The Nature and Structure of Space

by

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Curriculum Vitae

The author was born in Akron, Ohio on July 19, 1980. He attended the University of Akron from 1999 to 2003, and graduated with a Bachelor of Arts degree in 2003. He came to the University of Rochester in the Fall of 2003 and began graduate studies in philosophy. He received a University Fellowship in 2003 and 2004. He pursued his research in metaphysics under the direction of Edward Wierenga and received the Master of Arts degree from the University of Rochester in 2008.

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Abstract

In my dissertation, I address a variety of issues in the metaphysics of space and related areas. I begin by discussing the popular thesis that regions of space are identical to sets of points in space. I present three arguments against this thesis and conclude that we should be skeptical of it. In its place, I propose an axiomatic theory of regions of space that is consistent with both reductive accounts of their nature and with accounts that treat them as *sui generis* entities.

I next explore the consequences of the aforementioned considerations. In particular, I describe five different sorts of structure each of which is such that the claim that space could have that structure is consistent with the axiomatic theory previously proposed. I claim that this fact, together with the skepticism concerning reductive accounts argued for earlier, shows that we should take seriously the claim that space could have any of these structures.

Having argued that we should be skeptical of the thesis that regions of space are identical to sets of points in space and suggesting that space could have different sorts of structure, I discuss how best to analyze continuity. I present an analysis of continuity inspired by remarks of Richard Cartwright in his 1975 paper 'Scattered Objects'. I argue that this Cartwrightian analysis should be rejected because it identifies regions of space with sets of points in space, and I present a modified version of the analysis that does not do so. I note, however, that there is an intuitive notion of continuity that is not captured by this modified Cartwrightian analysis. I present and defend an analysis of continuity that better captures this intuitive notion.

I then turn to the issue of how to analyze what it is for a region of space to open and what it is for a region of space to be closed. Here I argue that Cartwright's analyses of these notions are incorrect. I then present a series of alternative analyses, revising each in response to objections. This process culminates with my proposed analyses of what it is for a region of space to be open and what it is for a region of space to be closed.

Finally, I discuss the Maximally Continuous Account of Simples (MaxCon), originally formulated and defended by Ned Markosian in his 1998 paper 'Simples'. I argue that Markosian's version of MaxCon, which identifies regions of space with sets of points in space and relies on the Cartwrightian analysis of continuity, should be rejected. I then formulate a new version of MaxCon that builds on the views defended earlier in my dissertation and defend this new version of MaxCon from objections.

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Chapter 1: Ways that Space Might Be

Introduction

In this chapter, I discuss two questions concerning the metaphysics of space. The first question, which I discuss in §§1 and 2, is this: What are regions of space? In §1, I argue against the popular reductive view that regions of space are sets of points in space[1] and one region is a subregion of another just in case the first is a subset of the second. I claim that although the arguments in this section are neither individually nor collectively conclusive, they do provide reasons to be skeptical of the view under discussion.

With this in mind, in §2 I present an axiomatic theory concerning regions of space that provides a systematic framework for reasoning about regions. Unlike the view discussed in §1, this theory is officially agnostic about what regions of space are. It is consistent both with reductive accounts of regions of space and with the view that regions of space are *sui generis* entities.

The second question I discuss concerns the possible structures of space. I discuss this question in §3, where I describe five different structures of space and note that the axiomatic theory presented in §2 is consistent with space having any of these structures. I argue that because that theory is consistent with space having any of these structures and because there are reasons to be skeptical of the reductive view discussed in §1 (which would rule out the possibility of space having some of these

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[1] In the remainder of the dissertation, I use ‘region of space’ and ‘region’ interchangeably, and I do the same with ‘point in space’ and ‘point’.

structures), we ought to take seriously the claim that space could have any of the five structures described.

1. Identifying Regions of Space with Sets of Points

According to one popular view concerning regions of space, they are identical to sets of points in space. Richard Cartwright (1987) explicitly endorses this view when he says, ‘By a region of space, or simply a region, let us agree to understand any set of points in space’ (p. 171), and Hud Hudson (2002) begins his paper by saying that ‘[a] region of space is a set of points in space’ (p. 432). Other philosophers also appear to be committed to this view. For example, Ned Markosian (1998a) offers the following definition of one of the key terms in his account of simples:

> x is a *maximally continuous object* =df x is a spatially continuous object and there is no continuous region of space, R, such that (i) the region occupied by x is a proper subset of R, and (ii) every point in R falls within some object or other. (p. 221)

Although Markosian is less explicit in his endorsement than Cartwright and Hudson, his use of the phrase ‘the region occupied by x is a proper subset of R’ indicates that he too takes regions of space to be sets of some sort. That he takes their members to be points is indicated in another definition he offers: ‘Object O occupies region R =df R is the set containing all and only those points that lie within O’ (p. 216).

The view endorsed by Cartwright, Hudson, and Markosian, among others, can be given a more precise statement as follows:

The Identity Hypothesis (IH):

(A) Necessarily, for all x, x is a region of space if and only if x is a set of points in space.

(B) Necessarily, for all regions of space x and y, x is a subregion of y if and only if x is a subset of y.

In this section, I offer three arguments against (IH). Although these arguments are not conclusive, they show that we should be skeptical of the proposed identification of regions of space with sets of points in space.

1.1. The First Argument from Analysis

One consequence of (IH) that the properties *being a region of space* and *being a set of points* are necessarily coextensive; if (IH) is true, then there couldn't be anything that has the first of these properties without having the second and there couldn't be anything that has the second but doesn't have the first. But suppose that *being a region of space* and *being a set of points* are necessarily coextensive. One might then think that it is very likely that this fact has an explanation.[2] One simple

[2] Why think that the fact that *being a region of space* and *being a set of points* are necessarily coextensive, supposing that it is a fact, has an explanation? Some philosophers maintain that there is no brute modality; that is, that there are no modal truths that do not have an explanation. (A paradigm example here is David Lewis (1986), who attempts to explain all modal truths in terms of the goings-on in spatiotemporally disconnected universes.) These philosophers will clearly maintain that the fact that *being a region of space* and *being a set of points are necessarily coextensive* has an explanation.

Even those who, like me, are skeptical of the claim that all modal truths have an explanation might have good reasons to think that the fact that *being a region of space* and *being a set of points* are necessarily coextensive, supposing again that it is a fact, has an explanation. For instance, we might maintain, along with Hud Hudson (2001) that the minimization of brutality (that is, the minimization of the number of facts taken to have no explanation) is a desideratum of metaphysical theorizing. If so, then we will maintain that so and elegant explanation is that these properties are identical: *being a region of space* just is *being a set of points*. The claim that these two properties are identical can be expressed as an analysis[3]:

The Identity Analysis (IdA): x *is a region of space* $=_{df}$ x is a set of points.

There is some reason, then, to think that if (IH) is true, then (IdA) is true.

However, there is also some reason to think that (IdA) is false. In particular, there is some reason to think that the property of being a point is to be analyzed as follows:

The Intuitive Analysis (InA): x *is a point* $=_{df}$ (i) x is a region of space, (ii) x has no proper subregions[4], and (iii) x is unextended.

If (InA) is true, then (IdA) is false. For if (InA) is true, then *being a region of space* is a constituent of *being a point*; and if (IdA) is true, then *being a point* is a constituent

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long as there is a potential explanation of the fact that *being a region of space* and *being a set of points in space* are necessarily coextensive, it is very likely that fact has an explanation.

[3] I accept an account of analysis developed by Jeffrey King (1998). According to this account, analyses have two functions. First, they express identities between properties (or relations). Second, they state the constituents of properties (or relations). The analysis stated above, then, expresses that *being a region of space* is identical to *being a set of points* and states that *being a region of space* has the following properties as constituents: *being a set* and *being something such that every member of it is a point*. Furthermore, since the latter is a complex property that has *being a member of* and *being a point* as constituents, the analysis also has the consequence that *being a region of space* has *being a member of* and *being a point* as constituents.

Arguing in favor of King's account would take me too far afield. Furthermore, King has already offered persuasive arguments in favor of his account in the work cited above. Anything I said in favor of his account would merely be a rehashing of his arguments. Thus, I simply assume here that King's account is correct.

[4] x *is a proper subregion of* y $=_{df}$ (i) x is a subregion of y and (ii) x is not identical to y.

of *being a region of space*. But constituenthood is asymmetric, however: for all x and y, if x is a constituent of y, then y is not a constituent of x.[5]

Thus, if (InA) is true, then (IdA) is false. But why think that (InA) is true? I think that linguistic considerations support (InA). Consider the term 'point'. It seems quite likely that this term was introduced into our language by way of a stipulative definition. The main alternative to this account of the origins of 'point' is that it was introduced into our language the way that terms like 'tree', 'rock', etc., were (probably) introduced into our language: some objects were noted to have something in common—some common property—and a term was introduced to apply to all and only those things that have that property. This alternative account seems unlikely, however. We were able to note that trees and rocks, for instance, have some common property because they are objects of our perceptual awareness; and points certainly are not.

Suppose, then, that 'point' was introduced into our language by way of a stipulative definition. One might then wonder what the content of this definition was. My suggestion is that our best guide to answering this question is to be found in the manner that we are nowadays introduced to the term 'point'. If we are introduced to this term by way of a stipulative definition, then (absent other considerations) it is

[5] One might accept a distinction between constituenthood and proper constituenthood. The idea would be that every property (and every relation) is a constituent of itself and that one property (or relation) is a proper constituent of another just in case the first is a constituent of the second and the first is not identical to the second. If one accepts this distinction, then one will deny that constituenthood is asymmetric. However, it is clear that this distinction is of no help in avoiding a conflict between (InA) and (IdA). After all, given this distinction, if *being a region of space* is a constituent of *being a point* and *being a point* is a constituent of *being a region of space*, then *being a region of space* is identical to *being a point*; and it is clear that these properties are not identical.

likely that the term was introduced into our language by way of that definition. Speaking for myself, I seem to remember being introduced to this term by way of a stipulative definition that looked very similar to (InA). So I think that there is some reason to think that ‘point’ was introduced into our language by way of such a definition.

If ‘point’ was introduced into our language by way of (InA), however, then that is good reason to think that (InA) is true. For if it was introduced into our language in this way, then the property expressed by ‘point’ simply is the property identified in (InA): it is simply is the property of being a region of space that has no proper subregions and is unextended. But if the property expressed by ‘point’ simply is that property, then (InA) is true.

I conclude, then, that there is some reason to think that (InA) is true. In particular, there is (i) some reason to think that ‘point’ was introduced into our language by way of a stipulative definition, (ii) some reason to think that (InA) was this definition, and (iii) some reason to think that if (i) and (ii) are true, then (InA) is true. However, if all of this is the case, then there is some reason to think that (InA) is true.

I have argued, then, that there is some reason to think that each of the premises of the following argument is true:

1. (InA) is true.
2. If (InA) is true, then (IdA) is not true.
3. Therefore, (IdA) is not true.
4. If (IH) is true, then (IdA) is true.

5. Therefore, (IH) is not true.

Thus, since we have some reason to think that each of the premises of this argument is true, we have some reason to think that its conclusion is true; that is, we have some reason to think that (IH) is not true.

1.2. The Second Argument from Analysis

In the last section, I argued in favor of the claim that the Intuitive Analysis is true and then I used this claim as a premise in an argument for the conclusion that the Identity Hypothesis is not true. This argument relied on the premise that if the Identity Hypothesis is true, then the Identity Analysis is true, which I also argued for in the last section. Here I would like to offer another argument for the conclusion that (IH) is not true, an argument that has the claim that (InA) is true as a premise. However, unlike the argument given in the previous section, the argument offered in this section does not have the claim that if (IH) is true, then (IdA) is true, as a premise. Thus, even if the latter claim is rejected, the claim that (InA) is true may still be used to show that there are reasons to think that (IH) is false.

According to the Identity Hypothesis, (a) necessarily, for all x, x is a region of space if and only if x is a set of points and (b) for all regions of space x and y, x is a subregion of y if and only if x is a subset of y. And according to the Intuitive Analysis, necessarily, for all regions of space x, x is a point iff x is a simple region of space and x is unextended. But now a problem arises.

Assume that both the Identity Hypothesis and the Intuitive Analysis are true and let p be a point. Given the Intuitive Analysis, p is a region of space. But given the

Identity Hypothesis, every region of space is a set of points. Thus, on the assumption that both the Identity Hypothesis and the Intuitive Analysis are true, p is a set of points. Now if p is a set of points, then p either has multiple points as members or it has only a single point as a member. I will show that each of these alternatives is unacceptable.

Suppose, for reductio, that p has multiple points as members. Then there is a subset of p, S, such that S is not identical to p and S is a set of points. But, given the Identity Hypothesis, every set of points is a region of space and one region of space is a subregion of another just in case the first is a subset of the other. So, S is a region of space and S is a subregion of p. Furthermore, S is a proper subregion of p, since S is distinct from p. However, given the Intuitive Analysis, points lack proper subregions. The reductio is complete. On the supposition that p has multiple points as members, p both has and lacks proper subregions, and this is impossible. So, p does not have multiple points as members.

Suppose, on the other hand, that p has only a single point, p*, as a member. Since p* is a point, the Identity Hypothesis and the Intuitive Analysis jointly entail that p* is a set of points. Furthermore, the argument given in the preceding paragraph shows that, given the Identity Hypothesis and the Intuitive Analysis, p* has only a single point, p**, as a member. And, by parallel reasoning, p** has a single point, p***, as a member; p***, has a single point, p****, as a member; and so forth. It follows that p has members, every member of p has members, every member of every member of p has members, every member of every member of every member of p has members, and so forth. If we introduce the two-place predicate ‘is a member* of’ to express the ancestral of the membership relation, we can make the point this way: there is an x such that x is a member* of p and for every member* y of p, there is a z such that z is a member* of y. But this result conflicts with the widely accepted claim that every set is well-founded. So, since every set is well-founded, it is not the case that p has only a single point as a member.

The reasoning outlined in the preceding paragraphs supports the following argument against the Identity Hypothesis:

1. If the (IH) is true and (InA) is true, then every point has a single point as a member.
2. If every point has a single point as a member, then not every set is well-founded.
3. Every set is well-founded.
4. Therefore, either (IH) is not true or (InA) is not true.
5. (InA) is true.
6. Therefore, (IH) is not true.

The argument is valid and we have reasons to think that each of its premises is true. In §1.1, I offered reasons to think that premise (5) of this argument is true. Premise (1) is supported by the reductio of the claim that p has multiple points as members, assuming (IH) and (InA), that was given above. Since p was an arbitrarily chosen point, we can generalize to reach premise (1). Premise (2) is supported by the reasoning given in the preceding paragraph along with the claim that there are points, a claim that proponents of (IH) will certainly not challenge.

What of premise (3)? Well, the claim that every set is well-founded is a widely accepted principle concerning sets. Furthermore, it is widely accepted for good reason; it helps to ensure that set theory does not fall prey to Russell's Paradox.[6] Thus, there are reasons to think that premise (3) is true as well.

I conclude that there are reasons to think that each premise of this argument is true. If so, however, there are reasons to think that its conclusion is true. Thus, there are reasons to think that (IH) is false.

1.3. The Argument from Alternative Structures

Let me now turn to the third argument against the Identity Hypothesis. According to (IH), necessarily, for all x, x is a region of space if and only if x is set of points. Consequently, if (IH) is true, then necessarily, if there are regions of space, then there are sets of points. But it is evident that necessarily, if there are sets of points, then there are points. So, if (IH) is true, then necessarily, if there are regions of space, then there are points. However, some have held that possibly, there are regions of space but there are no points. Thus, the following argument can be given against (IH):

1. If (IH) is true, then: necessarily, if there are regions of space, then there are points in space.
2. Possibly, there are regions of space but there are no points.
3. Therefore, (IH) is not true.

[6] There are other ways to avoid Russell's Paradox, to be sure. However, accepting the well-foundedness of sets is a very popular way of doing so. The claim that every set is well-founded should not be denied unless there are very strong reasons for doing so.

Premise (1) is supported by the reasoning given above. So, in the following, I consider reasons to think that premise (2) is true.

One reason to believe that possibly, there are regions of space but there are no points derives from considerations involving conceivability. It is conceivable that there are regions of space but there are no points, perhaps because (a) every region of space has proper subregions or (b) every region of space is extended. But if it is conceivable that there are regions of space but there are no points, then possibly, there are regions of space but there are no points. So, possibly, there are regions of space but there are no points.

A proponent of the Identity Hypothesis might object to this argument in favor of the claim that possibly, there are regions of space but there are no points, by denying that there is any reason to believe that if it is conceivable that there are regions of space but there are no points, then possibly, there are regions of space but there are no points. In support of his denial, he might argue that the fact that something is conceivable is no guarantee that it is possible and conclude on this basis that there is no reason to believe the claim in question.

Let us grant the proponent of the Identity Hypothesis that the fact that something is conceivable is no guarantee that it is possible. It does not follow from this claim that there is no reason to believe that if it is conceivable that there are regions of space but there are no points, then possibly, there are regions of space but there are no points. The claim that something is conceivable does, it seems, provide defeasible evidence in favor of the claim that it is possible. Thus, the claim that it is conceivable that there are regions of space but there are no points provides defeasible evidence in favor of the claim that possibly, there are regions of space but there are no points. Therefore, so long as this evidence is not defeated, there is reason to believe that if it is conceivable that there are regions of space but there are no points, then possibly, there are regions of space but there are no points. Since, so far as I can tell, that evidence is not defeated, there is reason to believe the claim in question.

1.4. Concluding Remarks

In this section, I have argued against the Identity Hypothesis. According to the (IH), (i) necessarily, for all x, x is a region of space if and only if is a set of points in space, and (ii) for all regions of space x and y, x is a subregion of y if and only if x is a subset of y. In summary, (IH) conflicts with the Intuitive Analysis in at least two ways, as well as with the possibility that there be regions of space although there are no points. These arguments do not, individually or collectively, conclusively refute (IH). However, they provide some reason to think that (IH) is false, and we should thus be skeptical of (IH).

2. An Axiomatic Theory Concerning Regions of Space

In this section, I articulate an axiomatic theory concerning regions of space. Unlike the Identity Hypothesis, this theory is officially agnostic about what regions of space are. It is consistent both with reductive accounts of regions of space, such as (IH), and with the view that regions of space are *sui generis* entities.

I present this theory for two reasons. First, even if there are good reasons to be skeptical of reductive accounts of regions of space, we will still want to reason about regions. The theory presented here provides a systematic framework for doing so. Second, as I show in §3, this can be used to formulate different claims concerning the structure of space. Since there are reasons to be skeptical of reductive accounts like (IH) that would rule out the possibility that space has some of these structures, I argue that the fact that each of these structures is consistent with the theory presented here is some reason to think that space could have any of these structures.

The axiomatic theory concerning regions of space I develop here employs only two non-logical primitives: the monadic predicate 'is a region of space' and the dyadic predicate 'is a subregion of'.[7] All other non-logical terminology that appears in its axioms is defined in terms of these primitives. In addition to these primitives, the theory employs both singular and plural quantification. In fact, the theory is similar in some respects to the exposition of classical extensional mereology given in Lewis 1991, which also employs both singular and plural quantification.[8]

My statement of this axiomatic theory begins with the following axiom:

A1. For all x, if there is a y such that x is a subregion of y or y is a subregion of x, then x is a region of space.

This axiom ensures that only regions of space are within the domain and range of the subregionhood relation; that is, it ensures that the only things that are or have subregions are regions of space. To this axiom, I add the following:

A2. For any region of space x, x is a subregion of x.

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[7] In §3, I will introduce other primitives needed to express different (seeming) possibilities concerning the structure of space. However, these other primitives are not primitives of the bare system concerning regions of space that I develop in this section.

[8] For a good introduction to plural quantification, see chapter 2 of van Inwagen 1990.

A3. For any regions of space x and y, if x is subregion of y and y is a subregion of x, then x is identical to y.

A4. For any regions of space x, y, and z, if x is a subregion of y and y is a subregion of z, then x is a subregion of z.

These three axioms jointly ensure that the subregionhood relation is reflexive (when restricted to regions of space), antisymmetric, and transitive.

The remaining axiom is best stated using the definienda of the following definitions:

D1. x *region-overlaps* y $=_{df}$ there is a region of space z such that z is a subregion of x and z is a subregion of y.

D2. x *region-fuses* yys $=_{df}$ (i) for all z such that z is one of yys, z is a subregion of x and (ii) for all z such that z is a subregion of x, there is a w such that w is one of yys and z region-overlaps w.

Given these definitions, we can state the following final axiom:

A5. For any regions of space xxs and for any y, if y region-fuses xxs, then for all z such that z region-fuses xxs, y is identical to z.

This axiom should be relatively uncontroversial. Informally, it says that if a region of space is made up entirely of some other regions of space, then it is the only region made up entirely of those regions. Alternatively, it says that no two regions of space can be entirely made up of the very same regions. This is extremely plausible.

The axiomatic theory concerning regions of space that I propose, then, consists of axioms (A1)-(A5). One might wonder why I do not include the following as an axiom in my theory:

A6. For any regions of space xxs, there is a y such that y region-fuses xxs.

(Including this axiom would introduce even greater structural similarities between the axiomatic theory presented and classical extensional mereology.) I do not do so because (A6) is somewhat controversial. Josh Parsons (2007), for example, takes seriously the possibility of knuggy regions of space, where:

x *is a proper subregion of* y $=_{df}$ (i) x is a subregion of y and (ii) x is not identical to y.

x *is a knuggy region of space* $=_{df}$ (i) x is a region of space, (ii) there is a y such that x is a proper subregion of y, and (iii) for all y such that x is a proper subregion of y, there is a z such that y is a proper subregion of z.

But (A6) rules out the possibility of such regions.

To see that (A6) rules out the possibility of knuggy regions, suppose that (A6) is true and that there is a knuggy region of space R. Given (A6), there is a region of space S such that S region-fuses the regions of space of which R is a proper subregion. Thus, every region of space of which R is a proper subregion is a subregion of S. So, given that every region of space of which R is a proper subregion is a region of space of which R is a subregion and the transitivity of the subregionhood relation, R is a subregion of S. But if R is a subregion of S, then (given the antisymmetry of the subregionhood relation) either (i) R is identical to S or

(ii) R is a proper subregion of S. As we shall see, however, neither (i) nor (ii) can be true.

Suppose that (i) is true; that is, suppose that R is identical to S. Then since S region-fuses the regions of space of which R is a proper subregion, R region-fuses the regions of space of which R is a proper subregion. But if R region-fuses the regions of space of which R is a proper subregion, then every region of space of which R is a proper subregion is a subregion of R. But every region of space of which R is a proper subregion is a region of space of which R is a subregion. Thus, every region of space of which R is a proper subregion is both a subregion of R and is a region of space of which R is a subregion. But then (given the antisymmetry of the subregionhood relation) every region of space of which R is a proper subregion is identical to R. But no region of space of which R is a *proper* subregion is identical to R, and we have reached a contradiction. Therefore, R is not identical to S.

Suppose, on the other hand, that (ii) is true; that is, suppose that R is a proper subregion of S. Then since R is a knuggy region of space, there is a region of space T such that S is a proper subregion of T. But then S is a subregion of T. So, since, as we saw above, R is a subregion of S, R is a subregion of T. However, if R is a subregion of T, then either R is identical to T or R is a proper subregion of T. R is not identical to T, though. For S is a subregion of T and so, if R is identical to T, S is a subregion of R. As we saw above, however, R is a subregion of S. Thus, if R is identical to T, then R is a subregion of S and S is a subregion of R. But it is not the case that R is a subregion of S and S is a subregion of R, for if R is a subregion of S and S is a subregion of R, then (given the antisymmetry of the subregionhood relation) R is identical to S and, as we saw in the previous paragraph, R is not identical to S. So, if R is a proper subregion of S, then R is a proper subregion of T. But then T is a subregion of S, since S region-fuses the regions of space of which R is a proper subregion. However, S is also a subregion of T, since S is a proper subregion of T. It follows that S is identical to T, since T is a subregion of S, S is a subregion of T, and the subregionhood relation is antisymmetric. But S is a proper subregion of T, and thus is not identical to T. Again, we have reached a contradiction. We can conclude, then, that (ii) is not true; R is not a proper subregion of S.

To summarize, we have seen that if (A6) is true and it is possible that there is a knuggy region of space, then either (i) or (ii) is true. But we have also seen that neither (i) nor (ii) is true. Therefore, either (A6) is not true or it is not possible that there is a knuggy region of space; that is, we have seen that (A6) precludes the possibility of knuggy regions of space.

Thus, I do not include (A6) as an axiom because whether knuggy regions of space are possible is controversial and (A6) rules out their possibility. Although I do not include (A6) as an axiom, I am inclined to think that it is true. Thus, in what follows, I will sometimes ask the reader to consider the region that region-fuses some specified regions without arguing for the claim that there is such a region. Readers should keep in mind that they may, if they'd like, reject the claim that there is such a region.[9]

In this section, I have presented an axiomatic theory concerning regions of space. This theory, which does not attempt to identify regions of space with sets of points in space and which is fully characterized by (A1)-(A6), provides the background for the discussion of regions of space in the remainder of this dissertation.

3. Possible Structures of Space

In this section, I discuss the issue of the possible structures of space in light of the discussion in §§1 and 2. I begin by making some general comments concerning the impact of the preceding discussion on this issue.

Given the Identity Hypothesis, it is necessary that every region of space is "made up" of points. However, as I argued in §1, there are reasons to be skeptical of (IH). Furthermore, in §2, I presented an axiomatic theory concerning regions of space that is consistent with the possibility that not every region of space is "made up" of points. I would suggest, then, that this possibility should be taken seriously. We should take seriously the claim that it is possible that space has a certain structure

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[9] Kris McDaniel has pointed out (in correspondence) that the following principle is consistent with the possibility of knuggy regions:

> For any regions of space xxs, if there is a y such that each of xxs is a subregion of y, then there is a z such that x region-fuses xxs.

Furthermore, in many of the cases in which I ask the reader to consider the region that region-fuses some specified regions, this principle implies that there is such a region. Thus, even readers who accept the possibility of knuggy regions should not for that reason reject the claim that there is such a region. They must also find fault with McDaniel's principle.

even when the claim that space has this structure is inconsistent with the claim that every region of space is "made up" of points.

In this section, then, I will describe some different structures of space. Each of the structures I describe is such that the claim that it is possible that space has that structure is consistent with the account of regions of space developed in the previous section. Thus, in light of the skepticism concerning (IH) that I have argued for, I think that we should take seriously the claim that space could have each of these structures.

3.1. The Standard Structure

The first structure I will discuss is what I will call 'the Standard Structure'. As its name suggests, the Standard Structure is the structure that space is standardly held to have. In addition, it is often, although not quite standardly, held that necessarily, space has the Standard Structure.

The Standard Structure can be succinctly characterized by three claims such that necessarily, space has the Standard Structure if and only if each of them is true. The first of these claims is:

Points (P): There are points,

where the relevant notion of a point (both here and below) is that captured by the Intuitive Analysis: a point is a region of space that is unextended[10] and has no proper subregions.

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[10] Note that 'is unextended' is not a primitive of the axiomatic theory presented in §2, nor is 'is extended', which I employ in §3.3 when characterizing the Tile Structure. Furthermore, so far as I can see, neither 'is unextended' nor 'is extended' can be defined in terms of the primitives of that theory. Thus, although those primitives are employed when characterizing the structures discussed in this section, other primitives must be employed as well.

The second claim by which the Standard Structure can be characterized is a claim concerning the structure of each region of space. Informally, it is the claim that every region of space is "made up" of points. More formally, it can be expressed as follows:

> **Non-Pointy Regions (NPR)**: For any region of space x, there are points yys such that x is a region-fusion of yys.

Together with (P), (NPR) guarantees that if it has the Standard Structure, space is much like it would be if the Identity Hypothesis were true. In particular, necessarily, if space has the Standard Structure, then there are points and every region of space is "made up" of such points, which claims would also be true if (IH) were true and there were regions of space. However, despite the fact that the space would have some of the same features whether it had the Standard Structure or whether, rather, (IH) is true, the claim that space has, or could have, the Standard Structure does not commit its proponent to (IH). Someone who holds that space in fact has the Standard Structure can consistently maintain that it is *possible* that not every region of space is "made up" of points. Furthermore, even one who maintains that it is necessary that space has the Standard Structure, and thus holds that it is not even possible for there to be regions of space that are not "made up" of points, can consistently deny (IH). For it is consistent with her view that although it is necessary that every region of space is "made up" of points, the points that a region is "made up" of are not members of that region.

The third and final claim by which the Standard Structure can be characterized specifies a feature of space that is not entailed by (IH). It is a claim concerning the spatial relations between points. In particular, it is the following claim:

> **Betweenness (B)**: For any points x and y such that x and y are distinct, there is a point z such that z is between x and y.

Necessarily, space has the Standard Structure, then, if and only if (P), (NPR), and (B) are all true.

3.2. The Gritty Structure

The second structure of space that I will discuss is the Gritty Structure. The Gritty Structure is very similar to the Standard Structure. In particular, necessarily, space has the Gritty Structure only if (P) and (NPR) are true—only if, that is, there are points and every region of space is "made up" of such points. Thus the Gritty Structure, like the Standard Structure, mimics in important ways what space would be like if the Identity Hypothesis were true and there were regions of space.

The main difference between the Gritty Structure and the Standard Structure concerns the spatial relations between points. Necessarily, space has the Standard Structure, remember, only if (B) is true. In contrast, necessarily, space has the Gritty Structure only if (B) is false; that is, necessarily, space has the Gritty Structure only if there are points x and y such that y and y are distinct and there is no point z such that z is between x and y.[11] However, the conjunction of the negation of (B) with (P) and

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[11] This assumes that there is more than one point. For a world containing only a single point is one in which space has both the Standard Structure and the Gritty Structure. Strictly speaking, then, the claim that space has the Gritty Structure is consistent with (B), but the

(NPR) does not suffice to characterize the Gritty Structure. For the conjunction of these three claims is consistent with the claim that, roughly, there are two points that have infinitely many points between them. It only rules out that this is true of any two distinct points that one picks; according to the negation of (B), at least two points must have no points between them. But space has the Gritty Structure only if no two points have infinitely many points between them. So in order to adequately characterize the Gritty Structure, we must formulate a stronger claim that captures this entailment of space having the Gritty Structure.

To formulate this stronger claim, it will be useful to first state more precisely the claim that is ruled out by the Gritty Structure but is consistent with (P), (NPR), and the negation of (B). This can be done as follows:

> **The Problematic Claim**: There are points x and y such that (i) x and y are distinct, (ii) there is a point z between x and y, and (iii) for any distinct points v and w each of which is either identical to x, identical to y, or between x and y, there is a point between v and w.

Then stronger claim needed in order to characterize the Gritty Structure can be formulated as follows:

> **Grittiness (Gr)**: For any points x and y, if x and y are distinct and there is a point z between x and y, then there are distinct points v and w each of which is either identical to x, identical to y, or between x and y, such that there is no point between v and w.

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claim that space has the Gritty Structure, (B), and the claim that there is more than one point are inconsistent.

So necessarily, space has the Gritty Structure if and only if (P), (NPR), and (G) are all true.

3.3. The Tile Structure

The Tile Structure is the third structure of space I would like to discuss. In some ways, the Tile Structure is very similar to the Standard Structure and the Gritty Structure. Like the latter two structures, space has the Tile Structure only if there are regions of space that have no proper subregions and every region of space is a region-fusion of such regions. However, the Tile Structure differs from the Standard Structure and the Gritty Structure with respect to whether these regions are extended. On the Standard Structure and the Gritty Structure, these regions are points and thus are unextended. But on the Tile Structure, they are tiles, where:

> x *is a tile* $=_{df}$ (i) x is a region of space, (ii) x has no proper subregions, and (iii) x is extended.

Necessarily, then, space has the Tile Structure if and only if the following two claims are both true:

> **Tiles (T)**: There are tiles.
>
> **Non-Tiley Regions (NTR)**: For any region of space x, there are tiles yys such that x is a region-fusion of yys.

It is important to note that whether space has the Tile Structure doesn't settle anything concerning the relations between tiles. This is because it is not partially characterized by an analogue of (B) or of (G). I leave out such analogues deliberately, for there is more than one way for space to have the Tile Structure. One way for space to have the Tile Structure is the following: (i) There is more than one tile, (ii) for each tile t, there is a tile t’ such that there is no tile between t and t’, and (iii) for each tile t, there is no tile t’ such that there are infinitely many tiles between t and t’. (Space would have the Tile Structure in this way, for instance, if space were a 10’ x 10’ x 10’ cube “made up” of a thousand 1’ x 1’ x 1’ tiles arranged in a grid.) On the other hand, another way for space to have the tile structure is the following: (i) There is more than one tile and (ii) there are a tile t and a tile t’ such that there are infinitely many tiles between t and t’. (For instance, imagine that space is a 10’ x 10’ square made up of smaller 1’ x 1’ squares and that each of these smaller squares is “made up” of parallel 1’ long line-segment-shaped tiles between any two of which there is another. In this case, space would have the Tile Structure in the second way.)[12]

3.4. The Gunky Structure

The characterization of each of the preceding structures of space has included claims that guarantee that there are regions of space that do not have proper subregions and that every region of space is “made up” of such regions. In contrast, necessarily, if space has the structure I now wish to characterize, there are no such regions and thus no regions of space are “made up” of regions that do not have proper subregions.

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[12] Notice that I don’t claim that these are the only two ways for space to have the Tile Structure. They certainly aren’t, since each of them is one in which there is more than one tile. But the claim that space has the Tile Structure is consistent with the claim that there is only one tile. Thanks to Joshua Spencer for pointing out to me that space might have the Tile Structure in the second way.

This fourth structure of space, the Gunky Structure, is quite easy to characterize with the help of the following definition:

> x *is a gunky region of space* $=_{df}$ (i) x is a region of space and (ii) every subregion of x has a proper subregion.

Given this definition, necessarily, space has the Gunky Structure if and only if the following two claims are both true:

> **Regions Requirement (RR)**: There are regions of space.

> **Gunkiness (Gu)**: Every region of space is a gunky region of space.

Since it is necessary that space has the Gunky Structure only if (Gu) is true, space cannot have the Gunky Structure if there are points or tiles. Necessarily, if space has the Gunky Structure, then there are no regions of space (unextended or extended) that have no proper subregions.

3.5. Mixed Structures

There is no guarantee that space must have one of the four structures characterized above. Perhaps it is possible that there are both points and tiles, possible that there are both tiles and gunky regions of space, possible that there are both points and gunky regions of space, or possible that there are points, tiles, and gunky regions of space. Perhaps possibly, there are regions of space and every region of space is a region-fusion of points but (B) is true when the universal quantifier it contains is restricted to some points and (Gr) is true when the universal quantifier it contains is restricted to other points. In such cases, space would have a *mixed structure*; that is, when our attention is (and our universal quantifiers are) restricted to some regions, those regions behave just as they would if space had one of the structures described above, but when restricted to other regions, those regions behave just as they would if space had another of those structures. In the remainder of the dissertation, however, I will focus primarily on non-mixed structures.

Conclusion

In this chapter, I have investigated the nature of regions of space and the possible structures of space. In §1, I offered four arguments against the relatively popular Identity Hypothesis, according to which regions of space are sets of points in space and a region of space is a subregion of another if and only if it is a subset of the other. I claimed that although these arguments are neither individually or collectively conclusive, they provide some reason to think that (IH) is false, and that, in light of these reasons, we ought to be skeptical of (IH). Then, in §2, I developed an axiomatic theory concerning regions of space that is officially agnostic between reductive accounts of regions, like (IH), and accounts that take regions to be sui generis entities. The interest in this theory lies in the fact that it provides a framework for reasoning about regions of space that does not presuppose an account of their nature. Finally, in §3, I characterized four different structures of space. I noted that each of these structures is such that the claim that space has that structure is consistent with the axiomatic theory presented in §2, although it is impossible for space to have two of these structures if (IH) is true. I suggested, though, that because the axiomatic theory concerning regions developed in §2 does not rule out the possibility that space have any of these structures and the arguments presented in §1 provide reasons to be skeptical of (IH), we should take seriously the claim that it is possible for space to have any of these structures.

Chapter 2: Continuity

Introduction

In this chapter, I will discuss the property of being continuous. In §1, I present an analysis of this property inspired by Richard Cartwright's (1987) paper 'Scattered Objects'. Then, in §2, I note that this Cartwrightian analysis presupposes that the Identity Hypothesis is true and I present a modified version of the analysis that does not presuppose this. However, I argue in §3 that there is an intuitive notion of what it is for a region of space to be continuous that is not adequately captured by the modified Cartwrightian analysis. Finally, in §4, I develop an alternative analysis that does capture this notion.

1. A Cartwrightian of Continuity

Cartwright (1987) presents an analysis of a property he calls '*being connected*'. Although it is not clear that this was his intention, some later authors—Ned Markosian (1998a and 2004), for example—have taken Cartwright to have given an analysis of continuity. Although I do not wish to take a stand on Cartwright's intentions, in this section I will present an analysis of continuity inspired by Cartwright's discussion, so it will be helpful for the purposes of exposition to assume that these authors are correct. In deference to the fact that this may not have been his intention, however, I do not call the analysis presented in this section 'Cartwright's analysis of continuity'. Instead, I call it 'the Cartwrightian analysis of continuity'.[13]

[13] I thank Kris McDaniel for pushing me to be explicit about Cartwright's intentions.

Cartwright's discussion is a bit complicated. He analyzes *being continuous* in terms of *being discontinuous*, which he further analyses in terms of other notions. Cartwright begins his discussion by introducing the notion of an open sphere about a point. He says: 'Let *p* be any point of space. By an *open sphere about p* is meant a region the members of which are all and only those points that are less than some fixed distance from *p*' (p. 171). More formally, we can say:

> x *is an open sphere about* y $=_{df}$ (i) y is a point in space and (ii) there is a distance z such that for all w, w is a member of x if and only if w is a point in space whose distance from y is less than z.

Cartwright next introduces the notion of the complement of a region. '[T]he *complement* of a region,' says Cartwright, 'is the set of points of space not in the region' (pp. 171-2); that is:

> x *is the complement of* y $=_{df}$ (i) y is a set of points in space and (ii) for all z, z is a member of x if and only if z is a point in space that is not a member of y.

Having thus introduced the notions of an open sphere about a point and of the complement of a region, Cartwright explains what a boundary point of a region is as follows: 'A point *p* is said to be a boundary point of a region *A* if and only if every open sphere about *p* has a non-null intersection with both *A* and the complement of *A*' (171). More formally:

> x *is a boundary point of* y $=_{df}$ (i) x is a point in space, (ii) y is a set of points in space, and (iii) every open sphere about x has a member of y as a member and a member of the complement of y as a member.

With the notion of a boundary point of a region in hand, Cartwright explains what it is for something to be the closure of a region and what it is for two regions to be separated. He says that '[t]he closure of a region is the union of the region with the set of all its boundary points' (173). Thus, we can explain what the closure of a region is as follows:

> x *is the closure of* y $=_{df}$ (i) y is a set of points in space and (ii) for all z, z is a member of x if and only if z is a member of y or z is a boundary point of y.

And he says that 'two regions are *separated* if and only if the intersection of either with the closure of the other is null' (174); or, in other words:

> x and y *are separated* $=_{df}$ (i) x is a set of points in space, (ii) y is a set of points in space, (iii) no member of x is a member of the closure of y, and (iv) no member of y is a member of the closure of x.

Finally, Cartwright addresses what it is for a region to be discontinuous and what it is for a region to be continuous. According to Cartwright, 'a region is said to be [*discontinuous*] if and only if it is the union of two non-null separated regions and a region is [*continuous*] if and only if it is not [discontinuous]' (174). That is:

> x *is discontinuous* $=_{df}$ (i) x is a set of points in space and (ii) there are two sets of points in space y and z such that something is a member of x if and only if it is either a member of y or it is a member of z, and y and z are separated.
>
> x *is continuous* $=_{df}$ (i) x is a set of points in space and (ii) x is not discontinuous.

This completes my summary of Cartwright's discussion, culminating in the Cartwrightian analysis of *being continuous*. In the next section, I discuss a way to modify this analysis so that it does not presuppose that regions of space are sets of points. That the analysis can be modified in this way is important, since I argued in the previous chapter that we should be skeptical of the claim that regions of space are sets of points.

2. Modifying the Cartwrightian Analysis of Continuity

If correct, the Cartwrightian analysis of *being continuous* should say what it is for a region of space to be continuous. But according to that analysis, necessarily, a region of space is continuous only if it is a set of points in space. However, as argued in §1 of Chapter 1, there are reasons to be skeptical of the claim that regions of space are sets of points in space. Thus, we should be skeptical of the claim that the Cartwrightian analysis of *being continuous* is correct.

It would be better not to presuppose that the regions of space are sets of points in space when analyzing *being continuous*. Luckily, there is a straightforward way to modify the Cartwrightian analysis so that it does not presuppose this. The following definition will be helpful when presenting this modified analysis:

> x *is a pointy region-fusion* $=_{df}$ there are some points in space xxs such that x region-fuses xxs.[14]

Then Cartwright's discussion can be modified as follows:

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[14] I reproduce here the definition of 'region-fuses' given in Chapter 1:

> x *region-fuses* yys $=_{df}$ (i) for all z such that z is one of yys, z is a subregion of x and (ii) for all z such that z is a subregion of x, there is a w such that w is one of yys and z region-overlaps w

x *is an open sphere about* y $=_{df}$ (i) x is a point in space and (ii) there are some xxs and a distance z such that for all w, w is one of xxs if and only if w is a point in space whose distance from y is less than z, and x region-fuses xxs.

x *is the complement of* y $=_{df}$ (i) y is a pointy region-fusion and (ii) there are some xxs such that for all z, z is one of xxs if and only if z is a point in space that does not region-overlap y, and x region-fuses xxs.

x *is a boundary point of* y $=_{df}$ (i) x is a point in space, (ii) y is a pointy region-fusion, and (iii) every open sphere about x region-overlaps both y and the complement of y.

x *is the closure of* y $=_{df}$ (i) y is a pointy region-fusion and (ii) there are some xxs such that z is one of xxs if and only if z is a point in space that region-overlaps y or z is a boundary point of y, and x region-fuses xxs.

x and y *are separated* $=_{df}$ (i) x is a pointy region-fusion, (ii) y is a pointy region-fusion, (iii) no point in space that region-overlaps x region-overlaps the closure of y, and (iv) no point in space that region-overlaps y region-overlaps the closure of x.

x *is discontinuous* $=_{df}$ (i) x is a pointy region-fusion and (ii) there are two pointy region-fusions y and z such that a point in space region-overlaps x if and only if it either region-overlaps y or it region-overlaps z, and y and z are separated.

x *is continuous* $=_{df}$ (i) x is a pointy region-fusion and (ii) x is not discontinuous.

This modified Cartwrightian analysis of *being continuous* is better than the original. For it does not require that a region of space be a set of points in space to be continuous. Thus, although we should be skeptical of the claim that regions of space are sets of points in space, we needn't be skeptical of the modified Cartwrightian analysis for that reason. However, I will argue in the next section that, despite its advantages, there is an intuitive notion of what it is for a region of space to be continuous that the modified Cartwrightian analysis does not adequately capture.

3. An Intuitive Notion of Continuity

This intuitive notion of what it is for a region of space to be continuous that, I claim, is not adequately captured by the modified Cartwrightian analysis can be roughly formulated in a variety of ways. One rough formulation is as follows: A region of space is continuous just in case it is possible to get from any proper subregion of that region to any other proper subregion of it without leaving the region and without "jumping" from any region to any other region. Or, according to another rough formulation of the intuitive notion in question, a region of space is continuous just in case there is a path from any proper subregion of that region to any other proper subregion of it that does not leave the region and does not contain any gaps.

With a rough grasp of the intuitive notion described above in hand, I will now argue that it is not adequately captured by the modified Cartwrightian analysis of *being continuous*. For consider two 10' x 10' square-shaped tiles[15] that "butt up" against one another and whose region-fusion[16] is a 10' x 20' rectangular region of space. According to the modified Cartwrightian analysis of *being continuous*, the

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[15] x *is a tile* $=_{df}$ (i) x is a region of space, (ii) x has no proper subregions, and (iii) x is extended. See Chapter 1.
[16] x is a region-fusion of yys $=_{df}$ x region-fuses yys.

region-fusion of the tiles is not a continuous region of space, since it is not a pointy region-fusion. However, according to the intuitive notion of what it is for a region of space to be continuous that I described above, it is a continuous region of space. For the region-fusion of the two tiles has only the tiles themselves as proper subregions and the tiles “butt up” against one another, so that it is possible to get from any proper subregion of the region-fusion to any other proper subregion of it without leaving it and without “jumping” from any region to any other region. Therefore, the modified Cartwrightian analysis of *being continuous* does not adequately capture the intuitive notion of what it is for a region of space to be continuous.

There are different ways of making this argument more precise. Here is one. Let us stipulate that necessarily, a region of space is $continuous_C$ if and only if it satisfies the modified Cartwrightian analysis. And let us stipulate that that a region of space is $continuous_I$ if and only if it satisfies the intuitive notion of what it is for a region of space to be continuous described above. Then the following argument can be formulated:

1. Possibly, there is a region-fusion of tiles that is $continuous_I$.
2. Necessarily, there is no region-fusion of tiles that is $continuous_C$.
3. If (1) and (2), then the modified Cartwrightian analysis of *being continuous* does not adequately capture the intuitive notion of what it is for a region of space to be continuous.
4. Therefore, the modified Cartwrightian analysis of *being continuous* does not adequately capture the intuitive notion of what it is for a region of space to be continuous.

Premises (2) and (3) are uncontroversial. The former is uncontroversial because necessarily, a region of space is continuous$_C$ only if it satisfies the modified Cartwrightian analysis. But necessarily, a region of space satisfies the modified Cartwrightian analysis only if it is a pointy region-fusion. However, necessarily, there is no region-fusion of tiles that is a pointy region-fusion. Thus, necessarily, there is no region-fusion of tiles that is continuous$_C$; that is, premise (2) is true.

Premise (3) is also uncontroversial. For suppose that premises (1) and (2) are true; that is, suppose that possibly, there is a region-fusion of tiles that is continuous$_I$, and necessarily, there is no region-fusion of tiles that is continuous$_C$. Then possibly, there is a region-fusion of tiles that is continuous$_I$ but is not continuous$_C$. That is, possibly, there is a region-fusion of tiles that satisfies the intuitive notion of what it is for a region of space to be continuous but that does not satisfy the modified Cartwrightian analysis. But if the modified Cartwrightian analysis adequately captures the intuitive notion of what it is for a region of space to be continuous, then necessarily, something satisfies the intuitive notion of what it is for a region of space to be continuous if and only if it satisfies the modified Cartwrightian analysis. So, on the supposition that (1) and (2) are true, the modified Cartwrightian analysis of *being continuous* does not adequately capture the intuitive notion of what it is for a region of space to be continuous. Therefore, if (1) and (2), then the modified Cartwrightian analysis of *being continuous* does not adequately capture the intuitive notion of what it is for a region of space to be continuous; that is, premise (3) is true.

However, premise (1) is controversial. It says, remember, that possibly, there is a region-fusion of tiles that is $continuous_I$. One might reject this claim for one of two reasons. First, one might hold that necessarily, there are no tiles, and hence no region-fusions of tiles. Second, one might hold that although possibly, there are tiles (and hence possibly, there are region-fusions of tiles), necessarily, no region-fusion of tiles is $continuous_I$.

The second of these reasons for rejecting premise (1) is unconvincing. For it seems clear that if possibly, there are tiles, then possibly, there are two 10’ x 10’ square-shaped tiles that “butt up” against one another and whose region-fusion is a 10’ x 20’ rectangular region of space. But if there were such tiles, then their region-fusion would be $continuous_I$, since it would be possible to get from any proper subregion of that region-fusion to any other without leaving it and without “jumping” from any region to any other region. Thus, if possibly, there are tiles, then possibly, there is a region-fusion of tiles that is $continuous_I$, which is just what the first reason for rejecting premise (1) denies.

On the other hand, many philosophers have denied that possibly, there are tiles, and thus accept the first reason for rejecting premise (1). I do not wish to go into a detailed defense of the claim that possibly, there are tiles. However, I will note again that that claim is consistent with the axiomatic theory concerning regions of space presented in Chapter 1 and, furthermore, that it is conceivable that there are tiles and this provides defeasible evidence for that claim that it is possible.

In summary, premises (2) and (3) of the argument given above are uncontroversially true. Premise (1), on the other hand, is controversial. However, there is some reason to think that it is true, or at least to take seriously the claim that it is. Thus, the argument presented above provides some reason to think that the modified Cartwrightian analysis of *being continuous* does not adequately capture the intuitive notion of what it is for a region of space to be continuous, or at least to take seriously the claim that it does not.

Since premise (1) of the above argument is controversial, that argument will not persuade some of its conclusion. For this reason, I will now discuss another argument in favor of the claim that the modified Cartwrightian analysis of *being continuous* does not adequately capture the intuitive notion of what it is for a region of space to be continuous, an argument whose soundness is consistent with the denial of the claim that possibly, there are tiles.

The argument is rather simple. Notice first that the intuitive notion of what it is for a region of space to be continuous does not, by itself, rule out the possibility that there is a continuous region of space that is not a pointy region-fusion. However, the modified Cartwrightian analysis of *being continuous* does, by itself, rule out this possibility. Thus, the modified Cartwrightian analysis of *being continuous* does not adequately capture the intuitive notion of what it is for a region of space to be continuous.

The idea behind this argument is that there is a property of regions of space that we intend to pick out with the intuitive notion of what it is for a region of space to be continuous gestured at above and that the rough formulations of that notion presented there, although likely not entirely accurate, aptly characterize many of the features of that property. In particular, those rough formulations correctly indicate that being a pointy region-fusion is not part of what it is to have the property in question. Thus, if it is not possible that there is a continuous region of space that is not a pointy region-fusion, that is due to independent modal facts such as that necessarily, every region of space is a pointy region-fusion rather than to the structure of the property picked out by our intuitive notion of what it is for a region of space to be continuous. So, the intuitive notion of what it is for a region of space to be continuous does not, by itself, rule out the possibility that there is a continuous region of space that is not a pointy region-fusion.

On the other hand, the modified Cartwrightian analysis of *being continuous* does, by itself, rule out the possibility that there is a continuous region of space that is not a pointy region-fusion. For according to that analysis, what it is for a region of space to be continuous is for it to be a pointy region-fusion that is not discontinuous, and thus part of what it is for a region of space to be continuous is for it to be a pointy region-fusion. Thus, simply in virtue of the structure it attributes to the property of being continuous, the modified Cartwrightian analysis rules out the possibility that there is a continuous region of space that is not a pointy region-fusion, regardless of any independent modal facts.

This completes my case for the claim that the modified Cartwrightian analysis of *being continuous* does not adequately capture the intuitive notion of what it is for a region of space to be continuous. In the next section, I will formulate and defend an alternative analysis of the property picked out by that intuitive notion.

4. An Alternative Analysis

At the beginning of §3, I gave two rough formulations of the intuitive notion of what it is for a region of space to be continuous. They went like this:

> **Rough Formulation #1 (RF1):** A region of space is continuous just in case it is possible to get from any proper subregion of that region to any other proper subregion of it without leaving the region and without "jumping" from any region to any other region.

> **Rough Formulation #2 (RF2):** A region of space is continuous just in case there is a path from any proper subregion of that region to any other proper subregion of it that does not leave the region and does not contain any gaps.

I intended these rough formulations to show readers that they had a prior grasp of this notion or, at least, to help them acquire such a grasp if they previously had none (whether or not readers of either sort would classify the notion as a notion of continuity). Furthermore, once readers acquired or realized they already had a grasp on this notion, I hoped that they would assent to the claim that there is a property of regions of space picked out by the notion in question. If my intentions have been fulfilled, then readers should have a grasp on the notion I have in mind, which I have been describing (and will continue to describe) as an 'intuitive notion of what it is for a region of space to be continuous'.[17] I proceed on the assumption that my intentions have been fulfilled and the relevant notion has been grasped.

In this section, I would like to present an analysis of the property that is picked out by the intuitive notion of what it is for a region of space to be continuous. First, however, I must make some preliminary remarks. I do not deny that the modified Cartwrightian analysis is an adequate analysis of *some* property, nor do I deny that there could be regions of space that have that property. In fact, it is clear that if there could be points, then there could be regions of space having the property of which the modified Cartwrightian analysis is an adequate analysis, since necessarily, any point has that property. Perhaps more importantly, I do not deny that it is correct to think of the property in question as continuity. Maybe we have more than one notion that is properly called 'a notion of continuity' and the modified Cartwrightian analysis is an adequate analysis of the property picked out by one of these notions, although not, as I have argued, of the property picked out by the intuitive notion of continuity with which I have been concerned.[18]

Since the modified Cartwrightian analysis is an adequate analysis of some property and it may be that the property in question is picked out by a notion of

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[17] The careful reader will have noted some of the qualifications included in this paragraph. In particular, I acknowledge that some readers may not think that the notion under discussion is appropriately described as a notion of continuity. Although, as noted, I will continue to describe it in that way, I am not really interested in whether it is appropriately so described. Instead, I am interested in the notion itself and in the property picked out by it, since it seems to be an important property even if it is not a notion of continuity. I call it a notion of continuity only because the property picked out by it seems to be similar in important respects to the property whose correct analysis is given by the analysans of the Cartwrightian analysis, which property some authors have taken to be the property of being continuous.

[18] Although see the previous footnote.

continuity, let's call this property 'the property of being continuous$_1$' or 'continuity$_1$'. On the other hand, let's call the property picked out by the intuitive notion of what it is for a region of space to be continuous, which is roughly formulated in (RF1) and (RF2), 'the property of being continuous$_2$' or 'continuity$_2$'.[19] Then the question with which I am concerned in this section is: What is the correct analysis of the property of being continuous$_2$?

It is clear that neither (RF1) nor (RF2) provides a satisfactory analysis of continuity$_2$. For suppose that (RF1) and (RF2) were taken to be analyses of continuity$_2$. Then (RF1) would have the consequence that a region of space is continuous$_2$ only if it is possible to get from any of its proper subregions to any other. Continuity$_2$, however, is a structural feature of regions of space. Thus, even if, *per impossible*, motion were impossible, it would still be possible for a region of space to be continuous$_2$ so long as it had the right structural features. Worse, it may be that certain changes in an object's size and shape are impossible. For instance, suppose

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[19] In §3, I introduced the terms 'continuous$_C$' and 'continuous$_I$', as follows: 'Let us stipulate that necessarily, a region of space is continuous$_C$ if and only if it satisfies the modified version of Cartwright's analysis. And let us stipulate that that a region of space is continuous$_I$ if and only if it satisfies the intuitive notion of what it is for a region of space to be continuous described above.' Thus, the property of being continuous$_C$ is the property of satisfying the modified version of Cartwright's analysis and the property of being continuous$_I$ is the property of satisfying the intuitive notion of what it is for a region of space to be continuous. Now presumably (i) it is necessary that something satisfies the modified version of Cartwright's analysis if and only if it has the property of which the modified version of Cartwright's analysis is an adequate analysis and (ii) it is necessary that something satisfies the intuitive notion of what it is for a region of space to be continuous if and only if it has the property picked out by that intuitive analysis. Thus, the properties of being continuous$_C$ and being continuous$_1$ are necessarily coextensive, as are the properties of being continuous$_I$ and being continuous$_2$. On some views concerning properties, then, the properties of being continuous$_C$ and being continuous$_1$ are identical, as are the properties of being continuous$_I$ and being continuous$_2$. However, on other views, neither of these identity claims is true. For this reason, it was important for me to introduce different terminology here than I used in §3.

that space has the Standard Structure and consider a spherical region R. R has a point, p, at its center, and there is also a region, R’, that is a pointy region-fusion of the remaining points in R. If (RF2) is an analysis of continuity$_2$, then it is possible to get from p to R’—that is, it is possible for something to occupy p at some time and to later occupy R’—if R is continuous$_2$. Is that possible? Maybe not. But even if not, R is continuous$_2$. Therefore, (RF1) is not a satisfactory analysis of the property of being continuous$_2$.

Furthermore, (RF1) and (RF2) are not satisfactory analyses of continuity$_2$ because they employ obscure unexplained terminology. (RF1), for instance, includes talk of *getting from* one region to another without *“jumping”* from one region to another. On the other hand, (RF2) employs the notion of *a path* from one region to another region that does not *leave* a specified region and does not contain any *gaps*. It would be best if our official analysis of continuity$_2$ does not employ such terminology; in particular, it would be best if any unexplained terminology in our official analysis is, at the very least, rather clear.

Based on these constraints, my proposal is that the official analysis of continuity$_2$ should employ the notion of two regions being next to one another, where:

> x *is next to* y $=_{df}$ (i) x is a region of space, (ii) y is a region of space, (iii) x and y do not region-overlap, and (iv) there is no region of space z such that z is between x and y.

Here is one simple, initial attempt at an analysis of continuity$_2$ that employs this notion:

Proposed Analysis 1 (PA1): x *is continuous*$_2$ $=_{df}$ (i) x is a region of space and (ii) for any proper subregions y and z of x, if y is distinct from z, then y is next to z.

(PA1) does a better job of capturing the intuitive notion of continuity than does the modified Cartwrightian analysis. Consider the case of the 10’ x 20’ region-fusion of two 10’ x 10’ square-shaped tiles that “butt up” against one another discussed in §3. (PA1) correctly yields the verdict that this region-fusion is continuous$_2$. On the other hand, as noted in that section, this region-fusion is not continuous according to the modified Cartwrightian analysis.

However, (PA1) is unsatisfactory. To see this, suppose that space has the Standard Structure and consider a square-shaped region of space R, where R is a region-fusion of three rectangular regions of space—R1, R2, and R3—with the same area and shape.[20] Now suppose that R1 is to the left of R2 and R2 is to the left of R3, so that R2 is between R1 and R3. Then R is a region of space, R1 and R3 are proper subregions of R, and R1 is distinct from R3, but R1 is not next to R3. So, if (PA1) is true, then R is not continuous$_2$. But R is continuous$_2$. Therefore, (PA1) is not true.

The problem with (PA1) is that it needn’t be true of any two (distinct) proper subregions of a continuous$_2$ region of space that they are next to one another. R1 and R3 are two (distinct) subregions of R, which is a continuous$_2$ region of space, but they are not next to one another.

[20] Since space is being supposed to have the Standard Structure, at least one of R1-R3 is open or partially open and partially closed. See §2 above and Chapter 3.

The example involving R and R1-R3 suggests a more satisfactory analysis of continuity$_2$, however. For notice that although R1 is not next to R3, R1 is next to R2 and R2 is next to R3. Thus, there is a sequence of proper subregions of R whose first member is R1 and whose last member is R3 such that every member of the sequence is next to the next member of the sequence (if any). This suggests the following analysis:

> **Proposed Analysis 2 (PA2):** x *is continuous$_2$* $=_{df}$ (i) x is a region of space and (ii) for any proper subregions y and z of x, if y is distinct from z, then there is a sequence S of proper subregions of x whose first member is y and whose last member is z such that for every member w of S, if there is a member of S that immediately succeeds w in S, then w is next to the member of S that immediately succeeds w in S.

One might think that (PA2) is subject to an obvious objection.[21] For suppose that space has the Standard Structure and let L be a line-segment-shaped region of space that contains its endpoints. Call its endpoints p1 and p2. p1 is distinct from p2 and, since space has the Standard Structure, there is a point between p1 and p2. Thus, p1 is not next to p2. But furthermore, for any other point p that is a subregion of L, there is a point between p1 and p, so that p1 is not next to p. Thus, one might think, there is no proper subregion of L that p1 is next to. So, there is no sequence S whose first member is p1 and whose second member is another proper subregion of L such

[21] Thanks to Joshua Spencer for discussion.

that the first member of S is next to the second member of S. Hence, if (PA2) is true, then L is not continuous$_2$. But L is continuous$_2$. Therefore, (PA2) is not true.

This objection to (PA2) rests on a mistake, however. For although it is true that there is no sequence of *points* that are subregions of L connecting p1 and p2 that is such that each member is next to the subsequent member (if any), the points that are subregions of L are not the only proper subregions of L. Let pps be such that for all x, x is one of pps if and only if (i) x is one of the points that is a subregion of L and (ii) x is distinct from p1 and p2. Then there is a region of space, L*, that fuses pps and L* is a proper subregion of L. But p1 is next to L* and L* is next to p2. Thus, there is a three-membered sequence of proper subregions of L whose first member is p1, whose second member is L*, and whose third member is p2 such that each member in the sequence is next to the subsequent member in the sequence, if any. And so the objection fails.

(PA2) is a plausible analysis of continuity$_2$. One attractive feature of (PA2) is that it can be used to explain the fact that (RF1) and (RF2) can be employed as rough formulations of the intuitive notion of what it is for a region of space to be continuous. Consider (RF2) first. In light of (PA2) we can explain the obscure terminology contained in (RF2), as follows:

x *is a path from* y *to* z $=_{df}$ (i) y and z are regions of space and (ii) x is a sequence of regions of space whose first member is y and whose last member is z.

x *does not leave* y $=_{df}$ (i) y is a region of space, (ii) x is a sequence of regions of space, and (iii) every member of x is a proper subregion of y.

x *contains gaps* $=_{df}$ (i) x is a sequence of regions of space and (ii) there are adjacent members of x that are not next to one another.

Given these analyses, (RF2) is equivalent to (PA2). Thus, it is no surprise that (RF2) can be employed as a rough formulation of the intuitive notion of what it is for a region of space to be continuous, since that intuitive notion picks out the property of being $continuous_2$, the same property (PA2) provides an analysis of.

A similar explanation can be given in the case of (RF1). Suppose that it is possible for an object to get from any region to any other region if they are joined by a sequence of regions the adjacent members of which are next to one another.[22] Then a sufficient condition for the satisfaction of the clause of (RF1) that reads 'it is possible to get from any proper subregion of R to any other proper subregion of R' is that the first proper subregion be joined to the second proper subregion by a sequence of regions (not necessarily proper subregions of R) the adjacent members of which are next to one another. (RF1) also requires that a region is continuous only if it is possible to get from any proper subregion of R to any other proper subregion of R *without leaving R* and *without "jumping" from any region to another region*. We may understand the requirement that it is possible to get from any proper subregion of R to any other proper subregion of R *without leaving R* so that it is equivalent to the requirement that it is possible for an object to get from any proper subregion of R to any other proper subregion of R by successively occupying the members of a

[22] I do not claim that this supposition is true. I do claim, however, that it is plausible that we treated as though it were true when we use (RF1) to formulate and to acquire the intuitive notion of continuity.

sequence of regions whose first member is the first of these proper subregions, whose last member is the last of these proper subregions, and whose intervening members (if any) are all proper subregions of R. Furthermore, we may understand the requirement that it is possible to do so without "jumping" from any region to any other region so that it is equivalent to the requirement that the adjacent members of such a sequence are next to one another.

Putting everything together, then, a necessary condition on its being possible to get from any proper subregion of R to any other proper subregion of R without leaving R and without "jumping" from any region to any other region is that there be a sequence of proper subregions of R whose first member is the first of these proper subregions of R, whose last member is the second of these proper subregions of R, and whose adjacent members are next to one another. Furthermore, given the supposition that it is possible for an object to get from any region to any other region if they are joined by a sequence of regions the adjacent members of which are next to one another, this is also a sufficient condition. Thus, on (RF1), a region of space R is continuous if and only if for any two distinct proper subregions x and y of R, there is a sequence of proper subregions of R whose first member is x, whose last member is y, and whose adjacent members are next to one another. Since this is equivalent to the analysis of the property of being continuous_2 given in (PA2), then, it is no surprise that (RF1) can be employed as a rough formulation of the intuitive notion that picks out that property if (PA2) is true.

So (PA2) can be used to explain the fact that (RF1) and (RF2) can be employed as rough formulations of the intuitive notion of what it is for a region of space to be continuous. But it has other attractive features as well. In particular, (PA2) seems to be consistent with the possibility that there is a region of space that is continuous$_2$ but is not a pointy region-fusion. This is important since the arguments presented in §3 for the conclusion that the modified Cartwrightian analysis of *being continuous* does not adequately capture the intuitive notion of what it is for a region of space to be continuous relied on the fact that the modified Cartwrightian analysis is not consistent with such a possibility whereas the intuitive notion is. Thus, if (PA2) were not consistent with such a possibility, similar arguments could be used to show that (PA2) does not adequately capture the intuitive notion of what it is for a region of space to be continuous.

In this connection, consider two 10’ x 10’ square-shaped tiles that “butt up” against one another and whose region-fusion is a 10’ x 20’ rectangular region of space. In §3, I noted that the region-fusion of these tiles seems to satisfy the intuitive notion of what it is for a region of space to be continuous. But it also appears that, given (PA2), this region-fusion is continuous$_2$. For it is natural to read “butt up” as ‘are next to one another’, and given this reading, the region-fusion of these tiles satisfies the analysans of (PA2). The region-fusion of the two tiles, after all, has as its only two proper subregions the two 10’ x 10’ square-shaped tiles that it region-fuses and, given the reading of “butt up” suggested, for either of these tiles there is a two-membered sequence whose first member is that tile and whose second member is the other tile in which each member is next to the member that immediately succeeds it in the sequence, if any.

Thus, (PA2)'s results concerning which region-fusions of tiles are continuous$_2$ appears, at least on preliminary inspection, to be correct. (PA2) also appears to give correct results concerning the possibility of continuous$_2$ gunky regions of space. For there seems to be no bar to two gunky regions of space being next to one another, nor does there seem to be any bar to any two proper subregions of a gunky region of space being connected to one another by a sequence of proper subregions of that region any two adjacent members of which are next to one another. Thus, (PA2) has the consequence that gunky regions of space can be continuous$_2$, which seems to be correct.

Despite these attractive features of (PA2), its consequences concerning continuous$_2$ regions of space in a space with the Gritty Structure might give one pause. For space having the Gritty Structure entails that there are two distinct points that are next to one another.[23] Thus, if space has the Gritty Structure, then there is a region of space that region-fuses two distinct points that are next to one another. Given (PA2), such a region of space is continuous$_2$. One might think, however, that this is the wrong result, since although there is no region of space between any two points in a space with the Gritty Structure, they are at a non-zero distance from one another. More generally, one might think that even if any two proper subregions of some region of space are connected to one another via a sequence of that region's

[23] Strictly speaking, it only entails this on the assumption that there are at least two points.

proper subregions whose adjacent members are next to one another, that region is not guaranteed to be continuous$_2$ unless the adjacent members of the series are also at zero distance from one another.

Suppose that this objection is successful. Then we may formulate the following analysis of continuity$_2$:

Proposed Analysis 3 (PA3): x *is continuous$_2$* $=_{df}$ (i) x is a region of space and (ii) for any proper subregions y and z of x, if y is distinct from z, then there is a sequence S of proper subregions of x whose first member is y and whose last member is z such that for every member w of S, if there is a member of S that immediately succeeds w in S, then w is next to and at a distance of zero from the member of S that immediately succeeds w in S.

Furthermore, (PA3) has the same attractive features as (PA2). And assuming that the objection to (PA2) is successful, it also has the advantage of avoiding that objection.

But is the objection successful? My inclination is to say that it is unclear, and perhaps even indeterminate, whether the intuitive notion of what it is for a region of space to be continuous picks out a property that is best analyzed as in (PA2) or a property that is best analyzed as in (PA3). In particular, I'm inclined to say in forming that intuitive notion, we formed a notion of a region that requires no "jumping" to get through and contains no gaps, but we failed to be explicit concerning whether the "jumping" in question was jumping between two regions that have a region between them or jumping between two regions that are at a distance from one another and we failed to be explicit concerning whether the gaps in question were what we might call 'regional gaps' or were instead gaps of distance.

Officially, then, I think that we should make our intuitive notion determinate by distinguishing between two notions, one of which picks out a property that is correctly analyzed in (PA2) and the other of which picks out a property that is correctly analyzed in (PA3). We may call the former property 'the property of being continuous$_{2.1}$' or 'continuity$_{2.1}$', and we may call the latter 'the property of being continuous$_{2.2}$' or 'continuity$_{2.2}$'. In the remainder of this dissertation, however, I will continue to use 'the property of being continuous$_{2}$' and 'continuity$_{2}$' to indeterminately refer to the properties of being continuous$_{2.1}$ and being continuous$_{2.2}$.

Conclusion

In this chapter, I argued that we should not accept the Cartwrightian analysis of the property of being continuous because it presupposes that regions of space are sets of points in space and, as I argued in Chapter 1, we should be skeptical of that claim. But the Cartwrightian analysis can, I claimed, be modified so as not to presuppose this. The modified Cartwrightian analysis, however, does not adequately capture a particular intuitive notion of what it is for region of space to be continuous, as I showed in §3. Thus we should distinguish between the property of being continuous$_{1}$, which is correctly analyzed by the modified Cartwrightian analysis, and the property picked out by that intuitive notion, which I attempted to analyze in §4. There, however, it was argued that it is unclear whether the property picked out by the intuitive notion is continuity$_{2.1}$, which is correctly analyzed in (PA2), or continuity$_{2.2}$, which is correctly analyzed in (PA3). It was hypothesized that this lack of clarity may be due to indeterminacy in what property is picked out by the intuitive notion. Regardless of the reason, however, it is important to recognize the difference between continuity$_1$, continuity$_{2.1}$, and continuity$_{2.2}$. Each is an interesting structural property that regions of space can have and, depending on what sorts of structure space could have, they may very well diverge in certain cases.

Chapter 3: Open and Closed Regions

Introduction

In this chapter, I will discuss the distinction between open and closed regions of space. I will argue that the traditional account of this distinction, formulated by Richard Cartwright (1987), should be rejected. The chapter will proceed as follows. In §1, I present Cartwright's analyses of the properties of being open and being closed; then I note that, like Cartwright's analysis of the property of being continuous, these analyses presuppose that regions of space are sets of points in space; and finally I show how they may be modified so as not to presuppose this. Next, in §2, I argue for the rejection of the modified versions of Cartwright's analyses. Finally, in §3, I present new analyses of the properties of being open and being closed. I conclude that these analyses are superior to Cartwright's.

1. Cartwright's Analyses

Cartwright analyzes openness and closedness in terms of the notion of a boundary point of a region.[24] He says: 'Now, a region… is said to be *open* just in case none of its boundary points is a member of it and *closed* just in case all of its

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[24] Cartwright's analysis of *being a boundary point of* was presented in Chapter 2, so I do not reproduce it in the main text. However, I do reproduce it here, as well as his analyses of those properties in terms of which he analyzes it:

> x *is a boundary point of* y $=_{df}$ (i) x is a point in space, (ii) y is a set of points in space, and (iii) every open sphere about x has a member of y as a member and a member of the complement of y as a member.
>
> x *is an open sphere about* y $=_{df}$ (i) y is a point in space and (ii) there is a distance z such that for all w, w is a member of x if and only if w is a point in space whose distance from y is less than z.
>
> x *is the complement of* y $=_{df}$ (i) y is a set of points in space and (ii) for all z, z is a member of x if and only if z is a point in space that is not a member of y.

boundary points are members of it' (p. 172). These analyses can be expressed more formally as follows:

> x *is open* $=_{df}$ (i) x is a set of points in space and (ii) no boundary point of x is a member of x.
>
> x *is closed* $=_{df}$ (i) x is a set of points in space and (ii) every boundary point of x is a member of x.

Cartwright takes *being open* and *being closed* to be properties of regions of space. However, in order for something to satisfy his analyses of those properties, it must be a set of points. Thus, those analyses presuppose that regions of space are sets of points. But, as I argued in Chapter 1, we should be skeptical of the claim that regions are sets of points. We should thus also be skeptical of his analyses of being open and being closed. It would be better to have analyses of these properties that do not presuppose that regions are sets of points.

Luckily, it is easy to modify Cartwright's analyses to avoid this presupposition. This can be done as follows:

> x *is open* $=_{df}$ (i) x is a pointy region-fusion[25] and (ii) no boundary point of x is a subregion of x.
>
> x *is closed* $=_{df}$ (i) x is a pointy region-fusion and (ii) every boundary point of x is a subregion of x.[26]

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[25] I reproduce the following definition from Chapter 2:

> x *is a pointy region-fusion* $=_{df}$ there are some points in space xxs such that x region-fuses xxs.

[26] Note that these modified versions of Cartwright's analyses still analyze *being open* and *being closed* in terms of the notion of a boundary point of a region. As should be clear from

As I argue in the next section, however, these modified versions of Cartwright's analyses, though better than the originals, should be rejected.

2. Against the Modified Analyses

In this section, I argue that the modified versions of Cartwright's analyses should be rejected. I first argue, in §§2.1 and 2.2, that those analyses yield somewhat unnatural classifications of regions of space. Then, in §2.3, I argue that they make the distinction between open and closed regions ill-suited for performing theoretical work to which some philosophers have put it. Finally, in §2.4, I argue that if the modified versions of Cartwright's analyses have these consequences, then they should be rejected.

2.1. Intrinsicality and Natural Classifications

Notice the following consequence of the modified versions of Cartwright's analyses. Given those analyses, any pointy region-fusion that doesn't have any boundary points is both open and closed. To see this, let R be an arbitrary pointy region-fusion that doesn't have any boundary points. Since R doesn't have any boundary points, it is vacuously true that no boundary point of R is a subregion of R. Thus, R is a pointy region-fusion and no boundary point of R is a subregion of R. So,

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footnote 24, however, Cartwright's analysis of *being a boundary point of* will not do, since his analysis of that relation reintroduces the presupposition that regions of space are sets of points. Rather, I suggest the following analyses, also presented in Chapter 2:

> x *is a boundary point of* y $=_{df}$ (i) x is a point in space, (ii) y is a pointy region-fusion, and (iii) every open sphere about x region-overlaps both y and the complement of y.
>
> x *is an open sphere about* y $=_{df}$ (i) y is a point in space and (ii) there are some xxs and a distance z such that for all w, w is one of xxs if and only if w is a point in space whose distance from y is less than z, and x region-fuses xxs.
>
> x *is the complement of* y $=_{df}$ (i) y is a pointy region-fusion and (ii) there are some xxs such that for all z, z is one of xxs if and only if z is a point in space that does not region-overlap y, and x region-fuses xxs.

given the modified version of Cartwright's analysis of *being open*, R is open. But since R doesn't have any boundary points, it is also vacuously true that every boundary point of R is a subregion of R. Thus, R is a pointy region-fusion and every boundary point of R is a subregion of R. So, given the modified version of Cartwright's analysis of *being closed*, R is closed. Therefore, given the modified versions of Cartwright's analyses, R is both open and closed. But R was an arbitrary pointy region-fusion that doesn't have any boundary points. Therefore, given the modified versions of Cartwright's analyses, any pointy region-fusion that doesn't have any boundary points is both open and closed.

That any pointy region-fusion that doesn't have any boundary points is both open and closed is a somewhat odd consequence of the modified versions of Cartwright's definitions. Some, however, might claim that it is unobjectionable. After all, why expect something different? Perhaps that's just how *being open* and *being closed* work.

I am sympathetic to this response. The mere fact that the modified versions of Cartwright's analyses have this consequence does not seem to me to be an objectionable feature of those analyses. However, as I will argue, the fact that they have this consequence can be used to show that the modified versions of Cartwright's analyses yield somewhat unnatural classifications of regions of space, which *is* an objectionable feature of those analyses.

To see that the modified versions of Cartwright's analyses yield somewhat unnatural classifications of regions of space, assume that space has the Standard

Structure[27] and let p be a point exactly two feet from the tip of my nose. Next, consider two regions of space, R1 and R2. R1 region-fuses those points whose distance from p is less than one foot. R2, on the other hand, region-fuses those points whose distance from p is less than or equal to one foot. Then, according to the modified versions of Cartwright's analyses, R1 is open but not closed and R2 is closed but not open. R1 is open but not closed because there are points that are exactly one foot from p, all and only those points are boundary points of R1 (since every open sphere about any of those points region-overlaps both R1 and the complement of R1 and no open sphere about any other point does so), and none of those points is a subregion of R1. R2, though, is closed but not open: there are points that are exactly one foot from p, all and only those points are boundary points of R2 (since every open sphere about any of those points region-overlaps both R2 and the complement of R2 and no open sphere about any other point does so), and each of those points is a subregion of R2.

So far, so good. The modified versions of Cartwright's analyses have the consequence that R1 is open but not closed and R2 is closed but not open. But now

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[27] As discussed in Chapter 1, necessarily, space has the Standard Structure if and only if each of the following three theses is true:

Points (P): There are points.

Non-Pointy Regions (NPR): For any region of space x, there are points yys such that x is a region-fusion of yys.

Betweenness (B): For any points x and y such that x and y are distinct, there is a point z such that z is between x and y.

I should note that the issue of whether space in fact has the Standard Structure is, strictly speaking, irrelevant to the argument I give in this section, which could be formulated entirely in terms of possible but non-actual worlds in which space has the Standard Structure. However, assuming that space has the Standard Structure has the advantages of making the argument more vivid and of simplifying the presentation of the argument, which is why I make that assumption here.

consider the world, W1, that results from simply “deleting” from the actual world all and only those points whose distance from p is greater than or equal to one foot. It would seem that such a world is possible. In addition, consider the world, W2, that results from simply “deleting” from the actual world all and only those points whose distance from p is greater than one foot. Again, it would seem that such a world is possible. However, R1 doesn’t have any boundary points in W1 and R2 doesn’t have any boundary points in W2. Thus, since the modified versions of Cartwright’s analyses have the consequence that any region of space that doesn’t have any boundary points is both open and closed, those analyses have the consequence that R1 is both open and closed in W1 and that R2 is both open and closed in W2.

The modified versions of Cartwright’s analyses thus classify R1 in the actual world and R1 in W1 together, since they are both open according to those analyses; classify R2 in the actual world and R2 in W2 together, since they are both closed according to those analyses; classify R1 in the actual world and R2 in W2 together, since they are both open according those analyses; and classify R2 in the actual world and R1 in W1 together, since they are both closed according to those analyses. Whereas the former two classifications seem quite natural to me, however, the latter two seem somewhat unnatural.

Suppose R1 in the actual world, R2 in the actual world, R1 in W1, and R2 in W2 were described to you and you were asked to classify them into pairs in the most natural way. Assuming that you were cooperative, I suspect that you would classify R1 in the actual world and R1 in W1 together and that you would classify R2 in the actual world and R2 in W2. However, I suspect that you wouldn't classify R1 in the actual world and R2 in W2 together and that you wouldn't classify R2 in the actual world and R1 in W1 together. This suggests to me that the latter classifications two are somewhat unnatural. But the modified versions of Cartwright's analyses yield those classifications. So, the modified versions of Cartwright's analyses yield somewhat unnatural classifications.

I think we can bolster the case for the conclusion that the modified versions of Cartwright's analyses yield somewhat unnatural classifications. Consider R1 in the actual world and R1 in W1. R1 in the actual world and R1 in W1 have the very same intrinsic features. After all, since W1 was simply the result of "deleting" from the actual world those regions that do not overlap R1 in the actual world, so R1 in the actual world and R1 in W1 differ only in their extrinsic features, features that depend on how things external to them are. Similar remarks apply to R2 in the actual world and R2 in W2: R2 in the actual world and R2 in W2 have the very same intrinsic features.

On the other hand, R1 in the actual world and R2 in W2 do not have the same very same intrinsic features, nor do R2 in the actual world and R1 in W1. Notice that every point that is a subregion of R1 in the actual world is between two other points that are subregions of R1 in the actual world, but not every point that is a subregion of R2 in the actual world is between two other points that are subregions of R2 in the actual world. Thus, R1 in the actual world has, while R2 in the actual world lacks, the following intrinsic feature: *being such that every point that is a subregion of it is* *between two other points that are subregions of it*. Thus, R1 in the actual world and R2 in the actual world do not have the very same intrinsic features. So, since R2 in the actual world and R2 in W2 have the very same intrinsic features but R1 in the actual world and R2 in the actual world do not, R1 in the actual world and R2 in the actual world do not have the very same intrinsic features. Similarly, since R1 in the actual world and R1 in W1 have the very same intrinsic features but R1 in the actual world and R2 in the actual world do not, R2 in the actual world and R1 in W1 do not have the very same intrinsic features.

Now the more similar two objects are with respect to their intrinsic features, the more natural it is to classify those two objects together. Thus, since R1 in the actual world and R1 in W1 have the very same intrinsic features and R2 in the actual world and R2 in W2 have the very same intrinsic features, it is very natural to classify R1 in the actual world and R1 in W1 together and to classify R2 in the actual world and R2 in W2 together. On the other hand, since R1 in the actual world and R2 in W2 do not have the very same intrinsic features and R2 in the actual world and R1 in W1 do not have the very same intrinsic features, it is somewhat unnatural to classify R1 in the actual world and R2 in W2 together and it is somewhat unnatural to classify R2 in the actual world and R1 in W1 together. But the modified versions of Cartwright's analyses yield the latter classifications. Therefore, those analyses yield somewhat unnatural classifications.[28]

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[28] One potential response to my arguments in this section is to deny that W1 and W2 are possible worlds. This response will be attractive, for instance, to structuralists concerning space. Structuralists concerning space hold that for any regions of space x and y and distance

In this section, I offered two arguments for the claim that the modified versions of Cartwright's analyses yield somewhat unnatural classifications of certain regions of space. In the next section, I will consider other regions of space concerning which the modified versions of Cartwright's analyses yield somewhat unnatural classifications.

2.2. Subdimensional Regions and Natural Distinctions

Let p* and p** be the points at the center of my right and left pupils, respectively, and let Plane be the plane determined by p, p*, and p**.[29] Next, consider R3, the region-fusion of those points that lie on Plane whose distance from p is less than one foot, and R4, the region-fusion of those points that lie on Plane whose distance from p is less than or equal to one foot.

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d such that the distance between x and y is d, it is necessary that if x exists, then the distance between x and y is d. (See, for instance, Esfeld and Lam 2008 and Ladyman and Ross 2007: pp. 141-5.) If this view is correct, W1 and W2 are impossible because although p and some point that region-overlaps neither R1 nor R2 stand in a distance relation in the actual world and p exists in both W1 and W2, p and that point stand in that distance relation in neither W1 nor W2.

There are a few problems with this response. First, even one who is a structuralist concerning space should want analyses of *being open* and *being closed* that are acceptable even to non-structuralists. Second, my argument could be reformulated in terms of worlds W1*, which is exactly like W1 except that it contains an intrinsic duplicate of R1 rather than R1 itself, and W2*, which is exactly like W2 except that it contains an intrinsic duplicate of R2 rather than R2 itself. This reformulating version of the argument would, I think, be no less convincing as the original, and structuralism concerning space provides no reason to deny that W1* and W2* are possible worlds.

[29] As above, 'p' here (and in the remainder of this chapter) refers to a point exactly two feet from the tip of my nose. (I assume that p, p*, and p** do not all lie on the same line, since otherwise they do not determine a plane. Be sure to choose p (from among all the points exactly two feet from my nose) carefully!) Furthermore, as in the previous section, I am assuming here that space has the Standard Structure. Just as the argument presented in that section could be formulated without that assumption, however, so could the argument presented here. See footnote 27 above.

The modified versions of Cartwright’s analyses classify R2 in the actual world with R4. We saw in the last section that R2 in the actual world is closed according to those analyses. The same is true of R4. To see this, notice first that R4 has boundary points. There are points that lie on Plane whose distance from p is less than one foot and each of these points is such that every open sphere about it region-overlaps both R4 and the complement of R4. There are also points that lie on Plane whose distance from p is exactly one foot and each of these points is also such that every open sphere about it region-overlaps both R4 and the complement of R4. (The main difference between the former points and the latter points is that every open sphere about one of the latter points region-overlaps points that lie on Plane that are not subregions of R4, whereas not every open sphere about one of the former points region-overlaps such points.) Thus, the former points and the latter points are all boundary points of R4. Furthermore, they are the only boundary points of R4. So, since R4 has boundary points and every boundary point of R4 is a subregion of R4, R4 is closed according to the modified versions of Cartwright’s analyses. Therefore, R2 in the actual world and R4 are classified together by those analyses, since both are closed according to those analyses.

Contrast R4 with R3. Whereas the modified versions of Cartwright’s analyses classify R2 in the actual world with R4, those analyses do not classify R1 in the actual world with R3. We saw in the last section that R1 is open but not closed according to the modified versions of Cartwright’s analyses. However, according to those analyses, R3 is neither open nor closed. To see this, notice that there are points that lie on Plane whose distance from p is less than one foot and each of these points is such that every open sphere about it region-overlaps both R3 and the complement of R3. There are also points that lie on Plane whose distance from p is exactly one foot and each of these points is also such that every open sphere about it region-overlaps both R3 and the complement of R3. (Again, the main difference between the former points and the latter points is that every open sphere about one of the latter points region-overlaps points that lie on Plane that are not subregions of R4, whereas not every open sphere about one of the former points region-overlaps such points.) Thus, the former points and the latter points are all boundary points of R3. But whereas the former points are subregions of R3, the latter are not. So, since R3 has boundary points some of which are and some of which are not subregions of R3, R3 is neither open nor closed according to the modified versions of Cartwright's analyses. Therefore, R1 in the actual world and R3 are not classified together by those analyses, since it is neither the case that both are open according to those analyses nor is it the case that both are closed according to those analyses.

I think that these classifications are (relatively) unnatural. Suppose R1 in the actual world, R2 in the actual world, R3, and R4 were described to you. Suppose further that you were told to classify R2 in the actual world with R4. Suppose finally you were asked to classify the remaining pairs together in the most natural way consistent with classifying R2 in the actual world with R4. Assuming that you were cooperative, I suspect that you would classify R1 in the actual world and R3 together. This suggests to me that either classifying R2 in the actual world with R4 is somewhat unnatural, with the result that your instructions required you to somewhat unnaturally classify R1 in the actual world and R3 together, or that failing to classify R1 in the actual world with R4 is somewhat unnatural. Either way, though, the modified versions of Cartwright's analyses yield somewhat unnatural classifications.

There are a couple of ways to bolster the case that the modified versions of Cartwright's analyses yield unnatural classifications of these regions. As stated in §2.1, the more similar two objects are with respect to their intrinsic features, the more natural it is to classify those two objects together. Now R1 in the actual world and R3 certainly differ with respect to their intrinsic features. However, the differences between the intrinsic features of R1 in the actual world and R3 are mirrored by the differences between the intrinsic features of R2 in the actual world and R4. Furthermore, the similarities between the intrinsic features of R2 in the actual world and R4 are mirrored by the similarities between the intrinsic features of R1 in the actual world and R3. For example, R2 in the actual world and R4 both lack the intrinsic feature *being such that every point that is a subregion of it is between two other points that are subregions of it*, whereas both R1 in the actual world and R3 have it. Thus, to the degree that it is natural to classify R2 in the actual world and R4 together, it is also natural to classify R1 in the actual world and R3 together. So, it is either somewhat unnatural to classify R2 in the actual world and R4 together or it is somewhat unnatural to fail to classify R1 in the actual world and R3 together. Either way, though, the modified versions of Cartwright's analyses yield somewhat unnatural classifications, since they classify R2 in the actual world and R4 together but do not classify R1 in the actual world and R3 together.

In addition, consider the world, W3, that results from simply “deleting” from the actual world all and only those points that do not lie on Plane. According to the modified versions of Cartwright’s analyses, R3 in W3 is like R1 in the actual world: each is open but not closed. R3 is open in W3 because in W3, the only points that are boundary points of R3 are those points that lie on Plane whose distance from p is exactly one foot and none of those points is a boundary point of R3. Thus, the modified versions of Cartwright’s analyses classify R1 in the actual world and R3 in W3 together.

Now R3 in the actual world and R3 in W3 have the very same intrinsic features. Thus, to the degree that it is natural to classify R1 in the actual world and R3 in W3 together, it is also natural to classify R1 in the actual world and R3 in the actual world together. (Again, I am relying on the principle that the more similar two objects are with respect to their intrinsic features, the more natural it is to classify those two objects together.) So, it is either somewhat unnatural to classify R1 in the actual world and R3 in W3 together or it is somewhat unnatural to fail to classify R1 in the actual world and R3 in the actual world together. Either way, though, the modified versions of Cartwright’s analyses yield somewhat unnatural classifications, since they classify R1 in the actual world and R3 in W3 together but do not classify R1 in the actual world and R3 in the actual world together.

2.3. Receptacles and the Open/Closed Distinction

In §§2.1 and 2.2, I argued that the modified versions of Cartwright's analyses yield somewhat unnatural classifications of regions of space. This may not, however, indicate that those analyses should be rejected. Compare the classifications of regions of space yielded by the modified versions of Cartwright's analyses with classifications of people based on income. Although it is quite unnatural to classify a pair of people whose income differs by one dollar together while failing to classify another pair of people whose income also differs by one dollar together, let alone to classify people based on income at all, it is quite useful to do so for tax purposes. Similarly, then, perhaps the classifications of regions of space yielded by the modified versions of Cartwright's analyses, while somewhat unnatural, are useful. In particular, perhaps the distinction between open and closed regions is well-suited for certain theoretical work.[30]

I argue here, though, that the modified versions of Cartwright's analyses actually make that distinction ill-suited for certain theoretical work to which some philosophers have put it. In particular, some philosophers have employed the distinction between open and closed regions when formulating their views concerning which regions are receptacles. I first explain what a receptacle is and formulate a question about receptacles. I then present some views that have been offered in response to this question that employ the distinction between open and closed

[30] Whether the distinction between open and closed regions can perform theoretical work is, of course, the standard for whether it is useful, since it seems quite unlikely that it can perform any important practical work (unlike distinction between people who fall within different income brackets).

regions. Finally, I argue that given the modified versions of Cartwright's analyses, these views are problematic.

A receptacle is simply a region of space that could be occupied by a material object. More formally:

> x *is a receptacle* $=_{df}$ (i) x is a region of space and (ii) possibly, x is occupied by a material object.

Given this understanding of what a receptacle is, one might wonder which regions of space are receptacles. More precisely, one might wonder: What are the properties, pps, such that (i) necessarily, a region of space is a receptacle if and only if it has one of the pps and (ii) necessarily, for any receptacle, there is a property that is one of the pps such that the receptacle in question is a receptacle in virtue of having that property? Call this question 'the Receptacle Question' or 'RQ'.

Of course, the Receptacle Question need not have a correct answer.[31] Perhaps the fact that a region of space is a receptacle is a brute fact about that region.[32] However, others think that RQ has a correct answer and have endorsed particular answers to it. Two answers that are often discussed on the literature on RQ appeal to the distinction between open and closed regions.[33] These are:

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[31] Some have held that questions similar to RQ do not have answers. For instance, Markosian (1998b) maintains that the Special Composition Question does not have an answer and Kris McDaniel (2007) maintains that the Simple Question does not.

[32] A brute fact is a fact that does not obtain in virtue of another fact.

[33] See Hud Hudson (2002) and Gabriel Uzquiano (2006) for a consideration of views similar to (OV) and (CV). Cartwright (1975) endorses an answer to RQ according to which being an open domain is the relevant property. The arguments I give for the conclusion that (OV) and (CV) are problematic given the modified versions of Cartwright's analyses apply to this view as well.

The Open View (OV):

(A) Necessarily, a region of space is a receptacle if and only if it is open.

(B) Necessarily, every receptacle is a receptacle in virtue of being open.

The Closed View (CV):

(A) Necessarily, a region of space is a receptacle if and only if it is closed.

(B) Necessarily, every receptacle is a receptacle in virtue of being closed.

In the remainder of this section, I will argue that given the modified versions of Cartwright's analyses, (OV) and (CV) are problematic and thus that those analyses make the distinction between open and closed regions ill-suited for certain theoretical work to which some philosophers have put it.

(OV) and (CV) are each the conjunction of two theses. For the sake of clarity, I will refer to the theses that (OV) is the conjunction of as '(OV-A)' and '(OV-B)' and to the theses that (CV) is the conjunction of as '(CV-A)' and '(CV-B)'.[34] I will first argue that (OV-B) and (CV-B) are problematic given the modified versions of Cartwright's analyses, after which I will argue that (OV-A) and (CV-A) are as well.

(OV-B) and (CV-B) are problematic given the modified versions of Cartwright's analyses because given those analyses, *being open* and *being closed* are not intrinsic features of regions of space. To see that *being open* is not an intrinsic feature of regions of space given the modified versions of Cartwright's analyses, begin by assuming that the modified versions of Cartwright's analyses are true. Then whether a pointy region-fusion is open depends on whether none of that region's

[34] It should, I hope, be clear that I intend '(OV-A)' to refer to thesis (A) of (OV) and '(OV-B)' to refer to thesis (B) of (OV), and similarly for '(CV-A)' and '(CV-B)'.

boundary points is a subregion of it: if none of its boundary points is a subregion of it, then it is open; otherwise, it is not. But whether none of a pointy region-fusion's boundary points is a subregion of it depends on whether it has boundary points: if it does not have boundary points, then none of its boundary points are subregions of it; on the other hand, if it does have boundary points, then whether none of its boundary points is a subregion of it depends on whether those points are subregions of it. However, whether a pointy region-fusion has boundary points depends on whether it has a complement: if it does, then it has boundary points; otherwise, it does not. Finally, whether a pointy region-fusion has a complement depends on whether there are points that it does not region-overlap: if there are, then it has a complement; otherwise, it does not. Thus, whether a pointy region-fusion is open depends on whether there are points that it region-overlaps. But if that is so, then whether a pointy region-fusion is open depends on facts concerning how things external to it are. Therefore, *being open* is not an intrinsic feature of regions of space given the modified versions of Cartwright's analyses, since, given those analyses, whether a region of space is open depends on facts concerning how things external to it are. And a similar argument shows, mutatis mutandis, that *being closed* is not an intrinsic feature of regions of space given the modified versions of Cartwright's analyses.

But if *being open* and *being closed* are not intrinsic features of regions of space, then (OV-B) and (CV-B) are problematic. Consider (OV-B). According to (OV-B), a receptacle is a receptacle in virtue of being open. Thus, given (OV-B), if *being open* is not an intrinsic feature of regions of space, a receptacle is not a receptacle (solely) in virtue of its intrinsic features. Rather, it is a receptacle (partially) in virtue of how things external to it are. It is difficult to believe, however, that a region of space is a receptacle—that a region of space is possibly occupied—(partially) in virtue of how things external to it are. It is difficult to believe that how things external to a region are is at all relevant to its being possibly occupied.[35] At the very least, then, if *being open* is not an intrinsic feature of regions of space, it is a cost of (OV-B) that it requires that how things external to a region are *is* relevant to that region's being possibly occupied. It is this cost that makes (OV-B) problematic if *being open* and *being closed* are not intrinsic features of regions of space. And here, of course, a similar argument shows, mutatis mutandis, that (CV-B) is problematic if *being open* and *being closed* are not intrinsic features of regions of space.

Putting together the conclusions of the preceding two paragraphs, *being open* and *being closed* are not intrinsic features of regions of space given the modified versions of Cartwright's analyses and if *being open* and *being closed* are not intrinsic features of regions of space, then (OV-B) and (CV-B) are problematic. Therefore, (OV-B) and (CV-B) are problematic given the modified versions of Cartwright's analyses.

I will now argue that (OV-A) and (CV-A) are problematic given the modified versions of Cartwright's analyses. To see that this is so, consider the region of space

[35] Imagine, for instance, a proponent of the Open View describing to you two intrinsic duplicates of R2, one of which (like R2 in the actual world) is embedded in a larger space while the other (like R2 in W2) is not. Next, suppose that proponent told you that the first region isn't possibly occupied by a material object but that the second is, simply because the first is embedded in a larger space but the second is not. At least if you're like me, you'll find such a claim rather implausible.

R1 introduced in §2.1. (Remember, R1 region-fuses those points whose distance from p is less than one foot, where p is a point exactly two feet from the tip of my nose.) Given the modified versions of Cartwright's analyses, R1 is open but not closed. However, possibly, R1 is both open and closed given those analyses. For there is a possible world that results from simply "deleting" from the actual world all and only those points whose distance from p is greater than or equal to one foot, which I have called 'W1'. And given the modified versions of Cartwright's analyses, R1 is both open and closed in W1. Thus, given the modified versions of Cartwright's analyses, R1 is open but not closed and possibly, R1 is both open and closed.

Now suppose that (CV-A) is true. Then given the modified versions of Cartwright's analyses, R1 is not a receptacle (since R1 is not closed) and possibly, R1 is a receptacle (since possibly, R1 is closed). But if R1 is not a receptacle and possibly, R1 is a receptacle, then it is not possible that R1 is occupied by a material object but it is possible that possibly, R1 is occupied by a material object. Thus, given the modified versions of Cartwright's analyses, if (CV-A) is true, then the characteristic axiom of the modal logic S4 (according to which everything that is possibly possible is possible) must be denied.

However, the characteristic axiom of S4 is very plausible. Furthermore, it is widely (though not universally) accepted.[36] Thus, given the modified versions of Cartwright's analyses, (CV-A) is problematic. For either the characteristic axiom of S4 is correct or, at the very least, whether it should be denied is very controversial.

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[36] Nathan Salmon (1986), for instance, denies the axiom.

But if it is correct, then (CV-A) is false given the modified versions of Cartwright's analyses and falsity is certainly a major problem for a thesis to face. On the other hand, even if the characteristic axiom of S4 is incorrect, whether it should be denied is very controversial. Thus, given the modified versions of Cartwright's analyses, a proponent of (CV-A) must take a stand on a very controversial issue in modal logic. But few would be willing to take such a stand, I think, on the basis of an answer to the Receptacle Question, which is again a problem. Therefore, (CV-A) is problematic given the modified versions of Cartwright's analyses.

A similar argument shows that (OV-A) is problematic given the modified versions of Cartwright's analyses. For consider the region of space R2 introduced in §2.1. (R2, remember, region-fuses those points whose distance from p is less than or equal to one foot.) Given the modified versions of Cartwright's analyses, R2 is closed but not open. However, again given those analyses, possibly R2 is both open and closed, since there is a possible world (which I have called 'W2') that results from simply "deleting" from the actual world all and only those points whose distance from p is greater than one foot and R2 is both open and closed in that world given those analyses. Thus, given the modified versions of Cartwright's analyses, if (OV-A) is true, then it is not possible that R2 is occupied by a material object but it is possible that possibly, R2 is occupied by a material object. Therefore, given the modified versions of Cartwright's analyses, if (OV-A) is true, then the characteristic axiom of S4 must be denied. And just as this shows that (CV-A) is problematic given the modified versions of Cartwright’s analyses, so too it shows that (OV-A) is problematic given the modified versions of Cartwright’s analyses.

In this section, I have argued that (OV) and (CV) are problematic given the modified versions of Cartwright’s analyses. (OV) is problematic given the modified versions of Cartwright’s analyses since it consists of (OV-A) and (OV-B), each of which is problematic given those analyses. Similar remarks apply to (CV), which consists of (CV-A) and (CV-B), each of which is problematic given the modified versions of Cartwright’s analyses. Since the formulation of responses to the Receptacle Question is one area in which philosophers have tried to put the distinction between open and closed regions to theoretical work and these responses are problematic given the modified versions of Cartwright’s analyses, this shows that those analyses make the distinction between open and closed regions ill-suited for certain theoretical work to which philosophers have put it.

2.4. Rejection of the Modified Versions of Cartwright’s Analyses

In §§2.1-2.3, I have argued that the modified versions of Cartwright’s analyses yield somewhat unnatural classifications of regions of space and that they make the distinction between open and closed regions ill-suited for certain theoretical work to which some philosophers have put it. In this section, I argue that given those analyses have these consequences, they should be rejected.

My argument is simply this. There are alternative analyses that deliver verdicts concerning whether a region is open and whether it is closed that are quite similar to the verdicts delivered by the modified versions of Cartwright’s analyses, similar enough that they too can reasonably be taken to be analyses of *being open* and *being closed*. Furthermore, these analyses yield more natural classifications of regions of space and do not make the distinction between open and closed regions ill-suited for use in answering the Receptacle Question. But if there are alternative analyses having these features, then the modified versions of Cartwright's analyses should be rejected. So, the modified versions of Cartwright's analyses should be rejected.

Notice that the claim that there are alternative analyses is, thus far, only a promissory note. I have yet to formulate such analyses. I turn to that task in the next section.

3. Alternative Analyses

In this section, I present my proposed alternative analyses of the properties of being open and being closed and I explain why they are superior to the modified versions of Cartwright's analyses. In §3.1, I present first-pass alternative analyses of *being open* and *being closed* and argue that, in spite of their attractive features, each is subject to counterexamples. Then, in §3.2, I consider how the analyses presented in §3.1 might be modified and I introduce second-pass alternative analyses. These second-pass analyses avoid the counterexamples to the first-pass analyses and are very similar to my proposed analyses. Finally, in §3.3, I argue that these second-pass analyses are unsuccessful as well. I then present my proposed analyses, show that they are not subject to the problems faced by the second-pass analyses, and argue that they are to be preferred to the modified versions of Cartwright's analyses.

3.1. First-Pass Alternative Analyses

Each of the alternative analyses of *being open* and *being closed* that I will consider, including my proposed analyses of those properties, employ the notion of a point being an exterior point of a region:

> x *is an exterior point of* y $=_{df}$ (i) x is a point, (ii) x is a subregion of y, and (iii) there are no points p and p' such that p and p' are subregions of y and x is between p and p'.

I will first discuss a few reasons why employing this notion in alternative analyses of *being open* and *being closed* is promising, then I will introduce first-pass analyses of those properties that employ this notion.[37]

One reason that employing the notion of a point being an exterior point of a region is promising is that analyses that employ this notion have the potential to yield the natural classifications that I argued, in §§2.1 and 2.2, the modified versions of Cartwright's analyses don't yield. For instance, neither R1 in the actual world nor R1 in W1 have an exterior point, whereas both R2 in the actual world and R2 in W2 have an exterior point. Thus, alternative analyses of *being open* and *being closed* that employ the notion of a point being an exterior point of a region have the potential to yield the natural classification of R1 in W1 with R1 in the actual world and not with R2 in the actual world, as well as the natural classification of R2 in W2 with R2 in the

[37] To avoid confusion, I should note that the notion of being an exterior point of a region is *not* meant to perform the same function in the alternative analyses I consider below as the notion of being a boundary point of a region performs in the modified versions of Cartwright's analyses. The reader should distinguish sharply between these two notions and the uses to which they are put.

actual world and not with R1 in the actual world. Similarly, neither R3 in the actual world nor R3 in W3 have an exterior point, so such analyses also have the potential to yield the natural classification of R3 in the actual world with both R1 in the actual world and R3 in W3.

Second, *having an exterior point* and *having no exterior point* are intrinsic features of regions of space. (Whether a region of space has the property *having an exterior point* depends entirely on facts concerning that region, its subregions, and the relations between its subregions, none of which depend on facts concerning how things external to that region are; and similarly for *having no exterior point*.) Now I argued in §2.3 that one reason the Open View and the Closed View are problematic given the modified versions of Cartwright's analyses is that given those analyses, the Open View and the Closed View each have the implausible consequence that a receptacle is not a receptacle (solely) in virtue of its intrinsic features. Since *having an exterior point* and *having no exterior point* are intrinsic features of regions of space, on the other hand, the Open View and the Closed View may not be problematic for this reason given analyses of *being open* and *being closed* that employ the notion of a point being an exterior point of a region.

Finally, in §2.3, I noted another reason that the Open View and the Closed View are problematic given the modified versions of Cartwright's analyses: given the modified versions of Cartwright's analyses, the Open View and the Closed View each conflict with the characteristic axiom of S4 modal logic. The reason that the Open View and the Closed View each conflict with that axiom given those analyses is that given those analyses, a region of space can be possibly open even though it is not open and a region of space can be possibly closed even though it is not closed. However, it would seem that a region of space cannot possibly have an exterior point unless it has an exterior point and that a region of space cannot possibly have no exterior point unless it has no exterior point. Thus, given analyses of being open and being closed that employ the notion of a point being an exterior point of a region, the Open View and the Closed View may not conflict with the characteristic axiom of S4 modal logic and thus may not be problematic for this reason either.

These considerations suggest the following first-pass alternative analyses of *being open* and *being closed*:

First-Pass Analysis of Openness (FAO): x *is open* $=_{df}$ (i) x is a pointy region-fusion and (ii) x does not have an exterior point.

First-Pass Analysis of Closedness (FAC): x *is closed* $=_{df}$ (i) x is a pointy region-fusion and (ii) x has an exterior point.

(FAO) and (FAC) have many attractive features. For one thing, they deliver the same verdicts concerning many regions of space as the modified versions of Cartwright's analyses. For instance, according to (FAO) and (FAC), R1 in the actual world is open but not closed and R2 in the actual world is closed but not open; and the modified versions of Cartwright's analyses deliver the same verdicts. Furthermore, (FAO) and (FAC) yield more natural classifications than the modified versions of Cartwright's analyses: Given (FAO) and (FAC), R1 in the actual world and R1 in W1 are both classified as open but not closed, R2 in the actual world and R2 in W2 are both classified as closed but not open, and R3 in the actual world and R3 in W3 are both classified as open but not closed. Finally, given (FAO) and (FAC), neither the Open View nor the Closed View has the consequence that a receptacle is not a receptacle (solely) in virtue of its intrinsic features, nor does either conflict with the characteristic axiom of S4 modal logic.

Despite these attractive features of (FAO) and (FAC), neither is a satisfactory analysis. I will first explain why (FAO) isn't a satisfactory analysis of *being open* by presenting two counterexamples to it. I will then present two counterexamples to (FAC) as well.

First counterexample to (FAO):[38] Let p1.5 be a point that is exactly a foot-and-a-half from p. Now consider R5, the region-fusion of those points whose distance from p is less than one foot except for those whose distance from p1.5 is also less than one foot. (R5 looks like a doughnut hole with a bite taken out of it. Each of R5's boundary points "within the bite" is a subregion of R5, but none of the rest of its boundary points is a subregion of R5. See Figure 1 below.) R5 does not have an exterior point. (Although each of R5's boundary points "within the bite" is a subregion of R5, none of them is an exterior point of R5 because each of them is between two other points that are subregions of R5. On the other hand, none of the other boundary points of R5 is an exterior point of R5 because none of them is a

[38] Thanks to John Bennett for drawing my attention to regions of space of the sort that figure in the first counterexample to (FAO) and the first counterexample to (FAC) presented below, as well as for suggestions concerning how to analyze being open and being closed in a way that is not subject to these objections. The discussion in §3 has been significantly influenced by his comments.

subregion of R5.) Thus, given (FAO), R5 is open. But R5 isn't open.[39] Therefore, (FAO) isn't a satisfactory analysis of *being open*.

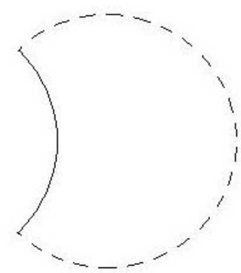

Figure 1: First Counterexample to (FAO)

Second counterexample to (FAO): Let p1 be one of those points whose distance from p is exactly one foot; f_1 be a function such that for every point p*

[39] The reader might wonder why I say that R5 isn't open. The reason is this. In §2.4, I argued that the modified versions of Cartwright's analyses should be rejected because there are alternative analyses of *being open* and *being closed* that deliver verdicts close enough to the verdicts delivered by the modified versions of Cartwright's analyses that they can reasonably be taken to be analyses of the same properties as the modified versions of Cartwright's analyses but that do not yield the somewhat unnatural classifications yielded by the modified versions of Cartwright's analyses and do not make the distinction between open and closed regions ill-suited for use in answering the Receptacle Question. This was basically a promissory note. For my argument against the modified versions of Cartwright's analyses to succeed, I must produce alternative analyses of *being open* and *being closed* with these features. Of particular relevance here is the first feature: I must produce alternative analyses of *being open* and *being closed* that deliver verdicts close enough to the verdicts delivered by the modified versions of Cartwright's analyses that they can reasonably be taken to be analyses of the same properties as the modified versions of Cartwright's analyses. Thus, the fact that alternative analyses of *being open* and *being closed* deliver different verdicts than those delivered by the modified versions of Cartwright's analyses is a reason to reject them *unless* the verdicts delivered by the modified versions of Cartwright's analyses reflect the fact that they yield somewhat unnatural classifications or the fact that they make the distinction between open and closed regions ill-suited for use in answering the Receptacle Question. But the modified versions of Cartwright's analyses deliver the verdict that R5 isn't open and this verdict does not reflect the fact that they yield somewhat unnatural classification or the fact that they make the distinction between open and closed regions ill-suited for use in answering the Receptacle Question. Thus, the fact that (FAO) delivers a different verdict is a reason to reject it. This is what I intend to convey when I say 'R5 isn't open'. Similar remarks apply to (the relevant portions of) the discussions of the second counterexample to (FAO) and the first and second counterexamples to (FAC).

whose distance from p is less than one foot, f_1(p*) is the point exactly one foot from p1 such that p1 is between p* and it; and f_2 be a function such that for every point p* whose distance from p is less than one foot, f_2(p*) is the point exactly two feet from p1 such that p1 is between p* and it. Now consider R6, the region-fusion of the following points: those points whose distance from p is less than one foot, p1, and those points each of which is such that there is a point p* whose distance from p is less than one foot such that it is between f_1(p*) and f_2(p*). (See Figure 2 below.) R6 does not have an exterior point. (Every point whose distance from p is less than one foot is between two other such points; every point such that there is a point p* whose distance from p is less than one foot such that it is that is between f_1(p*) and f_2(p*) is between two other such points; and for each point p* whose distance from p is less than one foot, p1 is between p* and each of those points that is between f_1(p*) and f_2(p*).) Thus, given (FAO), R6 is open. But R6 isn't open. Therefore, (FAO) is not a satisfactory analysis of *being open*.

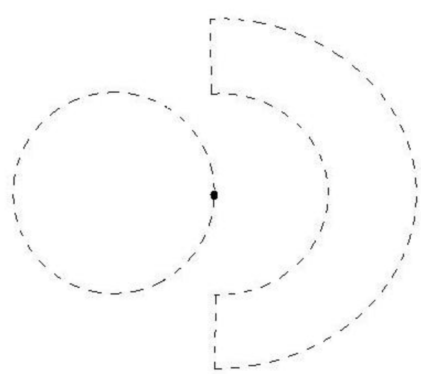

Figure 2: Second Counterexample to (FAO)

First counterexample to (FAC): Let p1.5 be as in the first counterexample to (FAO). Now consider R7, the region-fusion of those points whose distance from p is less than or equal to one foot except for those whose distance from p1.5 is also less than or equal to one foot. (Like R5, R7 looks like a doughnut hole with a bite taken out of it. Unlike R5, none of R7's boundary points "within the bite" is a subregion of R7, but each of the rest of its boundary points is a subregion of R7. See Figure 3 below.) R7 has an exterior point. (None of R7's boundary points "within the bite" is a subregion of R7, and so none of them is an exterior point of R7. On the other hand, each of the other boundary points of R7 is an exterior point of R7 because each of them is a subregion of R7 and none of them is between two other points that are subregions of R7.) Thus, given (FAC), R7 is closed. But R7 isn't closed. Therefore, (FAC) isn't a satisfactory analysis of *being closed*.

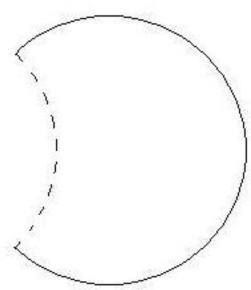

Figure 3: First Counterexample to (FAC)

Second counterexample to (FAC): Let p1, f_1, and f_2 be as in the second counterexample to (FAO); let p2a and p2b be those points exactly one foot from p1 that lie on the tangent to R2[40] that intersects p1; and let p3a and p3b be those points exactly two feet from p1 that lie on the tangent to R2 that intersects p1.[41] Now consider R8, the region-fusion of the following points: those points, except for p1, whose distance from p is less than or equal to one foot; those points each of which is such that there is a point p* whose distance from p is less than one foot such that it is f_1(p*); those points each of which is such that there is a point p* whose distance from p is less than one foot such that it is f_2(p*); those points each of which is such that there is a point p* such that whose distance from p is less than one foot such that it is between f_1(p*) and f_2(p*); p2a; p3a; those points each of which is between p2a and p3a; p2b; p3b; and those points each of which is between p2b and p3b. R8 has an exterior point. (See Figure 4 below.) All and only the following points are exterior points of R8: those points, except for p1, whose distance from p is exactly one foot; those points each of which is such that there is a point p* whose distance from p is less than one foot such that it is f_2(p*); p3a; and p3b.) Thus, given (FAC), R8 is closed. But R8 isn't closed. Therefore, (FAC) is not a satisfactory analysis of *being closed*.[42]

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[40] R2, remember, is the region-fusion of those points whose distance from p is less than or equal to one foot.

[41] I intend p2a and p3a to lie on one side of p1 and p2a and p2b to lie on the other side of p1. In other words, I intend it to be the case that each of the following is the case: p2a is between p3a and p1; p2b is between p3b and p1; and p1 is between p2a and p2b.

[42] There are other serious problems for (FAC) as well. For instance, consider the region-fusion of the following points: p1 and those points whose distance from p is less than one foot. p1 is an exterior point of this region, and it is the only exterior point of this region. Thus, given (FAC), this region is closed. But this region isn't closed. Therefore, (FAC) isn't a satisfactory analysis of *being closed*.

In response to this counterexample, one might propose the following modification of (FAC):

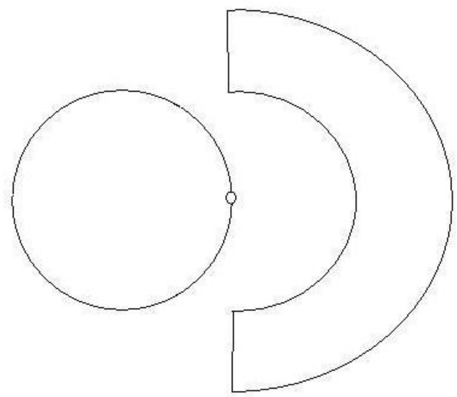

Figure 4: Second Counterexample to (FAC)

These counterexamples show that, despite their attractive features, (FAO) and (FAC) are unsatisfactory analyses of *being open* and *being closed*, respectively. Each of them delivers incorrect verdicts concerning certain regions, so that each is subject to counterexample. In the next section, I discuss how to modify (FAO) to avoid these counterexamples and, on the basis of this discussion, I introduce second-pass alternative analyses of *being open* and *being closed*.

3.2. Second-Pass Alternative Analyses

(FAO) and (FAC) can be modified so as to avoid the counterexamples discussed in the previous section. The result of making these modifications are the

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(FAC*): x is closed $=_{df}$ (i) x is a pointy region-fusion and (ii) every point that is a subregion of x is either an exterior point of x or between two exterior points of x.

(FAC*) isn't subject to the counterexample under discussion because not every point that is a subregion of the region mentioned there—the region-fusion of those points whose distance from p is less than one foot and p1—is either an exterior point of that region or between two exterior points of that region.

However, the first and second counterexamples to (FAC) are also counterexamples to (FAC*) because, like (FAC), (FAC*) delivers the verdict that R7 is closed and the verdict that R8 is closed.

second-pass alternative analyses presented below. Before presenting these second-pass analyses, though, I would like to motivate the modifications they contain.

The first fact to note when considering how to modify (FAO) and (FAC) is that the first counterexample to (FAO) and the first counterexample to (FAC) are structurally very similar, as are the second counterexample to (FAO) and the second counterexample to (FAC). This suggests that the modifications made to (FAO) and the modifications made to (FAC) should be parallel to one another. It also suggests that when considering how to modify (FAO) and (FAC), the first counterexample to (FAO) and the first counterexample to (FAC) should be considered together, as should the second counterexample to (FAO) and the second counterexample to (FAC). I will begin by considering how to modify (FAO) and (FAC) in response to the former two counterexamples. I will then consider how to modify them in response to the latter counterexamples.

The first counterexample to (FAO) concerns R5. (See Figure 1 on p. 79.) R5 is similar to R1 in some respects: each of them is a pointy region-fusion and neither of them has an exterior point. However, R1 is open and R5 is not. What accounts for the fact that they differ in this respect despite their similarities?

Maybe the answer is to be found by looking at their subregions. Let p.5 be the point that is exactly half-a-foot from p and is between p and p1.5. p.5 is a subregion of R5 because it is less than one foot from p but is not less than one foot from p1.5. Now consider the region-fusion, R9, of p.5 and those points that are between p and p.5. R9 is a subregion of both R1 and R5 and R9 has one and only one exterior point:

p.5. However, there are important differences between the relationship between R9 and R1, on the one hand, and the relationship between R9 and R5, on the other.

Perhaps the best way to see these differences is to imagine yourself walking the length of R9 from p to p.5 and then continuing along the same line past p.5. Once you have passed p.5, you are no longer walking within R5 and, so long as you continue along that line, you will never walk within R5 again. On the other hand, once you have passed p.5, you will once again walk within R1 if you continue along the same line. (In fact, in this case, you will continue to walk within R1 immediately after you have passed p.5.) Of course, at some point in your journey along this line, you will never again walk within R1. But it will be true that you will never again walk within R1 (by continuing along the same line) beginning immediately after you have walked the length of a region that does not have an exterior point. This contrasts with R5. In the latter case, it is true that you will never again walk within R5 (by continuing along the same line) beginning immediately after you have walked the length of a region (namely, R9) that has an exterior point.

This indicates an important difference between R1 and R5. Although R1 has (what we might call) “linear subregions” that have exterior points, each of these subregions can be, as it were, “extended” to reach a linear subregion of R1 that does not have an exterior point. On the other hand, R5 has linear subregions that have exterior points as well. But some of these subregions cannot be extended to reach a linear subregion of R5 that does not have an exterior point. Perhaps it is this important difference between R1 and R5 that accounts for the fact that R1 is open but R5 is not.

Based on these considerations, one might propose an analysis of *being open* according to which *being open* is *being a pointy region-fusion such that every linear subregion of that region-fusion that has an exterior point can be extended to reach a linear subregion of that region-fusion that does not have an exterior point*. A different way to think about the proposal is this. The fact that R1 is a pointy region-fusion such that every linear subregion of R1 that has an exterior point can be extended to reach a linear subregion of R1 that does not have an exterior point guarantees that R1 does not have an exterior point.[43] On the other hand, although R5

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[43] To see this, let R be a pointy region-fusion such that every linear subregion of R that has an exterior point can be extended to reach a linear subregion of R that does not have an exterior point. Now suppose, for reductio, that R has an exterior point, p_e. Since p_e is an exterior point of R, p_e is a subregion of R. But if p_e is a subregion of R, either p_e is identical to R or p_e is a proper subregion of R. Suppose first that p_e is identical to R. Then p_e is a linear subregion of R. (This may seem strange, but it follows from the definitions given below. See the next footnote.) But p_e can't be extended to reach a linear subregion of R1 that does not have an exterior point. Contradiction.

Suppose, on the other hand, that p_e is a proper subregion of R. Then there is at least one other point, p*, that is a subregion of R. Now consider R*, the region-fusion of p_e and p*. R* is a linear subregion of R that has an exterior point. (Again, this may seem strange. But it too follows from the definitions given below.) In fact, R* has exactly two exterior points: p_e and p*. Since every linear subregion of R that has an exterior point can be extended to reach a linear subregion of R that does not have an exterior point, R* can be extended to reach a linear subregion of R that does not have an exterior point. However, if R* can be so extended, then there are points that are subregions of R each of which is such that p_e is between it and p*. (Otherwise, either R* can't be extended to reach another linear subregion of R at all or R* can be to reach another linear subregion of R but can only be extended past p*, in which case the result of extending R* will also have p_e as an exterior point.) But if there are points that are subregions of R each of which is such that p_e is between it and p*, then p_e isn't an exterior point of R. Thus, p_e isn't an exterior point of R. Contradiction.

R, then, does not have an exterior point, since such an exterior point of R would either be identical to R or a proper subregion of R and both options results in a contradiction. But R was an arbitrary pointy region-fusion such that every linear subregion of R that has an exterior point can be extended to reach a linear subregion of R that does not have an exterior also does not have an exterior point, the fact that R5 does not have an exterior point is not guaranteed by the corresponding fact about R5. Thus, on the alternative analysis under consideration, the fact that a pointy region-fusion does not have an exterior point is not sufficient for it to be open; that fact must be guaranteed in the right way.

This alternative analysis is promising, since it delivers the correct verdicts that R1 is open and that R5 is not open. However, it has yet to be stated precisely. Instead, it was stated in terms of the unexplained notions of one region being a linear subregion of another and of extending a linear subregion of a region to reach another linear subregion of that region. Although I think that the reflection on R1 and R5 engaged in above provides an intuitive grasp of these notions, this grasp is rather weak. Stating the analysis precisely requires explaining these notions rather than relying on this weak intuitive grasp.

I begin with the notion of one region being a linear subregion of another. I think this notion is most easily grasped in terms of another notion: the notion of a region being a *maximal* linear subregion of another. I offer the following definition of the latter notion:

> x *is a maximal linear subregion of* y $=_{df}$ there are a point z and a point z' such that z and z' are subregions of y and x is the region-fusion of the following points: z, z', those points each of which is both a subregion of y and is between z and z',

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point. Therefore, for all x, if x is a pointy region-fusion such that every linear subregion of x that has an exterior point can be extended to reach a linear subregion of x that does not have an exterior point, then x does not have an exterior point. So, the fact that R1 is a pointy region-fusion such that every linear subregion of R1 that has an exterior point can be extended to reach a linear subregion of R1 that does not have an exterior point guarantees that R1 does not have an exterior point.

those points each of which is both a subregion of y and is such that z is between it and z’, and those points each of which is both a subregion of y and is such that z’ is between it and z.

(Roughly, one region is a maximal linear subregion of another just in case there is a line such that the first “consists” of all and only those points that both lie on that line and are subregions of the second.) With the notion of a region being a maximal linear subregion of another in hand, one can then introduce the following definition:

x *is a linear subregion of* y $=_{df}$ x is a subregion of a maximally linear subregion of y.[44]

(Roughly, one region is a linear subregion of another just in case there is a line such that the first “consists” of some of those points that both lie on that line and are subregions of the second.)

The notion of one region being a linear subregion of another has now been explained. However, stating the alternative analysis under discussion precisely requires yet more work. According to that analysis, a region of space is open only if every linear subregion of that region that has an exterior point can be extended to reach a linear subregion of that region that does not have an exterior point. So, in order to state the analysis precisely, what it is meant by saying of linear subregion of a region that has an exterior point that it *can be extended to reach* a linear subregion of that region that does not have an exterior point must be explained.

[44] A necessarily extensionally equivalent definition of this notion can be given directly, rather than in terms of the notion of a region being a maximal linear subregion of another. However, beginning with the latter notion, I believe, makes the former easier to grasp.

Let L be a linear subregion of R that has an exterior point. What conditions must be met in order for it to be the case that L can be extended to reach a linear subregion of R that does not have an exterior point? It is important when answering this question to notice that if L is a linear subregion of R that has an exterior point, then either L is a point, L is a non-point that has exactly one exterior point, or L has exactly two exterior points. So, when saying what conditions must be met in order for it to be the case that L can be extended to reach a linear subregion of R that does not have an exterior point, each of the following must be taken into account: (i) the conditions that must be met if L is a point, (ii) the conditions that must be met if L is a non-point that has exactly one exterior point, and (iii) the conditions that must be met if L has exactly two exterior points.

The case in which L is a non-point that has exactly one exterior point is the easiest. L is then like R9, which can be extended to reach a linear subregion of R1 that does not have an exterior point. In this case, in order for it to be the case that L can be extended to reach a linear subregion of R that does not have an exterior point, there must be another linear subregion of R, L*, such that L and L* are co-linear, L* doesn't surround L, and the region-fusion of L and L* does not have an exterior point. (Consider R9 and R1 again and suppose that p1, the point whose distance from p is exactly one foot that was mentioned in the second counterexample to (FAO) and the second counterexample to (FAC), is between p and p1.5. Now let R10 be the region-fusion of those points that are between p.5 and p1. R10 is a linear subregion of

R1, R9 and R10 are co-linear, R10 doesn't surround R9, and the region-fusion of R9 and R10 does not have an exterior point.)

Let's be a little more precise here. It should be clear from earlier definitions what it takes for L* to be a linear subregion of R and what it takes for the region-fusion of L and L* not to have an exterior point. But what does it take for L and L* to be co-linear and what does it take for L* not to surround L? Here are some definitions:

> x and y *are co-linear* $=_{df}$ (i) x is a pointy region-fusion, (ii) y is a pointy region-fusion, and (iii) for all distinct z, z', and z'' such that z is a point that is a subregion of x or of y, z' is a point that is a subregion of x or of y, and z'' is a point that is a subregion of x or of y, either z is between z' and z'', z' is between z and z'', or z'' is between z and z'.
>
> x *surrounds* y $=_{df}$ (i) x is a pointy region-fusion, (ii) y is a pointy region-fusion, and (iii) there are a z, z', and z'' such that z is a point that is a subregion of y, each of z' and z'' is a point that is a subregion of x, and z is between z' and z''.

With these definitions in hand, we can say precisely that in the case in which L is a non-point that has exactly one exterior point, L can be extended to reach a subregion of R that does not have an exterior point if and only if there is linear subregion of R, L*, such that L and L* are co-linear, L* doesn't surround L, and the region-fusion of L and L* does not have an exterior point.

The cases in which L is a point and in which L has exactly two exterior points can be treated together. In these cases, L must be extendable *in two directions*. To see what I mean be this, let L* be some other linear subregion of R* such that L and L* are co-linear and L* doesn't surround L. If L is a point or L has exactly two exterior points, then it is not the case that the region-fusion of L and L* does not have an exterior point. Thus in these cases it will not do to say that L can be extended to reach a subregion of R that does not have an exterior point if and only if there is linear subregion of R, L*, such that L and L* are co-linear, L* doesn't surround L, and the region-fusion of L and L* does not have an exterior point, since in these cases there is no such linear subregion of R as L*.

So, if L is a point or L has exactly two exterior points, what conditions must be met in order for it to be the case that L can be extended to reach a subregion of R that does not have an exterior point? The answer is simple. First, there must be two other linear subregions of R, L* and L**. Second, L, L*, and L** must all be co-linear; that is, it must be the case that L and L* are co-linear, L and L** are co-linear, and L* and L** are co-linear. Third, it must be the case that neither L* nor L** surrounds L. Fourth, and finally, it must be the case that the region-fusion of L, L*, and L** does not have an exterior point.

We can now generalize over the cases in which L is point, in which L is a non-point that has exactly one exterior point, and in which L has exactly two exterior points. Where L is a linear subregion of R that has an exterior point, L can be extended to reach a linear subregion of R that does not have an exterior point just in case there are an L* and an L** such that each of L* and L** is a linear subregion of R; L and L*are co-linear, L and L** are co-linear, and L* and L** are co-linear;

neither L* nor L** surrounds L; and the region-fusion of L, L*, and L** does not have an exterior point. (In the case in which L is a non-point that has exactly one exterior point, it just so happens that L* and L** may be the same region.)

Given these remarks, the alternative analysis of *being open* under discussion, which was suggested by considering the differences between R1 and R5, can be formulated more precisely as follows:

> **Not-Quite-Second-Pass Analysis of Openness (NSAO)**: x is open $=_{df}$ (i) x is a pointy region-fusion and (ii) for all y such that y is a linear subregion of x that has an exterior point, there are a z and a z' such that each of z and z' is a linear subregion of x; y and z' are co-linear, y and z'' are co-linear, and z and z' are co-linear; neither z nor z' surrounds y; and the region-fusion of y, z, and z' does not have an exterior point.

This analysis of *being open* is not quite the second-pass analysis of being open that I am building up to in this section since I have yet to take into account the modifications to (FAO) required by the second counterexample to (FAO). But it is an improvement over (FAO), since it is not subject to the first counterexample to (FAO).

I noted above that the first counterexample to (FAO) and the first counterexample to (FAC) are structurally very similar, and that this suggests that the modifications made to (FAO) in response to the former and the modifications made to (FAC) in response to the latter should parallel one another. This suggestion is borne out by considering how to modify (FAC) in response to the first counterexample to (FAC), as I will now show.

Remember that the first counterexample to (FAC) concerns R7, the region-fusion of those points whose distance from p is less than or equal to one foot except for those whose distance from p1.5 is also less than or equal to one foot (where p1.5 is a point whose distance from p is exactly one-and-a-half feet). (See Figure 3 on p. 81.) Now consider R11, the region-fusion of those points that are between p and p.5 (where p.5 is a point exactly half-a-foot from p that is between p and p1.5). R11 is a linear subregion of R7 and R11 is a linear subregion of R2. However, R11 cannot be extended to reach a linear subregion of R7 such that every point that is a subregion of it is either an exterior point of it or between two exterior points of it but R11 can be extended to reach a linear subregion of R1 such that every point that is a subregion of it is either an exterior point of it or between two exterior points of it.

This difference between the relationship between R11 and R2, on the one hand, and R11 and R7, on the other, suggests the following alternative analysis of *being closed*:

> **Not-Quite-Second-Pass Analysis of Closedness (NSAC)**: x *is closed* $=_{df}$ (i) x is a pointy region-fusion and (ii) for all y such that y is a linear subregion of x that does not have an exterior point, there are a z and a z' such that each of z and z' is a linear subregion of x; y and z' are co-linear, y and z'' are co-linear, and z and z' are co-linear; neither z nor z' surrounds y; and every point that is a subregion of

> the region-fusion of y, z, and z’ is either an exterior point of that region-fusion or between two exterior points of that region-fusion.[45]

Again, this is not quite the second-pass analysis of *being closed* that I am building up to in this section because I have yet to take into account the modifications to (FAC) required by the second counterexample to (FAC). But just as (NSAO) is an improvement over (FAO) because it is not subject to the first counterexample to (FAO), so too (NSAC) is an improvement over (FAC) since it is not subject to the first counterexample to (FAC).

I now turn to the second counterexample to (FAO) and the second counterexample to (FAC). As noted above, these counterexamples are structurally similar and so we should expect that the modifications made to (FAO) in response to the former and the modifications made to (FAC) in response to the latter should be parallel to one another. Below we will see that this expectation is borne out: The second counterexample to (FAO) requires yet further modifications to clause (ii) of (FAO), and thus requires modifications to clause (ii) of (NSAO); the second counterexample to (FAC) requires yet further modifications to clause (ii) of (FAC), and thus requires modifications to clause (ii) of (NSAC); and these modifications are parallel to one another.

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[45] (NSAO) requires that the region-fusion of y, z, and z’ does not have an exterior point. (NSAC) requires that every point that is a subregion of the region-fusion of y, z, and z’ is either an exterior point of that region-fusion or between two exterior points of that region-fusion. One might think that these two requirements aren’t parallel. But they are. Loosely speaking, the first requirement is the requirement that the region-fusion of y, z, and z’ isn’t “bounded on either side” by an exterior point of that region-fusion, whereas the second requirement is the requirement that the region-fusion of y, z, and z’ is “bounded on both sides” by an exterior point.

The second counterexample to (FAO) concerns R6. R6 is the region-fusion of the following points: those points whose distance from p is less than one foot, p1, and those points each of which is such that there is a point p* whose distance from p is less than one foot such that it is between f_1(p*) and f_2(p*) (where p1 is a point exactly one foot from p; f_1 is a function such that for every point p* whose distance from p is less than one foot, f_1(p*) is the point exactly one foot from p1 such that p1 is between p* and it; and f_2 is a function such that for every point p* whose distance from p is less than one foot, f_2(p*) is the point exactly two feet from p1 such that p1 is between p* and it). (See Figure 2 on p. 80.) R6 isn't open. However, according to (FAO), R6 is open because R6 is a pointy region-fusion that does not have an exterior point.

More importantly at this point, R6 is open according to (NSAO) as well. Thus, the second counterexample to (FAO) requires yet more modifications to (FAO), modifications beyond those required to avoid the first counterexample to (FAO). But exactly what further modifications are required?

Before answering this question, I must first draw a few parallels between R1, which is open, and R6, which isn't open, and then discuss how they differ. Consider R12, the region-fusion of p1 and those points that are between p and p1. R12 is a linear subregion of R6 that has an exterior point (namely, p1). Furthermore, it is the case, as (NSAO) requires for R6 to be open, that there are a z and a z' such that each of z and z' is a linear subregion of R6; R12 and z' are co-linear, R12 and z'' are co-linear, and z and z' are co-linear; neither z nor z' surrounds R12; and the region-fusion of R12, z, and z’ does not have an exterior point. In particular, consider R13, the region-fusion of those points that are between f_1(p) and f_2(p). R13 is a linear subregion of R6. Furthermore, R12 and R13 are co-linear and R13 and R13 are co-linear. In addition, R13 does not surround R12. Finally, the region-fusion of R12 and R13 does not have an exterior point.

As we saw above, similar remarks apply to R1 and R9, the region-fusion of p.5 and those points that are between p and p.5 (where R1 is the region-fusion of those points whose distance from p is less than one foot and p.5 is a point whose distance from p is exactly half-a-foot). R9 is a linear subregion of R1 that has an exterior point, but there are a z and a z’ such that each of z and z’ is a linear subregion of R1; R9 and z’ are co-linear, R9 and z’’ are co-linear, and z and z’ are co-linear; neither z nor z’ surrounds R9; and the region-fusion of R9, z, and z’ does not have an exterior point. In particular, assuming that p.5 is between p and p1, consider R10, the region-fusion of those points that are between p.5 and p1. R10 is a linear subregion of R1. Furthermore, R9 and R10 are co-linear and R10 and R10 are co-linear. In addition, R10 does not surround R9. Finally, the region-fusion of R12 and R13 does not have an exterior point.

It was the fact that every linear subregion of R1 that has an exterior point is related to R1 in the way that R9 is—every linear subregion of R1 that has an exterior point is such that there are a z and a z’ such that each of z and z’ is a linear subregion of R1; it and z’ are co-linear, it and z’’ are co-linear, and z and z’ are co-linear; neither z nor z’ surrounds it; and the region-fusion of it, z, and z’ does not have an exterior point—but not every linear subregion of R5 that has an exterior point is related to R5 in the way that R9 is related to R1 that led us to (NSAO). However, we now see that (NSAO) is incorrect, since every linear subregion of R6 that has an exterior point (for instance, R12) is related to R6 in the way that R9 is related to R1 but R6 isn't open. What, then, is the difference between R1 and R6 that accounts for the fact that R1 is open but R6 isn't? An answer to this question can, I think, be found by considering certain differences between R9 and R10, on the one hand, and R12 and R13, on the other.

Notice first that neither R10 nor R13 has an exterior point. R10, however, takes advantage of the fact that it does not have an exterior point to get right up next to R9; there is nothing between R9 and R10. On the other hand, R13 doesn't take advantage of the fact that it does not have an exterior point to get right up next to R12; there is nothing between R12 and R13.

This difference can be used to motivate further modifications to (FAO). According to (NSAO), a region, x, is open only if every linear subregion, y, of x that has an exterior point is such that there are a z and a z' such that each of z and z' is a linear subregion of x; y and z' are co-linear, y and z'' are co-linear, and z and z' are co-linear; neither z nor z' surrounds y; and the region-fusion of y, z, and z' does not have an exterior point. What the difference between R9 and R10, on the one hand, and R12 and R13, on the other, suggests is that further conditions should be put on z and z'. In particular, each of them should be required to have no exterior points and each of them should be required to be next to y.

The following alternative analysis of *being open* results from these modifications:

> **Second-Pass Analysis of Openness (SAO)**: x *is open* $=_{df}$ (i) x is a pointy region-fusion and (ii) for all y such that y is a linear subregion of x that has an exterior point, there are a z and a z’ such that each of z and z’ is a linear subregion of x; y and z’ are co-linear, y and z’’ are co-linear, and z and z’ are co-linear; neither z nor z’ surrounds y; neither z nor z’ has an exterior point; z is next to y[46] and z’ is next to y; and the region-fusion of y, z, and z’ does not have an exterior point.

(SAO) is a better analysis of *being open* than (FAO) because (SAO), like (NSAO), has the consequence that R5 isn’t open, whereas (FAO) has the consequence that R5 is open. Furthermore, (SAO) is also a better analysis of *being open* than (NSAO) because (SAO) has the consequence that R6 isn’t open whereas (NSAO) has the consequence that R6 is open. Thus, (SAO) appears to be a promising alternative analysis of *being open*. In fact, (SAO) is very similar to my proposed analysis of *being open*. I explain in the next section why I ultimately reject (SAO) in favor of my proposed analysis.

I now turn to (FAC), (NSAC), and the second counterexample to (FAC). Remember that the second counterexample to (FAC) concerns R8, which is the region-fusion of the following points: those points, except for p1, whose distance from p is less than or equal to one foot; those points each of which is such that there

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[46] As stated in Chapter 2, x *is next to* y $=_{df}$ (i) x is a region of space, (ii) y is a region of space, (iii) x and y do not region-overlap, and (iv) there is no region of space z such that z is between x and y.

is a point p* whose distance from p is less than one foot such that it is f_1(p*); those points each of which is such that there is a point p* whose distance from p is less than one foot such that it is f_2(p*); those points each of which is such that there is a point p* such that whose distance from p is less than one foot such that it is between f_1(p*) and f_2(p*); p2a; p3a; those points each of which is between p2a and p3a; p2b; p3b; and those points each of which is between p2b and p3b (where p1, f_1, and f_2 are as in the second counterexample to (FAO); p2a and p2b are those points whose distance from p1 is exactly one foot that lie on the tangent to R2 that intersects p1; and p3a and p3b are those points whose distance from p1 is exactly one foot that lie on that tangent). (See Figure 4 on p. 83.) R8 isn't closed. However, according to (FAC), R8 is closed because R8 is a pointy region-fusion that has an exterior point.

More importantly, R8 is closed according to (NSAC) as well. Thus, the second counterexample to (FAC) requires yet more modifications to (FAC), modifications beyond those required to avoid the first counterexample to (FAC). But exactly what further modifications are required?

I noted previously that the second counterexample to (FAO) and the second counterexample to (FAC) are structurally very similar and that this suggests that the modifications to (FAO) required by the second counterexample to (FAO) and the modifications to (FAC) required by the second counterexample to (FAC) should be parallel to one another. To see that this is in fact the case, consider R14, the region-fusion of those points that are between p and p1. R14 is a linear subregion of R8 that does not have an exterior point. Thus, according to (NSAC), R8 is open only if there are a z and a z’ such that each of z and z’ is a linear subregion of R8; R14 and z’ are co-linear, R14 and z’’ are co-linear, and z and z’ are co-linear; neither z nor z’ surrounds R14; and every point that is a subregion of the region-fusion of R14, z, and z’ is either an exterior point of that region-fusion or between two exterior points of that region-fusion. And there are. In particular, consider R15, the region-fusion of f_1(p), f_2(p), and those points that are between f_1(p) and f_2(p). R15 is a linear subregion of R8. Furthermore, R14 and R15 are co-linear and R15 and R15 are co-linear. In addition, R15 does not surround R14. Finally, every point that is a subregion of the region-fusion of R14 and R15 is either an exterior point of that region-fusion or between two exterior points of that region-fusion.

Notice, however, that the relationship between R14 and R15 parallels the relationship between R12 and R13. Remember that R13 doesn’t take advantage of the fact that it does not have an exterior point to get right up next to R12; there are points between R12 and R13. Similarly, R15 doesn’t take advantage of the fact that every point that is a subregion of it is either an exterior point of it or between two exterior points of it to get right up next to R14; there are points between R14 and R15.

This parallel suggests modifications to (NSAC) that parallel the modifications to (NSAO) made in response to the second counterexample to (FAO). Clause (ii) of (NSAO) was modified to include the following conditions on z and z’: that neither of them have an exterior points and that each of them be next to y. Similarly, clause (ii) of (NSAC) should be modified to include the following conditions on z and z’: that each of them is such that every point that is a subregion of it is either an exterior point of it or between two exterior points of it and that each of them is next to y. The resulting alternative analysis of *being closed* is as follows:

Second-Pass Analysis of Closedness (SAC): x *is closed* $=_{df}$ (i) x is a pointy region-fusion and (ii) for all y such that y is a linear subregion of x that does not have an exterior point, there are a z and a z' such that each of z and z' is a linear subregion of x; y and z' are co-linear, y and z'' are co-linear, and z and z' are co-linear; neither z nor z' surrounds y; every point that is a subregion of z is either an exterior point of z or between two exterior points of z and every point that is a subregion of z' is either an exterior point of z' or between two exterior points of z'; z is next to y and z' is next to y; and every point that is a subregion of the region-fusion of y, z, and z' is either an exterior point of that region-fusion or between two exterior points of that region-fusion.

Just as (SAO) is a better analysis of *being open* than are (FAO) and (NSAO), so too (SAC) is a better analysis of *being closed* than are (FAC) and (NSAC). Unlike (FAC) but like (NSAC), (SAC) has the consequence that R7 isn't closed. Furthermore, unlike both (FAC) and (NSAC), (SAC) has the consequence that R8 isn't closed either. Thus, (SAC) is a promising alternative analysis of *being closed*. In addition, just as (SAO) is very similar to my proposed analysis of *being open*, (SAC) is very similar to my proposed analysis of *being closed*. I turn now to explaining why I reject (SAO) and (SAC) in favor of my proposed analyses of *being open* and *being closed*, respectively.

3.3. The Proposed Analyses

This section concerns my proposed analyses of *being open* and *being closed*. I first review the consequences of the modified versions of Cartwright's analyses on which I based my earlier argument that those analyses should be rejected. I then argue that the cases I used to illustrate that those analyses have these consequences do not also serve to illustrate that (SAO) and (SAC) have those consequences; this shows that, in some respects, (SAO) and (SAC) are preferable to the modified versions of Cartwright's analyses. However, I present other cases designed to show that (SAO) and (SAC) have similar consequences and thus should also be rejected. Finally, I state my proposed analyses of being open and being closed and note that they do not have similar consequences. This completes my case for accepting my proposed analyses rather than the modified versions of Cartwright's analyses or (SAO) and (SAC).

My argument that the modified versions of Cartwright's analyses should be rejected, presented in §2, was based upon two main consequences of those analyses. First, I argued that the modified versions of Cartwright's analyses yield somewhat unnatural classifications of regions in some cases. Second, I argued that they make the distinction between open and closed regions ill-suited for use in answering the Receptacle Question. Having argued that they have these consequences, I claimed that since there are alternative analyses of *being open* and *being closed* that don't, the modified versions of Cartwright's analyses should be rejected.

I argued that the modified version of Cartwright's analyses have the first of these consequences in §§2.1 and 2.2. In those sections, I discussed certain cases in which those analyses yield somewhat unnatural classifications. In particular, I noted that if the modified versions of Cartwright's analyses are true, then: R1 is open but not closed in the actual world but is both open and closed in W1, R2 is closed but not open in the actual world but is both open and closed in W2, and R3 is neither open nor closed in the actual world but is open but not closed in W3. I argued, however, that these classifications are somewhat unnatural: it is more natural to classify both R1 in the actual world and R1 in W1 as open but not closed, to classify both R2 in the actual world and R2 in W2 as closed but not open, and to classify both R3 in the actual world and R3 in W3 as open but not closed.

(SAO) and (SAC) do not yield the same classifications as the modified versions of Cartwright's analyses. It is clear that if (SAO) and (SAC) are true, then R1 is open but not closed in both the actual world and W1, R2 is closed but not open in both the actual world and W2, and R3 is open but not closed in both the actual world and W3. Thus, the cases I used to illustrate the fact that the modified versions of Cartwright's analyses yield somewhat unnatural classifications do not similarly illustrate that (SAO) and (SAC) yield such classifications.

It is also clear that the cases I used in §2.3 to illustrate the fact that the modified versions of Cartwright's analyses make the distinction between open and closed regions ill-suited for use in answering the Receptacle Question do not similarly illustrate that (SAO) and (SAC) make that distinction ill-suited for that use. Remember that I argued that the modified versions of Cartwright's analyses make the distinction between open and closed regions ill-suited for that use on two grounds.

First, I argued that if the modified versions of Cartwright's analyses are true, then the Open View and the Closed View each have the consequence that a receptacle is not a receptacle (solely) in virtue of its intrinsic features. Second, I argued that if the modified versions of Cartwright's analyses are true, then the Open View and the Closed View each conflict with the characteristic axiom of S4 modal logic.

I did not use any cases to illustrate that the modified version of Cartwright's analyses have the former consequence, but I did use cases to show that they have the latter consequence. In particular, I considered the verdicts those analyses yield concerning R1 and R2. If the modified versions of Cartwright's analyses are true, then R1 is open but not closed in the actual world and is both open and closed in W1 and R2 is closed but not open in the actual world and is both open and closed in W2. Thus, given the modified versions of Cartwright's analyses, if (CV) is true, then it is possibly possible that R1 is occupied by a material object even though it is not possible that R1 is occupied by a material object. Similarly, given the modified versions of Cartwright's analyses, if (OV) is true, then it is possibly possible that R2 is occupied by a material object even though it is not possible that R2 is occupied by a material object.

As noted above, however, (SAO) and (SAC) do not yield the same verdicts concerning R1 and R2 as do the modified versions of Cartwright's analyses. Instead, given (SAO) and (SAC), R1 is open but not closed in both the actual world and W1 and R2 is closed but not open in both the actual world and W2. It would also seem that if (SAO) and (SAC) are true, then R1 is essentially open but not closed and R2 is essentially closed but not open. Thus, given (SAO) and (SAC), if (CV) is true, then it is neither possible nor possibly possible that R1 is occupied by a material object. Similarly, given (SAO) and (SAC), if (OV) is true, then it is neither possible nor possibly possible that R2 is occupied by a material object. Therefore, the cases I used to illustrate that the Open View and the Closed View conflict with the characteristic axiom of S4 modal logic if the modified versions of Cartwright's analyses are true do not also illustrate that those views conflict with the characteristic axiom of S4 modal logic if (SAO) and (SAC) are true.

I have just argued that the cases I used when arguing that the modified versions of Cartwright's analyses should be rejected cannot be used in a similar argument for the conclusion that (SAO) and (SAC) should be rejected. Nonetheless, I think that (SAO) and (SAC) should be rejected in favor of my proposed analyses of *being open* and *being closed*. I will now explain why.

I will first explain why (SAO) should be rejected. Consider R6, the region that figured in the second counterexample to (FAO).[47] I argued against (FAO) in §3.1 by noting that R6 isn't open but that it is open if (FAO) is true. However, as discussed in §3.2, R6 isn't open if (SAO) is true. Thus, the second counterexample to (FAO) isn't also a counterexample to (SAO). Despite this, (SAO) still has untoward consequences concerning R6. To see this, we must consider the world, W4, that results from simply "deleting" from the actual world all and only those points that are not subregions of R6 in the actual world.

[47] See §3.1 for a description of R6. Also see Figure 2 on p. 80.

Although (SAO) yields the verdict that R6 isn’t open in the actual world, things are different in W4. (SAO) yields the verdict that R6 is open in W4. I argued in §3.2 that given (SAO), R6 isn’t open in the actual world. In particular, I drew attention to R12 and R13. R12 is a linear subregion of R6 that has an exterior point. Furthermore, in the actual world, R13 is a linear subregion of R6, R12 and R13 are co-linear and R13 and R13 are co-linear, R13 does not surround R12, R13 does not have an exterior point, and the region-fusion of R12 and R13 does not have an exterior point. However, R13 is not next to R12 in the actual world because in the actual world, although there are no regions of space that are subregions of R6 and are between R13 and R12, there are regions of space that are not subregions of R6 and are between R13 and R12. In fact, in the actual world, for all z and z’, if (a) each of z and z’ is a linear subregion of R6, (b) R12 and z are co-linear, R12 and z’ are co-linear, and z and z’ are co-linear, (c) neither z nor z’ surrounds R12, (d) neither z nor z’ has an exterior point, and (e) the region-fusion of R12, z, and z’ does not have an exterior point, then: *either* (f) z is not next to R12 because although there are no regions of space that are subregions of R6 and are between z and R12, there are regions of space that are not subregions of R6 and are between z and R12, *or* (g) z’ is not next to R12 because although there are no regions of space that are subregions of R6 and are between z’ and R12, there are regions of space that are not subregions of R6 and are between z’ and R12. Thus, some linear subregions of R6 that have an exterior point do not satisfy the second clause of (SAO) in the actual world. This is why, given (SAO), R6 is not open in the actual world.

Notice, however, that every region of space that is not a subregion of R6 in the actual world has been “deleted” from W4. Thus, in W4, R12 satisfies clause (ii) of (SAO). Since the same applies in W4 to every other linear subregion of R6 that has an exterior point, R6 is open in W4 according to (SAO).

These remarks concerning R6 in the actual world and R6 in W4 show that (SAO) has consequences very similar to consequences of the modified versions of Cartwright’s analyses that led me to reject the latter view. First, they show that (SAO) yields somewhat unnatural classifications. Given (SAO), R6 in the actual world isn’t open but R6 in W4 is. Since R6 in the actual world and R6 in W4 have the same intrinsic features, however, it is more natural to either classify both as not open or to classify both as open. Second, they show that given (SAO), the Open View is problematic. For they show that given (SAO), the Open View has the consequence that a receptacle is not a receptacle (solely) in virtue of its intrinsic features. After all, R6 in the actual world and R6 in W4 have the same intrinsic features but given (SAO), R6 in the actual world is not open and R6 in W4 is open. Thus, given (SAO), *being open* is not an intrinsic feature of regions of space. But the Open View has the consequence that a receptacle is a receptacle in virtue of being open. So, if (SAO) is true, then the Open View has the consequence that a receptacle is not a receptacle (solely) in virtue of its intrinsic features. Finally, these remarks concerning R6 in the actual world and R6 in W4 show that given (SAO), the Open View conflicts with the characteristic axiom of S4 modal logic. Given (SAO), R6 in the actual world is not open but R6 in W4 is open. Thus, given (SAO), the Open View has the consequence that it is not possible that R6 is occupied by a material object although it is possibly possible that R6 is occupied by a material object, which consequence is in clear conflict with the characteristic axiom of S4 modal logic.

Mutatis mutandis, similar remarks show that (SAC) yields unnatural classifications; that given (SAC), the Closed View has the consequence that a receptacle is not a receptacle (solely) in virtue of its intrinsic features; and that given (SAC), the Closed View conflicts with the characteristic axiom of S4 modal logic. In particular, consider R8, the region of space that figured in the second counterexample to (FAC).[48] As I argued in §3.2, (SAC) yields the correct verdict that R8 is not closed in the actual world. However, (SAC) also yields the verdict that R8 is closed in W5, the world that results from simply "deleting" from the actual world all and only those points that are not subregions of R8 in the actual world. The reason is similar to the reason that (SAO) yields the verdict that R6 is open in W4. In the actual world, some linear subregions of R8 that do not have exterior points don't satisfy clause (ii) of (SAC). But in W5, those regions do satisfy clause (ii) of (SAC) because all regions that are not subregions of R8 in the actual world have been "deleted" from W5.

Someone might reply to these considerations involving R6 in the actual world and R6 in W4 and R8 in the actual world and R8 in W5 by denying that W4 and W5 are *possible* worlds. There are a couple of problems with this response. First, even if W4 and W5 are not possible worlds, it is still the case that the Open View has the consequence that a receptacle is not a receptacle (solely) in virtue of its intrinsic

[48] See §3.1 for a description of R8. Also see Figure 4 on p. 83.

features given (SAO) and that the Closed View has the consequence that a receptacle is not a receptacle (solely) in virtue of its intrinsic features given (SAC). For even if W4 and W5 are not possible worlds, (AAO2) and (AAC3) each have the consequence that whether a region of space is closed depends upon how things external to that region are. It's just that, if W4 is not a possible world, then it is impossible for R6 to have the intrinsic features it has in the actual world without also having those extrinsic features upon which it being non-open in the actual world depends; similarly for W5 and R8.[49]

Second, and more importantly, it is not at all clear that W4 and W5 are not possible worlds. One who claims that they are not will likely do so because they hold that it is impossible for a point p' to be a certain distance d from another point p'' without it being the case that for any distance d' less than d, there is a point that p' is d' from and which is between p' and p''. But this is a controversial claim about the structures that space could have. It would be nice to have analyses of *being open* and *being closed* that don't yield somewhat unnatural classifications even if such a controversial claim about the structures space could have is false and that don't have the consequence that the Open View and the Closed View conflict with the characteristic axiom of S4 modal logic unless such a controversial claim is true.

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[49] Some accounts of intrinsicality define that notion in terms of metaphysical necessity. See, for instance, Langton and Lewis 1998. If these accounts are correct and W4 and W5 are impossible, then *being open* and *being closed* are intrinsic features of regions of space. I reject such accounts, however, in favor of an account that defines intrinsicality in terms of metaphysical dependence.

In light of the fact that (SAO) and (SAC) have similar consequences to the consequences of the modified versions of Cartwright's analyses that I used to argue that the latter analyses should be rejected, I offer my proposed analyses of *being open* and *being closed*:

Proposed Analysis of Openness (PAO): x *is open* $=_{df}$ (i) x is a pointy region-fusion and (ii) for all y such that y is a linear subregion of x that has an exterior point, there are a z and a z' such that each of z and z' is a linear subregion of x; y and z' are co-linear, y and z'' are co-linear, and z and z' are co-linear; neither z nor z' surrounds y; neither z nor z' has an exterior point; the distance between y and z is zero and the distance between y and z' is zero; and the region-fusion of y, z, and z' does not have an exterior point.

Proposed Analysis of Closedness (PAC): x *is closed* $=_{df}$ (i) x is a pointy region-fusion and (ii) for all y such that y is a linear subregion of x that does not have an exterior point, there are a z and a z' such that each of z and z' is a linear subregion of x; y and z are co-linear, y and z' are co-linear, and z and z' are co-linear; neither z nor z' surrounds y; every point that is a subregion of z is either an exterior point of z or between two exterior points of z and every point that is a subregion of z' is either an exterior point of z' or between two exterior points of z'; the distance between y and z is zero and the distance between y and z' is zero; and every point that is a subregion of the region-fusion of y, z, and z' is either an exterior point of that region-fusion or between two exterior points of that region-fusion.

(PAO) and (PAC) do not yield the somewhat unnatural classifications yielded by the modified versions of Cartwright's analyses, nor do they yield the somewhat unnatural classifications yielded by (SAO) and (SAC). Given (PAO) and (PAC), R1 is open but not closed in both the actual world and W1, R2 is closed but not open in both the actual world and W2, and R3 is open but not closed in both the actual world and W3. These are the same classifications yielded by (SAO) and (SAC), and they are more natural than the classifications yielded by the modified versions of Cartwright's analyses. Thus, (PAO) and (PAC) are preferable to the modified versions of Cartwright's analyses, just as (SAO) and (SAC) are.

In addition, however, given (PAO) and (PAC), R6 is not open in both the actual world and W4 and R8 is not closed in both the actual world and W5. R6 is not open in the actual world given (PAO) because in the actual world, R12 is a linear subregion of R6 that has an exterior point but for all z and a z', if (a) each of z and z' is a linear subregion of R6, (b) R12 and z' are co-linear, R12 and z'' are co-linear, and z and z' are co-linear, (c) neither z nor z' surrounds y, (d) neither z nor z' has an exterior point, and (e) the region-fusion of y, z, and z' does not have an exterior point, then: *either* (e) the distance between y and z is zero *or* (f) the distance between y and z' is zero. Thus, some linear subregions of R6 that have an exterior point don't satisfy clause (ii) of (PAO) in the actual world. Similarly, however, some linear subregions of R6 that have an exterior point don't satisfy clause (ii) of (PAO) in W4, since "deleting" the regions of space that are not subregions of R6 in the actual world doesn't change the distances between two linear subregions of R6; it merely changes whether there are regions of space between two linear subregions of R6. Furthermore, similar remarks show that given (PAC), R6 is not closed in both the actual world and W5.

Not only do (PAO) and (PAC) fail to yield the somewhat unnatural classifications yielded by (SAO) and (SAC), they also do not make the distinction between open and closed regions ill-suited for use in answering the Receptacle Question. For, first, given (PAO) and (PAC), *being open* and *being closed* are intrinsic features of regions of space. Given (PAO), *being open* is an intrinsic feature of regions of space because whether a region of space is open depends entirely on facts concerning the relations borne by its subregions to it and to one another, facts that do not themselves depend on how things external to that region are. In particular, none of the following depends on how things external to a region are: whether that region is a pointy region-fusion, whether something is a linear subregion of that region, whether a linear subregion of that region has an exterior point or has no external point, whether two linear subregions of that region are co-linear, whether one linear subregion of that region surrounds another, whether the distance between two linear subregions of that region is zero, and whether the region-fusion of some linear subregions of that region has an exterior point. Contrast this with (SAO), where whether a region is open depends on whether one linear subregion of it is next to another, which does depend on facts concerning how things external to that region are, as consideration of R6 in the actual world and R6 in W4 show.

Similar remarks show that given (PAC), *being closed* is an intrinsic feature of a region of space. We must simply add to the previous remarks that neither of the following depends on facts about how things external to a region are: whether every point that is a subregion of a linear subregion of that region is either an exterior point of that linear subregion or between two exterior points of it and whether the region-fusion of some linear subregions of that region is such that every point that is a subregion of that region-fusion is either an exterior point of it or between two exterior points of it.

Since *being open* and *being closed* are intrinsic features of regions of space given (PAO) and (PAC), if (PAO) and (PAC) are true, then it is not a consequence of the Open View that a region of space is not a receptacle (solely) in virtue of its intrinsic features nor is that a consequence of the Closed View. In fact, if (PAO) and (PAC) are true, then it is a consequence of the Open View that a region of space is a receptacle solely in virtue of its intrinsic features and that is also a consequence of the Closed View. Thus, my first reason for claiming that the modified versions of Cartwright's analyses make the distinction between open and closed regions ill-suited for use in answering the Receptacle Question does not apply to (PAO) and (PAC).

Second, it is plausible that my second reason for claiming that the modified versions of Cartwright's analyses make the distinction between open and closed regions ill-suited for use in answering the Receptacle Question does not apply to (PAO) and (PAC) either. My second reason for claiming this was that if the modified versions of Cartwright's analyses are true, then it is possible that a region of space is open even though that region of space is not open and it is possible that a region of space is closed even though that region of space is not closed. Thus, given the modified versions of Cartwright's analyses, the Open View has the consequence that it is possibly possible that a region of space is occupied by a material object even though it is not possible that it is occupied by a material object, as does the Closed View. Therefore, given the modified versions of Cartwright's analyses, the Open View and the Closed View both conflict with the characteristic axiom of S4 modal logic, according to which whatever is possibly possible is possible.

On the other hand, it is plausible that given (PAO), if a region of space isn't open, then it is necessary that it isn't open, and that given (PAC), if a region of space isn't closed, then it is necessary that it isn't closed. Let R be an actual region of space. Then whether R is possibly open (or possibly closed) depends entirely on facts concerning the possible sizes (i.e., unextendedness) of its subregions, the possible betweenness relations obtaining between points that are subregions of it, and the possible distance relations obtaining between subregions of it. But it would seem that these facts do not vary across worlds. If it is possible that R has subregions with certain sizes, then those possible subregions of R are actually subregions of R with those sizes. In addition, if it is possible that a point that is a subregion of R is between two other points that are subregions of R, then those three points are actually subregions of R and the first is between the other two. Finally, if it is possible that a region that is a subregion of R is a certain distance from another region that is a subregion of R, then those two regions are actually subregions of R and the first is that distance from the second. At the very least, these claims are plausible. But then it is plausible that given (PAO) and (PAC), no region of space that isn't open is possibly open and no region of space that isn't closed is possibly closed. But then, given (PAO) and (PAC), neither the Open View nor the Closed View conflicts with the characteristic axiom of S4 modal logic.

(PAO) and (PAC) have important virtues. First, unlike (FAO) and (FAC), the verdicts they deliver correspond closely enough to the verdicts delivered by the modified versions of Cartwright's analyses that they can reasonably be taken to be analyses of the same properties as the latter. Second, unlike the modified versions of Cartwright's analyses and unlike (SAO) and (SAC), they do not yield somewhat unnatural classifications nor do they make the distinction between open and closed regions ill-suited for use in answering the Receptacle Question. I conclude that (PAO) and (PAC) should be accepted as the correct analyses of *being open* and *being closed*, respectively.

Conclusion

In this chapter, I discussed how to correctly analyze the properties of being open and being closed. In §1, I noted that Cartwright's analyses presuppose that regions of space are sets of points in space, a view I argued that we should be skeptical of in Chapter 1, and I explained how to modify them so that they do not presuppose this. However, in §2, I argued that these modified versions of Cartwright's analyses should be rejected. Finally, in §3, I discussed a number of alternative analyses of *being open* and *being closed*, each of which employed the notion of a point being an exterior point of a region. There I presented my proposed analyses and argued in favor of them. On the basis of my arguments, I conclude that we should accept (PAO) and (PAC) rather than the modified versions of Cartwright's analyses.

Chapter 4: MaxCon and the Possibility of Gunk[*]

Introduction

Ned Markosian (1998a) introduced the following question into the philosophical literature:

> **The Simple Question (SQ)**: What are the necessary and jointly sufficient conditions for an object's being a simple? [50,51]

In the same paper, Markosian surveyed a variety of responses to SQ and decided in favor of the following:

> **The Maximally Continuous View of Simples (MaxCon)**: Necessarily, for all x,[52] x is a simple if and only if x is a maximally continuous object.

Unlike Markosian, I am not a proponent of MaxCon. (I am inclined to accept Kris McDaniel's (2007) Brutal View of Simples.) However, I find MaxCon to be a very interesting response to SQ. Unlike some other responses to SQ, MaxCon is consistent with the intriguing thesis that extended simples, objects that have no proper parts but that nonetheless have spatial extension in one or more dimensions, are possible. In fact, the conjunction of MaxCon with the very plausible claim that

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[*] This chapter is based on my 2008.

[50] x *is a simple* $=_{df}$ x has no proper parts. x *is a proper part of* y $=_{df}$ x is a part of y and x is not identical to y.

[51] One might wonder why the correct answer to SQ isn't simply: Necessarily, for all x, x is a simple if and only if x has no proper parts. However, Markosian makes it clear that he does not take this to be an answer to the question when he says: 'It is important to note that the Simple Question is not a request for an analysis of the concept of a simple… For we already know the correct analysis of the concept of a simple—simples are objects with no proper parts. The Simple Question is, rather, a question about how the concept of a simple is linked up with other, preferably non-mereological, concepts' (pp. 214-215).

[52] I have added the "for all x" clause to Markosian's formulation of MaxCon.

extended maximally continuous objects[53] are possible entails this thesis. MaxCon deserves our attention.

In this chapter and the next, I discuss MaxCon. In particular, I consider whether certain arguments against MaxCon succeed. My intention is neither to endorse these arguments nor to conclusively refute them. Instead, I will focus my attention on chains of reasoning in favor of key premises of these arguments and I will argue that a proponent of MaxCon can reasonably reject these chains of reasoning. My goal is limited. Even if I am successful, there may be other chains of reasoning in favor of these premises that a proponent of MaxCon cannot reasonably reject. Nonetheless, I believe that the project is worth undertaking, since the chains of reasoning I consider involve independently interesting metaphysical principles and it is worthwhile discussing whether and how these principles can reasonably be rejected.

I now turn to the topic of this chapter. Recently, Hud Hudson (2001) has argued that if MaxCon is true, then there couldn't be gunky objects.[54] This suggests the following argument against MaxCon[55]:

1. If MaxCon is true, then there couldn't be gunky objects.
2. There could be gunky objects.
3. Therefore, MaxCon isn't true.

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[53] I explain what a maximally continuous object is in §1 of this chapter.
[54] x *is a gunky object* $=_{df}$ every part of x has proper parts.
[55] Note that Hudson (2001) does not offer this argument. Rather, he argues as follows: (1) Either MaxCon is true or the Pointy View of Simples is true. (2) If MaxCon is true, then there couldn't be gunky objects. (3) If the Pointy View of Simples is true, then there couldn't be gunky objects. (4) Therefore, there couldn't be gunky objects.

This argument effectively dramatizes the upshot of Hudson's argument: A proponent of MaxCon who finds Hudson's argument compelling must deny the possibility of gunky objects; and, similarly, a proponent of the possibility of gunky objects who finds it compelling must deny MaxCon.

I argue in this chapter, however, that Hudson's argument is not compelling. Its premises include substantive metaphysical claims that can reasonably be denied by a proponent of MaxCon. Thus, in the absence of any independently compelling reasons to accept premise (1) of the above argument, a proponent of MaxCon needn't deny the possibility of gunky objects nor need a proponent of the possibility of gunky objects deny MaxCon.

The chapter proceeds as follows. In §1, I present and explain Markosian's formulation of MaxCon, discuss why I think it should be reformulated in light of the discussion in Chapters 1 and 2, and make some remarks concerning how to properly interpret both MaxCon and Hudson's argument for (1) above. In §2, I present Hudson's formulation of his argument and state the premises of my reconstruction of his argument. (The reconstruction itself is contained in the appendix to the chapter.) I identify which of these premises I take to be substantive metaphysical claims in §3, where I argue that a proponent of MaxCon can reasonably deny them. I conclude that Hudson's argument does not compel a proponent of MaxCon to deny the possibility of gunky objects and does not compel a proponent of that possibility to deny MaxCon, and I consider what sorts of gunky objects a proponent of MaxCon can reasonably countenance.

1. Preliminaries

Clearly, one cannot understand MaxCon unless one understands the phrase 'maximally continuous object'. Markosian explicates this notion as follows:

> x *is a maximally continuous object* $=_{df}$ x is a spatially continuous object and there is no continuous region of space, R, such that (i) the region occupied by x is a proper subset of R, and (ii) every point in R falls within some object or other. (p. 221)

By itself, this definition does little to illuminate matters, since it makes use of the technical notions of a spatially continuous object, a continuous region of space, an object occupying a region, and a point falling within an object. However, Markosian offers explanations of each of these notions except the last, which he takes as primitive.

Employing this primitive, Markosian offers the following explication of what it is for an object to occupy a region:

> x *occupies* R $=_{df}$ R has as members all and only those points that fall within x. (p. 216)

He is then able to say what it is for something to be a spatially continuous object:

> x *is spatially continuous* $=_{df}$ x occupies a continuous region of space. (ftnt. 21)

Given these definitions, and assuming that we have a prior grasp on the notion of a point falling within an object, the only further notion that must be understood in order to grasp Markosian's definition and thereby comprehend MaxCon is the notion of a continuous region of space. Here Markosian employs the Cartwrightian analysis of *being continuous* (ftnt. 21). (See Chapter 2, §1 for this analysis.)

Markosian's explanation of MaxCon is clear. Nonetheless, I think that MaxCon should be reformulated. Throughout his explanation, Markosian presupposes that regions of space are sets of points in space, and I argued in Chapter 1 that we should be skeptical of this claim. This is particularly clear in his explications of what it is for something to be a maximally continuous object and what it is for an object to occupy a region. In the former, Markosian speaks of one region of space being a subset of another. In the latter, he speaks of a region having members. Furthermore, Markosian also employs the Cartwrightian analysis of *being continuous* and, as discussed in Chapter 2, that analysis itself presupposes that regions are sets of points.

There is a simple way to reformulate MaxCon so that it does not make this presupposition. First, Markosian's explications of what it is for something to be a maximally continuous object and what it is for an object to occupy a region may be replaced with the following:

> x *is a maximally continuous object* $=_{df}$ x is a spatially continuous object and there is no continuous region of space, R, such that (i) the region occupied by x is a proper subregion of R, and (ii) every point in R falls within some object or other.
>
> x *occupies* R $=_{df}$ all and only those points that fall within x are subregions of R.

Second, the Cartwrightian analysis of *being continuous* may be replaced with the modified Cartwrightian analysis presented in §2 of Chapter 2.

These modifications suffice to rid MaxCon of the presupposition that regions of space are sets of points in space. However, I think that a further modification of

MaxCon is in order, inspired by certain remarks Markosian makes towards the end of his paper in an attempt to provide an intuitive motivation for MaxCon:

> Imagine what it would be like to shrink to increasingly smaller and smaller sizes, so that other objects became increasingly larger and larger in comparison to yourself. Now imagine shrinking in this way and being able to dive into the insides of other objects, so that you can see what they are made of. Imagine shrinking in this way and somehow diving into the insides of Spero, our perfectly solid sphere, as you become smaller and smaller. You are trying to find some sign that Spero is not in fact a spatially continuous object. That is, you are trying to find an "island of matter" inside of Spero that is disconnected from the rest of the matter constituting Spero. And since there is no limit on how much you can shrink, there is no lower limit on the size that such an island inside of Spero would have to be in order for you to be able to detect it. If there is such an island in there, you will find it. But as you shrink down smaller and smaller, examining the insides of Spero on a smaller and smaller scale, you find no such disconnected island. Spero truly is, as advertised, a perfectly solid sphere, which means that Spero is indeed a spatially continuous object.
>
> It seems to me that your activity in this scenario could be correctly described as a failed attempt to identify the proper parts of Spero. It seems to me that at each stage along your journey, if you paused to reflect on whether you then knew that Spero had proper parts, you would be right if you were to say to yourself, "Well, I haven't found any proper parts yet. Spero might turn out to be a simple, for all I know." (pp. 226-227)

To see why these remarks indicate that a further modification of MaxCon is in order, consider performing the sort of task Markosian describes in a world in which space has the Gunky Structure. In this world, there is a spherical gunky region of space that has as subregions all and only those regions that fall within a particular material object, Spero*. Imagine now that you are in this world and shrinking to smaller and smaller sizes while investigating the insides of Spero* in an attempt to find an “island of matter” inside Spero* that is disconnected from the rest of the matter constituting Spero*, but that you find no such island as you shrink down smaller and smaller.

If Markosian is correct that finding no such island when performing the task he imagines could be correctly described as a failed attempt to identify the proper parts of Spero, it would seem that finding no such island when performing the task I just imagined could be correctly described as a failed attempt to identify the proper parts of Spero*. Now Markosian believes that reflection on the case he describes provides intuitive support for MaxCon. In particular, he thinks that it provides intuitive support for the claim that Spero is a simple (that is, that Spero has no proper parts) and, according to MaxCon, that claim is true: if MaxCon is true, then Spero is a simple. If this is so, however, then reflection on the case I have described provides intuitive support for the claim that Spero* is a simple. According to MaxCon, however, Spero* is not a simple. After all, according to MaxCon, any simple must occupy a continuous region of space and Spero* does not occupy a continuous region of space, at least if the Cartwrightian analysis of continuity or the modified version of that analysis is employed. According to those analyses, there are no continuous regions if there are no points, and there are no points in Spero*'s world. Thus, the intuitive motivation Markosian gives for MaxCon also motivates holding that Spero* is a simple, a claim with which MaxCon is inconsistent. The intuitive motivation Markosian gives for MaxCon conflicts with MaxCon![56]

Markosian might respond to this difficulty by denying the possibility that space has the Gunky Structure and thus denying the possibility of Spero*. However, I think that it would be more attractive to modify MaxCon so that it yields both the

[56] Neal Tognazzini (2006) offers a similar motivation for modifying MaxCon, although his argument is concerned with the Tile Structure rather than the Gunky Structure.

claim that Spero is a simple and the claim that Spero* is a simple. If it should turn out that it is impossible that space has the Gunky Structure and thus that Spero* is impossible, no harm done. On the other hand, should it turn out that it is possible that space has the Gunky Structure and that Spero* is possible, modifying MaxCon in this way will allow us to maintain both the intuitive motivation Markosian gives for MaxCon and the modified version of that view without conflict.

I suggest the following modifications. (1) Replacing Markosian's primitive notion of a point falling within an object with the primitive notion of a region falling within an object. (2) Employing the following account of what it is for something to be a maximally continuous object:

> x *is a maximally continuous object* $=_{df}$ x is a spatially continuous object and there is no continuous region of space, R, such that (i) the region occupied by x is a proper subregion of R, and (ii) every subregion of R region-fuses some regions each of which falls within an object.

(3) Employing the following account of what it is for an object to occupy a region:

> x *occupies* R $=_{df}$ all and only those regions that fall within x are subregions of R.

(4) Continuing to employ Markosian's account of what it is for something to be a spatially continuous region, but interpreting 'continuous' therein and in the preceding as 'continuous$_2$' rather than employing the Cartwrightian analysis of continuity or the modified version of that analysis.[57] These modifications yield a version of MaxCon

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[57] See Chapter 2, §4, where I introduced 'continuity$_2$' to refer to the property picked out by the intuitive notion of continuity I discussed in that chapter. There I presented two analyses of continuity$_2$ and suggested that it may be indeterminate which of these provides the correct

according to which both Spero and Spero* are simples. This new version of MaxCon neither presupposes that regions of space are sets of points in space, like Markosian's version, nor does it conflict with the intuitive motivation that Markosian gives for MaxCon, like both Markosian's version and the modified version I proposed earlier. The new version of MaxCon is thus superior to these other versions, and when I use the word 'MaxCon' in the remainder of this chapter, it is the new version to which I refer.[58]

Having stated my reformulated version of MaxCon, I now turn briefly to a different issue: the domain for which MaxCon and the conclusion of Hudson's argument are intended to hold. The first point I would like to make concerning the domain for which MaxCon and the conclusion of Hudson's argument are intended to

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analysis the property picked out by that intuitive notion. Thus, I introduced '$continuity_{2.1}$' to refer to the property that the first of these analyses correctly analyzes and '$continuity_{2.2}$' to refer to the property that the second does. I am unsure whether it would be best to interpret 'continuous' in the modified version of MaxCon I am proposing as '$continuous_{2.1}$' or '$continuous_{2.2}$', so here I simply say that it should be interpreted as '$continuous_2$'. Strictly speaking, this makes the modified version of MaxCon indeterminate between two views. However, the differences between these views is irrelevant to my discussion of Hudson's argument, so I don't distinguish between them in what follows.

[58] Joshua Spencer (2008) suggests a more radical modification of MaxCon in which the defined notions of occupation discussed above are replaced by a primitive notion. Spencer motivates this modification by arguing that a version of MaxCon employing the primitive notion is more plausible than versions employing the defined notions due to their consequences concerning certain cases of time travel. I find Spencer's argument persuasive and agree that the most plausible version of MaxCon will employ a primitive notion of occupation rather than a defined notion. Thus, I think that the new version of MaxCon I suggest in the text is not the most plausible version of MaxCon. There are two reasons I focus on it rather than on a version that employs a primitive notion of occupation. First, I am primarily concerned with whether Hudson's argument is compelling and my critique of that argument applies equally well whichever of these versions of MaxCon is in question. Second, the issue of whether the defined notions of occupation are replaced by a primitive notion is tangential to the issue of whether 'continuous' should be interpreted as '$continuous_2$' and I am more interested in the latter issue because I would like to show that $continuity_2$ can perform useful theoretical work.

hold is that Markosian intends the quantifier in MaxCon to be restricted to *physical* objects (ftnt. 10). Hudson similarly restricts his argument to *material* objects, stating that his ‘goal is to demonstrate that (MaxCon) rules out the possibility of material atomless gunk’ (p. 86). Here I will assume that these restrictions are the same: the domain of physical objects is necessarily coextensive with the domain of material objects. Thus, strictly speaking, the conclusion of Hudson’s argument is that if MaxCon is true, then there couldn’t be gunky material/physical objects. However, in what follows I will leave the restriction to material/physical objects tacit because it unnecessarily complicates the discussion and formulation of Hudson’s argument.

In addition, I will assume that something is a material/physical object just in case it is spatially located. Markosian (2000) argues for this claim and Hudson (2005) endorses it. In addition, Hudson’s argument seems to presuppose it. Although a proponent of MaxCon might wish to reject this claim, this issue is distinct from those I would like to raise. Thus, I grant the claim for the sake of argument.

In this section I have argued that MaxCon should be reformulated, explained how to reformulate it, and addressed preliminaries concerning the correct interpretation of MaxCon and of Hudson’s argument. Before concluding, let me introduce a formulation of the new version of MaxCon that is equivalent to the formulation I presented above. First, let us say that a region of space is matter-filled if and only if every subregion of it region-fuses some regions each of which falls within an object. Second, let us say that a region of space is a maximally continuous matter-filled region of space if and only if it is continuous, it is matter-filled, and it is not a proper subregion of a continuous, matter-filled region of space.[59] Then the new version of MaxCon turns out to be equivalent to the claim that necessarily, for all x, x is a simple if and only if x occupies a maximally continuous matter-filled region of space. It is this formulation that I employ in my reconstruction of Hudson's argument and in what follows.

2. Hudson's Argument

In this section, I present Hudson's formulation of his argument for the conclusion that if MaxCon is true, then there couldn't be gunky objects. Then I identify the premises of my reconstruction of that argument. However, because my reconstruction of Hudson's argument is quite complicated, I do not present it here. Rather, I relegate it to the appendix.

Hudson presents his argument in the following passage:

> Let us assume (toward reductio) that there is some hunk of material atomless gunk, H. Now, since any hunk of material atomless gunk exactly occupies some region or other and since any region has at least one (possibly point-sized) continuous subregion, there is some (possibly point-sized) continuous subregion of the region exactly occupied by H--hereby named 'S'. Now S itself is either a proper subregion of some extended, continuous region, every point in which falls within some object or other--or not. If not, then (by MaxCon) it follows that there is a simple at S (which would then be a part of H) and since ([by the definition of 'is a gunky object']) it also follows that H does not have any simple as a part, we have contradicted our assumption. Consequently, S is a proper subregion of some extended, continuous region, every point in which falls within some object or other. But every such region (i.e., every region such that every point in it falls within some object or other) either contains a maximally continuous object or else is a subregion of a region that contains a maximally continuous object. Since we are now committed to such a region, we are therefore committed to some maximally continuous object, M. Let R name the region exactly occupied by M. Now (by MaxCon) M is a material simple, and thus ([by the definition of 'is a gunky object']) we may derive

[59] 'Continuous' here should be interpreted as 'continuous$_2$'.

> (P) M is neither a part of H nor identical to H.
>
> Recall that M exactly occupies R. But this fact, together with the fact that M is a simple, guarantees that no subregion of R is a subregion of any region that is exactly occupied by a material object (unless that material object has M as a part or is identical to M). But, earlier we secured the result that S, which is a subregion of R, is a subregion of the region exactly occupied by H. So, we may derive
>
> (~P) M is either a part of H or identical to H.
>
> Consequently, we have arrived at (P & ~P), and our reductio is complete. Accordingly, since the truth-value of (MaxCon) is not a contingent matter, it would seem that if (MaxCon) is the right view about simples, then we have a simple demonstration of the impossibility of material atomless gunk. (pp. 86-7)

Unfortunately, although Hudson's formulation is relatively clear, it is not particularly helpful for my purposes here. Because it is presented in paragraph form, it is difficult to identify the premises of the argument and thus to determine whether these include any substantive metaphysical claims that a proponent of MaxCon can reasonably deny. A reconstruction whose premises are separated from one another and numbered would make these tasks much easier. Such a reconstruction is presented in the appendix. Here I simply state the premises of that reconstruction:

(i) Necessarily, for all x, if x is a gunky object, then there is an R such that R is a region of space and x occupies R.[60]

(ii) Necessarily, for all regions of space R such that there is something that occupies R, R is a matter-filled region of space.

(iii) Necessarily, for all matter-filled regions of space R, R is a continuous matter-filled region of space or R is a discontinuous matter-filled region of space.

[60] Remember that we are restricting our attention to material/physical objects and assuming that each material/physical object is spatially located. See §1 above.

(iv) Necessarily, for all discontinuous matter-filled regions of space R, there is an S such that S is a maximally continuous matter-filled region of space and S is a subregion of R.

(v) Necessarily, for any maximally continuous matter-filled region of space R, there is an x such that x occupies R.

(vi) Necessarily, for all x, y, R, and S, if R and S are regions of space, R is a subregion of S, x occupies R, and y occupies S, then x is a part of y.

(vii) Necessarily, for any simple x and gunky object y, x is not a part of y.

(viii) Necessarily, for all continuous matter-filled regions of space R, there is an S such that S is a maximally continuous matter-filled region of space and R is a subregion of S.

(ix) Necessarily, for any gunky object x and simple y, x is not a part of y.

3. Substantive Metaphysical Claims

In the previous section, I presented Hudson's formulation of his argument and stated the premises that appear in my reconstruction of it. Of these premises, (i)-(iv) and (vii)-(ix) are uncontroversial.[61] (v) and (vi), however, are substantive metaphysical claims, or so I will argue here. Let us call these claims 'Maximally Continuous Matter-Filled Regions to Objects' (for short, 'MCMRO') and 'Subregions to Parts' (for short, 'SP'), respectively. In this section I will discuss MCMRO and SP and argue that a proponent of MaxCon can reasonably deny them.

I begin with MCMRO. First let me clarify the relationship between MCMRO and MaxCon. Notice that MCMRO and MaxCon jointly entail the following thesis:

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[61] Or, at the very least, I am not concerned with them here. After all, all I must do to show that Hudson's argument is not compelling is to show that some of the premises of that argument can reasonably be denied by a proponent of MaxCon. If others are also controversial and can reasonably be denied by a proponent of MaxCon, that much the better for me.

Maximally Continuous Matter-Filled Regions to Simples (MCMRS):

Necessarily, for any maximally continuous matter-filled region of space R, there is an x such that x is a simple and x occupies R.

Notice, however, that MaxCon does not by itself entail MCMRS. It is consistent with MaxCon that there is a maximally continuous matter-filled region of space that is not occupied by anything at all. So, since MaxCon and MCMRO jointly entail MCMRS but MaxCon does not itself entail MCMRS, MaxCon does not entail MCMRO.

It is worth noting in connection with this point that although MaxCon does not entail MCMRO, Markosian endorses the latter:

> …MaxCon is consistent with there being a continuous, matter-filled region of space that is not occupied by any physical object. It might be desirable to add to MaxCon the following thesis, in order to have a theory of physical simples that rules out the possibility of matter without physical objects.
>
> **Against Matter Without Objects (AMWO):** Necessarily, if R is a continuous, matter-filled region of space, and there is no continuous, matter-filled region of space, R', such that R is a proper [subregion] of R', then there is a physical object that occupies R'.
>
> While I personally endorse AMWO, I have not officially conjoined it with MaxCon in my discussion because I want to consider MaxCon, as an answer to the Simple Question, independently of other, related issues. (ftnt. 23)[62]

Markosian's AMWO seems to be equivalent to our MCMRO.

Despite the fact that Markosian endorses MCMRO, I think that it can reasonably be denied by a proponent of MaxCon. Notice first that some may endorse MCMRO primarily because they endorse the following principle:

Matter-Filled Regions to Objects (MRO): Necessarily, for any matter-filled region of space R, there is an x such that x occupies R.[63]

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[62] Markosian (2004) calls the conjunction of MaxCon and AMWO 'MaxCon+' and endorses it (pp. 409-10).

However, this reason for endorsing MCMRO will likely not motivate a proponent of MaxCon, since she is unlikely to endorse MRO. For she is likely to believe that it is possible for there to be a simple that occupies an extended region of space none of whose proper subregions are occupied by any object. So since the proper subregions of such a region would be matter-filled, a proponent of MaxCon will likely deny MRO.

Thus, one of the primary reasons for *endorsing* MCMRO will not motivate many proponents of MaxCon. However, it is worth asking whether a proponent of MaxCon might have reasons to *deny* MCMRO. I think that there are at least two such reasons. First, if she believes that there could be gunky objects, then she might perform a so-called G.E. Moore-shift, denying MCMRO on the grounds that it is a premise in an argument for the conclusion that if MaxCon is true, there couldn't be such objects. In other words, a proponent of MaxCon who reasonably believes MaxCon and who reasonably believes that there could be gunky objects can reasonably deny MCMRO on that basis.

There is another, more interesting, reason a proponent of MaxCon might have to deny MCMRO. Suppose that MaxCon and MCMRO are both true and that at a certain time a cat, Cat, occupies a discontinuous region of space, Cat-Region, that

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[63] MRO has some of the same consequences as the conjunction of the liberal view of receptacles (see Hudson 2002 and Uzquiano 2006)—that is, the claim that every region of space is occupiable—with van Inwagen's (1981) Doctrine of Arbitrary Undetached Parts:

> **The Doctrine of Arbitrary Undetached Parts (DAUP)**: For every material object M, if R is the region of space occupied by M at time t, and if sub-R is any occupiable sub-region of R whatever, there exists a material object that occupies the region sub-R at t. (p. 123)

Both entail that any subregion of a region of space that is occupied by a material object is occupied as well.

consists entirely of a continuous tail-shaped region, Tail-Region, and a continuous tail-remainder-shaped region, Remainder-Region. Given MaxCon and MCMRO, at that time there is a simple, Tail, that occupies Tail-Region and a simple, Remainder, that occupies Remainder-Region and these simples are distinct.[64] Now it might happen that at some later time Tail is annihilated. But if that did happen, both Cat and Remainder would survive and each would occupy Remainder-Region. So, if MaxCon and MCMRO are both true, it might happen that two distinct objects, Cat and Remainder, occupy the very same region of space.[65,66] Thus, a proponent of MaxCon who believes that it is impossible for two distinct objects to occupy the very same region of space has a reason to deny MCMRO. I conclude, then, from the considerations adduced in this paragraph and in the preceding paragraph that a proponent of MaxCon can reasonably deny MCMRO.

I will now argue that a proponent of MaxCon can also reasonably deny SP. SP, remember, is the following claim:

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[64] Tail and Remainder are distinct because an object occupies a region of space just in case that region of space has as subregions all and only those regions that fall within that object. So, since Tail occupies Tail-Region, Tail-Region has as subregions all and only those regions that fall within Tail. But Tail-Region has none of the subregions of Remainder-Region as subregions (since Cat-Region is discontinuous and consists entirely of Tail-Region and Remainder-Region). Thus, none of the subregions of Remainder-Region fall within Tail and so Tail does not occupy Remainder-Region. But Remainder occupies Remainder-Region. Therefore, Tail and Remainder are distinct.

[65] This is, of course, a variant of van Inwagen's (1981) argument against DAUP.

[66] I should note that the argument makes assumptions that some might wish to deny. For instance, it assumes that a region such as Cat-Region is a receptacle. However, as long as a proponent of MaxCon can reasonably accept such assumptions, I will still have successfully shown that a proponent of MaxCon can reasonably reject MCMRO.

Subregions to Parts (SP): Necessarily, for all x, y, R, and S, if R and S are regions of space, R is a subregion of S, x occupies R, and y occupies S, then x is a part of y.

Again, there is a rather uninteresting reason a proponent of MaxCon might have to deny SP. If she reasonably believes MaxCon and reasonably believes that there could be gunky objects, then she may reasonably deny SP via a G.E. Moore-shift.

There are, in addition, more interesting reasons that a proponent of MaxCon might have to deny SP. First, a proponent of MaxCon might believe that there could be coincident objects—objects, that is, that are distinct yet occupy the very same region of space.[67] According to some metaphysicians, a statue and the lump of clay from which it is made, for instance, are distinct and yet occupy the very same region of space. Furthermore, some among these metaphysicians hold that the arm of the statue is a part of the statue but is not a part of the lump of clay although it occupies a subregion of the region occupied by the lump of clay. Thus, a proponent of MaxCon who reasonably accepts these claims can reasonably deny SP.

Not all proponents of MaxCon will accept the possibility of coincident objects, of course. Indeed, as we saw above, one reason a proponent of MaxCon might have for denying MCMRO is that she rejects the possibility of such objects. In addition, not all of those proponents of MaxCon who accept the possibility of coincident objects will accept that they could differ with respect to their parts. Thus, it

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[67] Hudson explicitly notes that his argument does not compel a proponent of MaxCon who accepts the possibility of coincident entities to reject the possibility of gunky objects (pp. 87-8).

is worth considering whether there are reasons a proponent of MaxCon might have to deny SP that do not commit her to the possibility of coincident objects.

There are. In a recent paper, Raul Saucedo (forthcoming) presents an argument against SP.[68] Thus, a proponent of MaxCon who reasonably accepts the premises of Saucedo's argument can reasonably deny SP. Saucedo's argument proceeds as follows. Any pattern of instantiation of any pairwise wholly distinct fundamental properties and relations is possible.[69,70] But, Saucedo says, the relations of being a part of, occupying, and being a subregion of are fundamental and pairwise wholly distinct (pp. 7-8).[71] Therefore, any pattern of instantiation of *being a part of, occupying*, and *being a subregion of* is possible. One pattern of instantiation of *being a part of, occupying,* and *being a subregion of* is the following: there are an x, y, R, and S such that R is a subregion of S, x occupies R, y occupies S, and x is not a part

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[68] Technically, the principle he considers is worded slightly differently. However, given certain plausible assumptions, it is equivalent to SP.

[69] Saucedo says that a property or relation is fundamental just in case it is one of David Lewis' perfectly natural properties and relations; that is, just in case it is among the 'properties and relations that explain cases of genuine similarity, and that constitute the supervenience base of the world' (p. 6). (See Lewis 1986.) It is more difficult to state what it is for two properties or relations to be wholly distinct. Here are some examples from Saucedo: the properties of being round and having a mass of one gram are wholly distinct, as are the properties of being round and being yellow, the properties of being round and having mass, and the properties of being round and being water; the properties of having mass of one gram and having mass of two grams are not wholly distinct, nor are the properties of being round and being round and yellow, the properties of being round and being shaped, and the properties of being water and being oxygen (p. 7).

[70] Saucedo calls this principle 'Pattern-to-Possibility' (p. 7).

[71] Actually, Saucedo does not explicitly claim that *being a subregion of* is fundamental nor that it is wholly distinct from being a part of and occupying. However, it is clear that he needs either this claim or the claim that being a subregion of can be analyzed in terms of parthood and/or occupation in order to argue against SP in the manner he does.

of y. So, possibly, there are an x, y, R, and S such that R is a subregion of S, x occupies R, y occupies S, and x is not a part of y. Therefore, SP is false.

Saucedo's argument raises many interesting issues that are beyond the scope of this chapter to address. However, it is clear that a proponent of MaxCon who reasonably accepts each of the premises of that argument can reasonably reject SP.[72] What is less clear is that she can do so without committing herself to the possibility of coincident objects. After all, the relation of identity is, plausibly, both a fundamental relation and wholly distinct from the relation of occupying. And anyone who accepts each of the premises of Saucedo's argument is committed both to the claim that occupying is a fundamental relation and to the claim that any pattern of instantiation of any pairwise wholly distinct fundamental properties and relations is possible. So it seems that a proponent of MaxCon who accepts each of the premises of Saucedo's argument is committed to the claim that the following pattern of instantiation of identity and occupation is possible: There are an x, y, and R such that x occupies R, y occupies R, and x is not identical to y. Thus, it would seem that a proponent of MaxCon who accepts each of the premises of Saucedo's argument is committed to the possibility of coincident objects.

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[72] Or, at least, it is relatively clear. On the formulation of MaxCon that we are considering, *being a region that falls within an object* is taken as primitive and occupation is defined in terms of it. Thus, it is unlikely that one who endorses that formulation of MaxCon will think that occupation is a fundamental relation. So she can not quite accept Saucedo's argument as it stands. However, if she thinks that *being a region that falls within an object*, *being a subregion of*, and *being a part of* are fundamental and pairwise wholly distinct, she can offer an argument against SP that is very similar to Saucedo's.

However, due to certain technical features of Saucedo's notion of a pattern of instantiation, a proponent of MaxCon who accepts each of the premises of Saucedo's argument need not be committed to the possibility of coincident objects. To see why this is so, let L be a quantified first-order language such that each predicate of L expresses a fundamental property or relation and each fundamental property or relation is expressed by a predicate of L. Then, according to Saucedo, something is a pattern of instantiation of some pairwise wholly distinct fundamental properties and relations $P_1 \ldots P_n$ just in case it is expressed by a sentence S of L such that (i) S contains only the logical vocabulary of L and predicates that express $P_1 \ldots P_n$ and (ii) there is a model M such that S is true in M and every sentence S' of M that contains only the logical vocabulary of L and only one of the predicates that express $P_1 \ldots P_n$ and that expresses a necessary truth is true in M (p. 10). But the identity relation will be expressed by part of the logical vocabulary of L.[73] So a sentence of L that

[73] If the claim that any pattern of instantiation of any pairwise wholly distinct fundamental properties and relations is possible is to be at all plausible, the identity relation will have to be expressed by part of the logical vocabulary of L. To see why this is so, suppose that it is not. Then consider the following sentence of L in which '=' expresses the identity relation and 'P' is another predicate of L that expresses fundamental property that is wholly distinct from the identity relation:

(S) $\exists x \exists y\ (x=y\ \&\ Px\ \&\ {\sim}Py)$

Given the assumption that the identity relation is not expressed by part of the logical vocabulary of L, (S) contains only the logical vocabulary of L and predicates that express pairwise wholly distinct fundamental properties and relations. In addition, given that assumption, there is a model M such that (S) is true in M and every sentence S' of M that contains only the logical vocabulary of L and only one of the predicates that express the identity relation and the property expressed by 'P' and that expresses a necessary truth is true in M. So, if the identity relation is not part of the logical vocabulary of L, (S) expresses a pattern of instantiation of the identity relation and the property expressed by 'P'. But then, according to the principle that any pattern of instantiation of any pairwise wholly distinct fundamental properties and relations is possible, the proposition expressed by (S) is possible. The proposition expressed by (S) is a violation of the Principle of the Indiscernibility of expresses the claim that it is not the case that there are an x, y, and R such that x occupies R, y occupies R, and x is not identical to y contains only the logical vocabulary of L and a predicate of L that expresses the occupation relation. Thus, since one who denies the possibility of coincident objects will claim that such a sentence expresses a necessary truth, she will hold that the claim that there are an x, y, and R such that x occupies R, y occupies R, and x is not identical to y is not a pattern of instantiation of the identity relation and occupation. Therefore, a proponent of MaxCon who denies the possibility of coincident objects is not committed to that possibility by affirming the claim that any pattern of instantiation of any pairwise wholly distinct fundamental properties and relations is possible.

I conclude that a proponent of MaxCon can reasonably deny SP. She can do so either via a G.E. Moore-shift, by accepting the possibility of coincident objects of a certain sort, or by accepting the premises of Saucedo's argument against SP. In addition, as we saw earlier, a proponent of MaxCon can reasonably deny MCMRO. Thus, Hudson's argument for the conclusion that if MaxCon is true, then there couldn't be gunky objects includes among its premises substantive metaphysical claims that a proponent of MaxCon can reasonably deny.

Conclusion

This chapter has been concerned with the following argument inspired by Hudson:

Identicals (PII), though. So, since a proponent of the principle that any pattern of instantiation of any pairwise wholly distinct fundamental properties and relations is possible will certainly not want to deny PII, she will have to hold that the identity relation is part of the logical vocabulary of L.

1. If MaxCon is true, then there couldn't be gunky objects.
2. There could be gunky objects.
3. Therefore, MaxCon isn't true.

In the previous section, however, I showed that Hudson's argument in favor of premise (1) includes substantive metaphysical claims that a proponent of MaxCon can reasonably deny. As a result, Hudson's argument is not compelling. I conclude that in the absence of any independently compelling reasons to accept premise (1) of the above argument, a proponent of MaxCon needn't deny the possibility of gunky objects nor need a proponent of the possibility of gunky objects deny MaxCon.

There is still one interesting question I would like to address: What sorts of gunky objects can a proponent of MaxCon reasonably countenance? Since MaxCon entails that any object that occupies a maximally continuous matter-filled region is a simple, and hence not a gunky object, a proponent of MaxCon cannot reasonably countenance the possibility of a gunky object that occupies such a region. Consider, however, the following sorts of gunky object:

Sort 1: A gunky object that occupies a discontinuous region of space

Sort 2: A gunky object that occupies a proper subregion of a maximally continuous matter-filled region of space

A proponent of MaxCon can, I claim, reasonably countenance the possibility of gunky objects of both Sort 1 and Sort 2. Consider first gunky objects of Sort 1. To countenance the possibility of such objects, a proponent of MaxCon must deny either MCMRO or SP. For if she conceded MCMRO, she would have to accept that every maximally continuous matter-filled subregion of the region occupied by such a gunky object would be occupied by a simple and if she conceded SP, she would have to accept that any such simple would be a part of that gunky object. So, since the region of space occupied by such a gunky object would have at least one maximally continuous matter-filled subregion and it is impossible for a simple to be a part of a gunky object, a proponent of MaxCon must deny either MCMRO or SP to accept the possibility of gunky objects of Sort 1.

Consider now gunky objects of Sort 2. Again, a proponent of MaxCon who countenances the possibility of such objects must deny either MCMRO or SP. For if she conceded MCMRO, she would have to accept that a gunky object of Sort 2 would occupy a proper subregion of a region occupied by a simple; and if she conceded SP, she would have to accept that such a gunky object would be a part of that simple. So, since it is impossible for a gunky object to be a part of a simple, a proponent of MaxCon must deny either MCMRO or SP to accept the possibility of gunky objects of Sort 2.

Thus, to countenance the possibility of gunky objects of either Sort 1 or Sort 2, a proponent of MaxCon must either deny MCMRO or SP. This is a pleasing symmetry. Further, since we have seen in this section that a proponent of MaxCon can reasonably deny these claims, we have shown that a proponent of MaxCon can reasonably accept the possibility of gunky objects of Sort 1 and of Sort 2.

Can a proponent of MaxCon reasonably accept the possibility of gunky objects that are of neither Sort 1 nor Sort 2? No. To see this, notice that any region occupied by a gunky object will be either a discontinuous region or a continuous region of space. Now any gunky object that occupies a discontinuous region of space will be a gunky object of Sort 1. On the other hand, any gunky object that occupies a continuous region of space will occupy a matter-filled region of space. So, since every region of space is a subregion of itself, any gunky object that occupies a continuous region of space will occupy a subregion of a continuous matter-filled region of space. But every continuous matter-filled region of space is a subregion of a maximally continuous matter-filled region of space. Consequently, any gunky object that occupies a continuous region of space will occupy a subregion of a maximally continuous matter-filled region of space. However, a proponent of MaxCon cannot accept the possibility of gunky objects that occupy maximally continuous matter-filled regions of space. Thus, given MaxCon, any gunky object that occupies a continuous region of space will occupy a proper subregion of a maximally continuous matter-filled region of space, and thus be a gunky object of Sort 2. Therefore, any gunky object will be of either Sort 1 or Sort 2, given MaxCon.

Appendix: A Reconstruction of Hudson's Argument

In this appendix, I present my reconstruction of Hudson's argument. I should note, however, that in my reconstruction, I do not adhere slavishly to the argument's surface structure nor to the exact phrasing of its premises. Instead, I attempt to uncover the reasoning that underlies it. For instance, my reconstruction does not contain a premise corresponding exactly to the inference Hudson makes when he says: '…M exactly occupies R. But this fact, together with the fact that M is a simple, guarantees that no subregion of R is a subregion of any region that is exactly occupied by a material object (unless that material object has M as a part or is identical to M)' (p. 87). It does not include such a premise because, I suspect, one who believes the claim that no subregion of a region occupied by a simple is a subregion of a region occupied by a material object that neither has that simple as a part nor is identical to that simple believes it because she accepts other, more basic, claims from which it follows.[74] In my reconstruction, I attempt to uncover these more basic claims.

Having said this, let me now turn to my reconstruction, which begins by assuming, for reductio, that:

1. For all x, x is a simple iff x occupies a maximally continuous matter-filled region of space, and there is a gunky object. [Assumption for reductio]

By conjunction elimination on (1) and existential instantiation, we derive:

2. h is a gunky object.

Now from (1), (2), and necessitated premises, we argue as follows:

[74] One can capture the inference in question by employing the following principle:

> **Simples, Subregions, and Parts (SSP)**: Necessarily, for all x and R, if x is a simple, R is a region of space, and x occupies R, then for all y, S, and T, if S is a subregion of R, S is a subregion of T, and y occupies T, then x is a part of y.

Acceptance of SSP, however, can be justified by appeal to the following principle, which I discuss above in §3 and which appears as premise (18) in my reconstruction:

> **Subregions to Parts (SP)**: Necessarily, for all x, y, R, and S, if R and S are regions of space, R is a subregion of S, x occupies R, and y occupies S, then x is a part of y.

Suppose that o is a simple, R is a region of space, and o occupies R. Now let S be a subregion of R, T be a region of space such that S is a subregion of T, and o* be an object that occupies T. There are two possibilities: (i) T is a subregion of R and (ii) R is a subregion of T. If SP is true, then given either (i) or (ii), o is a part of o*. To see this, suppose first that (i) is true. Then, if SP is true, o* is a part of o. However, since o is a simple, o is identical to every part of o. So, if SP is true, o is identical to o*. But everything identical to o* is a part of o*, by the reflexivity of parthood. So, given SP, if (i) is true, o is a part of o*. Now suppose that (ii) is true. Then, if SP is true, o is a part of o*, since o occupies a subregion of the region occupied by o*. So, if SP is true, o is a part of o*. Therefore, if SP is true, then SSP is true. Acceptance of SP justifies acceptance of SSP.

3. Necessarily, for all x, if x is a gunky object, then there is an R such that R is a region of space and x occupies R.[75]
4. Therefore, there is an R such that R is a region of space and h occupies R. [From (2) and (3)]
5. Therefore, T is a region of space and h occupies T. [From (4) by existential instantiation]
6. Necessarily, for all regions of space R such that there is something that occupies R, R is a matter-filled region of space.
7. Therefore, T is a matter-filled region of space. [From (5) and (6)]
8. Necessarily, for all matter-filled regions of space R, R is a continuous matter-filled region of space or R is a discontinuous matter-filled region of space.
9. Therefore, T is a continuous matter-filled region of space or T is a discontinuous matter-filled region of space. [From (7) and (8)]

We have now reached a dilemma. If we can show that neither horn of the dilemma is true, we will have shown that our original assumption is false. We do so by first assuming for reductio the second horn of the dilemma and showing that it is false, as follows:

10. T is a discontinuous matter-filled region of space. [Assumption for reductio]
11. Necessarily, for all discontinuous matter-filled regions of space R, there is an S such that S is a maximally continuous matter-filled region of space and S is a subregion of R.
12. Therefore, there is an S such that S is a maximally continuous matter-filled region of space and S is a subregion of T. [From (10) and (11)]
13. Therefore, U is a maximally continuous matter-filled region of space and U is a subregion of T. [From (12) by existential instantiation]

[75] Remember that we are restricting our attention to material/physical objects and assuming that each material/physical object is spatially located.

14. Necessarily, for any maximally continuous matter-filled region of space R, there is an x such that x occupies R.
15. Therefore, there is an x such that x occupies U. [From (13) and (14)]
16. Therefore, i occupies U. [From (15) by existential instantiation]
17. Therefore, i is a simple. [From (1), (13), and (16)]
18. Necessarily, for all x, y, R, and S, if R and S are regions of space, R is a subregion of S, x occupies R, and y occupies S, then x is a part of y.
19. Therefore, i is a part of h. [From (5), (13), (16), and (18)]
20. Necessarily, for any simple x and gunky object y, x is not a part of y.
21. Therefore, i is not a part of h. [From (2), (17), and (20)]

But now we have a contradiction between (19) and (21). So we can conclude that the second horn of the dilemma is false; that is:

22. It is not the case that T is a discontinuous matter-filled region of space. [By reductio]

Let us turn, then, to showing that the first horn of the dilemma is also false:

23. T is a continuous matter-filled region of space. [Assumption for reductio]
24. Necessarily, for all continuous matter-filled regions of space R, there is an S such that S is a maximally continuous matter-filled region of space and R is a subregion of S.
25. Therefore, there is an S such that S is a maximally continuous matter-filled region of space and T is a subregion of S. [From (23) and (24)]
26. Therefore, V is a maximally continuous matter-filled region of space and T is a subregion of V. [From (25) by existential instantiation]
27. Therefore, there is an x such that x occupies V. [From (14)[76] and (26)]
28. Therefore, j occupies V. [From (27) by existential instantiation]

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[76] Note that although (14) originally appeared in a different reductio argument, it is legitimate to appeal to it here since it was a premise of that reductio, not derived from the assumption for reductio. Similar remarks apply to use of (18) to derive (30), below.

29. Therefore, j is a simple. [From (1), (26), and (28)]
30. Therefore, h is a part of j. [From (5), (18), (26), and (28)]
31. Necessarily, for any gunky object x and simple y, x is not a part of y.
32. Therefore, h is not a part of j. [From (2) and (29)]

(32) contradicts (30), however. So the first horn of our dilemma is false:

33. It is not the case that T is a continuous matter-filled region of space. [By reductio]

Having thus shown that neither horn of our dilemma is true, we can conclude that our original assumption is false:

34. Therefore, it is not the case that (for all x, x is a simple iff x occupies a maximally continuous matter-filled region of space, and there is a gunky object). [By reductio]

My reconstruction of Hudson's argument is not quite complete yet, for we have yet to derive the desired conclusion that if necessarily, for all x, x is a simple iff x occupies a maximally continuous matter-filled region of space, then it is not the case that possibly, there is a gunky object. To reach this desired conclusion, notice first that (34) is equivalent to:

35. Therefore, either it is not the case that for all x, x is a simple iff x occupies a maximally continuous matter-filled region of space or it is not the case that there is a gunky object.

Which entails:

36. Therefore, if for all x, x is a simple iff x occupies a maximally continuous matter-filled region of space, then it is not the case that there is a gunky object.

Now since we have reached (36) using only valid inference rules and necessitated claims, we can infer:

37. Therefore, necessarily, if for all x, x is a simple iff x occupies a maximally continuous matter-filled region of space, then it is not the case that there is a gunky object.

From which it follows by the modal inference rule K that:

38. Therefore, if necessarily, for all x, x is a simple iff x occupies a maximally continuous matter-filled region of space, then necessarily, it is not the case that there is a gunky object.

And from this, given that possibility is the dual of necessity (and valid inference rules concerning the use of double negation within the possibility operator), we can derive our desired conclusion:

39. Therefore, if necessarily, for all x, x is a simple iff x occupies a maximally continuous matter-filled region of space, then it is not the case that possibly, there is a gunky object.

Which is simply a slightly more formal way of stating the intended conclusion of Hudson's argument: that if MaxCon is true, there couldn't be gunky objects.

In this appendix, I presented my reconstruction of Hudson's argument. In this reconstruction, the premises are separated from one another and numbered, making it easier to identify the premises of the argument and thus to determine whether these include any substantive metaphysical claims that a proponent of MaxCon can reasonably deny. I identified the premises in §2 above. Here they are stated in claims (3), (6), (8), (11), (14), (18), (20), (24), and (31). Furthermore, in §3 above, I argued that two of these premises are substantive metaphysical claims that a proponent of

MaxCon can reasonably deny. These two premises are stated in (14) and (18). Thus, this reconstruction has been helpful in showing that Hudson’s argument is not compelling.

Chapter 5: An Elegant Picture of Simplicity

Introduction

In the previous chapter, I argued that MaxCon should be reformulated in light of the discussion in Chapters 1 and 2 and defended a suitably modified version of MaxCon against an objection based on Hud Hudson's argument for the claim that if MaxCon is true, then there couldn't be gunky objects. In particular, I argued that Hudson's argument is not compelling and thus, unless there is some independently compelling argument in favor of its conclusion, neither is the objection that is based upon it. In this chapter, I continue my discussion of (the modified version of) MaxCon.[77]

Remember that MaxCon simply says that necessarily, for all x, x is a simple if and only if x is a maximally continuous object—or, alternatively, that necessarily, for all x, x is a simple if and only if x occupies a maximally continuous matter-filled region of space. This is certainly an interesting claim, one that is worthy of discussion. But notice that, by itself, MaxCon tells us very little.[78] For example, MaxCon doesn't tell us what shapes or sizes simples could be. In other words, for any shape, MaxCon doesn't entail that there could be a simple with that shape, and similarly for sizes. Furthermore, MaxCon doesn't tell us much about what regions of space simples could occupy; in particular, it doesn't tell us anything about the

[77] In the remainder of this chapter, I will use 'MaxCon' to refer to the modified version of MaxCon introduced in the previous chapter, dropping 'the modified version of'.
[78] In fact, MaxCon says even less than it appears to say, since the quantifier that appears therein is restricted to physical objects!

structure of such regions except that they must be continuous. Finally, MaxCon doesn't even entail that there could be simples!

This is nothing to complain about, of course. MaxCon is not intended to tell us these things. Rather, its purpose is to identify those properties that are characteristic of simples, those properties such that necessarily, all and only simples have them. Furthermore, MaxCon clearly does have consequences concerning whether there could be simples, concerning the structures of the regions of space that could be occupied by simples, and concerning what shapes or sizes they could be. After all, answers to those questions are entailed by the conjunction of MaxCon with certain very plausible claims about the possibility of objects occupying maximally continuous matter-filled regions with various structures and with various shapes and sizes. Thus, the claim that there could be simples is entailed by the conjunction of MaxCon with the claim that there could be an object that occupies a maximally continuous matter-filled region; the claim that there could be a simple that occupies a region of space with proper subregions is entailed by the conjunction of MaxCon with the claim that there could be an object that occupies a maximally continuous matter-filled region with subregions; and the claim that there could be a triangular simple is entailed by the conjunction of MaxCon with the claim that there could be an object that occupies a triangular maximally continuous matter-filled region.

Not all alleged consequences of MaxCon are entailed by the conjunction of MaxCon with claims about the possibility of objects occupying maximally continuous matter-filled regions with various structures and with various shapes and sizes, however. In this chapter, I discuss four alleged consequences of MaxCon that are not so entailed, each of which is alleged to be a consequence of MaxCon in an objection to that view.

In §1, I present these four objections. Then, in §2, I introduce two theses concerning simples. These theses are interesting for three reasons. First, although they are not entailed by MaxCon, MaxCon follows from their conjunction. Second, conjoining them results in an elegant picture of simplicity. (I discuss these first two reasons in §2.) Third, as I argue in §3, each of the alleged consequences of MaxCon introduced in §1 follows from these two theses along with some very plausible assumptions. The fact that these theses have these features suggests the following way of arguing for the claim that MaxCon has these consequences. In particular, let C be one of the alleged consequences of MaxCon introduced in §1. Then one can formulate the following argument for the claim that C is a consequence of MaxCon:

1. If MaxCon is true, then the two principles introduced in §2 are true.
2. If the two principles introduced in §2 are true, then C.
3. Therefore, if MaxCon is true, then C.

In §4, I discuss the first premise of this argument. In particular, I discuss whether there are independently plausible metaphysical principles from whose conjunction with MaxCon the two theses introduced in §2 follow and whether a proponent of MaxCon can reasonably deny those principles. I conclude that although there are such principles, a proponent of MaxCon can reasonably deny them. Thus, (i) absent other reasons to think that it is true, a proponent of MaxCon can reasonably deny the first premise of the argument above; and (ii) absent other reasons to think that C is a consequence of MaxCon, a proponent of MaxCon can reasonably deny that it is and can reasonably reject the objections to her view presented in §1.

1. Some Alleged Consequences of MaxCon

In this section, I identify some alleged consequences of MaxCon by considering four objections to MaxCon according to which it has those consequences. Three of these objections were first discussed in Markosian 1998a. The fourth is inspired by the objection that Kris McDaniel (2007) takes to refute MaxCon. I note that none of these alleged consequences is entailed by the conjunction of MaxCon with claims about the possibility of objects occupying maximally continuous matter-filled regions with various structures and with various shapes and sizes.

Markosian (1998a) considers several objections to MaxCon. He formulates the first such objection as follows:

> …MaxCon entails that a perfectly solid sphere, for example, would be a simple, even if it were rather large and even if it were physically divisible…
>
> Someone might object to the... above mentioned [consequence] of MaxCon by giving the following argument. (Let 'Spero' refer to some perfectly solid sphere.)
>
> **An Argument Against MaxCon**
>
> (i) If any object has some extension, then it has two halves.
> (ii) If any object has two halves, then it has at least two proper parts.
> (iii) Spero has some extension…
> (iv) [Therefore,] Spero has at least two proper parts. (p. 222)

Unfortunately, given this formulation of the objection, it is not entirely clear what objectionable consequence MaxCon is being alleged to have. For this reason, I propose the following reformulation:

1. If MaxCon is true, then: possibly, there is an object that occupies a region that has a left half and right half although that object has neither a left half nor a right half.
2. Necessarily, every object that occupies a region that has a left half and a right half has either a right half or a left half.
3. Therefore, MaxCon isn't true.

Here the allegedly objectionable consequence is clear. According to the objection, MaxCon has the consequence that possibly, there is an object that occupies a region that has a left half and right half although that object has neither a left half nor a right half.

This alleged consequence is not entailed by the conjunction of MaxCon with claims about the possibility of objects occupying maximally continuous matter-filled regions with various structures and with various shapes and sizes, however. The conjunction of MaxCon and the claim that there could be an object that occupies a maximally continuous matter-filled region that has a left half and a right half entails that possibly, there is a simple that occupies a region that has a left half and a right half. But their conjunction does not seem to entail that there could be such a simple that has neither a left half nor a right half. For it is consistent with their conjunction that necessarily, if there is a simple that occupies a region that has a left half and a right half, then there is an object that occupies the left half of that region and an object that occupies its right half. And this would seem to rule out the possibility of a simple occupying a region that has a left half and a right half without that simple having a left half and a right half.

After responding to this objection, Markosian considers another:

> Imagine a statue in the shape of Joe Montana. Let the statue be perfectly solid, so that it, like Spero, occupies a continuous region of space. Now it would seem very natural to say that such a statue had a right arm. But according to MaxCon, there is no such thing as a part of the statue that can accurately be described as "the statue's right arm." That's because MaxCon entails that such a statue would have no parts at all; it would be a simple. (p. 224)

Here a version of this objection designed to make the allegedly objectionably consequence of MaxCon explicit:

1. If MaxCon is true, then: possibly, there is a Joe Montana-shaped object[79] that has no right arm.
2. Necessarily, there is no Joe Montana-shaped object that has no right arm.
3. Therefore, MaxCon isn't true.

Again, the alleged consequence isn't entailed by the conjunction of MaxCon with claims about the possibility of objects occupying maximally continuous matter-filled regions with various structures and with various shapes and sizes. The claim that possibly, there is a Joe Montana-shaped simple, *is* entailed by MaxCon and the claim that possibly, there is an object that occupies a Joe Montana-shaped maximally continuous matter-filled region. But, like above, the conjunction of these two claims is consistent with the claim that necessarily, any such simple has a right arm.

These first two objections are somewhat similar to one another. Here is another, rather different, objection considered by Markosian:

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[79] Markosian speaks of the possibility of a Joe Montana-shaped statue whereas I speak of the possibility of a Joe Montana-shaped object. I take it that this is not an important difference, however, since I take Markosian's talk of a statue to be merely illustrative: Insofar as it is implausible that it is possible for there to be a Joe Montana-shaped statue that has no right arm, it is also implausible that it is possible for there to be a Joe Montana-shaped object that has no right arm.

> Suppose that there are two qualitatively similar, maximally continuous objects, A and B. Each one will of course count as a simple, according to MaxCon. Suppose that A and B move together until eventually they are actually touching. At that point, according to MaxCon, a strange thing happens to A and B: they go out of existence. And in the general vicinity of the regions occupied by A and B just before the "merger" there will come into being a new object, C, constituted by the matter that previously constituted A and B. (p. 225)

Here is an attempt to formulate this objection more precisely and in such a way that it is clear what consequence MaxCon is alleged to have. Begin by considering the following situation:

> *Situation S:* At 3 AM, there are two qualitatively similar, maximally continuous objects, A and B. A occupies a continuous open spherical region, R1, at 3 AM, while B occupies a continuous closed spherical region, R2, then, and the distance between A and B at 3 AM is 10 feet. Between 3 AM and 3:10 AM, A and B move continuously towards one another at a rate of 1 foot per minute. Furthermore, for each time t between 3 AM and 3:10 AM, A occupies a continuous region of the same shape and size as R1 at t and B occupies a continuous region of the same shape and size as R2 at t. Finally, neither A nor B moves discontinuously at 3:10 AM and neither changes its size or its shape then.

Then the objection can be formulated more precisely as follows:

1. If MaxCon is true, then: necessarily, if Situation S obtains, then A and B go out of existence at 3:10 AM.
2. Possibly, Situation S obtains but A and B do not go out of existence at 3:10 AM.
3. Therefore, MaxCon isn't true.

The alleged consequence here is a bit harder to grasp than the alleged consequences in the previous cases. However, it does not appear to be entailed by the conjunction of MaxCon with the sort of claims under consideration. What is clear is that necessarily, if Situation S obtains, then either A and B go out of existence at 3:10 AM or A and B occupy proper subregions of a maximally continuous matter-filled region then. But MaxCon doesn't appear to rule out the possibility of the second disjunct. Rather, what MaxCon does seem to entail is that necessarily, if Situation S obtains, then either A and B go out of existence at 3:10 AM or A and B are not simples then. Furthermore, conjoining MaxCon with a claim about the possibility of objects occupying maximally continuous matter-filled regions with various structures and with various shapes and sizes won't help, since the alleged consequence concerns a necessity rather than a possibility.

The final objection to MaxCon that I would like to consider is inspired by an objection due to Kris McDaniel (2007). According to McDaniel:

> The main argument against [MaxCon] is based on the possibility of co-located point-sized objects. Two objects are co-located if they occupy the same region of space (at the same time). The argument is as follows: (1) co-located point-sized objects are possible; (2) if co-located point-sized objects are possible, then mereologically complex point-sized objects are also possible. But then… MaxCon [is] false. (p. 239)

Again, I would like to consider a formulation of this objection that makes it clear what the allegedly objectionable consequence of MaxCon is:

1. If MaxCon is true, then: necessarily, there are no co-located point-sized objects.
2. Possibly, there are co-located point-sized objects.
3. Therefore, MaxCon isn't true.[80]

[80] Kris McDaniel has claimed (in correspondence) that this is an uncharitable reconstruction of his argument. I disagree. McDaniel clearly endorses each of the premises of this reconstruction in the passage in which he presents his argument. Of course, whereas he explicitly states premise (2) of the reconstruction, he does not explicitly state premise (1). However, premise (1) is entailed by two claims he clearly endorses. One of these is the claim that if co-located point-sized objects are possible, then mereologically complex point-sized objects are possible. Since this claim is a premise of McDaniel's argument, it is a claim that he clearly endorses. The other is the claim that if MaxCon is true, then mereologically

Here the alleged consequence of MaxCon is not entailed by MaxCon, which is consistent with the possibility of co-located point-sized objects. By itself, all MaxCon tells us is that if there are such objects, then either each of them is a simple or each of them occupies a proper subregion of a maximally continuous matter-filled region. And, again, conjoining MaxCon with a claim about the possibility of objects occupying maximally continuous matter-filled regions with various structures and with various shapes and sizes won't help, for the same reasons it won't help in the case of the previous objection.

In this section, I considered four objections to MaxCon, each of which alleges that MaxCon has a certain consequence. I have argued that, in each case, the alleged consequence is not entailed by the conjunction of MaxCon with claims about the possibility of objects occupying maximally continuous matter-filled regions with various structures and with various shapes and sizes.

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complex point-sized objects are not possible. It is clear that McDaniel endorses this claim as well, since otherwise the conclusion of his argument would not follow from its premises.

I conclude, then, that my reconstruction of McDaniel's argument is not uncharitable. I note, however, that it *would* be uncharitable for me to claim that the reasons for accepting premise (1) of the reconstruction that are explored in this chapter are the reasons McDaniel gives for accepting that premise. It is clear that this is *not* so. Luckily, though, I never claim that it *is* so. Remember, my sole concern in this chapter is with *one* particularly interesting chain of reasoning in favor of the claim that MaxCon has the consequence indicated in premise (1) of the reconstruction:

> If MaxCon is true, then the two theses to be introduced in §2 are true. If the two theses to be introduced in §2 are true, then: necessarily, there are no co-located point-sized objects. Therefore, if MaxCon is true, then: necessarily, there are no co-located point-sized objects.

I argue that a proponent of MaxCon can reasonably deny the first claim in this chain of reasoning and so need not find this chain of reasoning compelling. However, I leave it open whether other chains of reasoning in favor of the claim that MaxCon has the consequence indicated in premise (1) of the reconstruction are compelling. These other chains of reasoning include McDaniel's.

2. An Elegant Picture of Simplicity

Although the conjunction of MaxCon with such claims does not entail the alleged consequences discussed in the previous section, these consequences follow from the conjunction of the following two theses, along with some plausible assumptions[81]:

> **Maximally Continuous Matter-Filled Regions to Simples (MCMRS)**: Necessarily, for any maximally continuous matter-filled region of space R, there is an x such that x is a simple and x occupies R.
>
> **The Loneliness of Simples (LS)**: Necessarily, for all x, y, R, and S, if R and S are regions of space, R is a subregion of S, x occupies R, y occupies S, and y is a simple, then x is identical to y.

In addition, MCMRS and LS have some other interesting features. First, although neither of them is entailed by MaxCon, their conjunction entails MaxCon. Second, together they yield a very elegant picture of simplicity. In this section, I will first show that MCMRS and LS have these features. I will then show that each of the alleged consequences of MaxCon discussed in the previous section follows from the conjunction of these two theses along with plausible assumptions.

It is clear that MaxCon entails neither MCMRS nor LS. I discussed why MaxCon does not entail MCMRS in §3 of Chapter 4. As I noted there, MaxCon does entail that necessarily, for any maximally continuous matter-filled region of space R, if there is an x such that x occupies R, then there is an x such that x is a simple and x

[81] Note: Those consequences do not follow from MaxCon along with these same assumptions.

occupies R; in other words, MaxCon entails that it is impossible for a maximally continuous matter-filled region to be occupied without being occupied by a simple. But MaxCon does not entail that necessarily, for any maximally continuous matter-filled region R, there is an x such that x occupies x. Thus, MaxCon does not entail MCMRS.

MaxCon also does not entail LS. Now it is true that necessarily, for all R and S, if R and S are regions of space and R is a subregion of S, then R is either a proper subregion of S or R is identical to S. Thus, it is also true that necessarily, for all x, y, R, and S, if R and S are regions of space, R is a subregion of S, x occupies R, y occupies S, y is a simple, and x is distinct from y, then either x occupies a proper subregion of a region occupied by a simple that is distinct from x or x occupies a region occupied by a simple that is distinct from x. Given this, MaxCon entails:

(A) Necessarily, for all x, y, R, and S, if R and S are regions of space, R is a subregion of S, x occupies R, y occupies S, y is a simple, and x is distinct from y, then either x occupies a proper subregion of a maximally continuous matter-filled region that is occupied by simple that is distinct from x or x occupies a maximally continuous matter-filled region that is also occupied by a simple that is distinct from x.

Now it is also true that necessarily, no proper subregion of a maximally continuous matter-filled region is a maximally continuous matter-filled region. Furthermore, MaxCon simply is the claim that necessarily, for all x, x is a simple if and only if x occupies a maximaly continuous matter-filled region of space. Thus, MaxCon entails:

(B) Necessarily, for all x, y, R, and S, if R and S are regions of space, R is a subregion of S, x occupies R, y occupies S, y is a simple, and x is distinct from y, then either x is a non-simple that occupies a proper subregion of a region occupied by a simple that is distinct from x or x is a simple that occupies a region that is also occupied by a simple that is distinct from x.

However, MaxCon does not entail that necessarily, no non-simple occupies a proper subregion of a region occupied by a simple that is distinct from it, nor does it entail that necessarily, no simple occupies a region that is also occupied by a simple that is distinct from it. Thus, MaxCon does not entail LS.

Although MaxCon entails neither MCMRS nor LS, MaxCon does follow from them. MaxCon is equivalent to the conjunction of two theses. The first thesis, which I will call 'Thesis 1', is that necessarily, for all x, if x occupies a maximally continuous matter-filled region of space, then x is a simple. The second thesis, 'Thesis 2', is that necessarily, for all x, if x is a simple, then x occupies a maximally continuous matter-filled region of space. I will consider each of these theses in turn and show that both follow from MCMRS and LS.

To see that MCMRS and LS entail Thesis 1, first suppose that MCMRS and LS are true and let o be an object that occupies a maximally continuous region of space. Next, let R be the maximally continuous region of space occupied by O. By MCMRS, there is a simple that occupies R. Call it 's'. Since necessarily, every region is a subregion of itself, it follows that R is a region of space, R is a subregion of R, o occupies R, s occupies R, and s is a simple. But then, by LS, o is identical to s, and thus o is a simple. But o was an arbitrarily chosen object occupying a maximally continuous region of space. Thus, for all x, if x occupies a maximally continuous region of space, then x is a simple. Furthermore, each of the claims used to derive this result was a necessitated claim. Thus, necessarily, for all x, if x occupies a maximally continuous region of space, then x is a simple. Therefore, on the supposition that MCMRS and LS are true, Thesis 1 follows.

Now to see that Thesis 2 follows from MCMRS and LS, assume that MCMRS and LS are true and suppose for reductio that there is a simple that does not occupy a maximally continuous matter-filled region of space. Let s be such a simple and let R be the region of space occupied by s. Necessarily, for all x and R, if R is a region of space and x occupies R, then R is matter-filled. Thus, R is matter-filled. But necessarily, every matter-filled region of space is a proper subregion of a maximally continuous matter-filled region of space, is a maximally continuous matter-filled region of space, or has maximally continuous matter-filled region as a proper subregion. R, then, is a proper subregion of a maximally continuous matter-filled region of space, is a maximally continuous matter-filled region of space, or has maximally continuous matter-filled region as a proper subregion. But, by supposition, R is not a maximally continuous matter-filled region. So R is a proper subregion of a maximally continuous matter-filled region of space or R has maximally continuous matter-filled region as a proper subregion. Let's take each case in turn.

Suppose that R is a proper subregion of a maximally continuous matter-filled region of space. Call the maximally continuous matter-filled region of space of which

R is a proper subregion ‘R+’. By MCMRS, there is a simple that occupies R+. Let s+ be such a simple. Thus, R and R+ are regions of space, R is a subregion of R+, s occupies R, s+ occupies R+, and s+ is a simple. By LS, then, s is identical to s+. But necessarily, for all x, y, R, and S, if R and S are regions of space, x occupies R, y occupies S, and R is distinct from S, then x is not identical to y. Thus, since R is distinct from R+, s is not identical to s+. Contradiction. Therefore, our supposition is false: R is not a proper subregion of a maximally continuous matter-filled region of space.

Suppose, on the other hand, that R has a maximally continuous matter-filled region of space as a proper subregion. Let R- be such a proper subregion of R. By MCMRS, there is a simple that occupies R-. Let s- be such a simple. Thus, R- and R are regions of space, R- is a subregion of R, s- occupies R-, s occupies R, and s is a simple. By LS, then, s- is identical to s. But necessarily, for all x, y, R, and S, if R and S are regions of space, x occupies R, y occupies S, and R is distinct from S, then x is not identical to y. Thus, since R- is distinct from R, s- is not identical to s. Contradiction. Therefore, this supposition is false as well: R does not have a maximally continuous matter-filled region of space as a proper subregion.

Thus, neither the claim that R is a proper subregion of a maximally continuous matter-filled region of space nor the claim that R has a maximally continuous matter-filled region of space as a proper subregion is true. We saw above, however, that one of these must be true given our supposition that there is a simple that does not occupy a maximally continuous matter-filled region of space. So, that supposition must be false: for all x, if x is a simple, then x occupies a maximally continuous matter-filled region of space. Each of the claims used to derive this result was a necessitated claim, however. Thus, necessarily, for all x, if x is a simple, then x occupies a maximally continuous matter-filled region of space. Therefore, assuming that MCMRS and LS are true, Thesis 2 follows.

I have shown that Thesis 1 and Thesis 2 both follow from the conjunction of MCMRS and LS. However, MaxCon is equivalent to the conjunction of Thesis 1 and Thesis 2. Thus, MaxCon follows from the conjunction of MCMRS and LS.

The conjunction of MCMRS and LS yields an elegant picture of simplicity. Since MaxCon follows from these two theses, part of this picture is that all and only those objects that occupy maximally continuous matter-filled regions of space are simples. But there is more to the picture. In particular, according to this picture, every maximally continuous matter-filled region is occupied by a simple (by MCMRS). Furthermore, no two simples can occupy the very same region (by LS). (It follows that, according to this picture, the simples correspond one-one to the maximally continuous matter-filled regions of space.) In addition, no subregion of a region occupied by a simple is occupied by any distinct object (by LS). From the preceding it can be inferred that no mereologically complex object occupies a maximally continuous matter-filled region or proper subregion of a maximally continuous matter-filled region. But every region that is occupied by an object is a matter-filled region and every matter-filled region that is neither a maximally continuous matter-filled region nor a proper subregion of a maximally continuous matter-filled region is a region-fusion of some maximally continuous matter-filled regions. Thus, by MCMRS, every mereologically complex object occupies a region-fusion of some regions that are occupied by simples.

In this section, I have shown that MaxCon follows from MCMRS and LS although neither of them is entailed by MaxCon. I have also described the elegant picture of simplicity obtained by conjoining MCMRS and LS. In the next section, I note that each of the alleged consequences of MaxCon introduced in §1 follows from this picture along with some very plausible assumptions.

3. Deriving the Alleged Consequences from the Elegant Picture

In §1, I discussed four arguments against MaxCon. Each of these arguments claimed that MaxCon was false because MaxCon has a certain consequence and that consequence is incorrect. The alleged consequences of MaxCon mentioned in §1 were the following:

a. Possibly, there is an object that occupies a region that has a left half and a right half although that object has neither a left half nor a right half.
b. Possibly, there is a Joe Montana-shaped object that has no right arm.
c. Necessarily, if Situation S obtains, then A and B go out of existence at 3:10 AM.
d. Necessarily, there are no co-located point-sized objects.

However, I noted there that none of these alleged consequences follows from MaxCon. In the remainder of this section, though, I show that each of these consequences does follow MCMRS and LS along with some very plausible assumptions. I consider (a)-(d) in turn.

To see that (a) follows from MCMRS and LS along with some very plausible assumptions, assume that MCMRS and LS are true. Now notice that it is plausible that possibly, there is a maximally continuous matter-filled region of space that has a left half and a right half. Thus, by MCMRS, it is possible that there is a simple that occupies a region that has a left half and a right half. However, it is plausible that necessarily, for all x, if x has a left half and a right half, then the left half of x occupies a proper subregion of the region occupied by x and the right half of x occupies a proper subregion of the region occupied by x. However, as we saw in the previous section, it follows from LS that necessarily, no object occupies a proper subregion of a region occupied by a simple. It follows from the preceding two claims that necessarily, no simple has a left half and a right half. So, since it is possible that there is a simple that occupies a region that has a left half and a right half, it is possible that there is a simple that occupies a region that has a right half and a left although that simple has neither a left half nor a right half. Therefore, on the assumption that MCMRS and LS are true, it is possible that there is an object that occupies a region that a left half and a right half although that object has neither a left half nor a right half. (a) follows from MCMRS and LS along with plausible assumptions.

(b) also follows from MCMRS and LS along with very similar assumptions. Suppose that MCMRS and LS are true. It is plausible that possibly, there is a Joe

Montana-shaped maximally continuous matter-filled region of space. Thus, by MCMRS, it is possible that there is a simple that occupies a Joe Montana-shaped region. Now it is plausible that objects inherit their shapes from the regions they occupy, so that necessarily, for all x, if x occupies a Joe Montana-shaped region of space, then x is Joe Montana-shaped. So, possibly, there is a Joe Montana-shaped simple. But plausibly, it is necessarily the case that for any Joe Montana-shaped object x, if x has a right arm, then the right arm of x occupies a proper subregion of the region occupied by x. As mentioned above, however, it follows from LS that necessarily, no object occupies a proper subregion of a region occupied by a simple. It follows from the preceding two claims that necessarily, no Joe Montana-shaped simple has a right arm. So, since it is possible that there is a Joe Montana-shaped simple, it is possible that there is a Joe Montana-shaped simple that has no right arm and therefore possible that there is a Joe Montana-shaped object that has no right arm. Therefore, given MCMRS and LS along with plausible assumptions, (b) follows.

Let's turn now to (c) and again assume that MCMRS and LS are true. As I noted in §1, it is very plausible that necessarily, if Situation S obtains, then either A and B go out of existence at 3:10 AM or A and B occupy proper subregions of a maximally continuous matter-filled region of space at 3:10 AM. By MCMRS, however, it is necessarily the case that if A and B occupy proper subregions of a maximally continuous matter-filled region of space at 3:10 AM, then A and B occupy proper subregions of a region of space occupied by a simple at 3:10 AM. And, as we have seen, LS has the consequence that necessarily, no object occupies a proper subregion of a region occupied by a simple. Thus, necessarily, A and B do not occupy proper subregions of a maximally continuous matter-filled region of space at 3:10 AM. It follows from this claim and the claim that I noted was plausible in §1 that necessarily, if Situation S obtains, then A and B go out of existence at 3:10 AM. (c) too follows from MCMRS and LS given plausible assumptions.

Showing that (d) follows from MCMRS and LS along with plausible assumptions is bit harder, but is still relatively straightforward. (d) says that necessarily, there are no co-located point-sized objects; that is, it says that necessarily, for all x, y, and R, if R is a point in space, x occupies R, and y occupies R, then x is identical to y. Assume that MCMRS and LS are true and suppose for reductio that there are an x, y, and R such that R is a point in space, x occupies R, y occupies R, and x is not identical to y. Let a, b, and p be such an x, y, and R, respectively. Thus, p is a point in space, a occupies p, b occupies p, and a is not identical to b. It is plausible that necessarily, every point in space is a region of space, and that necessarily, every region of space that is occupied is a matter-filled region of space. So, p is a matter-filled region of space. But, plausibly, it is necessarily the case that every matter-filled region of space is a proper subregion of a maximally continuous matter-filled region of space, is a maximally continuous matter-filled region of space, or has maximally continuous matter-filled region as a proper subregion. Furthermore, p is a point and, plausibly, it is necessarily the case that no point has a proper subregion. It follows that p is a proper subregion of a maximally continuous matter-filled region of space or p is a maximally continuous matter-filled region of space. Let's take each case in turn.

Suppose for reductio that p is a proper subregion of a maximally continuous matter-filled region of space. Call that maximally continuous matter-filled region of space 'M'. By MCMRS, M is occupied by a simple. Thus, a and b both occupy a proper subregion of a region occupied by a simple. However, as we saw in the previous section, it follows from LS that necessarily, no object occupies a proper subregion of a region occupied by a simple. So, neither a nor b occupies a proper subregion of a region occupied by a simple. Contradiction. Therefore, our supposition is false: p is not a proper subregion of a maximally continuous matter-filled region of space.

Suppose, on the other hand, that p is a maximally continuous matter-filled region of space. Then, by MCMRS, there is a simple that occupies p. Call this simple 's'. Now it is plausible that necessarily, every region of space is a subregion of itself. Thus, p is a subregion of p. It follows that p is a region of space, p is a subregion of p, a occupies p, b occupies p, s occupies p, and s is a simple. But then, by LS, a is identical to s and b is identical to s. By the symmetry and the transitivity of identity, then, it follows that a is identical to b. But, by our original supposition for reductio, a is not identical to b. Contradiction. Thus, p is not a maximally continuous matter-filled region of space.

We have just seen that p is neither a proper subregion of a maximally continuous matter-filled region nor a maximally continuous matter-filled region. But we saw previously that (on our assumption that MCMRS and LS are true) if our supposition for reductio is true, then p is either a proper subregion of a maximally continuous matter-filled region or a maximally continuous matter-filled region. Thus, our supposition is false: it is not the case that there are an x, y, and R such that R is a point in space, x occupies R, y occupies R, and x is not identical to y. Equivalently, for all x, y, and R, if R is a point in space, x occupies R, and y occupies R, then x is identical to y. But we derived this conclusion from MCMRS and LS using only necessitated claims. Therefore, on the assumption MCMRS and LS are true, it is necessarily the case that for all x, y, and R, if R is a point in space, x occupies R, and y occupies R, then x is identical to y. Like (a)-(c), then, (d) too follows from MCMRS and LS along with plausible assumptions.

In this section, I have shown that each of the alleged consequences of MaxCon discussed in §1 follows from MCMRS and LS along with plausible assumptions. I discuss whether there are independently plausible metaphysical principles such that MCMRS and LS follow from their conjunction with MaxCon in the next section.

4. Deriving MCMRS and LS from MaxCon

As noted in the Introduction to this chapter, the features of MCMRS and LS discussed in the previous two sections suggests the following way of arguing for the claim that MaxCon has each of the alleged consequences introduced in §1. In particular, letting C be one of those alleged consequences, one can argue as follows:

1. If MaxCon is true, then MCMRS and LS are true.

2. If MCMRS and LS are true, then C.
3. Therefore, if MaxCon is true, then C.

In this section, I consider the first premise of this argument. In particular, I consider whether there are any independently plausible metaphysical principles from whose conjunction with MaxCon MCMRS and LS follow. I argue that there are, but that a proponent of MaxCon can reasonably deny these principles. I thus conclude that absent other reasons to think that the first premise of this argument is true, a proponent of MaxCon can reasonably deny that premise; and that, furthermore, absent other reasons to think that MaxCon has the alleged consequences introduced in §1, a proponent of MaxCon can reasonably deny that it does and need not find the objections to MaxCon presented there compelling.

With this in mind, I proceed. Consider MCMRS first. Does MCMRS follow from the conjunction of MaxCon with any plausible metaphysical principles? In §3 of Chapter 4, I considered the following thesis:

> **Matter-Filled Regions to Objects (MRO)**: Necessarily, for any matter-filled region of space R, there is an x such that x occupies R.

MCMRS follows from the conjunction of MRO and MaxCon. To see this, assume that MRO and MaxCon are true. Now notice that necessarily, for any maximally continuous matter-filled region of space R, R is a matter-filled region of space. Thus, by MRO, necessarily, for any maximally continuous matter-filled region of space R, there is an x such that x occupies R. But MaxCon entails that necessarily, for all x, if x occupies a maximally continuous matter-filled region of space, then x is a simple.

Thus, given MRO and MaxCon, MCMRS follows: Necessarily, for any maximally continuous matter-filled region of space R, there is an x such that x is a simple and x occupies R.

I think that MRO is an independently plausible, though controversial, metaphysical principle. Thus, MCMRS does follow from the conjunction of MaxCon with an independently plausible metaphysical principle. When I previously discussed MRO, however, I noted that a proponent of MaxCon will likely reject it. For notice that necessarily, every proper subregion of a maximally continuous matter-filled region of space is a matter-filled region of space. Given this, however, it follows from MaxCon and MRO that necessarily, every proper subregion of a region of space occupied by a simple is occupied by a material object. A proponent of MaxCon will likely reject this claim, however.

Furthermore, some proponents of LS will also reject MRO. Consider a proponent of LS who thinks that possibly, a simple occupies a region of space that has proper subregions. Now necessarily, every region of space that is occupied is a matter-filled region of space, and necessarily, every proper subregion of a matter-filled region of space is a matter-filled region of space. Thus, given MRO, if it is possible that a simple occupies a region of space that has proper subregions, then it is possible that an object occupies a proper subregion of a region of space occupied by a simple. As we saw previously, however, it follows from LS that the latter is impossible. Thus, some proponents of LS will reject MRO.

For these reasons, it is worth investigating whether there are any other independently plausible metaphysical principles from whose conjunction with MaxCon MCMRS follows. It turns out that there are. As I noted in §3 of Chapter 4, MCMRS follows from the conjunction of the following principle with MaxCon:

> **Maximally Continuous Matter-Filled Regions to Objects (MCMRO)**: Necessarily, for any maximally continuous matter-filled region of space R, there is an x such that x occupies R.

Like MRO, MCMRO is an independently plausible metaphysical principle. However, as I argued Chapter 4, a proponent of MaxCon can reasonably deny it.

I conclude that MRO and MCMRO are independently plausible metaphysical principles each of which is such that MCMRS follows from its conjunction with MaxCon. But I can think of no other such principles. Since a proponent of MaxCon could reasonably accept these principles, some proponents of MaxCon may be committed to MCMRS. However, a proponent of MaxCon can also reasonably deny MRO and MCMRO. Thus, a proponent of MaxCon need not be committed to MCMRS. Insofar, then, as a proponent of MaxCon has no independent reasons to accept the alleged consequences of MaxCon discussed in §1, a proponent of MaxCon need not accept them and so needn't be persuaded by those objections.

I now turn to discussing whether there are any plausible metaphysical principles such that LS follows from their conjunction with MaxCon. Consider the following principle discussed in Chapter 4:

> **Subregions to Parts (SP)**: Necessarily, for all x, y, R, and S, if R and S are regions of space, R is a subregion of S, x occupies R, and y occupies S, then x is a part of y.

SP is an independently plausible metaphysical principle. Furthermore, LS follows from the conjunction of SP and MaxCon. To see this, assume that SP and MaxCon are true and suppose for reductio that there are an x, y, R, and S such that R and S are regions of space, R is a subregion of S, x occupies R, y occupies S, y is a simple, and x is not identical to y. Let a, b, R1, and R2, be such an x, y, R, and S, respectively. Thus, R1 and R2 are regions of space, R1 is a subregion of R2, a occupies R1, b occupies R2, b is a simple, and a is not identical to b. By SP, a is a part of b. But necessarily, for all x and y, if x is a part of y and x is not identical to y, then x is a proper part of y. So, a is a proper part of b. However, necessarily, for all x and y, if x is a simple, then y is not a proper part of x. So, a is not a proper part of b. Contradiction. Thus, our supposition for reductio is false: it is not the case that there are an x, y, R, and S such that R and S are regions of space, R is a subregion of S, x occupies R, y occupies S, y is a simple, and x is not identical to y. Equivalently, for all x, y, R, and S, if R and S are regions of space, R is a subregion of S, x occupies R, y occupies S, and y is a simple, then x is not identical to y. Since each of the claims used to derive this result was a necessitated claim, it follows that necessarily, for all x, y, R, and S, if R and S are regions of space, R is a subregion of S, x occupies R, y occupies S, and y is a simple, then x is not identical to y. Therefore, if SP and

MaxCon are true, then LS is true; LS follows from the conjunction of SP and MaxCon.

Now I argued in the Chapter 4 that a proponent of MaxCon can reasonably deny SP. Since this is so, a proponent of MaxCon who denies SP and has no independent reasons to accept LS need not accept LS. Furthermore, unless such a proponent of MaxCon has independent reasons to accept the alleged consequences of MaxCon discussed in §1, she also need not accept those consequences.

It is worthwhile, then, asking whether there are any other plausible metaphysical principles from whose conjunction with MaxCon LS follows. In fact, this is worthwhile asking for another reason as well. In particular, notice that MaxCon plays no role in the argument offered above for the conclusion that LS follows from the conjunction of MaxCon and SP; that is, LS follows directly from SP. This important because an opponent of MaxCon may wish to argue that if MaxCon is true, then MCMRS and LS are true, and thus MaxCon has each of the alleged consequences discussed in §1, without thereby committing herself to LS. However, any opponent of MaxCon who employs SP in her argument for the claim that if MaxCon is true, then LS is true, *is* committed to MaxCon. Thus, it is worthwhile asking whether there are any independently plausible metaphysical principles from whose conjunction LS does not follow but from whose conjunction with MaxCon LS does follow.

I believe that there are. Consider the following principles:

No Co-Location (NC): Necessarily, for all x, y, and R, if R is a region of space, x occupies R, and y occupies R, then x is identical to y.

Continuous Matter-Filled Regions (CMR): Necessarily, for any continuous matter-filled region of space R, it is possible that R is a maximally continuous matter-filled region of space that is occupied by an object.

The Restricted Doctrine of Arbitrary Undetached Parts (RDAUP): Necessarily, for all x, R, and S, if x has proper parts, x occupies R, and S is a proper subregion of R, then a proper part of x occupies S.[82]

Strong Supervenience of Mereological Structure (SSMS): Necessarily, for all x, y, R, S, w, and v,[83] if w and v are possible worlds, R is a region of space in w, S is a region of space in v, x occupies R in w, y occupies S in v, and R in w has the same spatial structure as S in v[84], then x in w has the same mereological structure as y in v.[85]

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[82] Van Inwagen (1981) introduced the Doctrine of Arbitrary Undetached Parts into the literature. This principle can be stated as follows:

> **The Doctrine of Arbitrary Undetached Parts (DAUP)**: Necessarily, for all x, R, and S, if x occupies R and S is a subregion of R, then a part of x occupies S.

(This formulation of the principle is slightly different from van Inwagen's.) DAUP, however, has the consequence that any object that occupies a region of space that has proper subregions has proper parts. Thus, it is inconsistent with the possibility of a simple occupying a region of space that has proper subregions. RDAUP, however, is not inconsistent with this possibility. Thus, RDAUP represents an advance over DAUP in this context since (i) a proponent of MaxCon will likely hold that it is possible for a simple to occupy a region of space that has proper subregions and (ii) RDAUP otherwise has exactly the same consequences as DAUP.

[83] The quantifiers in this principle and the next are intended to be possibilist quantifiers.

[84] Necessarily, for all R, S, w and v such that w and v are possible worlds, R is a region of space in w, and S is a region of space in v, R in w has the same spatial structure as S in v if and only if there is a function *f* that satisfies the following constraints:

a. For all x, if there is a y such that *f*(x) = y, then x is a subregion of R in w.
b. For all x, if there is a y such that *f*(y) = x, then x is a subregion of S in v.
c. For all x, if x is a subregion of R in w, then there is a y such that *f*(x) = y.

Essentiality of Spatial Structure (ESS): Necessarily, for all R, w, and v, if R is a region of space in w and R is a region of space in v, then R in w has the same spatial structure as R in v.

LS, I claim, follows from the conjunction of these principles with MaxCon.

To see this, assume that NC, CMR, RDAUP, SMS, ESS, and MaxCon are true.[86] Now suppose for reductio that LS is false; that is, suppose that possibly, there are an x, y, R, and S such that R and S are regions of space, R is a subregion of S, x occupies R, y occupies S, y is a simple, and x is not identical to y. From this supposition it follows that there are an x, y, R, S, and w, such that w is a possible world, R and S are regions of space in w, R is a subregion of S in w, x occupies R in w, y occupies S in w, y is a simple in w, and x is not identical to y in w. Let a, b, R1, R2, and u be such an x, y, R, S, and w, respectively. Thus, u is a possible world, R1 and R2 are regions of space in u, R1 is a subregion of R2 in u, a occupies R1 in u, b occupies R2 in u, b is a simple in u, and a is not identical to b in u.

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d. For all x and y, if *f*(x) = *f*(y), then x=y.
e. For all x and y such that x and y are subregions of R in w, x is a subregion of y in w if and only *f*(x) is a subregion of *f*(y) in v.

[85] Necessarily, for all m, n, w and v such that w and v are possible worlds, m exists in w, and n exists in v, m in w has the same spatial structure as n in v if and only if there is a function *f* that satisfies the following constraints:

a. For all x, if there is a y such that *f*(x) = y, then x is a part of m in w.
b. For all x, if there is a y such that *f*(y) = x, then x is a part of n in v.
c. For all x, if x is a part of m in w, then there is a y such that *f*(x) = y.
d. For all x and y, if *f*(x) = *f*(y), then x=y.
e. For all x and y such that x and y are parts of m in w, x is a part of y in w if and only *f*(x) is a part of *f*(y) in v.

[86] I will assume S5 modal logic in the following argument. A variant of the argument can be given using weaker modal logics, but formulating the argument is much more difficult.

Now R1 is either identical to R2 in u or R1 is a proper subregion of R2 in u. Suppose first that R1 is identical to R2 in u. Then since a occupies R1 in u, a occupies R2 in u. It follows that a occupies R2 in u and b occupies R2 in u. But then, by NC, a is identical to b in u, contrary to our assumption. Therefore, R1 is not identical to R2 in u.

Thus, on the supposition that LS is false, R1 is a proper subregion of R2 in u. Since b is a simple that occupies R2 in u, it follows by MaxCon that R2 is a maximally continuous matter-filled region of space in u. But necessarily, no proper subregion of a maximally continuous matter-filled region is a maximally continuous matter-filled region. Thus, R1 is not a maximally continuous matter-filled region in u. However, a occupies R1 in u. Therefore, by MaxCon, a is not a simple in u; that is, a has proper parts in u.

Now either R1 is a continuous region of space in u or R1 is a discontinuous region of space in u. Suppose that R1 is a continuous region of space in u. Since R1 is occupied by a in u, R1 is a matter-filled region of space in u. Thus, by CMR, it is possible that R1 is a maximally continuous matter-filled region of space that is occupied by an object. So, there are an x and a w such that w is a possible world, R1 is a maximally continuous matter-filled region of space in w, and x occupies R1 in w. Let c and v be such an x and w, respectively. Thus, v is a possible world, R1 is a maximally continuous matter-filled region of space in v, and c occupies R1 in v. By MaxCon, it follows that c is a simple in v. Now it follows from ESS that R1 in u has the same spatial structure as R1 in v. Thus, u and v are possible worlds, R1 is a region of space in u, R1 is a region of space in v, a occupies R1 in u, c occupies R1 in v, and R1 in u has the same spatial structure as R1 in v. By SSMS, then, it follows that a in u has the same mereological structure as c in v. But a has proper parts in u and c does not have proper parts in v, so a in u does not have the same mereological structure as c in v. Thus, our supposition is false: R1 is not a continuous region of space in u.

Suppose, on the other hand, that R1 is a discontinuous region of space in u. Since a occupies R1 in u, it follows from MaxCon that a is not a simple in u; that is, a has proper parts in u. So, by RDAUP, every proper subregion of R1 is occupied by a proper part of a in u. But necessarily, every discontinuous region of space has a continuous proper subregion. Thus, R1 has a continuous proper subregion that is occupied by a proper part of a in u. Let R3 be such a continuous proper subregion of R1 in u and let c be the proper part of a that occupies R3 in u. Since R3 is a proper subregion of R1 in u and R1 is a proper subregion of R2 in u, R3 is a proper subregion of R2 in u. However, b is a simple that occupies R2 in u. So, by MaxCon, R2 is a maximally continuous matter-filled region of space in u. Necessarily, though, no proper subregion of a maximally continuous matter-filled region is a maximally continuous matter-filled region. Thus, R3 is not a maximally continuous matter-filled region of space in u. But R3 is occupied by c in u. Again by MaxCon, it follows that c is not a simple in u; that is, it follows that c has proper parts in u. So, R3 is a continuous region of space in u, c occupies R3 in u, and c has proper parts in u. But now the same sorts of considerations (involving CMR, SSMS, and ESS) that showed that R1 could not be a continuous region of space in u also show that this cannot be.

Roughly, by CMR, R3 could be occupied by a simple. But, by ESS, R3 would have the same spatial structure if it were occupied by a simple. So, by SSMS, c has the same mereological structure as that simple. But c does not have the same mereological structure as that simple, since c has proper parts and that simple does not. Contradiction. Therefore, R1 is not a discontinuous region of space in u.

Having assumed that NC, CMR, RDAUP, SMS, ESS, and MaxCon are true, we have shown that the supposition that LS is false leads to contradiction. For if LS is false, then either R1 is identical to R2 in u, R1 is a continuous proper subregion of R2 in u, or R1 is a discontinuous proper subregion of R2 in u. Given NC, CMR, RDAUP, SMS, ESS, and MaxCon, however, we have shown that each of these possibilities leads to contradiction. Thus, LS follows from the conjunction of NC, CMR, RDAUP, SMS, and ESS with MaxCon. Furthermore, NC, CMR, RDAUP, SMS, and ESS appear to be independently plausible metaphysical principles. Therefore, LS follows from the conjunction of independently plausible metaphysical principles with MaxCon.

A proponent of MaxCon, though, can reasonably deny some of these principles. I will focus on NC, RDAUP, and SMS. That a proponent of MaxCon can reasonably deny NC is illustrated by the fact that so many philosophers do reasonably deny NC. Whether co-location is possible is a very controversial question.[87]

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[87] Even if a proponent of MaxCon denies NC, however, all is not lost. For the following weakening of LS follows from the conjunction of CMR, RDAUP, SMS, and ESS with MaxCon:

Now a proponent of MaxCon can also reasonably deny RDAUP. Consider a proponent of MaxCon who holds that it is possible for a simple to occupy a region of space that has proper subregions and that is itself a proper subregion of a region of space occupied an object that has proper parts without any objects occupying the proper subregions of the region occupied by that simple. RDAUP rules this out. If RDAUP is true, then: if a simple occupies a region of space that has proper subregions and that is itself a proper subregion of a region of space occupied by an object that has proper parts, then every proper subregion of the region occupied by the simple is occupied by an object. Thus, since it seems that a proponent of MaxCon can reasonably accept the possibility in question, a proponent of MaxCon can reasonably deny RDAUP.[88]

> **The Modified Loneliness of Simples (MLS)**: Necessarily, for all x, y, R, and S, if R and S are regions of space, R is a *proper* subregion of S, x occupies R, y occupies S, and y is a simple, then x is identical to y.

Although MaxCon does not follow from the conjunction of MCMRS and MLS, alleged consequences (a)-(c) of MaxCon do follow from their conjunction along with plausible assumptions.

[88] One might wonder whether RDAUP could be modified to avoid the consequence mentioned here in such a way that LS still follows from the conjunction of NC, CMR, the modified version of RDAUP, SMS, and ESS with MaxCon. This may be possible. Consider the following modification of RDAUP:

> **The Modified Restricted Doctrine of Arbitrary Undetached Parts (MRDAUP)**: Necessarily, for all x, R, and S, if x has proper parts, x occupies R, and S is a proper subregion of R, then a proper part of x occupies S *unless* a proper subregion of R is occupied by a simple, in which case a proper part of x occupies S if S is not a proper subregion of a region occupied by a simple.

This avoids the consequence of RDAUP mentioned in the text. Furthermore, LS follows from the conjunction of NC, CMR, MRDAUP, SMS, and ESS with MaxCon, since none of the objects with proper parts that we were concerned with in our earlier argument were such that the region of space they occupied had a proper subregion occupied by a simple.

One might worry that MRDAUP, unlike RDAUP, is not an independently plausible metaphysical principle. I think that this is incorrect, however. As noted in footnote 82, RDAUP represents an advance over DAUP because RDAUP, but not DAUP, is consistent with the possibility of a simple occupying a region of space that has proper subregions but

Finally, let's consider SMS. A proponent of MaxCon can also reasonably deny this thesis. For consider a proponent of MaxCon who holds that each human being occupies a region of space that is also occupied by a distinct object. Furthermore, this proponent of MaxCon holds that while a human being has parts like arms, legs, a torso, etc., no human being has "arbitrary undetached parts". For instance, no human being has a part consisting of all of them but there right arm. Finally, the proponent of MaxCon in question holds that the other object that occupies the region of space occupied by the human being does have arbitrary undetached parts; in fact, for every proper subregion of the region occupied by this other object, there is a proper part of that object that occupies that region. I take it that a proponent of MaxCon could reasonably hold this view. However, if SMS is true, this view must be false, since, according to the view, the human being and the other object occupy the same region of space and thus occupy regions of space with the same spatial structure although they do not have the same mereological structure. Thus, a proponent of MaxCon can reasonably deny SMS.

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otherwise has the very same consequences as DAUP. Similarly, MRDAUP represents an advance over RDAUP since MRDAUP, but not DAUP, is consistent with the possibility mentioned in the text but otherwise has the very same consequences as RDAUP. Thus, insofar as it is the consequences that RDAUP and MRDAUP share that make RDAUP plausible, those consequences also make MRDAUP plausible.

So, LS follows from the conjunction of NC, CMR, MRDAUP, SMS, and ESS with MaxCon and, I suspect, MRDAUP is an independently plausible metaphysical principle. So why have I focused on RDAUP in the text? First, RDAUP is easier to work with than MRDAUP. Second, a proponent of MaxCon can also reasonably deny MRDAUP for precisely the sorts of reasons that DAUP is usually reasonably denied. But the reasons for someone to deny RDAUP considered in the text are relatively unique, and thus considering them is more interesting than a simple rehashing of the reasons usually given to deny DAUP.

Notice, however, that a proponent of MaxCon who denies SMS for these reasons also denies both NC and RDAUP. I think, however, that a proponent of MaxCon can reasonably deny SMS without denying NC. In §3 of Chapter 4, I argued that a proponent of MaxCon who accepts a principle of recombination discussed in Saucedo (forthcoming) can reasonably deny SP without denying NC. But such a proponent of MaxCon can also reasonably deny SMS.[89] Here's the argument. Begin with the claims that any pattern of instantiation of any pairwise wholly distinct fundamental properties and relations is possible and that the relations of being a part of, occupying, and being a subregion of are fundamental and pairwise wholly distinct. It follows that any pattern of instantiation of *being a part of*, *occupying*, and *being a subregion* of is possible. But now consider two claims, one according to which there is an object with mereological structure M1 that occupies a region of space with spatial structure S and another according to which there is an object with mereological structure M2 that occupies a region of space with spatial structure S, where M1 is distinct from M2. Since the mereological structure of an object can be described using solely the parthood relation and the identity relation and the spatial structure of a region can be described using solely the subregionhood relation and the identity relation, these two claims each describe a pattern of instantiation of *being a part of*, *occupying*, and *being a subregion of*. Furthermore, each of these claims is logically consistent with any necessarily true claim describing a pattern of

[89] Notice, however, that such a proponent of MaxCon must also deny RDAUP, since a similar argument employing Saucedo's principle can also be given for the conclusion that RDAUP is false.

instantiation of only one of these relations.[90] Thus, each of these claims describes a possible pattern of instantiation. If so, however, then SMS is false because there are an x, y, R, S, w, and v such that w and v are possible worlds, R is a region of space in w, S is a region of space in v, x occupies R in w, y occupies S in v, and R in w has the same spatial structure as S in v (in particular, both have spatial structure S), but x in w does not have the same mereological structure as y in v, since x has mereological structure M1 in w and y has mereological structure M2 in v. A proponent of MaxCon can thus reasonably deny SMS without denying NC.

I conclude that MCMRS follows from the conjunction of independently plausible metaphysical principles with MaxCon, as does LS. In particular, MCMRS follows from the conjunction of MRO with MaxCon and also follows from the conjunction of MCMRO with MaxCon. Similarly, LS follows from the conjunction of SP with MaxCon and from the conjunction of NC, CMR, RDAUP, SMS, and ESS with MaxCon. In each case, however, the proponent of MaxCon can reasonably deny some of these principles. In particular, I have argued that a proponent of MaxCon can reasonably deny MRO, MCMRO, SP, NC, RDAUP, and SMS.

Conclusion

I began by discussing some alleged consequences of MaxCon, noting that none of them are entailed by MaxCon. I then showed that each of these consequences follows from MCMRS and LS along with plausible assumptions. MCMRS and LS are interesting because, as I argued, MaxCon follows from their conjunction although

[90] See §3 of Chapter 4 for an explanation of why this claim needs to be made given certain technical features of Saucedo's notion of a pattern of instantiation.

neither is entailed by MaxCon and because together they yield an elegant picture of simplicity. I then considered whether there are plausible metaphysical principles whose conjunction with MaxCon entail MCMRS and LS and whether a proponent of MaxCon can reasonably deny these principles. I argued that there are such principles but that a proponent of MaxCon can reasonably deny them. I conclude that a proponent of MaxCon can reasonably deny MCMRS and LS absent independent reasons to think that they are true and thus that she needn't be persuaded by the objections to her view presented in §1 absent independent reasons to think that her view has the consequences it is alleged to have in those objections.

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