Higher-Order Metaphysics: An Introduction

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Higher-order metaphysics uses the formal languages of higher-order logic to formulate metaphysical views and arguments. This chapter provides an introduction to the field and an overview of this volume. The chapter is divided into five sections, which correspond to the five parts of the volume. Section 1 motivates the use of higher-order languages in metaphysics with a number of examples, before discussing the interpretation of such languages and their relationship to natural languages. Section 2 is concerned with questions which arise from the logical resources of higher-order languages alone, either about what kind of higher-order language to use, or about matters which can be expressed in purely logical terms. Section 3 considers applications of higher-order languages to particular topics of metaphysics. Section 4 discusses the history of the subject. Section 5 addresses the controversy concerning alternative first-order approaches. Along the way, we situate the other contributions to this volume within these debates. We focus especially on sections 1 and 2; these sections introduce the core ideas of higher-order metaphysics which provide a basis for engaging with current work in higher-order metaphysics, such as the other chapters of this volume.

1 Motivation and Meaning

As elsewhere in philosophy, logical formalisms are useful in metaphysics. They provide precise and systematic tools with which to formulate and evaluate theses, theories, and arguments. In some cases, this may be done just in propositional logic, using only truth-functional connectives, with propositional letters standing for particular claims. Often, first-order predicate logic is used, with predicate symbols for particular relations of metaphysical interest, such as the parthood relation of mereology, as well as quantifiers ranging over individuals that can be said to stand in those relations. Part I of this volume, to which this chapter belongs, motivates and introduces the expansion of these formal tools of metaphysics to include higher-order logics. We begin (section 1.1) with some motivating examples. We then turn (section 1.2) to the interpretation of these further resources and their relationship to ordinary talk, such as talk about properties and propositions.

1.1 Motivation

Not every interesting argument in metaphysics can be captured in a natural way in the standard frameworks of propositional and predicate logic. The most obvious cases may arise from modal notions, which are ubiquitous in metaphysics. To formalize talk about metaphysical necessity, for example, these languages are typically expanded by a non-truth-functional operator $\Box$, with a formula of the form $\Box\phi$ used to formalize the claim that it is metaphysically necessary that ϕ. In such a modal expansion, various specific modal arguments can be formulated, and the formal tools of modal logic can be used to evaluate which general modal principles underwrite a given argument. Such modal principles then become questions of modal metaphysics themselves. An example is the following principle, according to which what is possible is necessarily possible:

(5) $\Diamond p \rightarrow \Box\Diamond p$

For an example of the metaphysical controversy surrounding this principle, (Salmon, 1981) presents a putative counterexample to it.

Propositional and predicate logics can also be expanded by other connectives capturing notions used in metaphysics. Prominent examples include the counterfactual conditional, temporal operators, and operators for relations of metaphysical ground. Many interesting metaphysical questions can be formalized using these resources. But even with non-truth-functional operators, there are limitations to propositional and first-order logics.

Example 1. One important limitation has to do with existential generality. The question whether a principle like (5) is correct for metaphysical necessity is implicitly universal, since p may be taken to stand for any claim. But it is sometimes important to generalize existentially as well as universally.

For example, consider the notion of (strict partial) ground of (Fine, 2012). This can be formalized using a binary sentential connective $\prec$, with $p \prec q$ expressing that p grounds q. It is natural to ask, as Gideon Rosen (2010) does, whether every truth is grounded in some truth. There is no obvious way of formalizing this unless propositional letters like p can be treated as variables and bound by universal and existential quantifiers. If we can do so, then the thesis that every truth is grounded in some truth can be formalized as:

(1a) $\forall p(p \to \exists q(q \land (q \prec p)))$

Similarly, a natural question concerning necessity is whether there is a truth which strictly implies every truth, thereby encoding every detail of actuality. Kit Fine (1970) and David Kaplan (1970) formalize this as follows, using quantifiers binding propositional variables:

(1b) $\exists p(p \land \forall q(q \to \Box(p \to q)))$

A corresponding principle in the setting of temporal logic is discussed by Arthur Prior (1967, p. 79). The necessitation of (1b) is important for the view that possible worlds are propositions, since it helps to ensure that each world corresponds to some proposition. See (Fritz, forthcoming a) for detailed development of this view.

Example 2. The need for explicit quantification extends to other types of expressions. In first-order predicate logic, uninterpreted predicate constants can be used to formulate general principles which hold for all properties. But there is no way to capture analogous claims of existential generality, nor to embed claims of universal generality. Another limitation is that every predicate takes only singular terms as arguments; in particular, every occurrence of an identity predicate must be flanked by singular terms.

To illustrate these limitations, consider the idea of singular propositions, i.e. of propositions that are singularly *about* a specific individual. An attractive—although controversial—principle is that in general, the proposition that Fc is about c. The most natural way of formalizing this claim employs a binary expression $\mathcal{A}$ for aboutness which can take both the sentence Fc and the singular term c as arguments, as in:

(2a) $\mathcal{A}(Fc, c)$

Note that because Fc is a sentence and c is a singular term, the aboutness expression $\mathcal{A}$ is neither a binary sentential operator nor an ordinary binary relational predicate.

In (2a), universal generality can again be achieved implicitly, by letting F stand for any property and c for any individual. The truth of (2a) on this implicitly universal interpretation requires that, for every individual and property, the proposition attributing the property to the individual is about the individual.

Other natural principles to consider in this context require explicit existential quantification. Consider the stronger claim that a proposition is about an individual just in case the proposition attributes *some* property to the individual. To see how this might be formalized, begin with the claim that p attributes F to c. The most natural formalization of that claim employs an identity predicate taking sentences as arguments: $p = Fc$. To say that p attributes some property to c, we then replace the predicate F with a variable Y bound by an existential quantifier: $\exists Y(p = Yc)$. The target thesis concerning what it takes for a proposition to be about an individual can then be formalized as:

(2b) $\forall p \forall x(\mathcal{A}(p, x) \leftrightarrow \exists Y(p = Yx))$

A metaphysician might naturally intend (2b) as more than a mere material or even necessary equivalence between p being about x, and p attributing some property to x. On this reading, (2b) is intended to *identify* a proposition being about an individual with the proposition attributing some property to the individual. To make this metaphysical analysis of aboutness explicit, we can replace the material biconditional in (2b) with our identity predicate that takes sentences as arguments:

(2c) $\forall p \forall x(\mathcal{A}(p, x) = \exists Y(p = Yx))$

Example 3. Apparent counterexamples to the simple theory of aboutness (2b) motivate the introduction of further resources. For example, the proposition that Rab is about a because it attributes the relation R to a and b, but it doesn't seem to attribute any monadic property to a. Similarly, the proposition that $Fa \vee Gb$ is about a because it disjoins two propositions Fa and Gb, one of which is about a; it also doesn't seem to attribute any monadic property to a.

One natural response to these apparent counterexamples is to retain the simple theory of aboutness and deny that the propositions in question attribute no monadic property to a. Since we don't want to deny that the proposition that Rab attributes relation R to a and b, proponents of this view will reject the implicit assumption that each proposition has a unique decomposition into properties and attributions thereof to individuals. For example, the proposition that Rab may both attribute relation R to a and b, *and also* attribute to a the monadic property of bearing R to b. Similarly, the proposition that $Fa \vee Gb$ may both disjoin the propositions that Fa and that Gb, *and also* attribute to a the monadic property of being F or Gb.

To formalize these ideas, we need a way to take sentences such as Rab and form new expressions for properties from them. We can do so by introducing a new variable-binder λ, with the resulting λ-*terms* being predicates whose argument positions correspond to the variables bound by λ. For example, $\lambda x.Rxb$ is a predicate with one argument position corresponding to x; this formalizes talk of the property of bearing R to b. Similarly, $\lambda x.Fx \vee Gb$ is a predicate with one argument position corresponding to x; this formalizes talk of the property of being an x such that Fx or Gb.

We can now use our identity predicate which takes sentences as arguments to say explicitly that Rab and $Fa \vee Gb$ have the decompositions mentioned above:

(3a) $Rab = (\lambda x.Rxb)a$

(3b) $Fa \vee Gb = (\lambda x.Fx \vee Gb)a$

By existential generalization into predicate position from (3a) and (3b), we can infer that each of the propositions that Rab and that $Fa \vee Gb$ attributes some monadic property to a:

(3c) $\exists Y(Rab = Ya)$

(3d) $\exists Y((Fa \vee Gb) = Ya)$

Given (3c) and (3d), the simple theory of aboutness (2b) entails that Rab and $Fa \vee Gb$ are both about a, as desired:

(3e) $\mathcal{A}(Rab, a)$

(3f) $\mathcal{A}(Fa \vee Gb, a)$

In this way, predicate abstraction using λ allows us to precisely formulate metaphysical views that respond to the apparent counterexamples to the simple theory of aboutness (2b). To do so, it is essential that λ-terms can be used to instantiate quantifiers binding variables in predicate position.

Example 4. The need for explicit quantification extends also to sentential connectives. Consider Saul Kripke's (1980 [1972], p. 99) suggestion that metaphysical necessity is necessity in the highest degree. This requires there to be a highest degree of necessity, which is not a trivial matter. For example, Justin Clarke-Doane (2019) and Agustín Rayo (2020; this volume, ch. 17) both argue that the possibilities are indefinitely extensible. So, how can we formalize the disputed Kripkean claim that there is a highest degree of necessity?

First, we need to be able to distinguish necessities from other notions, such as negation. Given that necessities are expressed by unary sentential operators (e.g. the $\Box$ of metaphysical necessity), expressing this distinction requires a unary constant N taking such operators as arguments. We can now formalize the true claim that metaphysical necessity is a necessity as $N\Box$, and the false claim that negation is a form of necessity as $N\neg$.

Next, we need to be able to say that one necessity is of at least as high degree as another. For this, we need a binary connective $\sqsubseteq$ taking unary sentential operators as arguments. The claim that metaphysical necessity is of at least as high degree as some other necessity (expressed by $\Box'$) is then formalized as:

(4a) $\Box \sqsubseteq \Box'$

Finally, we need to be able to generalize over necessities. For this, we need to be able to replace unary sentential operators with variables m and n that can be bound by quantifiers, including an existential quantifier. Given this apparatus, the Kripkean claim that there is a highest degree of necessity can be formalized as:

(4b) $\exists m(Nm \land \forall n(Nn \to m \sqsubseteq n))$

We can now say what it is to be a necessity of highest degree, and also that there is such a necessity. It would be nice to be more informative still, to say what necessity occupies this role. Ideally, we would explicitly define the necessity in question. One natural suggestion motivates a further extension of predicate abstraction, allowing λ to bind non-singular-term variables. It also involves occurrence of λ-terms as arguments to suitable predicates.

Suppose we have a theory of propositions according to which there is a unique weakest proposition $\top$, i.e. a unique proposition which is entailed by every proposition. For example, standard possible worlds theories of propositions have this feature: according to them, propositions true in the same possible worlds are identical; $\top$ is then the proposition true in all possible worlds. Assuming there is such a weakest proposition $\top$, one natural idea is that only $\top$ is necessary in the highest degree. This suggests the hypothesis, explored by Andrew Bacon (2018), that necessity in the highest degree is *being* $\top$. To formalize this hypothesis, we need to let λ bind variables that occupy the position of sentences so that we can formalize the property of being $\top$ as $(\lambda p.p = \top)$. We can now formalize the claim that this defined operator is a highest degree of necessity thus:

(4e) $N(\lambda p.p = \top) \wedge \forall n(Nn \rightarrow (\lambda p.p = \top) \sqsubseteq n)$

Whereas λ-terms occurred as predicates in the previous example, this principle uses λ-terms as arguments to the predicates N and $\sqsubseteq$.

1.2 Meaning

These examples show how metaphysical theorizing naturally employs tools beyond the resources of standard propositional and predicate logic: First, predicate-like expressions—including symbols for identity—that take as arguments expressions that are not singular terms, such as sentences and sentential operators. Second, variables that can replace not merely singular terms but also sentences, predicates, and sentential operators. Third, quantifiers that can bind these variables to enable embedding of quantified claims inside the scope of other operators. Fourth, a device of predicate abstraction that can bind variables—including non-singular-term variables—to form complex predicates. Fifth, the ability for the resulting complex predicates to instantiate quantifiers and occur as arguments to other predicates.

Variables that can replace expressions other than singular terms are *higher-order variables*. Quantification and abstraction on higher-order variables are *higher-order quantification and abstraction*. Predicate-like expressions that take as arguments expressions that are not singular terms are *higher-order predicates*. A *higher-order language* is any language that provides any of these higher-order resources. Our opening examples motivate the use of higher-order languages within metaphysics. This is the research program of higher-order metaphysics.

What do higher-order quantifiers and λ-terms mean? In the following, we focus primarily on quantification rather than abstraction, both to simplify exposition and because quantification has been the primary focus of most published discussion of these questions. However, we intend our remarks to apply to abstraction too.

Above, higher-order quantifiers were introduced as natural ways of formalizing talk of propositions, properties, relations, modalities, and similar entities. (To simplify exposition, we henceforth count relations, modalities, and all other similar entities as properties.) A natural first hypothesis is therefore that the intended interpretation of higher-order quantifiers is determined by the meaning of natural language talk of properties and propositions. However, as Bacon notes in chapter 2, on closer inspection there appear to be some important differences between natural language talk of properties and propositions, and the corresponding higher-order statements. First, talk about properties gives rise to questions that don't themselves have higher-order counterparts. Examples include questions about the locations and abstractness of properties, as emphasized by (Jones, 2018). Secondly, claims about properties can have different truth-values from the corresponding higher-order statements. One (but not the only) kind of example comes from the paradoxes. There is plausibly no property instantiated by all and only the properties which do not instantiate themselves; see section 5. As a result, some meaningful conditions in the language of properties do not determine a coextensive property, such as 'property which does not instantiate itself'. By contrast, higher-order logics often consistently include an unrestricted principle of comprehension, according to which the following existential claim is true for every formula ϕ in which y but not X may be free:

$$\exists X \forall y (Xy \leftrightarrow \phi)$$

Intuitively, this is a higher-order formalization of the claim that ϕ determines a coextensive property. A witness for it is given by $\lambda y.\phi$.

There are two natural concerns one might have about dissociating higher-order quantifiers from talk of properties, propositions and so on. Firstly, if the meaning of higher-order quantifiers comes apart from natural language talk of properties, propositions and so on, one might be concerned that higher-order metaphysics changes the subject (Liggins, 2021, Hofweber, 2022). One might say: pre-existing metaphysical questions are about properties, propositions and so on; the analogous higher-order questions—when there are such questions, at least—are about a different, higher-order subject matter. One may thus conclude that the higher-order questions are simply irrelevant to the original questions. Relatedly, there is a mismatch in our initial examples (section 1.1) between our motivating informal talk about properties, propositions and so on, and our higher-order formalizations thereof. However, a change of subject matter sometimes yields an improvement in subject matter. The questions and resources that animate past theorizing need not be those that animate future theorizing. One aspect of an improved understanding is often a better grasp of what questions to ask, and consequent replacement of old questions with new. We find this eminently plausible in the present case. Indeed, in chapter 2, Bacon motivates higher-order metaphysics by arguing that talk of properties and propositions introduces metaphysical "noise" not associated with the corresponding higher-order questions, with the higher-order questions being "closer to the metaphysical action" than the corresponding questions about properties.

Secondly, if the meaning of higher-order quantifiers is not given in terms of translations into natural language talk of properties, propositions and so on, then what does determine their intended interpretation? One might consider ways of translating higher-order quantification into natural language without invoking talk of properties and propositions. For example, the second-order existential quantification $\exists X(Xa)$ might be translated as the claim that a is *somehow*. Proposals along these lines can be found in (Prior, 1971, chapter 3) and (Rayo and Yablo, 2001); see also (Sainsbury, 2018, ch. 2). Moreover, Jeremy Goodman argues in chapter 3 that higher-order abstraction can be at least partially understood in terms of our pre-existing practice of definition. Nevertheless, the resources available for idiomatic English higher-order quantification and abstraction are limited. They do not appear to include the full variety of higher-order quantifiers and λ-terms present in the formal languages most commonly deployed in higher-order metaphysics. (We introduce those languages in more detail shortly, in section 2.1.)

As a consequence, an influential view within higher-order metaphysics denies that translatability into any other language is necessary for the meaningfulness of higher-order quantification and abstraction. Following Timothy Williamson (2003, 2013), higher-order quantifiers are taken to have intended interpretations that make formulas containing them meaningful in the same way in which English sentences are meaningful (assuming interpretations for the rest of the formula's vocabulary). This *primitivist* view about higher-order quantifiers regards them as new theoretical vocabulary expressing new *sui generis* kinds of quantification. Like other theoretical notions, the best way to understand them is not by translating them into some antecedently understood idiom, but by immersion into the practice of using them. This does not mean that nothing can be said about their metasemantics; we return to this point in section 3 in connection with chapter 9 by Harvey Lederman. A primitivist view of higher-order quantification is developed by both Bacon in chapter 2 and Goodman in chapter 3; see also (Fritz, forthcoming a), as well as (Clarke-Doane and McCarthy, forthcoming) for a skeptical perspective.

According to primitivism about higher-order quantification, higher-order languages provide a novel tool for theorizing about reality. The meaningfulness of these languages is independent of whether their claims can be re-expressed in natural or first-order languages. We want to mention three points in support of this view.

Firstly, our motivating examples show how naturally higher-order languages arise within metaphysics. Yet various limitative results seem to show that the intended interpretations operative in these metaphysical applications cannot be understood in first-order terms. Bacon argues for this in chapter 2; see also (Williamson, 2003).

Secondly, primitivists about higher-order quantifiers treat them in the same way as others treat novel theoretical notions like ground, essence, ontological dependence, naturalness, structure, truth-making, exact verification, metaphysical necessity, tropes, and universals. We see no obvious reason to regard expansion of our metaphysical toolkit with primitive higher-order resources as any more dubious than these other expansions.

Thirdly, Goodman argues in chapter 3 that the meaningfulness of even first-order languages does not depend on them being translatable into English. Rather, constructions like 'for some x' are better understood as expanding English to enable pronunciation of independently understood formulas of first-order languages. The key conceptual leap to primitively meaningful formal languages has arguably already been made when metaphysicians theorize using first-order languages.

An alternative formalization of talk of properties and propositions employs first-order logic. This approach is independently natural. The English constructions 'every property' and 'some proposition' are plausibly understood as restrictions of 'everything' and 'something' to properties and propositions respectively. The unrestricted English 'something' and 'everything' are naturally formalized by the unrestricted first-order quantifiers $\exists x$ and $\forall x$ (although see Sainsbury, 2018, ch. 2 for a dissenting view). So the restricted quantifiers 'every property' and 'some proposition' are naturally formalized as explicitly restricted first-order quantifiers $\forall x(Fx \to \dots)$ and $\exists x(Gx \wedge \dots)$, where F and G formalize 'is a property' and 'is a proposition' respectively.

We return to such alternative first-order approaches in section 5. For now, it suffices to note that they can be combined with primitivism about higher-order quantification. This opens up theoretical space for an attractive form of nominalism. According to this view, there are no properties or propositions. Yet there are true higher-order counterparts of many claims about properties and propositions. For example, although Peter does not instantiate any properties, it is true that $\exists Y(Y\text{Peter})$. And although Nick does not believe any false proposition, it is true that $\exists p(\neg p \wedge$ Nick believes that $p)$. These higher-order generalizations can be used for much of the metaphysical work that has traditionally motivated metaphysicians to postulate properties and propositions. One important question for this proposal is whether there is any important metaphysical work for properties and propositions that both cannot be done—or cannot be done as well—using higher-order quantification, and yet really ought to be done; see (Liggins, 2021) for discussion.

Useful as they may be, higher-order languages are not without their limitations. In particular, they are less straightforward to speak, read, and write than natural languages. They are not designed for natural philosophical conversation. It is therefore convenient to have a way of pronouncing higher-order formulas in English. To this end, Goodman emphasizes in chapter 3 that we can introduce a new practice of using informal talk of properties and propositions to pronounce higher-order formulas. For example, we can pronounce the formula

(2b) $\forall p \forall x(\mathcal{A}(p, x) \leftrightarrow \exists Y(p = Yx))$

from Example 2 thus: a proposition p is about an individual x if and only if, for some property y, p is the proposition that x instantiates y. This licenses a convenient and familiar way of speaking by providing it with a new higher-order intended interpretation. We can pronounce $\exists Y(Y\text{Peter})$

as ‘Peter instantiates some property’ and thereby speak truthfully, while remaining compatible with the nominalist view that there are no properties (understood in first-order terms as individuals) and so without introducing the metaphysical noise that Bacon discusses. Talk about properties and relations in the remainder of this chapter (and much of this volume) should be understood in this nominalistically acceptable way.

Before moving on, we should say a word about model theories for higher-order languages. Like other formal languages, higher-order languages are often provided with a model theory formulated in a (first-order) set-theoretic metalanguage. Such a model theory is often called a “semantics”. However, a model theory is merely a piece of set theory and so does not alone endow any expression with meaning. Any attempted use of a model theory in a metasemantic capacity needs supplementing with an account of how purely set-theoretic constructions determine the intended interpretation of the relevant formal language. Model theory therefore often plays a much more mundane role in higher-order metaphysics: it provides an instrument for establishing facts about underivability: To show that a given formula *can* be derived in some axiom system, it suffices to exhibit a derivation of the formula in the system. Showing that the formula *cannot* be so derived is more difficult. Using suitable models, this can be done by showing that on the relevant class of models, the axioms are valid and the rules preserve validity, and by constructing a model which fails to validate the relevant formula.

2 Pure Higher-Order Metaphysics

Part II of this volume concerns pure higher-order metaphysics. We understand this as concerning questions which arise from the logical resources of higher-order languages alone. These questions can be roughly divided into two kinds. Firstly, questions about what kind of higher-order language to use within metaphysics. Secondly, questions that can be expressed using higher-order languages without employing non-logical constants; most prominently, questions about identity and existence. We begin this section by introducing the most prominent kinds of higher-order languages employed in higher-order metaphysics (sections 2.1–2.4). We then turn to the kinds of questions that can be asked in those languages without employing non-logical constants (section 2.5).

2.1 Type Theories

As already emphasized, a paradigmatic feature of the higher-order languages best suited to metaphysics is the ability to bind (using quantifiers and λ) variables which take the place of expressions other than singular terms. Starting from the languages of propositional and first-order predicate logic, there are two quick routes to such languages. The first starts from the language of propositional logic and adds quantifiers binding propositional variables, i.e. variables taking the position of sentences. Such quantifiers have been prominently discussed in the context of modal logic; see (Fine, 1970) and (Fritz, forthcoming b). The second starts from first-order predicate logic and adds variables in the position of predicates and quantifiers binding them. This is the language of second-order logic, which plays a prominent role in the philosophy of mathematics; see (Shapiro, 1991). These two quick routes to higher-order languages can be unified as follows. Propositional logic can be seen as a fragment of first-order predicate logic by understanding propositional letters as nullary predicates, i.e. predicates with zero argument positions. Similarly, propositional quantifiers can be understood as the special case of second-order quantifiers binding nullary variables, i.e. variables that can replace predicates with zero argument positions.

Propositional quantifiers suffice to regiment examples 1a and 1b from section 1.1. The other examples require resources which go beyond the language of second-order logic. They require predicates which take other predicates as arguments and quantifiers binding variables in the position of these higher-order predicates. We also saw the need in examples 3 and 4 for λ abstraction.

Although our examples are specific, they illustrate a general way in which higher-order resources may be introduced: for any kind of expressions, admit (i) variables of that kind which can be bound by quantifiers and λ, and (ii) predicates which take expressions of that kind as arguments. This leads to a recursive specification of kinds of expressions, otherwise known as syntactic categories or *types*. Consequently, the resulting higher-order languages are often called *type theories*. Although there is a rich variety of type theories, most of those currently employed in higher-order metaphysics belong to one of two groups: relational type theories and functional type theories. We now introduce them in turn.

When discussing type theories, all expressions are often called terms. We adopt this practice below, using ‘term’ and ‘expression’ interchangeably. We also use ‘singular term’ to refer to first-order variables as well as constants.

2.2 Relational Type Theories

Relational type theories typically specify the types using one basic type and the following recursive clauses:

> e is a type, the type of singular terms.
>
> If $\tau_1, \ldots, \tau_n$ are types, then $\langle \tau_1, \ldots, \tau_n \rangle$ is a type, the type of relational expressions taking as arguments n expressions of types $\tau_1, \ldots, \tau_n$ respectively.

The second clause admits the case of $n = 0$, according to which there is a type $\langle\rangle$ of nullary relation terms, which are identified with formulas. Alternatively, the nullary case may be excluded and a second basic type t for formulas used instead, as Goodman does in chapter 3.

A particular language in such a type theory is based on two things. Firstly, a choice of constants and variables of each type. Secondly, a specification of what counts as an expression of each type using the following two recursive clauses:

(R1) Any constant or variable of a type is a term of that type.

(R2) If A is a term of a type $\langle \tau_1, \ldots, \tau_n \rangle$ and $b_1, \ldots, b_n$ are terms of types $\tau_1, \ldots, \tau_n$, respectively, then $Ab_1 \ldots b_n$ is a term of type $\langle\rangle$.

These formation rules can be illustrated using example 2a. c is a singular term constant, hence of type e. F is a unary predicate constant taking a singular term as argument, hence of type $\langle e \rangle$. From R2, it follows that Fc is a term of type $\langle\rangle$, i.e. a formula. Now, the aboutness predicate $\mathcal{A}$ takes two arguments, a formula (type $\langle\rangle$) and a singular term (type e). So $\mathcal{A}$ should be treated as a constant of type $\langle\langle\rangle, e\rangle$. From R2 again, $\mathcal{A}$ can be applied to Fc and c (in that order) to form a formula (type $\langle\rangle$) $\mathcal{A}Fcc$. To aid readability, parentheses and commas may be added to produce example 2a as a stylistic variant: $\mathcal{A}(Fc, c)$.

No logical connectives have yet been introduced. There are two ways of doing so. The first adds a new formation rule for each logical connective, as in standard presentations of first-order logic. To illustrate, conjunction may be introduced using the following rule:

> If ϕ and ψ are terms of type $\langle\rangle$, then $\phi \wedge \psi$ is a term of type $\langle\rangle$.

Similarly, we can introduce existential quantifiers binding variables of any type, and an identity connective taking any terms of the same type as arguments:

> If x is a variable and ϕ is a term of type $\langle\rangle$, then $\exists x\phi$ is a term of type $\langle\rangle$.
>
> If a and b are terms of the same type, then $a = b$ is a term of type $\langle\rangle$.

So, for variables p of type $\langle\rangle$, Y of type $\langle e\rangle$, and x of type e respectively: Yx is a formula, so $p = Yx$ is a formula, whence $\exists Y(p = Yx)$ is a formula too. With universal quantifiers and the other Boolean connectives introduced analogously, and with $\mathcal{A}$ our non-logical aboutness constant of type $\langle\langle\rangle, e\rangle$, example 2b becomes a (stylistic variant of a) well-formed formula:

(2b) $\forall p\forall x(\mathcal{A}(p, x) \leftrightarrow \exists Y(p = Yx))$

Note that on this approach, the logical connectives are not assigned types, and the formation rules governing them are not instances of the clause R2 that governs formation of formulas from typed expressions.

The alternative way of introducing logical connectives treats them as constants of specific types. In the case of Boolean connectives, this is straightforward. For example, conjunction $\wedge$ takes two formulas as arguments to form a formula, and so may be assigned type $\langle\langle\rangle, \langle\rangle\rangle$. By R2, this makes $\wedge\phi\psi$ a formula for any formulas ϕ and ψ. The more common infix notation $\phi \wedge \psi$ may then be considered a stylistic variant of $\wedge\phi\psi$. Identity can be treated similarly, by including a constant $=_\tau$ of type $\langle\tau, \tau\rangle$ for each type τ.

In order to assign types to quantifiers, the variable-binder λ for forming complex predicates is commonly used. So let's turn now to λ before returning to the quantifiers.

Following our motivating examples, if x is a variable of type e, $\lambda x.\mathcal{A}(p, x)$ should be a complex predicate of type $\langle e\rangle$ we can use to formalize the property of being an individual which p is about. In the examples so far, λ has always bound one variable to form a unary predicate. A more general treatment permits formation of complex polyadic predicates by binding a sequence of different variables potentially of different types:

(R3) Where $n > 0$, if $x_1, \ldots, x_n$ are pairwise distinct variables of types $\tau_1, \ldots, \tau_n$ respectively, and ϕ is a term of type $\langle\rangle$, then $\lambda x_1 \ldots x_n.\phi$ is a term of type $\langle\tau_1, \ldots, \tau_n\rangle$.

This allows us to formalize relations by binding more than one variable at once. For example, let variables p, Y, and x have types $\langle\rangle$, $\langle e\rangle$, and e respectively. Then $\lambda pY.\exists x(p = Yx)$ is a predicate of type $\langle\langle\rangle, \langle e\rangle\rangle$. It formalizes the following relation between propositions p and properties Y: *p attributes Y to some individual.*

Back to the quantifiers. We can use λ to recast a quantifier binding a variable of a type τ as a constant of type $\langle\langle\tau\rangle\rangle$. The trick is to use λ rather than the quantifier to bind the variable, which delivers a complex predicate of type $\langle\tau\rangle$ (since the variable has type τ). This complex predicate then serves as argument to the quantifier, understood now as a predicate constant of type $\langle\langle\tau\rangle\rangle$.

To illustrate, consider the claim $\exists x\mathcal{A}(p, x)$ that p is about some individual. Since x here has type e, (R3) implies that $\lambda x.\mathcal{A}(p, x)$ has type $\langle e\rangle$. So a predicate $\exists_e$ of type $\langle\langle e\rangle\rangle$ can be applied to it to produce a formula $\exists_e\lambda x.\mathcal{A}(p, x)$. Think of $\exists_e$ as expressing the higher-order property of being a property of individuals which is instantiated by some individual. We can then read $\exists_e\lambda x.\mathcal{A}(p, x)$ as attributing this property to the property of being an individual which p is about, which amounts to saying that there is an individual which p is about.

The usual treatment of quantifiers as variable-binders can now be considered an abbreviating notation, with $\exists x$ standing for $\exists_\tau\lambda x$, where τ is the type of x (not the type of $\exists_\tau$, which is $\langle\langle\tau\rangle\rangle$). Note that if quantifiers are treated as typed constants in this way, a separate quantifier is needed for each type of quantifiable variable: a quantifier constant of type $\langle\langle\tau\rangle\rangle$ corresponds to a quantifier binding variables of type τ. Similarly, the above treatment of identity as typed requires, for each type τ, a different identity predicate $=_\tau$ of type $\langle\tau, \tau\rangle$. In this way, the quantifiers too can be assigned types. The construction of a relationally typed language is then exhausted by a choice of constants and variables for each type alongside rules (R1), (R2), and (R3).

2.3 Functional Type Theories

Relational type theories provide a natural way of generalizing the language of first-order predicate logic with singular terms and relational constants: singular terms are retained as terms of type e and relational constants are retained as constants having types of the form $\langle e, \ldots, e\rangle$. One distinctive feature of relational type theory is that any application of a typed expression to suitable arguments produces a formula. As a result, the function symbols included in some formulations of first-order predicate logic have no natural equivalent in relational type theory, because they combine with singular terms to form singular terms not formulas. Functional type theory provides an alternative approach which generalizes the notion of a function symbol. In this setting, the function symbols of first-order logic and typed expressions of relational type theory can all be represented as function terms.

A typical formulation of functional type theory specifies the types of expressions by the following recursive clauses:

> e is a type, the type of singular terms.
>
> t is a type, the type of formulas.
>
> If σ and τ are types, then $\sigma \to \tau$ is a type, the type of function terms taking one term of type σ as argument to form a complex term of type τ.

Note that unlike the case of relational theory, a second primitive type t is required as the type of formulas, instead of the relational type $\langle\rangle$.

As with relational type theory, logical connectives can either be introduced via primitive formation rules, or treated as constants of specific types. In the latter case, a particular language can again be based on a choice of constants and variables of each type alongside three recursive clauses specifying what counts as a term of each type. Here are the recursive clauses:

(F1) Any constant or variable of a type is a term of that type.

(F2) If a is a term of type $\sigma \to \tau$ and b is a term of type σ, then ab is a term of type τ.

(F3) If x is a variable of type σ and a is a term of type τ, then $\lambda x.a$ is a term of type $\sigma \to \tau$.

Unary logical connectives, including quantifiers, can then be treated as in the relational case. For example, the unary predicate of individuals F featured in example 2 may be construed as a constant of type $e \to t$. Taking c as a constant of type e, it follows from (F2) that Fc is a formula (type t). By (F3), $\lambda x.Fx$ is then of type $e \to t$. Taking the first-order existential quantifier $\exists_e$ as a constant of type $(e \to t) \to t$, (F2) gives us the quantificational formula (type t) $\exists_e \lambda x.Fx$. The more familiar notation $\exists x Fx$ can again be understood as a stylistic variant.

Binary connectives like $\wedge$ provide a challenge, as do polyadic expressions more generally. The problem is that functionally typed terms of the present system take only one term as argument, whereas polyadic expressions take more than one argument.

One solution is to modify the above specification of types and formation rules to accommodate polyadic functional types. For example, we might adopt clauses like these:

> Where $n > 0$, if $\sigma_1, \ldots, \sigma_n, \tau$ are types, then $\sigma_1, \ldots, \sigma_n \to \tau$ is a type, the type of function terms taking n terms of types $\sigma_1, \ldots, \sigma_n$ respectively as arguments to form a term of type τ.

(F2*) If a is a term of type $\sigma_1, \ldots, \sigma_n \to \tau$, and $b_1, \ldots, b_n$ are terms of types $\sigma_1, \ldots, \sigma_n$ respectively, then $ab_1 \ldots b_n$ is a term of type τ.

Binary sentential connectives like $\wedge$ can now be assigned type $t, t \to t$; so by (F2*), they take two formulas as arguments to form a formula. More generally, each n-ary relational type can be represented by a functional type of terms taking n arguments of appropriate types to form a formula.

However, there is often no need to complicate things by introducing polyadic functional types, because each such type can be represented by a unary functional type, using a technique known as *Schönfinkeling* or *Currying*. The idea is as follows. Consider, for example, the polyadic type $t, t \to t$ of binary sentential connectives like $\wedge$. The characteristic syntactic feature of this type is that its terms combine with two formulas to form a formula. There is a sense in which terms of the higher-order functional type $t \to (t \to t)$ also have this feature. Taking $\wedge$ to have this type, we can combine it first with one formula a to yield a term $\wedge a$ of functional type $t \to t$, which can then be combined with a second formula b to yield a formula $\wedge ab$ (with $a \wedge b$ again treated as a stylistic variant). Both the binary functional type $t, t \to t$ and the binary relational type $\langle t, t \rangle$ can in this way be represented using the unary higher-order functional type $t \to (t \to t)$. The technique generalizes, so that each relational type and polyadic functional type can be represented by a unary functional type.

To illustrate this idea further, recall example (2a):

(2a) $\mathcal{A}(Fc, c)$

This can be formalized in unary functional type theory as follows. We saw above that Fc can be understood as a term of type t by taking F and e to be constants of types $e \to t$ and e respectively, and applying (F1) and (F2). Taking $\mathcal{A}$ as a constant of type $t \to (e \to t)$, it can by (F2) be applied to Fc to form a term $\mathcal{A}Fc$ of type $e \to t$. This can by (F2) again then be applied to c to form a term $\mathcal{A}Fcc$ of type t. For readability, brackets and commas can be added to obtain 2a as a stylistic variant.

We've indicated how relational types can be represented as functional types. It's also worth noting that some functional types don't naturally represent any relational type. For example, the type $e \to e$, corresponding to functions from singular terms to singular terms, has no natural counterpart within relational type theory. This is in effect the type of unary function symbols in first-order logic, such as the symbol for the successor function in arithmetical theories. Another example comes from the type $(e \to t) \to e$ corresponding to functions from unary predicates to singular terms. This type is naturally used to formalize definite descriptions, using a symbol for the function which maps any property had by a unique object to that object.

It is sometimes convenient to focus only on functional types that represent relational types in the manner indicated above. When one's primary goal is to theorize only about properties and propositions, for example, the other functional types may not play a central role. For this purpose, a more restricted system of functional types is sometimes used, with complex types generated only by the following recursive clause:

If σ and τ are types and τ is not e, then $\sigma \to \tau$ is a type.

Each functional type generated from e and t by this clause represents some relational type in the manner indicated above. In chapter 4, Bacon and Cian Dorr work primarily with this restricted functional type system.

2.4 Type-Theoretic Diversity

The type-theoretic variants sketched here illustrate the rich diversity of type-theoretic languages which can be found in the literature. Let us briefly mention four other important cases not covered so far.

First, the combinatory logic of (Curry et al., 1958). In this setting, the types of terms are not fixed at the start but assigned in a given context, and all variable binding is dispensed with. These variants of higher-order languages have been influential in computer science, though they have as yet played less of a role in philosophy. See (Bacon, forthcoming a), as well as (Mitchell, 1996) and (Hindley and Seldin, 2008).

Second, cumulative type theory. In the type theories described above, terms of each type combine with terms of exactly one other type. Cumulative type theories relax this constraint. These systems have been primarily discussed in the setting of monadic relational type theory: the version of relational type theory whose only types are e, $\langle e \rangle$, $\langle\langle e \rangle\rangle$, and so on. Consider the natural ordering $<$ on these types: $e < \langle e \rangle < \langle\langle e \rangle\rangle < \ldots$. According to rule (R2) for forming formulas, Ab is a formula only if the type of b immediately precedes the type of A in this ordering. Cumulative type theory relaxes this constraint so that b may have any type preceding the type of A, not just the immediately preceding type:

(R2*) If A is a term of a type σ, b is a term of type τ, and $\tau < \sigma$, then Ab is a term of type $\langle\rangle$.

This significantly expands the class of formulas, allowing properties to be applied to a wider range of arguments. Cumulative type theory thus embodies a permissive conception of the structure of reality, on which properties combine with a wider range of arguments to yield propositions than under any of the other type theories so far discussed. Although this has not yet been widely employed within metaphysics, (Williamson, 2013, section 5.7) makes some use of the idea. For further discussion, see (Degen and Johanssen, 2000), (Florio and Jones, 2021), and (Button and Trueman, 2022); (Krämer, 2017) discusses a generalization of the idea to full relational type theory.

Third, plural logic. Whereas the higher-order languages discussed so far expand first-order logic with new types of predicates, plural logic adds to the *singular* individual terms of first-order logic new *plural* individual terms. One simple implementation adds just a single new type of plural constants (e.g. aa) and variables (e.g. xx) that can be bound by quantifiers. One may also add predicates taking these plural terms as arguments, including a plural membership predicate $\prec$ formalizing 'is one of'. This allows English plural claims like 'Beth spilled all the cheerios' to be formalized as formulas like:

$$\exists xx(\forall y(Cy \to y \prec xx) \wedge S(b, xx))$$

This apparatus has played a prominent role in contemporary metaphysics, especially concerning material objects, mereology, and sets; see (van Inwagen, 1990), (Lewis, 1991), and section 3. We hypothesize that it also played a causal role in alleviating scepticism about higher-order languages. Firstly, George Boolos (1984) and others explored ways of using plural logic to interpret second-order logic. Secondly, plural logic provides a form of higher-order quantification that many were happy to regard as primitively intelligible. Those factors helped to create a hospitable environment for other forms of primitive higher-order quantification in the late twentieth and early twenty-first centuries. For further discussion of plural logic, see for example (Linnebo, 2022), (Oliver and Smiley, 2013), and (Florio and Linnebo, 2021).

Various extensions of this basic plural language are possible. One can add a new "super-plural" type for pluralizing plural terms, analogously to how ordinary plural terms pluralize singular terms. Further types for pluralizing those super-plural types can also be added, and so on. One can then add predicate types with argument positions reserved for terms of these new super-plural types. Finally, and least familiarly, one can introduce types for pluralizing (and super-pluralizing etc.) types other than just e, as in (Fritz et al., 2021) and (Fritz, 2022, forthcoming a). (See also (Fine, 1977) for a closely related type theory involving types for 'relations-in-extension'.) These types allow us to talk of pluralities (and super-pluralities etc.) of properties and propositions.

Finally, the apparatus of constructive type theory, which Laura Crosilla discusses in chapter 6. This complex and elegant formal theory has become popular in the foundations of mathematics and computer science. More recently, it has been applied in natural language semantics (Chatzikyriadis and Luo, 2020). It has, however, not yet received significant attention within formal metaphysics, although see (Klev, 2022) and (Wilhelm, unpublished). Crosilla's contribution provides an important early step in this direction, emphasizing two central differences between constructive type theory and the type theories discussed above. Firstly, constructive type theory allows new ways of forming types to be incorporated. As well as types corresponding to functions from entities of one type to another, other operations on types can also be used to form new types. The theory is open-ended in that it readily permits the inclusion of new ways of forming types. Secondly, constructive type theory allows for *dependent types*, where the type of value returned by a function depends on what argument is supplied to the function. These two features allow for construction of new formulas within constructive type theory, without counterparts in the other type theories considered in this volume. Metaphysicians have barely even begun to consider what theoretical and expressive gains this might yield. As well as introducing constructive type theory as a framework for theorizing about properties and propositions, Crosilla discusses whether the apparatus can be used to make sense of one central tool of formal metaphysics, namely unrestricted quantification over absolutely everything whatsoever.

In some cases, there are mappings between different type-theoretic languages which allow one to associate each term of one language with a corresponding term of the other language. We have seen some examples already, in the use of functional types to represent relational types, and unary functional types to represent polyadic functional types. Often, these mappings can be proven to preserve important logical properties, such as derivability in suitable deductive systems. For a discussion of such mappings and results concerning the languages sketched above, see (Dorr, 2016, pp. 86–94). However, no matter what formal properties are established about these mappings, it is important to remember that it remains a substantial claim that they preserve intended meaning. That is, assuming that the relevant formulas have an intended interpretation, it remains a substantial claim that any formula has the same intended interpretation as the one to which it is mapped. The diversity of type-theoretic languages thus generates a cluster of important foundational questions for higher-order metaphysics. Which, if any, of these languages should be used in metaphysics? Can they be used interchangeably, using mappings like those sketched above? Or should one such language be preferred to the others, perhaps due to the range of claims it allows (or prevents) us to make sense of, or because its syntactic structure more closely matches the deep structure of reality?

2.5 Logical Questions

As well as questions about which higher-order language to use, pure higher-order metaphysics also concerns questions that can be asked without using non-logical constants; we call them *logical questions*. The true answers to logical questions are naturally called logical truths. One central part of higher-order metaphysics seeks to identify logical truths, in order to answer logical questions.

What exactly is the import of "logical" here? Logic is sometimes conceived as neutral between substantial metaphysical disputes. No logical truth would then take a stand on any metaphysically substantial matter. This conception of logic plays little role in higher-order metaphysics as currently practiced; it is explicitly rejected in (Williamson, 2013). It is often hard to draw an intuitive line between "purely logical" and "genuinely metaphysical" principles. It is also unclear what benefits such a division might bring. For example, it does not obviously help attain knowledge of the relevant claims. In order for deductive arguments to yield knowledge, it is important that the premises are known (and so true), and the rules of inference employed are known to preserve truth. It does not obviously matter whether, in addition, any of the premises are (known to be) *logically* true or the rules (known to) preserve truth *in virtue of logic*, or some similar status.

We do better to understand the logical vocabulary as a certain relatively small collection of expressions that are particularly amenable to formal investigation, and which are usefully applicable under an especially wide range of assumptions. Standard examples of logical vocabulary all have these features, such as quantification, negation, conjunction, disjunction, and identity. Likewise for the apparatus of λ-abstraction. Other examples are sometimes but not always treated as logical, most obviously metaphysical necessity. Higher-order metaphysics requires no deep or absolute distinction here. We can instead be pragmatic in what vocabulary we count as logical, and hence what questions we count as logical, depending on what best serves our theoretical purpose at the time.

On this view, the logical truths are privileged only by the fact that they can be expressed in languages whose only constants are logical in the sense just outlined. Logical truths are paradigmatically somewhat general and abstract, but not necessarily so. Moreover, logical truths may well be metaphysically controversial: many substantive metaphysical questions can be expressed using higher-order languages without non-logical constants. As Goodman observes in chapter 3, the same is true of first-order languages, although the substantive questions that can be asked in those languages—e.g. $\exists x \exists y(x \neq y)$?—are rather boring. For first-order languages, things get more interesting if we count the $\Box$ of metaphysical necessity as logical. We can then ask, following Williamson (2013), whether it is contingent what there is: $\Box \forall x \Box \exists y(x = y)$? Higher-order languages allow us to ask a yet richer supply of interesting and metaphysically substantial logical questions, even without the $\Box$ of metaphysical necessity.

One cluster of questions concern identity among properties and propositions, as emphasized by Cian Dorr (2016). In the terminology used by Goodman in chapter 3, *grain science* investigates general principles about identity among propositions, properties and so on. Higher-order languages provide a natural setting in which to conduct grain science, which indeed forms one central part of pure higher-order metaphysics.

Identity is often taken to be equivalent to a demanding form of indistinguishability. This form of indistinguishability is naturally formalized in a relationally typed higher-order language. In the case of individuals, it is formalized by the following λ-term where x and y have type e:

$$\lambda xy.\forall Z(Zx \leftrightarrow Zy)$$

Intuitively, for individuals x and y to stand in this relation is for them to have exactly the same properties, including identity-properties like $\lambda x.x = a$. (This informal gloss presupposes classical logic; (Jones, 2020) discusses alternative formulations in a non-classical setting. See also (Fritz, forthcoming) for variant formulations in modal settings which admit contingency in what there is at all types.)

Higher-order counterparts of this relation of indistinguishability are readily defined by allowing x and y to have a type τ other than e, and modifying the type of Z accordingly (to $\langle\tau\rangle$). The resulting relations are sometimes called *Leibniz equivalence*. The λ-terms expressing relations of Leibniz equivalence can be defined without non-logical constants. Questions about Leibniz equivalence may therefore count as logical questions. Many such questions have clear metaphysical interest. And as Goodman observes in chapter 3, this interest is independent of any particular connection between Leibniz equivalence and identity. Leibniz equivalence is metaphysically interesting in its own right. For further discussion of the relationship between identity and Leibniz equivalence, see (Bacon and Russell, 2019), (Dorr, 2016), and (Caie et al., 2020).

Here's an illustrative example of a metaphysically substantial logical question about identity and Leibniz equivalence. Consider the view that propositions are structured in a way analogous to the structure of the sentences that express them. On this kind of view, predications express the same propositions only if their predicates express the same property. This underwrites the following claim in relational type theory, where p has type $\langle\rangle$ and X and Y have type $\langle\langle\rangle\rangle$:

$$\forall X\forall Y\forall p(Xp = Yp \rightarrow X = Y)$$

As it turns out, this formula is inconsistent in classical logic, if (a) the higher-order quantifiers satisfy principles analogous to those governing first-order quantifiers in classical predicate logic, (b) identity is materially equivalent to Leibniz equivalence, and (c) λ-terms satisfy a natural principle of extensional

β-conversion or the higher-order quantifiers satisfy an unrestricted comprehension principle. The details of this important argument are provided by Øystein Linnebo in chapter 5, which also locates the argument within a class of similar results. The discovery of this inconsistency traces back to Bertrand Russell (1903) and John Myhill (1958), and so it is often known as the *Russell-Myhill argument*. The argument has received significant recent interest in higher-order metaphysics, following (Hodes, 2015), (Uzquiano, 2015), (Dorr, 2016), and (Goodman, 2017).

In response to the Russell-Myhill argument, one might question the principles governing the quantifiers, identity, and λ-terms; see Linnebo's chapter 5 and Christopher Menzel's chapter 13, as well as (Yu, 2017) and (Kment, 2022). One might also question whether the inconsistent sentence captures the idea of propositions being structured (Hofweber, 2022). One may even question the higher-order language itself, as Bacon (forthcoming b) does using a language which blocks the Russell-Myhill argument by restricting the term formation rules; this provides another example of a variant type theoretic language. One might also accept the conclusion of the argument, taking it to show that propositions are not structured. Settling the matter will require a detailed comparison of these options.

Questions about propositional structure are not the only metaphysically interesting questions of grain science. In chapter 4, Bacon and Dorr provide a wealth of other examples. They examine a systematic theory of identity for all types, which they call *Classicism*. One key idea driving their theory is that whenever one can prove material equivalence in classical higher-order logic, there is a corresponding identity between properties and propositions. For example, one can classically prove this equivalence:

$$\forall x Fx \leftrightarrow \neg\exists x \neg Fx$$

So Classicism yields this identity between propositions:

$$\forall x Fx = \neg\exists x \neg Fx$$

And also this identity between properties:

$$\forall = \lambda Y.\neg\exists x \neg Yx$$

Bacon and Dorr provide several different axiomatizations of Classicism, as well as a model theory for it. They also investigate the (sometimes surprising) relationships between Classicism and many other substantial metaphysical questions, for example:

Are necessarily equivalent propositions identical?

Is it contingent what there is?

Are identity and necessity non-contingent?

Do certain world-propositions play the theoretical role of possible worlds?

Are fundamental entities freely recombinable?

A second cluster of metaphysically interesting logical questions concern existence. The two clusters are connected, since existence is often identified with being identical to something (as we effectively did in examples 2 and 3 of section 1.1). The distinction between identity questions and existence questions is thus not wholly precise or exclusive. Yet we find it conceptually helpful nonetheless.

Central principles to consider here are so-called comprehension principles. A comprehension principle for properties is a schematic principle which says, for certain formulas, that there is a property corresponding to the formula. Here is an example, already mentioned in section 1.2, where ϕ is any formula in which y but not X may be free:

$$\exists X \forall y (Xy \leftrightarrow \phi)$$

Intuitively, this principle says that there is a property X with a certain extension: exactly the entities y that satisfy the defining formula ϕ possess X. Such a principle can be weakened by restricting the formulas ϕ which can be used to form an instance. It can also be strengthened by replacing the material biconditional $\leftrightarrow$ with an identity predicate $=$, or by inserting the $\Box$ of metaphysical necessity after the initial quantifier.

Similar principles can also be formulated for propositions and polyadic relations. For example, where ϕ is any formula in which p is not free:

$$\exists p (p = \phi)$$

Intuitively, we might gloss this as saying that ϕ expresses a proposition. Other variations are also possible. For example, say that a property is *rigid* if it could not possibly apply to different entities than it in fact does. Then we might wish to require that every property is coextensive with some rigid property:

$$\forall X \exists Y(\text{rigid}(Y) \wedge \forall z(Xz \leftrightarrow Yz))$$

Such a comprehension principle is discussed by Bacon and Dorr in chapter 4; see also (Gallin, 1975).

In chapter 5, Linnebo discusses existence principles such as these comprehension principles, focussing on their role in the Russell-Myhill argument. Historically, there has been significant discussion of whether comprehension principles should be restricted to avoid certain forms of circularity. Linnebo discusses a historically popular implementation of this idea, known as *predicativity*, which prohibits the defining formula on the right from quantifying over domains which include the defined entity on the left. He both responds to the most pressing arguments against predicative restrictions on comprehension, before arguing that such restrictions are superfluous nonetheless, in that a predicative version of the Russell-Myhill argument still renders a structured conception of propositions inconsistent. Linnebo instead proposes two different ways of restricting comprehension to block the Russell-Myhill argument. Both ways are motivated by a hierarchical conception of reality, on which higher levels are grounded by lower levels. (For more on ground, see Schaffer 2009, Rosen 2010, Fine 2012.) Interestingly, which restriction is required depends on the kind of higher-order entity governed by the comprehension principle. The existence of a plurality is fully grounded in the existence of its members; this motivates one way of restricting comprehension. By contrast, the existence of a property is not typically grounded in the existence of its instances; but on Linnebo's view its existence is grounded in some level of the hierarchy, which motivates a different restriction on comprehension. This makes the appropriate restriction on comprehension sensitive to the different natures of the higher-order entities concerned.

3 Applied Higher-Order Metaphysics

One way for higher-order languages to earn their keep in metaphysics is to prove fruitful in applications to traditional metaphysical topics. Formalization in these debates typically requires languages containing non-logical constants. Such applied higher-order metaphysics is the topic of part III of this volume. We have already mentioned some examples (section 1.1). Another instructive example arises in the context of mereology, the metaphysical theory of parthood.

Mereology begins with the observation that some material objects are parts of others, e.g. your elbow is part of your arm, which in turn is part of your body. Theorizing about parthood is often formalized using a first-order language which includes a two-place predicate $\leq$ regimenting 'is part of'. Various other mereological notions can be defined using parthood, such as overlap: x and y *overlap* if they have a part in common. Formally:

$$x \circ y := \exists z(z \leq x \wedge z \leq y)$$

Using parthood and overlap, we can define a very general notion of *fusion*: for y to be a fusion of the things x satisfying a condition $\phi(x)$ (a formula in which x but neither y nor y' may occur freely) is for every x that satisfies $\phi(x)$ to be part of y, and every part y' of y to overlap some x satisfying $\phi(x)$. This can be regimented as follows:

$$\mathrm{Fu}(y, \phi(x)) := \forall x(\phi(x) \rightarrow x \leq y) \wedge \forall y'(y' \leq y \rightarrow \exists x(\phi(x) \wedge y' \circ x))$$

The question now arises of what it takes for the things satisfying a condition $\phi(x)$ to have a fusion. One natural idea is that there are no restrictions on the existence of fusions: whatever things we might care to specify have a fusion. Someone who endorses such unrestricted fusion would endorse every instance of the following schematic principle:

(U1) $\exists y \, \mathrm{Fu}(y, \phi(x))$

Here, $\phi(x)$ may either be assumed to contain no free variables apart from x, or the instances of the schema may be assumed to be implicitly prefixed by a string of universal quantifiers binding the variables free in $\phi(x)$ other than x.

However, there are limits on the extent to which U1 captures the idea of unrestricted fusion. First, the strength of U1 depends on the language under consideration: the existence of a fusion is asserted only for things which can be delineated with a formula of the relevant language. Second, as in some of our initial examples (section 1.1) , U1 is essentially schematic, and so cannot be properly negated.

Staying within first-order logic, one might attempt to overcome these limitations by formulating unrestricted fusion using a theory of collections, such as set theory. The resulting principle says that every set has a fusion. Using $\in$ for the set-theoretic relation of membership, this can be formalized as follows:

(U2) $\forall z \exists y \, \mathrm{Fu}(y, x \in z)$

What exactly follows from this principle depends on what set-theoretic principles govern $\in$. Whatever those principles may be, however, U2 does not succeed in generalizing U1. For, as shown by Russell, there cannot be a set which contains just those sets x which are not members of themselves (in the sense that $\neg x \in x$). Thus, no consistent set theory allows us to derive from U2 the existence of a fusion of the individuals which are not members of themselves. But this is an instance of U1.

Second-order logic provides a way out of this difficulty, by replacing quantification over sets with monadic second-order quantification:

(U3) $\forall Z \exists y \, \mathrm{Fu}(y, Zx)$

This has the intended effect only if every instance of U1 follows from U3, which requires an unrestricted principle of comprehension for second-order quantifiers, as discussed above, which guarantees the existence of a property Z possessed by exactly the individuals x satisfying $\phi(x)$. Whereas the corresponding ("naive") unrestricted comprehension principle for sets is inconsistent, this second-order principle is consistent, and included in standard deductive systems for higher-order logic.

In sum, U3 says that every property has a fusion of its instances. We can consistently combine this in higher-order logic with an unrestricted comprehension principle which ensures that every condition determines a property. Every instance of U1 then becomes derivable. Higher-order languages thereby allow us to properly and consistently express unrestricted mereological fusion, unlike first-order languages.

There is something unnatural about appealing to properties when formulating unrestricted fusion. Intuitively, unrestricted fusion just says that any things—however delineated—have a fusion. A more direct formalization is therefore available using the plural quantifiers discussed earlier (section 2.4). We used $xx, yy, \ldots$ for plural variables, and $x \prec yy$ to say that x is one of yy. The claim that any things have a fusion can then be formalized as:

(U4) $\forall zz \exists y \, \mathrm{Fu}(y, x \prec zz)$

This plural version of mereology is now extremely widespread, following its influential use in (van Inwagen, 1990) and (Lewis, 1991). The example illustrates how different kinds of higher-order quantifiers may be better suited to different metaphysical applications.

Moving beyond mereology, we have already mentioned (section 1.2) another application of higher-order metaphysics: to the nominalist view that there are no properties, understood as individuals over which first-order quantifiers range. Since we're using 'property'-talk to pronounce higher-order quantification, we'll follow the helpful practice adopted by Tim Button and Robert Trueman in chapter 7 of using 'universal' for the first-order notion. On this way of speaking, the present nominalist view says that there are properties but no universals. For further discussion of this view and its theoretical benefits, see (Jones, 2018), (Trueman, 2021), and Bacon's chapter 2 in this volume.

One prominent problem for this view arises as follows. English and other natural languages contain nominalizing devices for converting predicates into names for universals. For example, 'is wise' can be nominalized into 'wisdom'. Now consider the predication 'wisdom is a virtue', which has a nominalized universal-name as subject. Plausibly, this predication is true only if 'wisdom' refers to a universal. Since the predication seems true, we have a problem for nominalism. And this clearly isn't an isolated example.

Button and Trueman's chapter develops a detailed response to this problem. On their view, simple predications like 'wisdom is a virtue' containing nominalized names for universals are all false, and so universal names like 'wisdom' needn't refer. We talk as if there were universals, even though there really are none. Button and Trueman develop the underlying theory of universals and nominalization governing this practice before proving a conservativeness result about it, roughly: adding their theory of nominalization to a theory that does not mention universals does not affect the theory's consequences about properties; the new consequences only concern universals. This makes available for Button and Trueman a fictionalist attitude to ordinary talk about universals: we talk in accordance with a literally false yet harmless fiction. The fiction is harmless in that its false consequences concern only universals, not properties. Although reasoning in accord with the fiction can lead from truth to falsity, the resulting false claims must concern universals, not just properties, and so cannot lead to false claims concerning only what really (outside the fiction) exists.

Another fruitful application of higher-order metaphysics has been to modality. As noted earlier, vocabulary expressing metaphysical modality is sometimes treated as logical, sometimes not; it's arguably a borderline case. Vocabulary expressing other modalities is not usually treated as logical. The higher-order metaphysics of modality thus exhibits the vagueness of the boundary between pure and applied higher-order metaphysics.

Williamson's 2013 book *Modal Logic as Metaphysics* provided an influential catalyst to higher-order metaphysics. Williamson applied higher-order modal logic to metaphysical modality, arguing for *necessitism*: necessarily, it is not contingent what there is. Formally:

(NNE) $\Box\forall x\Box\exists y(y = x)$

Strikingly, Williamson's necessitism was not just about individuals, the version of NNE in which the variables have type e. He advocated versions of NNE in which the variables may be assigned any (relational) types whatsoever (with the type of $=$ adjusted accordingly).

At one key point in *Modal Logic as Metaphysics*, Williamson argued from necessitism about non-individuals to necessitism about individuals. One line of argument goes via the existence of haecceities, i.e. identity properties such as *being identical to Socrates*; see (Skiba, 2022) for recent discussion. Another line of argument goes via opposition to an asymmetric treatment of first-order existence and higher-order existence, on which NNE holds for higher-order quantifiers but not for first-order quantifiers: the challenge for opponents of necessitism about individuals is to say what explains this asymmetry.

Maegan Fairchild takes up this challenge in chapter 8, focussing on a difference between two argumentative routes to NNE. For each type, the corresponding version of NNE is derivable from certain appropriately typed principles of the classical logic of quantification, necessity, and identity; call this the *classical route to NNE*. For types other than e, Williamson also offers a different argument for NNE, which makes essential appeal to an appropriately typed unrestricted comprehension principle (we introduced these principles in section 2); call this the *comprehension route to NNE*. There isn't an obvious principle for type e which can play the same role as comprehension in the comprehension route to NNE. Fairchild's response to Williamson makes use of this observation. She endorses a restriction of classical logic to free logic, which blocks the classical route to NNE by weakening universal instantiation (at each type). Yet the comprehension route to NNE still succeeds: NNE (for types other than e) is still derivable from unrestricted comprehension in free logic. The resulting view explains the asymmetry between first-order existence and higher-order existence as arising from two factors. First, an underlying free quantificational logic which is the same at all types. Second, a structural difference between type e and all other types concerning how free logic interacts with unrestricted comprehension.

The final paper in part III concerns propositional attitudes. Already in (Prior, 1971, chapter 3), we find the view that belief (and other propositional attitudes) is a relation between thinkers and propositions, rather than a relation between thinkers and other individuals. Adopting relational type theory for concreteness, this means that belief is a relation of type $\langle e, \langle\rangle\rangle$ rather than type $\langle e, e\rangle$. On this view, generalising about the contents of attitudes requires higher-order quantification. The true claim that Peter believes something, should be formalized as:

$$\exists p(\text{Peter believes that } p)$$

where p has type t, not as:

$$\exists x(\text{Peter believes } x)$$

where x has type e. In the former but not the latter, the quantified variable can be instantiated for the sentence 'modality reduces to logic'. Conversely, in the latter but not the former, the quantified variable has to be instantiated with a singular term, typically a nominalization of a sentence like 'that modality reduces to logic'. In chapter 9, Harvey Lederman labels the view that propositional attitudes can be formalized as such higher-order relations *naive (higher-order) relationism*. Naive relationism has been popular in recent higher-order metaphysics (Jones, 2019), (Trueman, 2021).

Lederman develops a problem for naive relationism in his chapter, from which he generates two more foundational problems about the interpretation of higher-order languages. The initial problem arises from Frege Puzzles. Intuitively, although Hesperus is Phosphorous, Plato believed that Hesperus is visible in the morning, and did not believe that Phosphorous is visible in the morning. Formally:

$$\text{(FP)}\quad h =_e p \wedge B(Vh) \wedge \neg B(Vp)$$

Here, B has type $\langle\langle\rangle\rangle$ and formalizes 'Plato believed that'; V has type $\langle e\rangle$ and formalizes 'is visible in the morning'; h and p have type e and formalize 'Hesperus' and 'Phosphorous' respectively. Lederman observes that (FP) is inconsistent in standard higher-order logics, since they contain the following principle of *Atomic Congruence*:

$$a = b \rightarrow Fa = Gb \text{ (where } a, b \text{ have any type } \tau \text{ and } F, G \text{ have type } \langle\tau\rangle)$$

Higher-order metaphysicians have three options: (i) reject Frege Puzzle claims like (FP), (ii) reject naive relationism and adopt a different treatment of propositional attitudes, or (iii) reject Atomic Congruence. Lederman argues that rejection of naive relationism or Atomic Congruence gives rise to two foundational problems for the widespread primitivist interpretations of higher-order languages.

The first problem is metasemantic. On a primitivist view of higher-order quantifiers, their meaning is not determined by a translation into English or any other antecedently understood language. Yet something must be done to bestow this novel theoretical vocabulary with meaning: some metasemantic constraints must be in place to select an intended interpretation and rule out unintended interpretations. Those constraints plausibly come from the theory taken to govern the novel vocabulary and its connections with other antecedently understood notions. In the case of higher-order languages, these constraints come from two sources. Firstly, logical principles provide the theory. Lederman argues that Atomic Congruence is a central such principle. Secondly, antecedently understood notions are connected to the higher-order formalizm by formalizing them using formal constants. Lederman argues that the naive relationist formalization of propositional attitudes is central here too. Rejecting naive relationism or Atomic Congruence thus threatens to leave higher-order languages highly indeterminate in meaning.

The second problem is epistemological. Lederman argues that we have two primary sources of evidence for the meaningfulness of primitively interpreted higher-order languages, and also about the existence and character of entities of various types. As with his first problem, those sources include the logical principles governing the language as well as formalizations of antecedently understood notions into it. Rejecting naive relationism or Atomic Congruence thus threatens to leave us with little evidence for either the meaningfulness of higher-order languages or the existence and character of the higher-order entities over which their variables range.

These examples give some indication of how higher-order metaphysics offers new insights about the topics of mainstream metaphysics. We conclude this section with some further references to recent work in this rapidly expanding field.

Propositional attitudes and Frege Puzzle principles like (FP) are also discussed in the context of higher-order metaphysics by (Bacon and Russell, 2019), (Caie et al., 2020), and (Yli-Vakkuri and Hawthorne, 2022). Aside from Frege Puzzles, discussion of the attitudes within higher-order metaphysics has largely focussed on some seemingly paradoxical results tracing back to (Prior, 1961) and Kaplan (1995). For example, these results show that in a relatively weak higher-order system, one can prove that there is some proposition which cannot be uniquely entertained. This is sometimes called the *Prior-Kaplan paradox*; for recent discussion, see (Bacon et al., 2016) and (Bacon and Uzquiano, 2018).

For further applications of higher-order logic to the metaphysics of modality, see (Bacon, 2018) and (Bacon and Zeng, 2022) on necessities and the existence of a broadest necessity, (Dorr et al., 2021) on puzzles of modal variation, (Fritz and Goodman, 2016, 2017) on necessitism, (Jacinto, 2019) on serious actualism, (Roberts, 2022) on the relation between metaphysical and physical necessity, and (Roberts, 2023) on the necessity of identity.

Higher-order resources have also been usefully applied to fundamentality and free recombination (Bacon, 2020), time (Banfi and Deasy, 2022), essence (Ditter, 2022, Litland, forthcoming), ground (Krämer, 2013, Fritz, 2022, Goodman, forthcoming), truth (Künne, 2003, Trueman, 2021), and the metaphysics of abstraction (Litland, 2022).

4 The History of Higher-Order Metaphysics

The need for quantifiers binding variables other than those taking the place of singular terms was already noted at the inception of modern formal logic, by Gottlob Frege (1879), who developed an early form of second-order logic. Frege did not rigorously define the syntax of his language, but clearly included atomic formulas composed of a predicate and some individual terms as arguments, as in modern predicate logic. Without feeling the need to remark on it, he allowed variables to take the place not just of these arguments but also the place of predicates, and allowed quantifiers to bind these variables. Further, he considered his logical language not just as a formal construct, but as a meaningful language. Can we already think of these earliest uses of higher-order quantifiers as instances of higher-order metaphysics? The answer depends crucially on how Frege understood higher-order quantifiers. Kevin Klement addresses this question in chapter 10.

Klement explains that Frege thought of higher-order quantifiers as ranging over functions. To illustrate this, consider an atomic sentence of first-order logic, such as Fa. According to Frege, what Fa represents can be obtained by applying the function represented by F to the individual represented by a. Eventually, Frege argued that sentences represent truth-values; a predicate like F therefore represents a function from individuals to truth-values. Unary second-order quantifiers thus range over such functions. Furthermore, Frege's functions come in a hierarchy similar to the type hierarchies introduced in section 2. For example, Frege already suggested an understanding of quantifiers over individuals on which they combine with a predicate like F to form a sentence $\exists F$ stating that the function represented by F maps some individual to truth; we introduced this kind of view in section 2.2. The quantifier can then be understood as representing a higher-order function which takes functions from individuals to truth-values as arguments, and maps them to truth-values.

Frege's work predates the ascendancy of set theory as a widely accepted foundational theory of mathematics. So we shouldn't assume that he identifies functions with sets of pairs as is common today. We do better to take his notion of a function as primitive, and see what features they must have to play the role Frege wants them to play. Klement notes that Frege developed logic in order to carry out his *logicist* project of reducing arithmetic to logic, and that this requires an abundant metaphysics of functions. This suggests a strongly non-linguistic conception of functions, with the functions over which we quantify outstripping the functions we can express using our linguistic resources. Frege went on to carry out his logicist project in great detail in (Frege, 1884, 1893/1903). In doing so, he found himself compelled to assume that what higher-order quantifiers range over is reflected in the individuals: for every X there is an individual $e(X)$, the extension of X. Fatally, he assumed his *Basic Law V*, which entails that having the same extension entails being coextensive, in the following sense:

$$e(X) = e(Y) \rightarrow \forall z(Xz \leftrightarrow Yz)$$

Frege needs extensions for his logicist ambitions, in order to ensure that numbers are individuals. However, as Russell famously showed in a letter to Frege from 1902 (van Heijenoort, 1967), the resulting system is inconsistent. Two features of this result are important to note. First, the appeal to extensions and Basic Law V is essential since Frege's basic higher-order logic is demonstrably consistent. Second, Frege's adoption of a higher-order language with its hierarchy of expressions and quantifiers predates the discovery of the inconsistency, and was therefore not motivated by it.

Russell tried to rescue logicism by avoiding the appeal to extensions. In carrying out this work, he developed a type theory more complicated than the "simple" type theories we have been discussing: the so-called "ramified" type theory, also discussed in chapter 5. This theory sub-divides each simple type into many orders, and uses these orders to impose predicative restrictions on comprehension principles. Klement explains how, like Frege, Russell also thought of higher-order quantifiers as ranging over functions. However, Russell thought of these functions not as functions to truth-values, but as functions to propositional complexes which contain individuals and properties as constituents. Klement traces the different conceptions of propositional functions which Russell adopted as his views developed.

Russell's mature view of higher-order quantification can be found in the culmination of his efforts to establish logicism, the three-volume (Whitehead and Russell, 1910–1913). Klement notes that on this view, the truth and falsity of higher-order quantified statements can always be explained in terms of the truth and falsity of their instances. Moreover, any statement involving higher-order quantifiers expresses a proposition which just attributes properties and relations to individuals, thereby freeing us from making any room in our metaphysics for propositional functions. As Klement discusses, this outlook is complicated by (Whitehead and Russell, 1910–1913) including in its system an axiom of reducibility intended to overcome a weakness introduced by ramification. Although not leading to outright inconsistency, this axiom shares some similarities with Frege's Basic Law V. Moreover, Klement argues that reducibility runs the risk of inconsistency if we want to accommodate general facts.

For Frege, Russell, and many other early analytic philosophers, higher-order logic was simply logic. The fragment we now know as first-order predicate logic emerged only after several decades of development; according to (Moore, 1988), it was first explicitly formulated in unpublished lectures by David Hilbert in 1917. For several further decades, higher-order logic continued to play an important role. Later milestones include more rigorous presentations of higher-order logic and its set-theoretic model theory, including the functional type theory of Alonzo Church (1940) and completeness result of Leon Henkin (1950). Corresponding work for relational type theory can be found in (Orey, 1959); see also (Myhill, 1958).

Thoralf Skolem argued for a restriction of logic to first-order logic as early as 1923 (Moore, 1988); although see (Eklund, 1996). This was particularly successful in the context of developing set theory, and mathematical logic more generally. In philosophy, skepticism about higher-order logic is more recent in origin. The chief originator of this skepticism in philosophy is Willard

Van Orman Quine, e.g. in (Quine, 1961). In chapter 11, Fraser MacBride lays out and assesses Quine's arguments against second- (and higher-)order logic, drawing principally on Quine (1970). MacBride focuses on two lines of argument.

The first line of argument aims at establishing that second-order logic, even if it is intelligible, does not count as logic proper. This conclusion is based on Quine's contention that logic must be obvious, topic neutral, and universally applicable. Quine rejects an interpretation of second-order logic as quantifying over entities like properties and propositions, since according to him, the latter lack sufficiently clear individuation conditions. He therefore assumes that second-order quantifiers range over sets, whence second-order logic becomes a fragment of set theory. It follows that second-order logic violates the requirements to count as logic. For example, it is not obvious that there is an empty set—indeed, any set—even though classical second-order logic proves $\exists X \forall y \neg Xy$.

For higher-order metaphysics as a research program, the conclusion that higher-order logic does not count as logic proper is not necessarily fatal: if we can still use higher-order languages to articulate and investigate metaphysical questions, we can still carry out higher-order metaphysics. We just wouldn't be doing higher-order *logic*. It would, however, still be important that higher-order quantifiers are not interpreted as ranging over sets; for otherwise, we might as well employ a first-order set theoretic language. A non-set-theoretic interpretation of higher-order quantification is required. In his second line of argument, MacBride develops a problem for such interpretations of higher-order language. Although the argument was not made by Quine himself, it is based on Quine's philosophy. The conclusion of the argument is that higher-order quantifiers cannot be understood as ranging over universals, although they might be interpreted either substitutionally, or primitively (see section 1.2).

Despite Quine's opposition, various forms of higher-order logic continued to play an important role in regimenting talk of properties and propositions. In metaphysics, a prominent voice which explicitly rejected Quine's skepticism can be found in the work of Prior. Prior's understanding of higher-order quantification is the topic of chapter 12 by Adriane Rini. Propositional and other higher-order quantifiers play a key role in many of Prior's works. Most immediately relevant for metaphysics is his work on time and modality (Prior, 1957, 1967) (recall Example 1b in section 1.1). Prior also made essential use of such quantifiers in talking about propositional attitudes (recall the Prior-Kaplan paradox mentioned in section 3). Rini uncovers two motivations which are important for understanding Prior's appeal to higher-order languages. First, Prior wanted to avoid a commitment to universals such as properties and propositions conceived of as individuals. Yet he still wanted to be able to formalize, e.g., the claim that one person believes everything someone else says. His solution was to use propositional quantifiers. Second, Prior was influenced on the one hand by the logical approach to philosophy pioneered by Russell, also drawing on influences like the work of Frank Ramsey and Stanisłewski Leśniewski, and on the other hand by the ordinary language philosophy contemporary with Prior. This motivated him to find instances of higher-order quantification in English, as mentioned in section 1.2.

Prior's most explicit defense of higher-order logic can be found in (Prior, 1971), which includes a reply to Quine's criticism. Rini explains that this work was published posthumously, based on notes circulated in 1964. Between then and his death in 1969, Prior visited UCLA, where Richard Montague was developing his own application of higher-order logic to the semantics of natural language (Montague, 1974). Rini notes that Montague employed much more sophisticated mathematical methods than Prior, and wonders whether Prior abandoned his work which later was published as (Prior, 1971) as a consequence, maybe considering it outdated. The logical underpinnings of Montague's theory were developed by Daniel Gallin (1975), and its linguistic applications were developed further by Barbara Partee (1975). Through these and further developments, Montague's work has had an enduring impact on formal semantics. One consequence is that in mainstream formal semantics, unlike mainstream metaphysics, higher-order logic continued to play an important role throughout the late twentieth century and into the present day.

5 Debating Higher-Order Metaphysics

The main alternative to the use of higher-order quantifiers in regimenting talk of propositions, properties and relations in metaphysics is the use of first-order quantifiers, restricted by certain predicates. For example, instead of regimenting universal quantification over propositions using $\forall p \ldots$, where p is a variable taking the place of sentences, it may be regimented using $\forall x(Px \to \ldots)$, where x is a first-order variable and P is a predicate for propositions. The final part V of this volume is devoted to a discussion of the reasons for and against higher-order metaphysics, given this alternative way of regimenting metaphysical discourse.

We will be comparing the conceptions of properties and propositions arising from first-order and higher-order formalizations of metaphysical discourse. For clarity, we will use different terminology for properties and propositions as understood on each approach, adopting conventions from Button and Trueman in chapter 7 and Lederman in chapter 9. We use 'universal' and '*e*-proposition' for properties and propositions understood as individuals, following the first-order approach. 'Property' and '*t*-proposition' we reserve for the higher-order conception.

On the first-order approach, several additional notions are required. This is best illustrated using some examples. Recall our first example of propositional quantification in section 1:

(1) Every truth is grounded in some truth

We formalized (1) as:

(1a) $\forall p(p \rightarrow \exists q(q \wedge (q \prec p)))$

Note how propositional variables occur here as arguments to the sentential connectives $\wedge$ and $\rightarrow$. Since first-order variables cannot do so, a first-order formalization of (1) requires a truth-predicate T to combine with first-order variables to produce formulas. Where x and y are first-order variables and $\prec$ is now a regular binary predicate, we can formulate this first-order counterpart to (1a):

(1a*) $\forall x(Px \wedge Tx \rightarrow \exists y(Py \wedge Ty \wedge y \prec x))$

The interpretation of (1a*) depends on the interpretation of the truth-predicate T and *e*-proposition predicate P. Given primitivism about higher-order quantification, the interpretation of (1a) does not so depend.

The need for further notions can be illustrated using another example from section 1 concerning aboutness:

(2) A proposition is about an individual just in case the proposition attributes some property to that individual

We formalized (2) as:

(2b) $\forall p \forall x(\mathcal{A}(p, x) \leftrightarrow \exists Y(p = Yx))$

To formalize (2) in a first-order setting, we need a predicate U for (unary) universals and a regular two-place predicate A for aboutness. We can use these to construct a sentence of the following form:

$$\forall z(Pz \to \forall x(Azx \leftrightarrow \exists y(Uy \land z = \dots))$$

The ... will be filled with a singular term for the e-proposition that x instantiates universal y. To obtain such a term, two more devices are needed. First, a binary instantiation predicate I, with xIy stating that x instantiates universal y. Second, a propositional abstraction device $[\dots]$ which turns a formula into a singular term standing for the e-proposition expressed by the formula. With this, $[xIy]$ stands for the e-proposition that x instantiates universal y. We can then formalize (2) as:

(2b*) $\forall z(Pz \to \forall x(Azx \leftrightarrow \exists y(Uy \land z = [xIy]))$

The interpretation of (2a*) depends on the interpretation of the e-proposition predicate P, universal predicate U, instantiation predicate I, and the term-forming square bracket operator. Given primitivism about higher-order quantification, the interpretation of (2a) does not so depend.

The square bracket notation allows us to talk about e-propositions specified using complex formulas, like $[Fx \land Gx]$, the e-proposition that x is F and G. Similarly, we would like to talk about universals specified using complex formulas, like the universal of being both F and G. In higher-order languages, λ serves the analogous purpose by providing complex predicates such as $\lambda x.Fx \land Gx$. On a first-order approach, the natural way to obtain singular terms for such universals is to extend the square bracket notation so that, e.g., $[x.Fx \land Gx]$ denotes the relevant universal of being both F and G.

The differences between the first-order and higher-order approaches are not merely notational. Consider the question of what it takes to have a property specified by a complex term. In higher-order logic, a widely endorsed principle is that of β-conversion, according to which, e.g., any y has the property of being both F and G just in case y is F and y is G:

$$\forall y((\lambda x.Fx \land Gx)y \leftrightarrow Fy \land Gy)$$

More generally, writing $\phi[y/x]$ for the result of replacing every free occurrence of x in ϕ by y (and assuming that no occurrence of y becomes bound), a weak form of β-conversion states:

(β) $\forall y((\lambda x.\phi)y \leftrightarrow \phi[y/x])$

This principle is very natural. It is also included in many standard theories in higher-order logic based on classical principles of quantification, which are demonstrably consistent. Now consider the analogous first-order principle that any y has universal $[x.\phi]$ just in case $\phi[y/x]$:

(β^*) $\forall y(yI[x.\phi] \leftrightarrow \phi[y/x])$

This is classically inconsistent, by a version of Russell's paradox, the argument mentioned in the previous section, by which Russell showed Frege's Basic Law V to be inconsistent. To see why, consider the instance of (β^*) in which ϕ is $\neg xIx$. Abbreviating $[x.\neg xIx]$ as r, that instance is:

$$\forall y(yIr \leftrightarrow \neg yIy)$$

Instantiating y with the singular term r, we obtain:

$$rIr \leftrightarrow \neg rIr$$

Which is a classical contradiction.

Assuming classical propositional logic, the first-order approach therefore has to restrict universal instantiation or the counterpart (β^*) of (β). No analogous problem arises for the higher-order approach because there is no λ-term corresponding to $[x.\neg xIx]$. In standard higher-order languages, the role of I is played by predication. No variable x can occupy its own argument to form a self-predication xx, whatever the type of x. So although there are first-order self-instantiation formulas xIx, there are no counterpart self-predication formulas; hence no negations thereof; hence no contradictory λ-term $\lambda x.\neg xx$.

Providing a consistent and substantial theory of universals and e-propositions is a difficult challenge for the first-order approach. In contrast, the higher-order approach can include unrestricted β-conversion alongside the classical laws of quantification for quantifiers of all types. However, the higher-order approach comes with difficult challenges of its own, which conversely are not faced by the first-order approach. Christopher Menzel discusses these challenges in chapter 13, where he also develops a first-order theory of e-propositions.

One kind of example comes from nominalizations like 'wisdom', in sentences such as 'wisdom is a virtue'. We mentioned these in section 3, in the context Button and Trueman's fictionalist proposal in chapter 7. They formalize nominalizations using terms of type e. A natural alternative in higher-order metaphysics formalizes 'is wise' and 'wisdom' using the same predicate W of type $\langle e \rangle$. To say that wisdom is a virtue, one combines W with a predicate V of type $\langle\langle e \rangle\rangle$ that formalizes 'virtue' to yield $V(W)$. This formalization runs into difficulties when we consider the claim that one loves wisdom.

The verb 'to love' is most naturally formalized using a predicate L of type $\langle e, e \rangle$. We can use this to say one individual loves another. The predicate W cannot serve as the second argument of L, since it has type $\langle e \rangle$ not e. So we cannot say using only L and W that one loves wisdom.

One solution is to formalize 'to love' using multiple predicates of different types, depending on the type of the second argument. But we still cannot then say that someone loves loving, i.e. that very same loving relation: two different loving predicates of different types must be involved. This seems to miss the point of the claim to be formalized. It also delivers unattractive distinctions between different versions of that claim, made using different pairs of loving predicates.

Difficult questions thus arise for both the first-order and the higher-order approach. An informed assessment requires detailed developments of both approaches. Menzel provides one such development of the first-order approach, laying out a theory of e-propositions in first-order logic. Besides avoiding the need to complicate first-order logic by the introduction of higher-order quantifiers, he argues that it also has the advantage of keeping logic *pure*: the paradoxes of universals and e-propositions essentially involve non-logical predicates like I and T, and so are not purely logical matters. Menzel's theory also individuates propositions very finely, a feature he motivates using attitude ascriptions. Recall that due to the Russell-Myhill argument, discussed in section 2.5, many proponents of higher-order logic are committed to individuating propositions relatively coarsely.

In chapter 14, Williamson replies to Menzel in defense of higher-order metaphysics. Williamson scrutinizes the two aspects just mentioned of Menzel's first-order approach. First, he puts pressure on the distinction between logical and non-logical matters, and so on the sense in which Menzel's approach keeps logic pure. For example, Williamson challenges Menzel's treatment of the identity relation $=$ but not the instantiation relation I as purely logical. Second, Williamson questions whether attitude ascriptions really motivate fine-grained distinctions between e-propositions. Based on a variation on the Frege Puzzle cases (discussed also in Lederman's chapter 9), he argues that some such cases cannot be explained in terms of distinctions between the relevant *e*-propositions. According to Williamson, this undermines the case for fine-grained *e*-propositions using attitude ascriptions.

The second critical chapter in part V is chapter 15 by Bryan Pickel. Pickel addresses the question of how higher-order logic is to be interpreted. He begins by noting an apparent problem for those proponents of higher-order metaphysics, like Williamson (2003, 2013), who think that first-order quantifiers can be given an unrestricted reading: how can first-order quantifiers be unrestricted, if they don't range over what the second-order quantifiers range over? Pickel focuses on second-order quantifiers, but the point extends to all other higher-order quantifiers too.

In standard higher-order languages, an identity statement $x = y$ is not well-formed if the variables x and y are not of the same type. This suggests a possible response, which is to say that it does not even make sense to say that first-order quantifiers range over what second-order quantifiers range over. Pickel probes the motivations for this response. He notes that the motivation cannot be purely syntactic, since the syntactic limitations of one language do not prevent a syntactically more inclusive language from being fully meaningful. The motivation must therefore be semantic, and establish a kind of semantic incommensurability between first- and second-order variables. This brings us back to the question discussed in section 1.2: what do higher-order quantifiers mean?

Pickel considers the primitivism about higher-order quantification, and notes that it does not alone suffice for incommensurability. In particular, he asks why—assuming second-order quantifiers are in good standing—we cannot further extend second-order logic so that second-order variables can meaningfully occupy the argument positions of the predicates of first- and second-order logic, thereby rendering first- and second-order variables commensurable. He considers a number of proposals, including (Florio and Jones, 2021), and finds them wanting.

Williamson responds to Pickel in chapter 16. He agrees with Pickel that a semantic explanation of second-order quantification in terms of first-order quantification over universals is a natural hypothesis, which initially is as plausible as the hypothesis that the syntactic differences between first- and higher-order quantifiers indicate semantic differences which render their variables incommensurable. However, Williamson argues that the Russellian arguments considered above decide the matter, showing that second-order quantifiers can't function as restricted first-order quantifiers. In response to the question why we cannot allow second-order variables as arguments of regular predicates, Williamson notes that Pickel's proposal involves positing a deep syntactic and semantic divide between second-order variables and first-order predicates, and questions the reasons for this divide.

If higher-order quantifiers are understood as restricted first-order quantifiers, higher-order metaphysics loses much of its appeal. Menzel and Pickel both argue that higher-order quantifiers should be understood in this way, and so argue against higher-order metaphysics. In the final chapter 17, Agustín 5Rayo notes that Menzel and Pickel nevertheless agree with many proponents of higher-order metaphysics, such as Williamson, that we can make sense of an unrestricted interpretation of first-order quantifiers. Rayo develops a more radical alternative: a view which employs higher-order quantifiers without conceding that they can be understood as restricted first-order quantifiers, while also denying that we can make sense of an unrestricted interpretation of first-order quantifiers.

On Rayo's view, the domains over which quantifiers, including higher-order quantifiers, can range are open-ended. In particular, it is open-ended what propositions and modalities there are. Rayo applies this feature of his view to provide accounts of some of the puzzling commitments of higher-order metaphysical theorizing, such as the Prior-Kaplan paradox mentioned in section 3.

As the discussions in this part of the volume indicate, it is a substantial and difficult question whether metaphysical theorizing about properties and propositions is better formalized in first-order or higher-order terms. An evaluation of the options must be based on their overall fruitfulness, which requires developing both higher-order metaphysics and first-order competitors as well as possible. This volume is intended to contribute in an open-minded spirit to this attempt to determine what is ultimately the better framework.

Although the first-order and the higher-order approach are naturally considered as competitors, it is important to note that they can be combined into a single system which provides higher-order quantifiers as well as first-order quantifiers restricted to universals and e-propositions. Indeed, as we have seen, the inconsistent system of (Frege, 1884, 1893/1903) was of roughly this form, and a modern such combination can be found in the 'object theory' of (Zalta, 1983, 1988).

Further critical discussion of higher-order metaphysics can be found in chapter 9 by Lederman and chapter 11 by MacBride. An influential development of the first-order approach can be found in (Bealer, 1982), and an example of an explicit debate of the merits of this approach compared to the higher-order approach can be found in (Anderson, 1987) and (Bealer, 1994). A recent criticism of higher-order metaphysics is (Sider, unpublished). And (Florio, forthcoming) critically examines arguments from expressibility considerations to higher-order languages, such as those we used to motivate higher-order metaphysics in section 1.1.

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