Mathematics, last bastion of Reason

by James Franklin

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The old century - and we hardly need a longer perspective to see this - was beset by, and in cultural life almost defined by, an unteachable *enfant terriblisme*. From Dadaism to The Sixties to postmodernism, it was sufficient to throw tomatoes at tradition to get a full-page spread from the intellectual paparazzi.

Let us not spend more time recalling any more of the isms and idiots of the culture of complaint, as the process of forgetting them is already well advanced and it may be unethical to impede it. Instead let us remember the possibilities that existed for those who wished to retain their sanity. Where to escape to? In the humanities world, there was always the past, and many a cultural refugee from various Modernisms recuperated through communion with Monteverdi, or Vermeer, or Jane Austen. But for those who preferred their culture still living and breathing, the most extensive vandal-free space was science.

Not quite all of science escaped the spirit of the age, unfortunately, and a few of the parts most visible from outside the scientific world caught some unpleasant philosophical diseases. High theory in physics was good science, but in its journey to popularisation acquired some German idealism that left it coated in prose about `reality dependent on the observer'. The achievements of genetics suffered a similar fate, becoming known largely through the snide inverted Panglossianism of `selfish gene' explanations of sociobiology. Real science, the kind that thinks hard and finds out what is the truth, became relatively hidden from view. It was still going on, though, and keeping happy several generations of dedicated researchers, almost all of them cheerfully oblivious to the cultural commentators' manifold demonstrations to the wider community that the pursuit of truth is impossible.

Two regions of science were particularly free of any modern nervousness about themselves. One was engineering, for the obvious reason that bridge construction on cultural relativist principles is forbidden by the laws of nature as strictly as by those of man. The other was mathematics.

Mathematics has several advantages as a cultural counterweight to relativisms and scepticisms. Everyone knows something about it - in fact quite a lot about it - so it is not necessary to take the word of experts about everything in it, as it is for, say, quantum physics. Secondly, the truths in it are subject to proof, and what is proved does not become unproved (though it can be proved better). For these reasons mathematics has always been an unfailing support for rationalist views, views which exalt the capacity of the human mind to find out the truth. Conversely, mathematics has been a perennial thorn in the side of opinions that abase human knowledge, and claim it is limited by sense experience, cultural experience or one's personal education and perspective. Any culture or person that can count to 4 has discovered that 2 + 2 = 4, and should any fear arise of losing a grasp of that truth, resort to counting stones will quickly relieve any anxiety.

Naturally, most cultural relativists have given mathematics a wide berth. Just a few have been prepared to claim that 2 + 2 = 4 *is* culturally relative, and that a different culture might find it more convenient to negotiate another answer, such as 5, or 37. It needs a certain strength of mind to fill a book with *arguments* to the effect that the arguments found in mathematics cannot reach an absolute truth, but it is certainly possible, as demonstrated by the existence of Paul Ernest's *Social Constructivism as a Philosophy of Mathematics*. This work has not so far made much impact outside faculties of education.

A single work did, however, succeed in undermining confidence in mathematical proof. It was the slim and compulsively readable 1976 book *Proofs and Refutations*, by Imre Lakatos.

The idea of the book is to follow the historical development of the proof of a single mathematical theorem, Euler's Theorem, and to `show' that it did not really succeed in establishing the theorem beyond doubt. One of several ironies in the affair was that the success of the book was partly due to the fact that the theorem being undermined was so interesting. It is easy to appreciate even for those whose school mathematics has virtually disappeared, and is in any case quite unlike the recipes for symbol manipulation that make school arithmetic so boring in the first place. Imagine a cube. Count its faces (6: top, bottom and four sides), its edges (12: four around the top, four around the bottom and four down the sides) and its vertices or corners (8: four around the top and four around the bottom). We now calculate - for reasons to be explained - the number V - E + F, meaning the number of vertices minus the number of edges plus the number of faces. For the cube it is 8 - 12 + 6, which is 2. The remarkable thing is that V - E + F is also exactly 2 for almost any solid one can think of, even those which vast numbers of faces, in the style of a Buckminster Fuller dome. (Even more generally, the flatness of the faces has nothing to do with the result, so it works for a network of lines drawn on the surface of a sphere as well.) A simpler case is a pyramid (including the bottom), for which V - E + F is (please try out your mental visualization facility one more time) is 5 - 8 + 5, which is 2 again. The secret lies, somehow, in the minus in front of the number of edges E: for solids with many faces, the extra vertices and edges are balanced by the extra edges. To begin to understand why this balance is exact - why extra vertices and faces are always exactly balanced by extra edges - imagine (last time) a square face on some perhaps large solid. What happens if we draw a diagonal of the square? The number of vertices V does not change. F goes up by one (since the square face has become two triangles), and E goes up by one (since we have added one new edge, the diagonal). So V - E + F for the whole solid is unchanged. The standard proof of the theorem proceeds by using this idea in reverse: one gradually removes edges, faces and vertices, showing that V - E + F is unchanged in the process, until one reaches a simple figure where one can count and find that V - E + F is 2.

So what does Lakatos have to say? He points out that there are certain solids for which the theorem is not true. One example is a cube with a hole drilled through it. *Proofs and Refutations* is structured as a classroom encounter between a teacher and a crowd of besieging students, who suggest further counterexamples to the theorem each time the teacher tries to amend the theorem to exclude them. There are many historical footnotes intended to show that the classroom encounter is a `rational reconstruction' of the real history of the theorem. Lakatos' conclusion is that this theorem, and by implication mathematical theorems in general, are never really proved, but always subject to refutation by further counterexamples.

It needs mathematical advice to explain why Lakatos' project is fundamentally dishonest. Most of the many mathematicians who read *Proofs and Refutations*, it is true, were not very alarmed by it. But that is because of the touchingly mole-like blindness to matters philosophical for which they are known, and which led them to believe Lakatos’ conclusion was merely that the teaching of mathematical theorems ought to be more interesting. Better informed mathematicians were not deceived. The distinguished Princeton mathematician John Conway points out that, for one thing, Euler's Theorem is atypical in that the terms in it turn out to be difficult to define. `Face' might seem clear, but do ring-shaped faces count? The vast majority of mathematical theorems, for example about numbers or algebra, do not have any such problems. Lakatos also refused to admit that the theorem has been proved of *some* solids - for example cubes - until it is established for *all* the solids for which it is true. But most misleading of all was his failure to describe the final state of the theorem. All the problems about ring-shaped faces, holes and so on can in fact be cleared up, but doing so requires that one should have the full classification of surfaces correct. This is a difficult and complicated, but not infinitely difficult and complicated, mathematical result. Lakatos stops his story just before getting to it, in order to leave the impression that the truth will never be pinned down. It is simply impossible that he should have stopped at exactly that point honestly.

There is more to complain about. For example, there are doubts about one's claim to be supported by history, when one has admitted to writing a `rational reconstruction' of history, that is, the history that would have supported one if it had happened. And Lakatos is one of the worst offenders in the compulsive scattering of quotation mark to `neutralise success words'. The word `proof' is a success-word, that is, it is part of its meaning that something proved is true, and that the proof has succeeded in showing it is true. So when Lakatos has his teacher say things like `I am not perturbed at finding a counterexample to a "proved" conjecture; I am even willing to set out to "prove" a false conjecture!' it is impossible to understand what it being said, though the reader is certainly left unsettled about the notion of proof. As David Stove explains in *Anything Goes: the origins of the cult of scientific irrationalism*, Lakatos puts quotation marks around words with success grammar like `show', `facts', `discovered' and `proof' so often that it is impossible for the reader to know when, or if, he thinks knowledge ever advances. Stove writes of the use of the word `proof' in Lakatos's book:

> In the book it is subjected countless times to neutralisation or suspension of its success-grammar by quotation marks. Often, of course, equally often, Lakatos uses the word without quotation marks. But what rule he goes by, if he goes by any rule, in deciding when to put quotation marks around "proof" and when to leave them off, it is quite impossible for the reader of that book to discover. Nor does the reader know what meaning the writer means to leave in this success-word. He knows that the implication of success is often taken out of it; or rather, he knows that on any given occurrence of the word in quotation marks, this implication may have been taken out of it. But what meaning has on those occasions been left in it, he is entirely in the dark.

It can hardly have seemed likely, in the glorious days when Euclid finally delivered the manuscript of his *Elements* and Plato required the study of geometry of entrants to the Academy, that the distant future would find mathematicians defending their access to the realm of forms by polemics on the misuse of quotation marks. It did not seem any more likely in 1900, but the Twentieth Century had a number of surprises in store of a similar nature. The forms, unfortunately, cannot defend themselves, as they do not have a causal action on the physical world. Neither ethical nor mathematical truths and ideals can fight tanks, or blizzards of quotation marks (though again, neither can they be liquidated by those enemies). They depend on human minds in tune with them to act on their behalf – to implement those ideals and teach them to the next generation. And the necessary defensive action that this implies has to be against whatever attacks have actually appeared, however ridiculous.

Who, then, was the real Lakatos? From the effect - the works of Lakatos - it would be natural to rationally reconstruct the cause as a typical Sixties irresponsible leftist guru. Nothing could be less true, as revealed now in full in Matteo Motterlini's *For and Against Method*. [1] This consists mostly of the correspondence between Lakatos and Paul Feyerabend, from 1968 up to Lakatos' sudden and untimely death in 1974. Feyerabend was a philosopher of science at the extreme irrationalist end of the spectrum, known principally for his *Against Method: outline of an anarchistic theory of knowledge*, which maintained that witchcraft was as rational as science. Feyerabend really was a typical Sixties irresponsible etc. He writes to Lakatos from Berkeley on 28 Feb 1970:

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[1] *For and Against Method: including Lakatos' lectures on scientific method and the Lakatos-Feyerabend correspondence*, by Imre Lakatos and Paul Feyerabend, edited and with an introduction by Matteo Motterlini; University of Chicago Press, 451 pages, $34.

Future events in my philosophy of science class:
Tomorrow: discussion on the *New Left*.
Next meeting: hidden variables.
Then: Genesis and Evolution.
Then: St Thomas's angelology as a contribution to the theory of light.

Searle is again trying to get me fired ... The only bright things are the little chicks here, and the freshmen altogether. They are very naive, but nice and undogmatic.

Lakatos was fascinated by this not from any sympathy but because, at least by this late stage of his life, it was his repressed Other. Lakatos had spent three years in jail in Hungary in the early 1950s, after the Party had investigated an incident during the War in which he had apparently taken the lead in encouraging the suicide of a fellow-member of a Communist cell, whose Jewishness was thought to render her silence under interrogation unlikely. He fled Hungary in 1956, confessing he had reported politically on colleagues, and found refuge at the London School of Economics. By the time of the correspondence with Feyerabend, he was in favour of U.S. bombing of North Vietnam, and was urging the authorities to firmer action against student radicals at LSE, the scene of some of the worst student disturbances in Britain.

Lakatos was a dangerous customer, in other words, because he was not a one-sided and ignorant ideologue trapped in a single thoughtworld. It was the same with his intellectual activities. He was no ignorant postmodernist attempting to undermine reason from a distance with some general-purpose argument to the effect that `everything is social'. He knew modern logic, and the technical investigations of the Vienna Circle on the logic and language of science. *Proofs and Refutations* has the impact it does because Lakatos knows the mathematics - or as much of it as he chooses to know - and can take the reader through it. The question is how he came to jettison so much of his intellectual past, and to believe himself a friend of reason, while making his living out of a project fundamentally irrational. What exactly had he taken on in youth that he found it impossible to discard?

The answer to this question appears in Larvor's recent introduction to Lakatos's thought, [2] and even more clearly in a remarkable article by Dusek [3] on how the debate between Lakatos and Feyerabend in science recapitulates that of their mentors, Lukács and Brecht, on aesthetics and on methodology generally. The childhood intellectual trauma that Lakatos could never throw off was Hegel – in particular the primacy of history. Lakatos himself never quite lost Hegel's faith that there was some rational direction to history. But his work served to reinforce a much more modern version of historicism. The idea that any given phenomenon should be explained primarily by stating the history of how it got to be that way (with the implication that it could equally easily have got to be some other way) is one has done immense damage in every branch of learning. Lakatos' `achievement' was to give this thesis a colour of plausibility in the discipline most resistant to it, mathematics. It is in mathematics that is it most obvious that the fundamental reason why the theorems are as they are is that they agree with the absolute truth. To use one's mathematical talent to make it appear otherwise, as Lakatos did, is to take the route described by Marlowe:

All: God forbid!
Faustus: God forbade it, indeed, but Faustus hath done it.

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[2] *Lakatos: An Introduction*, by Brendan Larvor, Routledge, 1998.
[3] V. Dusek, Brecht and Lukács as teachers of Feyerabend and Lakatos: the Feyerabend-Lakatos debate as scientific recapitulation of the Brecht-Lukács debate, *History of the Human Sciences* 11 (2) (May 1998).

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