrm{P}, \mathrm{W_{classical}})$ iff $\mathrm{X} \in \mathrm{P}$.

Poposition. $\mathrm{X} \in (\mathrm{NP}, \mathrm{W_{classical}})$ iff $\mathrm{X} \in \mathrm{NP}$.

Poposition. $\mathrm{SAT} \in (\mathrm{P}, \mathrm{W_{classical}})$ then $\mathrm{P} = \mathrm{NP}$.

The concepts $\mathrm{m} - \mathrm{reucibility}$ and $(\mathrm{NP}, \mathrm{W_{classicl}}) - \mathrm{compelte}$ is defined similar to the classical case.

Poposition. $X \in (\mathrm{NP}, \mathrm{W}_{\mathrm{classical}}) - \mathrm{complete\ iff}$
$\mathrm{X} \in \mathrm{NP} - \mathrm{compelete.}$

The concepts like seed and pseudorandom generator are defined analogues to the classical definition.

Corollary. If pseudo random generator exists, $(\mathrm{P}, \mathrm{W}_{\mathrm{classical}}) \neq (\mathrm{NP}, \mathrm{W}_{\mathrm{classical}})$, (i.e $\mathrm{P} \neq \mathrm{NP}$)

Proof. If $\mathrm{P} = \mathrm{NP}$ we are able to guess the seeds non deterministically, so pseudo random generator does not exist.

$\mathrm{W}_{\mathrm{classical}}$ is similar to the classical world of computation nevertheless time is a fuzzy concept. Due to fuzziness of time in this model of computational world, we have random generator, consequently $(\mathrm{P}, \mathrm{W}_{\mathrm{classial}}) \neq (\mathrm{NP}, \mathrm{W}_{\mathrm{classical}})$ **.** By the above proposition we have the following corollary.

Corollary. $\mathbf{P} \neq \mathbf{NP}$.

Conclusion. In the above proof, our presumption is the existence of a model for $\mathbf{TC}^{*}$. So we have,
$\mathrm{TC} + \mathrm{CON}(\mathrm{TC}^{*}) \vdash \mathrm{P} \neq \mathrm{NP}.$

Therefore, we have $\mathrm{P} \neq \mathrm{NP}$ so we have $\mathrm{P}^{*} \neq \mathrm{NP}^{*}$[1].

3. Polynomial Hierarchy

The second point is about PH. Independent of the oracle we use, the fuzzy time remains Fuzzy time and hence the supposed random generator remains random generator respect to the oracle Turing machines. In this case, analogues to the above argument we have arguments in all levels of hierarchy, consequently, the hierarchy never collapses.

(PSPACE^* is defined similar to P^*. ${\Sigma_n}^*$, ${\Pi_n}^*$

${\Sigma_n}^* - \mathrm{Compelete}$ ${\Pi_n}^* - \mathrm{Compelete}$, are defined similar to

(NP^*, $\mathrm{Co}-\mathrm{NP}^*$, $\mathrm{NP}^*-\mathrm{Compelete}$, $\mathrm{Co}-\mathrm{NP}^*-\mathrm{Compelete}$).

$\mathrm{P} \subsetneq \mathrm{NP} \subsetneq \mathrm{PH}$ and $\mathrm{P} \subsetneq \mathrm{NP} \subsetneq \mathrm{PSPACE}$

($\mathrm{P}^* \subsetneq \mathrm{NP}^* \subsetneq \mathrm{PH}^*$ and $\mathrm{P}^* \subsetneq \mathrm{NP}^* \subsetneq \mathrm{PSPACE}^*$).

So, $\mathrm{PH} \subsetneq \mathrm{PSPACE}$ a parallel proof shows, $\mathrm{PH}^* \subsetneq \mathrm{PSPACE}^*$.

To do more exactly, we show, there exists $\mathrm{PSPACE}^* - \mathrm{Compelete}$, ${\Sigma_n}^* - \mathrm{Compelete}$, ${\Pi_n}^* - \mathrm{Compelete}$ problem. Actually, it is easy to show by theorems in [2]

> **Poposition**. $\mathrm{X} \in \mathrm{PSPACE} - \mathrm{Compelete}$ then $\mathrm{X} \in \mathrm{PSPACE}^* - \mathrm{Compelete}$.
>
> **Poposition**. $\mathrm{X} \in \Sigma_n - \mathrm{Compelete}$ ($\Pi_n - \mathrm{Compelete}$) then $\mathrm{X} \in {\Sigma_n}^* - \mathrm{Compelete}$ (${\Pi_n}^* - \mathrm{Compelete}$).

Remark. In the above conclusion, some seems to be theorems in TC but actually, we need $\mathrm{CON}(\ \mathrm{TC}^{*})$ and existence of a model for TC^{*} to prove it. It is noticeable that, our language is not first order. More exactly, we have

3. $\mathrm{TC}\ +\ \mathrm{CON}(\ \mathrm{TC}^{*}) \vdash \mathrm{P} \neq \mathrm{NP}, \mathrm{P} \subsetneq \mathrm{NP} \subsetneq \mathrm{PH} \subsetneq \mathrm{PSPACE}$

The second type of conclusions, needs TC^{*} as premises too,

2. $\mathrm{TC}\ +\ \mathrm{CON}(\ \mathrm{TC}^{*}) + \ \mathrm{TC}^{*} \vdash \mathrm{P}^{*} \neq \mathrm{NP}^{*}, \mathrm{P}^{*} \subsetneq \mathrm{NP}^{*} \subsetneq \mathrm{PH}^{*} \subsetneq \mathrm{PSPACE}^{*}$

In above, by $\mathrm{CON}(\mathrm{T})$ we mean theory T is consistent and has a model.

As a corollary, $\mathrm{TC}\ +\ \mathrm{CON}(\ \mathrm{TC}^{*}) + \ \mathrm{TC}^{*}$ deduces graph isomorphism is not a NP-Complete problem.

(Lost flash , app two weeks ago. Farvardin, Ramadan)

References

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