Protective Measurement and the Meaning of the Wave Function

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Abstract

This article analyzes the implications of protective measurement for the meaning of the wave function. According to protective measurement, a charged quantum system has mass and charge density proportional to the modulus square of its wave function. It is shown that the mass and charge density is not real but effective, formed by the ergodic motion of a localized particle with the total mass and charge of the system. Moreover, it is argued that the ergodic motion is not continuous but discontinuous and random. This result suggests a new interpretation of the wave function, according to which the wave function is a description of random discontinuous motion of particles, and the modulus square of the wave function gives the probability density of the particles *being* in certain locations. It is shown that the suggested interpretation of the wave function disfavors the de Broglie-Bohm theory and the many-worlds interpretation but favors the dynamical collapse theories, and the random discontinuous motion of particles may provide an appropriate random source to collapse the wave function.

Contents

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It has even been doubted whether what goes on in an atom can be described within a scheme of space and time. From a philosophical standpoint, I should consider a conclusive decision in this sense as equivalent to a complete surrender. For we cannot really avoid our thinking in terms of space and time, and what we cannot comprehend within it, we cannot comprehend at all (Schrödinger 1926)

The wavefunction gives not the density of stuff, but gives rather (on squaring its modulus) the density of probability. Probability of what exactly? Not of the electron being there, but of the electron being found there, if its position is measured. Why this aversion to being and insistence on finding? The founding fathers were unable to form a clear picture of things on the remote atomic scale (Bell 1990)

1 Introduction

Quantum mechanics, on its Schrödinger picture, is a non-relativistic theory about the wave function and its evolution. The first interpretive problem is the physical meaning of the wave function. Notwithstanding more than eighty years' developments of the theory, this is still an unsolved issue. It has been widely argued that Born's probability interpretation is not wholly satisfactory because of resorting to the vague concept - measurement (Bell 1990), though it is still the standard interpretation in textbooks nowadays. On the other hand, the meaning of the wave function is also in dispute in the alternative interpretations of quantum mechanics such as the de Broglie-Bohm theory and the many-worlds interpretation (de Broglie 1928; Bohm

1952; Everett 1957). In view of this unsatisfactory situation, it seems that we need a new starting point to solve the fundamental interpretive problem of quantum mechanics.

The meaning of the wave function in quantum mechanics is often analyzed in the context of conventional impulse measurements. Even though the wave function of a quantum system is in general extended over space, an ideal impulse position measurement can only detect the system in a random position in space. Thus it seems natural to assume that the wave function is only related to the probability of these random measurement results as in the standard probability interpretation. However, it has been realized that impulse measurement is only one kind of quantum measurement, for which the coupling interaction between the measured system and the measuring device is of short duration and strong. There also exist other kinds of measurements in quantum mechanics, one of which is the protective measurement (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996). Protective measurement also uses a standard measuring procedure, but with a weak and long duration coupling interaction. Besides, it adds an appropriate procedure to protect the measured wave function from being changed (in some situations the protection is provided by the measured system itself). These differences permit protective measurement to be able to gain more information about the measured quantum system and its wave function, and thus it might unveil more physical content of the wave function. In this paper, we will analyze the possible implications of protective measurement for the meaning of the wave function.

The plan of this paper is as follows. In Section 2, we first introduce the principle of protective measurement. It is stressed that protective measurement can measure the expectation values of observables for a single quantum system, and these expectation values are the physical properties of the system, not the properties of an ensemble of identical systems. Section 3 then analyzes a typical example of such properties, the mass and charge density. According to protective measurement, the mass and the charge of a charged quantum system are distributed throughout space with density proportional to the modulus square of its wave function. In Section 4, the physical origin of the mass and charge density is investigated. It is shown that the mass and charge density of a quantum system is not real but effective, and it is formed by the time average of the ergodic motion of a localized particle with the total mass and charge of the system. Moreover, it is argued that the classical ergodic models, which assume continuous motion of particles, have serious drawbacks and can hardly be consistent with quantum mechanics and experiments. Based on this negative result, we suggest that the effective mass and charge density of a quantum system is formed by random discontinuous motion of a localized particle with mass and charge. Section 5 gives a detailed analysis of the random discontinuous motion of particles.

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It is argued that the wave function on configuration space can be interpreted as a description of the random discontinuous motion of particles in the real three-dimensional space. On this interpretation, the modulus square of the wave function not only gives the probability density of the particles *being found* in certain locations as the probability interpretation holds, but also gives the objective probability density of the particles *being* there. In Section 6, we analyze the possible implications of the suggested interpretation of the wave function for the solution to the measurement problem. It is argued that the de Broglie-Bohm theory and the many-worlds interpretation are inconsistent with the picture of random discontinuous motion of particles, and the collapse of the wave function is probably a real physical process. It is further guessed that the wavefunction collapse may result from the random discontinuous motion of particles. Conclusions are given in the last section.

2 Protective measurements

According to standard quantum mechanics, if a physical system is not in an eigenstate of the measured observable, then its state will collapse to one of the eigenstates after a conventional impulse measurement, and moreover, the result can only be the eigenvalue corresponding to the collapsed eigenstate. One way to prevent the collapse is to make the coupling with the measuring device sufficiently weak so that the change of the measured state due to the measurement can be neglected. This is the idea of weak measurements (Aharonov, Albert and Vaidman 1988). However, although the state is not changed appreciably by a weak measurement, the pointer of the measuring device hardly moves either, and in particular, its shift due to the measurement is much smaller than its position uncertainty, and thus little information can be obtained from individual measurements.

A possible way to remedy the weakness of weak measurements is to increase the time of the coupling between the measured system and the measuring device. If the state is almost constant during the measurement, the total shift of the pointer, which is proportional to the duration of the interaction, will be large enough to be identified. However, under normal circumstances the state of the system is not constant during the measurement, and the weak coupling also leads to a small rate of change of the state. As a result, the reading of the measuring device will correspond not to the state which the system had prior to the measurement, but to some time average depending on the evolution of the state influenced by the measuring procedure. Therefore, in order to be able to measure the state of a single system, we need, in addition to the standard weak and long-duration measuring interaction, a procedure which can protect the state from changing during the measuring interaction. A general method is to let the measured

system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction, and then make the measurement adiabatically so that the state of the system neither changes nor becomes entangled with the measuring device appreciably. In the following, we will introduce this principle of protective measurement in more detail (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996; Vaidman 2008)[1].

2.1 Measurements with natural protection

As a typical example of protective measurement, we consider a quantum system in a discrete nondegenerate energy eigenstate $|E_n\rangle$. In this case, the system itself supplies the protection of the state due to energy conservation and no artificial protection is needed.

The interaction Hamiltonian for a protective measurement of an observable A in this state involves the same interaction Hamiltonian as the standard measuring procedure:

$$H_I = g(t)PA, \tag{1}$$

where P is the momentum conjugate to the pointer variable X of an appropriate measuring device. Let the initial state of the pointer at $t = 0$ be $|\phi(x_0)\rangle$, which is a Gaussian wave packet of eigenstates of X with width w_0, centered around the eigenvalue x_0. The time-dependent coupling strength $g(t)$ is also a smooth function normalized to $\int dt g(t) = 1$. But different from conventional impulse measurements, where the interaction is very strong and almost instantaneous, protective measurements make use of the opposite limit where the interaction of the measuring device with the system is weak and adiabatic, and thus the free Hamiltonians cannot be neglected. Let the Hamiltonian of the combined system be

$$H(t) = H_S + H_D + g(t)PA, \tag{2}$$

where H_S and H_D are the Hamiltonians of the measured system and the measuring device, respectively. The interaction lasts for a long time T, and $g(t)$ is very small and constant for the most part, and it goes to zero gradually before and after the interaction.

The state of the combined system after T is given by

$$|t = T\rangle = e^{-\frac{i}{\hbar}\int_0^T H(t)dt} |E_n\rangle |\phi(x_0)\rangle. \tag{3}$$

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[1]For a review of earlier objections to the validity and meaning of protective measurements and the answers to them see Aharonov, Anandan and Vaidman (1996), Dass and Qureshi (1999) and Vaidman (2009).

By ignoring the switching on and switching off processes[2], the full Hamiltonian (with $g(t) = 1/T$) is time-independent and no time-ordering is needed. Then we obtain

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} |E_n\rangle |\phi(x_0)\rangle, \tag{4}$$

where $H = H_S + H_D + \frac{PA}{T}$. We further expand $|\phi(x_0)\rangle$ in the eigenstate of H_D, $\left|E_j^d\right\rangle$, and write

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} \sum_j d_j |E_n\rangle \left|E_j^d\right\rangle, \tag{5}$$

Let the exact eigenstates of H be $|\Psi_{k,m}\rangle$ and the corresponding eigenvalues be $E(k, m)$, we have

$$|t = T\rangle = \sum_j d_j \sum_{k,m} e^{-\frac{i}{\hbar}E(k,m)T} \langle \Psi_{k,m}|E_n, E_j^d\rangle |\Psi_{k,m}\rangle. \tag{6}$$

Since the interaction is very weak, the Hamiltonian H of Eq.(2) can be thought of as $H_0 = H_S + H_D$ perturbed by $\frac{PA}{T}$. Using the fact that $\frac{PA}{T}$ is a small perturbation and that the eigenstates of H_0 are of the form $|E_k\rangle \left|E_m^d\right\rangle$, the perturbation theory gives

$$\begin{aligned} |\Psi_{k,m}\rangle &= |E_k\rangle \left|E_m^d\right\rangle + O(1/T), \\ E(k,m) &= E_k + E_m^d + \frac{1}{T}\langle A\rangle_k \langle P\rangle_m + O(1/T^2). \end{aligned} \tag{7}$$

Note that it is a necessary condition for Eq.(7) to hold that $|E_k\rangle$ is a non-degenerate eigenstate of H_S. Substituting Eq.(7) in Eq.(6) and taking the large T limit yields

$$|t = T\rangle \approx \sum_j e^{-\frac{i}{\hbar}(E_n T + E_j^d T + \langle A\rangle_n \langle P\rangle_j)} d_j |E_n\rangle \left|E_j^d\right\rangle. \tag{8}$$

For the special case when P commutes with the free Hamiltonian of the device, i.e., $[P, H_D] = 0$, the eigenstates $\left|E_j^d\right\rangle$ of H_D are also the eigenstates of P, and thus the above equation can be rewritten as

$$|t = T\rangle \approx e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T - \frac{i}{\hbar}\langle A\rangle_n P} |E_n\rangle |\phi(x_0)\rangle. \tag{9}$$

It can be seen that the third term in the exponent will shift the center of the pointer $|\phi(x_0)\rangle$ by an amount $\langle A\rangle_n$:

$$|t = T\rangle \approx e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T} |E_n\rangle |\phi(x_0 + \langle A\rangle_n)\rangle. \tag{10}$$

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[2]The change in the total Hamiltonian during these processes is smaller than PA/T, and thus the adiabaticity of the interaction will not be violated and the approximate treatment given below is valid. For a more strict analysis see Dass and Qureshi (1999).

This shows that at the end of the interaction, the center of the pointer shifts by the expectation value of the measured observable in the measured state.

For the general case when $[P, H_D] \neq 0$ and $[A, H_S] \neq 0$, we can introduce an operator $Y = \sum_j \langle P \rangle_j \left| E_j^d \right\rangle \langle E_j^d |$ and rewrite Eq.(8) as

$$|t = T\rangle \approx e^{-\frac{i}{\hbar} E_n T - \frac{i}{\hbar} H_D T - \frac{i}{\hbar} \langle A \rangle_n Y} |E_n\rangle |\phi(x_0)\rangle . \tag{11}$$

Then by rechoosing the state of the device so that it is peaked around a value x_0' of the pointer variable X' conjugate to Y, i.e., $[X', Y] = i\hbar^3$, we can obtain

$$|t = T\rangle \approx e^{-\frac{i}{\hbar} E_n T - \frac{i}{\hbar} H_D T - \frac{i}{\hbar} \langle A \rangle_n Y} |E_n\rangle \left| \phi(x_0') \right\rangle = e^{-\frac{i}{\hbar} E_n T - \frac{i}{\hbar} H_D T} |E_n\rangle |\phi(x_0' + \langle A \rangle_n)\rangle . \tag{12}$$

Thus the center of the pointer also shifts by $\langle A \rangle_n$ at the end of the interaction. This demonstrates the generic possibility of the protective measurement of $\langle A \rangle_n$ without disturbing the measuring state $|E_n\rangle$.

It is worth noting that since the position variable of the pointer does not commute with its free Hamiltonian, the pointer wave packet will spread during the long measuring time. For example, the kinematic energy term $P^2/2M$ in the free Hamiltonian of the pointer will spread the wave packet without shifting the center, and the width of the wave packet at the end of interaction will be $w(T) = [\frac{1}{2}(w_0^2 + \frac{T^2}{M^2 w_0^2})]^{\frac{1}{2}}$ (Dass and Qureshi 1999). However, the spreading of the pointer wave packet can be made as small as possible by increasing the mass M of the pointer, and thus it will not interfere with resolving the shift of the center of the pointer in princple.

2.2 Measurements with artificial protection

Protective measurements can not only measure the discrete nondegenerate energy eigenstates of a single quantum system, which are naturally protected by energy conservation, but also measure the general quantum states by adding an artificial protection procedure in principle (Aharonov and Vaidman 1993). For this case, the measured state needs to be known beforehand in order to arrange a proper protection.

For degenerate energy eigenstates, the simplest way is to add a potential (as part of the measuring procedure) to change the energies of the other states and lift the degeneracy. Then the measured state remains unchanged, but is now protected by energy conservation like nondegenerate energy eigenstates. Although this protection does not change the state, it does change the physical situation. This change can be brought to a minimum by adding

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[3]Note that it may not always be possible to physically realize the operator Y, and an operator canonically conjugate to Y need not always exist either. For further discussions see Dass and Qureshi (1999).

strong protection potential for a dense set of very short time intervals. Then most of the time the system has not only the same state, but also the original potential.

The superposition of energy eigenstates can also be measured by a similar procedure. One can add a dense set of time-dependent potentials acting for very short periods of time such that the state at all these times is the nondegenerate eigenstate of the Hamiltonian together with the additional potential. Then most of the time the system also evolves under the original Hamiltonian. A stronger protection is needed in order to measure all details of the time-dependent state. The simplest way is via quantum Zeno effect. The frequent impulse measurements can test and protect the time evolution of the quantum state. For measurement of any desired accuracy of the state, there is a density of the impulse measurements which can protect the state from being changed due to the measuring interaction. When the time scale of intervals between consecutive protections is much smaller than the time scale of the original state evolution, the system will evolve according to its original Hamiltonian most of the time, and thus what's measured is still the property of the system and not of the protection procedure (Aharonov and Vaidman 1993).

2.3   Further discussions

According to the standard view, the expectation values of observables are not the physical properties of a single system, but the statistical properties of the ensemble of identical systems. This seems reasonable if there exist only conventional impulse measurements. An impulse measurement can only obtain one of the eigenvalues of the measured observable, and thus the expectation value can only be defined as a statistical average of the eigenvalues for an ensemble of identical systems. However, there exist other kinds of quantum measurements, and in particular, protective measurements can measure the expectation values of observables for a single system. Therefore, the expectation values of observables should be considered as the physical properties of a single quantum system, not those of an ensemble (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996)[4].

A major objection to this conclusion is that the adiabatic measuring procedure (where the measuring time T is infinite) in a protective measurement is only an ideal situation, and a realistic protective measurement can never be performed on a single quantum system with absolute certainty because of the tiny unavoidable entanglement in the final state (Dass and Qureshi 1999). For example, we can only obtain the exact expectation value $\langle A \rangle$ with a probability very close to one, and the measurement result may also

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[4]For a detailed and insightful analysis of the implications of this result for quantum realism see Dickson (1995).

be the expectation value $\langle A \rangle_\perp$ with a probability proportional to $\sim 1/T^2$, where $\perp$ refers to the normalized state in the subspace normal to the initial state as picked out by the first-order perturbation theory (Dass and Qureshi 1999). Therefore, an ensemble, which may be considerably small, is still needed for a realistic protective measurement.

Before answering this objection, it is worth pointing out that since the expectation values of observables can be measured by a realistic protective measurement for a very small ensemble of identical systems, the standard view seems already untenable, according to which a very large ensemble of identical systems is needed to determine the expectation values. In the following, we will answer this objection from several angles. First of all, one can point out that even in classical mechanics where a single system has its objective properties, a realistic measurement of the system can never be absolutely reliable. The key point here is that a protective measurement can measure the properties of a single quantum system with certainty in principle, using an adiabatic measuring procedure. Secondly, the inherent uncertainty in the realistic measurement of a single quantum system may be taken as the nature of the quantum system. Thus one can still associate physical reality with the state of a single system even though protective measurements are not absolutely reliable in realistic situations. Thirdly, one has the freedom of constructing a realistic model of the quantum world insofar as it is consistent with experience. Indeed, before the method of protective measurement was proposed there already appeared several realistic interpretations of quantum mechanics, such as the de Broglie-Bohm theory and the many-worlds interpretation (de Broglie 1928; Bohm 1952; Everett 1957).

A special rebuttal relating to protective measurements is as follows. Although a realistic protective measurement can never be performed on a single quantum system with absolute certainty, the measurement is distinct from the conventional one: at no stage of the measurement do we obtain the eigenvalues of the measured observable. Each system in the small ensemble contributes the shift of the pointer proportional not to one of the eigenvalues, but to the expectation value. This important point has been repeatedly stressed by the inventors of protective measurements (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996). Moreover, although the measured value might not be the expectation value of an observable in the measured state, which happens with an extremely low probability proportional to $\sim 1/T^2$ for large T, it is still the expectation value of the observable in the state normal to the measured state (Dass and Qureshi 1999). In any case, the measurement result is always the expectation value of an observable for the protective measurement of a single system[5]. In addition, when the state of the system

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[5]It is worth noting that the special scheme of Alter and Yamamoto (1996) can not only

is not destroyed during a series of protective measurements, the results of these measurements are always the same. When the state of the system is destroyed with a very low probability, the same result cannot be repeated indeed, but it is because the state is no longer the original one (but a new state normal to it). This also indicates that what protective measurements measure, namely the expectation values of observables, are the objective properties of the measured system.

It can be further argued that the expectation values are not the time-averaged properties of the evolution of a quantum system during a long period of time (Aharonov, Anandan and Vaidman 1996). Although a protective measurement cannot measure the expectation values at a precise instant, for an arbitrarily short period of time the measuring device always shifts by an amount proportional to the expectation value of the measured observable. Therefore, the expectation values of observables should be the instantaneous properties of a quantum system, which are defined during an infinitesimal time interval at a given instant.

3 On the mass and charge density of a quantum system

According to protective measurement, the expectation values of observables are properties of a single quantum system. One typical example of such properties is the mass and charge density of a quantum system. In this section, we will present a detailed analysis of this property, as it may have important implications for the physical meaning of the wave function.

3.1 A heuristic argument

The mass and charge of a classical system always localize in a definite position in space at each moment. For a charged quantum system described by the wave function $\psi(x, t)$, how do its mass and charge distribute in space then? We can measure the total mass and charge of the quantum system by the gravitational and electromagnetic interactions and find them in some region of space. Thus it seems that the mass and charge of a quantum system must also exist in space with a certain distribution. Before we discuss the answer given by protective measurement, we will first give a heuristic argument.

The Schrödinger equation of a charged quantum system under an external electromagnetic potential may provide a clue to the answer. The

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measure the expectation values of observables in a single squeezed coherent state, but also avoid entanglement exactly. Thus it provides an indirect rebuttal to the above objection and a direct support for the conclusion that the expectation values of observables are the physical properties of a single quantum system.

equation is

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = [-\frac{\hbar^2}{2m}(\nabla - \frac{iQ}{\hbar c}A)^2 + Q\varphi]\psi(x,t), \tag{13}$$

where m and Q are the mass and charge of the system, respectively, φ and A are the electromagnetic potential, and c is the speed of light. The electrostatic interaction term $Q\varphi\psi(x,t)$ in the equation indicates that the interaction exists in all regions where the wave function of the system, $\psi(x,t)$, is nonzero, and thus it seems to suggest that the charge of the system also distributes throughout these regions. If the charge does not distribute in some regions where the wave function is nonzero, then there will not exist an electrostatic interaction there. Furthermore, since the integral $\int_{-\infty}^{\infty} Q|\psi(x,t)|^2 d^3x$ is the total charge of the system, the charge density in space, if indeed exists, will be $Q|\psi(x,t)|^2$. Similarly, the mass density can be obtained from the Schrödinger equation of a quantum system under an external gravitational potential:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = [-\frac{\hbar^2}{2m}\nabla^2 + mV_G]\psi(x,t). \tag{14}$$

The gravitational interaction term $mV_G\psi(x,t)$ in the equation also suggests that the (passive gravitational) mass of the quantum system distributes throughout the whole region where its wave function $\psi(x,t)$ is nonzero, and the mass density in space is $m|\psi(x,t)|^2$.

3.2   The answer of protective measurement

Protective measurement provides a more convincing argument for the existence of mass and charge density. The mass and charge density of a single quantum system, as well as its wave function, can be measured by protective measurement as expectation values of certain observables (Aharonov and Vaidman 1993). For example, a protective measurement of the flux of the electric field of a charged quantum system out of a certain region will yield the expectation value of its charge inside this region, namely the integral of its charge density over this region. Similarly, we can also measure the mass density of a quantum system by a protective measurement of the flux of its gravitational field in principle.

Consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. We take the measured observable A_n to be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A_n = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{15}$$

11

The protective measurement of A_n then yields

$$\langle A_n \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 dv = |\psi_n|^2, \qquad (16)$$

where $|\psi_n|^2$ is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ everywhere in space.

Since the physical realization of the observable A_n and the corresponding interaction Hamiltonian must always resort to the electromagnetic or gravitational interaction between the measured system and the measuring device, what the above protective measurement measures is in fact the charge or mass density of the quantum system, and its result indicates that the mass and charge density is proportional to the modulus square of the wave function of the system, namely the density $\rho(x)$. In the following, we will give a concrete example to illustrate this important result (see also Aharonov, Anandan and Vaidman 1993).

3.3 A specific example

Consider the spatial wave function of a single quantum system with negative charge Q

$$\psi(x, t) = a\psi_1(x, t) + b\psi_2(x, t), \qquad (17)$$

where $\psi_1(x, t)$ and $\psi_2(x, t)$ are two normalized wave functions respectively localized in their ground states in two small identical boxes 1 and 2, and $|a|^2 + |b|^2 = 1$. An electron, which initial state is a small localized wave packet, is shot along a straight line near box 1 and perpendicular to the line of separation between the boxes. The electron is detected on a screen after passing by box 1. Suppose the separation between the boxes is large enough so that a charge Q in box 2 has no observable influence on the electron. Then if the system were in box 2, namely $|a|^2 = 0$, the trajectory of the electron wave packet would be a straight line as indicated by position "0" in Fig.1. By contrast, if the system were in box 1, namely $|a|^2 = 1$, the trajectory of the electron wave packet would be deviated by the electric field of the system by a maximum amount as indicated by position "1" in Fig.1.

We first suppose that $\psi(x, t)$ is unprotected, then the wave function of the combined system after interaction will be

$$\psi(x, x', t) = a\varphi_1(x', t)\psi_1(x, t) + b\varphi_2(x', t)\psi_2(x, t), \qquad (18)$$

where $\varphi_1(x', t)$ and $\varphi_2(x', t)$ are the wave functions of the electron influenced by the electric fields of the system in box 1 and box 2, respectively, the trajectory of $\varphi_1(x', t)$ is deviated by a maximum amount, and the trajectory of

$\varphi_2(x', t)$ is not deviated and still a straight line. When the electron is detected on the screen, the above wave function will collapse to $\varphi_1(x', t)\psi_1(x, t)$ or $\varphi_2(x', t)\psi_2(x, t)$. As a result, the detected position of the electron will be either "1" or "0" in Fig.1, indicating that the system is in box 1 or 2 *after* the detection. This is a conventional impulse measurement of the projection operator on the spatial region of box 1, denoted by A_1. A_1 has two eigenstates corresponding to the system being in box 1 and 2, respectively, and the corresponding eigenvalues are 1 and 0, respectively. Since the measurement is accomplished through the electrostatic interaction between two charges, the measured observable A_1, when multiplied by the charge Q, is actually the observable for the charge of the system in box 1, and its eigenvalues are Q and 0, corresponding to the charge Q being in box 1 and 2, respectively. Such a measurement cannot tell us the charge situation of the system in each box *before* the measurement.
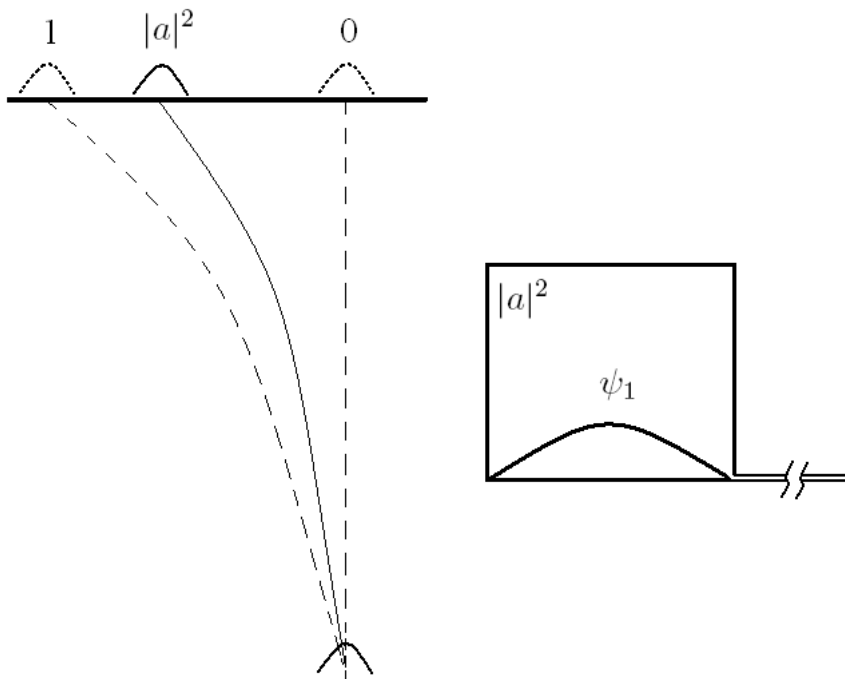

Fig.1 Scheme of a protective measurement of the charge density of a single quantum system

Now let's make a protective measurement of A_1. Since $\psi(x, t)$ is degenerate with $\psi'(x, t) = a\psi_1(x, t) - b\psi_2(x, t)$, we need an artificial protection procedure to remove the degeneracy, e.g. joining the two boxes with a long tube whose diameter is small compared to the size of the box. By this protection $\psi(x, t)$ will be a nondegenerate energy eigenstate. The adiabaticity condition and the weakly interacting condition, which are required for a

protective measurement, can be further satisfied when assuming that (1) the measuring time of the electron is long compared to $\hbar/\Delta E$, where ΔE is the smallest of the energy differences between $\psi(x,t)$ and the other energy eigenstates[6], and (2) at all times the potential energy of interaction between the electron and the system is small compared to ΔE. Then the measurement of A_1 by means of the electron trajectory is a protective measurement, and the trajectory of the electron is only influenced by the expectation value of the charge of the system in box 1. When the sizes of the electron wave packet and box 1 are small compared with the separation between them, the trajectory of the center of the electron wave packet, $\vec{r}(t)$, will in a very good approximation satisfy the following equation:

$$m_e \frac{d^2\vec{r}}{dt^2} = k\frac{e \cdot |a|^2 Q}{|\vec{r} - \vec{r_1}|(\vec{r} - \vec{r_1})}, \tag{19}$$

where m_e is the mass of electron, k is the Coulomb constant, $\vec{r_1}$ is the position of the center of box 1 or the average position of the system in $\psi_1(x,t)$, and $|a|^2 Q$ is the expectation value of the charge Q in box 1. Then the electron wave packet will reach the position "$|a|^2$" between "0" and "1" on the screen as denoted in Fig.1. This shows that the result of the protective measurement is the expectation value of the projection operator A_1, namely the integral of the density $|\psi(x)|^2$ in the region of box 1. When multiplied by Q, it is the expectation value of the charge Q in the state $\psi_1(x,t)$ in box 1, namely the integral of the charge density $Q|\psi(x)|^2$ in the region of box 1. In fact, as Eq. (19) clearly shows, this is what the protective measurement really measures.

As we have argued in the last section, the result of a protective measurement reflects the objective property of the measured system. Thus the result of the above protective measurement, namely the expectation value of the charge Q in the state $\psi_1(x,t)$, $|a|^2 Q$, will reflect the actual charge situation of the system in box 1. In other words, the result indicates that there exists a charge $|a|^2 Q$ in box 1[7]. In the following, we will give another two special arguments for this conclusion.

First of all, let's analyze the result of the protective measurement. Suppose we can continuously change the measured state $\psi_1(x,t)$ from $|a|^2 = 0$ to $|a|^2 = 1$. When $|a|^2 = 0$, the single electron will reach the position "0" of the screen one by one, and it is incontrovertible that no charge is in box 1. When $|a|^2 = 1$, the single electron will reach the position "1" of the screen one by one, and it is also incontrovertible that there is a charge Q in box 1. Then when $|a|^2$ assumes a numerical value between 0 and 1 and the single

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[6]Since the deviation of the trajectory of the electron wave packet is the same for $a\psi_1(x,t) + b\psi_2(x,t)$ and $a\psi_1(x,t) - b\psi_2(x,t)$, the detection of the electron on the screen will not collapse the measured state $a\psi_1(x,t)+b\psi_2(x,t)$ to $a\psi_1(x,t)-b\psi_2(x,t)$. Therefore, the other energy eigenstates may not include the state $a\psi_1(x,t) - b\psi_2(x,t)$.

[7]Whether the charge is effective or real will be investigated in the next section.

electron reaches the position "$|a|^2$" between "0" and "1" on the screen one by one, the results should similarly indicate that there is a charge $|a|^2Q$ in the box by continuity. The point is that the definite deviation of the trajectory of the electron will reflect that there exists a definite amount of charge in box 1. Next, let's analyze the equation that determines the result of the protective measurement, namely Eq. (19). It gives a more direct support for the existence of a charge $|a|^2Q$ in box 1. The r.h.s of Eq. (19) is the formula of the electric force between two charges located in different spatial regions. It is incontrovertible that e is the charge of the electron, and it exists in the position $\vec{r}$. Then $|a|^2Q$ should be the other charge that exists in the position $\vec{r_1}$. In other words, there exists a charge $|a|^2Q$ in box 1.

In conclusion, protective measurement shows that a quantum system with mass m and charge Q, which is described by the wave function $\psi(x,t)$, has mass density $m|\psi(x,t)|^2$ and charge density $Q|\psi(x,t)|^2$, respectively.

4   The physical origin of mass and charge density

We have argued that a charged quantum system has mass and charge density proportional to the modulus square of its wave function. In this section, we will further investigate the physical origin of the mass and charge density. Is it real or only effective? As we will see, the answer may provide an important clue to the physical meaning of the wave function.

4.1   The mass and charge density is effective

If the mass and charge density of a charged quantum system is real, that is, if the density at different locations exist at the same time, then there will exist gravitational and electrostatic self-interactions of the density. Interestingly, the Schrödinger-Newton equation, which was proposed by Diosi (1984) and Penrose (1998), just describes the gravitational self-interaction of the mass density. The equation for a single quantum system can be written as

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t) - Gm^2\int\frac{|\psi(\mathbf{x}',t)|^2}{|\mathbf{x}-\mathbf{x}'|}d^3\mathbf{x}'\psi(\mathbf{x},t) + V\psi(\mathbf{x},t),$$
(20)

where m is the mass of the quantum system, V is an external potential, G is Newton's gravitational constant. Much work has been done to study the mathematical properties of this equation (Moroz, Penrose and Tod 1998; Moroz and Tod 1999; Harrison, Moroz and Tod 2003; Salzman 2005). Several experimental schemes have been also proposed to test its physical validity (Salzman and Carlip 2006). As we will see below, although such gravitational self-interactions cannot yet be excluded by experiments, the existence of the electrostatic self-interaction for a charged quantum system already contradicts experimental observations.

If there is also an electrostatic self-interaction, then the equation for a free quantum system with mass m and charge Q will be

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t) + (kQ^2 - Gm^2)\int\frac{|\psi(\mathbf{x}',t)|^2}{|\mathbf{x}-\mathbf{x}'|}d^3\mathbf{x}'\psi(\mathbf{x},t). \quad (21)$$

Note that the gravitational self-interaction is attractive, while the electrostatic self-interaction is repulsive. It has been shown that the measure of the potential strength of the gravitational self-interaction is $\varepsilon^2 = (\frac{4Gm^2}{\hbar c})^2$ for a free system with mass m (Salzman 2005). This quantity represents the strength of the influence of the self-interaction on the normal evolution of the wave function; when $\varepsilon^2 \approx 1$ the influence is significant. Similarly, for a free charged system with charge Q, the measure of the potential strength of the electrostatic self-interaction is $\varepsilon^2 = (\frac{4kQ^2}{\hbar c})^2$. As a typical example, for a free electron the potential strength of the electrostatic self-interaction will be $\varepsilon^2 = (\frac{4ke^2}{\hbar c})^2 \approx 1 \times 10^{-3}$. This indicates that the electrostatic self-interaction will have a remarkable influence on the evolution of the wave function of a free electron[8]. If such an interaction indeed exists, it should have been detected by precise interference experiments on electrons. On the other hand, the superposition principle of quantum mechanics, which denies the existence of the observable electrostatic self-interaction, has been verified for microscopic particles with astonishing precision. As another example, consider the electron in the hydrogen atom. Since the potential of the electrostatic self-interaction is of the same order as the Coulomb potential produced by the nucleus, the energy levels of hydrogen atoms will be remarkably different from those predicted by quantum mechanics and confirmed by experiments. Therefore, the electrostatic self-interaction cannot exist for a charged quantum system.

In conclusion, although the gravitational self-interaction is too weak to be detected presently, the existence of the electrostatic self-interaction for a charged quantum system such as an electron already contradicts experimental observations. Accordingly, the mass and charge density of a quantum system cannot be real but be effective. This means that at every instant there is only a localized particle with the total mass and charge of the system, and during a time interval the time average of the ergodic motion of the particle forms the effective mass and charge density[9]. There exist no gravitational and electrostatic self-interactions of the density in this case. Moreover, since protective measurement implies that the mass and charge

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[8]By contrast, the potential strength of the gravitational self-interaction for a free electron is $\varepsilon^2 = (\frac{4Gm_e^2}{\hbar c})^2 \approx 4 \times 10^{-89}$.

[9]Note that even if there are only two masses and charges in space at a given instant, the densities formed by their motion also have gravitational and electrostatic interactions. Therefore, the mass and charge density of a quantum system can only be formed by the ergodic motion of one localized particle with the total mass and charge of the system.

density is an instantaneous property of a quantum system, the ergodic motion of the particle must form the effective mass and charge density during an infinitesimal time interval (*not* during a finite time interval) at a given instant.

4.2   The ergodic motion of a particle is discontinuous

Which sort of ergodic motion then? If the ergodic motion of the particle is continuous, then it can only form the effective mass and charge density during a finite time interval, which contradicts the above implication of protective measurement. Thus it seems that the ergodic motion of the particle cannot be continuous. This is at least what the existing quantum mechanics says. However, there may exist a possible loophole here. Although the classical ergodic models that assume continuous motion are inconsistent with quantum mechanics due to the existence of a finite ergodic time, they may be not completely precluded by experiments if only the ergodic time is extremely short. After all quantum mechanics is also an approximation of a more fundamental theory of quantum gravity, in which there may exist a minimum time scale such as the Planck time. Therefore, we need to investigate the classical ergodic models more thoroughly.

Consider an electron in a one-dimensional box in the first excited state $\psi(x)$ (Aharonov and Vaidman 1993). Its wave function has a node at the center of the box, where its charge density is zero. Assume the electron performs a very fast continuous motion in the box, and during a very short time interval its motion generates an effective charge density distribution. Let's see whether this density can assume the same form as $e|\psi(x)|^2$, which is required by protective measurement[10]. Since the effective charge density is proportional to the amount of time the electron spends in a given position, the electron must be in the left half of the box half of the time and in the right half of the box half of the time. But it can spend no time at the center of the box where the effective charge density is zero; in other words, it must move at infinite velocity at the center. Certainly, the appearance of velocity faster than light or even infinite velocity may be not a fatal problem, as our discussion is entirely in the context of non-relativistic quantum mechanics, and especially the infinite potential in the example is also an ideal situation. However, it seems difficult to explain why the electron speeds up at the node and where the infinite energy required for the acceleration comes from. Moreover, the sudden acceleration of the electron near the node may also result in large radiation (Aharonov, Anandan and Vaidman 1993), which is

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[10]Note that in Nelson's stochastic mechanics the electron, which is assumed to undergo a Brownian motion, moves only within a region bounded by the nodes (Nelson 1966). This ensures that the theory can be equivalent to quantum mechanics in a limited sense. Obviously this sort of motion is not ergodic and cannot generate the required charge density distribution.

inconsistent with the predictions of quantum mechanics. Again, it seems very difficult to explain why the accelerating electron does not radiate here.

Let's further consider an electron in a superposition of two energy eigenstates in two boxes $\psi_1(x) + \psi_2(x)$. In this example, even if one assumes that the electron can move with infinite velocity (e.g. at the nodes), it cannot *continuously* move from one box to another due to the restriction of box walls. Therefore, any sort of continuous motion cannot generate the effective charge density $e|\psi_1(x) + \psi_2(x)|^2$. One may still object that this is merely an artifact of the idealization of infinite potential. However, even in this ideal situation, the model should also be able to generate the effective charge density by means of some sort of ergodic motion of the electron; otherwise it will be inconsistent with quantum mechanics. On the other hand, it is very common in quantum optics experiments that a single-photon wave packet is splitted into two branches moving along two well separated paths in space. The wave function of the photon disappears outside the two paths for all practical purposes. Moreover, the experimental results are not influenced by the environment and setup between the two paths of the photon. Thus it is very difficult to imagine that the photon performs a continuous ergodic motion back and forth in the space between its two paths.

In view of these serious drawbacks of the classical ergodic models and their inconsistency with quantum mechanics, we conclude that the ergodic motion of particles cannot be continuous but be discontinuous. If the motion of a particle is discontinuous, then the particle can readily move throughout all regions where the wave function is nonzero during an arbitrarily short time interval at a given instant. Furthermore, if the probability density of the particle appearing in each position is proportional to the modulus square of its wave function there at every instant, the discontinuous motion can also generate the right effective mass and charge density. This will solve the above problems plagued by the classical ergodic models. The discontinuous ergodic motion requires no existence of a finite ergodic time. Moreover, a particle undergoing discontinuous motion can also move from one region to another spatially separated region, no matter whether there is an infinite potential wall between them, and such discontinuous motion is not influenced by the environment and setup between these regions either. Besides, discontinuous motion can also solve the problems of infinite velocity and accelerating radiation. The reason is that no classical velocity and acceleration can be defined for discontinuous motion, and energy and momentum will require new definitions and understandings as in quantum mechanics.

In conclusion, we have argued that the mass and charge density of a quantum system, which can be measured by protective measurement, is not real but effective. Moreover, the effective mass and charge density is formed by the discontinuous motion of a localized particle, and the probability density of the particle appearing in each position is proportional to the modulus

square of its wave function there. As a result, the wave function can be regarded as a description of the discontinuous motion of the localized particle. In the next section, we will give a detailed analysis of this suggested interpretation of the wave function.

4.3   An argument for the randomness of discontinuous motion

Although the above analysis demonstrates that the ergodic motion of a particle is discontinuous, it doesn't say that the discontinuous motion must be random. In particular, the randomness of the result of a quantum measurement may be only apparent. In order to know whether the motion of particles is random or not, we need to analyze the cause of motion. For example, if motion has no *deterministic* cause, then it will be random, only determined by a probabilistic cause. This may also be the right way to find how particles move. Since motion involves change in position, if we can find the cause or instantaneous condition determining the change[11], we will be able to find how particles move in reality.

Let's consider the simplest states of motion of a free particle, for which the instantaneous condition determining the change of its position is a constant during the motion. In logic the instantaneous condition can only be deterministic or indeterministic. That the instantaneous condition is deterministic means that it leads to a deterministic change of the position of a particle at a given instant. That the instantaneous condition is indeterministic means that it only determines the probability of the particle appearing in each position in space at a given instant. If the instantaneous condition is deterministic, then the simplest states of motion of the free particle will have two possible forms. The first one is continuous motion with constant velocity, and the equation of motion of the particle is $x(t+dt) = x(t)+vdt$, where the deterministic instantaneous condition v is a contant[12]. The second one is discontinuous motion with infinite average velocity; the particle performs a finite jump along a fixed direction at every instant, where the jump distance is a constant, determined by the constant instantaneous condition[13]. On the other hand, if the instantaneous condition is indeterministic, then the simplest states of motion of the free particle will be random discontinuous motion with even position probability density. At each instant the

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[11]The word "cause" used here only denotes a certain instantaneous condition determining the change of position, which may appear in the laws of motion. Our analysis is irrelevant to whether the condition has causal power or not.

[12]This deterministic instantaneous condition has been often called intrinsic velocity. It is different from the standard velocity, though they are equal in numerical values (Tooley 1988).

[13]In discrete space and time, the motion will be a discrete jump along a fixed direction at each time unit, and thus it will become continuous motion with constant velocity in the continuous limit.

probability density of the particle appearing in every position is the same.

In order to know whether the instantaneous condition is deterministic or not, we need to determine which sort of simplest states of motion are the solutions of the equation of free motion in quantum mechanics (i.e. the free Schrödinger equation). According to the analysis in the last subsection, the momentum eigenstates of a free particle, which are the solutions of the free Schrödinger equation, describe the discontinuous motion of the particle with even position probability density in space. Therefore, the simplest states of motion with a constant probabilistic instantaneous condition are the solutions of the equation of free motion, while the simplest states of motion with a constant deterministic instantaneous condition are not.

When assuming that (1) the simplest states of motion of a free particle are the solutions of the equation of free motion; and (2) the instantaneous condition determining the position change of a particle is always deterministic or indeterministic for any state of motion, the above result then implies that motion, no matter it is free or forced, has no deterministic cause, and thus it is random and discontinuous, only determined by a probabilistic cause. The argument may be improved by further analyzing these two seemingly reasonable assumptions, but we will leave this for future work.

5   The wave function as a description of random discontinuous motion of particles

In classical mechanics, we have a clear physical picture of motion. It is well understood that the trajectory function $x(t)$ in classical mechanics describes the continuous motion of a particle. In quantum mechanics, the trajectory function $x(t)$ is replaced by a wave function $\psi(x,t)$. Since quantum mechanics is a more fundamental theory of the physical world, of which classical mechanics is only an approximation, it seems natural that the wave function should describe some sort of more fundamental motion of particles, of which continuous motion is a only an approximation in the classical domain. The analysis in the last section provides a strong support for this conjecture, and it suggests that what the wave function describes is the more fundamental motion of particles, which is essentially discontinuous and random. In this section, we will give a more detailed analysis of this suggested interpretation of the wave function.

5.1   An analysis of random discontinuous motion of particles

The physical definition of random discontinuous motion of a particle is as follows. The position of the particle at each instant is only determined by a certain instantaneous condition at the instant in a probabilistic way, and this probabilistic instantaneous condition gives the probability density of the

particle appearing in every position in space. As a result, the trajectory of the particle is essentially discontinuous, i.e., that the trajectory function $x(t)$ of the particle is not continuous at every instant t[14]. Unlike the deterministic continuous motion, the trajectory function $x(t)$ no longer provides a useful description for random discontinuous motion. In the following, we will give a strict description of random discontinuous motion based on the measure theory in mathematics (see, e.g. Cohn 1993). For simplicity but without losing generality, we will mainly analyze the one-dimensional motion that corresponds to the point set in two-dimensional space and time. The results can be readily extended to the three-dimensional situation.
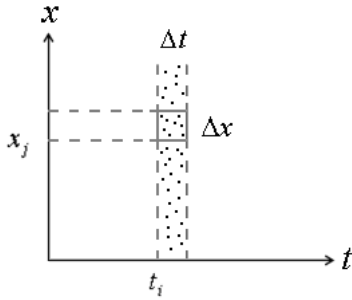

Fig.2 The description of random discontinuous motion of a single particle

We first analyze the random discontinuous motion of a single particle. Consider the state of motion of the particle in finite intervals Δt and Δx near a space-time point (t_i, x_j) as shown in Fig. 2. By the definition of random discontinuous motion, the positions of the particle form a random, discontinuous trajectory in this square region. We study the projection of this trajectory in the t-axis, which is a dense instant set in the time interval Δt. Let W be the discontinuous trajectory of the particle and Q be the square region $[x_j, x_j + \Delta x] \times [t_i, t_i + \Delta t]$. The dense instant set can be denoted by $\pi_t(W \cap Q) \in \Re$, where π_t is the projection on the t-axis. According to the measure theory, we can define the Lebesgue measure:

$$M_{\Delta x, \Delta t}(x_j, t_i) = \int_{\pi_t(W \cap Q) \in \Re} dt. \tag{22}$$

Since the sum of the measures of all such dense instant sets in the time interval Δt is equal to the length of the continuous time interval Δt, we have:

$$\sum_j M_{\Delta x, \Delta t}(x_j, t_i) = \Delta t. \tag{23}$$

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[14]Recall that a function $x(t)$ is continuous if and only if for every t and every real number $\varepsilon > 0$, there exists a real number $\delta > 0$ such that whenever a point t_0 has distance less than δ to t, the point $x(t_0)$ has distance less than ε to $x(t)$.

Then we can define the measure density as follows[15]:

$$\rho(x,t) = \lim_{\Delta x, \Delta t \to 0} M_{\Delta x, \Delta t}(x,t)/(\Delta x \cdot \Delta t). \tag{24}$$

We call it position measure density or position density in brief. This quantity provides a strict description of the position distribution of the particle in an infinitesimal space interval dx near position x during an infinitesimal interval dt near instant t. In other words, $\rho(x,t)$ provides a strict description of the state of random discontinuous motion of the particle at instant t. From Eq. (23) we can see that $\rho(x,t)$ satisfies the normalization relation, namely $\int_{-\infty}^{+\infty} \rho(x,t)dx = 1$.

Since the position density will change with time in general, we can further define the position flux density $j(x,t)$ through the relation $j(x,t) = \rho(x,t)v(x,t)$, where $v(x,t)$ is the velocity of the local position density. It describes the change rate of the position density. Due to the conservation of measure, $\rho(x,t)$ and $j(x,t)$ satisfy the continuity equation:

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial j(x,t)}{\partial x} = 0. \tag{25}$$

The position density $\rho(x,t)$ and position flux density $j(x,t)$ provide a complete description of the state of random discontinuous motion of a single particle.

It is direct to extend the description of the motion of a single particle to the motion of many particles. For the random discontinuous motion of N particles, we can define joint position density $\rho(x_1, x_2, ...x_N, t)$ and joint position flux density $j(x_1, x_2, ...x_N, t) = \rho(x_1, x_2, ...x_N, t)v(x_1, x_2, ...x_N, t)$. They also satisfy the continuity equation:

$$\frac{\partial \rho(x_1, x_2, ...x_N, t)}{\partial t} + \sum_{i=1}^{N} \frac{\partial j(x_1, x_2, ...x_N, t)}{\partial x_i} = 0. \tag{26}$$

When these N particles are independent, the joint position density can be reduced to the direct product of the position density for each particle, namely $\rho(x_1, x_2, ...x_N, t) = \prod_{i=1}^{N} \rho(x_i, t)$. Note that the joint position density $\rho(x_1, x_2, ...x_N, t)$ and joint position flux density $j(x_1, x_2, ...x_N, t)$ are not defined in the real three-dimensional space, but defined in the 3N-dimensional configuration space.

5.2 Interpreting the wave function

Although the motion of particles is essentially discontinuous and random, the discontinuity and randomness of motion is absorbed into the state of motion, which is defined during an infinitesimal time interval, by the descriptive

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[15]The existence of this limit relies on the continuity of the evolution of the probabilistic instantaneous condition determining the random discontinuous motion.

quantities of position density $\rho(x,t)$ and position flux density $j(x,t)$. Therefore, the evolution of the state of random discontinuous motion of particles can be a deterministic continuous equation. By assuming that the nonrelativistic equation of random discontinuous motion is the Schrödinger equation in quantum mechanics, both $\rho(x,t)$ and $j(x,t)$ can be expressed by the wave function in an unique way[16]:

$$\rho(x,t) = |\psi(x,t)|^2, \tag{27}$$

$$j(x,t) = \frac{\hbar}{2mi}[\psi^*(x,t)\frac{\partial\psi(x,t)}{\partial x} - \psi(x,t)\frac{\partial\psi^*(x,t)}{\partial x}]. \tag{28}$$

Correspondingly, the wave function $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$ (except for a constant phase factor):

$$\psi(x,t) = \sqrt{\rho(x,t)}e^{im\int_{-\infty}^{x} \frac{j(x',t)}{\rho(x',t)}dx'/\hbar}. \tag{29}$$

In this way, the wave function $\psi(x,t)$ also provides a complete description of the state of random discontinuous motion of particles[17]. For the motion of many particles, the joint position density and joint position flux density are defined in the 3N-dimensional configuration space, and thus the many-particle wave function, which is composed of these two quantities, is also defined in the 3N-dimensional configuration space.

Interestingly, we can reverse the above logic in some sense, namely by assuming the wave function is a complete description for the motion of particles, we can also reach the random discontinuous motion of particles, independent of our previous analysis. If the wave function $\psi(x,t)$ is a description of the state of motion for a single particle, then the quantity $|\psi(x,t)|^2 dx$ not only gives the probability of the particle being found in an infinitesimal space interval dx near position x at instant t (as in standard quantum mechanics), but also gives the objective probability of the particle being there. This accords with the common-sense assumption that the probability distribution of the measurement results of a property is the same as the objective distribution of the property in the measured state. Then at instant t the particle may appear in any location where the probability density $|\psi(x,t)|^2$ is nonzero, and during an infinitesimal time interval near instant t the particle will move throughout the whole region where the wave function $\psi(x,t)$ spreads. Moreover, its position density is equal to the probability density

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[16]Note that the relation between $j(x,t)$ and $\psi(x,t)$ depends on the concrete evolution under an external potential such as electromagnetic vector potential. By contrast, the relation $\rho(x,t) = |\psi(x,t)|^2$ holds true universally, independent of the concrete evolution.

[17]The picture of random discontinuous motion may exist not only for position but also for other dynamical variables such as momentum and energy etc, and thus this interpretation may also apply to the wave function in momentum space etc.

$|\psi(x,t)|^2$. Obviously this kind of motion is essentially random and discontinuous.

One important point needs to be stressed here. Since the wave function in quantum mechanics is defined at an instant, not during an infinitesimal time interval, it should be regarded not as a description of the state of random discontinuous motion of particles, but as a description of the instantaneous condition or instantaneous intrinsic property of the particles that determines their random discontinuous motion at a deeper level [18]. In particular, the modulus square of the wave function determines the probability density of the particles appearing in every position in space at a given instant. This intrinsic property may be called indeterministic disposition or propensity[19]. By contrast, the position density $\rho(x,t)$ and position flux density $j(x,t)$, which are defined during an infinitesimal time interval, are only a description of the state of the resulting random discontinuous motion of particles, and they are determined by the wave function. In this sense, we may say that the motion of particles is "guided" by the wave function in a probabilistic way.

The suggested interpretation of the wave function in terms of random discontinuous motion of particles might be taken as a natural realistic extension of the orthodox view. The naturalness of the extension lies in that it still makes particles ontological and the wave function epistemological[20]. That the extension is realistic is obvious. According to Born's probability interpretation, the modulus square of the wave function of a particle gives the probability density of the particle *being found* in certain positions, while according to the suggested interpretation, the modulus square of the wave function also gives the objective probability density of the particle *being* there. Certainly, the transition process from "being" to "being found", which is closely related to the quantum measurement problem, needs to be further explained. We will discuss this important issue in the next section.

6 Implications for the solution to the measurement problem

In standard quantum mechanics, it is postulated that when a wave function is measured by a macroscopic device, it will no longer follow the lin-

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[18]From a logical point of view, for the random discontinuous motion of particles, the particles should also have an intrinsic property that determines their discontinuous motion in a probabilistic way, otherwise they would not "know" how frequently they should appear in every position in space. See also the definition of random discontinuous motion given in the last section.

[19]It is worth noting that this kind of propensity relates to the objective motion of particles, not to the measurement on the particles (cf. Suárez 2004).

[20]By contrast, the de Broglie-Bohm theory and the many-worlds interpretation both attach reality to the wave function itself (Bohm 1952; Everett 1957).

ear Schrödinger equation, but instantaneously collapse to one of the wave functions that correspond to definite measurement results. However, this collapse postulate is only a makeshift, and the theory does not tell us why and how the definite measurement result appears (Bell 1990). There are in general two ways to solve the measurement problem. The first one is to integrate the collapse evolution with the normal Schrödinger evolution into a unified dynamics, e.g. in the dynamical collapse theories (Ghirardi 2008). The second way is to reject the postulate and assume that the Schrödinger equation completely describes the evolution of the wave function. There are two main alternative theories along this avoiding-collapse direction. The first one is the de Broglie-Bohm theory (de Broglie 1928; Bohm 1952), which takes the wave function as an incomplete description and adds some hidden variables to explain the definite measurement results. The second one is the many-worlds interpretation (Everett 1957), which assumes the existence of many worlds to explain our definite experience in one of these worlds and still regards the wave function as a complete description of the whole worlds. In this section, we will analyze the possible implications of our suggested interpretation of the wave function for these solutions to the measurement problem.

At first sight, the above three theories seem apparently inconsistent with the suggested interpretation of the wave function. They all attach reality to the wave function, e.g. taking the wave function as a real physical entity on configuration space or assuming the wave function has a field-like spatiotemporal manifestation in the real three-dimensional space (see, e.g. Ghirardi 1997, 2008; Wallace and Timpson 2009). But according to our suggested interpretation, the wave function is not a field-like physical entity on configuration space; rather, it is a description of the random discontinuous motion of particles in three-dimensional space (and at a deeper level a description of the instantaneous intrinsic property of the particles that determines their random discontinuous motion)[21]. Anyway, in spite of the various views on the wave function in these theories, they never interpret the wave function as a description of the motion of particles in three-dimensional space. However,

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[21]It has been argued that the wave function living on configuration space can hardly be considered as a real physical entity due to its multi-dimensionality (see, e.g. Monton 2002, 2006). However, it seems that this common objection is not conclusive, and one can still insist on the reality of the wave function living on configuration space by some metaphysical arguments (Albert 1996; Lewis 2004; Wallace and Timpson 2009). For example, a general strategy is to show how a many-dimensional world can appear three-dimensional to its inhabitants, and then argue on that basis that a wavefunction ontology is adequate to explain our experience (Lewis 2004). As we have argued earlier, the existence of the effective mass and charge density of a quantum system, which is measurable by protective measurement, poses a more serious objection to the wavefunction ontology; even for a single quantum system the wave function cannot be taken as a field-like entity in three-dimensional space either. Moreover, the reason is not metaphysical but physical, i.e., that the field-like interpretation contradicts both quantum mechanics and experimental observations.

on the one hand, the interpretation of the wave function in these theories is still an unsettled issue, and on the other hand, these theories may be not influenced by the interpretation in a significant way. Therefore, they may be consistent with the suggested interpretation of the wave function after certain revision.

6.1   The de Broglie-Bohm theory

Let's first analyze the de Broglie-Bohm theory (de Broglie 1928; Bohm 1952). According to the theory, a complete realistic description of a quantum system is provided by the configuration defined by the position of its particle together with its wave function. The wave function follows the linear Schrödinger equation and never collapses. The particle, often called Bohmian particle, is guided by the wave function via the guiding equation to undergo continuous motion. The result of a measurement is indicated by the position of the Bohmian particle describing the pointer of the measuring device, and thus it is always definite. Moreover, it can be shown that the de Broglie-Bohm theory gives the same predictions of measurement results as standard quantum mechanics by means of a quantum equilibrium hypothesis (so long as the latter gives unambiguous predictions). In this way, it seems that the de Broglie-Bohm theory can succeed in avoiding the collapse of the wave function.

However, although the de Broglie-Bohm theory is mathematically equivalent to standard quantum mechanics, there is no clear consensus with regard to its physical interpretation. To begin with, the interpretation of the wave function in the theory is still in dispute. For example, the wave function has been regarded as a field similar to electromagnetic field (Bohm 1952), an active information field (Bohm and Hiley 1993), a field carrying energy and momentum (Holland 1993), a causal agent more abstract than ordinary fields (Valentini 1997), and a component of physical law (Dürr, Goldstein and Zanghì 1997; Goldstein and Teufel 2001) etc. Notwithstanding the differences between these interpretations, they are inconsistent with the picture of random discontinuous motion of particles. To say the least, they can hardly explain the existence of mass and charge density for a charged quantum system, which is measurable by protective measurement. Our previous analysis suggests that the mass and charge density of a quantum system, which is proportional to the modulus square of its wave function, is effective and formed by the ergodic motion of a localized particle with the total mass and charge of the system, and thus the wave function is a description of the ergodic motion of particles.

A more pivotal issue concerns the guiding responsibility of the wave function. In the de Broglie-Bohm theory, the wave function of a quantum system is assumed to guide the deterministic continuous motion of its Bohmian particle, while the wave function and the Bohmian particle are

two different physical entities. According to our suggested interpretation of the wave function, the wave function of a quantum system indeed guides the motion of a localized particle in some sense. However, what the wave function describes is not a physical entity independent of the particle but an intrinsic property of the particle. Moreover, the motion of the particle is not continuous and deterministic but discontinuous and random, and the wave function guides the discontinuous motion in a probabilistic way; the modulus square of the wave function determines the probability density of the particle appearing in each position in space. This reveals a deeper discrepancy between the de Broglie-Bohm theory and our suggested interpretation of the wave function.

Next, let's analyze the hypothetical Bohmian particles in the de Broglie-Bohm theory. It has been a controversial issue what properties the Bohmian particles should have. On the one hand, the theory seems to require that the Bohmian particles have mass and charge. For example, for a many-body system, the guiding equation of a Bohmian particle obviously contains the mass of a sub-system, and the mass is usually regarded as the mass of the Bohmian particle (Goldstein 2009). This attribution seems inevitable, as the sub-system does not have its own wave function, and as a result the mass cannot be attributed to its wave function. Moreover, in the quantum potential formulation of the theory (Bohm 1952), the second-order equation of motion of a Bohmian particle contains a Coulomb force term when an electrostatic interaction is involved, which indicates that the Bohmian particle of a charged quantum system also has charge. Endowing mass and charge to the Bohmian particles seems quite natural, as for the theory the particles are primary or primitive, while the wave function is only secondary or derivative (Goldstein 2009).

On the other hand, it has been argued that the mass and charge of a quantum system should be possessed by its wave function, not by its Bohmian particle (see, e.g. Brown, Dewdney and Horton 1995). It is even claimed that a Bohmian particle has no properties other than its position (Hanson and Thoma 2011). As our previous analysis suggests, protective measurement may provide a more convincing argument for the "bareness" of the Bohmian particles. The existence of mass and charge density for a charged quantum system, which is proportional to the modulus square of its wave function and measurable by protective measurement, implies that mass and charge are attributes of the wavefunction and not of the hypothetical Bohmian particle. When the wave function is further interpreted as a description of the random discontinuous motion of particles as we have suggested, it becomes more obvious that the mass and charge (and other properties) of a quantum system belong to these particles, not to the added Bohmian particles.

There is a possible way to avoid the above inconsistency. One can only use the first-order guiding equation to formulate the theory. There is no

27

apparent appearance of charge in the equation even when an electrostatic interaction is involved; the charge information is absorbed into the wave function in some sense. Moreover, even if mass still appears in the guiding equation, one can attribute the masses of all sub-systems of a many-body system to its wave function. This seems to require a particle interpretation of the wave function. For example, when interpreting the wave function as a description of the random discontinuous motion of particles, the masses appearing in the guiding equations can be attributed to these particles. In this way, it seems that the Bohmian particles can be consistently regarded as bare.

However, the "bareness" of the Bohmian particles is at least a worrisome issue. According to the common-sense view, a real particle should have its intrinsic properties such as mass and charge etc, and its total energy cannot be zero either. If a particle has no properties other than its position, then in what sense it can be regarded as physically real? It seems that a bare Bohmian particle has no difference with a mathematical point. Furthermore, if the Bohmian particles are deprived of all intrinsic properties, then how can they be guided by the wave function? and how can the wave function "know" its existence and guide its motion? This also reminds us another debatable aspect of the de Broglie-Bohm theory, the interaction between the wave function and the Bohmian particles. In the final analysis, the influence of the wave function on the Bohmian particles is in want of a *physical* explanation.

Lastly, we analyze a possible combining picture of the de Broglie-Bohm theory and our suggested interpretation of the wave function. Even if the wave function describes the random discontinuous motion of particles, one may also add the bare Bohmian particles undergoing continuous motion to explain the emergence of definite measurement results. This is one of the main merits of the de Broglie-Bohm theory after all. This hybrid theory, however, has more drawbacks. First of all, the double-particle picture seems clumsy and unnatural. For example, an ordinary electron will contain two distinct particles; one is a real localized electron, and the other is a bare particle without any properties of the electron. Moreover, these two particles move in two essentially different ways; the real electron undergoes random discontinuous motion, while the bare particle undergoes deterministic continuous motion. The coexistence of continuous motion and discontinuous motion seems inconsistent with the general expectation that motion can only be continuous or discontinuous in nature.

Secondly, the continuous motion of the bare particle needs to be guided by the real particle, but this added guiding responsibility can hardly be explained. It is natural that the wave function as an intrinsic property of the real particle determines the motion of the particle, but it seems difficult to explain why and how this property also determines the motion of another bare particle. Moreover, the determining ways are essentially different. The

wave function guides the motion of the real particle in a probabilistic way, while it guides the motion of the bare particle in a deterministic way. Last but not the least, the trajectories of the bare particles as added hidden variables seems redundant[22]. In some sense, there are already hidden variables in the picture of random discontinuous motion of particles. They are the properties (e.g. position, momentum, energy etc) of the particles at each instant. Though these variables are not continuous and deterministic, their random motion may just lead to the stochastic collapse of the wave function and further account for the emergence of definite measurement results. We will discuss this seemingly more natural possibility in detail later on.

To sum up, when taking into account of the implications of protective measurement and our suggested interpretation of the wave function based on them, the de Broglie-Bohm theory seems to be not a satisfactory solution to the measurement problem[23]. Although the theory can be mathematically equivalent to standard quantum mechanics, it seems lack of a reasonable physical interpretation. The added hidden variables, which are used to explain the emergence of definite measurement results, can only be carried by bare particles without any intrinsic properties of the involved quantum system such as mass and charge. Moreover, the theory can hardly explain why the evolution of the hidden variables is guided by the wave function in the way it requires. In particular, when the wave function is interpreted as a description of the random discontinuous motion of particles (and at a deeper level a description of the intrinsic property of the particles that determines their discontinuous motion in a probabilistic way), it seems impossible that the wave function belonging to these particles also guides the motion of other particles, especially when these particles are bare and the guiding way is deterministic.

6.2 The many-worlds interpretation

Now let's turn to the second approach to avoid wavefunction collapse, the many-worlds interpretation. Although this theory is widely acknowledged as one of the main interpretations of quantum mechanics, its many fundamental issues have not yet been solved (see Saunders et al 2010 and references

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[22]The reality of the trajectories of the Bohmian particles has been questioned based on the analysis of weak measurement and protective measurement (Englert, Scully, Sssmann and Walther 1992; Aharonov and Vaidman 1996; Aharonov, Englert and Scully 1999; Aharonov, Erez and Scully 2004). Although these objections may be answered by noticing that what protective measurement measures is the Ψ-field, not the Bohmian particles (see also Drezet 2006), they do reveal a seemingly unnatural feature of the theory, namely that the motion of the Bohmian particles is not ergodic, and the time averages of the Bohmian particles positions typically differ remarkably from the ensemble averages (Aharonov, Erez and Scully 2004). By contrast, the random discontinuous motion of particles is ergodic in our suggested interpretation of the wave function.

[23]This conclusion also applies to the general hidden variables theories with added particle ontology.

therein). For example, the stuff of the many worlds, what they are made of, seems never adequately explained, nor are the worlds precisely defined. Moreover, no satisfactory role or substitute for probability has been found in the many worlds theories, and their consistency with quantum mechanics is still debatable. In the following, we will analyze whether there are many worlds according to the suggested interpretation of the wave function in terms of random discontinuous motion of particles.

In order to examine the validity of the many-worlds interpretation, it is crucial to know exactly what a quantum superposition is. No matter how to define the many worlds, they correspond to some branches of a quantum superposition after all (e.g. the branches where measuring devices obtain definite results, and in particular, observers have definite conscious experience). According to the picture of random discontinuous motion of particles, a quantum superposition exists in the form of random and discontinuous time division. For a superposition of two positions A and B of a quantum system, the system randomly and discontinuously moves between these two positions. At some random and discontinuous instants the system is in position A, and at the other instants it is in position B[24]. As a result, each position branch exists in a time sub-flow, and the sum of all these time sub-flows constitute the whole continuous time flow. In this picture of quantum superposition, it is obvious that there is only one system all along in the continuous time flow, which randomly and discontinuously moves throughout all branches of the superposition, no matter the system is a microscopic particle or a measuring device or an observer. In other words, there is only one world which instantaneous state is constantly changing in a random and discontinuous way.

This conclusion is also supported by a comparison between discontinuous motion and continuous motion. For a quantum particle undergoing discontinuous motion, the position of the particle changes discontinuously. For a classical particle, its position changes continuously. There is no essential difference between these two kinds of changes. For both cases the position of the particle is always definite at each instant, and the positions of the particle at different instants may be different. Moreover, the discontinuous change, like the continuous change, does not result in any branching process needed for creating the many worlds, because, among other reasons, the change happens all the while but the branching process only happens once. Therefore, if there is only one world in classical mechanics, so does in quantum mechanics according to the picture of random discontinuous motion of particles, no matter how the many worlds are defined.

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[24]That the system is in a definite position A or B at every instant already implies that there is only one world at any time.

6.3   A possible origin of wavefunction collapse

The above analysis suggests that the de Broglie-Bohm theory and the many-worlds interpretation are not satisfactory solutions to the measurement problem according to our suggested interpretation of the wave function. If there are neither hidden variables nor many worlds that can explain the emergence of definite measurement results, then the collapse of the wave function is probably a real physical process, which is responsible for the transition from microscopic uncertainty to macroscopic certainty. Accordingly, the dynamical collapse theories may be in the right direction by admitting wavefunction collapse (Ghirardi 2008)[25].

However, the existing dynamical collapse theories are still phenomenological models, and they are also plagued by some serious problems such as energy non-conservation etc (Pearle 2007, 2009). In particular, the physical origin of the wavefunction collapse, including the origin of the randomness of the collapse process, is still unknown, though there are already some interesting conjectures (see, e.g. Diosi 1987; Penrose 1996). In the following, we will briefly analyze the possible origin of wavefunction collapse in terms of the random discontinuous motion of particles, and a detailed analysis will be given in another separate paper.

According to our suggested interpretation of the wave function, the wave function of a quantum particle can be regarded as an instantaneous intrinsic property of the particle that determines its random discontinuous motion. However, the wave function is not a complete description of the instantaneous state of the particle. The instantaneous state of the particle at a given instant also includes its random position, momentum and energy etc at the instant[26]. As a result, these random variables may also appear in the complete evolution equation of the instantaneous state, or in other words, they may also play a role in determining the instantaneous states at later instants in the equation. Since these variables are essentially random, their values at an instant will not influence their values at other instants in any direct way. Then these random variables can only manifest themselves in the law of motion by their influences on the evolution of the wave function. This forms a feedback in some sense; the wave function determines the probabilities of these variables assuming a particular value, while the random values of these variables at each instant also influence the evolution of the wave

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[25]As noted earlier, the ontology of these theories, such as mass density ontology and flashes ontology (Ghirardi, Grassi and Benatti 1995; Ghirardi 1997, 2008; Allori et al 2008), is inconsistent with our suggested interpretation of the wave function in terms of random discontinuous motion of particles. Especially, the existence of the effective mass and charge density of a quantum system seems to already exclude the mass density ontology.

[26]Although the probabilities of these variables assuming a particular value are determined by the wave function, the random values of these variables at every instant are new physical facts independent of the wave function.

function in a stochastic way. Therefore, the evolution of the wave function will be governed by a revised Schrödinger equation in general, which includes the normal linear terms and a stochastic nonlinear term resulting from the influences of these random variables. It has been shown that a certain form of such stochastic evolution may lead to the right collapse of the wave function, which can explain the emergence of definite measurement results and the macroscopic world (Gao 2006).

To sum up, the existence of the collapse of the wave function seems natural according to our suggested interpretation of the wave function. On the one hand, the random discontinuous motion of particles may provide an appropriate random source to collapse the wave function, and thus it might be the physical origin of the wavefunction collapse. On the other hand, the collapse of the wave function just releases the randomness and discontinuity of motion, and as a result, the random discontinuous motion of particles can also manifest itself.

7   Conclusions

The meaning of the wave function is the first interpretative problem of quantum mechanics. A satisfactory solution to this problem may have important implications for solving the other fundamental problems of quantum mechanics such as the measurement problem. In this paper, we argue that the mass and charge density of a quantum system, which is measurable by protective measurement and proportional to the modulus square of its wave function, is formed by the random and discontinuous ergodic motion of a localized particle with the total mass and charge of the system. This result leads to a new interpretation of the wave function, according to which the wave function on configuration space is a description of random discontinuous motion of particles in the real three-dimensional space, and the modulus square of the wave function gives the probability density of the particles being in certain locations in space. It is shown that the suggested interpretation of the wave function gives a possible clue to the solution of the measurement problem, and in particular, the collapse of the wave function may have its origin in the random discontinuous motion of particles.

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