

From a Mereotopological Point of View: Putting the Scientific Magnifying Glass on Kant's First Antinomy



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Abstract

In his *Critique of Pure Reason* Immanuel Kant presents four antinomies. In his attempt to solve the first of these antinomies he examines and analyzes “thesis” and “antithesis” more thoroughly and employs the terms ‘part’, ‘whole’ and ‘boundary’ in his argumentation for their validity. According to Kant, the whole problem surrounding the antinomy was caused by applying the concept of the world to nature and then using both terms interchangeably. While interesting, this solution is still not that much more than a well thought out idea if it does not also include an adequate formal explication. Since the aforementioned terms all have counterparts in modern mereotopology, a discipline that has seen significant progress in recent times, we will apply these concepts to Kant’s analysis in an attempt to evaluate Kant’s solution in light of modern analytic philosophy.

1 Introduction

The first step of our analysis (section 2) will be an illustration of the first antinomy and a concise summary of Kant’s reaction to it. We will also examine how it ultimately led to the position of Transcendental Idealism. In this section we shall also argue motivations why mereotopology seems to be a suitable means for representing Kant’s thoughts in this matter.

In section 3 a mereotopological system (GEMT^M) adequate for representing extensional domains and the integral terms of Kant’s argumentation will be introduced, together with some explanations of those axioms and definitions which are more difficult to understand. In section 4 we will then apply said system to Kant’s views, draw the consequences from our results and will then summarize them to provide a future outlook in section 5. The appendix contains an overview of the relevant axioms, definitions and theorems of GEMT^M .

2 Taking a closer look at Kant's first antinomy

Thesis and antithesis of the first antinomy as it is found in Kant's *Critique of Pure Reason* (cf. [2, pp. B 454-455]):

Thesis

The world has a beginning in time and is enclosed in spatial boundaries.

Antithesis

The world has no beginning and is not enclosed in boundaries but is infinite regarding time as well as space.

It is possible to provide (at least intuitively) convincing arguments for both thesis and antithesis:¹

Thesis

REGARDING TIME For us the world is the whole of all spatial parts (or at least it is difficult to imagine it to be something different) in order of their temporal occurrence. This whole is built upon by successively adding one temporal part after another as it occurs in time. No matter how many parts are added to a whole it will always remain finite as long as its base or any of the parts added are not infinite. Since, as of yet, no one has ever experienced an infinite event, it is highly implausible that there are such things as temporally infinite parts, thus suggesting the presupposition of an infinite base provided the world is temporally infinite. As such, the only way to talk about a world with no temporal beginning would be to assume an "infinite beginning", which is simply counterintuitive and certainly not what anyone could mean when they talk about a beginning. Therefore a temporal boundary of the world, as expressed in the thesis, is highly plausible.

REGARDING SPACE The world encompasses all spatial parts at every point in time. Since there has never been a reported observation of an infinite spatial entity, it is reasonable to assume that spatial parts are always finite. It is difficult to imagine an entity consisting of all those spatial parts as being of a different structure than that of the parts it contains, therefore suggesting that the world itself is finite.

¹These are not the reasons given by Kant in his *Critique of Pure Reason*, but are intuitively easier to understand and less opaque while still being analogous enough to Kant's argumentation. For Kant's original argumentation for the validity of both thesis and antithesis of the first antinomy refer to [2, pp. B 455-457].

Antithesis

REGARDING TIME Assumed that time has a beginning, then there would be a corresponding boundary marking said beginning of time. All known meanings of ‘boundary’ have one basic intuition in common: the separation of one thing from another. This intuition, however, is hardly applicable to our natural understanding of ‘time’: If time had indeed a beginning, then there would have to be something beyond its border with no temporal attributes whatsoever. What strange kind of entity could this possibly be considering we only ever experience entities in time? Moreover, does the question even make sense, when we take into account that we are trying to speak of a non-temporal entity to exist *before* time? ‘Before’ and ‘after’ serve to distinguish events by their temporal order and are therefore not suitable for a non-temporal entity.

REGARDING SPACE A thought similar to the one regarding the implausibility of a beginning of time applies here: Spatial boundaries always serve to separate one spatial entity from another. What then would be beyond a boundary encompassing all spatial entities? What could possibly be beyond it that is supposedly not spatial but at the same time, by the very meaning of the word ‘spatial boundary’, hardly imaginable to be of anything else than spatial nature? This is not just counterintuitive, it is hardly comprehensible.

For thesis and antithesis to actually result in a contradiction, it is necessary for every term appearing in both sentences to do so with the same meaning. This holds true particularly for the word ‘world’ which, according to Kant, violates the aforementioned principle:

“Hieraus erhellt, dass der Obersatz des kosmologischen Vernunftschlusses das Bedingte in transzendentaler Bedeutung einer reinen Kategorie, der Untersatz aber in empirischer Bedeutung eines auf blosser Erscheinungen angewandten Verstandesbegriffes nehmen, folglich derjenige dialektische Betrug darin angetroffen werde, den man *sophisma figurae dictionis* nennt.” ([2, pp. B 527-528])

While typically convoluted and difficult to understand in this original formulation of Kant, Leonard Nelson offers a more comprehensible account of the underlying basic idea:²

The antinomy disappears the very moment in which we stop using ‘world’ and ‘nature’ synonymously. (cf. [3, p.243])

²For another more comprehensible account see the discussion of Bernard Bolzanos critique from Arto Siitonen in [4, p.89].

In his then newly introduced position called “Transcendental Idealism” Kant further argues that it is not even possible anymore to speak of the world as something spatially and/or temporally limited or unlimited beyond our experience, which would effectively eliminate any speculations of such nature. Further, he is convinced that nature, while being the embodiment of all that is spatial and temporal, does not exist independently of its subjects, suggesting that there is indeed no objective boundary of nature.³

Transcendental Idealism is not just some hypothesis which aids Kant in solving the antinomy but he is, without a doubt, guided to his solution by the antinomy itself. (cf. [3, p.243])

It can be said that Transcendental Idealism is Kant’s response to and solution of his first antinomy. In his analysis of the antinomy Kant brings the terms ‘part’, ‘whole’ and ‘boundary’⁴ into play – terms which have undergone significant development and clarification in modern mereotopology. Together with the fact that Kant and many of his contemporaries thought the first antinomy to be a major obstacle which had to be overcome, it would now be interesting to try and see in the light of modern mereotopology, whether the antinomy was justly perceived as a philosophical obstacle that ultimately led Kant to his Transcendental Idealism or whether it only seems to pose a problem and therefore cannot be an adequate motivation for epistemological alternatives such as Kant’s aforementioned solution.

3 *Whole, parts and boundaries*

In our analysis we will use a modified version (GEMT^M) of the mereotopological system GEMT (General Extensional Mereotopology) introduced by Roberto Casati and Achille Varzi⁵. We presuppose a first order logic with identity and begin with the basic mereological framework GEM. In addition to the standard quantifiers and sentential connectives of predicate logic with identity ‘ $\neg$ ’, ‘ $\wedge$ ’, ‘ $\vee$ ’, ‘ $\rightarrow$ ’, ‘ $\leftrightarrow$ ’, ‘ $=$ ’, ‘ $\forall$ ’ and ‘ $\exists$ ’ we shall also use ‘ $\leftrightarrow_{df}$ ’ and ‘ $=_{df}$ ’ to indicate definitions. ‘ $\prec$ ’ will signify our basic relation: *Proper Part*.

‘GEM’ stands for ‘General Extensional Mereology’ and denotes the mereological standard system for representing extensional (and with that

³Cf. [2, pp. B 518 ff].

⁴Cf. [2, pp. B 452-457]

⁵Cf. [1]

also spatial and temporal) structures. GEM allows, unlike many other mereological systems, introduction of an operator for the general sum, thus facilitating speech of the world as the sum of all extensional objects, which is why GEM seems especially suited to analyzing space and time in regards to Kant's first antinomy. GEM^M (General Extensional Mereotopology Modified) further expands GEM by the topological basic relation *Connectedness* by which a relation for x is a boundary of y (the focal point of this whole undertaking) can be expressed.

With that said, we shall begin our brief foray into the world of mereotopology. Our basic relation *Proper Part* is both transitive and asymmetric, as is expressed by the following two axioms:

$$(A.1) \quad x \prec y \wedge y \prec z \rightarrow x \prec z \quad (Transitivity)$$

$$(A.2) \quad x \prec y \rightarrow \neg y \prec x \quad (Asymmetry)$$

Also, the mereological standard relations *Improper Part* and *Overlap* as well as, for practical reasons, the relation *Disjointedness* can be defined:

$$(D.1) \quad x \preceq y \leftrightarrow_{df} x \prec y \vee x = y \quad (Improper Part)$$

$$(D.2) \quad x \circ y \leftrightarrow_{df} \exists z(z \preceq x \wedge z \preceq y) \quad (Overlap)$$

$$(D.3) \quad x[y] \leftrightarrow_{df} \neg x \circ y \quad (Disjointedness)$$

With these we can add the following two axioms⁶ and arrive at the aforementioned basic framework GEM:

$$(A.3) \quad x \prec y \rightarrow \exists z(z \preceq y \wedge z[x]) \quad (Weak Supplementation Principle)$$

$$(A.4) \quad \exists x A \rightarrow \exists z \forall y (y \circ x \leftrightarrow \exists x (A \wedge y \circ x)) \quad (Fusion Principle)$$

While the *Weak Supplementation Principle* is fairly self explanatory, it is a little bit more difficult at first glance, to grasp the meaning and benefits of the *Fusion Principle*. In a nutshell it says that for any number of objects there exists always one entity consisting of those objects (i.e. their sum). Therefore a definition of the *General Sum* seems to be the

⁶ 'A' in (A.4) and the following sentences (i.e. (T.1), (D.4) and (T.2)) is a schematic variable and stands for any formula of our system containing no free variables but x . Thus (A.4) is, strictly speaking, an axiom schema.

appropriate next step. Prior to defining our general sum, we must first ensure that there is exactly one and only one object that fulfills the requirements above. Fortunately there is a theorem in our system that tells us exactly that:

$$(T.1) \quad \exists x A \rightarrow \exists! z \forall y (y \circ z \leftrightarrow \exists x (A \wedge y \circ x))$$

And that is all we need for our partial definition of the operator σ for the *General Sum*:

$$(D.4) \quad \exists x A \rightarrow \forall z (\sigma x A = z \leftrightarrow_{df} \forall y (y \circ z \leftrightarrow \exists x (A \wedge y \circ x)))$$

(*General Sum*)

$$(T.2) \quad \exists x A \rightarrow \exists! z (z = \sigma x A)$$

(T.2) is a convenient theorem that shows that for any number of objects there is exactly one entity that is their sum. Following the initial intuition that the world is the sum of all things, the next step seems obvious. First one needs to guarantee that there is exactly one such sum (which, conveniently enough, another theorem (i.e. (T.3)) provides). We can then continue to introduce and define an individual constant W for the world:

$$(T.3) \quad \exists! z (z = \sigma x (x \preceq x))$$

$$(D.5) \quad W =_{df} \sigma x (x \preceq x)$$

(*World*)

There is one last useful tool for speaking of borders that can still be formulated without having to modify GEM: the *Relative Complement*:

$$(T.4) \quad x \prec z \rightarrow \exists! y (y = \sigma u (u \preceq z \wedge u] [x))$$

$$(D.6) \quad x \prec z \rightarrow x'z =_{df} \sigma u (u \preceq z \wedge u] [x)$$

(*Relative Complement*)

Given two entities x and z so that x is a proper part of z , then the relative complement $x'z$ is simply the sum of all parts of z that do not overlap x . Without the proper part prerequisite it would be necessary to introduce a null individual that is part of everything (i.e. the mereologist's equivalent to the empty set in set theory). While, from a systematical standpoint, the *Relative Complement* is a quite comfortable and powerful tool, the existence of such an entity is – at least when applied to extended domains like space and time – hardly imaginable. So in order to keep the system applicable to Kant's first antinomy (at least we do not think Kant had null individuals in mind when writing

his *Critique of Pure Reason*) we shall refrain from introducing such a null individual and partially define the relative complement $x'z$ instead.

This system (GEM) is already quite rich in content but not yet powerful enough to allow us to speak of boundaries (our initial goal) as the terms available through GEM are not sufficient for an adequate definition of ‘boundary’, which is why we need at least one other basic relation to achieve such a definition. We choose *Connectedness* as this basic relation, not only because it is intuitively plausible that spatial or temporal borders require at least some kind of connection between objects but also because the axioms needed are uncomplicated, easy to understand, plausible and they determine that the relation behaves well within the system. In this context we shall speak of two objects as connected ($\bowtie$) simply if they touch or overlap. In adding the following three axioms for *Connectedness* we arrive at GEMT^M:

$$(A.5) \quad x \bowtie x \quad (Reflexivity)$$

$$(A.6) \quad x \bowtie y \rightarrow y \bowtie x \quad (Symmetry)$$

$$(A.7) \quad x \preceq y \rightarrow \forall z(z \bowtie x \rightarrow z \bowtie y)$$

It is important to note that, while they seem quite similar, *Connectedness* and *Overlap* are not the same: Two objects that overlap are also connected but not necessarily vice versa (e.g. the right and left half of a ball, while touching each other, have not a single part in common).

The basic idea behind our attempt to define borders is the following: Given any object y of which we want to determine a boundary x , we can get said boundary x by first cutting off the interior of y and then cutting off all surroundings. Now any remaining x is a boundary of y . To express this intuition in GEMT^M we first need to define the relation *Internal Part*:

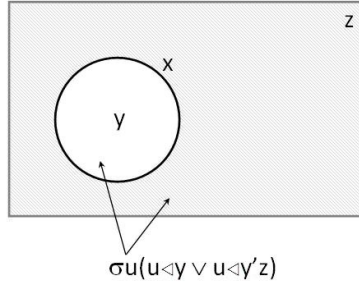
$$(D.7) \quad x \triangleleft y \leftrightarrow_{df} x \prec y \wedge \forall z(z \bowtie x \rightarrow z \circ y) \quad (Internal Part)$$

For any x to be an internal part of a y it is essential that (a) x is a part of y but not just any part but a *proper part* of y , as it is implausible that an object could be an *interior* part of itself or not something that is in at least some kind of way “smaller” than the object containing it; (b) every z that touches x must have at least one part with y in common (thus expressing the intuition that for every internal part x of an object y there is also a part of y that is between x and the surface of y).

Now we can finally try to find an adequate definition of the relation

Boundary ($\sqcap$) which can be used to represent (in GEMT^M) what we mean by saying that an object x is a boundary of an object y . Our aforementioned “cutting process” requires at least one object z containing y so that we can then eventually arrive at our desired boundary x of y . This can be graphically illustrated as follows:

Fig. 1



For an object x to be called a *boundary of y relative to z* ($x \sqcap [y^{(z)}]$) it has to fulfill the following requirements: it must not overlap any u that is an internal part of y or of its relative complement $y'z$ and the object has to actually be an internal part of z . In fig. 1 x fulfills these requirements, however, it is of note that fig. 1 hardly covers every case concerning boundaries and therefore we will limit our definition to only such cases where y is an internal part of z (as it is the case in fig. 1). With this we arrive at the following partial definition of the relation x is a boundary of y relative to z :

$$(D.8) \quad y \triangleleft z \rightarrow \forall x(x \sqcap [y^{(z)}] \leftrightarrow_{df} x \triangleleft z \wedge \forall u(u \triangleleft y \vee u \triangleleft y'z \rightarrow u)[x])$$

(*Boundary*)

Now we have everything we need to examine Kant's first antinomy from a mereotopological point of view.

4 Kant under the mereolognifying glass

The most important term in our analysis of Kant's first antinomy is certainly ‘boundary’, as such, let us take a closer look at the sentence expressing it (i.e. (D.8)). Since the definition makes use of the relative complement $y'z$ which was itself partially defined only for cases in which $y \prec z$, the same presupposition has to apply to (D.8). Having this as our sole presupposition of (D.8) could lead to problems when it comes

to objects y that lie at the edge of z .⁷ To avoid such problems we will further demand the stronger presupposition $y \triangleleft z$ (in which $y \prec z$ is included). This means that in a system such as GEMT^M , that is built on axioms that are as plausible as possible in order to keep it applicable to natural contexts, we can speak of something being a boundary of an object only if said object is an internal part of another object containing it, which is something to be kept in mind.

This leads us to another important term: ‘internal part’. Here it has to be noted that the usual approach is to define the internal part relation via the improper part relation.⁸ Such a definition, however, would pave the way for theorems such as $W \triangleleft W$ which is obviously not unproblematic and in dire need of justification. Because of this it is advisable to define it via the proper part relation instead.

Now that the two most important definitions ((D.7) and (D.8)) have been made sufficiently clear it is finally time to shift our focus to Kant. He speaks of the world as something spatially and temporally extended, which can be accounted for by giving separate interpretations of the mereotopological world for spatial and temporal contexts respectively. Since the whole problem revolves around the question of the world and its supposed boundaries, the crucial point of our analysis is determining whether one or both of the following sentences is/are a theorem/theorems of our mereotopological system GEMT^M :

$$(*1) \quad \exists x \exists z (x \lceil W^{(z)})$$

$$(*2) \quad \neg \exists x \exists z (x \lceil W^{(z)})$$

If one of these sentences would be a theorem in our system we could confirm the existence or nonexistence of the boundaries of space and time. In any case such a theorem would allow us to *speak* of the world’s boundaries (whether they are spatial or temporal, existent or non-existent).

Interestingly neither sentence is deducible in GEMT^M : The relation *Boundary* was partially defined via the internal part relation which means that at most the following two sentences could be (and actually are) theorems of GEMT^M :

$$(**1) \quad \exists z (W \triangleleft z) \rightarrow \exists x \exists z (x \lceil W^{(z)})$$

⁷E.g. presuming that one allows the border of an object to be a part of that same object, the sum of the border of z and the border of y would itself be a boundary of y , which is obviously not what one would expect from a boundary.

⁸Cf. [1]

$$(**2) \quad \exists z(W \triangleleft z) \rightarrow \neg \exists x \exists z(x \lceil W^{(z)})$$

Since there is no z containing the mereotopological world W there is no case in which our presupposition is fulfilled, which means that we cannot speak of the world as spatially or temporally limited and neither can we deny it, thus arriving at a surprisingly similar result as Kant who (as has been stated in section 2) regarded talk of the world and its supposed boundaries as meaningless. Seeing as our aforementioned presupposition does not allow us to deduce any of the desired sentences (i.e. (*1) and (*2)), why do we not simply build a more adequate system without such a seemingly clunky prerequisite? Simply because any definition of boundary requires reference to the relative complement which itself cannot be defined without the presupposition $x \prec z$ (see section 3) and there is no z fulfilling it for those cases in which x is the world.

5 Conclusion

Our analysis has led us to two very interesting results – the first regards Kant's analysis of the first antinomy, the second, the antinomy itself. Regarding Kant's view of space and time after his analysis of the first antinomy we can say that, if our analysis is adequate, it is indeed not reasonably possible to reach beyond our experience and speak of the world as something limited or unlimited regarding space and/or time, thus somewhat validating one's choice to pursue alternatives such as Transcendental Idealism from a mereotopological standpoint.

Regarding the antinomy itself, however, this means that Kant's claim that both thesis and antithesis can be convincingly argued for seems rather difficult to uphold, since such an argumentation would require the possibility to speak of the world and its existent or nonexistent spatial and/or temporal borders in a meaningful manner, which is simply not the case. This raises an important question for the transcendental idealist: If it is true that the first antinomy is an essential motivator for Transcendental Idealism, what else does Transcendental Idealism have going for it when it is robbed of said motivator? Is there any ground left that could validate this position, or does it inevitably crumble like the first antinomy itself?

Appendix - the system $GEMT^M$

$$(A.1) \quad x \prec y \wedge y \prec z \rightarrow x \prec z$$

$$(A.2) \quad x \prec y \rightarrow \neg y \prec x$$

$$(D.1) \quad x \preceq y \leftrightarrow_{df} x \prec y \vee x = y$$

$$(D.2) \quad x \circ y \leftrightarrow_{df} \exists z(z \preceq x \wedge z \preceq y)$$

$$(D.3) \quad x][y \leftrightarrow_{df} \neg x \circ y$$

$$(A.3) \quad x \prec y \rightarrow \exists z(z \preceq y \wedge z][x)$$

$$(A.4) \quad \exists x A \rightarrow \exists z \forall y(y \circ z \leftrightarrow \exists x(A \wedge y \circ x))$$

$$(T.1) \quad \exists x A \rightarrow \exists! z \forall y(y \circ z \leftrightarrow \exists x(A \wedge y \circ x))$$

$$(D.4) \quad \exists x A \rightarrow \forall z(\sigma x A = z \leftrightarrow_{df} \forall y(y \circ z \leftrightarrow \exists x(A \wedge y \circ x)))$$

$$(T.2) \quad \exists x A \rightarrow \exists z(z = \sigma x A)$$

$$(T.3) \quad \exists z(z = \sigma x(x \preceq x))$$

$$(D.5) \quad W =_{df} \sigma x(x \preceq x)$$

$$(T.4) \quad x \prec z \rightarrow \exists! y(y = \sigma u(u \preceq z \wedge u][x))$$

$$(D.6) \quad x \prec z \rightarrow x'z =_{df} \sigma u(u \preceq z \wedge u][x)$$

$$(A.5) \quad x \boxtimes x$$

$$(A.6) \quad x \boxtimes y \rightarrow y \boxtimes x$$

$$(A.7) \quad x \preceq y \rightarrow \forall z(z \boxtimes x \rightarrow z \boxtimes y)$$

$$(D.7) \quad x \triangleleft y \leftrightarrow_{df} x \prec y \wedge \forall z(z \boxtimes x \rightarrow z \circ y)$$

$$(D.8) \quad y \triangleleft z \rightarrow \forall x(x[y^{(z)} \leftrightarrow_{df} x \triangleleft z \wedge \forall u(u \triangleleft y \vee u \triangleleft y'z \rightarrow u][x))$$

Symbols

$x \prec y$ x is a proper part of y

$x \preceq y$ x is an improper part of y

$x \circ y$ x overlaps y

$x][y$ x is disjointed from y

$\sigma x A$ the sum of all x , that A

W the world

$x'z$ the complement of x relative to z

$x \boxtimes y$ x is connected to y

$x \triangleleft y$ x is an internal part of y

$x[y^{(z)}$ x is a boundary of y relative to z

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