

Can Bayesianism Solve Frege's Puzzle?

Jesse Fitts

Philosophia

Philosophical Quarterly of Israel

ISSN 0048-3893

Volume 49

Number 3

Philosophia (2021) 49:989-998

DOI 10.1007/s11406-020-00283-6

Your article is protected by copyright and all rights are held exclusively by Springer Nature B.V.. This e-offprint is for personal use only and shall not be self-archived in electronic repositories. If you wish to self-archive your article, please use the accepted manuscript version for posting on your own website. You may further deposit the accepted manuscript version in any repository, provided it is only made publicly available 12 months after official publication or later and provided acknowledgement is given to the original source of publication and a link is inserted to the published article on Springer's website. The link must be accompanied by the following text: "The final publication is available at link.springer.com".



Can Bayesianism Solve Frege's Puzzle?

Jesse Fitts¹

Received: 30 March 2020 / Accepted: 9 October 2020 /
Published online: 17 November 2020
© Springer Nature B.V. 2020

Abstract

Chalmers (*Mind*, 125, 499–510, 2016), responding to Braun (*Mind*, 125, 469–497, 2016), continues arguments from Chalmers (*Mind*, 120, 587–636, 2011a) for the conclusion that Bayesian considerations favor the Fregean in the debate over the objects of belief in Frege's puzzle. This short paper gets to the heart of the disagreement over whether Bayesian considerations can tell us anything about Frege's puzzle and answers, no, they cannot.

Keywords Bayesianism · Propositions · Propositional attitudes

1 Introduction

David Chalmers¹ has presented a novel argument for Fregeanism over referentialism, which, as Chalmers (2016, p.499) defines it, is “roughly ... the view that the objects of belief are constituted by the objects and properties that the belief is about.” If Bayesianism is true, Chalmers argues, then referentialism about belief is false. In response, Braun (2016) and Fitts (2014) argue that Bayesian considerations do not tell against the referentialist or in favor of the rival Fregean.

This short debate that Chalmers has initiated involves, according to Chalmers himself, some talking past in addition to some interesting issues that, in my view, aren't at the crux of the debate. In this short paper I identify what I think is the crux of Chalmers' argument that has been obscured. In Chalmers' argument from Bayesianism against referentialism, we can either identify the objects of credence and belief with the objects in the domain of credence functions, or we cannot make such an identification. Chalmers' argument is strongest *if* we make the identification, but as I

¹Chalmers (2011a, 2016)

✉ Jesse Fitts
jjfitts@illinois.edu

¹ University of Illinois Urbana-Champaign, 810 S Wright St - MC468 200 Gregory Hall, Urbana, IL 61801, USA

will argue, there are independent reasons to doubt the identification. But if we don't make the identification, Chalmers' argument is much less plausible.

The main thrust of my argument that I explore in §3 is that the domain of a credence function is part of a *model*, and as such we can't read off anything interesting about the *metaphysics* of partial and full belief from the model. If this is correct, then if we identify the objects of credence with the objects in the domains of credence functions, then we would mistakenly conflate a model with its target. If I'm right, then Bayesian considerations don't favor the Fregean, and the Frege's puzzle situation reverts to the status quo. While the focus of this paper is on the objects of belief and credence, the arguments below will allow us to discuss the perhaps under-discussed and interesting issue of the metaphysics of formal epistemology, if only for a moment.

Two points before moving on: First, the just-mentioned talking past has involved Chalmers' focus on the objects of belief and credence, a topic in the metaphysics of mind, and the objects of attitude reports, a topic in the philosophy of language. One of Chalmers' main complaints against Braun is that Braun conflates the two, resulting in counterarguments that misfire. My argument below *does not* conflate these two perspectives. Second, the literature on the objects of the attitudes is *massive*, many of the moves in the literature are well known, and one may think that the debate has reached a stalemate. My argument is focused squarely on the novel aspects of Chalmers' arguments, and my conclusion is that the Bayesian arguments do not move the debate forward. Below I defend one kind of referentialism, a version that has went by the names of "sophisticated referentialism," "neo-Russellianism," and sometimes "contextualism." The basic idea is that the objects of our beliefs (and other attitudes) are referentialist, but some non-referentialist component mediates our belief in these referentialist propositions. There are well-worn objections to this theory, and it's important to me in this paper that we bracket those objections and focus only on the novel aspects of Chalmers' argument. Thus, for example, one may think that there's no real difference between saying that the objects of belief themselves are Fregean or whether the objects of belief are referentialist but some Fregean component mediates our belief in these propositions. Fair enough, but if you think that, you should have thought that *before* Chalmers' new argument. My challenge to Chalmers' arguments is entirely focused on the argument from Bayesianism against referentialism, and I am not mounting a full-scale defense of referentialism in this paper. But before presenting my argument, let's first quickly rehearse Chalmers' terminology, argument, as well as its responses.

2 Background

First some terminology:

Referentialism The thesis that the object of an attitude—a proposition—about an individual (or individuals) and a property or relation is wholly determined by that individual (or individuals) and property or relation.

Bayesianism The thesis that agents have subjective degrees of belief that we can represent with a credence function and that an agent updates her credences via conditionalization: upon receiving some new evidence, her new credence in a given proposition will be her old credence in that proposition given that evidence, if defined. Sometimes there is a further requirement that a perfectly rational agent's credence function is a probability function and so conforms to the axioms of probability—we'll call this thesis "probablism"—but Chalmers doesn't require this.

For the sake of this dialectic, grant that Bayesianism is true. Referentialism, strictly speaking, is a thesis about the identity conditions of propositions—it says nothing about how we bear attitudes toward them. It entails that, e.g., the proposition that Hesperus is a planet is identical to the proposition that Phosphorus is a planet. Yet, as the intuition goes, we can bear different attitudes toward this object. To accommodate this intuition, some referentialists claim that a non-referentialist component, a *guise*, mediates our attitudes toward referentially individuated propositions.² The quintessential referentialist theory is Russellianism: the proposition p is the tuple $\langle \langle e_1, \dots, e_n \rangle, R^n \rangle$ where $\langle e_1, \dots, e_n \rangle$ is an n -ary tuple of *objects* (not senses thereof) and R^n is an n -ary *relation/property* (not senses thereof), and p is true just in case the entities stand in the relation/instantiate that property. Since "Hesperus" and "Phosphorus" are just two names for the same thing, that Hesperus is a planet and that Phosphorus is a planet both express the proposition $\langle v, P \rangle$, where Venus itself occupies the first slot and the property of being a planet itself occupies the second. It will be easier to refer to a particular referentialist theory, so below we'll use Russellianism.

Onto Chalmers' argument. The referentialist can either include her guises in the Bayesian framework or not. Both lead to untoward results. If she doesn't, then Bayesianism is false. Why? First, the credence "function" wouldn't be a function: that Hesperus is a planet and that Phosphorus is a planet—the same argument to the "function"—may get mapped to different credences. Second, conditionalization is false: someone ignorant of the Jekyll–Hyde identity may obtain evidence bearing on Hyde. She should update her credence in propositions involving Jekyll as well as those involving Hyde upon obtaining this evidence if referentialism is true—since all relevant Jekyll-involving propositions and Hyde-involving propositions are identical. But, according to Chalmers, this is wrong. To correct this problem, the referentialist may try to fit her guises into the Bayesian framework: instead of the credence function mapping propositions to credences, it now maps guise–proposition pairs to credences. This preserves the truth of Bayesianism, but now referentialism is false. There are two problems depending on how one interprets the relationship between the credence function's domain and the objects of belief and credence. The first is to identify them all. The problem here is simply that the objects in the credence function's domain and the objects of belief and credence are not referentialist

²See, e.g., adherents of the so-called "hidden indexical theory," such as Schiffer (2007/1992) and Crimmins and Perry (1989/2007).

propositions but rather guise–proposition pairs. The second is to not identify the objects in the domain with the objects of credence and belief but to require that the objects in the domain of the credence function have important properties if referentialism is true, namely that those objects behave referentially. The problem here is that guise–proposition pairs don't behave referentially. In my view, I think if we take the second route and don't make the identification, Chalmers' argument is implausible for reasons that have been made elsewhere, which I briefly note below. In this paper, I give reasons to doubt that we should identify the objects of belief and credence with the objects in the domain of the credence function.

3 Bayesians as Modelers

This is what a particular credence function with guises included looks like for the referentialist option under way:

$$\{ \langle \langle G_{p_1}, p_1 \rangle, r_1 \rangle, \langle \langle G_{p_2}, p_2 \rangle, r_2 \rangle, \dots \},$$

where G_{p_i} is a guise, p_i a Russellian proposition, and r_i some number in the interval $[0, 1]$. The domain of a credence function comprises the first members of the above pairs: the set containing each $\langle G_{p_i}, p_i \rangle$. Chalmers (2011a, p. 601) argues against this option as follows:

But this view now says that the objects of credence, as we are understanding them, are such ordered pairs. If so, the objects of credence behave in a non-referential way. . . , and referentialism will be false of the objects of credence.

Recall that “referential” expresses a property of propositions—the property that the identity conditions of an individual-involving proposition are entirely determined by the property/relation and the individuals involved in that proposition. What does it mean to say that a guise–proposition pair behaves referentially or not? It may mean that whether an agent has an attitude in a referentialist proposition is determined by more than just that proposition—that something up and above $\langle v, P \rangle$ determines whether an agent believes that Hesperus is a planet or Phosphorus is a planet. But this is just the previously mentioned second option in the introduction—that the objects in the domain of the credence function should behave referentially. Briefly, this shouldn't worry the referentialist for at least two reasons. First, the Bayesian context is superfluous if the objection is that non-referentialist attitude conditions impugn referentialism. Second, those conditions are *meant* to be non-referential (on some referentialist theories).³

If we instead identify the objects in the domain of the credence function with the objects of belief and credence, then the objection is this. The objects of the attitudes, for referentialists, are referential propositions. But, if the objects in the domain of the credence function are the objects of credence, then agents don't have credences in propositions. *They have credences in guise–proposition pairs, and whether we call*

³See Fitts (2014) for more on this.

these guise–proposition pairs referential (or referentially behaving) or not is beside the point. The objection really tells against anyone that thinks that the objects of credence, belief, hope, etc., are propositions of *any kind*, but, for reasons having to do with her theory of propositions, she is required to put something in addition to her propositions in the domain of the credence function. The referentialist claims that the objects of the attitudes are referentialist propositions, but guises mediate our attitudes. We believe, or partially believe, that Hesperus is a planet *via* a guise. The referentialist doesn't think we believe, partially believe, hope, etc., guise–proposition pairs. That doesn't make sense. We've reached a situation in which Chalmers' argument is implausible if we don't identify the objects in the domain of the credence function with the objects of belief and credence, but the argument seems more plausible if we do make the identification.

3.1 Bayesianism and Modeling

Should we make such an identification? To answer this question, we need to get clear on just what a credence function is. This is a messy project because different theorists likely have different views on the metaphysics of formal epistemology, and formal epistemology is put to diverse uses—in the study of ideal rationality, artificial intelligence, in the study of actual humans, etc. To focus our task, it's important to note that Chalmers is interested in the objects of *belief* and partial belief, so it's safe to assume that he's interested in the objects of credence in that he's interested in the partial belief states of actual humans—not ideal agents, robots, etc. So when a formal epistemologist considers credence functions vis-à-vis actual humans, what is it that she is considering? I think a plausible answer is that credence functions are *models*. Now if you pick up just about any article on formal epistemology and search for the word “model,” you will likely find it. What I mean is that credence functions are models in the sense often discussed in the philosophy of science literature,⁴ though modeling methodology is not limited to just the sciences.⁵

Bayesians, insofar as they are concerned with actual humans, model in a way that fits with other modelers that employ idealized formal devices to study some aspect of human psychology and behavior—modelers such as some syntacticians and formal semanticists. A sign that modeling is present, Bayesianism involves an enormous amount of idealization as well as a plurality of representational formats.⁶ There are various options on offer for what exactly a degree of belief is, whether they be betting behaviors, or some mental state we take as basic, or something else.⁷ Whatever the case may be, Bayesians study humans in an indirect way by studying a mathematized model rather than engaging in what Weisberg (2013) calls “abstract

⁴See, e.g., Frigg and Hartmann (2018).

⁵See Paul (2012) for the persuasive view that metaphysicians also model. More on this in a moment.

⁶For example, while we canonically represent conditionalization as an equation—upon receiving evidence e , an agent should update her credences to $cr_e(\cdot) = cr(\cdot|e)$ if defined—we can represent this idea in a multitude of ways, such as the muddy Venn diagram of van Fraassen (1989, p.178).

⁷Eriksson and Hájek (2007) explore the options for what credences are and conclude that we should take them as basic.

direct representation” in which theorists study their target directly. His paradigm for abstract direct representation is Mendeleev’s creation of the periodic table, in which he represented key properties of atoms directly.

To make this more concrete, let us apply a particular theory of scientific modeling to the foregoing discussion—the *semantic view*. Here, I am following and extending the view from Paul (2012), who argues that metaphysicians model, and we are in the context of the *metaphysics* of partial and full belief. According to the semantic view, scientific theories (those that model) are sets of abstract structures comprising entities with relations among those entities that when interpreted are the theory. Here are two examples from Paul (2012, §2.2) of metaphysical modeling. First, Paul considers a mereological composition theory view according to which some x s compose a y if but only if the x activity constitutes a life. The model in this case are abstract objects that stand in part–whole relations and these structures allegedly represent parts composing wholes that constitute a life. The metaphysical theory is the class of abstract structures, and the target of the model are the parts and wholes that those models represent. The theory is successful if these structures *in fact* represent the target correctly. Second, Paul considers a simple counterfactual theory of causation according to which c causes e if but only if e wouldn’t have occurred had c not occurred. The theory here consists of models that are structures that allegedly represent counterfactual relations between events, and if these models in fact represent actual world causal relations, then the theory is successful.

Let’s extend these ideas to Bayesianism. Models for Bayesianism consist of credence functions defined over a domain of suitable objects (more about this soon), a range of real numbers in $[0, 1]$, and various normative constraints on the credence function. These, as noted, typically include the synchronic constraint of probabilism. But as we’ve presented Bayesianism, there is only one constraint on $cr(\cdot)$ (in addition to its being indeed a function): conditionalization. These structures, presumably, on one interpretation, represent agents and the strength of their partial belief states and the rational *normative* (also more on this) constraint on how those agents react to evidence.

Let’s suppose that Bayesians are modelers. Chalmers himself offers his own non-referentialist objects that he defines the domain of the credence function over: sets of epistemically possible scenarios. Chalmers has written extensively about scenarios.⁸ Technically, scenarios are centered worlds, where a centered world is

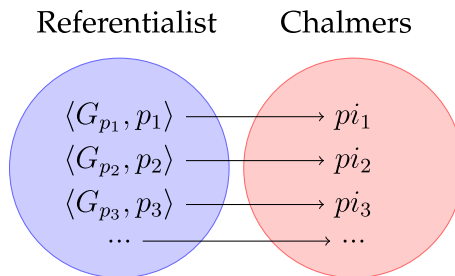
usually conceived as an ordered triple of a metaphysically possible world, an individual who is present in that world, and a time in that world (I discuss a refinement to this conception later). For any given subject s , at time t in (uncentered) world w , we can say that the centred world that s inhabits (at t in w) is $\langle w, s, t \rangle$. Chalmers (2011a, p.615)

And these scenarios represent epistemic possibility “in the sense where epistemic possibility goes with what cannot be ruled out a priori” (Chalmers 2011a, p.616).

⁸See e.g. Chalmers (2011b).

Importantly, Chalmers can characterize the domain of a credence function set theoretically.⁹

In this debate, a decisive reason in favor of Chalmers' objects of credence over the referentialist guise–proposition pairs would be if the referentialist proposal didn't provide *enough* objects in the domain of the credence function or if those objects didn't have the right similarity relation to their targets. In these cases, Chalmers' would provide a model that more accurately represents the relevant target. It's plausible that the referentialist does, however, provide for each of Chalmers' objects of credence a corresponding guise–proposition pair so that there is a bijection between the guise–proposition pairs and primary intensions, where each pi_i is a primary intension:



This is partially because modes of presentations just should be the entities that satisfy what Schiffer (2006, p.362) calls *Frege's Constraint*:¹⁰

- (A) x is believed by y to be such that... x ...iff $\exists m$ (m is a mode presentation of x & x is believed by y under m to be such that ... x ...).
- (B) For any modes of presentation m and m' of x , if x is rationally both believed by y under m to be such that ... x ... and either disbelieved or not believed by y under m' to be such that ... x ..., then y does not take m and m' to be modes of presentation of the same thing.

In other words, for the purposes of this paper, guises will just be the entities such that an agent can't rationally believe and disbelieve the same proposition under the same guise.¹¹

But for the sake of argument, suppose that the referentialist couldn't produce the needed guise–proposition pairs. What would this mean? This would mean that at the level of the target of the model, an agent could possibly have a partial belief in some proposition, but the referentialist couldn't capture that at the level of the Bayesian model. If that were the case, that means that there either wouldn't be enough Russellian propositions or guises to go around, or both. We've made the assumption that the objects of partial and full belief are identical, while questioning that the objects in the domain of the credence function are identical to both. Thus, *if* the critical objection under consideration succeeded—that the referentialist's model were

⁹See §11 of Chalmers (2011a) for a probabilistic understanding of scenarios.

¹⁰Schiffer originally called it this in Schiffer (1978).

¹¹This formulation is borrowed from Chalmers (2011c, p.607).

impoverished vis-à-vis Chalmers' model—then this would be a critical objection. However, *we could form this objection without the Bayesian context*.

Similar considerations hold for other aspects of our competing models. For example, suppose Chalmers' model vindicated the credence function's functionhood, allowed for rational credences less than 1 in propositions such as Hesperus is Phosphorus, and allowed for agents to rationally fail to update on evidence that they shouldn't properly recognize as evidence—all while the referentialist's model failed to vindicate these alleged *desiderata*. Again, though: *we could form this objection without the Bayesian context*. If the referentialist with her guises didn't provide the correct partial belief conditions, then she also wouldn't provide the right *full* belief conditions. To put this another way, some referentialists appeal to guises to capture the intuitive conditions under which agents bear attitudes toward what are, metaphysically, referentialist objects. If the referentialist's guise-proposition pairs didn't vindicate Bayesianism, then there would *already* be a problem at the level of full belief. But if that is the case, then the Bayesian context is superfluous: there would be independent arguments in the well-worn context of full-belief against this version of referentialism.

3.2 Further Model Considerations

If we understand Bayesians as modelers, then we need not understand the objects in the domain of the credence function as the objects of belief and credence. And as I've argued, either the referentialist will be able to provide enough objects in the model with the right structure or, if she weren't able to do this, this problem would have cropped up in the context of belief, making the Bayesian context superfluous. Yet there may still be further model-theoretic reasons to prefer the referentialist or the non-referentialist. In terms of modeling, there is something different about Bayesian epistemology that separates it from other theorists such as syntacticians and formal semanticists: the Bayesian's model is *normative*. Typical examples of scientific models are descriptive and explanatory. To take an example that commonly appears in the modeling literature,¹² consider the Lotka-Volterra model of population growth. In the model, a pair of differential equations describes and predicts population dynamics. The model involves different *kinds* of idealizations, as Colyvan (2013, p.1339) notes:

- (1) Population abundance is discrete and yet the model treats it as continuous.
- (2) The model treats the growth rates as constants.
- (3) The model treats the predator as a specialist (i.e. the predator eats only the prey).
- (4) The model treats the prey as having only one predator.
- (5) Responses to changes in population abundances are instantaneous.

Colyvan (2013) separates idealizations for *mathematical convenience*, such as (1), from *close enough* idealizations such as (3)—since the relevant predators are almost specialists but sometimes eat other things.

¹²See, especially, the work of Micael Weisberg, e.g., Weisberg (2013).

When we move to the normative realm, we simply treat the normative constraints as idealizations. In the formal epistemology case, the normative constraints are those that rationality require. Colyvan (2013, p.1341) uses the case of decision theory:

- (6) Beliefs come in (continuous) degrees.
- (7) Utilities are dense (and usually represented to be continuous as well).
- (8) Connectedness: there are no incommensurable outcomes.
- (9) Preferences are transitive: if p_1 is preferred to p_2 , and p_2 is preferred to p_3 , then p_1 is preferred to p_3 .
- (10) The Archimedean axiom: Whenever an agent has preferences $p_1 < p_2 < p_3$ there will be a lottery (or “mixture”) of p_1 and p_3 such that the agent is indifferent between p_2 and the lottery.

(6) is a mathematical, not normative, idealization—rationality doesn’t require this, and (8) is a close-enough idealization. (9), however, is a normative idealization such that, if one fails to live up to it, then one isn’t fully rational.

Let’s examine the Bayesian case in light of these considerations. Probablism and conditionalization are both normative constraints. As far as I can tell, there aren’t any close-enough idealizations as we’ve stated Bayesianism. I have argued above that if Chalmers can provide enough objects of credence with the desired identity conditions and the referentialist can’t, then the Bayesian context, and so the novel aspect of Chalmers’ argument, is superfluous.

At this point, if I’ve argued successfully, then any benefit that Chalmers may accrue for his theory would be along the lines of mathematical convenience. And when we recognize that an idealization is of the mathematically convenient kind, the only criticisms that stick are those to the effect that the idealization is not, indeed, convenient. As noted, Chalmers’ own account of the objects of credence lend themselves to a set-theoretic treatment. Given the set theoretic structure of primary intensions, Chalmers can construct the domain of his credence functions elegantly—with a σ -algebra on the space of possibilities Ω : i.e., a collection of subsets from Ω that is closed under complementation and union.¹³ Characterizing the standard axioms for $cr(\cdot)$ follows naturally. The objection, then, to the account given in this paper is that my domain consisting of guise-proposition pairs isn’t convenient. The current objection may be a fair point, but I doubt this point will move the needle in the debate over the objects of belief.

References

- Braun, D. (2016). The objects of belief and credence. *Mind*, 125, 469–497.
- Chalmers, D.J. (2011a). Frege’s puzzle and the objects of credence. *Mind*, 120, 587–636.

¹³Or, more technically: a σ -algebra F is a non-empty collection of subsets of X such that

- $X \in F$
- $a \in F \rightarrow a^c \in F$
- If $\{A_n\}$ is a collection of subsets of F , then $\bigcup\{A_n\} \in F$.

- Chalmers, D.J. (2011b). The nature of epistemic space. In Egan, A., & Weatherson, B. (Eds.) *Epistemic Modality*. Oxford: Oxford University Press.
- Chalmers, D.J. (2011c). Propositions and attitude ascriptions: A Fregean account. *Noûs*, 45, 595–639.
- Chalmers, D.J. (2016). Referentialism and the objects of credence: a reply to Braun. *Mind*, 125, 499–510.
- Colyvan, M. (2013). Idealisations in normative models. *Synthese*, 190, 1337–1350.
- Crimmins, M., & Perry, J. (1989/2007). The prince and the phone booth: reporting puzzling beliefs. In Davidson, M. (Ed.) *On Sense and Direct Reference* (pp. 356–379). Boston: McGraw-Hill.
- Eriksson, L., & Hájek, A. (2007). What Are Degrees of Belief? *Studia Logica*, 86, 183–213.
- Fitts, J. (2014). Chalmers on the objects of credence. *Philosophical Studies*, 170, 343–358.
- Frigg, R., & Hartmann, S. (2018). The Stanford Encyclopedia of Philosophy. Metaphysics Research Lab, Stanford University, summer 2018 edition. In Zalta, E.N. (Ed.)
- Paul, L.A. (2012). Metaphysics as modeling: the handmaiden's tale. *Philosophical Studies*, 160, 1–29.
- Schiffer, S. (1978). The basis of reference. *Erkenntnis*, 13, 171–206.
- Schiffer, S. (2006). A problem for a direct-reference theory of belief reports. *Noûs*, 40, 361–368.
- Schiffer, S. (2007/1992). In Davidson, M. (Ed.) *Belief ascription*, (pp. 402–420). Boston: McGraw-Hill.
- van Fraassen, B.C. (1989). *Laws and Symmetry*. Oxford: Oxford University Press.
- Weisberg, M. (2013). *Simulation and Similarity: Using Models to Understand the World*. Oxford University Press.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.