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Chapter Seven

Creating a New Mathematics

Arran Gare

In this chapter, my focus is on efforts to create a new mathematics, with my prime interest being the role of mathematics in comprehending a world consisting first and foremost of processes, and examining what developments in mathematics are required for this. I am particularly interested in developments in mathematics able to do justice to the reality of life. Such mathematics could provide the basis for advancing ecology, human ecology and ecological economics and thereby assist in the transformation of society and civilization so that we augment life rather than undermining the conditions for our existence. It was in the process of grappling with these problems that I was drawn to investigate the tradition of intuitionism in mathematics and the role of intuition in mathematics, science and philosophy, and then to consider Whitehead’s work on mathematics and its philosophy in relation to these.

This is part of a broader project. As I see it, the defense of process philosophy is a struggle against the nihilism of European civilization, brought about, as Nietzsche argued, by the will to power turned against itself. While it is usual to interpret this in relation to Christian morality, it is clear from Nietzsche’s *Philosophical Notebooks* that it was the transmogrification of this morality into the quest for scientific truth that most concerned him. It is this quest for truth that has led to the denial of reality to creative processes and life, since the reality of these stands in the way of comprehending the world as transparent to reason and thereby, predictable and controllable. As Nietzsche noted, 'To impose upon becoming the character of being - that is the supreme will to power.'[1] Nietzsche characterized this as 'Egyptianism'. As Nietzsche wrote of philosophers in *Twilight of the Idols*:

> There is ... their hatred of even the idea of becoming, their Egyptianism. They think they are doing a thing *honour* when they dehistoricise it, *sub specie aeterni* - when they make a mummy of it. All that philosophers have handled for millenia has been conceptual mummies; nothing actual has escaped their hands alive. They kill, they stuff, when they worship, these conceptual idolaters - they become a mortal danger to everything when they worship. Death, change, age, as well as procreation and growth, are for them objections - refutations even. What is, does not *become*; what becomes *is* not ... Now they all believe, even to the point of despair, in that which is.[2]

Nothing has contributed more to and is more closely associated with this Egyptianism than mathematics. Writing of the illusions associated with claims to truth associated with science, Nietzsche argued that language works to construct concepts. The outcome of this labour is that 'the great edifice of concepts displays the rigid regularity of a Roman columbarium[3] and exhales in logic that strength and coolness which is characteristic of mathematics.'[4] Pythagoras, embracing the principle of sufficient reason that underpinned Anaximander's cosmology, that there

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1 Friedrich Nietzsche, *The Will to Power*, trans. Walter Kaufman and R.J. Hollingdale (New York: Vintage, 1968, §617), 330.

2 Friedrich Nietzsche, *Twilight of the Idols*, trans. R.J. Hollingdale (Harmondsworth: Penguin Books, 1889/1968), 35.

3 A columbarium is a vault with niches for funeral urns containing the ashes of cremated bodies.

4 Friedrich Nietzsche, *Philosophy and Truth: Selections form Nietzsche's Notebooks of the Early 1870's,* ed. and trans. Daniel Breazeale (New Jersey: Humanities Press, 1979), 85.

must be a reason why things are the way they are and not otherwise, paved the way for Parmenides to deny the reality of change. The drive to understand the world entirely through mathematics, combined with the principle of sufficient reason, cannot allow for real creativity and the reality of life. All this was clearly evident in Descartes' mathematics and philosophy. The drive to power is now disguised, but it is evident in the dominant quest by physicists to impose on becoming the character of being. The same will to power is implicit.

Many philosophers and mathematicians appreciated where science was leading. It was his appreciation of the nihilistic implications of Newton's mathematical physics that led Kant to argue that the world understood through mathematics is only the world of appearances, not the noumenal world. Kant characterized mathematics as a construction by subjects, developing this conception of mathematics to highlight the paradox of taking the objective world as portrayed by mathematical physicists, the world that appears to have no place for subjects, as the real world. That is, constructivism was developed by Kant to circumscribe and delimit the claims to validity of science. Many philosophers and mathematicians have supported this argument, or some variation of it, most notably, neo-Kantians and phenomenologists. Others opposed it. David Hilbert promoted formalism, while Gottlob Frege and Bertrand Russell argued that mathematics should be reduced to logic and defended a purely objectivist semantics that had no place for subjects. Opposing Hilbert and Frege, Luitzen Brouwer, who was influenced by Nietzsche through his teacher, Gerrit Mannoury, defended the role of intuition in mathematics. In doing so, he was not simply defending a particular philosophy of mathematics; he was taking a stand against nihilism, as is evident from his early work, *Life, Art and Mysticism,* published in 1905.[5] Brouwer's intuitionism was another name for constructivism, and was really a development of a tradition of thought on mathematics that had begun with Kant's account of and defence of constructivism. As such, intuitionism was also defended by Henri Poincaré and Hermann Weyl, and to some extent by Edmund Husserl, in each case reacting at least in

5 Luitzen Ergbertus Jan Brouwer, "Life, Art and Mysticism," trans. Walter P. Van Stigt, *Notre Dame Journal of Formal Logic* 37.3 (1996): 391-429.

part to the nihilism of mathematical physics. My claim is that as a process philosopher concerned to overcome the nihilism of scientific materialism, Whitehead had more affinity with these intuitionists than with Frege and Russell, and examining the ideas that influenced him reveals more evidence to justify this view, although these also show the notions of construction and intuition were understood differently. They are more akin to the ideas of C.S. Peirce, and are best seen in conjunction with Peirce's philosophy. With this historical background, it should become clearer what characterizations of and developments within mathematics are required to further advance Whitehead's project.

WHITEHEAD (AND PEIRCE) IN HISTORICAL CONTEXT: WHITEHEAD AND GRASSMANN

Neither Whitehead nor Peirce originally took as their aim to develop philosophies that would overcome the Parmenidean tradition. They were predisposed to do so, however, because each had been exposed to the values that developed in reaction to the nihilism of the mechanistic world-view. This reaction began in Britain and France, but reached its high point in Germany towards the end of the Eighteenth and the beginning of the Nineteenth Centuries, with Kant's philosophy being a major source of inspiration for this reaction. Whitehead was exposed to these values through Romantic poetry and the work of the British Idealists. Peirce was exposed to this German influence through the study of Kant himself and through the Concord transcendentalists. However, these values were hardly central to their interests when they began their intellectual careers. There was another source connected to this German philosophical movement, however, a tradition within science and mathematics that, while being inspired by Kant, was much more radical and sought to overcome the whole tradition of Newtonian science. In the case of Whitehead, the crucial figure was the mathematician Hermann Grassmann, although William Hamilton's mathematics and James Clerk Maxwell's physics were also important. The significance of Whitehead's alignment with these thinkers was not fully appreciated by Whitehead or most of his interpreters. What we see in Whitehead is a tension between the influences on him of different traditions of thought, and it is through the working out of this tension in favor of German influences that Whitehead developed as a process philosopher. And in doing so, he exposed with great clarity the flawed assumptions of the dominant traditions of thought.

The tension in Whitehead's philosophy can be found in the 'Preface' to his first published work, *A Treatise on Universal Algebra.* Here he defines mathematics 'in its widest signification' as 'the development of all types of formal, necessary, deductive reasoning. … The reasoning is formal in the sense that the meaning of a proposition forms no part of the investigation. … Mathematical reasoning is deductive in the sense that it is based upon definitions which, so far as the validity of the reasoning is concerned (apart from any existential import), need only the test of self-consistency.'[6] Although there is a hint of Whitehead's theory of abstraction when he characterizes a 'mathematical definition with an existential import' as 'the result of an act of pure abstraction',[7] there is nothing here inconsistent with a logical empiricist's understanding of mathematics, and the construal of mathematics as a system of tautologies. It foreshadows his later effort with Russell to reduce all mathematics to logic, following Frege in this regard. This involved an allegiance to logicism as a distinct philosophy of mathematics defined through its opposition to Hilbert's formalism and to Platonism, but more fundamentally, to the intuitionism of Brouwer, Poincaré and Weyl. Frege, following Bernard Bolzano and Herman Lotze, rejected Kant's constructivism and its implications, and was concerned to eliminate any role for mental processes, whether ideas, images, imaginative projections, constructions or intuitions. As opponents of intuitionism, Russell denied the significance accorded by Kant to synthesis in perception and thought, and rejected Kant's claim that arithmetic is a *synthetic a priori* form of knowledge.

However, Whitehead also characterized *A Treatise on Universal Algebra* as an exhibition of 'the algebras both as systems of symbolism, and also as engines for the investigation of the possibilities of thought

6 UA vi

7 UA vii

and reasoning connected with the abstract general idea of space' providing '[a] natural mode of comparison between the algebras … by the unity of subject-matters of their interpretation' concerned to provide a 'detailed comparison of their symbolic structures'.[8] Whitehead acknowledged the source of these ideas in the work of Benjamin Peirce, C.S. Peirce's father. Both Benjamin and Charles Peirce characterized mathematics as 'the science that draws necessary conclusions' and regarded mathematics as useful for studying logic, supporting Boole's and de Morgan's conception of symbolic logic as 'an algebra of logic' in opposition to Frege's effort, followed by Russell and Whitehead, to reduce mathematics to logic understood as a universal language. Charles Peirce, together with his father, had made a thorough study of Kant, and later of Friedrich Schelling, who had embraced and further developed Kant's constuctivism. Peirce characterized himself in a letter to William James as a 'Schellingian of some stripe'.[9] Charles Peirce went on to characterize mathematics through semiosis as 'diagrammatic reasoning', treating mathematics as a system of indexical signs the study of which could yield new knowledge. This was a development of Kant's constructivist view of mathematics, not a rejection of it, and gave a central place to intuition associated with observation of diagrams.

After having acknowledged Peirce, Whitehead wrote that '[t]he greatness of my obligation in this volume to Grassmann will be understood by those who have mastered his two *Ausdehnungslehres*. The technical development of the subject is inspired chiefly by his work of 1862, but the underlying ideas follow the work of 1844.'[10] Whitehead is unlikely to have been aware of it, but Hermann Grassmann was developing a conception of mathematics advanced by his father, Justus, under the influence of Schleiermacher and Schelling. These philosophers were influenced by Kant, but radicalized and generalized Kant's notion of construction and his ideas on life developed in the *Critique of Judgment*. As Michael Otte argued, 'J. Grassmann defines mathematics in the spirit

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8 UA v

9 C.S. Peirce, *Collected Paper* (8 vols), ed. Charles Hartshorne, Paul Weiss and A. W. Burks (Cambridge, MA.: Harvard University Press, 1931-1966), 6.605.

10 UA x

of Schelling, not Kant, as pure constructivity.'[11] The Grassmanns' work was part of, and a further development of, the quest to develop a flowing, dynamic mathematics to overcome the Newtonian mechanistic view of the world. Justus Grassmann attempted to develop what he thought of as a 'fluid geometry', that is, a 'dynamist, morphogenetic mathematics' that would facilitate insight into the emergence and inner synthesis of patterns in nature.[12] It was crucial that this mathematics not be limited to a theory of quantity and be independent of all relations of quantity so that it could go beyond the extrinsic, mechanical behavior of matter and recognize the intrinsic possibilities within nature for structuring and organizing. Hermann Grassmann's work, which he characterized as the 'theory of extension', continued this project.

Grassmann presented this work as a survey of a general theory of forms, assuming, as he put it, 'only the general concepts of equality and difference, conjunction and separation.'[13] He argued that there are two branches in mathematics,

> the continuous form or magnitude [which] separates into the algebraic continuous form or intensive magnitude and the combinatorial continuous form of extensive magnitude. The intensive magnitude is thus that arising through generation of equals, the extensive magnitude or extension that arising through generation of the different.[14]

Grassmann claimed that this second branch was previously unknown, but it is this branch that provides the foundations for all

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11 Michael Otte, "Justus and Hermann Grassmann: philosophy and mathematics," in *Hermann Grassmann: From Past to Future: Grassmann's Work in Context*, Hans-Joachim Petsche, 61-70 (Basel: Springer, 2011), 67.

12 Marie-Luise Heuser, "The Significance of *Naturphilosophie* for Justus and Hermann Grassmann," in *Hermann Grassmann: From Past to Future: Grassmann's Work in Context*, edited by Hans-Joachim Petsche, 49-59 (Basel: Springer, 2011), 58.

13 Hermann Grassmann, *A New Branch of Mathematics: The "Ausdehnungslehre" of 1844 and Other Works,* trans. Lloyd C. Kannenberg (Peterborough, NH: Open Court, 1995), 33.

14 Hermann Grassmann, *Extension Theory*, trans. Lloyd C. Kannenberg (American Mathematical Society, 1862/2000), 27.

mathematics. As he characterized the aim of his 1844 version of his extension theory in his 1862 reworking of this, it

> extends and intellectualizes the sensual intuitions of geometry into general, logical concepts, and, with regard to abstract generality, is not simply one among other branches of mathematics, such as algebra, combination theory, and function theory, but rather far surpasses them in that all fundamental elements are unified under this branch, which thus as it were forms the keystone of the entire structure of mathematics.[15]

In relation to this, it is significant that William Lawvere, one of the major figures involved in the development of category theory, argued that Grassmann's work was a precursor to category theory.[16]

Extended magnitude was defined by Grassmann as the magnitude created by the generation of difference in which the elements separate and become fixed as separate. This was understood dynamically, as is evident in Grassmann's exposition of the concept of extension theory:

> Continuous becoming analysed into its parts, appears as a continuous production with retention of that which has already become. With the extensive form, that which is newly produced is always defined as different; if, during this process, we no longer always retain what has already become, then we arrive at the concept of *continuous evolution.* We call that which undergoes this evolution the generating element, and the generating element, in any of the states it assumes in its evolution, an element of the continuous form. Accordingly, the extensive form is the collection of all elements into which the generating element is transformed by continuous evolution.[17]

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15 Ibid., p.xiii.

16 F. William Lawvere, F. William, "Grassmann's Dialectics and Category Theory," in *Hermann Günther Grassmann (1809-1877): Visionary Mathematician, Scientist and Neohumanist Scholar*, edited by Gert Shubring, 255-264 (Dordrecht: Kluwer, 1996), 256.

17 Grassmann, *A New Branch of Mathematics*, p.28f.

Grassmann was concerned to apply this mathematics, and did so to the study of tides and to electrodynamics. In the tradition of Schellingian thought, the dynamic nature of construction in mathematics was meant to provide insight into the self-construction of nature.

Whitehead's work, insofar as it was influenced by Frege, was antithetical to the Schellingian tradition and supported the development of logical empiricism, a philosophy that cemented in place mainstream reductionist science and rendered any knowledge, apart from how to control the world, as almost unintelligible. However, Whitehead became increasingly dissatisfied with the whole project of *Principia Mathematica,* at least as this project had been understood by Russell. Whitehead saw logic as a means for clarifying mathematical reasoning and exposing defective arguments, but acknowledged that 'deductive logic has not the coercive supremacy which is conventionally conceded to it. When applied to concrete instances, it is a tentative procedure, finally to be judged by the self-evidence of its issues.'[18] In a late paper, he concluded that 'Logic, conceived as an adequate analysis of the advance of thought, is a fake.'[19] However, the more fundamental issue was that Whitehead's whole orientation was different from Russell's. While to use Leibnizian terminology, Russell, like Frege, was striving to develop a *Lingua Universalis* – a universal medium whose symbolic structure would reflect directly the structure of the world, Whitehead was concerned to create a *Calculus Ratiocinator*, a method of symbolic calculation which would mirror and refine the processes of human reasoning.[20]

Whitehead's rejection of the project to reduce mathematics to logic and the development of his mature philosophy, and along with it, a different conception of mathematics, was really a development of the early influence of the Schellingian tradition and a creative contribution to constructivist thought. Instead of reducing mathematics to logic, Whitehead argued in *Modes of Thought* that 'Mathematics is the most powerful technique for the understanding of pattern, and for the analysis of the

18 MT 106

19 ESP 96

20 These different orientations have been identified by Jaakko Hintikka in *Lingua Universalis vs. Calculus Ratiocinator* (Dordrecht: Kluwer, 1996).

relationships of pattern.'[21] When we say 'twice three is six', Whitehead proclaimed, 'we are not saying that these two sides of the equation mean the same thing, but that two threes is a fluent process which become six as a completed pattern.' So, for Whitehead, 'mathematics is concerned with certain forms of process issuing into forms which are components for further process.'[22] Aligned with Schelling, Charles Peirce's characterization of mathematics was also a development within this tradition, and a further advance of it. Combining Whitehead's and Peirce's conceptions of mathematics, we can characterize mathematics as diagrammatical reasoning, studying iconic signs as a way of studying patterns and their transformations, including the patterns of reasoning, and in terms of such patterns and transformations, the relationship between the different branches of mathematics. Diagramatic reasoning is really a form of intuition achieved through the construction and transformation of diagrams. Conceived of in this way, Whitehead's *Universal Algebra* can itself be seen not only as a form of constructivism giving a central place to intuition and understanding, but as a further precursor to category theory.

Robert Rosen and Category Theory

It is through category theory that this conception of mathematics can be further developed. Category theory was characterized by one of its proponents as 'a powerful language to develop a universal semantics of mathematical structures.'[23] The concept of structures is problematic, but Saunders Mac Lane, one of the founders of category theory, characterized mathematics as 'not so much about things (objects) as about form (patterns or structures)', virtually equating forms, structures and patterns. Structures are 'lists of operations and their required properties, commonly given as axioms, and often so formulated as to be properties, shared by a number of possibly quite different specific mathematical

21 ESP 109

22 MT 92

23 Andrée C. Ehresmann, and Jean-Paul Vanbremeersch, *Memory Evolutive Systems*: *Hierarchy, Emergence, Cognition* (Amsterdam: Elsevier, 2007), 26.

objects.'[24] Category theory enables us to see the universal components of a family of structures of a given kind, how structures of different kinds are interrelated, and to examine the mutability and admissible transformations of precisely defined structures. A category has been defined as 'a composite item consisting of a graph and an internal law which associates an arrow of the graph to each path of the graph, called its composite, and which satisfies some axioms given further on.'[25] Category theory began with the observation that many properties of mathematical systems can be unified and simplified through a presentation with a diagram of arrows between 'objects' (which can be sets, groups or rings, or can be unspecified), where each arrow represents a function. The most important property of these arrows is that they can be 'composed', that is, arranged in a sequence to form a new arrow. The focus is then not on 'objects', but on the structure preserving mappings or 'morphisms' between these 'objects'.[26] These mappings, which reveal the possible transformations of structures, can themselves be studied in this way. If the structures are themselves categories so that the morphisms revealing possible transformations are between categories, these are referred to as 'functors', and are represented as arrows between the categories. There can also be a category of functors. The morphisms that transform one functor into another while respecting the internal structure of the categories involved, thereby bringing into focus their mutability, are 'natural transformations'.

Rosen's conception of mathematics, and its relation to science, is based on his development of category theory as a general theory of modeling. He argued that in fact most mathematics has some referent to something external to the formalism itself, and so is 'applied' mathematics, with modeling being the judicious association of a formalism

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24 Saunders Mac Lane, "Structures in Mathematics," *Philosophia Mathematica* 4.3 (1996): 174-183, 174.

25 Ehresmann and Vanbremeersch, *Memory Evolutive Systems*, 25f.

26 Robert Rosen, *Life Itself: A Comprehensive Inquiry into the Nature, Origin, and Fabrication of Life* (New York: Columbia University Press, 1991), 143ff.

with such external referents.[27] However, category theory makes explicit and clarifies the nature of this modeling relation. Rosen characterized categories as formal systems, morphisms as entailment structures, and natural transformations as explicit modeling of one system by another.[28] He then argued that from material systems we can abstract out 'natural systems' which can be modeled in the same way as formal systems are modeled. Modeling natural systems in this way is really hypothesizing via abstractions about their elements and entailment structures to establish congruence between formal systems and these natural systems. This involves carefully delineating observables and linkage relations of the natural systems. There can be no mechanical algorithm for doing this, it is inescapably an art. Once this congruence has been established successfully, we can learn about the modeled system by studying the model of it. This involves using encoding and decoding arrows, along with 'dictionaries' to translate back and forth between the two systems, with measurement being a form of encoding, and tracing causal entailments being a form of decoding.

Examining the variety of entailment structures, Rosen argued that modern science, under the influence of Newton, has excluded the kinds of observations, relations and models with complex forms of entailments that are characteristic of living organisms. Rosen's main concern was to develop mathematics and to reconceive the goal of science to do justice to the reality of life itself. This involved advancing the tradition of natural philosophy inspired by Kant's *Critique of Judgment* and the work of Schelling. Category theory as conceived by Rosen can be interpreted as a major development of the Whiteheadian/Peircian conception of mathematics - as the study through abstraction of possible patterns of connectedness and their transformations utilizing iconic signs or diagrams.[29]

Based on this way of understanding mathematics, Rosen argued that Gödel's theorem is just another foundation crisis for mathematics

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27 Robert Rosen, *Essays on Life Itself* (New York: Columbia University Press, 2000), 359.

28 Rosen, *Life Itself*, 147.

29 Zalamea, Fernando, *Synthetic Philosophy of Contemporary Mathematics* (New York: Sequence Press, 2012), 219ff.

due to it having taken a fateful wrong turn with Pythagoras. Pythagoras had attempted to reduce geometry to arithmetic, equating effectiveness with an iteration procedure such as counting; that is, computation. It was this assumption that led to Zeno's paradoxes and the crippling of mathematics for millennia. The basic problem is assuming that the simple procedures adequate to simple domains of mathematics, which are adequate for modeling very limited domains of reality, are adequate to more complex domains and can define acceptable procedures. This underpins the quest for formalization, and over and over again, it has failed. More recent efforts in this direction involved efforts to eliminate semantics from mathematics and to reduce mathematics to syntactical operations without any outside reference. Rosen noted the consequence of this: 'once inside such a universe … we cannot get out again, because all the original external *referents* have presumably been pulled inside with us. … Once inside, we can claim "objectivity"; we can claim independence from any external *context*, because there *is* no external context anymore.'[30]

Most mathematics is not formalizable through axioms as Hilbert called for. For Rosen, what Gödel showed was that the model of arithmetic, developed by Frege, Russell and Whitehead using set theory and logic, is less rich than arithmetic. Arithmetic is 'soft science' relative to the 'hard science' of set theory and logic, just as arithmetic is less rich than what is modeled by it, the richness of which is better captured by the 'soft' disciplines of the humanities and by the arts.[31] With modeling, this will always be the case. The modeling relation, where something is learned about one system by studying another which is analogous to it, is ubiquitous and characteristic of everyday life as well as of both theoretical and experimental science.[32] It is the failure to appreciate this that has led to the belief that objectivity implies the reduction of biology to chemistry and physics. As Rosen diagnosed source of this problem:

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30 Rosen, *Essays on Life Itself*, 77.

31 Rosen, *Life Itself*, 9f.

32 Robert Rosen, *Anticipatory Systems: Philosophical, Mathematical, and Methodological Foundations* (New York: Springer, 1985/2012), 82.

> [T]hese ideas [that every model of a material process must be formalizable] have become confused with *objectivity* and hence with the very fabric of the scientific enterprise. Objectivity is supposed to mean observer independence, and more generally, context independence. Over the course of time this has come to mean only building from the smaller to the larger, and reducing the larger to the smaller. … In any large world, such as the one we inhabit, this kind of identification is in fact a mutilation, and it serves only to estrange most of what is interesting from the realm of science itself.[33]

Once this is realized, we can not only free ourselves from the spell of the Pythagorean/Parmenidean ideal of science, reveal further aspects of its incoherence and free science to acknowledge the reality of life and mind, but develop mathematics more adequate to life. We can also recognize the limits of mathematics and the role and validity of non-mathematical conceptualizations and models that acknowledge some measure of indeterminacy in the present and openness to the future. Rosen showed the real issue and the real problem to be how to develop a mathematical structure in which the logical entailments within the mathematical models adequately reflect the causal entailments in that which is being investigated. In biology, what is being investigated are living beings. Rather than invoke an inadequate surrogate universe, it is necessary to appreciate the full reality of life itself characterized by final causes and functionality of components.[34] Functional components cannot be fractionated and treated independently of the organism since they are aspects of and definable only through the whole organism.

Recognizing a place for final causes, Rosen set out to model anticipatory systems, systems which do not simply respond to their environments but anticipate and respond to what will happen in the future.[35] That physics at present has no place for the influence of future conditions and final causes indicates, Rosen argued, that it is too specific and

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33 Rosen, *Essays on Life Itself*, 80.

34 Rosen, *Life Itself*, 108ff.

35 See Robert Rosen, *Anticipatory Systems.*

conceptually limited, just as nineteenth century physics was too specific and conceptually limited to account for atomic spectra, radioactivity and chemical bonding. Just as to explain these required the conceptual revolutions of relativity theory and quantum theory, so a new conceptual revolution is required. Rosen questioned the primacy given to closed systems in science, arguing that open systems are generic, and a closed system is an extremely degenerate case of an open system. Along with this, he also questioned the notion of 'state' in science and reality, suggesting that it is a fiction. The conceptual revolution required to account for life will also require a new mathematics. The mathematics of Oliver Heaviside and Paul Dirac gave a place to discontinuous signals (the δ-function), which were initially discounted or denigrated (by John von Neumann among others) as not genuine mathematics. Nevertheless, they eventually had to be accepted and the old mathematics relegated to the status of a special case. Similarly, new mathematics will have to be developed that will relegate the old mathematics to the status of a special case.[36] This is what Rosen set out to do.

Modeling anticipatory systems involves modeling systems that produce their own components (in accordance with how Kant and Schelling understood living organisms). To do this, they require models of themselves (as von Neumann argued). Such systems, Rosen showed, can be represented through synthetic models in which functional components are the direct product of the system. In these models the components are context dependent, and cannot be reduced to fractional parts conceivable independently of the models. Such systems are complex, but not as mainstream complexity theory understands complexity. This theory, Rosen claimed, had not freed itself from Newtonian assumptions and dealt only with the complicated. Genuine complexity requires multiple formal descriptions which are not derivable from each other, to capture all their properties. The example Rosen produced to illustrate this was his metabolism, repair, reproduction models (the M-R systems). These models consist of three algebraic maps, one of which represents the efficient cause of metabolism in a cell, another, the efficient cause of

36 Rosen, *Life Itself*, 28ff.

repair (that repairs damage to the metabolic processes), and the third represents replication which repairs damage to the repair process.[37] Each of these maps has one of the other two as a member of its co-domain, and is itself a member of the co-domain of the remaining map. The maps thus form a loop of mutual containment. As Rosen put it: '*a material system is an organism if, and only if, it is closed to efficient causation*. That is, if f is any component of such a system, the question "why f" has an answer within the system, which corresponds to the category of efficient cause of f.'[38] On the basis of such models it is possible to appreciate the ability of complex systems to incorporate models of themselves in their environments into their behaviour, anticipating future events and correcting their behaviour as new information sheds light on the anticipatory process.[39]

Creating a New Mathematics

Rosen's work has freed mathematicians from Newtonian assumptions to explore the possibilities opened up by category theory. He has been a source of inspiration for an increasing number of mathematicians and theorists, beginning with his students. A.H. Louie, the most prominent of his students, subsequently published *More Than Life Itself: A Synthetic Continuation in Relational Biology,* and *The Reflection of Life: Functional Entailment and Imminence in Relational Biology.*[40] However, Rosen and his students are not the only mathematicians who have embraced this project of using category theory, and in doing so, have transcended Newtonian assumptions to develop a process relational view of reality. Andrée Ehresmann and Jean-Paul Vanbremeersch in *Memory Evolutive Systems* began by noting that while it is necessary for humans to distinguish objects and their relations, we should not allow ourselves to

37 Ibid., 248ff.

38 Ibid., 244.

39 Rosen, *Essays on Life Itself,* 199.

40 A.H. Louie, *More Than Life Itself: A Synthetic Continuation in Relational Biology* (Frankfurt: Ontos Verlag, 2009) and *The Reflection of Life: Functional Entailment and Imminence in Relational Biology* (Berlin: Springer, 2013).

be dominated by the very limited notion of objects as physical objects located in space; these should include 'a musical tone, an odour or an internal feeling. The word *phenomenon* (used by Kant, 1790) or *event* (in the terminology of Whitehead, 1925) would perhaps be more appropriate.'[41] An 'object' can be a body, property, event, process, conception, perception or sensation, and it is also necessary to take into account more or less temporary relations between such objects. As Ehresman and Vanbremeersch put it: 'Long ago, the Taoists imagined the universe as a dynamic web of relations, whose events constitute the nodes; each action of a living creature modifies its relations with its environment, and the consequences gradually propagate to the whole of the universe.'[42] They argued that while Rosen recognized the potential of category theory, he did not fully develop it. They suggest that the role of categories in Rosen's models of metabolism and repair and of organismic systems are often purely descriptive, and do not exploit the deep results of category theory. They claim that in their model, 'we make use of fundamental constructions, to give an internal analysis of the structure of the dynamics of the system.'[43] They then described their efforts to characterize this in a mathematical model in which 'the successive configurations of a system, as defined by its components and the relations among them around a given time, will be represented by *categories*; the changes among configurations by *functors*. The evolution of the system will mostly depend on the interactions between agents at various levels of complexity, acting with different time scales.'[44] Memory evolutive systems are multi-scale, multi-agent and multi-temporal and analyse changes, from an internal, or 'endo' perspective, through a net of internal agents acting as co-regulators. Involving a family of categories indexed over time, these are able to model a complexification process internally selected by the net of co-regulators capable of creativity.[45]

The work of Rosen and Ehresmann has stimulated further efforts

41 Ehresmann and Vanbremeersch, *Memory Evolutive Systems,* 21.

42 Ibid., 33.

43 Ibid., 33.

44 Ibid., 21 & 22.

45 Ibid.

to develop mathematics adequate to life, notably the non-reductionist biomathematics, or 'integral biomathics', exemplified in Plamen Simeonov. Simeonov led the work to produce a major anthology on integral biomathics which he edited along with Leslie Smith and Andrée Ehresmann,[46] and two special editions of *Progress in Biophysics & Molecular Biology,* the first which, published in 2013, he edited with Koichiro Matsuno and Robert Root-Bernstein,[47] and the second with Steven Rosen and Arran Gare, published in 2015.

CONCLUSION

These developments in mathematics are based on fundamentally different conceptions of what mathematics is and of its role in science than those that led philosophers to deny the reality of change, creativity and life. Mathematics is no longer assumed to be about what *is*, and only then about transformations, or that it is first and foremost about objects, and only in terms of these, about relations. Also abandoned is the assumption that success in understanding any item in the world is achieved when a largest model can be found from which all other models applicable to it can be deduced, and therefore that the ultimate goal of science is to find the equations modeling the whole universe through which all other features of the universe can be deduced. Furthermore, the Pythagorean assumption that mathematics by itself is capable of modeling every aspect of nature is abandoned. With the new conception of mathematics, we can now view mathematics as playing a major part in comprehending a creative universe rather than explaining away the appearance of creativity. Since living beings are seen to have models of themselves in their environments and can be modeled as such, we can now see more clearly through mathematics, final causes and activities as transformations, and how mathematical patterns, or forms of

46 Plamen L. Simeonov, Leslie L. Smith and Andrée C. Ehresmann, *Integral Biomathics: Tracing the Road to Reality* (Berlin: Springer-Verlag, 2012).

47 Simeonov, Plamen L., K. Matsuno and R.S. Root-Bernstein, "Can Biology Create a Profoundly New Mathematics and Computation?" Focussed Issue of *Progress in Biophysics & Molecular Biology* 113 (2013).

definiteness, can ingress in nature. All this requires recognition of the place of synthesis in experience, in mathematical work, in developing models of processes, and in what is modeled through mathematics, and all such synthesis involves constructive intuition. This recognition should free mathematicians to advance further this work of creating a new mathematics.