

# A Purely Recombinatorial Puzzle\*

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## Abstract

A new puzzle of modal recombination is presented which relies purely on resources of first-order modal logic. It shows that naive recombinatorial reasoning, which has previously been shown to be inconsistent with various assumptions concerning propositions, sets and classes, leads to inconsistency by itself. The context sensitivity of modal expressions is suggested as the source of the puzzle, and it is argued that it gives us reason to reconsider the assumption that the notion of metaphysical necessity is in good standing.

## 1 The Puzzle

It is natural to assume that what is possible is closed under certain ways of recombining individuals. The idea is hard to formulate in full generality, but it is easily explained by way of example. Lewis (1986, p. 88) uses the following:

“[...] if there could be a dragon, and there could be a unicorn, but there couldn't be a dragon and a unicorn side by side, that would be an unacceptable gap in logical space [...]”

Of course, following Kripke (1980 [1972], pp. 156–158), it might be impossible for there to be any unicorns or dragons. And Kripke's essentialist theses put other restrictions on recombination: it might be impossible for there to be Socrates without there being Socrates's parents (at some point in time). Nevertheless, many instances of recombination are not in conflict with any such essentialist claims. E.g., none of them seem to rule out the natural claim that if it is possible for there to be a gold sphere of a certain diameter  $d$ , then it is possible for there to be this sphere, made of gold and of diameter  $d$ , as well as another gold sphere of diameter  $d$ . Similarly, if for each diameter  $d$  among  $d_1, \dots, d_n$ , it is possible for there to be a gold sphere of diameter  $d$  such that no two of them share any matter, then it should be possible for there to be all of them, made of gold and of diameters  $d_1, \dots, d_n$ , respectively.

Recombination has mostly been discussed either in the context of particular combinatorial theories of modality such as Armstrong's (1989), or Lewis's modal realism. In the peculiar framework of Lewis (1968, 1986), where modal discourse is re-interpreted using counterpart theory, recombinatorial principles take on

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a very specific flavor. But the general idea of recombination is motivated by our pre-theoretic understanding of modality, and the term “recombination” is increasingly applied independently of any specific theory of modality, e.g., in Uzquiano (2015b). I will follow this wider usage, although nothing depends on this; any occurrence of, e.g., the term “recombinatorial principle” in the following might be replaced by a more neutral term, such as “principle of modal plenitude”. Furthermore, I will take modal discourse at face value, *pace* Lewis.

The puzzle to be stated shows that naive recombinatorial reasoning supports two principles which are jointly inconsistent. To motivate them, I will engage in the familiar talk of possible worlds and of individuals in a given world. In particular, I will allow myself to quantify over the individuals in various worlds as if from an outside perspective. The two principles themselves will be stated somewhat more carefully, and they will be formalized in first-order modal logic in the next section, in order to show that such problematic ways of speaking are inessential for the puzzle. For purposes of illustration, it will also be assumed that necessarily, it is possible for there to be an angel, and, with Hawthorne and Uzquiano (2011, p. 54), that necessarily, if angels have location at all, it is possible for distinct angels to be co-located.

To motivate the first principle, consider any possible world  $w$ . Take all the things in  $w$  which are angels in  $w$ . By recombinatorial reasoning, there is then a possible world  $v$  in which all of them are angels, and in which there is an extra angel, i.e., in which there is something which is an angel in  $v$  but which is not an angel in  $w$ . This can be motivated somewhat more carefully by distinguishing two cases: If there are no things in  $w$  which are angels in  $w$ , then the assumption that there could then be an angel entails that there is a world  $v$  as required. If there are some things in  $w$  which are angels in  $w$ , then the existence of a world  $v$  in which there is an extra angel is motivated by the recombinatorial idea of being able to duplicate a given individual to generate a further possibility, as in the above example of the gold sphere.

It is hard to sum up the conclusion of this train of thought in natural language without reference to possible worlds. A certain use of “actually” allows us to make the relevant claim at least for the actual world with relative ease. We can say:

- (1@) Whatever angels there are, it is possible for all of them to be angels and for there to be an angel which is actually not an angel.

But because of the indexical nature of “actually”, we can’t just prefix (1@) by “necessarily” in order to say that the relevant claim holds necessarily (on any easily accessible reading). The following construction, although somewhat awkward, makes the intended reading more natural:

- (1) The following is necessarily the case: whatever angels there are, it is possible for all of them to be angels and for there to be an angel which is actually not an angel.

This formulation is clearly ambiguous, but the intended reading can at least be inferred from the preceding motivating discussion in terms of possible worlds. Additionally, a statement of this principle in first-order modal logic will be given in the next section.

To motivate the second principle, consider all possible worlds. For each world  $w$ , consider the individuals in  $w$  which are angels in  $w$ . Now take all of them,

i.e., take all the individuals  $x$  such that for some world  $w$ ,  $x$  is in  $w$  and  $x$  is an angel in  $w$ . By recombinatorial reasoning, there should then be a world in which all of them are angels, just as in the case of golden spheres of various diameters above. The assumption that angels can be co-located if they are located at all guarantees that they won't require too much space to all fit into a single world.

This conclusion is also difficult to state without appeal to possible worlds, but we can do so if we allow ourselves to use the phrase “all possible angels” to universally quantify over all individuals which, for some possible world  $w$ , are in  $w$  and an angel in  $w$ . Such constructions are in fact relatively common. For example, it is natural to talk of “all possible people” without intending this to be restricted to things there actually are; see Fritz and Goodman (forthcoming) for further discussion. The second principle can then be stated as follows:

(2) Possibly, all possible angels are angels.

As with (1), this formulation is ambiguous, but again, the discussion in terms of possible worlds isolates the intended reading, and a formal statement will be given below.

(1) and (2) are jointly inconsistent: if, as (2) claims, it is possible for all possible angels to be angels, then in such a case it would be impossible for there to be an additional angel, contradicting (1). One can see the puzzle as revolving around the question whether it is possible for all possible angels to be angels. Recombination leads us into contradiction by giving us reasons for answering both yes and no.

This version of the puzzle is easily answered: there are no angels, and, following Kripke, there couldn't be any. If this is correct, then (2) is trivially true: since there could not be any angels, all possible angels are in fact angels. But analogous puzzles arise for certain kinds of elementary particles. E.g., Hawthorne and Uzquiano (2011, p. 55–56) suggest that large numbers of bosons may be co-located. Even if the actual laws of physics put restrictions on how many elementary particles of any actual kind can co-exist, presumably there could be kinds of elementary particles which are not subject to any such restriction. For vividness, the following continues on the assumption that there could be angels.

## 2 Formalization

There are a number of aspects of the discussion so far with which one might take issue: The motivation of (1) and (2) was couched in terms of possible worlds and individuals in various possible worlds. As Williamson (2013, chapter 3) argues, it is not obvious whether all such talk can be made sense of without incurring a commitment to necessitism, the contentious claim that it is necessary what there is. (2) contains the phrase “all possible angels”, which is not intended to be understood as a restricted universal quantifier – all things which are possibly angels – but rather as some kind of modalized quantifier which intuitively ranges not only over what there is but also over what there could be. Even (1), which contains no such phrase, was explicitly noted to be ambiguous.

One might therefore wonder whether the puzzle arises from an equivocation or non-sensical ways of speaking. In order to show that the puzzle is in fact genuine, the present section restates it in a language of first-order quantified modal logic. The formal language to be used contains atomic formulas constructed

from individual variables  $x, y, \dots$  and a unary predicate  $A$  for “angel”, the Boolean operators  $\wedge$  for conjunction,  $\neg$  for negation and  $\rightarrow$  for material implication, modal operators  $\Box$  and  $\Diamond$  for necessity and possibility, respectively, and a universal and an existential quantifier binding individual variables, respectively written  $\forall$  and  $\exists$ . As usual in metaphysics, the modal operators  $\Box$  and  $\Diamond$  will be interpreted as expressing a particular modality called “metaphysical modality”, at least until further notice – the assumption that there is such a modality will later be questioned. Similarly, quantifiers are intended to be read unrestrictedly, and the availability of such a reading will be discussed below. On these assumptions, all expressions apart from individual variables receive specific interpretations, and so closed formulas can be understood to be true or false *simpliciter*, not just relative to a model in a formal semantics.

Going back to Fine (2005 [1977]), there is a well-known strategy of formalizing a modalized phrase such as “all possible angels” in quantified modal logic. The basic idea is to paraphrase “all possible angels are such that  $\varphi$ ” with “necessarily all angels are such that  $\varphi$ ”. The difficulty in making this precise is that  $\varphi$  should be evaluated as if it were outside of the scope of “necessarily”; somehow the effect of this operator must be undone for the purposes of evaluating  $\varphi$ . Following the version presented in Correia (2007, section 4), add two unary operators  $\downarrow_i$  and  $\uparrow_i$  for each natural number  $i$  to the language. On the intended reading, an occurrence of  $\downarrow_i$  undoes the semantic effect of the modal operators in the scope of the previous occurrence of  $\uparrow_i$ . (The intended reading of  $\downarrow_i$  is sometimes described as exempting the subformula it operates on *from the scope of* the modal operators in the scope the previous occurrence of  $\uparrow_i$ , but this requires a controversial distinction between syntactic and semantic scope; see Forbes (1989, pp. 86–102).) In a possible world semantics,  $\uparrow_i$  can be thought of as storing the current world of evaluation under the label  $i$ , and  $\downarrow_i$  as retrieving it and evaluating the following subformula at it. With this, “all possible angels are such that  $\varphi$ ” can be formalized as follows:

$$\uparrow_1 \Box \forall x (Ax \rightarrow \downarrow_1 \varphi)$$

Correspondingly, (2) is now easily formalized:

$$(F2) \Diamond \uparrow_1 \Box \forall x (Ax \rightarrow \downarrow_1 Ax)$$

Formalizing (1) is only slightly more difficult:

$$(F1) \Box \uparrow_1 \Diamond \uparrow_2 (\downarrow_1 \forall x (Ax \rightarrow \downarrow_2 Ax) \wedge \exists x (Ax \wedge \downarrow_1 \neg Ax))$$

( $F1$ ) and ( $F2$ ) are easily seen to be inconsistent: If a conjunction is necessarily possible, then so are its conjuncts. So ( $F1$ ) entails the result of eliminating the first conjunct of its conjunction. In the resulting formula,  $\uparrow_2$  is redundant; eliminating it as well yields:

$$(F1') \Box \uparrow_1 \Diamond \exists x (Ax \wedge \downarrow_1 \neg Ax)$$

Informally, ( $F1'$ ) can be stated as follows:

(1') Necessarily, there could be an angel which is not an angel.

Like (1) and (2), this is ambiguous; here the formalization ( $F1'$ ) isolates the intended reading. Note that this reading does not claim that necessarily, there

could be something which is both an angel and not; as the use of the indicative suggests, the phrase “is not an angel” is intended to refer back to “necessarily” but not to “could be”.

( $F1$ ) entails ( $F1'$ ), and ( $F1'$ ) entails the negation of ( $F2$ ). Thus ( $F1$ ) and ( $F2$ ) are jointly inconsistent.

Entailment here is roughly understood as a familiar relation of logical consequence among closed formulas of first-order modal logic (with  $\uparrow_i$  and  $\downarrow_i$ ). As usual, such relations can be described in various ways. Appendix A specifies one such relation using variable domain Kripke models. It is noted there that this relation supports the entailment judgements above: according to it, the negation of ( $F2$ ) follows from ( $F1'$ ), which in turn follows from ( $F1$ ). Alternatively, a proof system could be specified along familiar lines in which the negation of ( $F2$ ) is derivable from ( $F1'$ ) which in turn can be derived from ( $F1$ ). For present purposes, what notion of consequence is appealed to is to a large degree irrelevant; all that is required to establish the inconsistency is that it guarantees truth-preservation.

Assuming that we have successfully endowed all expressions of the formal language unambiguously with meaning, the formalization presented here shows that the puzzle is genuine; it is not the result of, e.g., a scope confusion or the appeal to non-sensical ways of speaking. Thus, at least one of ( $F1$ ) and ( $F2$ ) is false, and so the recombinatorial reasoning which motivates them must be rejected.

Might there be a problem with the interpretation of the formal language? Presumably, there is nothing problematic about the use of Boolean connectives. This leaves atomic predications of the form  $Ax$ , the quantifiers, the operators  $\downarrow_i$  and  $\uparrow_i$ , and the operators  $\Box$  and  $\Diamond$ .  $Ax$  may exhibit some kind of indeterminacy if the English “angel” exhibits such a feature, maybe in virtue of being vague or context-sensitive. However, even if this is the case, it seems to be incidental to the puzzle, since plausibly, it is possible for there to be a language containing a predicate for the relevant kind of spiritual beings or elementary particles which does not exhibit the relevant kind of indeterminacy. Similarly, it is implausible that any potential indeterminacy in the quantifiers is responsible for an equivocation in ( $F1$ ) and ( $F2$ ). For in ( $F1$ ) and ( $F2$ ), quantifiers only occur restricted to  $A$ , i.e., in subformulas of the form  $\forall x(Ax \rightarrow \dots)$  and  $\exists x(Ax \wedge \dots)$ . Even if there is no distinguished unrestricted reading of quantifiers, e.g., because it is indefinitely extensible what sets there are (whatever that means – see Uzquiano (2015c) for some options), it is implausible that this kind of indeterminacy extends all of the relevant restricted quantifiers: even if on certain theological views, it is indefinitely extensible what angels there are, there is nothing to motivate the view that for any possible kind of elementary particles, it is indefinitely extensible what particles of this kind there are. Note that the present considerations concern the indefinite extensibility of what particles of a certain kind there *are*; the idea that it is indefinitely extensible what particles of a certain kind there *could be* will be taken up in section 6.

Although somewhat unfamiliar, the operators  $\downarrow_i$  and  $\uparrow_i$  are increasingly used in modal metaphysics; see, e.g., Williamson (2013). Going back to their origins in Vlach (1973), they can be seen as modal analogs of formalizations of “now” and “then”. Although there may be no entirely natural way of translating  $\downarrow_i$  and  $\uparrow_i$  directly into English, there is substantial evidence that they closely correspond to features found in everyday usage of English; see Cresswell (1990). However,

one might argue that the intelligibility of  $\downarrow_i$  and  $\uparrow_i$  in everyday contexts does not show that they are intelligible in discourses involving metaphysical necessity. Indeed, it has been argued that the use of operators like  $\downarrow_i$  and  $\uparrow_i$  is unavailable for those who take modal operators to be more fundamental than possible worlds talk; see Melia (1992) for an argument along these lines and Forbes (1992) for a reply. Whatever one thinks about  $\downarrow_i$  and  $\uparrow_i$ , it is of interest to formulate the present puzzle without appeal to them. Appendix B notes that no uncontroversial formalization of (1) and (2) is available in first-order modal logic without  $\downarrow_i$  and  $\uparrow_i$ , and develops a variant formalization using plural quantifiers. In the following,  $\downarrow_i$  and  $\uparrow_i$  will be assumed to be in good standing. This leaves the interpretation of the modal operators as the only remaining possible source of an equivocation in (F1) and (F2). This issue will be taken up again in section 5; until then, it will be assumed that there is a distinguished metaphysical modality with which we are concerned in metaphysics, and which serves as the interpretation of  $\Box$  and  $\Diamond$ .

### 3 Comparisons

The puzzle presented here is reminiscent of a number of similar puzzles. Although not all of these puzzles have been discussed under the label “recombination”, they all rely on sufficiently similar principles concerning the possibility of various situations to be usefully grouped under this label for the purpose of comparing them to the puzzle presented here.

A well-known example is the argument of Kaplan (1995), already discussed in Davies (1981, appendix 9). A simple version of it starts from the premise that for any proposition, it is possible for a given agent to uniquely entertain it, and derives a contradiction. Another well-known example is the argument in Forrest and Armstrong (1984), intended as an argument against Lewis’s modal realism. A variant of this puzzle presented in Nolan (1996, p. 246) assumes that for any cardinality  $\kappa$  and object  $o$ , it is possible for there to be  $\kappa$  duplicates of  $o$ , and concludes that there is no set of all possible objects. The last example to be mentioned here is less well-known but more closely related to the puzzle stated above. It is given in Fine (2005 [2003], p. 223); a version of it can be summed up as the following triad, which Fine argues to be inconsistent: First, it is possible for there to be an angel. Second, necessarily, whatever angels there are, it is possible that all of them are angels, and that for every class of them, there is a distinct guardian angel. Third, for any class of possible angels, it is possible that there are all of them.

All of these examples essentially rely on broadly logical resources which go beyond those of first-order modal logic: the first appeals to propositions, the second to sets and cardinalities, and the third to classes. But naive reasoning involving these notions has led to inconsistency before, without considering matters of recombination, or indeed any modal matters – recall Russell’s paradox of naive set-comprehension, which of course applies to classes as well, or the Russell-Myhill paradox of propositions. The three puzzles mentioned here are therefore naturally taken to put constraints on these resources, rather than to cast doubt on recombination in general.

Such conclusions are widely endorsed. E.g., Anderson (2009) shows how to recast Kaplan’s argument in a purely syntactic form, leading to a puzzle which

is purely about propositions, and closely related to one already discussed by Prior (1958, 1961); see also Kripke (2011) for a similar puzzle involving sets of times. Kaplan himself suggests adopting a ramified theory of propositions, and Lindström (2009) concludes that it is contingent what propositions there are. Concerning puzzles along the lines of Forrest and Armstrong, some recent writers have concluded that they motivate rejecting certain theses of impure set theory, such as the claim that there is a set of all and only the non-sets; see, e.g., Nolan (1996), Oksanen (1999) and Menzel (2014). Sider (2009) and Hawthorne and Uzquiano (2011) pose such puzzles as problems for particular metaphysical views, such as necessitism; Uzquiano (2015b) uses such a puzzle to argue, tentatively, against conceiving of propositions as objects. See also Uzquiano (2015a) for an overview of related arguments.

The puzzle which this paper puts forth differs fundamentally from these puzzles in that it does not appeal to propositions, sets, classes, or any such further resources. It only requires the resources of first-order modal logic, including the operators  $\downarrow_i$  and  $\uparrow_i$ , naive reasoning about which has not led to inconsistency – in this sense, it is a *purely recombinatorial* puzzle. Unless we are prepared to give up the principles of quantified modal logic from which the inconsistency of the two premises follows – an option I won’t consider here – we simply have no choice but to reject the recombinatorial reasoning which motivated the inconsistent principles. Furthermore, (1) and (2) seem to be motivated by a single underlying recombinatorial idea, rather than being motivated independently. The puzzle therefore casts doubt on this recombinatorial idea in general, not just the particular instances (1) and (2).

This conclusion has repercussions for the recombinatorial puzzles discussed in the literature. Many versions of them rely on instances of recombinatorial reasoning which are very similar to the trains of thought which motivated (1) and (2). Since we have to reject the latter, we have good reasons to mistrust the former as well. Thus the present purely recombinatorial puzzle undermines the recombinatorial premises of the (impurely) recombinatorial puzzles mentioned above, and so the conclusions concerning propositions, sets, and certain metaphysical views which various authors have drawn from them. Of course, this observation is not a blanket refutation of all arguments referred to above, but it does point to a need to re-evaluate the recombinatorial premises on which they are based.

What could such a re-evaluation look like? If, following Lewis (1973, p. 88), metaphysical theorizing consists in developing our pre-theoretic judgements into systematic theories, then a systematic theory which can accommodate some but not all of our pre-theoretic judgements concerning recombination may be better than one which cannot accommodate any such judgements. In weighing the costs of rejecting recombinatorial principles, we must consider how firmly we are attached to them on their own, as well as how well rejecting some but accepting other such principles can be motivated in a systematic theory. It is therefore the principles themselves and their comparative similarity or independence which we must take into account; that they are here all grouped under a very broad use of the term “recombinatorial principle” is to a large extent an irrelevant terminological choice.

In addition to the fact that the puzzle presented here does not depend on any particular theory of propositions, sets, or classes, it should also be noted that it is independent of a number of further assumptions which figure in some versions

of the other puzzles discussed in this section. In particular, it is independent of the question whether it is necessary or contingent what there is, of the question whether being an angel is an essential property of individuals, of any particular theory of modality or the semantics of modal operators, of the question whether being an angel entails being something, and, as discussed in more detail in appendix C, of the familiar iteration principles for modal operators.

It is also worth noting that Fine’s puzzle, which has not found much discussion in the literature on recombination, can be reformulated purely in terms of plural quantifiers rather than classes, similar to the formulation of the present puzzle developed in appendix B: Fine’s third premise may be reformulated along the lines of (2), and the second premise may be expressed by appeal to an unrestricted theory of ordered pairs. Such a version of Fine’s puzzle may be used to draw the same conclusions as I draw here from the puzzle set out above. Since the latter is considerably simpler in formulation and its premises are *prima facie* more compelling, this paper focuses on this version.

The purely recombinatorial puzzle presented here suggests that recombination is problematic itself, and that we should be suspicious of principles like (1) and (2). But leaving things at this conclusion would be theoretically unsatisfying; one would like to know why (1) and (2) seem so plausible despite being inconsistent. The next section develops a tentative answer.

## 4 Context Sensitivity

Everyday modal talk is notoriously context sensitive. Without knowing the context of utterance, it is impossible to tell which modality is expressed by an utterance of a sentence like “I can buy groceries today”. Further, uttering a sentence involving a modal expression can change the context of utterance, and thus the contextually salient modality, causing a subsequent utterance of the same modal expression to express a different modality. This opens up a way of saving our naive recombinatorial reasoning from inconsistency at least in everyday contexts, by showing that such reasoning involves context shifts, and so only supports readings of (1) and (2) on which different modal terms express different modalities.

How might this idea be fleshed out? A very rough story can be given on the basis of the following principle:

- (C) Typically, whatever (possible) individuals are contextually salient, the contextually salient modality is permissive enough to admit of certain ways of recombining them; in particular, as far as this does not conflict with essentialist theses like origin essentialism, it admits of there being all of them, and of there being all of them plus a duplicate of any one of them.

A few aspects of this principle need to be clarified: First, the principle is not intended to apply to epistemic or deontic modalities, but rather only to modalities which are metaphysical in a wide sense (not in the narrow sense of expressing what philosophers call “metaphysical modality”). In linguistics, such modalities are sometimes called “circumstantial”, but there is no consensus about how to classify the various uses of modal terms; see Portner (2009, sect. 4.1). Second, (C) is only intended to describe features of modal context dependence in *typical* contexts. E.g., if there are contexts in which, for all notions of possibility, all



possible individuals are salient, then by assumption, no modality admits of there being an extra individual. Third, the contextually salient (possible) individuals are not assumed to be all those which are possibly something according to the contextually salient modality; rather, it is assumed that the former is typically more restricted.

Let “possible<sub>c</sub>” express the modality expressed by “possible” in context *c*. We start from some context *c* which is typical in the sense of (*C*). Assessing the truth of (2), we consider the possible<sub>c</sub> angels. By (*C*), it follows that the contextually salient modality must admit of there being all of these angels. Of course, it might be that it is not possible<sub>c</sub> that all possible<sub>c</sub> angels are angels, in which case (*C*) forces a context shift to a context *d* such that it is possible<sub>d</sub> that all possible<sub>c</sub> angels are angels. Thus this reasoning may only support a reading of (2) on which its two modal terms express different modalities. A similar train of thought can be carried out for (1). On such non-uniform readings (1) and (2) need not be jointly inconsistent – the above arguments for the inconsistency of (1) and (2) relied on a uniform resolution of the context sensitivity of the modal terms occurring in them. Formally, if all four occurrences of modal operators in (*F1*) and (*F2*) are indexed by distinct indices and no further assumptions are made about the relations between modal operators with different indices, their consistency is easily demonstrated model-theoretically.

The story sketched in this section of course neither amounts to a full cognitive theory of the relevant processes of human reasoning nor a full semantic theory of the relevant linguistic data. In particular, it does not tell us whether in a given utterance of (1) or (2), the various modal terms express different modalities. It is also unclear whether it essentially relies on mid-sentence context-shifts, or whether a similar account can be given on which (1) and (2) each receive uniform interpretations, and the context only shifts between the two sentences. But the story does point the way to a compelling resolution of the puzzle, in the sense of explaining why we are pre-theoretically inclined to judge both (1) and (2) to be true, despite also being inclined to judge them as being incompatible: our naive recombinatorial reasoning only supports (1) and (2) on non-uniform readings of their modal operators, but the argument for their inconsistency relied on resolving their context sensitivity uniformly.

## 5 Metaphysical Necessity

The story about context sensitivity might explain why we are inclined to endorse both (1) and (2) despite also judging them to be incompatible. But it does not challenge the conclusions reached above. Recall the assumption that in metaphysics, modal terms are understood as expressing a distinguished modality, sometimes called “metaphysical modality”, thereby resolving the context sensitivity of modal expressions uniformly. Principles of recombination are intended to apply to this modality, and the usual treatment of recombination motivates (1) and (2), reading modal terms uniformly as expressing metaphysical modality. Nothing in the previous section changes the conclusion that the inconsistency of (1) and (2) shows that naive recombinatorial reasoning in metaphysics – where modal terms are read as uniformly expressing metaphysical necessity – leads to inconsistency.

With the pervasive context dependence of modal terms in everyday contexts,

this conclusion puts pressure on the assumption that we have managed to resolve the context sensitivity of modal terms by qualifying them as “metaphysical”. This might be a surprising conclusion to draw from the puzzle. But consider what reasons we have for believing that we have succeeded in singling out a particular notion of metaphysical necessity. Mostly, this is a dogma of current metaphysics, which is supported by the hope that some core theoretical roles of metaphysical necessity suffice to pick out a particular distinguished modality. Principles of recombination constitute one of these theoretical roles. If they cannot consistently be applied to any one modality, they cannot help to single out metaphysical necessity. Thus, the puzzle at least chips away at the support for the assumption that the notion of metaphysical necessity is in good standing.

Recombinatorial principles don’t have an especially central place among the theoretical roles of metaphysical necessity. It would be hasty to reject a notion as well-established as metaphysical necessity on the sole basis of a little puzzle about recombination. But the puzzle may serve to remind us that the wide acceptance which metaphysical necessity has gained among metaphysicians is no guarantee that this notion is in fact in good standing. Whether there is such a notion is not a terminological question we can easily settle by stipulation, but neither is it a dogma we should take on faith.

## 6 Solving the Puzzle

So far, this paper has focused on stating the puzzle and drawing some consequences for metaphysics from it. But how should the puzzle itself be solved? One option is to take the skeptical conclusions concerning metaphysical necessity suggested in the previous section seriously, and to claim that there is no distinguished metaphysical modality. Building on the discussion in section 4, one might then claim that for every uniform resolution of the context-sensitivity of modal operators, at most one of (1) and (2) is true, but that for any such reading on which one of these principles comes out as false, there is a stronger reading on which it comes out as true (where a reading of modal operators is stronger than another if everything which is necessary according to the former is necessary according to the latter, but not *vice versa*). Such a response is highly revisionary, as the assumption that there is a distinguished metaphysical modality is widely held in contemporary metaphysics. The assumption also features in several of the established arguments discussed in section 3, such as Kaplan’s argument. The response of rejecting this assumption therefore does not provide a general way of holding on to the conclusions concerning propositions, sets and classes drawn in the literature from impurely recombinatorial puzzles in the face of the present purely recombinatorial puzzle.

Consider now the more conservative option of holding on to the existence of a distinguished modality of metaphysical necessity. On pain of contradiction, this requires rejecting one of (1) or (2). Such a rejection might fall out of a general rejection of recombinatorial reasoning, although this response leaves open which, if any, of (1) and (2) is true. A more subtle response is to reject one of (1) and (2) on the basis of an argument for the claim that only one of these principles is genuinely motivated by recombinatorial reasoning. E.g., one might roughly trace (1) to the idea of being able to duplicate individuals, and (2) to the idea of being able to cut and paste individuals (subject to essentialist restrictions), and

argue that only one of these constitutes a safe recombinatorial principle. Such an argument might involve both intrinsic and abductive considerations, and could lead to a re-evaluation of the recombinatorial premises of the impurely recombinatorial arguments, as suggested in section 3. Particular metaphysical views might of course also provide their own specific reasons for rejecting one of (1) and (2).

Section 2 noted that the puzzle is not plausibly solved by claiming that it is indefinitely extensible what angels there *are*, but postponed a discussion of the idea that it is indefinitely extensible what angels there *could be*. There are a number of ways in which one can understand this idea. One is as the idea that the indefinite extensibility of what angels there could be results from a kind of open-endedness in what broadly metaphysical modalities there are, discussed in the paragraph before the last. Another idea is to claim that since it is indefinitely extensible what angels there could be, there could not be all possible angels, which provides a principled reason for rejecting (2). Finally, one might understand the indefinite extensibility of what angels there could be as the claim that there could have been possible angels which actually are not possibly angels. This requires us to deny the familiar iteration principle stating that what is possibly possible is possible, but it allows us to hold both that the possible angels could all have been angels and that necessarily, there could have been an extra angel (as in (1)) – the witness to this last existential claim need not *in fact* be a possible angel. It is important to note that this position does not deny the incompatibility of (1) and (2), for which no appeal to this iteration principle is necessary, but simply endorses (1) and a principle subtly different from (2). Appendix C discusses this idea in more detail, and argues that it is unpromising as a response to the puzzle.

The range of options in responding to the puzzle sketched here indicates that it is a difficult question how it should be solved. What the puzzle does clearly show is that appeals to judgements about recombination must be treated with more caution than they have received so far. More tentatively, it has been suggested that the puzzle also casts some small doubt on the assumption that there is a distinguished notion of metaphysical necessity.

## Appendices

### A Kripke Models

Let formulas be built up from individual variables  $x, y, \dots$  and a unary predicate  $A$  using the quantifier  $\forall$  binding individual variables, the binary operator  $\wedge$ , and the unary operators  $\neg$ ,  $\Box$  and  $\uparrow_i$  and  $\downarrow_i$  for each natural number  $i$  in the usual way.  $\rightarrow$ ,  $\Diamond$  and  $\exists$  are to be read as abbreviations as usual. Let a model be a tuple  $M = \langle W, R, D, d, a \rangle$ , where  $W$  is a non-empty set,  $R \subseteq W \times W$ ,  $D$  is a non-empty set, and  $d$  and  $a$  are functions mapping each  $w \in W$  to a subset of  $D$ . Informally,  $W$  represents the possible worlds,  $R$  represents a relation of accessibility among them,  $D$  represents a domain from which possible individuals are taken, and for each  $w \in W$ ,  $d(w)$  represents the individuals at  $w$  and  $a(w)$  the angels at  $w$ .

Truth is defined relative to such a model  $M$ , a world  $w \in W$ , an assignment function  $s$  mapping each individual variable to a member of  $D$ , and a function

$f : \mathbb{N} \rightarrow W$  which maps each index  $i$  to a world  $f(i)$ . Writing  $M, w, s, f \models \varphi$  for  $\varphi$  being true relative to these parameters, this is defined inductively as follows:

$M, w, s, f \models Ax$  iff  $s(x) \in a(w)$

$M, w, s, f \models \neg\varphi$  iff not  $M, w, s, f \models \varphi$

$M, w, s, f \models \varphi \wedge \psi$  iff  $M, w, s, f \models \varphi$  and  $M, w, s, f \models \psi$

$M, w, s, f \models \forall x\varphi$  iff  $M, w, s[o/x], f \models \varphi$  for all  $o \in d(w)$

$M, w, s, f \models \Box\varphi$  iff  $M, v, s, f \models \varphi$  for all  $v \in W$  such that  $Rwv$

$M, w, s, f \models \uparrow_i \varphi$  iff  $M, w, s, f[w/i] \models \varphi$

$M, w, s, f \models \downarrow_i \varphi$  iff  $M, f(i), s, f \models \varphi$

Here,  $s[o/x]$  is the function mapping  $x$  to  $o$ , and every other individual variable  $y$  to  $s(y)$ ;  $f[w/i]$  is defined analogously. A binary consequence relation among closed formulas is derived from this class of models: let  $\varphi \models \psi$  if for all  $M, w, s$ , and  $f$  as above,  $M, w, s, f \models \varphi$  only if  $M, w, s, f \models \psi$ . Plausibly,  $\varphi \models \psi$  entails that  $\varphi$  is true only if  $\psi$  is true (both on the intended interpretation).

As claimed in section 2, it is routine to show that  $(F1) \models (F1') \models \neg(F2)$ .

## B A Plural Formulation

How can the puzzle be formulated without  $\downarrow_i$  and  $\uparrow_i$ ? It turns out that there is no uncontentious formalization of the premises (1) and (2) in the fragment of the above first-order modal language excluding  $\downarrow_i$  and  $\uparrow_i$ . This follows from Hodes (1984, Theorems 7 & 9): If there were formulas in such a language model-theoretically equivalent to  $(F1)$  and  $(F2)$ , respectively, in the sense of being true in the same worlds of the models of the previous appendix, then replacing any atomic predication of the form  $Ax$  by an existence claim of the form  $\exists y(x = y)$  would yield formulas expressing properties which Hodes shows to be inexpressible. In fact, this observation can be strengthened by considering only models in which the accessibility relation is universal, and by adding a logical identity predicate to the language. These formal results do not conclusively show that there are no sentences in this fragment which express what the English sentences (1) and (2) express, since it is not obvious that the relevant notion of expressing the same is adequately captured by the relevant model-theoretic condition. But it is clear that no formalizations which fail to satisfy the condition of being equivalent to  $(F1)$  and  $(F2)$  on the models defined above uncontentiously succeed in expressing the claims expressed by (1) and (2). This suffices to conclude that there is no uncontentious formalization of (1) and (2) in a first-order modal language not containing  $\downarrow_i$  and  $\uparrow_i$ .

It will now be shown how to formulate a version of the puzzle in an extension of first-order modal logic by plural quantifiers instead of  $\downarrow_i$  and  $\uparrow_i$ . So, consider a language defined as in appendix A, except that instead of  $\downarrow_i$  and  $\uparrow_i$ , there is a quantifier  $\forall$  binding plural variables  $xx, yy, \dots$ , and there are atomic formulas of the form  $x < yy \dots$ , read as expressing that  $x$  is one of the  $yy$ s. Extend assignment functions to map plural variables to subsets of the domain  $D$ . Dropping the relativity to  $f$ , no longer necessary in the absence of  $\downarrow_i$  and  $\uparrow_i$ , define truth in a model as above, with the following extra clauses:

$M, w, s \models \forall xx\varphi$  iff  $M, w, s[X/xx] \models \varphi$  for all  $X \subseteq d(w)$

$M, w, s \models x \prec yy$  iff  $s(x) \in s(yy)$

For simplicity, we consider (1') instead of (1), and make the natural assumption that being an angel entails being something; model-theoretically, this means considering only models in which  $A$  is existence-entailing:  $a(w) \subseteq d(w)$  for all  $w \in W$ . The idea behind the plural formalizations can be motivated using the following variants of (1') and (2):

(P1') Necessarily, the angels are such that possibly, there is an angel which is not one of them.

(P2) Possibly, the angels are such that necessarily, each angel is one of them.

To state these in the plural language, a definite description operator  $\mathbb{A}xx$  will be used, which may be read as expressing “the angels are such that ...”. Analogous to the definition of  $\exists$  in terms of  $\forall$ , it is defined as the following syntactic abbreviation:

$\mathbb{A}xx\varphi := \forall xx(\forall y(Ay \leftrightarrow y \prec xx) \rightarrow \varphi)$

(P1') and (P2) are now easily regimented as follows:

(FP1')  $\Box \mathbb{A}xx \Diamond \exists y(Ay \wedge \neg y \prec xx)$

(FP2)  $\Diamond \mathbb{A}xx \Box \forall y(Ay \rightarrow y \prec xx)$

In all worlds of all models in which  $A$  is existence-entailing, (F1') is true iff (FP1') is true, and (F2) is true iff (FP2) is true. It is worth noting that this fails on the following variant interpretation of  $\prec$ :

$M, w, s \models x \prec yy$  iff  $s(x) \in s(yy)$  and  $s(yy) \subseteq d(w)$

However, (FP1') and (FP2) are still not jointly satisfiable in any world of any model on this interpretation of  $\prec$ , and the two formulas plausibly express principles which are motivated by the recombinatorial reasoning discussed in the main text, independently of whether they express the same as (F1') and (F2).

The model theory of plural quantifiers used here validates an unrestricted principle of plural comprehension; in particular, the formula  $\exists xx \forall y(y \prec xx \leftrightarrow Ay)$  is true in all worlds of all models. This is crucial for the equivalence of (FP1') and (FP2) to (F1') and (F2). In a variant model theory along the lines of the so-called general or Henkin semantics for second-order quantifiers, this instance of plural comprehension can be falsified. In worlds in which it is falsified, the equivalence can no longer be assumed to hold, as formulas of the form  $\mathbb{A}xx\varphi$  will be trivially true; indeed, in models with a universal accessibility relation containing such a world, (FP2) will itself be trivially true. This raises the philosophical question whether the plural formulation of the puzzle can be answered by denying the instance of plural comprehension for being an angel, i.e., by claiming that there are no such things as the angels.

The truth of unrestricted plural comprehension has rarely been questioned, but Yablo (2006) and Linnebo (2010) reject the instance of plural comprehension for being a set. Their rejection of this principle can be seen as a way of formulating the idea of the indefinite extensibility of what sets there are. Yablo

and Linnebo hold that any sets give rise to a new set: for any sets, there is the set of all and only those. Since they deny that there is a set containing all and only the sets, they have to deny that there are any things such as all and only the sets, i.e., the relevant instance of plural comprehension. As argued in section 2, an analogous rejection of plural comprehension for angels and kinds of elementary particles is implausible, as it is implausible that it is indefinitely extensible what individuals of these kinds there are.

If the present recombinatorial puzzle is formulated using plural quantifiers, does it still count as pure? There is no point in debating how to use “pure” in the present context. As usual in philosophy, nothing is completely uncontroversial – not even the use of classical propositional logic, which has not been questioned here. There is therefore no hope of formulating the puzzle completely purely, in the sense of formulating it using only resources which are completely uncontroversial. The reason why the puzzle formulated in first-order modal logic extended by  $\downarrow_i$  and  $\uparrow_i$  is called “pure” above is simply that the background assumptions required in this formulation are much less contentious than the background assumptions required for familiar recombinatorial puzzles formulated in terms of propositions, sets and classes. How to think of the present formulation in plural terms simply depends on how contentious one considers the necessary background theory of plural quantifiers to be. At least as far as versions of the puzzle for kinds of elementary particles are concerned, the required assumptions seem quite mild.

## C Iteration

Consider the following iteration principle for necessity:

(4) Necessarily, what is necessary is necessarily necessary.

Since the inconsistency of (1) and (2) does not rely on (4), even those who reject it cannot accept both of (1) and (2). But there is a way of using the rejection of (4) to make the rejection of (2) more palatable. The idea is that rejecting (4) allows us to accept (2a) instead of (2):

(2a) Possibly, all *actually* possible angels are angels.

(F2a)  $\uparrow_1 \diamond \uparrow_2 \downarrow_1 \Box \forall x (Ax \rightarrow \downarrow_2 Ax)$

To demonstrate that (1) and (2a) are jointly satisfiable in the model theory defined in appendix A, let  $M = \langle W, R, D, d, a \rangle$  be the following model:  $W$  is the set of finite sets of natural numbers;  $Rwv$  just in case  $\max(v) \leq \max(w) + 1$ ;  $D = \mathbb{N}$ ; and  $d(w) = a(w) = w$ . So for any world  $w = \{n_1, \dots, n_i\}$ , the domain of  $w$  as well as the extension of  $A$  at  $w$  are simply  $\{n_1, \dots, n_i\}$ . Now consider any world  $w$ . (F1) is true in  $w$ :  $v = w \cup \{\max(w) + 1\}$  is accessible from  $w$ , and at  $v$  there are all the angels from  $w$  plus an extra one. (F2a) is true in  $w$  as well: all worlds accessible from  $w$  are subsets of  $v = \{1, \dots, \max(w) + 1\}$ , and  $v$  is accessible from  $w$ . Contingentism, the claim that it is contingent what there is, is true in all worlds, but  $M$  is easily turned into a model where necessitism is true in all worlds by letting the domain function map each world to the set of all natural numbers.

Besides witnessing the model-theoretic consistency of (1) and (2a), this model also serves as a toy model illustrating one way of understanding the idea that it is indefinitely extensible what angels there could be, as mentioned in section 6. Think of angels as having ranks corresponding to the natural numbers, each rank being had by a single angel, and of the model as identifying angels with their ranks. Then the model can be understood as capturing the idea that for any angel, there could be one of the next rank, but not necessarily one of any higher ranks. As we consider more and more iterated possibilities, we may consider the existence of angels of higher and higher rank.

The consistency of (1) and (2a) essentially relies on rejecting (4). Assuming (4), (F2a) entails:

$$(F2b) \uparrow_1 \diamond \uparrow_2 \downarrow_1 \square \square \forall x (Ax \rightarrow \downarrow_2 Ax)$$

Which in turn entails:

$$(F2c) \uparrow_1 \diamond \uparrow_2 \square \forall x (Ax \rightarrow \downarrow_2 Ax)$$

Eliminating the redundant operator  $\uparrow_1$  and replacing the index 2 by 1, we obtain (F2), which was seen above to be inconsistent with (F1).

Model-theoretically, let  $\models_4$  be the strengthening of  $\models$  obtained by restricting the definition of this relation to models with a transitive accessibility relation. It is then routine to show that (F2a)  $\models_4$  (F2b) and (F2b)  $\models$  (F2c)  $\models$  (F2).

The response to the puzzle sketched here offers a trade: in exchange for (4), we get to keep (2a) alongside (1). It is important to note the limitations of this maneuver: First, we still have to give up (2). Second, (2a) tells us that possibly, all actually possible angels are angels. With (1), it follows that had there been all actually possible angels, it would have been possible that all of them are angels and that there is an angel which is not one of them. According to the proposed response, although this would then have been possible, it is not actually possible. However, the recombinatorial reasoning which supports (1) also supports that had there been all actually possible angels, it would *actually* have been possible that all of them are angels and that there is an angel which is not one of them. More generally, the recombinatorial reasoning which supports (1) also supports (1a):

(1a) Necessarily, whatever angels there are, it is *actually* possible that all of them are angels and that there is an angel which is not one of them.

$$(F1a) \uparrow_1 \square \uparrow_2 \downarrow_1 \diamond \uparrow_3 (\downarrow_2 \forall x (Ax \rightarrow \downarrow_3 Ax) \wedge \exists x (Ax \wedge \downarrow_2 \neg Ax))$$

But (1a) is inconsistent with (2a): as above, eliminating the first conjunct of (F1a), as well as the consequently redundant  $\uparrow_3$ , yields:

$$(F1a') \uparrow_1 \square \uparrow_2 \downarrow_1 \diamond \exists x (Ax \wedge \downarrow_2 \neg Ax)$$

which is straightforwardly inconsistent with (F2a); model-theoretically, it is routine to show that (F1a)  $\models$  (F1a')  $\models \neg$ (F2a).

To emphasize the motivation for (1a), consider again the example of the possible gold sphere of diameter  $d$ . Recombinatorial reasoning doesn't just support that *had there been such a sphere*, it would be possible for there to be it and another like it, but also that if it is possible for there to be a gold sphere of diameter  $d$ , then *actually*, it is possible for there to be this sphere, made of gold and of diameter  $d$ , as well as another gold sphere of diameter  $d$ .

The strategy of rejecting (4) is therefore limited: although it allows us to maintain both of (1) and (2a), we still have to reject (1a) and (2). Furthermore, it suffers from another, maybe more decisive problem:

If not everything that is possibly possible is possible, the modality of being necessarily necessary is stronger than that of being necessary, in the sense that everything necessarily necessary is necessary, but not everything necessary is necessarily necessary. Yet, it is one of the most central theoretical roles of metaphysical necessity that it is the strongest of the relevant kinds of necessity. Indeed, this may be *the* most central theoretical role of metaphysical necessity: More than anyone else, it was Kripke (1980 [1972]) who made the assumption popular that there is a distinguished metaphysical modality. He only briefly considers the issue of the multiplicity of modalities on p. 99, and intends to single out his intended reading of modal terms with the phrase “necessity in the highest degree”. The idea continues to play an important role; see, e.g., Hale (2013, section 4.3) for a recent discussion.

Thus, if there is any such modality as metaphysical necessity, it is the strongest of the relevant kinds of necessity, and presumably, this is not a metaphysically contingent fact. So it is metaphysically necessary that if something is metaphysically necessary, it is metaphysically necessary that it is metaphysically necessary, and thus the response developed in this appendix is unavailable. The issue is worth considering in much greater detail – maybe other roles override the strongest modality role after all – but this is not the place. For now, let me sum up the situation as it applies to the conclusions drawn from the purely recombinatorial puzzle above: If we were to accept (1) and (2a), we would have to give up (4). But giving up (4) gives us strong reasons to doubt that we are picking out a unique modality with our use of “metaphysical necessity”, thereby undermining the assumption that we have singled out unique readings of (1) and (2a), as well as unique readings of the principles appealed to in some of the recombinatorial arguments in the literature mentioned above. Thus, if we are to make sense of these recombinatorial arguments, we have to reject one of (1) and (2a). This means that our naive recombinatorial reasoning must be seriously restricted – even more so than was argued for in the main text. This should at least make us suspicious of recombinatorial reasoning in general, and the recombinatorial premises in some of the recombinatorial arguments in the literature in particular; it also puts some very light pressure on the assumption that we succeed in picking out a unique modality with the term “metaphysical necessity”.<sup>1</sup>

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