

Group Field Theories: Decoupling Spacetime Emergence from the Ontology of non-Spatiotemporal Entities

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Abstract

With the present paper I maintain that the group field theory (GFT) approach to quantum gravity can help us clarify and distinguish the problems of spacetime emergence from the questions about the nature of the quanta of space. I will show that the use of approximation methods can suggest a form of indifference between scales (or phases) and that such an indifference allows us to black-box questions about the nature of the ontology of the fundamental levels of the theory.

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1 Introduction

Many approaches to quantum gravity (QG) seem to agree that spacetime is not a fundamental entity and, as such, it should emerge from a different non-spatiotemporal structure —for a general overview of many such approaches see: (Oriti 2009).¹ The immediate consequence is that the fundamental

¹Notably, there are also non-emergent approaches such as string theory, and emergent approaches that posit fundamental entities that have spatiotemporal entities different form

ontology of QG seems to be non-spatiotemporal in nature, which is an issue that causes many headaches to the working philosophers. For example: is the emergence of spacetime an inter-theoretic property (Bain 2013b), or is it an ontological property that requires a metaphysical account (Huggett and Wüthrich 2013), (Lam and Wüthrich 2018)? Instead of making spacetime emergent, can we relinquish either space or time separately (Smolin 2020), (Gomes 2016)? What account of emergence should we expect (Oriti 2021a)?

Perhaps, for the sake of simplicity, we can divide some of the technical and philosophical problems of quantum gravity into two broad categories: on the one hand we have the challenge of accounting for the emergence of spacetime, that is, we need to provide a mechanism (mathematical or physical) that describes how non-spatiotemporal and pre-geometric entities can approximate the geometry of spacetime. On the other hand, we have to face the problem of accounting for the existence of fundamental entities that are pre-geometric and that do not live in spacetime. In what follows, I will focus on the group field theory approach and use the analogy with the Ising model to show that we can treat the two classes of problems as independent of one another.

It is well known that physical systems may undergo an abrupt change in their macroscopic behavior as certain quantities are varied smoothly. Such changes of behavior happen when those quantities (such as, for example, temperature and pressure) reach certain values called critical points, which mark a phase transition from one state of matter to another. These critical points are strongly related to the length scale at which the properties of the macroscopic system begin to substantially differ from the properties of the individual parts. Such a length scale is usually called correlation length, and it indicates the distance at which the fluctuations of the microscopic degrees of freedom of the system are correlated with each other.

There are two ways in which a phase transition may occur: (i) discontinuous (first-order) phase transitions in which the two phases coexist at the critical point, and the correlation length is finite; (ii) continuous (second-order) phase transitions in which as the critical point is approached both phases are identical, and the correlation length diverges.² The divergence of

those described by general relativity (for example: (Volovik 2006)). In this contribution I will focus on emergentist approaches to quantum gravity that posit non-spatiotemporal fundamental entities.

²The distinction between the two types of phase transitions can be further characterized in terms, for example, of latent heat. See: (Binney et al. 1992).

the correlation length close to a second order phase transition allows for a description of the behavior of the macroscopic system (near critical point) in terms of effective theories, involving only long-range collective fluctuations. It is in this sense that: “[m]any properties of a system close to a continuous phase transition turn out to be largely independent of the microscopic details of the interactions between the individual atoms and molecules” (Cardy 1996, p. 3).³ The description of these systems makes use of mathematical tools such as: renormalization group techniques, mean field theory, hydrodynamic approximations, and others.⁴

In what follows, I will use the Ising model, mean field theory, and the hydrodynamic description of many-body systems to remark and describe this independence between microscopic and macroscopic scales of a system under specific conditions. If a macroscopic system is indeed (at least partly) independent of its component parts, then the philosophical problem of addressing the properties of the individual components can be separated from the problem of addressing the macroscopic properties of the system. Afterward, I will apply such considerations to the group field theory (GFT) approach to quantum gravity. I shall argue that, if the emergence of spacetime is (partly) indifferent to the dynamics of the individual quanta of space, we can separate the problems related to spacetime emergence from those related to the fundamental ontology of the theory.

The reason as to why I focus on the group field theory approach is the immediate connection with the physics of phase transitions. As a matter of fact, the key idea in the GFT approach—which also led to the derivation of cosmological models (both homogeneous and inhomogeneous, see for example: (Gielen and Oriti 2018) and (Marchetti and Oriti 2022))—is to interpret the collective behavior of the quanta of space as a form of Bose-

³It is important to stress the fact that not all systems manifest the type of independence between macroscopic and microscopic scales described here. Indeed, for example, in Bose-Einstein condensates the Bose statistics (microscopic scale) is directly responsible for the superfluidity behavior at the system’s macroscopic scale. I thank an anonymous reviewer for stressing this point in a previous version of this manuscript.

⁴Another important feature that characterizes systems undergoing this type of phase transitions is that they can be grouped into different classes, each characterized only by global features, such as symmetries and number of spatial dimensions of the system. This feature is called universality, and it has been the subject of many debates in philosophy of physics—especially with respect to the possibility of giving a reductionist explanation to such an interesting feature. For example: (Franklin 2018), (Batterman 2013), (Butterfield 2014), (Butterfield and Bouatta 2012), (Morrison 2012), and others).

Einstein condensate (see: (Gielen, Oriti, and Sindoni 2014), (Gielen, Sindoni, et al. 2016), (Oriti 2017), (Pithis and Sakellariadou 2019), (Gielen and Polaczek 2020), (Gabbanelli and De Bianchi 2021)). The individual quanta of space of the theory are taken to be living in a pre-geometric microscopic phase, in that they are the fundamental entities of the theory and they have no spatiotemporal properties. By studying their collective behavior (within the thermodynamic limit) in the mean field approximation, one obtains the equations of a condensate from which it is possible to extract some properties such as, for example, the volume of the condensate.⁵

In Section 2 and 3, I present the philosophical problems of quantum gravity that I will be discussing in this paper. I will clarify notions such as emergence and reduction and review some of the recent literature. In section 4, I will provide a brief discussion of the Ising model to the purpose of showing how phase transitions offer us a case of indifference of the macroscopic properties of a given system from the dynamics of its individual components. Section 5 briefly introduces the group field theory approach to quantum gravity and applies the results of Section 4 to the emergence of spacetime. Section 6 offers some concluding remarks.

2 Setting the Stage: Levels of Emergence

Two recent papers, (Oriti 2021a) and (Oriti 2021b), set out a multilevel-ontology account of spacetime emergence in the context of some approaches to quantum gravity.⁶ The starting point of Oriti’s argument is the necessity (and corresponding difficulties) of identifying the fundamental degrees of freedom of a theory that aims at justifying the emergence of spacetime from entities that are fundamentally non-spatiotemporal. The problem, which stems from the common understanding of observables living in spacetime, has even led to doubts about the very possibility of verifying a theory of quantum gravity in the first place (see: (Huggett and Wüthrich 2013)). The atoms of space, the fundamental degrees of freedom of the theory, shall define

⁵One of the main results of the approach is that the equation of motion of the condensate is analogous to the Gross-Pitaevskii equation in canonical condensed matter (Gielen and Polaczek 2020). The geometric quantities defined by the condensate can be interpreted as the Friedmann equations for the GFT condensate and they can be shown to be consistent with their canonical counterpart (see: (Oriti, Sindoni, and Wilson-Ewing 2016), (Oriti, Sindoni, and Wilson-Ewing 2017)).

⁶An overview of such levels can also be found in: (Bianchi and Gabbanelli 2023).

not only a quantum dynamics, but also show how at some (continuum) limit general relativity (GR) becomes a good effective description of spacetime. In other words: one of the challenges faced by the community working on quantum gravity is to show how GR spacetime emerges from non-spatiotemporal fundamental entities.

The instance of emergence, in this case, is taken to be one that justifies novel properties that are missing from the properties of the underlying entities. See, for example: (Butterfield and Bouatta 2012), and (Butterfield 2011a). In this sense: “[e]mergence is understood to be the appearance, in a certain description of a physical system, of properties that are novel with respect to a different (more ‘fundamental’) description of the same system, robust, and thus stable enough to represent a characterization of the new description and to form part of new predictions stemming from it” (Oriti 2021a, p. 17). The emergence of such new phenomena from some underlying entities often requires the use of limiting procedures and approximations. These provide a new description of the system *via* novel quantities that, as we shall see below, are indifferent to the dynamics of the more fundamental levels.

In his account, Oriti defines four non-sequential levels of spacetime emergence, to be interpreted as describing the issues involved in the emergence of spacetime from the atoms of space of a theory of quantum gravity. The first level (listed as ‘Level -1’) emphasizes how general relativity already implies a disappearance of a notion of absolute space and time. That is: the continuous fields of GR are defined on a differentiable manifold but, because of diffeomorphism invariance, the manifold offers only global topological constraints on the fields that ‘constitute’ spacetime. As a consequence, the individual points of the manifold carry no physical meaning, and the general solutions to the dynamical equations do not single out a preferred direction of time or space. In quantum mechanics, and with respect to the choice of a preferred reference frame, things are not that dissimilar: “no preferred space or time direction is present in the theory, coordinate frames are unphysical and generic physical configurations of the quantum spacetime will also not select any” (Oriti 2021a, p. 5). A possible solution is the relational strategy (see, for example, (Gambini and Porto 2001)), for which there is no space and time, but only imperfect physical clocks and rods. That is, one can attempt to identify some internal degrees of freedom (some appropriate generic fields) of the system acting as approximate clocks and rods that thereby parametrize the spatial and temporal relations of the remaining degrees of freedom. The

notions of space and time thus defined do not match their usual Newtonian counterparts yet, since that is the case only for special kind of fields and approximations.

Then, (Oriti 2021b) further distinguishes a level of emergence that involves the canonical quantization of general relativity. Provided that one can proceed with canonical quantization, one would obtain a valid theory of quantum gravity insofar as “the quantum dynamical nature of the matter fields chosen as preferred reference frame can be neglected” (Oriti 2021b, p. 4). In such a theory of quantum gravity, space and time disappear in a sense that is even more radical than in the previous level. Indeed, the process of quantization implies that the field will be subject to quantum properties such as uncertainty, non locality (entanglement), and discreteness of the observables. Therefore, “[s]tarting from such quantum realm, the emergence of space and time as we know them from GR requires a number of approximations and restrictions, which together define the semiclassical limit of quantum gravity” (Oriti 2021b, p. 5). This level, which recovers space and time from generally covariant dynamical quantum fields is listed as Level 0.

Level 1, in Oriti’s taxonomy, constitutes a different sense in which space and time ought to emerge in some approximations in a theory of quantum gravity. This level implies the existence of quanta of space (or atoms of space) as non-spatiotemporal entities, and such new degrees of freedom constitute the theory’s new fundamental building blocks from which space, time, and geometry, are supposed to emerge in some continuum limit —although they are still different from the smooth continuous spacetime of general relativity. Level 1 is thus fundamentally different from the previous ones, in that space time and geometry do not emerge in the context of a (more or less) straightforward quantization of general relativity. Notably, several approaches seem to conceive of emergence as described in Level 1. For example, spin networks in loop quantum gravity (for example: (Ashtekar and Lewandowski 2004)): “with their dual functional dependence on group elements or group representations associated to graphs, and their histories labeled by the same algebraic data and associated to cellular complexes” (Oriti 2021a, p. 6). Another approach that is based on a new type of fundamental entities is causal set theory (for a review, see: (Surya 2019)), where discreteness is assumed as a core tenet of the theory and the elements of a causal set are in a causal relation (partial order) with one another. Another example are the quanta of group field theory (see: (Oriti 2012), (Krajewski 2012)): “which can be described both as generalized spin networks and as simplicial building blocks

of piecewise-flat geometries” (Oriti 2021a, p. 6). Although there are other approaches that fit in with this level of spacetime emergence, in this contribution I shall focus on group field theories only.

Since in these approaches the atoms of space are non-spatiotemporal in themselves, how can they give rise to spatiotemporal properties in the first place? A possible response is that the collective behavior of individual entities can lead to novel properties that are not possessed by the underlying components. Yet, some theoretical, if not ontological, bridging between the underlying and emergent components should be accounted for: “if spacetime has to be reconstructed at all, the more fundamental theory should allow for a dictionary, mapping its basic entities and some of their properties into continuum fields including those defining spatiotemporal notions” (Oriti 2021a, p. 25).

There is an important distinction that should be accounted for in the previous quote: whether the dictionary is to be taken as translating concepts from one theory to another, or whether the ontology tracks the respective theories. The first possibility implies a more timid perspective in that it would require an account of inter-theoretic reduction between the atoms of space of quantum gravity and the spacetime of general relativity. On the other hand, a stronger claim (and one that requires some metaphysical finessing) is that not only a dictionary between theories is possible, but also that the ontology of the respective theories follows such a reduction. With respect to these possibilities, (Oriti 2021b) emphasizes how the physical entities we endow with ontological status are defined within the contexts of either the theory, or models thereof. This is true especially for theories operating at scales beyond immediate sensory experience, such as quantum mechanics, quantum field theory, and, consequently, quantum gravity. It is therefore hard to imagine how the ontologies proper of each theory could be independent from the corresponding mathematical framework. This amounts to following a cautious scientific realism, and I emphasize the term ‘cautious’ because, while a complete separation between formal apparatus and ontology is unlikely, it is also too strong of a claim to say that all theoretical objects partake in the ontology of the theory. However, to provide a detailed analysis of how to separate the ontological wheat from the mathematical chaff goes far beyond the scope of the present contribution.

In addition, one could argue that I am confusing emergence and reduction. In the context that I will be discussing in this paper, i.e., that of phase transitions (and group field theory), to determine which concept should apply

is a matter open to debate. For example, (Butterfield 2011b) and (Butterfield and Bouatta 2012) maintain that phase transitions combine a form of inter-theoretic reduction and emergence at the thermodynamic limit, (Batterman 2011) and (Morrison 2012) maintain that some approximations and limit procedures imply the emergence of new phenomena. Alternatively, (Palacios 2019) argues for a sophisticated notion of inter-theoretic reduction between thermodynamics and statistical mechanics that involves logical deduction between theories and a form of limiting reduction (which makes use of approximations and idealizations). The bottom line is that notions such as reduction and emergence need not be incompatible if applied to different contexts. Indeed we can have inter-theoretic reduction via limiting procedures and emergent properties stemming from the collective behavior of fundamental entities.⁷

I shall conclude this section by summarizing the second and third level of emergence which focus on the atoms of space in terms of collective behavior (Oriti 2021a). Level 2 deals with issues related to the fact that there are more than one continuum phases for the same atoms of space. Indeed, by exploring the continuum limit of the collective behavior of the fundamental entities, one should expect that such a limit is not unique. That is, there might be different phases that are separated by different phase transitions, and this yields different kind of macroscopic properties and systems, one for each phase. However, of these macroscopic systems, not all of them are amenable to be described as space, time, and geometry governed by general relativity (and approximations thereof). It is thus of great importance to any theory of quantum gravity to identify such phases, and “it is the task of quantum gravity formalisms that suggest fundamental non-geometric atoms of space to show that there exists such geometric, spatiotemporal phase, in a continuum limit, in some approximation” (Oriti 2021a, p. 10). This again emphasizes that the transition from fundamental atoms of space to macroscopic phases is not ‘just’ a matter of some approximations, but rather the emergence of new properties starting from entities that are not spatiotemporal. That is: there is an ontological difference between the emergent phases and the

⁷Notably, while Oriti (2021a) seems to suggest a realist attitude about the fundamental entities postulated by the theory, Oriti (2021b) is more cautious about such an ontological commitment. Nonetheless, he maintains that ontological emergence might follow from the inter-theoretic one: “This intertheoretic (or epistemic) emergence amounts in fact to a relation between mathematical and conceptual models of the world, from which we imply a relation between natural phenomena described by those theories” (Oriti 2021b, p. 2).

underlying fundamental entities.

Finally, the third level of spacetime emergence involves the (physical) mechanism responsible for the transition from non-geometric to geometric phases. A possible interpretation to such mechanism might be found in cosmology, for example: (Magueijo, Smolin, and Contaldi 2007). A possible idea is to interpret the mechanism as a process of condensation of the atoms of space that gives rise to the universe described as a quantum field (for example: (Gielen, Sindoni, et al. 2016)).

In sum, Oriti’s taxonomy can be divided into two main classes: On the one hand we have issues concerned with the (canonical) quantization of gravity, that is, levels -1 and 0 are focused on recovering spacetime from generally covariant classical and quantum fields. On the other hand, levels 1, 2 and 3 discuss the problems of recovering space and time starting from new fundamental degrees of freedom. More specifically, Level 1 assumes that there is only one phase, Level 2 deals with problems related to the existence of different continuum phases and phase transitions, and Level 3 deals with issues related to the transition from non-geometric to geometric phases. In what follows, I will focus mostly on Level 1, 2, and 3, and I will emphasize the distinction between the problem of spacetime emergence—that is, how non-spatiotemporal entities can approximate spacetime structures—and the problem of the nature of non-spatiotemporal entities—that is, the apparent difficulty of accounting for a theory whose fundamental ontology does not live in spacetime.

3 The Twofold Problem

Having shed some light on concepts such as emergence and reduction, we can now move back to the original problem I intend on discussing here: the justification of spacetime emergence from non-spatiotemporal entities. The problem is twofold, and thus requires separate considerations. On the one hand, we want an account of how spacetime emerges from, or reduces to, more fundamental entities. On the other hand, we want a precise account, possibly endowed with a plausible physical interpretation, of the kinematics and dynamics of such fundamental entities (the atoms of space). The latter problem, as far as the current research goes, presents itself with the demand for the atoms of space to be fundamentally non-spatiotemporal. How to conceive of some fundamental physical entities to be non-spatiotemporal is

a philosophical conundrum, yet one that ought not to be confused with the question of how such entities can approximate spacetime.

Notably, that spacetime is not the sort of fixed background that allows us to absolutely identify objects and events is already questioned in general relativity. However, one of the challenges posed by quantum gravity is that even the individual rods and clocks used by GR to keep track of the dynamics between entities and events seem to vanish. Perhaps, one could relinquish the idea that being-in-spacetime is a necessary condition for existence.⁸ Alternatively, one can opt for an instrumentalist view and consider fields as the only truly physical entities, while the non-spatiotemporal basic structures of the theory are conceived of as mathematical artifacts.

In what follows, I will discuss how emergent properties in the context of phase transitions can be considered to be independent from their fundamental constituents. Afterward, I will review the analogy between GFT and hydrodynamics models (Kadanoff and Martin 1963) (Marchetti et al. 2022a), (Marchetti et al. 2022b), (Volovik 2006) and emphasize how GFT condensate offers a mechanism for spacetime emergence that relies on the independence of spacetime from the dynamics of the underlying physics.

4 Ising Model and Indifference

In this section I will briefly present the Ising model, and some approximation methods that allow us to define salient mesoscale quantities. The relevant feature of these quantities is that they are indifferent to the micro-dynamics of the more fundamental levels. This indifference will turn out to be central for our discussion on quantum gravity, since it will allow us to ‘black-box’ the questions about the nature of the atoms of space and focus on the emergence of spacetime.

The Ising model is a simplified lattice model that can be used to describe the total magnetization of a system composed of many individual atomic spins. In conjunction with the Ising model, I will introduce the mean field theory approximation, which derives an effective field by averaging over the behavior of the individual atomic spins. As a result, the effective field ‘ignores’ the interactions between individual spins and allows us to derive the

⁸Notably, this is not necessarily a new view. Mathematical and abstract objects do not exist in spacetime and yet they can be (for the most part) well-defined and individuated. See: (Linnebo 2018).

thermodynamic properties of the macroscopic system (for example, its magnetization). While I will emphasize that a change in the orientation of some of the individual atomic spins would not be relevant to the overall magnetization, it would be too strong of a claim to characterize such an indifference as a complete autonomy. Indeed, while we could modify some of the orientations of the individual spins without affecting the total magnetization, a change in the orientation of all spins would inevitably change the total magnetization.⁹ It is precisely because of this indifference that we have the emergence of novel (and as we will see, robust) properties at the macroscopic phase.

4.1 Ising Model

The Ising model (Onsager 1944), (Stanley 1971) is a mathematical model that can be used to represent the ferromagnetic behavior of a collection of atomic spins on a lattice. Each spin σ_i has values ± 1 and interacts with the neighbor sites on the lattice, and with an external magnetic field h . The Hamiltonian of the system reads:

$$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i \quad (1)$$

The first sum on the right hand side of the equation is taken over all the sites of the lattice $\langle i, j \rangle$ close to σ_i , and J is the coupling factor between spins. As I have mentioned in the introduction, phase transitions are characterized by critical points associated with some macroscopic quantities, in this case the critical temperature $T = T_c$. The two phases of the system (ferromagnetic and paramagnetic) can be described by the order parameter magnetization $M \equiv \frac{1}{N} \sum_{i=1}^N \langle \sigma_i \rangle$, where $\langle \sigma_i \rangle = \frac{Tr(\sigma_i \exp(-\beta H))}{Z}$, Z is the partition function $Z = Tr \exp(\beta H)$, and $\beta = 1/k_b T$.¹⁰ The exact solution of the two-dimensional Ising model was presented by (Onsager 1944), but higher dimensionality leads to untractable terms. As a consequence, scientists need to rely on approximation methods, such as mean field theory (MTF), to derive

⁹I leave the discussion about how many atomic spins we can change before affecting the macroscopic system to later works.

¹⁰A more thorough presentation of the mathematical details of the Ising model can be found in (among others): (Binney et al. 1992), (Cardy 1996).

the macroscopic properties of the modeled systems.¹¹

The idea behind MTF is that instead of accounting for all the interactions of the individual atomic spins, we can average their behavior and treat them as a single effective field. More precisely, consider the interaction between two spins $\sigma_i\sigma_j$ in the previous Hamiltonian, where each spin is $\sigma_i = \langle\sigma_i\rangle + \delta\sigma_i$, and $\delta\sigma_i$ denotes the fluctuations around the mean value of σ_i . Then, one obtains that: $\sigma_i\sigma_j = \langle\sigma_i\rangle\langle\sigma_j\rangle + \langle\sigma_i\rangle\delta\sigma_j + \langle\sigma_j\rangle\delta\sigma_i + \delta\sigma_i\delta\sigma_j$. The central assumption of the mean field theory approximation is that the fluctuations quadratic term is negligible: $\delta\sigma_i\delta\sigma_j = 0$.¹² In addition, since the system is invariant under translations, the averaged spin is independent of its specific location i in the lattice. Thus, $M = \langle\sigma_i\rangle = \langle\sigma_j\rangle$, and the interaction term in the original Hamiltonian becomes: $\sigma_i\sigma_j = M(\sigma_i + \sigma_j) - M^2$. Also, the sum over nearest neighbors $\sum_{\langle ij\rangle}$ can be re-written as a sum over the lattice locations times the nearest neighbors to each such locations: that is, $\sum_{\langle ij\rangle} \rightarrow 1/2 \sum_{i=1}^N \sum_{j \in \text{neigh}(i)}$.¹³ Thus, the latter summation term is simply accounting for the number of neighbors of each lattice site i and can be expressed as: $\sum_{j \in \text{neigh}(i)} = z$. Upon substitution, one derives the mean field theory Hamiltonian $H_{\text{eff}} = +JM^2Nz - (JMz + h) \sum_{i=1}^N \sigma_i$ and the mean field equation for the order parameter magnetization:

$$M = \frac{1}{N} \sum_{i=1}^N \frac{\text{Tr}(\sigma_i \exp(-\beta H_{\text{eff}}))}{Z} = \tanh[\beta(h + zJM)]$$

The equation means that we can ignore the interactions between particles and consider only those with the external field h and with the effective field Jzm . By solving the equation, one obtains that the description of the thermodynamic properties of the system is independent from the dynamics of

¹¹Mean field theory is but one method that involves forms of coarse-graining. Another example is represented by renormalization group techniques (or, renormalization group theory (RG), (Wilson 1975)). As pointed out in (Batterman 2013, p. 8): “In a mean field theory, the order parameter M is defined to be the magnetic moment felt at a lattice site due to the average over all the spins on the lattice. This averaging ignores any large-scale fluctuations that might (and, in fact, are) present in systems near their critical points. The RG corrects this by showing how to incorporate fluctuations at all length scales, from atomic to the macro, that play a role in determining the macroscopic behavior [...] of the system near criticality”.

¹²A more thorough exposition of mean field theory approximations and consequences thereof can be found in, for example, (Binney et al. 1992), (Cardy 1996).

¹³The term $1/2$ is to account for the double counting like: i -being-near- j and j -being-near- i .

the microscopic quantities (the orientation of the individual spins):¹⁴

$$m(T, h = 0) = \begin{cases} \pm \left(1 - \frac{T}{T_c}\right)^{1/2} & T \rightarrow T_c^- \\ 0 & T \rightarrow T_c^+ \end{cases} \quad (2)$$

The result is that, starting from a system composed of (infinitely) many atomic spins, it is possible to average over the degrees of freedom using some statistical methods and obtain a new quantity that was not present at the scale of the individual atoms. This new quantity can be used to describe the macroscopic properties of the system. The crucial point is that the total magnetization of the system seems to be independent from the orientation of the individual atomic spins, where such an independence is provided mean field theory approximations.

One could contend that the independence was built-in in the approximation method, that is, in the fact that mean field theory assumes that small fluctuations in the interaction between spins can be ignored. Indeed, if we were trying to demonstrate that mean field theory explains such an independence, the argument would not hold, for we would be assuming the very same thesis we are trying to prove. However, what I emphasize here is that one can derive macroscopic properties (and phenomenology) of a giving system by using a method that ignores some physical information about the microscopic scale of that same system.

Another approach that testifies the irrelevance of the microscopic degrees of freedom to the order parameter M is the hydrodynamic description of many-body systems. The general idea is to describe the behavior of a many-particles system—which may be too complicated to deal with—with a simpler and more tractable theory. More specifically: “[t]he simplification occurs because when all physical quantities vary slowly in space and time each portion of the system is almost in thermodynamic equilibrium. Under these conditions, the variation in the system is completely described by local values of the various thermodynamic variables—for example, by giving the pressure, density, and velocity as a function of space and time. The basis of fluid mechanics is the partial differential equations satisfied by these local thermodynamic quantities” (Kadanoff and Martin 1963, p. 3).

Let us consider, again, the Ising model and let us define $M(r, t)$ the magnetization of the system at site r and time t . At equilibrium, the variation

¹⁴The mathematical details of the solution can be found in, for example: (Selinger 2016), (Kadanoff 2000), (Goldenfeld 2018).

over time of the magnetization will be conserved: $\partial/\partial t \int M(r,t)dr = 0$. If we add a perturbation, the system will relax back to equilibrium state. This determines a flux $\langle j^M \rangle = -D\nabla \langle M(r,t) \rangle$ and a diffusion equation that uses averages to describe the diffusion of the long-lived disturbances to the system by the external field:¹⁵

$$\frac{\partial}{\partial t} \langle M(r,t) \rangle - D\nabla^2 \langle M(r,t) \rangle \quad (3)$$

Here, M indicates the magnetization, the brackets indicate averages and thus not individual spins, and D the transport coefficient (or spin-diffusion coefficient).¹⁶

It was Kadanoff and Martin (1963, p. 419) who pointed out how the hydrodynamic description of a system can be equivalent to that provided by correlation functions. That is, one can use equilibrium statistical mechanics to describe non-equilibrium behaviors. For instance, let us characterize an external field acting on the atomic spins as: $\langle M(r) \rangle = \chi h(r)$ where $\chi = \partial M/\partial h$ is the magnetic susceptibility. The relaxation back to equilibrium after the macroscopic disturbance by the external field h “[...] follows the same laws as the regression of microscopic fluctuations at equilibrium. These fluctuations are represented by correlation functions” (Batterman 2021, p. 59). This means that correlation functions such as $\langle C_{ij} \rangle = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle$ lead to thermodynamic information. For example, one can show that magnetic susceptibility can be expressed in terms of a sum of correlation functions over all sites of the lattice:¹⁷

$$\chi(T, H) = N \frac{m^2}{k_\beta T} \sum_i C_i(T, H) \quad (4)$$

From magnetic susceptibility one can obtain the thermodynamic properties of the system by using the free energy and appropriate variational principles, see: (Solé 2011), (Selinger 2016), (Kadanoff 2000), (Goldenfeld 2018).

The example, which I have reported here in a simplified form, shows that there is a connection between the linear response of a system to an

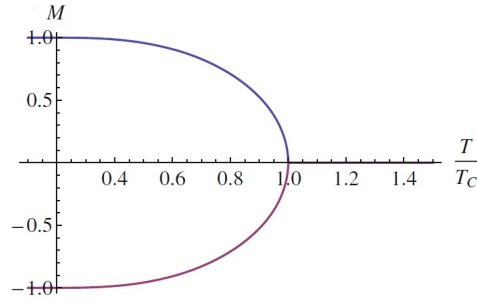
¹⁵Batterman (2021) notes that the averages do not indicate individual spins nor continuum systems, and therefore they are to be considered as mesoscale quantities.

¹⁶Because magnetization is expressed in terms of averages, (Batterman 2021) maintains that the equation describes mesoscale quantities.

¹⁷A rigorous derivation of the magnetic susceptibility from hydrodynamic equations is offered in (Kadanoff and Martin 1963)

external ‘macroscopic push’, and the internal fluctuations of a system in equilibrium —I shall say more on this in the next section. Such a connection allows us to use hydrodynamic equations (such as the diffusion equations) to describe the behavior of a many-body system without needing a description of the microscopic behavior. For example, to calculate the thermodynamic properties of the system we do not need the fine details of the individual spins —e.g., we do not need the interaction coupling constant between spins to calculate the spontaneous magnetization below critical temperature. This is made evident graphically in Figure 1: below the critical temperature, the system undergoes a symmetry-breaking corresponding to the ferromagnetic phase.

Figure 1: The parameter M (magnetization) as a function of parametrized temperature T/T_c (Selinger 2016, p. 18).



4.2 Neither Realism nor Instrumentalism

Models such as the Ising model have the capacity of representing correlations that are statistically representative at different scales. In this sense: “equilibrium statistical mechanics *itself* has the means to describe the non-equilibrium behavior of the transport properties in the slow, linear regime” (Batterman 2021, p. 58). This is considered by Batterman as a consequence of the fluctuation-dissipation theorem in (Kubo 1966, p. 256), for which there is: “a general relationship between the response of a given system to an external disturbance and internal fluctuation of the system in the absence of the disturbance [...] The internal fluctuation is characterized by a correlation function of relevant physical quantities of the system fluctuating in

thermal equilibrium, or equivalently by their fluctuation spectra”. In other words, the fluctuation-dissipation theorem proves a connection between the relaxation back to equilibrium of correlated atoms (or molecules) with some external perturbation of the system. This justifies the use of hydrodynamic equations, as shown in (Kadanoff and Martin 1963, p. 800):

The response of a system to an external disturbance can always be expressed in terms of time dependent correlation functions of the undisturbed system. More particularly the linear response of a system disturbed slightly from equilibrium is characterized by the expectation value in the equilibrium ensemble, of a product of two space -and time- dependent operators. When a disturbance leads to a very slow variation in space and time of all physical quantities, the response may alternatively be described by the linearized hydrodynamic equations.

Thus, as we have seen earlier for the Ising model, the hydrodynamics approach emphasizes the autonomy of the mesoscale from the (more) fundamental micro-dynamics. Because of this autonomy, (Batterman 2021) considers the correlation variables as ‘natural variables’ of the system at a given scale.

There are cases, though, where the suppressed fluctuations at all scales have indeed an effect on the macro dynamics of the system. Renormalization Group Theory (RG) solves this problem by iterating the process of averaging over different length scales: “Instead of using the ensemble to calculate an average, as in SM [statistical mechanics], we use RG to transform one ensemble into another one with different couplings. Each transformation increases the length scale so that the transformation eventually extends to information about the parts of the system that are infinitely far away” (Morrison 2014). This way, while the system loses information about the microscopic structure, it displays the new macroscopic correlations. With respect to our purposes, the crucial point remains: one can describe macroscopic properties using an appropriately defined mesoscale which is (partly) indifferent to the behavior of the more fundamental entities.

At this point, one might question whether the mesoscale level is to be considered as merely an instrumental tool for calculation purposes. For example, (Williams 2019) advocates a form of effective realism that includes entities derived from approximation methods such as renormalization group techniques: “focusing exclusively on fundamental ontology [...] leaves one

with an interpretation unequipped to support the theory in the performance of its explanatory duties [...] many explanatory affirmations made in the theory simply cannot be made true by including in one’s ontology only those entities at the fundamental scale” (Williams 2019, p. 19). Alternatively, one could argue that the indifference of the mesoscales from the fundamental ontology will one day be explained by a complete physical theory. The argument calls for a form of strong reductionism that attempts to ‘build the universe from fundamental entities’ (see: (Anderson 1972)). Other forms of reductionism set a less stringent requirement (Bain 2013a), but they still do not invalidate the fact that the explanatory power of approximate models is provided by quantities that lie at the mesoscale level: densities and gradients in flowing contexts, geometrical properties and topological features in static cases, effective fields in magnetic phenomena. The (more) fundamental levels of the theory remain irrelevant to the explanatory power of those models.

This is very clear in the case of Putnam’s pegs and board (“Philosophy and our mental life” 1975). Suppose we have a wooden board with two holes drilled on it. The first hole is circular with diameter 1cm and the second is squared with each side being 1cm long. Direct experience tells us that a cubical peg that perfectly fits the squared hole will not fit the circular one. We can offer two types of explanations to this fact. On the one hand, we can adopt a bottom-up approach and attempt to derive an explanation starting from the microscopic structure of the system. On the other hand, we can rely on the geometrical and topological structure of both pegs and holes, since the area of a circle with diameter d is smaller than the area of a square whose sides are of the same length as the diameter of the circle. The geometrical explanation is indifferent to the microscopic structure of the board (or of the cubical peg).

The crucial point is that independently of the philosophical attitude we assume towards geometric properties and microscopic structure, the irrelevance of the latter to the former seems to remain a brute fact when it comes to explaining why the square peg does not find the round hole. This fact can tell us something about the tension that I mentioned in the first section between realism of non-spatiotemporal entities and instrumentalism towards the atoms of space.

The discussion about the Ising model and approximation methods leaves us with the conclusion that we might separate the problem of accounting for spacetime emergence in QG from the discussion over the ontology of the corresponding fundamental entities. In the next section I will review a theory

of quantum gravity that interprets the emergence of spacetime in terms of phase transitions, thereby rendering the problem of the ontology of the atoms of space, at least partly, irrelevant.

5 GFT and Phase Transition

Originally, a theory of quantum fields of geometry was developed in the context of global quantum cosmology (see: (Giddings and Strominger 1989), (Banks 1988), and (Oriti 2009) for a review). The theory would construct a sum over possible topologies where each topology would correspond to a Feynman graph and corresponding quantum amplitude. However, the approach presented interpretative and mathematical problems which could be partially eased by adopting a local framework that generalizes dynamically to the whole universe (Oriti 2006). Thus, a partial solution was to use a simplicial description of spacetime obtained by gluing together many fundamental discrete building blocks (atoms of space).¹⁸ The complex simplicial structures would be expressed by the tensor product of the wave functions associated to each individual block, where the geometry of each building block of space is described in terms of group and representation variables (see: (Oriti 2016) and (Oriti 2012)). While more complex simplices can be realized by gluing simpler ones along a shared boundary, this is not a spacetime structure yet, since it lacks both a continuum limit and a metric structure.

The theory defines a complex scalar field $\varphi : G^{\times d} \rightarrow \mathbb{C}$ on a group manifold G which is usually taken to be either the Lorentz group $SO(3)$ or the rotation group $SU(2)$. The many wave functions are then promoted to operators and the field theory is thus “specified by a choice of action and by the definition of the quantum partition function expressed perturbatively in terms of Feynman Diagrams” (Oriti 2009, p. 311). The action is chosen so that the perturbative expansion of the partition function equals the discretized path integrals for quantum gravity of the form:

$$Z = \int \mathcal{D}\varphi \mathcal{D}\varphi^* e^{-S(\varphi, \varphi^*)} \quad (5)$$

Then, from the path integrals form, one can couple a scalar field to provide the dynamics for the structure of the tetrahedra: “in particular, we are

¹⁸The use of discrete structures is an approach taken also by simplicial quantum gravity (Hartle 2022) and spin foam models (Rovelli and Vidotto 2014)

interested in adding degrees of freedom that can be interpreted as discretized scalar matter, just like the group-theoretic variables can be interpreted as discrete geometric data” (Oriti 2021b, p. 10). Thus, the initial field defined on the $SU(2)$ group assumes the form: $\varphi(g_I, \phi^J) : SU(2)^4 \times \mathbb{R} \rightarrow \mathbb{C}$. The newly added free, massless, real-valued field ϕ will act as a relational clock, i.e., as an internal time variable with respect to which the other variables evolve.¹⁹

At this stage, the individual tetrahedra are analogous to the individual atomic spins of the Ising model I described above. They do not carry any spatiotemporal information in the sense of general relativity, similarly to how the orientation of the individual spins does not inform us about the overall magnetization. To be more precise, the distance between the fundamental entities of GFT and the macroscopic spacetime of general relativity is even larger than the one between individual spins and total magnetization in the Ising model. Indeed, the total magnetization of the Ising model is obtained from averaging over the individual orientation of the individual spins. In this sense mean field theory, for example, relies on the loss of some of the microscopic information to obtain a macroscopic property of the system. It is that lost information that allows us to maintain that the macroscopic property is independent from the corresponding microscopic dynamics. It would be more appropriate to characterize such an independence as only partial, in that the effective Hamiltonian still depends on the terms σ_i , and it only neglects the quadratic term $\delta\sigma_i\delta\sigma_j$. On the other hand, the individual tetrahedra in the GFT case do not carry any spatiotemporal properties of the relativistic fields. The continuum limit here corresponds to Level 1 of Oriti’s taxonomy, in that a spatial manifold is thought of as “a collection of (glued) building blocks, akin to many-particle state, and the field theory should be defined on the space of possible geometries of each such building block” (Oriti 2012, p. 8). Then, how does spacetime emerge in such a context?

In general, in quantum field theory, the evaluation of the full partition function incorporates all dynamical degrees of freedom and thus the continuum limit of the theory as well. In the GFT context, this amounts to: “[...] resumming the full perturbation series, thus the sum over triangulations weighted by a discrete gravity path integral [...] including infinitely refined lattices. In physical terms, this means being able to control the full collec-

¹⁹The strategy of adding a field to play the role of relational clock is not new, see: (Dittrich 2006) and (Brown and Kuchař 1995).

tive quantum dynamics of the QG atoms, looking for regimes in which the discrete picture can (and should) be replaced by one in terms of continuum spatiotemporal fields” (Oriti 2021b, p. 11).

That is, from the perspective of spacetime emergence, one needs to move from the atoms of space to the continuum phase, but this requires the control of the dynamics of the theory at all scales and regimes. Yet, the mathematical control of the theory does not discern between physical and mathematical phases. In addition, one needs to identify which phases amidst the ones allowed by the theory can be rewritten in terms of spatiotemporal fields and dynamics of general relativity. One can employ approximation methods (such as renormalization group techniques and mean field theory) to obtain a picture of the continuum phases from which to extract some physical insights:

If the emergence of space and time takes place due to the collective dynamics of the QG atoms, we need approximation schemes that capture such collective dynamics, that correspond to some form of coarse-graining of the fundamental ‘atomic’ dynamics, and that maintain visible the quantum nature of the same atoms (since the continuum limit is distinct from the classical one, and it could well be that quantum properties of the QG atoms are in fact responsible for key aspects of the spatiotemporal physics we want to reproduce) (Oriti 2021b, p. 12).

Oriti (2021b) uses mean field theory to approximate the full theory with quantum states expressed in terms of excitations of the Fock vacuum $|\sigma\rangle = \exp(\hat{\sigma})|0\rangle$ that are simplified with respect to the initial tetrahedra, for they now do not encode correlation information or quantum entanglement. The use of such a simplification shifts the theory to a new level of description: “we are then moving from the QG atoms to the full continuum description of quantum gravity, but within a specific regime of approximation, which remains quantum and focused on the collective properties of the same QG atoms, rather than their individual, pre-geometric features” (Oriti 2021b, p. 12). This way, the approximation methods allow us to obtain simplified states (in a quantum superposition) associated with a wave function σ of infinitely many degrees of freedom: the new wave function describes the collective behavior of infinitely many atoms of space.

The point is then to obtain from such collective behavior an (effective) dynamics that can be understood as quantum general relativity. One possibility is to individuate a phase of the quantum gravity system that resembles

a condensate phase, and to treat the dynamics of the fundamental atoms of space in terms of hydrodynamics regime. In sum, a quantum spacetime is conceived of as a very large number of small GFT fundamental entities close to a many-particles vacuum described collectively, and whose dynamics is provided by (continuum) large scale equations. This situation is similar to that of a fluid, where the quantum spacetime is analogous to a quantum fluid of GFT atoms-of-space governed by the GFT partition function at the microscopic level, and by some effective hydrodynamics at the macroscopic level. In this sense, the continuum limit and the classical approximations are different, where the former consists of the limit of QFT quanta governed by some collective dynamics equations, and the latter is needed to extract a specific dynamics in a given regime.

Now, granted the feasibility of such a strategy, what kind of spacetime physics can we expect? The most supported answer (see, for example: (Oriti and Sindoni 2011)) is cosmological dynamics; that's because the focus on macroscopic variables and maximal coarse-graining limit us to a dynamics close to equilibrium. In addition, it is only collectively that one can talk about geometries, since the atoms of space are strictly speaking non-geometrical and a notion of local geometric behavior is not available. Also, the condensate wave function and the mean field can only be treated statistically due to the coarse-graining, even though the fundamental degrees of freedom are treated quantum mechanically.

5.1 GFT Condensate

The idea of using some forms of coarse-graining to model relativistic effects in quantum theories is not new. For example, (Volovik 2006) studied the similarity of relations between classical and quantum hydrodynamics, and quantum hydrodynamics and quantum gravity. Originally, (Landau 1941) discussed the derivation of quantum hydrodynamics from its classical counterpart by expressing the quantum Hamiltonian as the energy of a liquid where the classical velocity \mathbf{v} and density ρ are replaced by the corresponding quantum operators $\hat{\mathbf{v}}$ and $\hat{\rho}$. The classical Hamiltonian reads (Volovik 2006, p. 2):

$$H_{hydro}(\rho, \mathbf{v}) = \int d^3 \left(\frac{1}{2} \rho v^2 + \tilde{\epsilon}(\rho) \right) \quad (6)$$

where $\tilde{\epsilon}(\rho) = \epsilon\rho - \mu\rho$ and $\epsilon(\rho)$ is the energy of the static liquid which will be related to the vacuum energy state (assumed that the temperature is $T = 0$) and μ is the constant chemical potential. The relation $P = -\tilde{\epsilon}$ between pressure and energy can be taken to be as the equation of state for the vacuum of any system, and it does not depend on underlying physics of the vacuum state. Similarly, one can obtain the hydrodynamics equations using Poisson brackets, which depend on symmetry conditions of the system, rather than on the underlying physics. By looking at cases in which quantum hydrodynamics and quantum gravity can be used to obtain fine corrections to classic hydrodynamics and general relativity, (Volovik 2006) concludes that such cases are not generalizable. This implies that the route from classical to quantum hydrodynamics does not lead to a theory that is completely faithful to the microscopic theory. This might be considered as a genuine instance of emergence in physics.

In a similar fashion, some approaches to GFT aim at using the condensate analogy to model relativistic behavior of quantum systems.²⁰ Indeed, a great advantage of the GFT formalism is that one can use quantum field theory (QFT) methods for treating many degrees of freedom: “condensation of many atoms into a common ground state can be viewed as a transition from a perturbative phase around the Fock vacuum (of zero atoms) into a condensed phase, with associated symmetry breaking of the U(1) symmetry of the theory” (Gielen, Sindoni, et al. 2016, p. 2). The aim is to approximate 3-d geometries and cosmological evolution in terms of some specific condensate states in the formalism of GFT. These states should come from the macroscopic quantum dynamics, in a way inspired by phase transitions.

The construction of the condensate state is analogous to the construction of the effective field we have seen for the Ising model. That is, one can coarse grain the many degrees of freedom of the theory represented by N -excitations of the Fock vacuum and define a new state (which now plays the role of order parameter) as a superposition of one-particle wave functions (Gielen, Sindoni, et al. 2016, p. 19):

²⁰There is though an important difference between the works of Volovik and the GFT approach. In the former, while gravity emerges in the hydrodynamics regime, spacetime is already present from the start, since the many-body system is defined in flat spacetime. On the other hand, GFT (and other quantum gravity approaches) is more ambitious since spacetime itself would be emergent.

$$|\sigma\rangle := \mathcal{N}(\sigma) \exp\left(\int dg \sigma(g_I) \varphi^\dagger(g_I)\right) |0\rangle \quad (7)$$

where \mathcal{N} is a normalization factor and $\int dg$ is the integral over the local gauge group. The state $|\sigma\rangle$ corresponds to a single particle condensate state which is an eigenstate of the field operator $\hat{\varphi}(g_I) |\sigma\rangle = \sigma(g_I) |\sigma\rangle$ with non vanishing expectation values $\langle \sigma | \hat{\varphi}(g_I) | \sigma \rangle \neq 0$, unlike for the Fock vacuum where $\langle 0 | \hat{\varphi}(g_I) | 0 \rangle = 0$. The condensate wave function, together with the massless scalar field ϕ^J can be interpreted as a continuum spacetime geometry (Gielen, Sindoni, et al. 2016) in a way analogous to how magnetization was defined over the effective field in the Ising model. This is because the condensate wave function ignores the fluctuations between individual quanta due to the mean field theory approximation which assumes that: “the system exhibits a separation of scales which allows to average over the microscopic details. [...] This leads to a model which only involves scales which extend from the mesoscale to the macroscale. The field variable is an averaged quantity (the order parameter) which only reflects general features of the system such as symmetries and the dimensionality of the domain” (Marchetti et al. 2022b, p. 5). Then, the thermodynamic limit corresponds to having $N \rightarrow \infty$ and it is described by states that are no longer in the GFT Fock space: “this is standard in quantum field theory, where in the limit corresponding to a phase transition one needs to change representation to a different, unitarily inequivalent, Hilbert space” (Gielen, Sindoni, et al. 2016, p. 19).²¹

The condensate approach to GFT rests on the fact that low-energy scale physics can be independent from its high-energy counterpart, and that one can study quantum gravity models in terms of collective behavior of fundamental entities. However, how should we obtain geometric quantities from a condensate function obtained as the thermodynamic limit of N-many non-geometric atoms of space? To answer the question, even though I will skimp on the details, we can consider (Oriti 2021b), (Gielen, Oriti, and Sindoni 2014) and use the simplest case of homogeneous and isotropic cosmology. To do so, explains (Oriti 2021b, p. 16), “we define the relational observables that we expect to be relevant for describing homogeneous cosmological evolution [...] the universe volume [the operator \hat{V}] (constructed from the matrix elements of the 1st quantized tetrahedra, with eigenvalues V_j , convoluted with

²¹Notably, $\sigma(g_I)$ is not an ordinary wave function: it is a superposition of states $\psi(g_I)$ but it is not linear $|\sigma\rangle + |\sigma'\rangle \neq |\sigma + \sigma'\rangle$.

field operators) [...] the operator adds the individual volume contributions from the GFT quanta populating the state:”

$$V(\chi_0) \equiv \left\langle \hat{V} \right\rangle_{\sigma; \chi_0, \pi_0} = \sum_j V_j \rho_j^2(\chi_0) \quad (8)$$

where $\rho_j(\chi)$ is the density of the fluid which is obtained from the decomposition of the condensate wave function in hydrodynamics variables. From the volume observable (and others that are calculated in (Oriti 2021b)), and from the description of the evolution of the condensate with respect to the relational clock χ_0 , (Oriti 2021b, p. 17) obtains the generalized Friedmann equations (in χ_0): “that our quantum gravity model gives for the emergent spacetime in the homogeneous case”.

Philosophically, the point I raised for the Ising model applies quite naturally to the case of group field theory and condensate models. The geometric properties of the condensate are indifferent to the individual tetrahedra, and that is warranted by the use of approximation methods such as the mean field theory.²² A caveat, similarly to how I have suggested that the independence between macroscopic and microscopic regimes in the Ising model should be addressed as *partial*, the same cautionary step should be taken here. Indeed, the use of approximation methods still relies on some features of the microscopic regime, for example, the combinatorial structure of the atoms of space and their group theoretic data.²³ It follows that more work is needed to properly spell out the conditions of independence between microscopic and macroscopic regimes in different approaches to quantum gravity, but I shall leave this investigation to later works.

In this contribution, I have looked into the case of group field theories to show that hydrodynamic description and mesoscale quantities are at least partly indifferent to the microscopic dynamics of the fundamental entities. This allows us to discuss higher-order properties with a moderate indifference with respect to their more fundamental counterparts. I added the term ‘moderate’ here because, although magnetization, spacetime, and geometric properties can be considered independently of their constituent parts, the question about the ontology of fundamental entities remains unattended. However,

²²Even further, the vacuum state of the condensate lives on a unitarily inequivalent Hilbert space from the Hilbert space of the tetrahedra (which is a common feature of phase transitions on quantum field theory).

²³The addition of group theoretic data in tensorial models is discussed, for example, in (Oriti 2014).

such an indifference allows us to separate between the previous question and that of spacetime emergence. In this sense, ‘how does spacetime emerge’ and ‘what does spacetime emerge from’ become separate problems that require separate analysis. For example, the former calls for further investigations about inter-theoretic reduction and the possibility of interpreting phase transitions as actual physical processes. The latter, looks at the very possibility of having experimental verifications of non-spatiotemporal entities, or at the possibility of relinquishing the property of ‘being-in-spacetime’ as necessary requirement for existence.

6 Conclusion

The philosophical discussion on the emergence of spacetime is very much alive and prolific. For example, the community still discusses issues related to specific accounts of spacetime (such as, for example, the recent functionalist view (Lam and Wüthrich 2018)), or re-definition of the problem of spacetime emergence (Jaksland and Salimkhani 2023), or the emergence of low-energy theories from high-energy ones (Crowther 2014), and many others. In this contribution, after discussing some of the literature on reduction, emergence, and corresponding philosophical problems, I divided the problem of spacetime emergence into two sub-problems: (i) to account for the ontology of non-spatiotemporal fundamental entities, and (ii) to provide a mechanism for the emergence of spacetime from such entities. Afterward, I argued that such a division is warranted in the context of the group field theory approach to quantum gravity and in the analogy with the physics of phase transitions. I presented the Ising model to show how the total magnetization of the system is indifferent to the microscopic dynamics of the individual atomic spins. This means that the macroscopic thermodynamic properties of the system can be considered independently of the corresponding microscopic dynamics—granted that specific conditions apply, such as those needed by the corresponding approximation methods. This conclusion is less interesting for the case of the Ising model, in that we already know the properties of the individual atomic spins. However, the same conclusion becomes relevant to the discussion on quantum gravity and, more specifically, in the context of group field theory. Since group field theory suggests to treat the collective behavior of the individual atoms of space in terms of condensate states after the application of mean field theory, we can warrant a very similar type of

indifference that we also find in the mean field theory and hydrodynamics approximation in the Ising model.²⁴

Therefore, if we accept that the physics of phase transitions can display genuine forms of emergence in the use of some approximation methods, then a theory of quantum gravity that makes use of these same (or analogous) approximations in deriving continuum and classical limits should display the same (or at least similar) type of independence. I have discussed the case of group field theory precisely because it makes use of such methods that warrant the independence between the fundamental quanta of the theory and some macroscopic quantities. It is because of this form of independence that one can black-box some of the (philosophical) problems related to the ontology of the fundamental quanta, even though this requires that we accept to work within the scope of group field theory (or similar approaches). Indeed, different types of quanta (and thus different QG approaches) might not allow for the use of those mathematical tools that warrant the above mentioned independence. Yet, a very similar argument as the one presented here might apply to other approaches (for example: spin foam models). It would be interesting to dig deeper and look at the commonalities between such approaches and their use of approximation methods. Similarly, it would be interesting to investigate whether and to what extent other approaches, that do not make use of phase transitions and approximation techniques, deal with the separation between mechanism of emergence and ontological considerations on their fundamental entities. Finally, the independence advocated here does not diminish the need for a serious philosophical account of the fundamental ontology of theories quantum gravity, nor it dispenses philosophers from having to investigate the problem(s).

References

- Anderson, P. W. (1972). “More Is Different”. In: *Science* 177.4047, pp. 393–396.
- Ashtekar, Abhay and Jerzy Lewandowski (July 2004). “Background independent quantum gravity: a status report”. In: *Classical and Quantum*

²⁴Notably, similar conclusions apply to the case of geometrogenesis, for which it is suggested that the Big Bang consisted of a phase transition from a disordered, non-spatiotemporal phase, to our current universe.

- Gravity* 21.15, R53–R152. ISSN: 1361-6382. DOI: 10.1088/0264-9381/21/15/r01. URL: <http://dx.doi.org/10.1088/0264-9381/21/15/R01>.
- Bain, Jonathan (2013a). “Emergence in Effective Field Theories”. In: *European Journal for Philosophy of Science* 3.3, pp. 257–273. DOI: 10.1007/s13194-013-0067-0.
- (2013b). “The emergence of spacetime in condensed matter approaches to quantum gravity”. In: *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 44.3, pp. 338–345.
- Banks, Thomas (1988). “Prolegomena to a theory of bifurcating universes: a nonlocal solution to the cosmological constant problem or little lambda goes back to the future”. In: *Nuclear Physics B* 309.3, pp. 493–512.
- Batterman, Robert (Feb. 2013). “255 The Tyranny of Scales”. In: URL: <https://doi.org/10.1093/oxfordhb/9780195392043.013.0008>.
- Batterman, Robert W (2011). “Emergence, singularities, and symmetry breaking”. In: *Foundations of physics* 41.6, pp. 1031–1050.
- (2021). *A middle way: A non-fundamental approach to many-body physics*. Oxford University Press.
- Bianchi, Silvia De and Luciano Gabbanelli (2023). “Re-thinking geometrogenesis: Instantaneity in quantum gravity scenarios”. In: *Journal of Physics: Conference Series* 2533.1, p. 012001. DOI: 10.1088/1742-6596/2533/1/012001. URL: <https://dx.doi.org/10.1088/1742-6596/2533/1/012001>.
- Binney, James J et al. (1992). *The theory of critical phenomena: an introduction to the renormalization group*. Oxford University Press.
- Brown, J. David and Karel V. Kuchař (1995). “Dust as a standard of space and time in canonical quantum gravity”. In: *Physical Review D* 51.10, pp. 5600–5629. DOI: 10.1103/physrevd.51.5600. URL: <https://doi.org/10.1103/physrevd.51.5600>.
- Butterfield, Jeremy (2011a). “Emergence, reduction and supervenience: A varied landscape”. In: *Foundations of physics* 41.6, pp. 920–959.
- (2011b). “Less is different: Emergence and reduction reconciled”. In: *Foundations of physics* 41.6, pp. 1065–1135.
- (2014). “Reduction, emergence, and renormalization”. In: *The Journal of Philosophy* 111.1, pp. 5–49.
- Butterfield, Jeremy and Nazim Bouatta (2012). “Emergence and reduction combined in phase transitions”. In: *AIP Conference Proceedings* 11. Vol. 1446. 1. American Institute of Physics, pp. 383–403.

- Cardy, John (1996). *Scaling and renormalization in statistical physics*. Vol. 5. Cambridge university press.
- Crowther, Karen (2014). “Appearing out of nowhere: The emergence of space-time in quantum gravity”. In: *arXiv preprint arXiv:1410.0345*.
- Dittrich, B (2006). “Partial and complete observables for canonical general relativity”. In: *Classical and Quantum Gravity* 23.22, pp. 6155–6184. DOI: 10.1088/0264-9381/23/22/006. URL: <https://doi.org/10.1088/0264-9381/23/22/006>.
- Franklin, Alexander (2018). “On the renormalization group explanation of universality”. In: *Philosophy of Science* 85.2, pp. 225–248.
- Gabbanelli, Luciano and Silvia De Bianchi (2021). “Cosmological implications of the hydrodynamical phase of group field theory”. In: *General Relativity and Gravitation* 53.7, pp. 1–36.
- Gambini, Rodolfo and Rafael A Porto (2001). “Relational time in generally covariant quantum systems: Four models”. In: *Physical Review D* 63.10, p. 105014.
- Giddings, Steven B and Andrew Strominger (1989). “Baby universe, third quantization and the cosmological constant”. In: *Nuclear Physics B* 321.2, pp. 481–508.
- Gielen, Steffen and Daniele Oriti (2018). “Cosmological perturbations from full quantum gravity”. In: *Physical Review D* 98.10, p. 106019.
- Gielen, Steffen, Daniele Oriti, and Lorenzo Sindoni (2014). “Homogeneous cosmologies as group field theory condensates”. In: *Journal of High Energy Physics* 2014.6, pp. 1–69.
- Gielen, Steffen and Axel Polaczek (2020). “Generalised effective cosmology from group field theory”. In: *Classical and Quantum Gravity* 37.16, p. 165004.
- Gielen, Steffen, Lorenzo Sindoni, et al. (2016). “Quantum cosmology from group field theory condensates: a review”. In: *SIGMA. Symmetry, Integrability and Geometry: Methods and Applications* 12, p. 082.
- Goldenfeld, Nigel (2018). *Lectures on phase transitions and the renormalization group*. CRC Press.
- Gomes, Henrique de A (2016). “Back to parmenides”. In: *arXiv preprint arXiv:1603.01574*.
- Hartle, James B. (2022). *Simplicial Quantum Gravity*. arXiv: 2201.00226 [gr-qc].
- Huggett, Nick and Christian Wüthrich (2013). “Emergent spacetime and empirical (in) coherence”. In: *Studies in History and Philosophy of Sci-*

- ence Part B: *Studies in History and Philosophy of Modern Physics* 44.3, pp. 276–285.
- Jaksland, Rasmus and Kian Salimkhani (2023). “The many problems of spacetime emergence in quantum gravity”. In.
- Kadanoff, Leo P (2000). *Statistical physics: statics, dynamics and renormalization*. World Scientific.
- Kadanoff, Leo P and Paul C Martin (1963). “Hydrodynamic equations and correlation functions”. In: *Annals of Physics* 24, pp. 419–469.
- Krajewski, Thomas (2012). *Group field theories*. arXiv: 1210.6257 [gr-qc].
- Kubo, R (1966). “The fluctuation-dissipation theorem”. In: *Reports on Progress in Physics* 29.1, p. 255. DOI: 10.1088/0034-4885/29/1/306. URL: <https://dx.doi.org/10.1088/0034-4885/29/1/306>.
- Lam, Vincent and Christian Wüthrich (2018). “Spacetime is as spacetime does”. In: *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 64, pp. 39–51.
- Landau, L. (1941). “Theory of the Superfluidity of Helium II”. In: *Phys. Rev.* 60 (4), pp. 356–358. DOI: 10.1103/PhysRev.60.356.
- Linnebo, Øystein (2018). “Platonism in the Philosophy of Mathematics”. In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Spring 2018. Metaphysics Research Lab, Stanford University.
- Magueijo, Joao, Lee Smolin, and Carlo R Contaldi (2007). “Holography and the scale invariance of density fluctuations”. In: *Classical and Quantum Gravity* 24.14, p. 3691.
- Marchetti, Luca and Daniele Oriti (2022). “Effective dynamics of scalar cosmological perturbations from quantum gravity”. In: *Journal of Cosmology and Astroparticle Physics* 2022.07, p. 004.
- Marchetti, Luca et al. (2022a). “Mean-field phase transitions in TGFT quantum gravity”. In: *arXiv preprint arXiv:2211.12768*.
- (2022b). “Phase transitions in TGFT: a Landau-Ginzburg analysis of Lorentzian quantum geometric models”. In: *arXiv preprint arXiv:2209.04297*.
- Morrison, Margaret (2012). “Emergent physics and micro-ontology”. In: *Philosophy of science* 79.1, pp. 141–166.
- (2014). “Complex Systems and Renormalization Group Explanations”. In: *Philosophy of Science* 81.5, 1144–1156. DOI: 10.1086/677904.
- Onsager, Lars (1944). “Crystal statistics. I. A two-dimensional model with an order-disorder transition”. In: *Physical Review* 65.3-4, p. 117.

- Oriti, Daniele (2006). *The group field theory approach to quantum gravity*. DOI: 10.48550/ARXIV.GR-QC/0607032. URL: <https://arxiv.org/abs/gr-qc/0607032>.
- (2009). *Approaches to quantum gravity: Toward a new understanding of space, time and matter*. Cambridge University Press.
- (2012). “The microscopic dynamics of quantum space as a group field theory”. In: *Foundations of space and time: reflections on quantum gravity*, p. 257.
- (2014). “Non-commutative quantum geometric data in group field theories”. In: *Fortschritte der Physik* 62.9-10, pp. 841–854.
- (2016). “Group field theory as the second quantization of loop quantum gravity”. In: *Classical and Quantum Gravity* 33.8, p. 085005.
- (2017). “The universe as a quantum gravity condensate”. In: *Comptes Rendus Physique* 18.3-4, pp. 235–245.
- (2021a). “Levels of Spacetime Emergence”. In: *Philosophy Beyond Spacetime: Implications from Quantum Gravity*, p. 16.
- (2021b). “TGFT condensate cosmology as an example of spacetime emergence in quantum gravity”. In: *The Foundations of Spacetime Physics*. Routledge, pp. 251–286.
- Oriti, Daniele and Lorenzo Sindoni (2011). “Toward classical geometrodynamics from the group field theory hydrodynamics”. In: *New Journal of Physics* 13.2, p. 025006.
- Oriti, Daniele, Lorenzo Sindoni, and Edward Wilson-Ewing (2016). “Emergent Friedmann dynamics with a quantum bounce from quantum gravity condensates”. In: *Classical and Quantum Gravity* 33.22, p. 224001.
- (2017). “Bouncing cosmologies from quantum gravity condensates”. In: *Classical and Quantum Gravity* 34.4, 04LT01.
- Palacios, Patricia (2019). “Phase transitions: A challenge for intertheoretic reduction?” In: *Philosophy of Science* 86.4, pp. 612–640.
- “Philosophy and our mental life” (1975). In: *Philosophical Papers*. Vol. 2. Cambridge University Press, 291–303. DOI: 10.1017/CB09780511625251.016.
- Pithis, Andreas GA and Mairi Sakellariadou (2019). “Group field theory condensate cosmology: An appetizer”. In: *Universe* 5.6, p. 147.
- Rovelli, Carlo and Francesca Vidotto (2014). *Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spinfoam Theory*. Cambridge University Press. DOI: 10.1017/CB09781107706910.

- Selinger, Jonathan V (2016). *Introduction to the theory of soft matter: from ideal gases to liquid crystals*. Springer.
- Smolin, Lee (2020). “Temporal relationalism”. In: *Beyond spacetime. The foundations of quantum gravity*, pp. 143–175.
- Solé, Ricard (2011). *Phase transitions*. Vol. 3. Princeton University Press.
- Stanley, H.E. (1971). “Introduction to phase transitions and critical phenomena”. In: *Introduction to Phase Transitions and Critical Phenomena*.
- Surya, Sumati (Sept. 2019). “The causal set approach to quantum gravity”. In: *Living Reviews in Relativity* 22.1. ISSN: 1433-8351. DOI: 10.1007/s41114-019-0023-1. URL: <http://dx.doi.org/10.1007/s41114-019-0023-1>.
- Volovik, GE (2006). “From quantum hydrodynamics to quantum gravity”. In: *arXiv preprint gr-qc/0612134*.
- Williams, Porter (2019). “Scientific Realism Made Effective”. In: *British Journal for the Philosophy of Science* 70.1, pp. 209–237. DOI: 10.1093/bjps/axx043.
- Wilson, Kenneth G (1975). “Renormalization group methods”. In: *Advances in Mathematics* 16.2, pp. 170–186.