Protective measurement and de Broglie-Bohm theory

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We investigate the implications of protective measurement for de Broglie-Bohm theory, mainly focusing on the interpretation of the wave function. It has been argued that de Broglie-Bohm theory gives the same predictions as quantum mechanics by means of quantum equilibrium hypothesis. However, this equivalence is based on the premise that the wave function, regarded as a Ψ-field, has no mass and charge density distributions. But this premise turns out to be wrong according to protective measurement; a charged quantum system has effective mass and charge density distributing in space, proportional to the square of the absolute value of its wave function. Then in de Broglie-Bohm theory both Ψ-field and Bohmian particle will have charge density distribution for a charged quantum system. This will result in the existence of an electrostatic self-interaction of the field and an electromagnetic interaction between the field and Bohmian particle, which not only violates the superposition principle of quantum mechanics but also contradicts experimental observations. Therefore, de Broglie-Bohm theory as a realistic interpretation of quantum mechanics is problematic according to protective measurement. Lastly, we briefly discuss the possibility that the wave function is not a physical field but a description of some sort of ergodic motion (e.g. random discontinuous motion) of particles.

Key words: wave function; de Broglie-Bohm theory; protective measurement; Ψ-field; mass and charge density; ergodic motion of particles

1. Introduction

De Broglie-Bohm theory is an ontological interpretation of quantum mechanics initially proposed by de Broglie and later developed by Bohm (de Broglie 1928; Bohm 1952)[1]. According to the theory, a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. Although the de Broglie-Bohm theory is mathematically equivalent to standard quantum theory, there is no clear consensus with regard to its physical interpretation. In particular, the interpretation of the wave function in this theory is still in hot debate even today. The wave function is generally taken as an objective physical field called Ψ-field[2]. As stressed by Bell (1981): "No one can understand this theory until he is willing to think of Ψ as a real objective field rather than just a probability amplitude". However, there are various views on exactly what field the wave function is. It has been regarded as a field similar to electromagnetic field (Bohm 1952), an active information field

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[1] Among other differences, de Broglie's dynamics is first order while Bohm's dynamics is second order.

[2] It should be pointed out that the wave function is also regarded by some authors as nomological, e.g. a component of physical law rather than of the reality described by the law (Dürr, Goldstein and Zanghì 1997; Goldstein and Teufel 2001). We will not discuss this view in this paper. But it might be worth noting that this non-field view may have serious drawbacks when considering the contingency of the wave function (see, e.g. Valentini 2009), and the results obtained in this paper seemingly disfavor this view too.

(Bohm and Hiley 1993), a field carrying energy and momentum (Holland 1993), and a causal agent more abstract than ordinary fields (Valentini 1997) etc[3].

In this paper, we will examine the validity of the field interpretation of the wave function in de Broglie-Bohm theory in terms of protective measurement (Aharonov, Anandan and Vaidman 1993; Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1996). It has been argued that the time averages of Bohmian particle's positions typically differ markedly from the ensemble averages, and this result based on weak measurement and protective measurement raises some objections to the reality of Bohmian particles (Englert, Scully, Süssmann and Walther 1992; Aharonov and Vaidman 1996; Aharonov, Englert and Scully 1999; Aharonov, Erez and Scully 2004). On the other hand, it seems that these objections can be answered by noticing that protective measurement is in fact a way of measuring the effect of the Ψ-field rather than that of the Bohmian particle (see, e.g. Drezet 2006). However, our new analysis will show that this answer cannot really save the de Broglie-Bohm theory from the "attack" of protective measurement; on the contrary, protective measurement will pose a more serious "threat" to the reality of the Ψ-field in the theory.

The plan of this paper is as follows. First, we will argue that there are good reasons to think, and in particular, protective measurement already implies that a quantum system with mass *m* and charge *Q*, which is described by the wave function $\psi(x,t)$, has effective mass and charge density distributions $m|\psi(x,t)|^2$ and $Q|\psi(x,t)|^2$ in space respectively. Then we investigate the implications of this result for de Broglie-Bohm theory. To begin with, taking the wave function as a Ψ-field will lead to the existence of an electrostatic self-interaction of the field for a charged quantum system. This not only violates the superposition principle of quantum mechanics but also contradicts experimental observations. Secondly, there will also exist an electromagnetic interaction between the field and the Bohmian particle, as they all have charge density distribution in space for a charged quantum system. This contradicts the predictions of quantum mechanics and experimental observations too. These results indicate that the field interpretation of the wave function cannot be right, and thus the de Broglie-Bohm theory, which takes the wave function as a Ψ-field, is problematic. Lastly, we briefly discuss the possibility that the wave function is not a physical field but a description of some sort of ergodic motion (e.g. random discontinuous motion) of particles.

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[3] Note that there is a common objection to the field interpretation of the wave function, which claims that the wave function can hardly be considered as a real physical field because it is a function on configuration space, not on physical space (see, e.g. Monton 2002, 2006). However, this common objection is not conclusive, and one can still insist on the reality of the wave function living on configuration space by some metaphysical arguments (see, e.g. Albert 1996; Lewis 2004; Wallace and Timpson 2009). Different from the common objection, I will in this paper propose a more serious objection to the field interpretation, according to which even for a single quantum system the wave function living in real space cannot be taken as a physical field either. Moreover, the reason is not metaphysical but physical, i.e. that the field interpretation contradicts both quantum mechanics and experimental observations.

2. How do mass and charge distribute for a single quantum system?

The mass and charge of a charged classical system always localize in a definite position in space at each moment. For a charged quantum system described by the wave function $\psi(x,t)$, how do its mass and charge distribute in space then? We can measure the total mass and charge of the quantum system and find them in some region of space. Thus the mass and charge of a quantum system must also exist in space with a certain distributions if assuming a realistic view. Although the mass and charge distributions of a single quantum system seem meaningless according to the orthodox probability interpretation of the wave function, it should have a physical meaning in a realistic interpretation of the wave function such as de Broglie-Bohm theory[4].

As we think, the Schrödinger equation of a charged quantum system under an external electromagnetic potential already provides an important clue. The equation is

$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\left(\nabla-\frac{iQ}{\hbar c}A\right)^2+Q\varphi+V\right]\psi(x,t) \tag{1}$$

where m and Q is respectively the mass and charge of the system, φ and A are the electromagnetic potential, V is an external potential, $\hbar$ is Planck's constant divided by 2π, c is the speed of light. The electrostatic interaction term $Q\varphi\psi(x,t)$ in the equation seems to indicate that the charge of the quantum system distributes throughout the whole region where its wave function $\psi(x,t)$ is not zero. If the charge does not distribute in some regions where the wave function is nonzero, then there will not exist any electrostatic interaction and the electrostatic interaction term will also disappear there. But the term $Q\varphi\psi(x,t)$ exists in all regions where the wave function is nonzero. Thus it seems that the charge of a charged quantum system should distribute throughout the whole region where its wave function is not zero. Furthermore, since the integral $\int_{-\infty}^{+\infty}Q|\psi(x,t)|^2dx$ is the total charge of the system, the charge density distribution in space will be $Q|\psi(x,t)|^2$. Similarly, the mass density can be obtained from the Schrödinger equation of a quantum system with mass m under an external gravitational potential V_G:

[4] Unfortunately it seems that the orthodox probability interpretation of the wave function still influences people's mind even if they already accept a realistic interpretation of the wave function. One obvious example is that few people admit that the realistic wave function has energy density (Holland (1993) is a notable exception). If the wave function has no energy, then it seems very difficult to regard it as physically real. Even if Bohm and Hiley (1993) interpreted the Ψ-field as "active information", they also admitted that the field has energy, though very little. Once one admits that the wave function has energy density, then it seems natural to endow it with mass and charge density, which are two common sources of energy density.

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + mV_G + V \right]\psi(x,t) \quad (2)$$

The gravitational interaction term $mV_G\psi(x,t)$ in the equation also indicates that the (passive gravitational) mass of the quantum system distributes throughout the whole region where its wave function $\psi(x,t)$ is not zero, and the mass density distribution in space is $m|\psi(x,t)|^2$.

The above result can be more readily understood when the wave function is a complete realistic description of a single quantum system as in dynamical collapse theories. If the mass and charge of a quantum system does not distribute as above in terms of its wave function $\psi(x,t)$, then other supplement quantities will be needed to describe the mass and charge distributions of the system in space, while this obviously contradicts the premise that the wave function is a complete description. In fact, the dynamical collapse theories such as GRW theory already admit the existence of mass density (Ghirardi, Grassi and Benatti 1995).

In addition, even in de Broglie-Bohm theory, which takes the wave function as an incomplete description and admits supplement hidden variables (i.e. the trajectories of Bohmian particles accompanying the wave function), there are also some arguments for the above mass and charge density explanation (Holland 1993; Brown, Dewdney and Horton 1995). It was argued that since the Ψ-field depends on the parameters such as mass and charge, it may be said to be massive and charged (Holland 1993, p.79). In addition, Brown, Dewdney and Horton (1995), by examining a series of effects in neutron interferometry, argued that properties sometimes attributed to the "particle" aspect of a neutron, e.g., mass and magnetic moment, cannot straightforwardly be regarded as localized at the hypothetical position of the particle in Bohm's theory. They also argued that it is hard to understand how the Aharonov-Bohm effect is possible if that the charge of the electron which couples with the electromagnetic vector-potential is not co-present in the regions on all sides of the confined magnetic field accessible to the electron (Brown, Dewdney and Horton 1995, p.332).

One may object that de Broglie-Bohm theory seemingly never admits the above mass density explanation, and no existing interpretation of quantum mechanics including dynamical collapse theories endows charge density to the wave function either. As we think, however, protective measurement provides a more convincing argument for the existence of mass and charge density distributions[5]. The wave function of a single quantum system, especially its mass and charge density, can be directly measured by protective measurement. Therefore, a realistic interpretation of quantum mechanics should admit the existence of mass and charge density in some way; if it cannot, then it will be at least problematic concerning its interpretation of the wave function.

3. Protective measurement and its answer

In this section, we will give a brief introduction of protective measurement and its

[5] It is very strange for the author that most supporters of a realistic interpretation of quantum mechanics ignore protective measurement and its implications. Admittedly there have been some controversies about the meaning of protective measurement, but the debate mainly centers on the reality of the wave function. If one insists on a realistic interpretation of quantum mechanics such as de Broglie-Bohm theory, then the debate will be mostly irrelevant and protective measurement will have strict restrictions on the realistic interpretation.

implication for the existence of mass and charge density distributions. Different from the conventional measurement, protective measurement aims at measuring the wave function of a single quantum system by repeated measurements that do not destroy its state. The general method is to let the measured system be in a non-degenerate eigenstate of the whole Hamiltonian using a suitable interaction, and then make the measurement adiabatically so that the wave function of the system neither changes nor becomes entangled with the measuring device appreciably. The suitable interaction is called the protection.

As a typical example of protective measurement (Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996), consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. The protection is natural for this situation, and no additional protective interaction is needed. The interaction Hamiltonian for measuring the value of an observable A in the state is:

$$H_I = g(t)PA \tag{3}$$

where P denotes the momentum of the pointer of the measuring device, which initial state is taken to be a Gaussian wave packet centered around zero. The time-dependent coupling $g(t)$ is normalized to $\int_0^T g(t)dt = 1$, where T is the total measuring time. In conventional von Neumann measurements, the interaction H_I is of short duration and so strong that it dominates the rest of the Hamiltonian (i.e. the effect of the free Hamiltonians of the measuring device and the system can be neglected). As a result, the time evolution $\exp(-iPA/\hbar)$ will lead to an entangled state: eigenstates of A with eigenvalues a_i are entangled with measuring device states in which the pointer is shifted by these values a_i. Due to the collapse of the wave function, the measurement result can only be one of the eigenvalues of observable A, say a_i, with a certain probability p_i. The expectation value of A is then obtained as the statistical average of eigenvalues for an ensemble of identical systems, namely $<A>=\sum_i p_i a_i$. By contrast, protective measurements are extremely slow measurements. We let $g(t)=1/T$ for most of the time T and assume that $g(t)$ goes to zero gradually before and after the period T. In the limit $T \to \infty$, we can obtain an adiabatic process in which the system cannot make a transition from one energy eigenstate to another, and the interaction Hamiltonian does not change the energy eigenstate. As a result, the corresponding time evolution $\exp(-iP<A>/\hbar)$ shifts the pointer by the expectation value $<A>$. This result strongly contrasts with the conventional measurement in which the pointer shifts by one of the eigenvalues of *A*.

It should be stressed that $T \to \infty$ is only an ideal situation[6], and a protective measurement can never be performed on a single quantum system with absolute certainty because of the tiny unavoidable entanglement (see also Dass and Qureshi 1999)[7]. For example, for any given values of *P* and *T*, the energy shift of the above eigenstate, given by first-order perturbation theory, is

$$\delta E = <H_I> = \frac{<A>P}{T} \tag{4}$$

Correspondingly, we can only obtain the exact expectation value $<A>$ with a probability very close to one, and the measurement result can also be the expectation value $<A>_\perp$, with a probability proportional to $1/T^2$, where $\perp$ refers to the normalized state in the subspace normal to the initial state $\psi(x)$ as picked out by first-order perturbation theory (Dass and Qureshi 1999). Therefore, an ensemble, which may be considerably small, is still needed for protective measurements.

Although a protective measurement can never be performed on a single quantum system with absolute certainty, the measurement is distinct from the standard one: in no stage of the measurement we obtain the eigenvalues of the measured variable. Each system in the small ensemble contributes the shift of the pointer proportional not to one of the eigenvalues, but to the expectation value. This essential novel point has been repeatedly stressed by the inventors of protective measurement (see, e.g. Aharonov, Anandan and Vaidman 1996). As we know, in the orthodox interpretation of quantum mechanics, the expectation values of variables are not considered as physical properties of a single system, as only one of the eigenvalues is observed in the outcome of the standard measuring procedure and the expectation value can only be defined as a statistical average of the eigenvalues. However, for protective measurements, we obtain the expectation value directly for a single system and not as a statistical average of eigenvalues for an ensemble. Since the expectation value of a variable can be directly measured for a single system, it must be a physical characteristic of a single system, not of an ensemble (e.g. as a statistical average of eigenvalues). This is a definite conclusion we can reach by the analysis of protective measurement.

In the following we will show that the mass and charge density can be measured by protective measurement as expectation values of certain variable for a single quantum system (Aharonov and Vaidman 1993). Consider again a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. The interaction Hamiltonian for measuring the value of an observable A_n in the state assumes the same form as Eq. (3):

$$H_I = g(t)PA_n \tag{5}$$

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[6] Note that the spreading of the wave packet of the pointer also puts a limit on the time of the interaction (Dass and Qureshi 1999).

[7] It can be argued that only observables that commute with the system's Hamiltonian can be protectively measured with absolute certainty for a single system (see e.g. Rovelli 1994; Uffink 1999).

where A_n is a normalized projection operator on small regions V_n having volume v_n, which can be written as follows:

$$A_n = \begin{cases} \frac{1}{v_n}, & x \in V_n \\ 0, & x \notin V_n \end{cases} \tag{6}$$

Then a protective measurement of A_n will yield the following result:

$$\langle A_n \rangle = \frac{1}{v_n} \int_{v_n} |\psi(x)|^2 dv = |\psi_n|^2 \tag{7}$$

It is the average of the density $|\psi(x)|^2$ over the small region V_n. When $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can find the whole density distribution $|\psi(x)|^2$. For a charged system with charge Q the density $|\psi(x)|^2$ times the charge yields the effective charge density $Q|\psi(x)|^2$. In particular, an appropriate adiabatic measurement of the Gauss flux of the electric field coming out of a certain region will yield the value of the total charge inside this region, namely the integral of the effective charge density $Q|\psi(x)|^2$ over this region (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1996). Similarly, we can measure the effective mass density of the system in principle by an appropriate adiabatic measurement of the flux of its gravitational field.

It can be shown that protective measurements can not only measure the nondegenerate energy eigenstates of a single quantum system, but also measure its time-dependent quantum states via Zeno effect by frequent conventional measurements in principle (Aharonov and Vaidman 1993). Thus the above results hold true for any given wave function. This provides a strong argument for associating physical reality with the wave function of a single quantum system. Although one may still object to this association, the objection will be irrelevant in a realistic interpretation of the wave function such as de Broglie-Bohm theory. Therefore, we can always test the realistic interpretations of quantum mechanics by the above results of protective measurement, which show that the mass and charge of a single quantum system described by the realistic wave function $\psi(x)$ is distributed throughout space with effective mass density $m|\psi(x)|^2$ and effective charge density $Q|\psi(x)|^2$ respectively.

4. Implications for de Broglie-Bohm theory

Now we will investigate the implications of the existence of mass and charge density for de Broglie-Bohm theory. For the sake of simplicity, we will restrict our discussions to the wave function of a single quantum system. The conclusion can be readily extended to many-body systems.

It has been argued that de Broglie-Bohm theory gives the precisely same predictions as quantum mechanics by means of quantum equilibrium hypothesis. Concretely speaking, the quantum equilibrium hypothesis provides the initial conditions for the guidance equation which make de Broglie-Bohm theory obey Born's rule in terms of position distributions. Moreover, since all measurements can be finally expressed in terms of position, e.g. pointer positions, this amounts to full accordance with all predictions of quantum mechanics. However, this equivalence is based on the premise that the wave function, regarded as a Ψ-field, has no mass and charge density distributions. If the wave function has mass and charge density distributions as protective measurement implies, then, as we will argue below, taking it as a Ψ-field will lead to some predictions (e.g. the existence of electrostatic self-interaction) that contradict both quantum mechanics and experimental observations.

If the wave function is a physical field such as a Ψ-field, then its mass and charge density will be *simultaneously* distributed in space. This has two disaster results at least. One is that charge will not be quantized; the total charge inside a very small region can be much smaller than an elementary charge for a single quantum system. This obviously contradicts the common expectation that charge should be quantized. But maybe our expectation needs to be revised. So this result is not fatal for the field interpretation of the wave function. The other is that the wave function will not satisfy the superposition principle of quantum mechanics. For example, for the wave function of a single electron, different spatial parts of the wave function will have gravitational and electrostatic interactions, as these parts have mass and charge simultaneously.

Let's analyze the second result in more detail. Interestingly, the so-called Schrödinger-Newton equation, which was proposed for other purposes (Diosi 1984; Penrose 1998), just describes the gravitational self-interaction of the wave function. The equation for a single quantum system can be written as

$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial^2 x}-Gm^2\int\frac{|\psi(x',t)|^2}{|x-x'|}d^3x'\,\psi(x,t)+V\psi(x,t) \qquad (8)$$

where m is the mass of the quantum system, V is an external potential, and G is Newton's gravitational constant. Much work has been done to study the mathematical properties of this interesting equation (see, e.g. Harrison, Moroz and Tod 2003; Moroz and Tod 1999; Salzman 2005). Some experimental schemes have been also proposed to test its physical validity (Salzman and Carlip 2006). As we will see, although such gravitational self-interactions cannot yet be excluded by experiments[8], the existence of electrostatic self-interaction already contradicts experimental observations.

If there is also an electrostatic self-interaction, then the equation for a free quantum system

[8] It has been argued that the existence of a self-interaction term in the Schrödinger-Newton equation does not have a consistent Born rule interpretation (Adler 2007). The reason is that the probability of simultaneously finding a particle in different positions is zero. However, in a realistic interpretation of quantum mechanics where the wave function is regarded as a real physical entity rather than as a probability amplitude, the existence of gravitational self-interaction term seems quite natural. For example, the field interpretation can be consistent with conventional quantum measurement via a dynamical collapse process. As we think, one convincing objection is that if there is a self-gravitational interaction for the wave function of a charged particle, then there will also exist an electrostatic self-interaction because the charge density always accompanies the mass density, while the existence of electrostatic self-interaction is already inconsistent with experimental observations (see below). If this objection is valid, then the Schrödinger-Newton equation will be wrong even as an approximation, and moreover, the approach of semiclassical gravity will also be excluded (cf. Salzman and Carlip 2006).

with mass *m* and charge *Q* will be

$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial^2 x}+(kQ^2-Gm^2)\int\frac{|\psi(x',t)|^2}{|x-x'|}d^3x'\psi(x,t) \qquad (9)$$

where *k* is the Coulomb constant. Note that the gravitational self-interaction is an attractive force, while the electrostatic self-interaction is a repulsive force. It has been shown that the measure of the potential strength of a gravitational self-interaction is $\varepsilon^2=\left(\frac{4Gm^2}{\hbar c}\right)^2$ for a free particle with mass *m* (Salzman 2005). This quantity represents the strength of the influence of self-interaction on the normal evolution of the wave function; when $\varepsilon^2\approx 1$ the influence is significant. Similarly, for a free charged particle with charge *Q*, the measure of the potential strength of the electrostatic self-interaction is $\varepsilon^2=\left(\frac{4kQ^2}{\hbar c}\right)^2$. As a typical example, for a free electron with charge *e*, the potential strength of the electrostatic self-interaction will be $\varepsilon^2=\left(\frac{4ke^2}{\hbar c}\right)^2\approx 1\times 10^{-3}$. This indicates that the electrostatic self-interaction will have significant influence on the evolution of the wave function of a free electron. If such an interaction indeed exists, it should have been detected by precise experiments on charged microscopic particles. As another example, consider the electron in the hydrogen atom. Since the potential of its electrostatic self-interaction is of the same order as the Coulomb potential produced by the nucleus, the energy levels of hydrogen atoms will be significantly different from those predicted by quantum mechanics and confirmed by experimental observations.

Therefore, taking the wave function as a Ψ-field will lead to the existence of electrostatic self-interaction that contradicts both quantum mechanics and experimental observations[9]. Moreover, de Broglie-Bohm theory makes the situation worse by adding the Bohmian particles. Inasmuch as the wave function has charge density distribution in space for a charged quantum system, there will exist an electromagnetic interaction between it and the Bohmian particles. This is inconsistent with the predictions of quantum mechanics and experimental observations either.

Certainly, one can eliminate the electromagnetic interaction between the Ψ-field and Bohmian particles by depriving the Bohmian particles of mass and charge. But they will be not real particles any more. Then in what sense the de Broglie-Bohm theory provides a realistic interpretation of quantum mechanics? One may also want to deprive the Ψ-field of mass and charge density to eliminate the electrostatic self-interaction. But, on the one hand, the theory will break its physical connection with quantum mechanics, as the wave function in quantum

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[9] One may object to the argument here with the example of classical electromagnetic field. Electromagnetic field is a field, but it has no self-interaction. Thus a field does not require the existence of self-interaction. However, this is a common misunderstanding. The crux of the matter is that the non-existence of electromagnetic self-interaction results from the fact that electromagnetic field itself has no charge. If the electromagnetic field had charge, then there would also exist electromagnetic self-interaction due to the nature of field, namely the simultaneous existence of its properties in space. In fact, although electromagnetic field has no electromagnetic self-interaction, it does have gravitational self-interaction; the simultaneous existence of energy densities in different spatial locations for an electromagnetic field must generate a gravitational interaction, though the interaction is too weak to be detected by current technology.

mechanics has mass and charge density according to our analysis, and on the other hand, since protective measurement can measure the mass and charge density for a single quantum system, the theory will be unable to explain the measurement results either[10]. Although de Broglie-Bohm theory can still exist in this way as a mathematical tool for experimental predictions (somewhat like the orthodox interpretation it tries to replace), it obviously departs from the initial expectations of de Broglie and Bohm, and as we think, it already fails as a physical theory because of losing its explanation ability.

To sum up, de Broglie-Bohm theory cannot accommodate the result that the wave function has mass and charge density distributions, which is implied by protective measurement. If the wave function, regarded as a Ψ-field, has charge density distribution in space for a charged quantum system, then there will exist an electrostatic self-interaction of the Ψ-field and an electromagnetic interaction between the field and Bohmian particle. This not only violates the superposition principle of quantum mechanics but also contradicts experimental observations. Therefore, de Broglie-Bohm theory as a realistic interpretation of quantum mechanics is problematic according to protective measurement.

5. Further discussions

If the wave function is not a description of physical field as de Broglie-Bohm theory assumes, then exactly what does the wave function describe? There is already an important clue. It is that the superposition principle in quantum mechanics permits no existence of the self-interaction of the wave function in real space for a single quantum system. This indicates that the mass and charge density do not exist in different regions *simultaneously*. How is this possible? It naturally leads us to the second view that takes the wave function as a description of some kind of ergodic motion of a particle. On this view, the effective mass and charge density are formed by time average of the motion of a charged particle, and they distribute in different locations at different moments. In other words, the mass and charge density exists in a time division way (by contraries a field exists throughout space simultaneously). At any instant, there is only a localized particle with mass and charge. Thus there will not exist any self-interaction for the wave function.

There are indeed some realistic interpretations of quantum mechanics that attempt to explain the wave function in terms of some sort of ergodic motion of particles. A well-known example is the stochastic interpretation of quantum mechanics (e.g. Nelson 1966). Nelson (1966) derived the Schrödinger equation from Newtonian mechanics via the hypothesis that every particle of mass *m* is subject to a Brownian motion with diffusion coefficient $\hbar/2m$ and no friction. In more technical terms, the quantum mechanical process is claimed to be equivalent to a classical Markovian diffusion process. On this interpretation, particles have continuous trajectories but no velocities, and the wave function is a statistical average description of their motion. However, it has been pointed out that the classical stochastic interpretations are inconsistent with quantum mechanics (Glabert, Hänggi and Talkner 1979; Wallstrom 1994). Glabert, Hänggi and Talkner (1979) argued that the Schrödinger equation is not equivalent to a Markovian process, and the various correlation functions used in quantum mechanics do not have the properties of the correlations of a classical stochastic process. Wallstrom (1994) further showed that one must add

[10] One cannot simply regard the results of protective measurement of mass and charge density as meaningless. These results are proportional to the module square of the wave function of a single quantum system at *every* location of space.

by hand a quantization condition, as in the old quantum theory, in order to recover the Schrödinger equation, and thus the Schrödinger equation and the Madelung hydrodynamic equations are not equivalent. In fact, Nelson (2005) also showed that there is an empirical difference between the predictions of quantum mechanics and his stochastic mechanics when considering quantum entanglement and nonlocality.

In addition, it has been generally argued that the classical ergodic models that assume continuous motion cannot be consistent with quantum mechanics (Aharonov and Vaidman 1993; Gao 2010). Classical ergodic models are plagued by the problems of infinite velocity and accelerating radiation (Aharonov and Vaidman 1993). In particular, a particle undergoing continuous motion, even if it has infinite velocity, cannot move throughout two spatially separated regions where the wave function of the particle may spread. Besides, the classical ergodic models entail the existence of a finite ergodic time, which is also inconsistent with the existing quantum theory (Gao 2010).

Based on these negative results, it has been suggested that the wave function may describe random discontinuous motion of particles (Gao 2006a, 2006b, 2010). This new interpretation of the wave function can avoid the problems of classical ergodic models, and it also provides a natural realistic alternative to the orthodox view. On this interpretation, the square of the absolute value of the wave function not merely gives the probability of the particle *being found* in certain locations, but also gives the objective probability of the particle *being* there. Moreover, it seems that the theory of random discontinuous motion can also provide a promising solution to the notorious quantum measurement problem (Gao 2006a, 2006b). However, the theory is still at its preliminary stage, and much study is still needed before a definite conclusion can be reached about the true meaning of the wave function.

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