Recursionist Theories of Knowledge

Frode Alfson Bjørdal

To my teacher, professor Anthony Brueckner (1953 – 2014).

1 Introduction

In the preliminaries of $ 1.1, an overview is given on some aims and prejudices.

1.1 Preliminaries

1.1.1 A framework for recursionist theories of knowledge is developed.

1.1.2 The main aim is to resolve Gettier style problems.

1.1.3 The approach is predicated upon the assumptions that:

   1.1.3.1 Gettier problems are genuine.

   1.1.3.2 There is no consensus that they cannot be resolved.

   1.1.3.3 There is no agreement upon how to resolve them.

   1.1.3.4 The Gettier problems can be resolved.

1.1.4 As the Gettier literature is so huge, it is an aim to not discuss other contributions.

1.1.5 The emphasis will be upon developing the theory, and explain its resolutions.

1.1.6 A distinct feature of the recursionist resolutions is that *recursive resources* are so prominent in the definitional apparatus, and it is assumed that these have a leading role in avoiding the problems.

1.1.7 It is a crucial aim to be as impartial as possible, on controversial issues. For that purpose we abide by the

NEUTRALITY MAXIM: *Do not commit unnecessarily to a point of view!*

1.1.8 An important advantage with following the Neutrality Maxim is that it helps secure access to a variety of recursionist theories of knowledge, each with their distinctive features.

1.1.9 By dint of $ *1.1.8*, the Neutrality Maxim helps us focus upon the most important problems producing the Gettier perplexities, for it implies abstraction from irrelevant features.

The ensuing subsections of this section are ordered as the corresponding sections further below, and partial accounts of the latter are related.

1.2 Presuppositions

In $ 2.1.1 it is clarified why the term "dictum" is taken to refer to an object of belief, and its plural "dicta" taken to refer to objects of belief. Properties of *belief* and *justification* are laid down in $$ 2.1.2–2.1.3. The Neutrality Maxim is put to work in $ 2.2, so that a number of noncommittals are expressed. Important properties of the essential notion *because* are postulated in $ 2.3. It is pointed out, in $ 2.4, that there is a wide variety of recursionist theories of knowledge, on account of their neutralities with respect to important epistemological discussions. $3.5 discusses whethether there are justified, or true, prime beliefs.

1.3   Recursionist Approaches to Knowledge

Formulations of a recursive definition, of S *knows that p*, are advanced in $ 3.1, using concepts discussed earlier. A notion of *consecution* is introduced in $ 3.2, to distinguish between beliefs in relevant ways. $ 3.3 extends $ 2.4, by giving more causes for varieties of recursionist epistemologies. Given the foregoing, an epistemological theorem is isolated in $ 3.4; arguably, it expresses the necessary and sufficient conditions for knowledge in a more intuitive way.

1.4 Gettier's Examples

Gettier's examples are analyzed, to show that recursionist epistemologies avoid them.

1.5 Conclusion

The essay, finally, reaches a justification of a result which indicates that theories of knowledge cannot be undermined by Gettier style beliefs.

2   Presuppositions

Several conditions, predominantly geared to maximize the exposition's agreement with the *Neutrality Maxim*, are laid down for key notions in $$ 2.1–2.2.

$ 2.3 sets forth provisional conditions upon the dyadic sentence operator *because*.

2.1 Assumptions with terminological impact

2.1.1 THE OBJECTS OF BELIEF

2.1.1.1 It is usually assumed that there are propositions, declarative sentences, judgments, elements of a language of thought, or ilks, which are the objects of belief.

2.1.1.2 The unusual term "dictum" is used here for objects of belief, to convey that the exposition does not commit to any one of more specific theories on objects of belief.

2.1.2 Beliefs and their dicta

2.1.2.1 S's belief that p is denoted by "*S believes p*".

2.1.2.2 If S believes p, the dictum p is the object of the belief.

2.1.2.3 S believes a dictum just if disposed to endorse it.

2.1.2.4 A belief is tacit (implicit) just if not occurrent (explicit).

2.1.3 Justification on

2.1.3.1 S is justified on p just if S is justified in believing p.

2.1.3.2 S may be justified on p without believing p.

2.1.3.3 S may neither be justified on p nor on *not-p*.

2.1.3.4 S may be justified on p and on *not-p*, so

2.1.3.5 S may be justified on p although p is false.

2.2 Neutralities

2.2.1 Neutrality on internalism and externalism

It is not assumed that justification of beliefs comes about by proper ligations between subject and surroundings, or by relations to other justified beliefs, or beliefs *simpliciter*.

2.2.2 Neutrality on deontologism

S may be justified on q without having a dentological *right* to believe q.

2.2.3 Neutrality on coherentism, foundationalism, and infinitarianism

Justifications, structured as recommended by coherentists, foundationalists as well as infinitarianists, are all provisionally accepted in the exposition.

2.2.4 Neutrality on truth and paradox:

.1 A subject knows a dictum only if it is true.

.2 Some quantification over dicta is allowed.

.3 The exposition does not commit to a theory on truth, or a theory on paradoxes.

2.3 Because

2.3.1 A dyadic sentence operator is *doubly veridical*, just if it is true of its two operands only when both of the latter are true.

2.3.2 The dyadic sentence forming sentence operating operator *because* is taken to be doubly veridical, so that S believes q *because* S believes p only if S believes p and S believes q.

2.3.3 Salient notions as *inference*, *explanation* and *cause*, and ilks, enter into necessary conditions for versions of the *because* operator to hold of two doxastic operands.

2.3.4 It may at least be presupposed that S believes q because S believes p only if S infers q from p, *or* the fact that S believes p (partly) explains the fact that S believes q, *or* the occurrence of the event that S believes p causes the occurrence of the event that S believes q, or similar conditions obtain.

2.4 Variety observation 1/2:

As a consequence of the various neutralities pointed to, a variety of recursionist theories of knowledge are generated, corresponding with ways to deal with problems, and be less neutral. But such varieties will not be elaborated here.

3 Recursionist approaches to knowledge

3.1 The recursive definitions of knowledge

Using notions discussed above, recursive definitions of S *knows that p* are advanced:

Definition 3.1.1. (Informal recursionist version)

S knows that q *just if* S is justified on q,   1

         q is true,   2

         S believes that q &   3

         if for some p, S is justified on p and S believes q 4

         because S believes p , then for some p, S believes

         q because S believes p , and S knows that p , and S

         knows that p only if q .

Definition 3.1.2. *Let* $\mathcal{K}_q^S$ *signifiy that* S knows that q, J_q^S that S is justified on q, $\mathcal{T}q$ that q is true, B_q^S that S believes q, and $\mathbb{S}_q^p$ that S believes q because S believes p.

Definition 3.1.3. (Formal recursionist version)

$$\mathcal{K}_q^S \leftrightarrow \mathcal{J}_q^S \wedge \mathcal{T}q \wedge \mathcal{B}_q^S \wedge (\exists p(\mathbb{S}_q^p \wedge \mathcal{J}_p^S) \to \exists p(\mathbb{S}_q^p \wedge \mathcal{K}_p^S \wedge \mathcal{K}_{(p \to q)}^S)).$$

$ *3.1.4* $\mathcal{K}$ occurs only positively in the definiens of Definition 3.1.3, so it is only seemingly circular, and can be expressed, equivalently, without circularity, under the assumption that one may make use of an impredicative higher order logic. The informal Definition 3.1.1 is only apparently circular in the same sense.

3.2 Consecutions

The following definition segregates prime, primal, cyclical, infinite and infinitary beliefs:

Definition 3.2.1.

1: *For* $1 < n \in \omega$ *and* $s = (s_0, \ldots, s_{n-1})$ *an n-tuple, s* terminates *with* s_0 *and* begins *with* s_{n-1} . *Moreover, for* $0 \le i < j < n$, s_i succeeds s_j , *and* s_j precedes s_i . *ω-tuples,*

as per 4, have a termination, but no beginning.

2: *A 2-consecution to q_0, for S, is a 2-tuple $\langle q_0, q_1 \rangle$, such that S believes q_0, S is justified on q_0, S believes q_0 because S believes q_1, and S is justified on q_1.*

3: *An $(n+1)$-consecution to q_0 for S, is an extension of an n-consecution, $(q_0, \ldots, q_{n-1})$, to q_0 for S, to an $(n+1)$-tuple $(q_0, \ldots, q_{n-1}, q_n)$, obtained by appending a dictum q_n, which is such that for $m < n$, S believes q_m, S is justified on q_m, S believes q_m because S believes q_{m+1}, S believes q_{m+1} and S is justified on q_{m+1}.*

4: *An ω-consecution for S is a function f, with the infinite domain ω of all natural numbers, such that for $m \in \omega$, $f(m)$ is a dictum q_m which S believes. Moreover, for any $m \in \omega$, S believes q_m, S is justified on q_m, S believes q_m because S believes q_{m+1}, S believes q_{m+1} and S is justified on q_{m+1}.*

5: *An ω-consecution is n-cyclical, with root r, just if $1 < n \in \omega$, and $1 < r \in \omega$ is the least natural number such that for all m, if $r \le m \in \omega$, $q_m = q_{m+n}$, and, moreover, such that $\forall o (m < o < m + n \rightarrow q_m \ne q_o)$.*

6: *An ω-consecution is cyclical just if n-cyclical with root r for some $n \in \omega$ & $r \in \omega$.*

7: *A tuple of dicta is a consecution to q_0 for S, just if it, for some natural number $n > 1$ is an n-consecution to q_0 for S.*

8: *A belief is prime just if its dictum has no consecution.*

9: *Prime beliefs may be false.*

10: *Prime beliefs may be unjustified.*

10: *A belief is primal just if terminates a consecution which begins with a prime belief.*

11: *A belief is cyclical just if it terminates a cyclical ω-consecution.*

12: *A belief is infinite just if it terminates an ω-consecution.*

13: *A belief is infinitary just if it is infinite, and neither is cyclical nor primal.*

13: *Prime beliefs are neither primal, cyclical nor infinite.*

14: *Some beliefs may be both primal, cyclical and infinite, lest further restrictions are*

imposed; abiding by the Neutrality Maxim, we do not adopt other restrictions.

3.3 Variety observation 2/2:

A variety of recursionist theories of knowledge may be engendered by varying restrictions upon consecutions, as above, in addition to the different theories of knowledge which may be generated according to the precepts of $ 2.4.

3.4 An epistemological theorem

The definitions of consecution, and knowledge, have the consequence that a prime belief is known just if it is true and justified, and other true and justified beliefs are known just if they have a consecution where all preceding dicta are known.

3.5 Are there justified, or true, prime beliefs?

3.5.1 Given Definitions 3.2.1.9–3.2.1.10, prime beliefs may be false, and not justified.

3.5.2 The author does not suggest that there are justified or true prime beliefs.

3.5.3 Justifications of prime beliefs may come about in various ways:

3.5.3.1 Perceptually: Maybe S believes that S sees that there is a tree, only if S is justified on believing that S sees that there is a tree.

3.5.3.2 Intuitively: Suppose S believes that S sees that 2 plus 3 equals 5. Some basic intuitions may behave like perception, so that, consequently, S is justified in believing that S sees that 2 plus 3 equals 5.

3.5.4 Cohesionally: Perhaps a set of prime beliefs may be severally justified by an inducing source, such as *cohesion* with each other, and with other sets of beliefs, without being entailed by any other member of that set.

3.5.5 Even a dictum of a prime belief, or of a justified prime belief, may be true.

4 Gettier's examples

The counter examples of Gettier 1963, against the classical definition of knowledge as justified, true belief, are discussed in light of recursionist theories of knowledge:

4.1 Smith and Jones and the coins in the pocket

In the first example of Gettier 1963, Smith concludes that the one who gets the job has ten coins in his pocket, based upon the supposedly justified beliefs that Jones gets the job and that Jones has ten coins in his pocket.

But Smith's belief that Jones gets the job is mistaken. Instead, it is Smith who gets the job, and, to Smith's surprise, Smith as well has ten coins in the pocket. So Smith has a justified true belief that the one who gets the job has ten coins in the pocket, but Smith does not know that the one who gets the job has ten coins in the pocket.

Recursionist theories of knowledge avoid the problematic conclusion, for the latter justified belief is held because Smith believes, falsely, that Jones gets the job and has ten coins in the pocket, and Smith does not, as required by Definition 3.1.1.4, have a belief p such that Smith believes that the one who gets the job has ten coins in the pocket because Smith believes p, and such that Smith knows p and knows that p only if the one who gets the job has ten coins in the pocket.

4.2 Smith, Jones and Brown in Barcelona

In the second example of Gettier 1963, Smith is justified on the false dictum that Jones owns a Ford, and concludes that Jones owns a Ford or Brown is in Barcelona. Without Smith having an inkling about Brown's whereabouts, the latter is in Barcelona.

In as much as justification is closed over known entailment, Smith has a justified true belief that Jones owns a Ford or Brown is in Barcelona, but, as Gettier points out, without knowing it.

With the recursionist definition of knowledge, we need to invoke Definition 3.1.1.4, as Smith believes and is justified on the dictum that Jones owns a Ford or Brown is in Barcelona, and believes it because S believes that Jones owns a Ford. But S does not have a belief p such that S believes that Jones owns a Ford or Brown is in Barcelona because S believes p, and such that S knows that p, and knows that p only if Jones owns a Ford or Brown is in Barcelona. So the recursionist teory does not support the implausible result that S knows that *Jones owns a Ford or Brown is in Barcelona.*

5 Conclusion

The author posits that S has a Gettier–style belief q only if S is justified on q, and there is a belief p, such that S is justified on p, and S believes q because S believes p.

Under the posit, the assumptions that S has a Gettier–style belief q, and Definition 3.1.1, it is provable that S knows that q just if S is justified on q, q is true, S believes q, and there is a dictum p such that S believes q because S believes p, S knows that p, and S knows that p only if q.

More formally, the assumption that S has a Gettier belief, and the posit, entail, under Definitions 3.1.2 and 3.1.3, that

$$\mathcal{K}_q^S \leftrightarrow \mathcal{J}_q^S \wedge \mathcal{T}q \wedge \mathcal{B}_q^S \wedge \exists p(\mathbb{S}_q^p \wedge \mathcal{K}_p^S \wedge \mathcal{K}_{(p \rightarrow q)}^S).$$

This suggests recursionist epistemologies cannot be ruined by Gettier–style beliefs.

References

Gettier, E. (1963). "Is Justified True Belief Knowledge?" *Analysis* 23.6, 121–123.