Mathematical Scepticism: the Debate between Hobbes and Wallis

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Contra Geometrtas (amice Lector) non Contra Geometriam, haec scribo.

Hobbes [1666]

1. Introduction

The label "mathematical scepticism" was coined in Floridi [1998] to refer to the body of sceptical arguments developed against mathematical knowledge, its methodology and nature (e.g. its certainty, reliability, necessity etc.), and hence to a sceptical philosophy of mathematics, not to some form of "mathematised" scepticism.

Little is known about mathematical scepticism in modern times.[1] Imre Lakatos once remarked that "in discussing modern efforts to establish foundations for mathematical knowledge one tends to forget that these are but a chapter in the great effort to overcome scepticism by establishing foundations for knowledge in general" (Lakatos [1978], 4). And in a sense he was clearly right: modern thought—with its new discoveries in mathematical sciences, the mathematisation of physics, the spreading of sceptical doctrines, the centrality of epistemological foundationalism, the diffusion of the geometrical method in philosophy and the foundationalist crisis in mathematics itself—was the most natural arena in which scepticism and mathematics could confront each other. The problem remains, however, that no full investigation of the whole topic has yet been attempted. Thus, as far as we know, mathematical certainties should have clashed with sceptical doubts, but whether and to what extent there was indeed a fierce debate on mathematical scepticism in modern thought remains to be ascertained.

The worst way to cope with the conceptual amnesia highlighted by Lakatos would be to implant an utterly new memory in our system of knowledge. Fortunately, there is no need to fabricate an ideal history of the *Zeitgeist*. We can be more moderately Platonist and work towards a recollection of our intellectual past by uncovering the actual roots of our knowledge. Apart from Montucla's *Histoire des mathematiques*,[2] there are several primary sources[3] that contain a direct discussion of mathematical scepticism in modern times. The debate between Hobbes and Wallis is one of them, and the following pages are devoted to its analysis. Here is a brief summary. Sections 2 and 3 provide the necessary background against which the remarks on mathematical scepticism, occurring in the debate between Hobbes and Wallis, need to be interpreted. Sections 4 and 5 introduce Hobbes' and Wallis' position respectively. Section 6 contains an analysis of the references to mathematical scepticism in the debate between Hobbes and Wallis. Section 7 offers an interpretation of the role played by scepticism in Hobbes. Section 8 provides a conclusion. For the sake of simplicity, I have confined the textual evidence to two appendices.

2. The Many Uses of Mathematical Scepticism

Until Descartes and his "malicious demon" argument, Sextus Empiricus provided the main source of information about mathematical scepticism and the most powerful battery of arguments in its favour. Since the recovery of Sextus' texts during the Renaissance (especially *Contra Geometras* and *Contra Arithmeticos*, see Floridi [1995] and Floridi [2002]; Popkin [1979]), his anti-mathematical arguments have been used in five main ways:

a) the philological use, made by readers interested in linguistic matters or in what Sextus says about other authors, Euclid included. In different ways, this is the case in Vossius (Vossius [1660]),[4] Bochner,[5] or, more interestingly, scholars such as Heiberg or Heath;

b) the polemic use. According to Henry Savile, for example (see section 3), to be a sceptic in mathematics is to be a fool;

c) the anti-intellectualist use, made for example by Gian Francesco Pico della Mirandola. Mathematics is attacked by means of Sextus' arguments, but with the principal intention of undermining the dogmatist's excessive faith in human knowledge. During the sixteenth and seventeenth centuries, the main polemic target was usually Aristotelianism;[6]

d) the epistemological use. Sextus and his followers challenge mathematics not as a goal in itself, but as part of a more encompassing strategy against knowledge in general; and finally

e) the foundationalist use, which employs sceptical arguments in order to investigate and test the solidity and reliability of mathematical knowledge. This is Montucla's interpretation, for example, who explicitly connects it to Descartes.

The difference between (c) and (d)-(e) is easily clarified once we realise that there are a number of problems Sextus never mentions in *Contra Geometras*. Although it would have been in keeping with the sceptical strategy of accumulating any sort of arguments in order to undermine the dogmatic position, Sextus never rejects what is stated by the postulates or the common notions, possibly because, whoever is the source of *Contra Geometras*, he did not mean to be cut off from discussion with other geometricians. He does not question either the fifth postulate or the use of superposition. And, more importantly for our present topic, Sextus completely disregards each of the three classic problems of Greek geometry, which were so well known in his time, namely the duplication of the cube (how to construct the edge of a cube having twice the volume of a given cube), the trisection of an angle (how to divide a given arbitrary angle into three equal angles) and the famous quadrature of the circle (how to construct a square having an area equal to that of a given circle). We know nowadays that none of these problems can be solved, except by approximation, with an unmarked straight edge and compasses, that is via algebraic methods, but the three problems were still discussed as open questions in the seventeenth century. Hobbes' attempts to solve both the quadrature of the circle and the duplication of the cube, although erroneous, were far from being unreasonable in principle. Now, a foundationalist attack against the roots of geometry, or more generally an epistemological challenge, had no great interest in investigating such issues. Considered simply as difficulties that had yet to be solved because they were particularly complex, their destiny would depend on the status of geometry as a science of space, not *vice versa*. On the contrary, a general denunciation of the intellectual ambitions of mathematicians could ingeniously exploit such clear cases of failure, presenting them as a reminder of the limits of human knowledge. In line with this interpretation, we observe Agrippa,[7] Sanchez and Guy de Brués all making use of the geometrician's incapacity to square the circle to stress the limits of mathematical knowledge. When Sextus (or his sources) criticised geometry, he had a more scientific aim in mind. For the same reason, in the debate between Hobbes and Wallis, we find mathematical scepticism mentioned only in the foundationalist sense listed under (e). No connection is ever drawn between the three classic problems and mathematical scepticism in the sense of (c). Hobbes himself, who had a foundationalist programme of research in geometry (more on this in section 4), seems to have decided to tackle the quadrature initially only as a *famous* problem, whose solution would have shown the value of his "materialist" approach in geometry. He did not mistake it as a *fundamental* issue, on whose solution the future of geometry should depend. Later, it became a crucial point of debate between him and Wallis, but only because they both managed to transform it into a criterion to evaluate Hobbes' mathematical competence, and this for extra-mathematical reasons, as we are going to see in section 4 and 5.

3. The Many Strategies of Mathematical Scepticism

Mathematical knowledge is usually appreciated for being, among other things: (1) consistent, (2) necessary, (3) *a priori*, (4) certain, (5) universal, (6) infallible and (7) epistemically informative about the world. Any philosophy of mathematics is faced by the task of ordering these (as well as other) positive features and accounting at least for the most fundamental ones. Of course, not all philosophies adopt the same priorities or must necessarily attempt to preserve the whole list, but in order to qualify as sceptical a philosophy of mathematics must deny at least (7). As Proclus remarked in his *Commentary on Euclid*: "Up to this point we have been dealing with the principles, and it is against them that most critics of geometry have raised objections, endeavouring to show that these parts are not firmly established. Of those in this group whose arguments have become notorious some, such as the Sceptics, would do away with all knowledge, like enemy troops destroying the crops of a foreign country, in this case a country that has produced philosophy [...]"(Proclus [1970], 199).

Thus, the negation of (7) is the necessary feature shared by the Pyrrhonian (P), the Cartesian (C), the Humean (H) and the Wittgensteinian (W) strategy.[8] There are then other affinities and, more importantly, contrasts between the four strategies, depending on what other features in the list they attack:

(P) is the most radical strategy, for it attempts to undermine (7) by moving a general attack against {(1), (2), (3), and (4)};

(C) is less radical than (P), for it attacks (7) indirectly, by challenging (4);

(H) is even more moderate, for it attempts to show that either {(1), (2), (3), (4), (5), (6)} is the case or (7) is the case, but since the former, then not the latter. In Hume, the sceptical challenge against mathematics acquires a new nature: it attempts to exclude mathematics from the epistemological debate as empirically/epistemically irrelevant.

For historical reasons, (P), (C) and (H) all address mainly geometry rather than arithmetic. Strategy (W) takes an orthogonal different approach, since it discusses mathematics as a body of human procedures, practices or activities of calculation and measurement, which are then challenged as inevitably fallible. Therefore, the rejection of (7) comes through the rejection of (6).

The strategy that interests Hobbes is (P), so let us analyse it at more length.

3.1 Mathematical Scepticism: the Pyrrhonian Strategy

The Pyrrhonist strategy is based on a thoroughly empiricist view.[9] Sextus treats geometrical entities as if they should maintain some resemblance to material objects in order to be meaningful at all. He adopts this line of reasoning because of an empiricist epistemology according to which:

i) the logical possibility of an object is equivalent to the possibility of conceiving it; but

ii) no conceivable object can be a purely mental object;[10] therefore

iii) a conceivable object cannot be completely void of empirical content but must preserve some mimetic feature.

The empiricist tone of Sextus' criticism suggests that the epistemological turn, i.e. a clear focus on what the mind can know about the mathematical realm, has already occurred. It is a criticism justified by the fairly concrete approach adopted by Euclid himself in the *Elements.*

I remarked above that (P), (C) and (H) discuss mathematical scepticism by concentrating above all on Euclidean geometry. For centuries, the *Elements* were the most popular and influential paradigm of a deductive body of knowledge, very often the only one with which most educated people might have had any acquaintance. Like the Bible, it is one of those texts that have shaped Western culture. In them, we encounter the classic metaphor of the building as a model for the structure of knowledge, a metaphor that, together with the image of the tree of knowledge, will become common currency within any foundationalist project. The unique style of the work, which contributed so significantly to its popularity throughout the centuries, is the result of an admirable balance between empirical intuition and logical postulates, visual thinking and purely rational deductions. The overall structure of the thirteen books bears witness to a remarkable effort made towards the systematic construction of an abstract, universal and logically rigorous body of mathematical knowledge, in which 465 theorems are formally inferred from a limited number of first principles explicitly stated at the outset.[11] And yet, a fundamental empiricism still pervades the whole work. For example, the criterion of existence, provided by the notion of geometrical constructability, is justified by an empiricist approach, which came to be perceived as too limited only in the nineteenth century. And one needs to mention only the first proposition of Book I, which requires an equilateral triangle to be constructed on a given finite straight line, to recall that the very notion of demonstration often relies on the visualisation of the theorem in question.

As the limited use of the fourth ("Things which coincide with one another are equal to one another") and fifth ("The whole is greater than the part") postulate shows, Euclid was at least partially aware of such "realistic" features of his geometry and perhaps not thoroughly happy with them.[12] If he could still regard them as unproblematic, it was because of a more fundamental assumption underlying the *Elements*. Classic geometry was thought to be the result of the correct *idealisation* of the properties of physical space and the corresponding behaviour of extended bodies in that space. And since "Euclidean" geometry was the abstract grammar of physical space, until the nineteenth century space was understood as intrinsically Euclidean, that is, as Poincaré [1905] clearly put it, three-dimensional, (at least potentially) infinite, continuous (no gaps), homogeneous (no privileged points), isotropic (no privileged directions through any point, i.e. equal in every direction), and such that any discrete object in it would satisfy the theorems of Euclidean geometry. Given such a strict relation between space and geometry, theorems were supposed to be true descriptions of actual features of physical space (*physicalisation of geometry*). The empirical truth of geometrical statements (*alethisation of geometry*) eclipsed the need for a tight verification of the formal consistency of the system and hence of the independence of its set of axioms. Sound proofs (valid inferences from true premises) rather than logically correct deductions (valid inferences in which it is never the case that the premise is affirmed and the conclusion is negated) represented the backbone of Euclidean geometry. Given such a moderate alethisation and physicalisation of geometry, empirical factors could not only be tolerated as useful aids to the understanding, but also appreciated as the semantic links whereby the geometrical system was tied to the natural world of empirical intuition. As we shall see, this was largely Hobbes' view as well. The interpretation of Euclidean geometry as the idealised model of physical space was explicitly conveyed by the characterisation of the most elementary geometrical objects in terms of abstract (in the strong sense of *abstracted*) entities. Insofar as points, lines and surfaces have unique properties they are no longer physical objects, but insofar as they are the result of an evident process of refinement and generalisation from particular objects of intuition they are not "mere" logical constructs either, which may not be amenable to physical (let alone visual) interpretation. Again, we find a strong echo of this interpretation in Hobbes' philosophy of geometry. Such abstract objects seem to enjoy a peculiar ontological status. They are not like other physical objects, but they are linked to perception and the real world via the criterion of conceivability in imagination, which is precisely the criterion exploited by Sextus Empiricus in the construction of his paradoxes. In *Book I*, Euclid provides 5 geometrical postulates and 5 more general "common notions" without any further justification. They are left unproved because of their self-evidence. Before this, Euclid lists 23 definitions that are supposed to clarify in terms that are more intuitive his technical vocabulary. The logical utility of such definitions is doubtful, since they use other undefined terms, but Euclid seems to have believed that they could serve to interpret the geometrical objects as referring to physical entities. They are not organised into primitive and derivative terms, but there is a tendency to accept this implicit distinction, and Sextus Empiricus attacks precisely those three terms that appear to be the most primitive in Euclid, namely (1st) "A point is that which has no part", (2nd) "A line is breadthless length" and (5th) "A surface is that which has length and breadth only". Hobbes concentrates his attention on the same notions.

3.2 Two Interpretations of the Pyrrhonian Strategy

The aim of the sceptical challenge is sufficiently straightforward: to show that geometrical and arithmetical statements cannot be claimed to provide actual knowledge about the world. In order to achieve such an end, Sextus relies on the usual weapon of logical possibility. Against the empirical truth of geometrical statements, he sets consistent counterfactuals leading to paradoxes or contradictions. In this way, he can highlight the physical content still pervading Euclidean geometry. Because of such empiricist criticism, two radically different interpretations of the Pyrrhonian strategy become possible, one slightly superficial and the other somewhat incorrect. Unfortunately, Hobbes adopted none of them, but his "third" position shared the limits of each (more on this in section 6).

On the one hand, the sceptic may be supposed simply to have failed to grasp the abstract nature of geometrical objects, and hence to have misunderstood Euclidean geometry. One may argue that Sextus is incapable of seeing that the geometrical objects discussed by Euclid are not physical points and physical lines at all, so no further attention should be wasted on Pyrrhonian arguments. A champion of this position was Sir Henry Savile (1549 – 1622). Savile owned a manuscript, now in the Bodleian Library, containing a late sixteenth or early seventeenth copy of *Contra Mathematicos*.[13] The text is in perfect condition apart from the book of *Contra Geometras*, which is underlined throughout and seems to have been studied by Savile. Savile does not appreciate Sextus' criticism. He mentions him only very briefly, in one of his manuscripts (Savilianus 37, fol. 11), where he refers to Sextus Empiricus' work without any further remark. And in his *Lectures on Euclid* (Savile [1621]), after having discussed the nature of geometrical definitions and axioms, he dedicates a few paragraphs to Epicureans and Pyrrhonists, but only to dismiss them because "their arguments against the principles of Geometry are thoroughly insignificant and indeed completely sophistic".[14] This does not mean that Savile himself failed to recognise that Geometry faced the major problem represented by the lack of full evidence, for example in the case of the 5th postulate.[15] The point was too crucial to escape his attention. Simply, Savile had no patience for Sextus' subtle arguments. When Riemann [1868] introduced his version of non-Euclidean geometry, in his famous lecture *On the hypothesis on which geometry ultimately lies*, he started by explaining the problems arising from the definitions of point and line in Euclid, the very issue Savile had been unable to grasp when reading Sextus.

On the other hand, the glass of Euclidean geometry is only half-empty of empirical presuppositions, as it were. Thus, the sceptical challenge can also be interpreted as a radical attempt to eliminate all the intuitive and physical residues in the geometrical system, that is, as a *reductio ad absurdum* of the empirical elements still present in classic geometry. This was Leibniz's position. In a letter to Varignon, he wrote that: "I even find that it means much in establishing sound foundations for a science that it should have such critics. It is thus that the sceptics, with as much reason, fought the principles of geometry; that father Gotignies, a Jesuit scholar, tried to throw out the best foundations of algebra; and that Mr. Cluver and Mr. Nieuwentijt have recently attacked our infinitesimal calculus, though on different grounds. Geometry and algebra have survived, and I hope that our science of infinities will survive too. [...] I have often thought that a reply by a geometrician to the objections of Sextus Empiricus and to the things that Francis Sanchez [...] sent to Clavius,[16] or to similar critics, would be something more useful than we can imagine. This is why we have no reason to regret the pains that are necessary to justify our analysis for all kinds of minds capable of understanding it."[17] Whether Leibniz appreciated the anti-empirical impact of Sextus' arguments, he certainly knew very well that "all the difficulties raised by the Pyrrhonians concern only the empirical truths (*veritez sensibles*)".[18]

Leibniz correctly understood that the sceptical challenge had a foundationalist nature. Yet, as I have anticipated, such a foundationalist interpretation of the Pyrrhonist challenge can be slightly incorrect, not conceptually, but in a scholarly sense. The main difficulty is that for Sextus Empiricus a non-empirical geometry was impossible. Insofar as Euclidean geometry provides information about physical space and the behaviour of objects in it, it must be true of the world and rely on a physicalisation of its primitive notions. One can then demonstrate that geometry provides no direct knowledge about the real nature of the world by determining what contradictions and paradoxes must necessarily arise from a physical interpretation of its elements. As we shall see, this point was completely missed by Hobbes. The process of de-physicalisation thus amounts to a process of de-alethisation of mathematics—mathematical statements in themselves are not necessarily true of the world. Many contemporary mathematicians and certainly Poincaré would have found little to object to this position. However, according to Sextus, a full de-alethisation of geometry amounts to showing that no geometry is possible at all. This is why, contrary to Huet's or Hume's, his mathematical scepticism should be interpreted as a radical use of empiricism only. Sextus uses mathematical scepticism in the (d) sense seen in section 2, without further implications for the foundation of axiomatic geometry in the (e) sense. For the Pyrrhonist, either mathematics counts as knowledge of the world or it is nothing at all, but not the former, therefore the latter. No appreciation of a purely a priori, hypothetico-deductive approach is envisaged. This appears very clearly in the first part of *Contra Geometras*, where the use of axioms is criticised because postulating does not amount to a justification of the hypotheses, i.e. cannot provide the hypothesis in question with a truth-content, whilst no notice is taken of the possibility of interpreting geometry as a purely hypothetico-deductive, ontologically non-informative science. Once again, it is worth remarking that Leibniz defended an interpretation of geometrical statements as only conditionally true in his correspondence with Simon Foucher Ishiguro [1978] on mathematical scepticism.

4. Hobbes' "Geometrical Foundationalism" and His "Materialist Philosophy of Geometry"

Hobbes is renowned as a philosopher and political theorist. Few students know that he spent an incredible amount of time and efforts working on geometry. Although he discovered geometry late in life, between 1629 and 1631 (he was born in 1588), more than a quarter of all his writings is devoted to the subject. Unfortunately, Hobbes never achieved the high standards of competence required by the very ambitious goals he set to himself. After a promising start, he soon became involved in a harsh and sterile polemic with John Wallis concerning his various alleged solutions of the quadrature problem. The diatribe lasted for decades and completely destroyed Hobbes' reputation as a mathematician.[19]

Despite his total and embarrassing failure, Hobbes' almost obsessive interest in geometry was fully justified by the foundationalist role he attributed to it within his philosophical system. His position can be briefly summarised as follows. According to Hobbes (*empirical thesis*) all sciences can be organized hierarchically, depending on how they analyse the two fundamental concepts of body and motion. This empirical thesis led Hobbes to see in mathematics the foundation of all knowledge. But, (*mathematical supremacy thesis*) geometry is the most important branch of mathematics and provides a foundation for arithmetic. For example, for Hobbes positive integers are the only admissible numbers, hence the square root of $\sqrt{2}$ is not a number but a line. So, (*logical supremacy thesis*) geometry is the first of all sciences in the "order of demonstration", that is, geometry is the science whose truths are the most general and on which the truths of all the other natural sciences somehow depend. And this means that (*geometrical foundationalism thesis*) geometry is the foundation of all philosophy. All "Elements of philosophy", from first philosophy and mechanics through ethics and politics, are based on it.

Given the foundationalist role of geometry, it was inevitable that Hobbes should dedicate to it a large part of his research. Indeed, the more Wallis was able to show his incompetence and discredit him as a mathematician, the more it must have seemed to Hobbes vital to regain a firm basis for his philosophical system. It soon became a vicious circle.

Because of Hobbes' comprehensive empiricism, his philosophy of geometry was bound to be based on his materialism, rather vice versa. The result was a strange combination of conservatism and innovation, each developed in the wrong direction. Hobbes the conservative always remained hostile to algebra, to the algebraization of geometry (analytic geometry) and to the extension of the domain of numbers to include decimal fractions, negative numbers, irrationals and imaginaries. Hobbes the innovator defended a materialist interpretation of geometry as an abstract physical science. He attempted to increase the physicalisation of geometry, with basic notions understood in terms of body and motion, a position he struggled but never managed to make fully coherent. For mainstream mathematics, geometrical objects are either bottom-up abstractions or top-down idealisations, but for Hobbes they were "special" material objects, somewhat in the middle. Thus, instead of freeing geometry from its empirical residues, Hobbes thought the solution rested in a fuller ontological commitment. A good example is provided by his *Fourteenth Objection* to Descartes' *Fifth Meditation*. Descartes is defending the Platonic nature of a triangle, whose essence, according to him, exists independently of the existence of its empirical implementation. But Hobbes objects that "If the triangle does not exist anywhere, I do not understand how it has a nature. For what is nowhere is not anything, and so does not have any being or nature. A triangle in the mind arises from a triangle we have seen, or else it is constructed out of things we have seen. […] Essence without existence is a mental fiction" (Descartes [1984], vol. II, 135-6). However, once the label "triangle" and the corresponding concept is formulated, then it remains available even if the triangle is destroyed. For Hobbes, geometrical objects come into existence but then have no "best before", no chronological upper limit.

Although Hobbes was right in arguing that his position was close to at least some empirical aspects still present in Euclid's *Elements*, he was mistaken in thinking that those were the ones worth preserving in the foundation of modern geometry.

Consider the three fundamental geometrical objects, point, line and surface. In Euclidean geometry, a point is "that which has not part", but for Hobbes "a point is a visible mark and so must have quantity and must be potentially divisible into parts, although such parts are not considered or negligible in demonstrations". In other words, a point is a physical body considered without its magnitude. Again, in Euclidean geometry a line is "breadthless length", but for Hobbes "lines are not drawn but by motion, and motion is of body only" (Hobbes [1839-45, 1997], VII, 211) so that a line must have a width, although this too is negligible in practice. We obtain a geometrical line by considering the path of a body/point through space. In Euclidean geometry, a surface is that which has only length and breadth, yet for Hobbes "A superficies [surface] is the space made by the motion of a body considered as a line [that is, considered without its depth]." (Hobbes [1839-45, 1997], I, 111-12).

Hobbes looked at geometry as Descartes looked at the *cogito*: he thought that without it, his whole philosophical system would have collapsed. No wonder he spent so much time working on it, especially once Wallis begun to show all its weakness. We now know he was mistaken. We still study and apply Hobbes' philosophical ideas despite the total failure of his mathematical efforts. Hobbes could have abandoned his mathematical projects by rejecting the underlying geometrical foundationalism. His geometrical works have become an appendix that can be fully disregarded as a mere historical accident, without any loss for his political thinking. Yet someone else took Hobbes' geometrical foundationalism seriously: John Wallis.

5. John Wallis's Anti-Hobbesianism

John Wallis (1616-1703) was a great mathematician and erudite (Scott [1938], Scriba [1966]). He was the third Savilian professor of geometry at Oxford, between 1649 and 1703 (note that the Savilian Chair of Geometry was founded in 1619 at the University of Oxford by that Sir Henry Savile whom we have already encountered in section 3.2). He worked on cryptograph and was employed by the parliament to decipher intercepted despatches (1642-5; he also worked for William III in 1690).[20] In mathematics, Wallis introduced the principles of analogy and continuity, contributed substantially to the origins of calculus, widened the range of the higher algebra, invented the symbol for infinity and edited classical mathematical authors, being an important early historian of mathematics. In 1655, he published his famous *Arithmetica Infinitorum*, which contained the germs of the differential calculus. Later (1693-9), he published a collection of his mathematical works. Contrary to Hobbes, Wallis seems to have considered arithmetic more fundamental than geometry.[21]

Many have wondered why Wallis, who is unanimously considered the most influential English mathematician before Newton, ever bothered to give more than passing thought to Hobbes' quadratures and spent so much time refuting and quarrelling with someone who must soon have appeared as an incompetent amateur. The explanation lies in Hobbes' geometrical foundationalism and in Wallis' extra-mathematical motives. As Douglas Jesseph (Jesseph [1999]) has well documented and convincingly argued, "Hobbes was led by a misplaced faith in the efficacy of his materialistic foundations for geometry to think that he could quickly dispatch all the great problems of that science. For his part, Wallis undertook the refutation of Hobbesian geometry primarily for the purpose of discrediting Hobbes' 'dangerous' metaphysical, theological, and political ideas. (p. 340)" Wallis, like Hobbes, believed that, by undermining the geometrical foundation, the whole Hobbesian system would have collapsed. It is in this partly mathematical and partly political context that their references to mathematical scepticism occur.

6. Mathematical Scepticism in the Dispute between Hobbes and Wallis

The endless controversy between Hobbes and Wallis on the quadrature problem has been fully analysed (Bird [1996], Probst [1993], Jesseph [1999]). In this context, it is useful to list some chronological references in order to map the scattered references to mathematical scepticism made by the two authors.

1643: Wallis briefly mentions mathematical scepticism in *Truth tried* (Wallis [1643]) This is not a mathematical treatise, predates the polemic between Wallis and Hobbes, and Hobbes seems to have disregarded it, although the discussion "whether *Quantity be divisible in semper divisibilia*", could count as an implicit criticism of Hobbes' position. In this context, Wallis writes: "And if I make it not evident (to those that are acquainted with Mathematical terms) that a *Continuum* consists not of Indivisible Points, by as certain and infallible Mathematical Demonstrations, as That 2 and 2 make 4, I will hereafter turn *Sceptick*, and affirm confidently That we are sure of nothing." (see Appendix I) There follows the analysis of some interesting counterexamples.[22]

1649: Wallis speaks again about mathematical scepticism in his *Oratio* (Wallis [1649]). He does so in terms that are not entirely negative. On p. 7, he regrets the poor state of mathematical studies, comparing the current situation to the glorious past, when Greek philosophers vigorously pursued mathematical knowledge. He then remarks that some philosophical schools have in fact neglected or been adverse to mathematics, and he mentions the Cynics and the Epicureans, but, very interestingly, not the Pyrrhonians or the Academic sceptics, contrary to what we have seen he could read in Proclus. Then, on p. 9, he adds: "Anyway, I shall add this: not only should mathematical studies be cultivated on account of the truth they possess (which, in itself already striking, and carefully considered by the extreme disputes of the sceptics, reinvigorates and delights very much the mind) but also because it greatly contributes to the knowledge of many other things, and this indeed for more than one reason." (see Appendix I). Wallis distances himself from the standard position that listed together Sceptics and Epicureans as both enemies of mathematical knowledge. In *De Algebra Tractatus* (Wallis [1657]), there is no reference to Pyrrhonism or Sextus Empiricus, despite the lenghty historical introduction.

1655: Hobbes publishes *De Corpore* (Hobbes [1655]), adding a chapter containing and alleged demonstration of the quadrature of the circle.

1655: Wallis promptly and correctly refutes Hobbes' quadrature in his *Elenchus Geometriae Hobbianae* (Wallis [1655]).

1656: Hobbes replies in the Appendix to the English version of *De Corpore* entitled, *Six Lessons to the Professors of Mathematiques* (Hobbes [1656]). He twice accuses Wallis of endorsing a form of mathematical scepticism (see Appendix II). According to Hobbes, not only does Wallis not understand the concepts of "*quantity*, *line*, *superficies*, *angle*, and *proportion*; without which you cannot have the science of any one proposition in geometry", but he also shares with Sextus Empiricus the same mistaken definition of the most fundamental concept of point, understood as "*that whereof there is no part*". According to Hobbes, this "de-physicalisation" of the basic notions is exactly what allows Sextus to dismantle the whole edifice of geometry. Hence, Wallis has "betrayed the most evident of the sciences to sceptics". According to Hobbes, the solution is to define a point as "*that whereof no part is reckoned*". From the brief and only comment he then attaches to this claim (if we use his definition Sextus' "arguments have no force at all, and geometry is redeemed. If a line have no latitude, how shall a cylinder rolling on a plane, which it toucheth not but in a line, describe a superficies?"), it seems that Hobbes had this in mind. Both Sextus and Wallis and misunderstand Euclid.[23] Depending on the context of application, geometrical objects (point, line and surface) may or may not be considered unextended in the relevant dimension. When talking to the sceptic, we must stress that geometry is a sort of purified physics, in which we merely disregard but do not deny extension and the relevant dimensions. Hobbes equivocates as to how far the "physicalisation" of geometrical entities should go. He never clarifies what he thinks the reasonable justification is for this "double truth" (geometrical objects do and do not have physical dimensions), nor does he ever question what sceptical paradoxes may emerge from his own conception of geometrical objects. But above all, Hobbes fails to realise that the sceptical arguments, based on the "dimensionless" nature of the geometrical objects, are only part of a more general strategy. They are actually meant to show that an empirical interpretation of geometry is inevitable. Once this step is taken, the sceptic is able to show that mathematical knowledge, as a kind of empirical knowledge, leads to contradictions and paradoxes. In other words, Hobbes willingly falls into the Pyrrhonian trap. He seems never to have realised the difficulties inherent in his materialist philosophy of geometry.

1656: Wallis further replies in *Due Correction for Mr. Hobbes*: *or Schoole Discipline, for not saying his Lessons right* (Wallis [1656]). Surprisingly, despite the very accurate and extremely meticulous analysis of Hobbes' *Lessons*, Wallis completely ignores his sceptical charge.

1657: Hobbes replies publishing the *Markes of the Absurd Geometry* (Hobbes [1657]). This time there is no reference to Sextus Empiricus and the charge of mathematical scepticism is not reiterated. Had Hobbes realised his mistake in interpreting Sextus?

1657: Wallis publishes *Mathesis Universalis* (Wallis [1657]) a general treatise on the nature of mathematics and its history.

1660: Hobbes's *Emendatio Mathematicae Hodiernae* (Hobbes [1660]) is a full criticism of Wallis' *Oratio* and *Mathesis Universalis*, which had been published together in 1657 as Wallis' *Operum Mathematicorum Pars Prima* (Wallis [1657]). Hobbes comments at length on Wallis' remark concerning scepticism and Sextus (see Appendix II). Somewhat inconsistently, Hobbes now writes "[…] mathematics possesses some undoubted principles of demonstration, namely definitions, axioms and assumptions, that politics, ethics s and even physics lack. That's why he [Wallis] is right [in the *Oratio*] in saying that only mathematics stands out [NB. Both in the sense of "emerges untouched" and in the sense of "it is left untouched"] from the sceptics' disputes." Apparently, Hobbes no longer considers Wallis a sceptic. Although the dialogue in the text proceeds to modify the initial statement, this admission is not changed.

1662: Wallis replies by publishing *Hobbius Heauton-timorumenos* (Wallis [1662]). Once again, he does not address the issue of mathematical scepticism. This is all he has to say about Hobbes' sceptical comment (p. 23) "He doth not think that *Geometry* is lesse litigious or more certain, than *Physicks*, *Ethicks*, and *Politicks*; but These are *Mathematicks*, as much as That; and may be as clearly *Demonstrated*. (He hath shewed us, How.)"

Further, polemical exchanges follow, but there are no more references to mathematical scepticism either in Hobbes or in Wallis.

Judging from the scarce evidence available, Wallis' position with respect to mathematical scepticism might have been akin to Leibniz's. Wallis probably did not know much about Sextus Empiricus, although he shows some knowledge of the Pyrrhonian challenge. It seems that he never owned a copy of his works. The manuscript (Savilianus 101, fol. 11) contains a list of books that belonged to Wallis and were left to the Bodleian and it does not include any edition of Sextus' writings. On the other hand, he was wellread in the history of mathematics and in communication with René-François de Sluse (see Bernes and Lefebvre [1986]), a mathematician and scholar who translated into Latin *Sexti Empirici De Philosophia libri duo adversus logicos* (see Floridi [2002], 58). As a pioneering mathematician, he might have appreciated Sextus' arguments as a step towards a better understanding of mathematics. Unless further evidence becomes available, this must remain a conjecture.

On the basis of the admittedly few but more substantial remarks made by Hobbes, he seems to have maintained an interpretation of mathematical scepticism that was somewhat in between Savile's and Leibniz's. Hobbes took the sceptical challenge seriously enough, like Leibniz's, yet unlike Leibniz he failed to see in it an argument against a materialist/empiricist philosophy of mathematics. He thought Sextus (and Wallis with him) had misunderstood the basic nature of geometry, like Savile's, but unlike Savile he made the mistake of opting for an alleged solution of the sceptical challenge that misunderstood the latter and merely provided more of the same problematic ingredient: an increase in the physicalisation of geometry that could only end in a complete failure of Hobbes' philosophy of mathematics.

7. Scepticism in Hobbes

The literature concerning the role played by scepticism in Hobbes ranges between two poles. The minimalist view denies that Hobbes had any substantial interest in sceptical problems or that he was significantly influenced by their discussion. Sorell seems to defend this view. He writes "But if answering scepticism about geometry is one of Hobbes' intentions in outlining the principles of geometry, it is one he keeps under wraps in *De Corpore*. He seems to announce his anti-sceptical leanings in the *Six lessons* as an afterthought, to take the sting out of Wallis' attack on his mathematics. In any case, it is clear that his defence against Pyrrhonism is question-begging." (Sorell [1986], 65-66; note that there is no reference to scepticism or Sextus Empiricus in Sorell [1996]). The shortcoming of this view is that it does not help to explain the available evidence in Hobbes' writings.

The maximalist view argues that scepticism played a major role in Hobbes' philosophy. It has been supported mainly by Richard Tuck: "As we saw in Part I, there are good grounds for supposing that Hobbes began his philosophical enquiries in the late 1630s because he was intrigued by the philosophical problems raised by modern natural science, and particularly by the possibility of replacing late Renaissance scepticism with a philosophy accommodated to the ideas (above all) of Galileo." (Tuck [1989], 40; see also Tuck [1988]). The shortcoming of this interpretation is that it fails to provide a corresponding body of evidence that would textually, or at least conceptually, support Hobbes' alleged fundamental engagement with the sceptical debate or with sceptical literature.

As we have seen, the analysis of mathematical scepticism supports a middle-ground alternative.[24] The debate between Hobbes and Wallis included some interesting references to mathematical scepticism but failed to ignite a more radical and widespread discussion of it. Hobbes' interest in the sceptical debate seems to have been a case of sparks without a fire.[25]

8. Conclusion: What Leibniz saw and Hobbes missed

The interpretation of mathematical scepticism I have offered in the previous pages could be summarised in a fistful of famous quotations: for Plato "God eternally geometries", and at the end of the sixteenth century Kepler still agreed with that. Yet, the algebraisation of geometry made Kroneker believe that "God made the integers; all else is the work of man". The discovery of non-Euclidean geometries and the development of Cantor's treatment of infinite sets convinced Hilbert that "No one shall expel us from the paradise which Cantor created for us". God and above all geometry had been replaced by the human construction of set theory, but the former was going to reappear in the mathematical imagination. For after Gödel's proof that consistency of number theory cannot be established by the narrow logic permissible in metamathematics, Weyl suggested that "God exists since mathematics is consistent, and the devil exists since we cannot prove its consistency". By the time geometry had been replaced by set theory and the de-physicalisation and the corresponding de-alethisation of mathematics had been completed, Russell wrote that "mathematics is the subject in which we never know what we are talking about, nor whether what we are saying is true", while Einstein believed that: "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." I suppose Sextus Empiricus would have found this series of remarks very reassuring. The history of reason is one of a constant striving of the mind for the achievement of intellectual freedom from reality. In the course of the history of thought, the distance between mind and being widens and, through such a constant process of detachment, reflection becomes epistemologically ever more responsible for its own constructs, while at the same time increasingly self-referential in its activities. From Proclus' invitation "to free geometry from Calypso's embrace", to Cantor's suggestion that "Mathematics is entirely free in its development and its concepts are restricted only by the necessity of being noncontradictory and coordinated to concepts previously introduced by precise definitions. [...] The essence of mathematics lies in its freedom" Cantor et al. [1932], the history of mathematical theories appears to be perfectly coherent with the previous view, which I acknowledge to be unashamedly metaphysical. A progressive mathematisation of our knowledge of the world in its most diverse aspects, from physical to social phenomena, and an equally impressive, if somewhat later, de-physicalisation of mathematics, which led, between the end of the last and the beginning of our century, to a full axiomatisation of the foundations of the discipline: these two movements are aspects of the same phenomenon. It was precisely the detachment of mathematics from its empirical models that made it possible to interpret and dominate more and more aspects of reality with the same mathematical theories. When Euclidean geometry disengaged itself from empirical interpretations via its arithmetisation and then axiomatisation, geometries only locally isomorphic to it became conceivable, geometries that could replace the fifth postulate with a different axiom and hence become capable of handling non-Euclidean spaces of n-dimensions. Only a purely algebraic approach allows us to define Weierstrass' curve, which is nowhere differentiable, or Peano's curve, which is capable of covering a whole surface. The same non-empirical approach to set theory makes it possible to understand how the part may not necessarily be smaller than the whole. Geometry has moved from the abstraction and idealisation of selected properties of physical objects to the hypostatisation of logical relations. The loss of intuitive certainty has been repaid by the acquisition of certain universality. As thought increasingly detached from what common sense offers up as apparently indisputable in ordinary experience, a kind of constructive scepticism has often been a fundamental driving force. As Reichenbach [1958] put it, discussing the fortune of Euclidean geometry, "Unless one was a [Pyrrhonian] skeptic, one was content with the fact that certain assumptions had to be believed axiomatically [indeed Sextus would say "dogmatically"]; analytical philosophy has learned through Kant's critical philosophy to discover genuine problems in questions previously utilized only by skeptics in order to deny the possibility of knowledge". Radical questioning is made possible by the capacity of the mind to conceive what is logically consistent but not actual, and the presentation of the conceivable is usually the best conceptual tool whereby thought can disengage itself from its momentary forms of more or less dogmatic realism, and hence move towards a better appreciation of its theoretical responsibilities. This is what Leibniz and perhaps Wallis saw but Hobbes missed.

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Notes

[1] For references to the available literature, see Floridi [1998; [2000]; Popkin [1996].

[2] Jean-Etienne Montucla (1725-1799) is the first great historian of mathematics. Floridi [1998] analyses Montucla's lengthy confutation of the sceptical theses in defence of the new analytical techniques, which he develops in the first book of his monumental *Histoire des mathematiques* (Montucla [1758]).

[3] To my knowledge, after Montucla's none of the subsequent histories of mathematics has ever again dedicated so much space to mathematical scepticism. Before Montucla, I know of only a few other texts which discuss Sextus' objections at some length, among which are Pico della Mirandola [1520] (about which, see below); Lange [1656]; Huet [1679; [1680; [1690; [1703]; a long section in Crousaz [1733] dedicated to the relation between sceptical doubts and mathematical certainty. Gianni Paganini has very kindly called my attention to Cartaud de la Vilate [1733]. Of course, to these texts one must add Descartes' discussion of mathematical scepticism in the Meditations and the debate he engendered, Bayle's *Dictionary*, and Hume's remarks on the nature of mathematics. I hope to study these sources in my future research.

[4] On p. 1, introducing the topic of the *scientiae mathematicae*, their nature and number, Vossius writes "Sic voce matematon utitur Sextus Pyrrhnonius, cùm libros xinscribit adverus Mathematicos. Nec enim disputat adversus Arithmeticen, & Geometriam; quàm Grammaticem, Historiam, Poëticen, Rhetoricen, Astrologiam judiciarim, Musicen, Logicen, Physicen, Ethicen." He then refers to Sextus a few other times in the work in order to explain some linguistic matters, but never actually discusses his sceptical arguments, even when he deals critically with Epicurus and Ramus.

[5] Bochner [1966], 363: "[...] [Sextus Empiricus'] works are boring, but important. For instance, the proemium in the poem of Parmenides comes from Sextus".

[6] Socrates, for example, objected to the utility of the study of mathematics on ethical grounds, and philosophers such as Giovanni Francesco Pico della Mirandola employed sceptical arguments for anti-intellectualist and theological purposes, see Pico della Mirandola [1520], lib. I, cap. vii, pp.750-751, which contain a brief summary of the Sextian issues with definitions of point, line and plane, and lib. III, cap. 5-6 against geometry and 7 against arithmetic.

[7] Agrippa von Nettesheim [1530; [1974; [1993]. On Agrippa von Nettesheim [1974], p. 58, chap. 11, *Of Mathematical sciences in general*, Agrippa writes that the mathematical sciences, thought to be the most certain, consist only of the opinions of teachers to whom great credit is given. Their objects, like a perfect sphere or a circle, do not and cannot exist. And even if mathematical theories have never been the cause of heresies, Augustine wrote that they do not further salvation but lead men into error and separate them from God, while Jerome says that they are not "sciences of Godlinesse". (A note by the editor suggests Augustine's *De actis cum Felice Manichaeo* I.10, or *Confessions* V.3 as possible sources). But on p. 75, *Of Geometry*, chap. 22, we read that Geometry is the Princess and mother of all learnings, as Philo Judaes has called it (the source is possibly *De Agricultura* 13). Geometricians agree on everything and discuss only points lines and other things. However, no geometrician has ever been able to discover how to square the circle, in spite of Archimedes' claims to the contrary.

[8] For the sake of simplicity, the strategies are labelled according to their most significant places of occurrence. "Cartesian" and "Wittgensteinian" mean "as discussed in Descartes" and "as discussed in Wittgenstein".

[9] Sextus' mathematical scepticism is analysed in details by Freytag [1995].

[10] On Greek philosophy of mathematics and the sceptical attack see Mueller [1982]. Mueller [1980] comments upon M III.37 thus "the force of this sceptical argument derives from the representation of mental apprehension as imagining or picturing and the imposition of severe limits on imagination (p. 13)." The point is that nothing can be apprehended unless it can somehow be imagined. See also p. 17: "For Proclus the mathematical imagination is quite like what later philosophers called intuition. Its images are produced by reason itself as a necessary condition of its mathematical knowledge: the images are a "projection" (*probole*) of concepts and principles contained in reason but not fully grasped by it."

[11] Euclid's geometry can be presented as a formal organisation, not axiomatic and not thoroughly syllogistic, of material previously accumulated. Mueller [1974] points out that "Euclid shows no awareness of syllogistic or even of the basic idea of logic, that validity of argument depends on its form. [p. 37]. [...] In his systematic presentation of the categorical syllogism in the first twenty-two chapters of the Prior Analytics, Aristotle never invokes mathematics. [p.48] [...] Stoic propositional logic, investigated most thoroughly by Chrysippus in the third century, shows no real connection with mathematical proof [p.66]." For a full analysis of Euclid's mathematical methods and a comparison with Hilbert's axiomatic approach see Mueller [1981].

[12] The fourth axiom, another clear case of empirical influence in the *Elements*, states that "things which coincide with one another are equal to one another", and this implies superposition, which in turn is necessary to prove congruence of figures. It is significant that Euclid tries to avoid its use whenever possible. Likewise, the fact that all Euclidean geometry is based on the avoidance of geometrical objects with actually infinite dimensions may not necessarily be due to the fact that the Elements present a geometry of touch or are even a tactile-muscular study of metric space, cf. Ivins [1964].

[13] Ms Savilianus Gr. 1, fol. 10v: "Extant Sexti Empirici libri decem pros mathematikos, adversus mathematicos, hoc est universam dogmaticorum nationem. Nec enim illis in libris tam Geometriae et Arithmeticae, quam Grammaticae Poeticae, Historiae, Rhetoricae, Astrologiae divinatricis, Musicae, Logicae, Physicae et Ethicae fundamenta conbellantus [*conbello* means literally *uproot*]".

[14] "[...] contra quae [i.e. Geometriae], totamque, adeò Geometriam acriter insurgunt duae philosophorum sectae, Pyrrhoniorum dico (qui sceptici & ephectici) & Epicureorum. Ac Ephecticorum quidem, qui quasi hostium more ex philosophiae agris fertilis cumprimis & foecundae frumenta populantes, & tanquam solem è mundo, sic ex animis nostris omnes scientiae non ramos modò, sed radicum fibras evellentes, totam evertunt philosophiam: horum, inquam, argumenta contra principia Geometriae perquàm levia sanè aut planè sophistica videre licet apud Sextum Empiricum lib. I cap. 19." The quotation comes from Savile Savile [1621], p. 157, see also the original manuscript in the Bodleian, Ms Savile 37 ff. 99v-100, which contains a brief, erased sentence not included in the printed text.

[15] Savile [1621], p. 140. Savile also mentions a second problem, the theory of proportion, which was discussed by Leibniz.

[16] Leibniz' reference to Clavius is interesting. The latter wrote "Ita ut ad Pyrrhoniorum fere (erant Pyrrrhonii Philosophi, qui nihil decernebant, sed de omnibus dubitabant) haesitantiam deventurus fuerit, nisi Arithmeticae, Geometriae, Dialecticaeque (quibus artibus ab avis & patre fuerat institutus) esset cognitione scientiaque revocatus. Unde suadet, sequendos esse characteres illos Arithmeticos, & linearum demonstrationes. Clavius [1612], vol. I, Prolegomena, Euclidis atque Geometriae Commendatio, p. 7.

[17] Leibniz to Varignon Hanover 2 Fevrier 1702, pp. 94-95 of Leibniz [1850], Erste Abtheilung, Band IV. See also Leibniz [1969], p. 544. Leibniz was not alone in appreciating Sextus Empiricus. Walther von

Tschirnhaus wrote to him that: “Sexti Philosophi Pyrrhoniarum hypotheseon libri tres, Parisiis 1569 in folio, habe mitt delectation gelessen.” see Bd. I, p. 397 of Leibniz [1899].

[18] Letter to Edmonde Mariotte, Hälfte 1676 in Leibniz [1923], pp. 268-269. In the letter, Leibniz presents geometry as the most fundamental of all mathematical branches.

[19] For an excellent reconstruction of the debate between Hobbes and Wallis see Jesseph [1999], which contains a full bibliography on Hobbes' philosophy of mathematics.

[20] It is in this role that he appears in Pears [1997], a murder mystery set in Oxford in the 1660's.

[21] On this and on Wallis' influence on Berkeley see Pycior [1987] and Pycior [1997].

[22] Unfortunately, there is no space for a detailed analysis of them in this context. I hope to be able to devote another paper to their critical discussion.

[23] Note that, contrary to what Hobbes seems to think, Sextus is usually considered a reliable source of information about the original text of Euclid's *Elements*, which he could still read, see Euclid [1956], I.63; Euclid [1883-1888], vol V, xciii; Heiberg [1882], pp. 194-197 and Russo [1998].

[24] Popkin [1979]. Paganini's contribution to this volume provides an analysis of Hobbes' interest in scepticims and the “continental” tradition, with further references.

[25] This paper is part of a long-term research on modern mathematical scepticism, from the recovery of Pyrrhonism during the Renaissance to Wittgenstein. Some results have already appeared in print (Floridi [1998; [2000]). Some sections in this paper contained revisions of material appeared in Floridi [1998]. A first version of the paper, in the form of a power point presentation (http://www.wolfson.ox.ac.uk/~floridi/pdf/ms3.htm) was given at the Colloquium *Scepticism as a Force in Renaissance and Post-Renaissance Thought: New Findings and New Interpretations of the Role and Influence of Modern Scepticism,* organised by Richard Popkin and hosted by the UCLA Center for Seventeenth- and Eighteenth-Century Studies (William Andrews Clark Memorial Library, Los Angeles, 8-9 March 2002. I wish to thank the participants for their useful questions and comments and especially Richard Arthur, Otavio Bueno, Sarah Hutton, Dick Popkin, Gianni Paganini, Maarten Ultee and Robert Westman for their suggestions and information.