THE LOGIC OF THE WHOLE TRUTH

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This paper, using truth-functional erotetic logic, gives a characterization of "the whole truth" sworn to by witnesses offering testimony.

Witnesses swear to tell the truth, the whole truth, and nothing but the truth. The last requirement, as the first, is easy to characterize using classical, modern truth-functional logic. The criterion of "the whole truth," however, is not as easily captured. This is because the truth the whole of which must be told is in reference to the question put by counsel or the court and any truth(s) which go(es) beyond the framework of the question will be found immaterial, rather than more truthful. It is thus necessary to develop a logic that can deal with both questions (not propositions) and answers (propositions) in order to formally characterize a notion as elusive as "the whole truth."

Such a logic is called an *erotetic logic* and one such logic was developed and elaborated on by Belnap and Steel in a monograph entitled *The Logic of Questions and Answers.*[1] That work, its 45-page bibliography, our own previous work,[2] and our insights developed in working out concepts relating logic to law have guided us in the application of erotetic logic to the characterization of "the whole truth" presented here.

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1. N. BELNAP & T. STEEL, THE LOGIC OF QUESTIONS AND ANSWERS (1976).

2. Fulda, *Meaningfulness from Logical Form*, 61 THOUGHT 482 (1986); Fulda, *Estimating Semantic Content: An A Priori Approach*, 3 INT'L J. INTELLIGENT SYS. 35 (1988).

INTRODUCTION TO EROTETIC LOGIC

We first introduce the principles and notation of the variant of erotetic logic that we will be using. The fundamental principle of erotetic logic is that every question is associated with the disjunction of all its possible answers, true or false. This disjunction is called *the pose of the question* and its disjuncts are termed *elements of the pose.* Those elements of the pose that are true are known as *the true answers.* The pose of a question may be given either extensionally or intensionally. Thus, the question, "What type of weapon was used to murder Frank Jones?" gives rise to "gun" v "knife" v "club" v ...[3] which can also be given intensionally as (?x)(Wx & Mxf), where (?x) is the quantifier-like notation used for the definite description—"the type of weapon used to murder Frank Jones."[4] In this case, there will only be one true answer, but that is neither a requirement of erotetic logic nor of all questions asked of witnesses. Nevertheless, we will at first assume that the questions we pose as examples have exactly one true answer.

A question the answer to which will be given in the propositional logic will be denoted *Q?* Q? is *logically true* if and only if all the elements of its pose are true answers. It will not do to characterize Q? as logically true if its pose is logically true or if its true answers are logically true. The former is the case whenever the question is exhaustive (e.g., a simple yes-or-no question, P?, with possible answers P, $\sim$ P).[5] The latter is true in the domain of testimony only if the question is bizarre. (Remember that tautologies are true regardless of their content by virtue of

3. This follows the natural language convention for answering questions with fragments. To be more precise, however, we have: "The type of weapon used to murder Frank Jones was a gun" v "The type of weapon used to murder Frank Jones was a knife" v . . . . Henceforth, we will be imprecise, unless that would cause a misunderstanding.

4. Actually, the status of descriptions as designators is one of the main issues in the contemporary philosophy of language and logic. *See* S. KRIPKE, NAMING AND NECESSITY (1972).

5. Thus, if $U = \{a,b\}$, Pa v Pb is the pose of the question (?x)Px, and is exhaustive. Given $(\exists x)Px$, the presupposition of the question, Pa v Pb is logically true.

their form, whereas questions are asked of witnesses to learn contingent (not known *a priori*) facts.) The definition we are using for logically true questions, moreover, is well-motivated, because as with propositions, such a question is true by virtue of its form. An example of a logically true question is "Choose a color."[6]

A *logically false* question is a question all of whose answers are false; this just follows the definition of logical truth for questions. One way a logically false question can arise in the courtroom is the standard "When did you shoot the victim"-type question. The pose is a disjunction of (propositions giving) times and none correctly answers the question, given the falsity of the *presupposition*, the proposition that must be true for the question to have a true answer (here, that the defendant did indeed shoot the victim). Note that a question may have several presuppositions. For example, "Whom did you run over with your car?" has at least two presuppositions: that the witness has a car and that he ran over someone with it.

Continuing with our elaboration of erotetic logic, a question the answer to which will be given in the many-sorted[7] predicate logic will be denoted $(?x \in S)Px$ (first-order logic) or $(?P \in F)Px$ (second-order logic). Usually, S and F will be understood from the context and left implicit. The pose of such questions is naturally given by the question's existentially quantified correlate, $(\exists x \in S)Px$ or $(\exists P \in F)Px$, respectively, which is at the same time its presupposition. Of course, existential quantification on a countable (here finite) domain is just disjunction writ large. It is thus no surprise that the pose of a question and the existential quantification are generally the same.

6. Strictly speaking, this is a request (imperative) rather than a question (interrogative). For the purposes of analyzing testimony, however, anything requiring a response and having alternative responses is a "question" that can be posed to a witness. Except perhaps as a rhetorical device when "warming up" the witness, I am unable to think of a rationale for asking a logically true question in a courtroom situation. If all possible proper answers are true, why ask the question?

7. Note that our use of many-sorted logic explains why "Choose a color," $(?x \in C)Cx$, is logically true. Below, we will see that, for our purposes, a many-sorted logic is essential for efficient use of courtroom time.

THE MODEL

We now present the simple and elegant characterization of "the whole truth" that erotetic logic enables. A witness tells the whole truth in answer to a question put to him if he testifies to a disjunction all of whose disjuncts are elements of the pose, called *a limitation of the pose*, provided he believes no further (i.e., a limitation of the) limitation of the pose. More generally, a witness may testify to *any* proposition from which a limitation of the pose can be derived using deductive methods. And, provided he believes no proposition that entails a further limitation of the pose, he is being wholly truthful.

There are several points worthy of note here. First, it may seem strange that one approaches "the whole truth" by peeling off disjuncts from a proposition, rather than, say, adding conjuncts. But one must recall that the informational content of a disjunction *increases* as disjuncts are removed and *decreases* as disjuncts are added.[8] Second, a witness who answers "I don't know"—and no characterization of the truthfulness of testimony would be complete without a discussion of this special case—in response to a question testifies, in effect, to the trivial limitation of the pose and is telling the whole truth provided he believes no nontrivial limitation of the pose. Third, a witness presented with a logically false question—another special case that must be discussed—must state so (in any reasonably understandable way). Perhaps a witness can be excused for answering "I don't know" to a logically false question, but technically, at least, that is not the proper answer. This is because "I don't know" returns *all* the elements of the pose, while a logically false question should be answered by a statement that makes clear that *none* of the elements hold! Fourth, *every* answer not ruled improper by the court will be a limitation of the pose or some proposition(s) from which a limitation of the pose can be deduced. Any other reply is simply not responsive to the question. Fifth, the limitation of the pose offered as testimony need not, in general, be the true answer, although unless the witness is mistaken, it will contain it. This is because "the whole truth" is relative to the witness' knowledge

8. *See supra* note 2.

and as long as he reveals all his (relevant) knowledge, he has faithfully discharged his obligation to be wholly truthful with the Court. Sixth, any limitation of the pose is consistent with any further limitation of the pose and, as a result, a witness who is withholding information will not also *thereby* be violating the criterion of "nothing but the truth." Seventh, the characterization of "the whole truth" using limitations of the pose does not speak to the issue of whether "the whole story" has been brought out in trial. A witness *is* obligated to answer questions put to him wholly truthfully; it is counsel's job, however, to ensure that a line of questioning—with each question answered wholly truthfully—brings out "the whole story" or as much of it as he feels is in his client's best interest. Thus, if a witness is asked whether he got a good look at the perpetrator's clothing and he answers "No" truthfully, he has fulfilled his obligation to the Court, even though he did get a good look at the perpetrator's car and failed to so testify. It is counsel's job to put the questions; the witness need only supply wholly truthful answers and need not concern himself with counsel's competence to elicit "the whole story" from those who testify. To use the terminology of a computer scientist, witnesses are intelligent databases which respond to queries and perform deductions in order to do so, but no more.

EXAMPLES

Following are several examples of increasing complexity applying the characterization we have given and bringing out important issues related thereto. Recall that we are confining our discussion, for the moment, to questions with one true answer.

(1) What color was the defendant's shirt? (asked in a case in which it has been established that the shirt was monochrome).[9] If the witness was some distance from the defendant on the day in question, then "blue or green," while not the true answer (say, "blue"), is that limitation of the pose that constitutes the whole truth. One must be very careful with erotetic logic, as with other logics, to make sure that the translation from natural language to

9. Formalization: $(?x \in C)Sx$. C: set of colors; Sx: the defendant's shirt has color x.
Pose: $(\exists x \in C)Sx$.

logical form is precise. Thus, while question (1) could be formulated in several different ways including, for example, with a second-order pose [$(?P \in C)Px$], it would be a mistake to write $(?x \in S)Dx$, where S is the set of shirts and Dx denotes "x is the shirt of the defendant." If symbolized this way, a witness who knows not only that the shirt was blue, but also that it is exhibit A would wrongly be convicted of perjury if he revealed only the former fact (because he believes a further limitation of the pose), even though he has fully answered the question put to him. Were the question phrased, however, "Identify the defendant's shirt," then the response "I can't identify it, but I do know that it was blue or green" is helpful, since it rules out most of the elements of S—namely, all shirts which are not blue or green—and thereby substantially limits the pose. This is a typical example of the case where a witness does not testify to a limitation of the pose, but rather to some proposition from which a limitation of the pose can be deduced.

(2) Who was running from the scene of the crime? (asked in a case in which it has been established that only one suspect was running from the scene of the crime).[10] Clearly, an answer naming an individual, say *a*, would be optimal. Typically, however, the most that a single witness will be able to do is incrementally help the jury (judge) discover the facts. Here the likely knowledge that the witness has is $\sim Rb$: *b* has an alibi and so could not have been the man running from the scene of the crime. By disjunctive syllogism, the pose is limited by the knowledge gained from the witness. This is another typical example of the case where a witness testifies not to a limitation of the pose, but to some proposition from which a limitation of the pose can be deduced. Suppose, now, that the witness and *b* attended a party that fateful night with 100 other attendees. Is the witness required, to be wholly truthful, to assert alibis for each and thus limit the pose still further (by 100 applications of disjunctive syllogism or one application of negative hyperresolution)? Obviously, the judge will find this time-consuming testimony

10. Formalization: $(?x \in S)Rx$. S: set of suspects; Rx: x was running from the scene of the crime.

Pose: $(\exists x \in S)Rx$.

intolerable because the pose is already limited—implicitly—to the suspects, as given by the many-sorted formulation in footnote 10.[11] This example shows why a many-sorted logic is essential for the application of logic to the domain of testimony: it limits the testimony to what is material to the case at hand.

(3) What was the license number of the getaway car? (asked in a state where a license number consists of six alphanumeric characters).[12] A partial answer, "JK1 something" (JK1 $(\exists x)(\exists y)(\exists z)\ (x,y,z \in \{\ 0..9\ \} \cup \{\ A..Z\ \})$) is not wholly truthful if the witness knows, in addition, that the last character was, say, a digit. In that case, the pose can be further limited by a further restriction on the sort: (JK1 $(\exists x)(\exists y)(x,y \in \{\ 0..9\ \} \cup \{\ A..Z\ \})(\exists z)(z \in \{\ 0..9\ \})$). This shows that the pose can be limited either explicitly (by pruning off disjuncts) or implicitly (by limiting the sort).

Questions can also call for numerical answers, and answers to such questions can be evasive by not providing the whole truth in just the manner we have described. (4) How many perpetrators were there?[13] If the witness knows that there were three or four culprits, then none of "between 2 and 5" (2 v 3 v 4 v 5) or "at least 3" (3 v 4 v . . .) or "at most 4" (1 v 2 v 3 v 4) is wholly truthful.

Questions and answers can also be expressed with quantifiers and be intensionally given. (5) Who was present at the meeting?[14] This question must have multiple true answers, since a

11. One of the difficulties that arises in translating natural language to logical form (regardless of the variant of logic used) is capturing implicit information that derives from the context of the conversation. *See* L. WOS, R. OVERBEEK, E. LUSK & J. BOYLE, AUTOMATED REASONING (1984). Courtroom "conversation" has the advantage that its high degree of structure reduces the opportunities for misapprehension. Implicit information—such as the limitation of the sort in question (2) to suspects—is thus more easily noticed.

12. Formalization: $(?u,v,w,x,y,z \in \{0..9\ \} \cup \{\ A..Z\ \})$ Luvwxyz. Luvwxyz: u,v,w,x,y,z constitute, in that order, the license number.

Pose: $(\exists u,v,w,x,y,z \in \{\ 0..9\ \} \cup \{\ A..Z\ \})$Luvwxyz.

13. Formalization: $(?x \in N)$Px. N: set of natural numbers (1,2,3,...); Px: x is the number of perpetrators.

Pose: $(\exists x \in N)$Px.

14. Formalization: $(?x \in P(H))$Px. H: set of persons; Px: x was present at the meeting.

Pose: $(\exists x \in P(H))$Px.

meeting must have at least two participants to be so called. Consider now the two responses: "Only teachers were present" $((\forall x)(Px \Rightarrow Tx))$ and "Only female teachers were present" $((\forall x)(Px \Rightarrow (Tx \& Fx)))$. The former limits the pose by cancelling all non-teacher elements; the latter further limits the pose by also cancelling all male teacher elements. To see this, recall that universal quantification over a countable (here finite) domain is just conjunction writ large. Thus for the first response, we have: $[(Pa \Rightarrow Ta) \& (Pb \Rightarrow Tb) \& \ldots]$. Now suppose we know $\sim Ta$, then by simplification we obtain $Pa \Rightarrow Ta$, by Modus Tollens we obtain $\sim Pa$ which, in turn, can be used to limit the pose by disjunctive syllogism. For the second response, we have: $[(Pa \Rightarrow (Ta \& Fa)) \& (Pb \Rightarrow (Tb \& Fb)) \& \ldots]$. Now suppose we know $\sim Fb$, then by addition we have $\sim Fb \vee \sim Tb$, by commutation we have $\sim Tb \vee \sim Fb$, by De Morgan's Law we have $\sim(Tb \& Fb)$, by simplification we have $Pb \Rightarrow (Tb \& Fb)$, by Modus Tollens we have $\sim Pb$, and by disjunctive syllogism we have a limitation of the pose. (Remember that as long as a limitation of the pose can be logically derived from the witness' response, the form of the testimony is proper—the witness need not, and often will not, testify directly to a limitation of the pose.)

Quantification can also be multiple. Consider (6) Who caused the mass suicide at Jonestown? (6a) $(?x \in L)(\forall y \in J)Cxy$, or, (6b) $(\forall y \in J)(?x \in L)Cxy$ where L is the set of leaders, J is the set of people at Jonestown (a superset of L), and where Cxy denotes "x caused the suicide of y." If the question is taken to mean that someone in particular caused all the suicides (if we read the question as having that presupposition), then (6a) results. If different leaders may have been responsible for different deaths then (6b) results. Bearing in mind that (?x) is akin to the existential quantifier, (6a) and (6b) are evidently seen to be inequivalents. Indeed, if (6b) is understood, then a witness' testimony that $(\exists x \in L)(\forall y \in J)Cxy$ is a substantial limitation of the pose of the question, which can then be further limited (by additional testimony or other evidence) to the true answer, $(\forall y \in J)Cjy$, where *j* refers to Reverend Jim Jones.

The methods advanced in our previous work[15] allow us to quantify the degree to which the whole truth has been revealed when the witness knows, say, $(\forall y \in J)Cjy$ but reveals only that, say, $(\exists x \in L)(\forall y \in J)Cxy$. A detailed exposition of these methods applied to erotetic logic is well beyond the scope of the present inquiry, but its potential use as a measure of the severity of an instance of perjury and its potential practical application, therefore, to sentencing decisions for perjury convictions are interesting.

Quantification of the degree of perjury can sometimes be precluded even when the perjury itself is undoubted, because of vague quantifier-like terms. Thus, the tenure candidate appearing before his university's faculty personnel board is asked (7) Name whom you dated while a student in one of your classes.[16] The pose of this question is a disjunction of propositions regarding all sets of students that the instructor has had, and the panel is looking for that one particular set of students to discover whether other-than-academic reasons have ever motivated the candidate's academic actions. Not wishing to lie outright but unwilling to accept what he regards as an invasion of his privacy, the candidate responds evasively, i.e. truthfully but not wholly truthfully, "I don't date current students," meaning in the vernacular, as a practice, habit, or norm, rather than *ever* as intended by his interrogators. Here, since the witness *must* know the true answers, his limitation of the pose by an unknown degree (norms, practices, and habits are hard to quantify) and in an unknown way (with which students did he follow his usual practice and with which students did he depart from it) is not the whole truth. Curiously but consistently, even if he *never* dated a current student, he *has* committed perjury.

Some questions call for the second-order logical apparatus, although they can be recast in first-order logic, if one is willing to sacrifice naturalness of representation for a more tractable

15. *See supra* note 2.

16. Formalization: $(?x \in S)Dx$. S: set of students that instructor had in a class; Dx: instructor in question dated x while x was a student in one of his classes.

Pose: $(\exists x \in S)Dx$.

calculus. Consider a homicide case in which the convicted defendant is about to be sentenced and is asked by the judge (8) What did the victim do to your wife immediately prior to your murdering him?[17] Here, clearly, the question revolves around the verb or predicate involved. The concepts developed here to characterize the notion of the whole truth using erotetic logic apply as well to second-order logic, *mutatis mutandis*.

Let us now drop the restriction that each question put to the witness has precisely one true answer. Question (1), however, What color was the defendant's shirt?, presupposes that there is exactly one true answer. This is evident by the use of the singular, "color." Question (2), on the other hand, Who was running from the scene of the crime?, may admit of several true answers. It is therefore properly formalized $(?x \in P(S))Rx$, where $P(S)$ is the powerset (set of all subsets) of S. Concretely, if $S = \{a,b,c\}$, then the pose of this question is: F v Ra v Rb v Rc v (Ra & Rb) v (Ra & Rc) v (Rb & Rc) v (Ra & Rb & Rc). Actually, we have substituted logical conjunction for set membership since the notation is easier to follow and the substitution creates no technical difficulties here. However, to be totally precise, the pose of the question is given by: $R\emptyset$ v $R\{a\}$ v $R\{b\}$ v $R\{c\}$ v $R\{a,b\}$ v $R\{a,c\}$ v $R\{b,c\}$ v $R\{a,b,c\}$, and Rx means x is the set of persons who were running from the scene of the crime. Note the slight modification in the meaning of Rx. Note that F corresponds to the empty (sub)set and indicates that the question is logically false—that there are no true answers because the question has a false presupposition (namely, $(\exists x)Rx$). Questions (3) and (4) must each have only one true answer: a car has only one license number and any crime has a fixed number of perpetrators. Multiple answers for questions (5) and (6) have already been dealt with, although we have assumed, in answering question (6), that any *one* person's suicide was caused by a single leader. We may presume that this was established earlier in the line of questioning. As a general rule, if a question has a presupposition,

17. Formalization: $(?P \in C)Pvw$. C: set of crimes against the person; Pvw: the victim perpetrated crime P on the person of the wife of the defendant. (P might be A (assault), B (battery), R (rape), etc.)
Pose: $(\exists P \in C)Pvw$.

then either the presupposition is established first in the line of questioning or the question will cause an objection to be made and sustained (question presupposes something not in evidence), in which case chances are it was asked out of sloppy technique or because it is logically false and the attorney is trying to prejudice the jury. In discussing question (7), we merely mentioned sets of students without elaboration. Here, as with question (2), the question is properly formalized using the powerset: $(?x \in P(S))Dx$, and Dx must be slightly modified to mean "x is the set of students . . . ." Likewise, if question (8) is understood to allow multiple crimes against the person to have been committed by the deceased victim on the person of the defendant's wife, the question is properly formalized $(?P \in P(C))Pvw$, and a similar modification of the predicate variable P is required.

We should briefly discuss the case where a true answer or answers are given in testimony, but there are additional true answers known to the witness, but not offered in testimony. Let us return to question (2), this time allowing for multiple true answers. Suppose there are three suspects: a, b, and c. (I.e., $S = \{a,b,c\}$.) Then the pose is: F v Ra v Rb v Rc v (Ra & Rb) v (Ra & Rc) v (Rb & Rc) v (Ra & Rb & Rc), as given above. Exactly how is this pose limited if a witness testifies "$\sim$Ra"? And what if a witness testifies "Ra"? The former case provides little difficulty: $\sim$Ra rules out Ra, Ra & Rb, Ra & Rc, and Ra & Rb & Rc. (For example, we rule out Ra & Rb & Rc as follows: by addition twice we have $\sim$Ra v $\sim$Rb v $\sim$Rc, by De Morgan's Law we have $\sim$(Ra & Rb & Rc), and by disjunctive syllogism the pose is limited as described.) The latter case, however, is not as clear-cut. Ra may or may not be a complete answer. Thus, we cannot settle on Ra as *the* true answer, but must rather content ourselves with ruling out F, Rb, Rc, and Rb & Rc. It is up to *a*'s counsel to suggest that perhaps some other suspect(s) was also present at and running from the scene of the crime, by asking the witness: Was anybody else running from the scene of the crime? Formally, $[(\exists x \in S)(Rx \;\&\; x \neq a)]$? If the answer to this question is "No," we may indeed settle on Ra as the true answer. If the answer is "I don't know" or something to that effect, we cannot further limit the pose. If the answer is "Yes," we proceed to ask that the other party be identified: Who else was running from the scene of the crime? Formally, $(?x \in S)(Rx \& x \neq a)$ or, if there may be multiple additional fleeing suspects, $(?x \in P(S))(Rx \& a \notin x)$. The answer "Ra" when it was known to the witness that both *a* and *b* were fleeing suspects clearly does not constitute perjury, since the witness testified to the whole truth about the fleeing of *a*, in contrast to the case where the witness testifies Ra v Rb in which case the truth value of Ra remains unknown. In both cases, to be sure, the whole truth was withheld in some sense, but only in the latter case is it clearly a violation of the witness' oath rather than inadequacy on the part of counsel. In any instance where the whole truth is held back in the sense that we defined it—using the concept of limitation of the pose—there is nothing further that the attorney can possibly do to elicit the whole truth, since repeating an answered question is not allowed (opposing counsel may object "asked and answered"). In the case of multiple true answers only some of which have been offered in testimony, however, we have shown that counsel can continue to pursue the line of questioning to its logical conclusion and can indeed thereby elicit the remaining true answers. The case where a true answer or answers are given in testimony but where there are also additional true answers known to the witness is thus seen to be an instance of counsel's responsibility to elicit "the whole story" as discussed earlier, rather than a case of "the whole truth" as was the principal concern of this paper.