Causal Set Theory and Growing Block? Not Quite

Abstract

In this contribution, I explore the possibility of characterizing the emergence of time in causal set theory (CST) in terms of the growing block universe (GBU) metaphysics. I show that although GBU seems to be the most intuitive time metaphysics for CST, it leaves us with a number of interpretation problems, independently of which dynamics we choose to favor for the theory —here I shall consider the Classical Sequential Growth and the Covariant model. Discrete general covariance of the CSG dynamics does not allow us to individuate a single history of the universe (defined by a causal history of different causal sets), thereby making the claim that 'the past exists' at best problematic. In addition, because the evolution of the universe in CSG dynamics leads to an outward branching causal tree, it becomes impossible to determine a proper 'line of becoming', thereby blurring the presentists' claim that only the present exists. Similarly, the covariant approach runs into the same, if not even more severe problems, since each configuration of the universe would amount to a set of possible causal sets, thereby making the individuation of a single configuration of the universe —and thus the physical interpretation of the theory— implausible.

1 Introduction

There are several reasons why some physicists believe developing a theory of quantum gravity would be a good idea. For example, the strong appeal that reductionism has oftentimes played in science. Many scientists think it is better to have a single theoretical framework that can explain many phenomena, rather than many theories each operating in a different domain. In this sense, it would be better to have a single theory capable of explaining phenomena such as black holes and particles' scattering, rather than having two different theories: one accounting for the existence of black holes, and the other providing transition amplitudes for the scattering of subatomic particles.

However, since reduction might not be enough of a reason to develop a theory of quantum gravity, one could argue that there are phenomena that neither of our best theories can explain properly. Indeed, while General Relativity (GR) tells us about the existence of exotic objects such as black holes, it does not tell us what happens inside of them at the so-called singularity. Similarly, quantum field theory (QFT) requires us to use some special techniques to tame the infinities that the theory seems to present us with, leaving us with the suspicion of some form of ad-hocness of some of our calculation methods —independently of how accurate they are.

Finally, one can hardly overlook the manifest incompatibility between GR and QFT: even though they are among the best empirically well-verified theories that science has ever produced, they also seem to describe a very different type of reality. On the one hand, GR presupposes that spacetime is a dynamical entity and that Einstein's equations constraint which specetimes are allowed given a certain matter distribution. In addition, the dynamical fields of the theory are continuous and local, that is, they undergo local interactions only. On the other hand, QFT describes a very different type of reality, one which relies on a non-dynamical background that constitutes the playground of the different quantum fields. In addition, the theory is fundamentally probabilistic (unlike GR), non-local, and discrete.

In recent years, many research groups have attempted to develop a unifying theory for such apparently incompatible descriptions of the world (for a general overview see: (Oriti 2009)). One common feature of many of such new approaches is that spacetime seems to become a derivative entity, an entity that does not partake to the fundamental structure of reality, but rather, it emerges from the collective behavior of some more fundamental atoms-of-space. The emergent behavior of spacetime has attracted the attention of philosophers of science who promptly started to inquire different notions of emergence, the epistemic possibilities of a theory of quantum gravity, and the possible mechanisms for the emergence of spacetime. For example, if, as many approaches seem to suggest, a theory of quantum gravity does not presuppose spacetime as a fundamental structure, it becomes unclear how to empirically probe the behavior of the new fundamental entities (Huggett and Wüthrich 2013). The emergence of spacetime seems to suggest that we will only be able to test the emergent properties derived by the theory, but if the theory is constructed so that it recovers the results of quantum field theory and general relativity, how should we assess the validity of the new quantum gravity framework?[1] In sum, it appears evident that a theory whose fundamental entities are not in-spacetime, but rather constitute-spacetime, poses not only some physics challenges, but also many philosophical ones.

Indeed, a second philosophical problem, which is the one I will be discussing in this paper, is one that concerns the emergence of time starting from more fundamental entities. In the absence of a spacetime background one could legitimately ask how to recover the dynamics of the physical systems, since dynamics is a concept that implies time evolution, but time evolution is something that requires some form of temporal coordinate —although not necessarily absolute. In this paper, I will consider the approach to quantum gravity named Causal Set Theory (CST) and I will explore the possibility of characterizing the emergence of time in terms of the growing block universe (GBU) metaphysics. Alas, I will show that although GBU seems to be the most intuitive time metaphysics for CST, it leaves us with a number of interpretation problems, independently of which dynamics we choose to favor for the theory —here I shall consider the Classical Sequential Growth (Rideout and Sorkin 1999) and the Covariant model (Dowker et al. 2020), (Zalel 2020). Indeed, I will argue that the discrete general covariance of CSG does not allow us to individuate a single history of the universe (defined by a causal history of different causal sets), thereby making the claim that 'the past exists' at best blurred. In addition, because the evolution of the universe in CSG dynamics leads to an outward branching causal tree, it becomes impossible to determine a proper 'line of becoming', thereby blurring the presentists' claim that only the present exists. Similarly, the covariant approach runs into the same, if not even more severe problems, since each configuration of the universe would amount to a set of possible causal sets, thereby making the individuation of a single configuration of the universe —and thus the physical interpretation of the theory— implausible.

The plan for the paper is the following: in Section 2 I will review the basics of causal set theory and two of the most common dynamics: classical

[1]Obviously, one still hopes that the new theory will be able to account for new phenomena (or new explanations) within the range of our working experiments.

sequential growth and covariant model. In Section 3, I will discuss the problem of time in causal set theory and argue that the growing block view is the most intuitive metaphysics for CST —some divergent opinions on the matter can be found in: (Wüthrich 2023), (Arageorgis 2016), and (Huggett 2014). I will focus on the distinction between the internal time of the theory —that is, the partial ordering of the individual causal sets— and the emergence of phenomenological time, associated with the transition between different configurations of the universe (or causal sets). Finally, Section 4 investigates the compatibility between the dynamics of theory (CSG and Covariant) with the growing block view. I will show that interpreting CSG in light of the growing block view leaves open the problem of giving a physical interpretation to the different histories of the universe. The covariant model does not lead to any better interpretative results, for each node in the causal tree is constituted by a set of covariant causets, thereby making a physical interpretation of the history of the universe even more challenging. Section 5 offers some concluding remarks.

2 Causal Set Theory Overview

Causal Set Theory, which was originally proposed by (Bombelli et al. 1987), is oftentimes appreciated for its 'simple' approach to understanding spacetime —it is perhaps because of this simplicity, together with a direct reference to set theory, that the theory is appreciated especially by philosophers (Wüthrich 2023). While the simplicity of the mathematical structure and fundamental principles warranted some appreciation, they also come with the difficulty of formulating a proper dynamics: "One of the primary difficulties in formulating a dynamics for causal sets is the sparseness of the fundamental mathematical structure. When all one has to work with is a discrete set and a partial order, even the notion of what we should mean by a dynamics is not obvious" (Rideout and Sorkin 1999, p. 1). Far from being a complete survey, this section offers a brief overview of the mathematical structure of the theory and of its fundamental principles. More detailed and comprehensive reviews can be found in (among others): (Surya 2019), (Henson 2009), (Wallden 2013), (Dowker 2006), (Bombelli et al. 1987).

Causal set theory is inspired by the sum-over-histories approach, which calculates the transition amplitude as a sum over the many possible histories of the system. One of the reasons that make the sum over-over-histories an appealing formulation is that there no need for a preferred foliation or external time parameter; instead, only the total history of the system is relevant. Then, various observables can be related to the histories, thereby making them independent of external observers. Causal set theory is inspired by this formulation in that the domain of the sum in the sum-over-histories approach corresponds to the sample space Ω of the theory, where the amplitudes are defined by the dynamical processes and the observables are set of histories.

The two fundamental pillars of causal set theory are discreteness and causal relations. With respect to the former, the theory assumes from the very beginning that its fundamental ontology is constituted by discrete elements (atoms-of-space) related to each other by causal relations. The coarse-grained approximation of such elements corresponds to (relativistic) spacetime. The assumption of discreteness as a fundamental property, rather than being derived from a canonical process of quantization, bears some relevant consequences —especially if we maintain relativistic spacetime as the guiding star for the emergent properties that CST should reproduce. For example, CST cannot be diffeomorphism invariant, and yet "the physical content of a causal set is independent of what mathematical objects the causal set elements are and is also independent of any additional labels those causal set elements might carry: only the order relation of the elements and the number of elements has a physical meaning" (Rideout and Sorkin 1999, p. 3).[2] I will discuss the principle of discrete general covariance below, since, before continuing, we need some basic definitions and properties of causal set theory.

- A causal set (causet) is an ordered pair $\mathcal{C} : \langle C, \prec \rangle$ where C corresponds to a set of discrete elements (atoms-of-space) and the binary relation $i \prec j$ is transitive ($\forall x, y, z\ x \prec y,\ y \prec z$, then $x \prec z$), acyclic ($\forall x, y$, $x \prec y$ and $y \prec x$ then $x = y$), and locally finite ($\mathrm{Card}(C) < \infty$).

The expression 'causal relation' stands for a partial order relation and thus it does not imply a metaphysics of time of any sort. Similarly, the definition of past of a given event does not imply a metaphysical commitment to the existence of the past, but it simply consists of the set of elements that precede the given event: $\mathrm{past}(x) := \{y \in C | y \prec x\}$. In this contribution I will be using the irreflexive formalism of CST, for which the causal relation

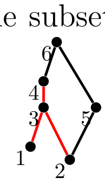
[2] Notably, the fact that only the number of elements and their order relation has physical meaning is amenable to a structuralist interpretation, see: (Wüthrich 2023).

is irreflexive and the past of a given event does not contain the event itself. Some other useful definitions in the context of CST are:

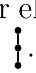
- A causal set is equipped with a discrete measure which assigns to each subset of the causal set a volume equal to the number of elements in fundamental units, up to Poisson-type fluctuations.

- A *chain* is a linearly ordered subset S of a given causet C in which every two elements are related by the relation $\prec$ such that, for example, is a chain of three elements.

- An *anti-chain* is a subset S of a given causet C in which there are no elements related by the causal relation $\prec$. For example, the following is a three elements anti-chain: • • •.

- A *stem* is a subset S of a causet C that contains its own past. A total stem is such that every element of the complement set of S lies to the future of an extremal element of S. For example, in the following causal set, the subset marked in red is a stem (yet, not a total stem) of the set C: 

- A *link* in a causet C is an irreducible relation between two events $a \prec b$ such that there is no event in between a and b. A *path* is a sequence of elements all related by a link.

- A *family* is the class of causets that can be formed by adding a single maximal element to a given causal set. One can define a *parent-child* relation as the transition from one causal set C_i to another causet C_n such that: $C \to C'$. A *gregarious child* and a *timid child* are formed by adding one element that is spacelike separated or to the future of any other elements of the parent set respectively, for example: $\to$  and $\to$ .

- An *isomorphism* is a bijective map between causal sets that preserves the relations of partial ordering: given two causets C and D, $f : C \to D$ such that $f(x) \prec_D f(y) \leftrightarrow x \prec_C y$, $\forall x, y$.

Despite the many definitions, the theory thus far presented does not warrant the recovery of the relativistic spacetime structure. To ensure this, one needs the Hawking-Malament theorem, which demonstrates the equivalence up to conformal factor between the prescriptive spacetime structure of relativity theory and the description of all causal relations taking place in spacetime (see: (Hawking, King, and McCarthy 1976) and (Malament 1977)) The condition that the equivalence holds up to a conformal factor means that causal relations do not provide us with the scale information on the relativistic manifold and, because of diffeomorphism invariance, we cannot retrieve such information from an external embedding manifold. The axiom of local finiteness and the measure μ make up for the lack of scale data by establishing that all ordered intervals in a causal set have finite cardinality. This corresponds to having a finite cut-off interpreted as a measure of volume. Finally, the continuum based geometry is recovered as a smoothing-out (coarse-graining) of the discrete causal set. One might ask how many elements of a causal set can fit in a unit of spacetime volume. (Dowker 2006) estimates that in a causal set underlying a spacetime volume of $1cm^3 \cdot s$ contains approximately 10^{143} elements.

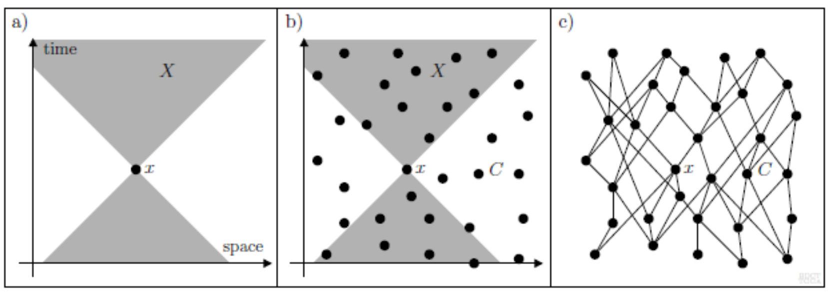


Figure 1: (a) A classical spacetime manifold X, (b) elements of C sprinkled on X, (c) a causal structure embedded on C, (Dribus 2013, p. 5)

The measure μ, the atoms-of-space, and the partial order, should now warrant a bottom-up reconstruction of spacetime. However, due to the numerous potential configurations of elements in a causal set, the theory makes use of an injective map $\phi : C \rightarrow (M, g)$ from a causal set to a pseudo-Riemannian manifold, such that given $x, y \in C$:

$$x \prec_{CS} y \longleftrightarrow \phi(x) \prec_M \phi(y) \tag{1}$$

That is, the injective map (also called *embedding*) preserves the causal relations of relativistic spacetime into the causal set, and it is constructed by a process of Poisson Sprinkiling, as depicted in Figure 1 and remarked by (Surya 2019, p. 16): "We say that a causal set C is approximated by spacetime (M, g) if C can be obtained from (M, g) via a high probability Poisson sprinkling [...]In a Poisson sprinkling into a spacetime (M, g) at density ρ_C one selects points in (M, g) uniformly at random and imposes a partial ordering on these elements via the induced spacetime causality relation". It remains to determine how a manifold-like causal structure can uniquely determine large-scale manifolds. Alas, the uniqueness of the continuum approximation is is warranted by a conjecture, the *Hauptvermutung* (fundmanetal conjecture of causal set theory):

> ***The Hauptvermutung of CST:*** C can be faitfully embedded at density ρ_C into two distinct spacetimes, (M, g) and (M', g') iff they are approximately isometric (Surya 2019, p. 19).

The conjecture is not proven yet, but some work in this direction has been put forward by, among others: (Bombelli 2000), (Noldus 2004), (Bombelli, Noldus, and Tafoya 2012).

Before moving on, it is useful to visualize the theory as composed of two levels: the individual causal set defined by all the elements in a partial order relation, and the space of possible growths. The latter pertains to the dynamics (as we shall see soon) and it represents the possible transitions from one causal set to another in a tree of partially ordered causal sets named *poscau*.

2.1 Classical Sequential Growth

The most common approach to the dynamics of causal set theory is called Classical Sequential Growth (CSG) and it was originally developed in (Rideout and Sorkin 1999). The main idea of the model is that a causal set is built one-element-at-a-time via transitions representing the evolutionary steps. Each transition is associated with a classical probability that encodes the likelihood of obtaining a causal set C_n among the many possibilities defined by the kinematic space of the theory. The many possible growths and the process of one-element-accretion give rise to what was defined earlier as *poscau*: the tree representing the possible transitions from one causal set to another according to the partial order relation. An example of the poscau up to four elements is represented in Figure 2.

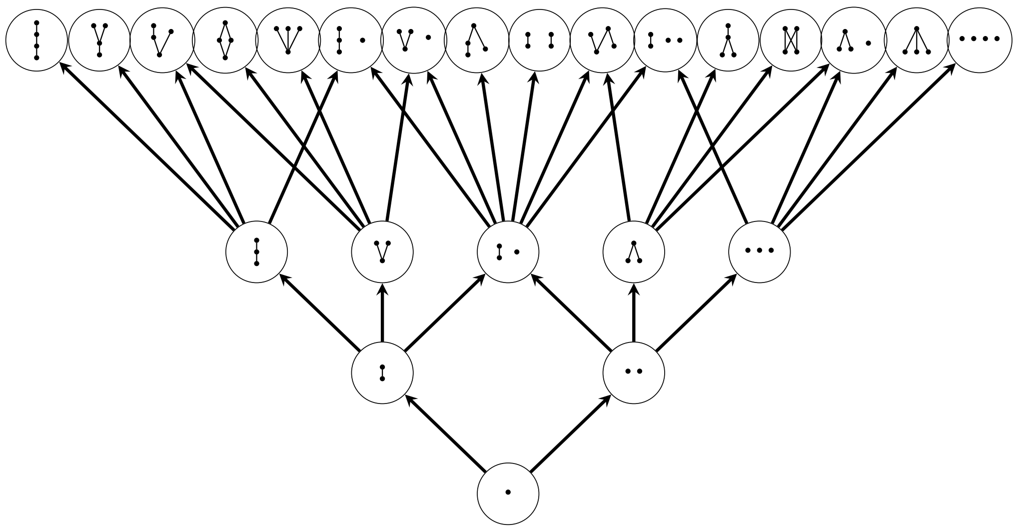


Figure 2: A poscau up to four elements.

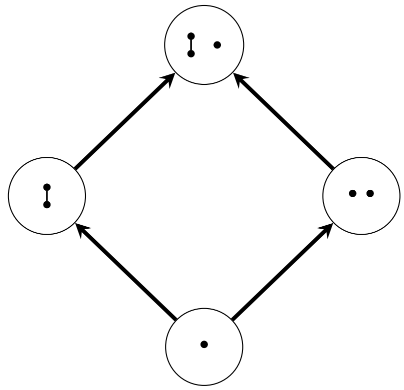
In the original formulation of the CSG model, the phenomenological passage of time was recovered from the process of accretion of the causal set, as remarked by (Rideout and Sorkin 1999, p. 3): "The phenomenological passage of time is taken to be a manifestation of this continuing growth of the causet. Thus, we do not think of the process as happening 'in time' but rather as 'constituting time', which means in a practical sense that there is no meaningful order of birth of the elements other than that implied by the relation $\prec$". The order relation and the birthing of new events seem to allow for a (natural) labeling that maps the growth of the causal sets to natural numbers. Consider the interval of natural positive numbers and let $\tilde{\Omega}(\mathbb{N})$ be the set of partial orders such that:

$$\tilde{\Omega}(\mathbb{N}) := \bigcup_{\tilde{C}(n)} \text{is the set of all finite labeled sets.} \tag{2}$$

A causal set $\tilde{C}(n) \in \tilde{\Omega}(\mathbb{N})$ is naturally labeled if there exist a map $L : \tilde{C}(n) \to \mathbb{N}$ that preserves the order relation in $\tilde{C}(n)$, that is: $\tilde{C}(n) \prec \tilde{C}(n+1) \to n < (n+1)$. However, discrete general covariance is one of the physical requirements of CSG that determine under what conditions a given history in poscau can be considered physically meaningful. The requirement, as we shall see, undermines the possibility of a physical interpretation of the natural labeling.

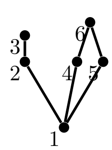
First, we want to know what questions we can ask of the dynamics, and this corresponds to asking “which classes of causal sets (the ‘histories’ of the theory) are measurable in a way compatible with general covariance” (Brightwell et al. 2002, p. 2). To do so, we need to identify the questions, and then to give an account of how to compute the answers in terms of probabilities. Second, we set a sample space Ω, a σ-algebra $\mathcal{R}$ on the sample space, and a probability measure μ with domain $\mathcal{R}$. That $\mathcal{R}$ “is a σ-algebra on Ω means that it is a family of subsets of Ω closed under complementation and countable intersection. That μ is a probability measure with domain $\mathcal{R}$ means that it takes members of $\mathcal{R}$ to non-negative real numbers and is σ-additive, with $\mu(\Omega) = 1$. Finally, σ-additivity means that μ assigns to the union of a countable collection of mutually disjoint sets in its domain the sum of the measures it assigns to the individual sets” (Brightwell et al. 2002, p. 6). The triad Ω, μ, and $\mathcal{R}$ constitutes a stochastic process that in causal set theory uses: $\tilde{\Omega}$ as the sample space of completed labeled causal sets, a measure $\tilde{\mu}$, and the domain $\tilde{\mathcal{R}}$. The domain $\tilde{\mathcal{R}}$ is constructed as the smallest σ-algebra containing all cylinder sets $\mathrm{cyl}(\tilde{C}_n)$ defined as: $\{\tilde{C} \in \Omega | \tilde{C}_n$ is a stem in $\Omega\}$, where $\tilde{C}_n$ indicates a labeled causal set.

The condition of discrete general covariance posits that the product of transition probabilities along different paths in the causal tree (or poscau) leading to a specific causal set C_n is equivalent to the transition probability of an alternative path in the poscau that also culminates in C_n. As a consequence, “different paths in P [the causal tree] leading to the same causet should be regarded as representing the same (partial) universe, the distinction between them being pure gauge” (Rideout and Sorkin 1999, p. 5). For example, in the causal tree represented below, the two paths that lead to the final causet  are physically indistinguishable.



Yet, this does not mean that all paths are the same, but rather that there is a class of equivalence of paths with the same initial and final causal sets. Indeed: "just because the 'arrival probability' at C is independent of path/labeling, that does not necessarily mean that it carries an invariant meaning [...] [r]ather, it limits the physically meaningful questions that we can ask of the dynamics" (Rideout and Sorkin 1999, p. 10). This means that a given path in a poscau does not necessarily define an individual causal history, but rather a class of isomorphic paths: "only the relations between elements have physical significance: the labels on causet elements are considered as physically meaningless. Thus, for a subset $A \subset \tilde{\Omega}$ to be covariant, A must also belong to $\tilde{\mathcal{R}}$" (Brightwell et al. 2003, p. 6). Then, as pointed out in (among others) (Brightwell et al. 2002), (Zalel 2023), (Rideout and Sorkin 1999), if we let $\mathcal{R}$ be the collection of all such sets, any of the elements of $\mathcal{R}$ corresponds to a covariant question that can be answered by the dynamics in the form of a probability measure. Alternatively, the condition of discrete general covariance implies that the probability of arriving at a certain labeled node of the causal tree depends on the sets in $\tilde{\mathcal{R}}$.

The condition of discrete general covariance works also for individual causal sets (the individual nodes in poscau). Indeed, we can label the individual elements of a causal set, change the labels, and obtain a causal set that is physically indistinguishable from the first one, as shown for example in the following three causets:



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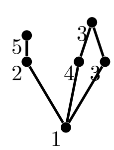

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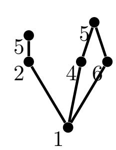


(3)

The internal structure of a causal set needs to follow another physical requirement of the dynamics: internal temporality. This imposes that: "each element [of a causal set] is born either to the future of, or unrelated to, all existing elements; that is, no elements can arise to the past of an existing element" (Rideout and Sorkin 1999, p. 9). Internal temporality sets some limits to label transformations and to the growth of a causal set in a way that given a new element x in a labeled causal set $\tilde{C}_n$, then $x \not\prec y$, $\forall y \in \tilde{C}_n$. One might notice already that the condition of internal temporality does add much to the theory in that it replicates the relation of partial order inside the structure of a causal set. Indeed, the partial order relation allows for the growth of a causal set one-element-at-a-time insofar as the child element is not born before its parent —stil allowing for the possibility that a newborn event can be unrelated to any other element of the set.

Therefore, to further limit the possible growths of causal sets, (Rideout and Sorkin 1999) adds the condition of Bell's causality, which introduces an idea of locality with respect to the causal influence exerted by subsets of $\tilde{C}_n$ onto newborn events: "The physical idea behind our condition is that events occurring in some parts of a causal set C should be influenced only by the portion of C lying to their past" (Rideout and Sorkin 1999, p. 10). The physical idea is then formalized in the principle that the ratio of a transition probability between two distinct paths from the same given causal set C is the same as the ratio of the transition probabilities of two different paths from the same set B without spectators:

$$\frac{\text{Prob}(C \to C_1)}{\text{Prob}(C \to C_2)} = \frac{\text{Prob}(B \to B_1)}{\text{Prob}(B \to B_1)} \tag{4}$$

where the transitions $C \to C_1$ and $C \to C_2$ are two paths of poscau starting from the same causal set, the causet B is the union between the precursor of the transition $C \to C_1$ with the precursor of the transition $C \to C_2$, and B_1 and B_2 are the union between the set B with the newborn element in C_1 and C_2 respectively.

Finally, I conclude this review of the CSG models by mentioning the last physical requirement, the Markov sum, and the formula for the transition probability at a given stage n of the causal tree. The Markov sum rule demands that the total sum of the transition probabilities of the causal tree must add to unity. One interesting aspect of this is that the transition probabilities should change with the progressive accretion of the causal set and they become finalized only when the causal set is 'run to completion'. This is made evident in the work by (Rideout and Sorkin 1999) where it is shown

(Lemma 2) that the addition of a disconnected element does not depend on the internal structure of the causal set, but only by its cardinality. Transitions can be calculated by associating to each newborn element a probability of being unrelated to any other element of the set. In general, the probability for a given transition is given by the formula: (Dowker et al. 2020)

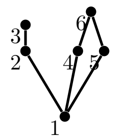
$$P(\tilde{C}_n \to \tilde{C}_{n+1}) = \frac{\lambda(\bar{\omega}, m)}{\lambda(n, 0)}, \text{ where: } \lambda(p,k) := \sum_{i=0}^{p-k} \binom{p-k}{i} t_{k+i} \tag{5}$$

where $\bar{\omega}$ is the cardinality of the ancestor set of the newborn element n, and m is the number of maximal elements of the ancestor set of n. What the equation emphasizes is that covariant events in CSG are combinations of stem events, and this implies that every physical statement in CSG dynamics is a combination of statements about which isomorphism classes of labeled causal (sub)sets are stems in the causal set representing the universe.

2.2 Covariant Growth

The general consensus is that the elements of a causal set are indistinguishable, thereby interpreting general covariance of general relativity as the label-invariance in causal set theory. This blurs a notion of a total order and thus of global becoming. While the labeling in CSG contains elements of gauge invariance, it also reflects the partial order postulated by the theory. Then, one could imagine to work with unlabeled causal sets and thus eliminate the problem of adding the invariance under label transformations. This amounts to asking whether it is possible to construct: "[...] a physically well-motivated measure on the stem algebra $R(S)$ directly, in a manifestly label-independent way that does not rely on any gauge dependent notion and which respects the heuristic of growth becoming" (Dowker et al. 2020, p. 12). Alas, after introducing the covariant dynamics of causal set theory, I will emphasize that an unlabeled partial order is an equivalence class of order-isomorphism partial orders. This is to say that the covariant models do not work with individual configurations that are indistinguishable under label transformations, but they work with sets of isomorphic causal set configurations. Hence, not only the unlabeled approach (i.e.,: covariant dynamics) does not retain a global becoming, but it complicates the physical interpretation of the covariant causets at a given node.

First of all, we define as *order* any unlabeled causal set, which is a class of equivalence of labeled causal sets.



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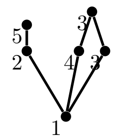

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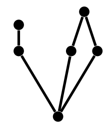
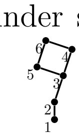

(6)

Then, from the definition of order, one can define S an *unlabeled stem* if there is a labeling of S that is a stem in a labeled causal set $\tilde{C}$. In CSG, the new elements grow one by one and for each labeled causal set one can individuate a cylinder set as a stem. Then, the set of cylinder sets defines a σ-algebra $\tilde{\mathcal{R}}$ which is also the domain of the probability measure $\tilde{\mu}$ induced by the causal histories —in this sense, histories $\tilde{\epsilon} \in \tilde{\mathcal{R}}$ are interpreted as observables of the theory (Zalel 2023), (Dowker et al. 2020). I have also briefly discussed how CSG adds discrete general covariance adds the invariance over label transformations to the theory, thereby limiting the number of possible causal sets. This limited collection of causal sets is the set of covariant events for which one cannot distinguish between order-isomorphic causal sets, and it corresponds to a sub-σ-algebra, $\mathcal{R} \subset \tilde{\mathcal{R}}$.[3]

If one wishes to interpret a random walk on the covariant tree as a physical process, then we need to give a physical interpretation to each node in the tree. Thus, for example, arriving at a given node D should correspond to the physical occurrence of the causal history $stem(D)$. But, one of the problems is that some nodes can be associated with different stem sets (unlike for cylinder sets): $stem(A) \cap stem(B) \neq \emptyset$. For example, consider the causal set: 

The node labeled (6) can be individuated by two labeled causal histories $[A = (1, 2, 3, 4)] \cap [B = (1, 2, 3, 5)] \neq \emptyset$. The non-empty intersection means that the identification of the stem sets with physical occurrences of causal histories does not warrant the possibility of having a unique physical causal history. The solution proposed by the covariant approach is that "a covariant dynamics can be defined as a walk on a tree formed of countably many levels in which the nodes in level n are not single n-orders, but sets of n-orders. Each set of n-orders in level n will correspond to the covariant

[3] A σ-algebra on a set X is a non-empty collection Σ of subsets of X that is closed under countable unions, complement, and countable intersections. The ordered pair $< X, \Sigma >$ is a measurable space.

event 'the n-stems of the growing causal set are the elements of this set'. We call this the *covtree*, short for covariant tree" (Zalel 2023, p. 10).

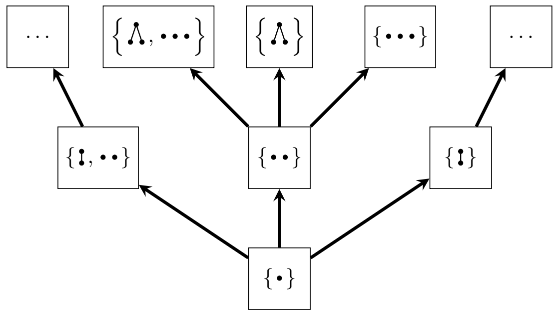
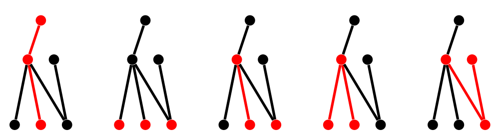


Figure 3: Example of a covariant tree.

A covariant tree (see: Figure 3) is a partial order whose elements are the collection of sets for which there is a certificate, and the relation is the usual causal relation $\prec$. A certificate Γ_n is an order C such that $\Gamma_n \subseteq \Omega_n$ is non-empty and Γ_n is the set of all n-stems in C. For example: given $\Omega(3) = \{ ⋮, •••, : •, Λ, V \}$ one of the certificates of $\Omega(3)$ is a causal set that contains all stems of $\Omega(3)$ as shown below:



That the nodes of the covtree are collections of sets for which there is a certificate guarantees that the nodes are not simply combinations of causal sets with cardinality n. For example, consider again the collection of 3-orders $\Omega(3)$. The subset $\Gamma_3 = \{ ⋮, ••• \} \subset \Omega(3)$ has no certificate because any order C that is a certificate of Γ_3 will contain the order : •. That is, one cannot build a certificate from the set Γ_3 without including the set : • as its stem. Finally, the map in (Dowker et al. 2020)

$$\mathcal{O}(\Gamma_n) \coloneqq \{B_{n-1} \in \Omega(n-1) | \exists A_n \in \Gamma_n \; s.t. \; B_{n-1} \text{ is a stem in } A_n\}$$

takes the set Γ_n to the set of $(n-1)$-stems of elements of Γ_n. For example:

$$\mathcal{O}\left(\left\{ : \,•, V, Λ \right\}\right) = \{ :, •• \} \tag{7}$$

We can now give a better definition of covtree as the partial order $< \Lambda, \prec>$, where $\Gamma_n \prec \Gamma_m$ if and only if $n < m$ and $\mathcal{O}^{m-n}(\Gamma_n) = \Gamma_m$.

Thus, the map $\mathcal{O}$ helps us navigate covtree backward, but: "[a]t stage n, we do not know which finite order has grown thus far nor its cardinality, only which n-stems it contains. While in the CSG models the growth is explicit, on covtree it is implicit or 'vague'. But if there is a process of growth which can be associated with a covtree walk, then it may be that it is this quality of vagueness which embodies asynchrounous becoming" (Zalel 2023, p. 20).[4] Here, the expression 'asynchronous becoming' refers to the fact that new elements of a causal set are born in a partial, order, see: (Sorkin 2007) (Dowker 2014), (Dowker 2020) (Bento and Zalel 2021).

Since the collection of stem and of certificate sets generates the same σ-algebra, we can define a measure μ on unlabeled causal sets similarly to how we defined the measure on random walks on labeled poscau in CSG.[5] However, similarly to CSG models, it is not granted that the dynamics defined on un-labeled causal sets is physically interesting. CSG models limited the number of possible causal sets by adding the conditions of Bell Causality and Markov Sum, but this same conditions cannot be applied to the covariant dynamics since both conditions require labeling. For example, Bell's causality applies insofar as the new-born element of a transition can be uniquely individuated —which is not possible in covtree due to the nature of orders as isomorphic classes.

Then, even though covtree allows us to define a dynamics that avoids the problem of introducing gauge elements (labels), it does not escape the problem of identifying which histories in covtree (paths) are physically salient. This raises a few pressing questions that make the model less appealing: "is there a condition on a random walk up covtree which expresses the physical condition of relativistic causality? [...] is this new condition enough to reduce the class to a physically interesting one or are other conditions needed and what are they?" (Dowker et al. 2020, p. 26).

[4]This is warranted by a theorem that proves that every path in covtree has at least one certificate, and it is proven by showing that there is a surjection from the set of infinite orders Ω to the set of different covariant paths. See: (Zalel 2023), (Zalel 2020) and (Dowker et al. 2020) for details.

[5]I shall leave the details of deriving the measure to more mathematically oriented reviews, for example: (Surya 2019), and (Dribus 2017).

3 The Problem of Time

3.1 Presentism and Block View

In a recent paper, (Wüthrich 2023) argues that the block view is the best approach to address time in causal set theory and classical sequential growth, and that structuralism is the best metaphysics for causal set theory in general. Here, I shall focus on the former point, and leave the latter to later works.

Let us first consider space and time separately. On the one hand, causal precedence is structurally similar to some temporal precedence with no time-extension, nor duration, nor flow. The similarity is so evident that (Dowker 2020, 136n) considered the fundamental relation of causal set as temporal and not causal. On the other hand, space is usually characterized by topological and metrical structures that define relations such as: nearby, between, far away, etc. In relativistic spacetime, space is built as a foliation of Lorentzian manifolds into 3-d hypersurfaces that are ordered by a time parameter. But, causal set theory has no such structure, and at best one can use inextensible anti-chains as equivalent to the foliation of relativistic spacetime. However, inextensible antichains are un-structured subsets of a causal set, and thus do not possess an internal structure that can mirror the one of Lorentzian spacetime hypersurfaces. It seems that not only spacetime is difficult to recover from causal set theory, but also space alone.

It has been widely discussed in the literature of philosophy of time how relativistic physics tends to make things difficult to the presentists (and growing block theorists), due to the lack of a sharp distinction between what is present and what is not.[6] As remarked in (Wüthrich 2023), some solutions have been attempted and thoroughly discussed in the literature. For example, light-cone presentism (Stein 1991) maintains that, given an event in spacetime, all events in the light-cone are to be considered as co-present.

However, the events on the null-geodesics, and thus those events that are sitting on the edge of the light-cone, cannot be ordered globally by a relation of temporal precedence. The reason for this impossibility is that these events are spacelike separated, and therefore, their ordering is dependent on the choice of reference frame. In other words, it is a consequence of the relativity of simultaneity in relativity theory. This is analogous to how the events on the antichains in causal set theory have no order, although the reason in that

[6] See, for example: (Ingram and Tallant 2023).

case is the lack of internal structure of the antichain subsets.

In addition, since a light-cone is always centered around an event, the present is always dependent on the arbitrary choice of the reference frame that takes the event as the central point of the light-cone. Yet, an attempt to make (Stein 1991)'s proposal objective was set forth in (Clifton and Hogarth 1995). With their account, the present becomes objective in that it is identified with a given worldline, but it also becomes inevitably local: "[w]orldline-dependent becoming is objective in that it only relies on the geometry of Minkowski spacetime. Furthermore, it is absolute in that it is frame-independent, i.e., is based only on Lorentz-invariant structures. However, it is local in that it depends on a particular given worldline. Thus, even though it does not privilege a particular frame of reference, it sanctions one particular worldline or observer. I will call this feature of worldline-dependent becoming *local*" (Wüthrich 2023, p. 14).

Special relativity is therefore a problem to the advocates of presentism, since they either give up on their metaphysical desiderata —that is a clear and well-defined notion of becoming—, or they give up on the compatibility with relativity. Similar problems arise with general relativity (GR) as well: on the one hand, some models (such as FLRW models) allow for a foliation of spacetime into hypersurfaces, but the principle of general covariance brings back the problems we have seen for light-cone presentism. On the other hand, other models of GR do not admit a foliation, thereby blurring the distinction between what is present and everything else. This means that in facing the theory of relativity, "the presentist has two main strategies available: either they forgo the idea of global present in favor of a more local notion, or else they make the case that those unfoliable spacetimes are, although formally models of GR, not physically reasonable possibilities" (Wüthrich 2023, p. 16).

What about causal set theory? The kinematics does not offer a proper notion of becoming in that it only consists of the space of the possible configurations of the causal sets. Therefore, real becoming could be based on the dynamics of the theory, which limits the configurations of the causal sets to those that are physically possible.

However, we have seen how the principle of discrete general covariance in causal set theory mirrors the diffeomorphism invariance of general relativity. It follows that, since general covariance in GR is problematic to the presentist view, the same difficulty manifests in causal set theory due to discrete general covariance. In addition, light-cone presentism, applied to general relativity, also fails to provide a proper global line of becoming since only the events on the surface of the light-cone (i.e.: edges excluded) can be temporally ordered. This implies that the ordering of all events on a light-cone corresponds to a partial order similar to that of causal set theory. The two partial orders are more than 'just similar' in that the Hawking-Malament theorem guarantees that the totality of causal relations in relativistic spacetime describes the structure of spacetime up to a conformal factor. Causal set theory is built from such a result, and thus it should be of no surprise that the two theories display an analogous causal structure. It is by construction that the local temporal becoming of general relativity resembles the asynchronous becoming of causal set theory.

It is thus explained why causal set theory does not solve the problems of temporal becoming that were set forth in (Wüthrich 2023), and related discussions in philosophy of time. The theory is constructed starting from the very causal structure of relativistic spacetime that makes the presentists' view at best problematic. In addition, Bell's causality, which is one of the requirements imposed to the theory to select physically significant causal sets, establishes that the growth of a new element does not depend on the global structure of the causal set. What this means is that there is a locality condition engrained in the CSG dynamics that affects the causal relation and the birth of new elements. By affecting the birth of elements, the locality condition excludes the possibility of having a global temporal coordinate indexing each birth. Also, the very definition of the causal condition as a partial order hinders the possibility of a global temporal coordinate, in that it would become impossible to give a temporal order to newborn un-linked events. It follows that the difficulties raised by general covariance to the growing block view and presentism are similar, if not the same, as the ones raised by the principle of discrete general covariance.

The conclusion of (Wüthrich 2023, p. 20) is that the proper temporal metaphysics for causal set theory can be determined only when causal sets run-to-completion: "it thus seems as if all events in a dynamically growing causal set, including 'past' ones, remain ontologically indeterminate until the growth process is completed. At that stage, finite or not, we have the full causal set, and the resulting ontology is indistinguishable from one based on the block universe metaphysics". It is indeed true that the proper transition probabilities in CSG can be assigned when the causal set has 'run-to-completion', that is, $\tilde{\Omega}(\mathbb{N}) = \tilde{\Omega}(\infty) = \tilde{\Omega}$, but this only avoids the need for updating all probabilities at each new birth in the causal set. It remains that a run-to-completion labeled causal set is still subject to the principle of discrete general covariance, and that a given causal history will be equivalent to all of the other causal histories that partake to the same isomorphism class.

Perhaps, the best analogy is with the sum-over-histories approach to quantum mechanics, where the transition amplitude of a given system is obtained by summing over the individual transition amplitudes of all the possible trajectories, each weighted by a phase factor that contains the classical action of the given path.[7] In the case of the sum-over-histories, the initial and final states of the system are fixed, thereby resembling the idea of a causal set that has run-to-completion. However, (Forgione 2020), for example, has shown that it is impossible to individuate a physical history of the quantum system among the ensemble of possible paths, and that the possible histories are subject to a mechanism of cancellation that involves paths that are mathematical artifacts, and thus have no physical interpretation. The CSG dynamics offers no such mechanism of cancellation, and the principle of discrete general covariance does not allow us to distinguish the physical history of the universe, even when run-to-completion.[8]

The eternalist view accommodates causal set theory with respect to the absence of a global line of becoming, and that is because the block view does not bestow a special ontological value to presentness and present entities. This is possible because of the run-to-completion condition, which by definition excludes the existence of a line of becoming. That is, without becoming, one does not need to identify a line that separates present from future, and the metaphysics of eternalism suddenly becomes a viable possibility to account for time in causal set theory. Yet, even if we accept the eternalist view, we still have no answers to what causal history represents the history of the universe, nor we have a clear physical interpretation of what it means to have a causal set that has run-to-completion.[9]

3.2 The Growing Block View

By investigating some philosophical questions about temporality and becoming, (Arageorgis 2016) compares aspects of general relativity with the causal set approach. The conclusion is that, despite some promising assumptions,

[7]See, for example: (Forgione 2020), (Hartle 1993).

[8]I will say more about arguments that are based on causal sets that have run-to-completion in the next section.

[9]the mathematical aspects of a causal set that has run-to-completion are discussed in (among others): (Brightwell et al. 2003) and (Brightwell et al. 2002).

causal set theory fails at identifying a clear 'line of becoming', and thus at fitting in with the metaphysics of the growing block view. In what follows, I shall integrate the argument presented in the previous section with some considerations from (Arageorgis 2016), and I will expand them to the covariant dynamics presented in Section 2.

The starting point of (Arageorgis 2016)'s argument is the recognition of how difficult it is to reconcile a clear notion of becoming and relativistic spacetime. The source of this difficulty is because of the 4-dimensional manifold that hosts the geometry of spacetime and that seems to favor an eternalist metaphysical view.

> What I advocate as 'the doctrine of the manifold' [...] is simply a philosophical acceptance, as an ultimate literal truth about the way things are in themselves, of the conception that nature, all there is, was, or will be, 'is' (tenslessly) spread out in a four dimensional scheme of location relations which intrinsically are exactly the same, and hence in principle commensurate, in all directions, but which hapen to be differentiated in our neighborhood at least, by the fact pattern of the things and events in them. [Cited in: (Savitt 2002, p. 2)].

The debate is again between two major camps: on one side there is the static notion of the world (block-view and eternalism) for which what exists does not depend on time and change, but rather it is a matter of what properties the universe has at a given time. On the other hand, there is the dynamical view (presentism and growing block view) for which the properties of the universe (and objects thereof) change with time. For example, the notion of cause often plays a role in the dynamical view, for the events of the universe are connected by causal relations in a way that the cause, which is prior-in-time, produces a temporally subsequent event.

From a philosophical point of view, the problems start to emerge at the junction point between cause and effect, that is, at the imaginary line of becoming. Indeed, to support the dynamical view, one needs a robust notion of becoming that applies to the spacetime manifold that hosts all events. As I recalled in the previous section, the common solution is to identify the line of becoming with the spacelike hypersurfaces that foliate spacetime, and to index the foliation with a global time-function. There are some problems related to this solution, such as: the fact that some models of general relativity do not admit a global time function, or, even if they did, they admit infinitely many.

While the causal set approach to quantum gravity might seem to provide a possible solution to the problems of a global time function, (Arageorgis 2016) suggests that such a hope is in the end misplaced. The hope to use CST to solve the problem of global time is due to the CSG models and to the growth of one element at-a-time. For example: "By providing a physical mechanism for producing growth and becoming, this approach promises to transmute Becoming from a piece of speculative metaphysics to one of naturalized methaphysics" (Earman 2008, pp. 159–160).

But, in causal set theory, the process of becoming is a stochastic process that ought to be defined as a family of random variables of a probability space. To define such a stochastic process with mathematical consistency, one would need the 'complete' causal set: "In order to define μ consistently, one must take $\tilde{\Omega}$ [the sample space] to be a space of infinite causets, ones for which the growth process has 'run to completion'. We meet here with an echo of the block-universe idea, that is in effect built into mathematicians' formalisation of the concept of stochastic process" (Sorkin 2007, 160, fn.8).

Let us imagine, for the sake of the argument, that the run-to-completion problem is solved. In this scenario, the probability space would provide answers to questions that do not account for the invariance under label transformation, and thus: "[t]he elements of a causal set are not intrinsically individuated and, consequently, for each labeled causal set the only representation endowed with physical significance is the isomorphism equivalence class it belongs to (Arageorgis 2016, p. 45)". Here, the argument presented by (Arageorgis 2016) is analogous to the one we have seen in the previous section about the possibility of stopping the process of birth at a given stage: "[...] stopping the process at a given stage has no objective meaning within the theory, because with a different choice of birth-order, the causet at the same stage of growth would look entirely different" (Sorkin 2007, p. 157).

Nonetheless, another possible way to ensure the compatibility between causal set theory and a well-defined line of becoming is to embrace a notion of localized becoming, for which: "the temporary locus of becoming at each stage of growth of a causet contains exactly one element" (Arageorgis 2016, p. 48). This seems surely appealing, especially because it is consonant with the condition of Bell's causality for which the growth of a new element is only local. Therefore, it seems that the precedence relation ($\prec$) can be good for distinguishing what has come into existence and what has not, but only locally. The idea is not new, as it was already suggested in (Dowker 2005, p. 458): "[t]here is growth and change. Things happen! But the general covariance means that the physical order in which they happen is a partial order, not a total order. This doesn't give any physical significance to a universal Now, but rather to events, to a Here-and-Now. I am not claiming that this picture of accumulating events (which will have to be reassessed in the quantum theory) would explain why we experience time passing, but it is more compatible with our experience than the Block Universe View". Similarly, (Dieks 2006, pp. 172–173) suggests that "[...] the natural view is that the history of our universe is realized by events that come into being; and that they come into being after and before each other as dictated by the partial ordering relation induced by the spacetime structure. According to this proposal the life of the universe is not one linear series of events, but a partially ordered set of events".[10]

However, to the possibility of a local becoming, (Arageorgis 2016) adds two objections: on the one hand the view is not new. It is basically coming from the literature on philosophy of special relativity in Minkowski spacetime. On the other hand, a local present is too weak to underpin a strong notion of becoming (see: (Savitt 2021)). The first objection is not much of a punch against causal set theory and the definition of a line of becoming. Rather, it is the recognition that the debate on time in special relativity is not over yet. Arageorgis (2016) reaches the same conclusion: "The only way out for a proponent of [causal set theory], who wishes to cling to a notion of Becoming that salvages a dynamic conception of the world, seems to be the stratagem of localizing Becoming and the present. But this move, whether on the right track or not, is not novel: it has been proposed and debated in the context of philosophical attempts to trace a viable notion of Becoming within relativistic spacetime theories on continuous (smooth) manifolds" (Arageorgis 2016, p. 51). Under such a perspective, which is not too dissimilar from the one that emerges in the debate on relativity and time, the heart of the matter seems to be in the hands of the presentists, and on the difficulty of accepting that the line of becoming can be defined only locally; but this is for the philosophy of time community to discuss.

In addition, Arageorgis (2016) maintains that causal set theory does not meet the challenge of the growing block metaphysics also because the dynam-

[10]The acceptance of a local becoming has also emerged in (Wüthrich 2023) as one of the only possibilities for the advocate of a metaphysics of presentism.

ics of the theory is not fully developed yet. The CSG dynamics: "cannot be defined with mathematical consistency but in the limit of infinite time 'when' causal set growth 'has reached completion'" (Arageorgis 2016, p. 51). As I mentioned earlier, one could hope for a dynamics that replicate the idea behind the sum-over-histories account, but at least two problems would still remain. On the one hand, one would need to develop a mechanism of paths cancellation between causal trajectories, and thus the measure used in CSG models would need some substantial revisions. On the other hand, it is not clear what the final state of the universe would amount to —which is also the problem of having a causal set that is run to completion.

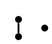
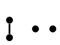
Finally, I wish to explore one more possibility before turning to covariant dynamics. The dual time view by (Huggett 2014) suggests the existence of a second time that is not identical to our physical time, but still related. In the paper, Huggett represents physical time by the term t_0, while t_T represents a second more fundamental time. Then, one can express change of one time with respect to the other and say things like: 't_0 = Monday is the present at $t_T = 0$, but not at $t_T = 1$'. This way, the t_0-present changes relative to the t_T-time. In causal set theory: the evolution in t_T generates a causal set and hence a spacetime region within which one has metric notions and temporal extensions. There: t_T is the time of the growth and t_t is the time internal to the causal sets. "In those terms [...] there are two kinds of temporal relations involving t_0 times: first, relations to other t_0 times, such as Monday being before Tuesday —these are the ordinary, unchanging B-series relations. But second, there are relations between t_0 and T times, between times in the causet and the external time: call these 'T-relations'. Potentially, $t_0 = 0s$ might be the present at $T = 1$, but $t_0 = 1s$ the present at $T = 100$. With respect to T, the present changes, time passes" (Huggett 2014, p. 10).

In the dual time model, it seems as if the time t_T is absolute and external. If that is the case, what about relativity and the problems mentioned earlier? Since CSG dynamics is generally covariant, the probability of the growth of a causal set is independent from the order in which the growth happens, and t_T is just a label in a theory that is invariant under label transformations:

> Put another way, there is no way to determine T beyond what follows from the causal order given by the effective, internal time: anything more is, in the general sense, pure gauge. Concretely, if p is in the absolute future of q, then it is physical that p is in the future of q with respect to T, since every covariant ordering

> has $T(p) < T(q)$, but $|T(p) - T(q)|$ is unphysical. If p and q are spacelike, then there is not even a fact about which comes first with respect to T; $T(p) < T(q)$ and $T(q) < T(p)$ are physically indistinguishable. [...] T-relations are fundamental, and hold just between the discrete nodes of the causet; the relations internal to spacetime are merely effective, and hold between the continuous points of the effective manifold. Of course the relations agree, but that is a consequence of the general covariance of the dynamics which produces the causet, and does not make them the same relations (Huggett 2014, p. 10).

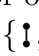
This is not a new formal feature of the theory: Dowker has pointed that out already when she claimed that things happen as a partial order and not as a total order. Huggett (2014)'s point here is that taking the partial order proposal requires a distinction between t-relations and phenomenal spacetime as distinct, but in agreement — and the agreement is explained by the general covariance of the dynamic.

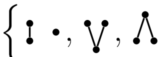
Thus, instead of having one single relational clock (the labeling of the internal elements of a causal set) that tracks the partial ordering of the newborn elements, (Huggett 2014) adds an external clock t_T that tracks the overall growth of the causal set. The labels of the elements of the causal set can be put in relation with this external clock, which, because of discrete general covariance, agrees with the internal partial ordering of the causal set. In other words, the dual time account gives a temporal interpretation to the two levels of causal set theory (see: (Dribus 2017)): on the one hand, an individual node corresponds to a possible configuration of a physical system with its internal time. On the other hand, there is the objective (external) growth of the causal set as represented by the causal tree (poscau). I do not wish to discuss whether the view suggested by Huggett offers a better temporal metaphysics for causal set theory. It remains that the incompatibility with presentism persists. For example, consider the causal set  to be the present at the external time $t_T = 1$. Then, while the causal set  is in the absolute future of $t_T = 1$, it is unphysical to ask whether the causal set  is in the absolute future of $t_T = 1$, or whether the causet  preceded the causet . As emphasized by (Huggett 2014), Dowker had already suggested that things happen in a partial order, but partial orders are incompatible with presentism (and with the growing block view).

3.3 Covariant View

What about covariant dynamics, does it solve or help solving some of the temporal problems we have been addressing so far? The identification of antichains as a physical line of becoming, or the use of maximal elements (and corresponding stems) in a causal set have proven to be implausible solutions. On the one hand, antichains are too unstructured to be identified as a demarcation between what is present and what is not. On the other hand, maximal elements are subject to the invariance under label transformations, and thus can be at best used to indicate a local present. The invariance under label transformations blurs the concept of an individual causal history, as each causal history belongs to an isomorphism class of histories that differ only in their labeling.

Since many of the problems of accounting for time in causal set theory stem from the invariance under label transformations, does a dynamics that has no labels solve, or help solving, some of such problems? Let us begin with the invariance under label transformation. Since covariant dynamics has no labels, this should set us on a good start to solve the problem. However, instead of having nodes of individual causal sets ordered in a poscau, the covtree of covariant dynamics uses classes of equivalence of labeled causal sets as the content of each node. This makes the physical interpretation of each node even more problematic, for each node does not distinguish between the individual sets of a given class of equivalence. For example, the order  corresponds to a class of equivalence that contains the two causets  and . The use of unlabeled sets as representative of a class of equivalence is due to the gauge invariance of the theory, which is meant to recover the general covariance of relativity, and it is thereby incompatible with presentism.

As I mentioned in section 2, one of the problems of interpreting the stems as genuine causal histories is the possible occurrence of non-empty intersections between different stem sets. Covariant dynamics solves this problem by using sets of orders as nodes in the covariant tree. Yet, the dynamics imposes some conditions on which orders are allowed in the covtree: only those sets for which there is a certificate. While certificates limit the possible causal sets in a node, it remains that many nodes will be constituted by a set of classes of equivalence (that is, sets of orders). This implies that, for example, we can have a transaction $\{\bullet\} \to \{$$, \bullet\,\bullet\}$ which renders the physical interpretation of which spacetime configuration one has at the given node impossible.

Furthermore, within the same node in covtree we can have orders that represent very different causal connections. For example, consider the node represented by the set of orders: {  }. Even if we were to identify a line of becoming with a given order, we would not be able to determine which order represents a genuine causal history of the universe. Also, because those different orders instantiate very different causal connections among the same number of elements, it is impossible to determine the temporal order of the individual elements.

Finally, that covariant dynamics is not compatible with presentism is also implied in the words of (Zalel 2023, p. 20): "While in the CSG models the growth is explicit, on covtree it is implicit or 'vague'". As I have argued earlier, one of the problems with the CSG dynamics and its compatibility with presentism is that the growth of the poscau follows the partial order relation of the causal sets —which was made evident also in the dual-time account by (Huggett 2014). It follows that if the covariant dynamics has an even vaguer mechanism of growth, which also implies the notion of asynchronous becoming, the incompatibility between presentism and CSG can only be reinforced in covariant dynamics.

4 Conclusions

In this paper, my aim has been to present additional objections to the possibility of pairing the causal set approach to quantum gravity with prevalent temporal metaphysics. In the first part of the paper, I have provided a concise overview of causal set theory, specifically delving into classical sequential growth and covariant dynamics. I emphasized the correlation between discrete general covariance, invariance under label transformations, and the general covariance of relativity theory.

The philosophical discussion of the paper focused on the temporal metaphysics of presentism, with a particular focus on the growing block view of the universe. I discussed, and expanded upon, the arguments presented in (Wüthrich 2023) and (Arageorgis 2016), highlighting the incongruities between causal set theory and those temporal metaphysics frameworks. Specifically, the primary challenge for a presentism metaphysics lies in establishing a clear line of becoming. Causal set theory complicates this endeavor due to the lack of structure in the anti-chains, which thus cannot constitute the equivalent of a sequence of hypersurfaces ordered by a temporal parameter (as seen in Lorentzian geometry). Other attempts, such as worldline presentism, fall short in that, while they solve the problem of the objective present, they also imply that the present is only local. In addition, I used the similarity between discrete general covariance and the general covariance principles of general relativity to argue that the very same challenges faced by presentism in the context of relativity theories manifest in causal set theory as well.

I briefly explored the prospect of a causal set employing a run-to-completion strategy and how it implies a block view metaphysics. However, this strategy fails to resolve the issue of invariance under label transformations, rendering the physical interpretation of a system's causal history somewhat ambiguous. Finally, I touched upon covariant dynamics and argued that despite the absence of labels and label transformations, the challenges for presentism and eternalism persist in covtree as well.

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