ACHIEVEMENTS AND FALLACIES IN HUME'S ACCOUNT OF INFINITE DIVISIBILITY

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Throughout history, almost all mathematicians, physicists and philosophers have been of the opinion that space and time are infinitely divisible. That is, it is usually believed that space and time do not consist of atoms, but that any piece of space and time of non-zero size, however small, can itself be divided into still smaller parts. This assumption is included in geometry, as in Euclid, and also in the Euclidean and non-Euclidean geometries used in modern physics. Of the few who have denied that space and time are infinitely divisible, the most notable are the ancient atomists, and Berkeley and Hume. All of these assert not only that space and time might be atomic, but that they must be. Infinite divisibility is, they say, impossible on purely conceptual grounds.

In the hundred years or so before Hume's *Treatise,* there were occasional treatments of the matter, in places such as the *Port Royal Logic,* and Isaac Barrow's mathematical lectures of the 1660's, [1] They do not add anything substantial to medieval treatments of the same topic. [2] Mathematicians certainly did not take seriously the possibility that space and time might be atomic; Pascal, for example, instances the Chevalier de Méré's belief in atomic space as proof of his total incompetence in mathematics. [3] The problem acquired a more philosophical cast when Bayle, in his *Dictionary,* tried to show that *both* the assertion and the denial of the infinite divisibility of space led to contradictions; the problem thus appears as a general challenge to "Reason". [4] The problem was still a live one for Kant, whose Second Antinomy includes the infinite divisibility of space as a premise. [5] The eighteenth century also felt a certain tension, largely unacknowledged, between the corpuscular hypothesis of matter and the infinite divisibility of space. Newton and most scientists supposed matter and light to be atomic, but unambiguous scientific evidence remained tantalizingly unavailable until Dalton's work after 1800; and while the atomic hypothesis remained essentially a philosophical one, there was an uncomfortable tension between the atomicity of matter and the continuity of space. Thus, Lord Stair in 1685 (in a scientific work reviewed by Bayle) defended the atomicity of matter against mathematical objections concerning infinite divisibility, and arrived at a position close to Hume's. [6]

Nevertheless, it is obviously hard to explain why space and time should be infinitely divisible, and how this could be known if it were true: surely knowing it requires that measurement should be able to follow nature into the infinitely small? The details of the argument in this period are not very relevant to what Hume says, so will not be discussed here. (Nor are the details interesting mathematically, since they just consist in extracting from Euclid the implicit assumption of infinite divisibility). Suffice it to say that by 1739, the problem of the infinite divisibility of space and time had the status of old chestnut, not unlike the problem of interpreting quantum mechanics today. Problems of this kind attract the attention of two kinds of philosopher: the technical expert who follows the scientists into the intricacies, and the Young Turk, eager to rush in where others fear to tread, and cut the Gordian knot with his brilliant new insight.

No further explanation seems necessary as to why Hume should have written on the question, nor why he should have given it such prominence at the beginning of the *Treatise.* To solve a long-running problem with his "experimental method of reasoning" would have been a simple demonstration of its value. But in omitting his treatment of space and time almost entirely from the later *Enquiry,* Hume seems to admit tacitly that it was not a success with its intended audience. It has had no better reception since. Almost all commentators, even ones who usually admire Hume, have judged his conclusion on infinite divisibility to be false, and his reasons so hopelessly confused as to be of no interest. [7]

Given that Hume was wrong about the supposed impossibility of infinite divisibility, there are two reasons why what he says could still be interesting. First, there might be something correct among the errors. Secondly, his mistakes might be, not one-off confusions, but things that systematically affect other parts of his thinking, or perhaps even the thinking of his age.

Both of these are the case.

To understand what is right about Hume, it will be necessary to review briefly what is now known to be the correct answer on the question of infinite divisibility. Anachronism threatens, of course, but at least we will avoid the error of ignorantly dismissing as impossible what experts with the benefit of all history presume true. In any case, we are dealing with mathematics, where knowledge *is* cumulative.

(1) The infinite divisibility of space and time is possible. (This is because there exists a consistent model which incorporates infinite divisibility, namely the set of infinite decimals). It follows that all supposed proofs of the impossibility of infinite divisibility, whether mathematical or philosophical, are invalid. There is a small cost to this, in that one must accept that an infinite number of points with zero length can add up to something with a positive length. This is odd, but no more than that; it just means that length is not constituted by counting. [8] (The points in a line can be paired off with the points in a line twice the length, so the two have the same — infinite — number of points, but different lengths. This matter was obscured up to and including Hume's time because all mathematicians, except possibly Galileo, accepted the Aristotelian orthodoxy that a line is not composed of its points). A further oddity is that the infinity of points needed to compose a line is a large one: a countably infinite number of points (that is, a number of points equal to the number of whole numbers) is not enough to constitute a positive length, so one needs a greater infinity. Bayle would have laughed.

(2) It is also possible that space should be discrete, or atomic, that is, composed of units and only finitely divisible. (This is because there is a consistent model in this case too: in one dimension, the integers, and in higher dimensions, the lattice of points with integer coordinates). It follows that all supposed proofs of infinite divisibility are invalid. The manner in which geometry is possible with finite numbers of points is perhaps best suggested by the calculations a computer makes in projecting an image onto its screen, or to a laser printer. These have a finite resolution, so the computer, in deciding how to rotate a screen image, for example, calculates with a finite precision arithmetic to decide the appropriate color for the finite number of screen pixels. Of course, the pixel itself has spatial parts, but that is not "known" to the computer, which does geometry as if the pixels were simple. A computer with a very high resolution screen, with pixel size equal to the human *minimum visibile*, is a good model of what Hume thinks real space is. Like anything in computing, it is finite. The computer produces geometrical results, like the rotation of images, that are indistinguishable from those that would be produced by a geometry that incorporated infinite divisibility. (The formal mathematics of finite precision arithmetic is harder than it looks, but work proceeds [9]).

(3) On present evidence from physics, space and time are most likely infinitely divisible. This could change at any moment, as the sub-microscopic world becomes better known; a few physicists are still actively engaged in investigating the possibility of atomic, or, as they say, "quantized", space. [10]

Where Hume was right

It is clear, then, where Hume was right, against the mathematicians of his day. They claimed to prove that space was infinitely divisible, so Hume was correct in claiming they could not have done so. As to his actual argument against them, there is again something which modern mathematics would approve:

> . . . this idea [of extension], as conceiv'd by the imagination, tho' divisible into parts or inferior ideas, is not infinitely divisible, nor consists of an infinite number of parts: For that exceeds the comprehension of our limited capacities. Here then is an idea of extension, which consists of parts or inferior ideas, that are perfectly indivisible: consequently this idea implies no contradiction: consequently 'tis possible for extension really to exist conformable to it: and consequently, all the arguments employ'd against the possibility of mathematical points are mere scholastick quibbles, and unworthy of our attention. [11]

The general idea is correct: to show the consistency of some notion, build a model of it; then any supposed proofs of its impossibility must be invalid. Mathematicians laboured to prove the inconsistency of non-Euclidean geometry, until a model of it was found. [12] A post-Fregean mathematician might take exception to Hume's building the model out of *ideas*, but it must be admitted that the attempts of philosophers to say what sets and the other materials used for mathematical modeling are, if not ideas, have not been an unqualified success. Sets are "abstract", perhaps even "mental" constructs; they are like ideas at least in being non-physical, and a case can be made that they are the nearest twentieth-century substitute for the eighteenth century's ideas. Frege's attacks on "psychologism" in the foundations of mathematics indicates that he hoped to replace talk about ideas with entities that would be non-mental, but otherwise perform the same task. [13] The finite model of space possessed by the computer that performs on-screen geometry is made out of numbers, or symbols: both notions suggesting ideality, though capable of being represented in electrical impulses in silicon (just as "real" ideas are presumably represented by impulses in neurons).

Hume can claim something of the same success that Berkeley had in the matter of infinitesimals. Berkeley said that the mathematicians' use of infinitesimals was contradictory, in that they regarded infinitesimals as non-zero when it suited them, and zero when that suited them. The mathematicians ignored Berkeley, as one ignorant of the subtleties of their art — until they solved the problem by eliminating infinitesimals in favor of multiple quantification, at which point they said, "Berkeley was right, which demonstrates the excellence of our new answer". [14] Hume was right too, but it took the discovery of non-Euclidean geometry to force mathematicians to distinguish between mathematical models and reality.

Another question on which Hume has been vindicated, at least up to a point, is the matter of the complexity of ideas. Hume writes without apology things like:

> the idea of a grain of sand is not distinguishable, nor separable into twenty, much less into a thousand, ten thousand, or an infinite number of different ideas. [15]

He speaks freely throughout of the divisibility of, and the parts of, ideas and images. Later philosophical commentators have professed themselves unable to understand what a part of an idea is. Typical is D.G.C. McNabb, the editor of the Collins edition of the *Treatise.* His first note to the text is at the passage just quoted, and his reaction is to dismiss it with as little argument as Hume used to support it. McNabb writes:

> This proposition is true, not because, as Hume thinks, the number of parts into which the idea can be divided is less than twenty, but because it does not make sense to talk of dividing ideas into parts at all. This mistake vitiates the whole section. Hume's admission that we may be able to "imagine" the grain of sand divided into tiny indivisible parts, suggests that something is wrong with the doctrine of impressions and ideas. [16]

While the simplicity of ideas has remained philosophical orthodoxy, those who have actually investigated images and ideas have found otherwise. Psychologists studying mental images, for example, have established that images have parts —an image of a ball on a box has a part corresponding to the ball and a part corresponding to the box. [17] (If psychology is sometimes accused of labouring to prove the obvious, its defense must be that someone will deny the obvious otherwise). The most famous experiments on images are those of Shepard and Metzler on mental rotations, of around 1970. They found that the time taken for subjects to decide whether one of two similar-looking figures could be got by rotating the other is proportional to the angle of rotation needed; this confirms what is clear from introspection, that the problem is solved by mentally rotating the figures, in a kind of mental space. [18] More generally, it has long been understood that the internal representations of space in rats, chimpanzees and so on, are "topographically organized", or map-like, in some sense. [19] That is, there is a kind of internal space. Indeed, what use would an internal representation be otherwise? For purposes like tracking prey, or navigating, one needs an internal map on which to simulate the places and likely paths of oneself, prey, obstacles and so on. And a map, or internal space, is complex —it has parts, as Hume says. These lessons have been taken to heart in that very Humean discipline, Artificial Intelligence, [20] but less so, unfortunately, in philosophy.

The same considerations cast doubt on Flew's analysis of what is wrong with Hume's arguments on infinite divisibility. [21] Flew sees two "mistakes" as crucial: the first, Hume's identifications of conceiving with imagining, and of imagining with forming a mental image, the second, that "whatever is capable of being divided *in infinitum*, must consist in an infinite number of parts". It is doubtful if the second of these is a clear mistake (arguments on "potential infinity" continue, while modern mathematics is happy to regard the real line as consisting of infinitely many points as its parts), and it is of doubtful relevance in any case (as the essence of Hume's conclusion is arrived at already if one concedes that there is a finite limit to conceivability, and hence possibility). So let us concentrate on the first point, as to whether conceiving should be identified with forming a mental image. The relevance of the point is obvious, and any denial of it will strike at the heart of Hume's doctrine of impressions and ideas. There *are* many reasons to think, in general, that conceiving is something more than imagining. But it is especially difficult to make the separation in geometry. One of Aristotle's most quoted sayings is, "There is no thinking without an image", [22] and his standard example is a geometer reasoning about triangles. Proclus argues that the pure understanding cannot do geometry, as its concepts are simple and there is only one kind of each, so that it cannot deal with circles of different size; it must therefore project images "distinctly and individually on the screen of the imagination". [23] Even in dealing with such things as four-dimensional objects, which cannot be conceived as a whole, one can proceed by imagining parts of them, or projections onto lower-dimensional spaces. Although mathematicians can claim that it is theoretically possible to do geometry purely by manipulating symbol strings ("'Tis usual with mathematicians, to pretend, that those ideas, which are their objects, are of so refin'd and spiritual a nature, that they fall not under the conception of the fancy" [24]), the modern evidence is that a spatial visualization capacity (read "imagination") is necessary in practice. [25] So Hume's project of beginning his "impressions and ideas" project with an extended consideration of our "ideas of space and time" can be awarded some belated praise on this score.

Hume's mistakes, and their consequences

Now for what is wrong with Hume's arguments against infinite divisibility. He makes various objections, but it is obvious that the central, and most controversial, step is that from the impossibility of infinitely dividing our *ideas* of space and time, to the impossibility of infinitely dividing space and time themselves. Hume writes:

> Wherever ideas are adequate representations of objects, the relations, contradictions and agreements of the ideas are all applicable to the objects; and this we may in general observe to be the foundation of all human knowledge. But our ideas are adequate representations of the most minute parts of extension; and thro' whatever divisions and subdivisions we may suppose these parts to be arriv'd at, they can never become inferior to some ideas, which we form. The plain consequence is, that whatever *appears* impossible and contradictory upon the comparison of ideas, must be *really* impossible and contradictory, without any further excuse or evasion. [26]

This thought is not in Hume's mathematical sources: Barrow, Bayle and the *Port Royal Logic.* [27] Fogelin rightly identifies this passage as something remarkable, and describes it as "a match for anything found in the writings of the rationalists". [28] Loosely, there is indeed a rationalist air about the passage, in that some close connection is asserted between what is within the mind and what is outside. But in another way, the comment is quite the reverse of the truth. Descartes and Leibniz, like the scholastics, attribute extraordinary powers to the mind, in knowing the world as it is. What they do not say is, that the world is restricted by the powers the mind has. The scholastics and rationalists were enthusiastic about the power of rationality, and that means they believed the mind could extend its knowledge. Descartes, for example, maintains that everything I conceive clearly and distinctly in material bodies (he means their mathematical properties) is in them. [29] But, though he does once say that what conflicts with our ideas is impossible, he does not really mean it: he says many times that what is inconceivable to us is possible through God's power, and our belief that two and two could not be other than four is in effect (though he does not use the words) a deceit perpetrated on us by God. [30] The attitude of sour grapes, that if something is beyond the reach of the mind, then it isn't there at all, is absolutely foreign to the men of reason.

Foreign to them, but quite at home with —indeed, constitutive of —idealism.

Hume's argument against infinite divisibility is the same as Berkeley's, and it has been noticed that Hume and Berkeley are probably at their closest at this point. [31] Berkeley writes in the *Principles of Human Knowledge*:

> Every particular finite extension which may possibly be the object of our thought is an *idea* existing only in the mind, and consequently each part thereof must be perceived. If, therefore, I cannot perceive innumerable parts in any finite extension that I consider, it is certain they are not contained in it. [32]

He then repeats himself a number of times, as usual, compares mathematicians to Papists, and so on, but adds no new argument of substance. Interestingly, he does consider the strictly mathematical question of whether it is possible to develop the science of geometry without assuming infinite divisibility. [33] He asserts that it must be possible, but wisely refrains from trying to show how to do it. Nevertheless, while some commentators have held it to be impossible, [34] modern computers have been able to do geometry with finite precision arithmetic perfectly well, as explained above.

Berkeley uses essentially the same argument in his attack on the calculus, the *Analyst* of 1734. Besides his correct arguments noted above, he writes of infinitesimals and fluxions:

> Now, as our Sense is strained and puzzled with the perception of objects extremely minute, even so the Imagination, which faculty derives from sense, is very much strained and puzzled to frame clear ideas of the least particles of time, or the least increments generated therein: and much more so to comprehend the moments, or those increments of the flowing quantities in *statu nascenti,* in their very first origin or beginning to exist, before they become finite particles . . . the clear conception of it will, if I mistake not, be found impossible; whether it be so or no I appeal to the trial of every thinking reader. [35]

Now with Berkeley, we know where we are. The arguments just quoted are essentially the same as his basic one for idealism, the argument called in Stove's study of idealism, "the Gem". [36] This is the argument: "It is impossible to think of anything existing outside the mind, without thinking of it; so, there cannot be things outside the mind." Here is Berkeley's version of it:

> But, say you, surely there is nothing easier than for me to imagine trees, for instance, in a park, or books existing in a closet, and nobody by to perceive them. I answer, you may say so, there is no difficulty in it. But what is all this, I beseech you, more than framing in your mind certain ideas which you call *books* and *trees*, and at the same time omitting to frame the idea of any one that may perceive them. But do not you yourself perceive or think of them all the while? This therefore is nothing to the purpose: it only shews you have the power of imagining, or forming ideas in your mind; but it does not shew that you can conceive it possible the objects of your thought may exist without the mind. To make out this, it is necessary that you conceive them existing un-conceived or unthought of; which is a manifest repugnancy. When we do our utmost to conceive the existence of external bodies, we are all the while only contemplating our own ideas. [37]

This has more wrong with it, certainly, than Hume's argument against infinite divisibility. But it seems inconceivable Berkeley would have entertained it, much less offered it in print, unless he had been, like Hume, susceptible to the fallacy, "It is not conceivable by the human mind, therefore it cannot be".

This is not the only argument for idealism in Berkeley. But Stove shows it is the one which survived, and, dressed up in various ways, provided the base for idealism during its whole life, up to 1900, when it became the first and only philosophy to capture the entire academic world, and then suddenly disappeared. Now if Berkeley is susceptible to the fallacy, "It is not conceivable, so it cannot be", then we are not surprised; it is precisely the grossness of his fallacies that makes Berkeley so useful as target practice for undergraduates. And no one is surprised at any fallacies that appear when the sentences of the German idealists are divided into atomic propositions. But what are we to make of it when Hume, the paragon of rationality in the century of "Reason", does the same?

We will make nothing of it, because we are too flabbergasted.

It is to be noted that more is being asserted here than the familiar thought that Hume sometimes insists so much on the primacy of experience that he tends to phenomenalism. That is a problem in Hume's philosophy, but it is a different one to the strictly logical problem being complained of here. In a writer who is trying to reduce everything to a single kind of entity, one expects such difficulties as a threatened collapse into a simple view like phenomenalism. One does not expect straight fallacies.

On the other hand, it is not being asserted that Hume actually agrees with Berkeley's idealism. The assertion is that Hume accepted an argument of the same logical form as Berkeley's argument for idealism, not that he applied the argument to reach Berkeley's conclusion. Again, we are discussing arguments, not conclusions.

One is at first tempted to defend Hume. Surely he means something more restrained, perhaps along the lines of "What we cannot conceive *because it contains a contradiction* cannot exist"? But, one must ask, what is Hume's account of contradiction? He does not think of contradictions as vacuous, nor as external constraints on thought (that is the view of the scholastic syllogistic that he continually attacks). The question, "Why cannot a contradiction be true?" is not one he poses clearly — though it is a serious problem for his thought — but his answer to it is quite explicit:

> The same is the case with *contrariety* [that is, it is a relation of ideas, discoverable at first sight] . . . No one can once doubt but existence and non-existence destroy each other, and are perfectly incompatible and contrary. [38]

So, far from appeal to contradiction saving Hume, we find that for him, contradiction itself is to be explained in terms of the "contrariety" of ideas. And the move in the last sentence quoted from "contrary" (of ideas) to "incompatible" (of things) is itself an instance of the fallacy, "It is inconceivable, so it cannot be".

There is, then, no evading the fact that Hume relies on this fallacy. Let us just note some connected facts. First, Hume makes the same move elsewhere, though perhaps never so crudely. For example, he argues that there is no time in unchanging things because, "since the idea of duration cannot be deriv'd from such an object, it can never in any propriety or exactness be apply'd to it". [39] And his discussion of the vacuum [40] contains the assumption that if there can be no idea of a vacuum (as he argues), then there can be no vacuum (although related remarks in the Appendix to the *Treatise* give evidence of a wish to qualify the principle, because of difficulties over the existence of absolute space [41]). These examples will at least suggest that in the main passage above we have not misread Hume. Secondly, there are hints of the fallacy in earlier philosophy. Zeno had suggested that one of the problems with an infinite series was that the mind would not be able to count it. [42] Aristotle and his followers deny there can be actually infinite multitudes, using an argument that sounds very like, "The mind cannot conceive an actual infinite, so there cannot be any". One of the most difficult passages of Aristotle's writings on the senses discusses whether there can be variations in sensible qualities smaller than those perceivable by the senses; it is hard to discern what his answer is. [43] Arguments that there can be no more than the five senses, and no more than three dimensions, sometimes hint at the same way of thinking. Still, they are no more than hints; with Aquinas, for example, one can be left with the impression that he is deliberately avoiding the appearance of committing this fallacy, even when he is really doing so. [44] Something of Hume's tone can be seen in Hobbes' thought that the "length without breadth" of the mathematicians is meaningless, since it makes no perceptual sense, [45] and his statement that, "SPACE is the phantasm of a thing existing without the mind simply". [46] Even on the side of the mathematicians, Isaac Barrow's lectures (delivered in the 1660's, printed in 1734), which are the standard defence of infinite divisibility, make a dangerous concession. Barrow says:

> For some assert an indivisible Quantity in Words, but none I believe can paint its Image in the Mind: Extension vanishes with Divisibility, and the whole with its parts. [47]

To thus positively invite "an attempt to introduce the experimental method" into mathematical subjects is, as they say, "asking for it".

Nevertheless, it was only with Berkeley and Hume that the fallacy became explicit, and, far from being immediately exposed, as one would expect, immediately carried all before it.

As to why the eighteenth century exhibited this weakness, when all previous centuries had been almost free of it, no convincing reason suggests itself. A possible speculation is that the generation of Locke and Leibniz was the last to be properly trained in scholastic thought, while Berkeley and Hume, lacking this steadying discipline, were free to make the crudest of logical errors. As there seems no way of confirming this suggestion, it must remain purely speculative.

Hume has another argument against infinite divisibility, independent of any considerations about ideas. The mistake he makes in it is also interesting, and reappears elsewhere in his work. This is the argument, attributed to Malezieu, that as a purely conceptual matter, anything composite must be composed of units:

> Twenty men may be said to exist; but 'tis only because one, two, three, four, *&c.* are existent; and if you deny the existence of the latter, that of the former falls of course. 'Tis therefore utterly absurd to suppose any number to exist, and yet deny the existence of unites; and as extension is always a number, according to the common sentiment of metaphysicians, and never dissolves itself into any unite or indivisible quantity, it follows that extension can never at all exist. 'Tis vain to reply, that any determinate quantity of extension is an unite . . . The whole globe of the earth, nay the whole universe *may be consider'd as an unite.* That term of unity is merely a fictitious denomination . . . [48]

Like Lucretius, in a passage quoted in Barrow's *Mathematical Lectures,* [49] Hume simply cannot understand how a complex thing can exist without its being made up of units. That is, he rules out *a priori* what seems to almost everybody a contingent matter of fact and existence, whether there are things that are infinitely complex.

This opinion of Hume's and the reasons for it just quoted resurface at a crucial point in Part IX of his *Dialogues Concerning Natural Religion,* where he considers the cosmological argument for the existence of God. What is the cause, the religious Demea asks, that accounts for there being something rather than nothing? What bestowed being on this particular succession of causes that we call the world? Cleanthes, for Hume, replies:

> In such a chain too, or succession of objects, each part is caused by that which preceded it, and causes that which succeeds it. Where then is the difficulty? But the WHOLE, you say, wants a cause. I answer, that the uniting of these parts into a whole, like the uniting of several distinct counties into one kingdom, or several distinct members into one body, is performed merely by an arbitrary act of the mind, and has no influence on the nature of things. Did I show you the particular causes of each individual in a collection of twenty particles of matter, I should think it very unreasonable, should you afterwards ask me, what was the cause of the whole twenty. This is sufficiently explained in explaining the cause of the parts.

As in the passage on infinite divisibility, Hume's "bottom-up" perception of the world as a heap of atoms blinds him to the opposite possibility. This mistake vitiates the whole section. The possibility that the world is a whole arbitrarily divided by the mind into parts, is symmetrical in all *a priori* respects with Hume's opinion that the world consists of particles arbitrarily grouped into wholes. [50] If Hume had been able to see past his *a priori* atomism, he would have had to admit that if ordinary-sized things need causes for their existence, so does the world, mere size and longevity being incapable of supplying a cause. And his *a priori* atomism is wrong, since its consequence, the denial of even the possibility of infinite divisibility, provides a *reductio* of it.

The same mistake is responsible also, of course, for the way Hume states the problem of personal identity. To see the problem as one of explaining how a bundle of entities can have unity is not the only possible view of the question (think of how Parmenides would see it), but it is the only one for an atomist. The way Hume speaks about causation also depends on his atomism about time: his assertion that a cause exists at the moment before and contiguous to the effect does not strictly make sense if time is infinitely divisible, since in that case points in time cannot be next to each other (since there are always other points in between). [51]

How big is an indivisible?

A final note: How big is a Humean indivisible? Strictly, this question has no answer, since indivisibles are extensionless. But there remains an almost identical question which must have an answer: How many indivisibles are there to the inch? "A finite number", according to Hume. But every finite number is some particular finite number; in our computer age, it has become better realized that one must not just assert that a computation, say, can be done in a finite time, but one must determine if this finite time is less than, say, the expected lifetime of the universe. The question is a perfectly reasonable one, on Hume’s view. Not only that, but he provides various hints towards answering it, though they are possibly not all consistent. His most definite pronouncement is one which will lead to a numerical answer: Put an ink blot on a piece of paper, he says, and take it out to where it is just visible; this produces the smallest possible impression, [52] which is necessarily the same size as the smallest atom of space. Comparing sizes in internal space with sizes in external space will not strike everyone as intelligible, but Hume plainly has no qualms on this score. What can be seen depends on such factors as how well the paper is lit; presumably we should take the minimum, achieved under the best possible conditions. So we may interpret Hume as saying that there are as many indivisibles in an inch as there are black dots in an inch at the distance when they are just visible. There seems no other choice, given Hume’s insistence that there should be a numerical answer. It is true that Hume adds that "There are bodies vastly more minute than those which appear to the senses", but in the context this appears to mean only that a body seen as a speck in the distance is known to have parts; it does not mean (for Hume immediately denies it) that there can be real things smaller than the smallest idea or impression. It should be possible, then, to find the size of an indivisible by performing the ink-drop experiment, and measuring the angle subtended at the eye by the drop when it is just visible. Hume does not say if he performed this experiment, but early experimental psychology spent a considerable amount of effort establishing the limits of perceptual acuity. Of course, one can see things that occupy an indefinitely small angle, if they are bright enough; this can be confirmed by going outside on a clear night and looking up at any star. But it does make sense to determine the smallest black dot that can be seen against a white background. Such measurements were carried out in Hume’s lifetime by Tobias Mayer, who found that under good conditions, the dot is visible if it occupies about 34 seconds of arc. [53] This corresponds to a mere 500 indivisibles to the inch. But it turns out that much better results can be got by testing very thin black lines against a white background; in perfect conditions one can see a line so thin it occupies half a second of arc at the eye. [54] This is a very small angle, approximately that subtended by a telephone wire at 2 km, or a telephone pole at over 100km. Given that the best visibility is obtained about 1 foot in front of the eye, it can be calculated how many half-seconds of arc are subtended by one inch at the distance of 1 foot. This is the number of indivisibles in an inch. ("As the ultimate standard of these figures is deriv'd from nothing but the senses and imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of." [55]). The answer is about 35,000.

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2. R. Sorabji, *Time, Creation and the Continuum: Theories in Antiquity and the Early Middle Ages* (London: Duckworth, 1983); J.E. Murdoch, 'Infinity and continuity', ch. 28 of *The Cambridge History of Later Medieval Philosophy* ed. N. Kretzmann, A. Kenny & J. Pinborg (Cambridge: Cambridge University Press, 1982), especially pp. 579-82.
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4. N. Kemp Smith, *The Philosophy of David Hume* (London: Macmillan, 1941) ch xiv, appendix C, especially pp. 284-8.
5. D. Jacquette, 'Kant's second antinomy and Hume's theory of extensionless indivisibles', *Kant-Studien* 84 (1993): 38-50.
6. James Dalrymple, Viscount Stair, *Physiologia nova experimentalis* (Leiden, 1685), reviewed in *Oeuvres diverses de Mr. Pierre Bayle* (The Hague, 1728); discussion in D. Raynor, 'Hume's *Treatise* and the debate over the infinite divisibility of matter', unpublished.
7. References in Jacquette, p. 48 n. 21.
8. See A. Grunbaum, 'A consistent conception of the continuum as an aggregate of unextended elements', *Philosophy of Science* 19 (1952): 288-306; cf. R.J. Trudeau, *The Non-Euclidean*

Revolution (Boston: Birkhauser, 1987), p. 23-30.

9. G. Barrett, ‘Formal methods applied to a floating-point number system’, *I.E.E.E. Transactions on Software Engineering* 15 (1989): 611-621; P.R. Turner, ‘Will the "real" real arithmetic please stand up?’, *American Mathematical Society Notices* 38 (1991): 298-304.

10. Various references in M.J. Duff & C.J. Isham, eds, *Quantum Structure of Space and Time* (Cambridge: Cambridge University Press, 1982); H. Bacry, *Localizability and Space in Quantum Physics* (Berlin: Springer, 1988).

11. Hume, *Treatise of Human Nature,* I.II.ii (ed. L.A. Selby-Bigge, Oxford: Clarendon, 1960, p. 32).

12. Trudeau, ch. 7.

13. M. Dummett, *Frege: Philosophy of Mathematics* (Cambridge, Mass: Harvard University Press, 1991), pp. 13-19.

14. E.g. Smith, *Source Book,* vol. 2, p. 632 n. 1; I. Grattan-Guinness, ‘Berkeley’s criticism of the calculus as a study in the theory of limits’, *Janus* 56 (1969): 213-27.

15. *Treatise,* I.II.i (p. 27).

16. *Treatise,* bk 1, ed. D.G.C. McNabb (London: Fontana, 1962), p. 357.

17. S.M. Kosslyn, *Image and Mind* (Cambridge, Mass: Harvard U. Press, 1980), pp. 31-34.

18. R.N. Shepard & L.A. Cooper, *Mental Images and Their Transformations* (Cambridge, Mass: MIT Press, 1982); M. Tye, *The Imagery Debate* (Cambridge, Mass: MIT Press, 1991); cf. M.J. Farah, ‘The neural basis of mental imagery’, *Trends in Neuroscience* 12 (1989): 395-399.

19. E.g., E.C. Tolman, B.F. Ritchie & D. Kalish, ‘Studies in spatial learning. I’, *J. of Experimental Psychology* 36 (1946): 13-24; E.W. Menzel, ‘Chimpanzee spatial memory organization’, *Science* 182 (1973): 943-945; B. Tversky, ‘Spatial mental models’, *The Psychology of Learning and Motivation* 27 (1991): 109-145.

20. E.g., J. Glasgow & D. Papadias, ‘Computational imagery’, *Cognitive Science* 16 (1992): 355-94. On another Humean note, the 1992 *Science Citation Index* lists 36 articles containing the words ‘associative’ and ‘memory’ in their titles, and 10 with ‘associative’ and ‘learning’. Perhaps the view that the psychological aspects of Hume’s philosophy are the worst parts of it is due for revision.

21. A. Flew, ‘Infinite divisibility in Hume’s *Treatise*’, in *Hume: A Re-evaluation,* ed. D.W. Livingston & J.T. King (N.Y.: Fordham University Press, 1976), pp. 257-269.

22. Aristotle, *De Anima* 431a17.

23. Proclus, *Commentary on the First Book of Euclid's Elements* trans. G.R. Morrow (Princeton: Princeton University Press, 1970), pp. 41-45.

24. *Treatise,* I.III.i (p. 72).

25. Many refs in K. Rochford, A.P. Fairall, A. Irving & P. Hurly, 'Academic failure and spatial visualization handicap of undergraduate engineering students', *International J. of Applied Engineering Education* 5 (1989): 741-9; A.J. Bishop, 'Review of research on visualization in mathematics education', *Focus on Learning Problems in Mathematics* 11 (1-2)(Win-Spr, 1989): 7-16.

26. *Treatise,* I.II.ii (p. 29).

27. Fogelin, 'Hume and Berkeley . . .', pp. 48-51; see also R.J. Fogelin, *Hume's Skepticism in the Treatise of Human Nature* (London: Routledge & Kegan Paul, 1985), pp. 25-7.

28. Fogelin, 'Hume and Berkeley . . .', p. 53.

29. R. Mattern, 'Descartes: "All things which I conceive clearly and distinctly in corporeal objects are in them"', in A.O. Rorty, ed, *Essays on Descartes' Meditations* (Berkeley: U. of California Press, 1986), pp. 473-489.

30. J.P. Wright, *The Sceptical Realism of David Hume* (Manchester: Manchester U. Press, 1983), pp. 100-1; M.D. Wilson, *Descartes* (London: Routledge & Kegan Paul, 1978), pp. 126-7.

31. D. Raynor, '"Minima sensibilia" in Hume and Berkeley', *Dialogue* (Canada) 19 (1980): 196-200.

32. Berkeley, *Principles of Human Knowledge,* § 124; cf. G. Moked, *Particles and Ideas: Bishop Berkeley's Corpuscularian Philosophy* (Oxford: Clarendon, 1988), Appendix A; M. Atherton, *Berkeley's Revolution in Vision* (Ithaca, N.Y.: Cornell University Press, 1990), pp. 133-5.

33. § 132.

34. E.g. R.J. Brook, *Berkeley's Philosophy of Science* (The Hague: Nijhoff, 1973), pp. 157-8; C.D. Broad, 'Hume's doctrine of space', *Proceedings of the British Academy* 47 (1961): 161-176, at 168-171.

35. Berkeley, *Analyst,* quoted in Smith, *Source Book,* p. 629.

36. D.C. Stove, *The Plato Cult and Other Philosophical Follies* (Oxford: Blackwell, 1991), p. 140.

37. Berkeley, *Principles of Human Knowledge,* par. 23; cf. *Three Dialogues* (*Berkeley's Philosophical Writings*, ed. D.M. Armstrong, N.Y.: Collier, 1965, pp. 163-4).

38. *Treatise* I.III.i (p. 70); see F. Zabeeh, 'Hume's scepticism with regard to deductive reason', *Ratio* 2 (1960): 134-43.

39. *Treatise* I.II.iii (p. 37).

40. I.II.v.

41. See Wright, pp. 103, 105.

42. [Aristotle], *On Indivisible Lines* 968b1-4.

43. Aristotle, *De Sensu* 445b21-446a20.

44. Thomas Aquinas, *Summa Theologiae,* Part I q. 7 art. 4.

45. Hobbes, *English Works,* ed. W. Molesworth (1839-45, repr. Aalen: Scientia, 1962), vol. 7, pp. 67, 200ff, 438.

46. Vol. 1, p. 94; see further G.H. Herbert, 'Hobbes's phenomenology of space', *J. of the History of Ideas* 48 (1987): 709-717; H.M. Pycior, 'Mathematics and philosophy: Wallis, Hobbes, Barrow, Berkeley', *J. of the History of Ideas* 48 (1987): 265-286; W. Sacksteder, 'Hobbes: geometrical objects', *Philosophy of Science* 48 (1981): 573-590.

47. I. Barrow, *The Usefulness of Mathematical Learning* Lecture IX (1734, repr. London, Cass, 1970, 151).

48. *Treatise,* I.II.ii (p. 30).

49. Lucretius, *De Rerum Natura,* bk I lines 615-622; Barrow, p. 158. It is interesting that when Leibniz saw the same argument coming, he retreated to one of the most obscure of the scholastic doctrines on form and matter to explain the unity of space: *The Leibniz-Arnauld Correspondence*, ed and trans H.T. Mason, (Manchester: Manchester University Press, 1967), p. 153; cf. P. Cummins, 'Bayle, Leibniz, Hume and Reid on extension, composites and simples', *History of Philosophy Quarterly* 7 (1990): 299-314, and T.M. Lennon, 'Locke's atomism', *Philosophy Research Archives* 9 (1983): 1-28.

50. J. Franklin, 'More on Part IX of Hume's *Dialogues*', *Philosophical Quarterly* 30 (1980): 69-71; cf. J. Dye, 'A word on behalf of Demea', *Hume Studies* 25 (1989): 120-140; D.L.M. Baxter, 'Hume on infi nite divisibility', *History of Philosophy Quarterly* 5 (1988): 133-140, at p. 137.

51. A.D. Kline, 'Hume on causation and the necessity of temporal contiguity', *Mind* 94 (1985): 550-556.

52. *Treatise,* I.II.i (p. 27).

53. O.J. Grüsser, 'Quantitative visual psychophysics during the period of the European enlightenment', *Documenta Ophthalmologica* 71 (1989): 93-111.

54. R.L. Gregory, *Concepts and Mechanisms of Perception* (London: Duckworth, 1974), p. 20; R.N. Haber & M. Hershenson, *The Psychology of Visual Perception* (N.Y.: Holt, Rinehart & Winston, 1973), p. 113.

55. *Treatise,* I.II.iv (p. 51).