Social Epistemology Review and Reply Collective, 2015
Vol. 4, No. 6, 52-61. http://wp.me/p1Bfg0-29f

Symbol Systems as Collective Representational Resources: Mary Hesse, Nelson Goodman, and the Problem of Scientific Representation

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This short paper grew out of an observation—made in the course of a larger research project—of a surprising convergence between, on the one hand, certain themes in the work of Mary Hesse and Nelson Goodman in the 1950/60s and, on the other hand, recent work on the representational resources of science, in particular regarding model-based representation. When it comes to scientific models, a number of recent authors have emphasized their status as 'concrete artefacts, which are built by various representational means' (Knuuttila 2011, 267), the importance of 'mature mathematical formalisms' which encompass 'locally applicable rules for the manipulation of [their] notation' (Gelfert 2011, 272), and model users' reliance on other ways 'to alleviate the cognitive load and increase the reliability of [their] inferences' (Kuorikoski and Ylikoski 2014, 7).

The convergence between these more recent accounts of representation in science and the earlier proposals by Hesse and Goodman consists in the recognition that, in order to secure successful representation in science, *collective representational resources* must be available. Such resources may take the form of (amongst others) mathematical formalisms, diagrammatic methods, notational rules, or—in the case of material models—conventions regarding the use and manipulation of the constituent parts.

More often than not, an abstract characterization of such resources tells only half the story, as they are constituted equally by the pattern of (practical and theoretical) activities—such as instances of manipulation or inference—of the researchers who deploy them. In other words, representational resources need to be sustained by a social practice; this is what renders them *collective* representational resources in the first place.

Symbol Systems and Scientific Representation

My main concern in what follows will be with *symbol systems* as representational resources, examples of which include mathematical formalisms as deployed in particular subdisciplines (e.g., the formalism of creation and annihilation operators in quantum field theory), diagrammatic methods (e.g., the Feynman diagrams of high-energy physics), visual symbol systems (e.g., structural formulas in organic chemistry), numerical notations, and so forth. While a full definition of what constitutes a symbol system is beyond the scope of this paper, typically such systems would require that well-formed arrangements (e.g., marks on paper, figures in a table, etc.) can be registered semantically as instances of a particular character (that is, such systems would be, in Goodman's terminology, 'syntactically articulate'), and that certain other features (such as compositionality) allow for the manipulation and interpretation of expressions within that system.

For the limited purposes of this paper, a working knowledge of the general character of symbol systems suffices—whether such general knowledge derives from one's familiarity with natural (or formal) languages, or from specific examples of symbol systems in the various sciences. In this sense, my usage of the term 'symbol system' is entirely generic; nothing much in what follows is likely to depend on the finer details of what constitutes a symbol system. In particular, I wish to emphasize that my discussion does not bear on the *physical symbol system hypothesis* put forward by Allen Newell and Herbert Simon in 1976, which asserts that a physical system that manipulates symbols, such as a digital computer, 'has the necessary and sufficient means for intelligent action' (Newell and Simon 1976, 116).

Newell and Simon's thesis implies that by providing a digital computer with suitable symbol-processing capabilities (e.g., by feeding it with an appropriate computer program) would render it capable of intelligent (human-like) action. (Furthermore, Newell and Simon also argued for the reverse: namely, that 'the symbolic behavior of man arises because he has the characteristics of a physical symbol system'; ibid.)

The symbol systems referred to in what follows are thoroughly *human* constructs: students learn them when being educated in their disciplines, scientists routinely deploy them (both individually, and in order to communicate with other scientists), and they are subject to—sometimes gradual, sometimes more radical—changes and modifications. Whether or not machines can be programmed to successfully deploy them is, for the purposes of this paper, of little importance.

Hesse on Mathematical Formalisms

Best-known for her view of scientific models as analogies (Hesse 1963), Mary Hesse situates her discussion within a broader project of analyzing how science and mathematics relate to one another. In her first major paper on the topic, published as 'Models in Physics' in *The British Journal for the Philosophy of Science* in 1953, Hesse sets out to defend two theses, the second of which is put modestly, in the form of a suggestion:

> I shall suggest that *most physicsts do not regard models as literal descriptions of nature, but as standing in a relation of analogy to nature* (Hesse 1963, 201; emphasis original).

It is this suggestion—suitably elaborated and qualified, given that Hesse is fully aware of the shifting meanings of the term 'analogy'—that has received considerable philosophical attention over the years. Hesse's work is best understood against the backdrop of a tradition that largely equated scientific models with either material or mechanical models. On this view, which derives from 19th-century scientific usage, a typical scientific model is 'a (real or imagined) concrete, material representation of something' (De Regt, 2005, 215). Obvious examples would include the billiard ball model of an ideal gas, or Maxwell's vortex model of the ether.

By the time Hesse embarked on her analysis of models and analogies, the term 'model' had proliferated beyond the realm of the mechanical or 'picturable' to also include, for example, quantum-mechanical models. Indeed, it was partly this proliferation of the use of the term 'model'—expressed in the claim that a model can be 'any system, whether buildable, picturable, imaginable, or none of these' (Hesse 1963, 21)—which motivated Hesse's project. Hence, it is not surprising that Hesse's intervention sparked a lively debate about the proposed analogical character of models, in classical as well as in quantum physics.

However, Hesse's thesis concerning the analogical character of models is preceded by another thesis, which she puts as follows:

> *Mathematical formalisms, when used as hypotheses in the description of physical phenomena, may function like the mechanical models of an earlier stage in physics, without having in themselves any mechanical or other physical interpretations* (Hesse 1953, 199; emphasis original).

Hesse here points to the significance of extant (established) mathematical formalisms, which are likened to the way mechanical models were used as tools of inquiry during earlier (presumably less theoretically-unified) stages in physics. Assuming, as seems legitimate, that the parallel with mechanical models is a deliberate one, one can then focus on specific features that both have in common.

One common feature is defined by the goal of *scientific representation*; this, after all, is what models—whether material or mathematical—aim at (which, needless to say, does not preclude that models often serve a variety of other epistemic and non-epistemic goals). Furthermore, mathematical formalisms may be employed in a suppositional mode: they may be 'used as hypotheses in the description of physical phenomena' and, like hypotheses, may only be vindicated in the long run. (Of course, particular choices of mathematical formalisms may also turn out to have been misguided, insofar as they fail to be fruitful.)

Hesse subsequently gives a fuller characterization of the contribution of mathematical formalisms to some of the newer scientific developments at the time:

> The question then arises, what takes the place in these physical theories of the pointers towards further progress which are provided by an easily pictured mechanical model? I shall suggest that what takes their place is provided by the nature of the mathematical formalism itself—any particular piece of mathematics has its own ways of suggesting modification and generalisation; it is not an isolated collection of equations having no relation to anything else, but is a recognisable part of the whole structure of abstract mathematics, and this is true whether the symbols employed have any concrete physical interpretation or not (Hesse 1953, 200).

Three points are worth highlighting in this passage. First, Hesse credits mathematical formalism with providing 'pointers towards further progress', not unlike how picturable mechanical models in the past have often been suggestive of new directions of research.

The tentative language is important here: picturable mechanical models as well as mathematical formalisms *point towards* progress and *suggest* new steps of modification and generalization, rather than *logically entailing* them or making them otherwise inevitable.

Second, mathematical formalisms do not merely accommodate whatever hypotheses one may already have come up with regarding the target system, but instead also *constrain* the ways in which a system can be represented. As a '*particular* piece of mathematics' each formalism 'has its *own* ways of suggesting modification and generalisation' (ibid; emphasis added); that is, every choice of a particular mathematical formalism involves a trade-off between certain affordances and constraints.

Third, though this is largely implicit, there is an undeniably pragmatic dimension to Hesse's characterization of mathematical formalisms, in that it acknowledges the user (and audience) of such theoretical tools. It falls to those employing mathematical formalisms for representational purposes to employ the right sorts of symbol systems (i.e., those that are fit for the intended purposes), to pick up on promising lines of research, and to recognize the place—as well as the limitations—of the deployed formalisms within both mathematics-at-large and the research programme in question.

Goodman on Representational Resources

In his seminal book *Languages of Art* (1976; first published in 1968), subtitled 'An Approach to a Theory of Symbols', Nelson Goodman aimed to bring together previously separate debates about the nature of representation in language, art, and science, by exploring how signs in the different domains allow us to refer to the various target systems. Philosophers, of course, have long distinguished between *natural* and *non-natural* signs. Whereas non-natural signs (e.g. words in a language) acquire their meaning largely arbitrarily, by way of convention, natural signs (e.g., smoke as a likely sign of fire) stand in certain causal relations that connect them to their target.

One might ask where we should locate representation along this spectrum ranging from natural to non-natural signs—not least since, surely, we cannot plausibly demand that a representation must be causally dependent on its target. Goodman, for this purpose, introduces the notion of 'denotation' in order to capture an important ingredient in any representational relationship, *viz.* the fact that a representation 'stands in for' its target.

Understood along those lines, denotation frees the representational relationship from the constraints of causality or resemblance, in that it may be entirely stipulative: 'almost anything may stand for almost anything else' (Goodman 1976, 5). It does not follow, of course, that any attempted act of denotation will automatically succeed at representing a target. For one, in order for there to be an instance of denotation, what is being denoted must exist. (I will leave aside Goodman's discussion of how we may nonetheless legitimately speak of representation-*types* when there are no actual *tokens* that can be denoted directly.)

Importantly, though, successful representation—even when the target exists—requires *more* than mere denotation. While, on occasion, Goodman gives the impression that anything may represent anything else, he is well aware of the fact that *successful* representation is de facto heavily constrained. Even before issues of faithfulness, accuracy, and truth arise, there is the question of whether a given representation makes relevant information salient and whether it can draw on entrenched denotative practices: 'Representation […] is apt, effective, illuminating, subtle, intriguing, to the extent that' its originator 'grasps fresh and significant relationships and devises means for making them manifest' (ibid, 32-33).

As Goodman remarks, 'there are countless alternative systems of representation and description', which themselves 'are the products of stipulation and habituation in varying proportions': we often find that we inherit established notational systems and representational formalisms, which—though in principle arbitrary—are no longer "up to us". While we have some degree of choice among such systems, given a particular system 'the question whether a newly encountered object is a desk or a unicorn-picture […] is a question of the propriety, under that system, of projecting the predicate "desk" or the predicate "unicorn-picture" […], and the decision both is guided by and guides usage for that system' (ibid, 30-31).

Representation of a target system thus arises from the interplay of denotation and the various factors that determine whether one thing can successfully 'stand in for' another. While denotation affords considerable (though not unlimited) latitude, the factors that determine the success of one's representation (or attempt at representation) are at least partly external, given that a successful representation must make relevant features of the target system manifest to its user.

It might seem natural to interpret Goodman's proposal along individualistic lines, for example by interpreting talk about 'encountering' an object, or about making a 'decision' to prefer some predicates over others, as referring to the autonomous judgments and actions of an individual cognizer. Whether a representation is successful then might seem to be largely a matter of the (individual) user's intentions and epistemic interests. Indeed, this seems to be the preferred view of some recent commentators who insist, as Craig Callender and Jonathan Cohen do, that 'the varied representational vehicles used in scientific settings (models, equations, toothpick constructions, drawings, etc.) represent their targets (the behavior of ideal gases, quantum state evolutions, bridges) by virtue of the mental states of their makers/users' (Callender and Cohen 2006, 75). (For a rebuttal, see Gelfert 2015, chapter 2.)

When analyzed more closely, Goodman's position goes far beyond such a 'thin' construal in terms of the user's mental states. While his proposal can accommodate the fact that different users might use the same vehicles for different representational purposes (and that mental states may be what makes the difference), Goodman does not believe that mental states on the part of their users (or makers) are all that matters for representational success. For one, individual decisions are themselves guided by the prior (collective) usage of the requisite system of representation and description. Indeed, our representational vehicles depend crucially on there being entrenched symbol systems—such as mathematical formalisms, diagrammatic notations, etc.—along with rules and conventions that set the requisite standards of correctness.

Entrenchment of this sort is something that results from repeated successful deployment as part of a progressive research programme. The stability that comes with entrenchment, as Harry Collins puts it, 'is the stability of the forms of life or taken-for-granted practices—ways of going on—in which they are embedded; it is the stability of cultures and their social institutions' (Collins 1985, 18). Catherine Elgin broadly concurs when she notes that, although 'Goodman has not studied the social forces that entrench particular predicates or those that favor one mode of representation over another', not only *could* such an investigation 'fruitfully take place within the theoretical framework he provides', but—via the centrality of the concept of 'entrenchment'—it may even be said to be 'woven into the fabric of Goodman's epistemology' (Elgin 1991, 90).

Mature Symbol Systems as a Form of Cognitive Scaffolding

The overall picture that emerges from this discussion portrays representational devices as collective resources that researchers routinely draw on in their attempts to represent reality, convince colleagues of the merits of their models, and generally grasp 'fresh and significant relationships' and devise 'means for making them manifest' (as Goodman puts it). Symbol systems, such as mathematical formalisms, are an important theoretical tool in this process.

At the individual level, for those of us who have mastered them, such systems aid us in carrying out inferences and allow us to fixate them in a medium that, when compared with natural language, reduces ambiguity. Indeed, as has been noted by others, '[u]sing a formal language forces the reasoner to explicate implicit assumptions, and makes the comparison of inferences from similar yet different assumptions easier' (Kuorikoski and Ylikoski 2014, 7). Similar considerations apply to the comparison of inferences and results *across different reasoners*—which is why instruction in the relevant notations and formalisms makes up a large part of technical and scientific training.

Hesse's point—that mathematical formalisms provide 'pointers towards further progress' and have their own 'ways of suggesting modification and generalisation'—makes sense only against the backdrop of a scientific community whose members are skilled in applying and modifying models and theories, and which, collectively, is able to arrive at determinations regarding the fruitfulness (or 'progress') of new theoretical proposals.

Elsewhere, I have discussed similar points specifically in relation to mathematical modelling, under the heading of *mature mathematical formalisms*, where this term refers to 'a system of rules and conventions that deploys (and often adds to) the symbolic language of mathematics; it typically encompasses locally applicable rules for the manipulation of its notation, where these rules are derived from, or otherwise systematically connected to, certain theoretical or methodological commitments' (Gelfert 2011, 272).

While such a 'mature formalism' will typically be set up in such a way that its output will automatically satisfy certain theoretical desiderata, it is not *just* an application of fundamental theory; indeed, some theoretically permissible scenarios lend themselves more easily to description in terms of a given formalism than others, while some scenarios may even be excluded from it. (Excluding certain types of situations in this way would be yet another way in which substantive theoretical constraints can be 'built into' a formalism.)

Neither does a mature mathematical formalism consist in the wholesale application of mathematics-at-large to specific scientific problems: the versatility of mathematics as a 'global' representational resource severely underdetermines the specific rules and notations that govern mature mathematical formalisms such as the operator formalism in quantum mechanics. (In this sense, Hesse's identification of mathematical formalisms with a 'particular piece of mathematics' may still be too global: mature mathematical formalisms also include rules, conventions, and theoretical assumptions that are not themselves part of pure mathematics.)

The emphasis on mathematical formalisms as representational vehicles, however, should not obscure the fact that similar considerations also apply to other (e.g. visual or diagrammatic) symbol systems. Consider the case of the notational system of Feynman diagrams, which was developed with the goal of representing a potentially indefinite number of physical processes in quantum electrodynamics. Each Feynman diagram consists of points ('vertices') and arrows (of different orientation) attached to the vertices, representing interacting electrons and positrons, as well as wavy lines signifying photons that may be emitted or absorbed.

Enshrined in the formalism of Feynman diagrams are both rules for the construction of new diagrams (e.g., 'At every vertex, conservation of energy and momentum among the interacting particles is required'), as well as for the interpretation of the diagrams thus generated (e.g., 'Lines in intermediate stages in the diagram represent "virtual particles", which may "temporarily" violate the relativistic energy-momentum relation, but which are in-principle unobservable if they do not'). While the formalism of Feynman diagrams was developed on the basis of an overarching theoretical conception—which takes each diagram to represent a contribution to the total amplitude for a (multiply realizable) quantum process—it has taken on 'a life of its own' in certain areas of high-energy physics, where it has developed from a mere shorthand to what one might call a notational *lingua franca*.

If the corresponding *Wikipedia* entry is any guide to what physicists (or at least those writing on physics) think about the value of Feynman diagrams, then the verdict is clear: Feynman diagrams are credited with 'provid[ing] deep physical insight into the nature of particle interactions', such as scattering processes, 'in addition to their value as a mathematical tool'. The historian of science David Kaiser characterizes the adoption of diagrammatic methods in high-energy physics as a process of 're-tooling' (Kaiser 2005,

163) of the discipline as a whole, and also notes the importance of social practices that led to the acceptance of the new methods.

Taking Princeton's Institute for Advanced Study in the late 1940s/early 1950s as his preferred example, Kaiser notes that its character as a 'focused yet informal haven for post-docs proved crucial for spreading Feynman diagrams around' (ibid, 162). Though the formalism of Feynman diagrams may have started off as an initially arbitrary notational system, it quickly acquired a degree of social and institutional reality that would shape the behaviour and cognitive processes of theoretical physicists, while at the same time being constituted by their actions and communications.

On the Character and Function of Symbol Systems

I want to close with a few general remarks about the character and function of symbol systems as collective representational resources, which, I hope, will open up space for discussion. The idea I wish to put forward is that the kinds of mature symbol systems I have discussed serve as a form of 'cognitive scaffolding', which allow their users to 'offload' or 'externalize' cognitive load, to fixate their inferences—both across time and in order to make them available to others—and to draw on a wide range of inferential resources.

Mature symbol systems serve as a way of outsourcing inferential work, for example by ensuring that results derived within the formalism satisfy the requisite criteria of validity. This is especially obvious in the case of mathematics, given the truth-preserving nature of logical and mathematical reasoning, but it also applies to physically interpreted formalisms such as Feynman diagrams and the operator formalism in quantum physics, where adherence to certain syntactic rules of the formalism often automatically ensures that certain physical constraints—conservation of energy and momentum, or conformity with Pauli's exclusion principle—will be satisfied. While it would be an exaggeration to say that mature symbol systems 'do the thinking' for their users, the preceding discussion also makes clear that they are by no means simply 'neutral tools' that function as mere conduits for their users' intentions and mental states.

There is a close affinity between this analysis of mature symbol systems and recent discussions in philosophy about the concept of extended cognition. In Andy Clark's and David Chalmers's classic thought experiment, Otto, who suffers from Alzheimer's disease, always carries with him a notebook to record information for future reference, relying on it much as he would on his memory (if the latter were to be functioning properly). According to Clark and Chalmers (1998), Otto's mind has been extended to include the (external) notebook, given that the latter fulfills the same functional role as the (internal) memory does for a normally functioning cognizers. Thus, the *extended mind hypothesis*: A human subject's cognitive processes can come to include, at a constitutive level, external artefacts such as a computer or a notebook. An alternative take on extended cognition is offered by Kim Sterelny.

According to Sterelny's *scaffolded mind hypothesis*, irrespective of whether one accepts Clark's and Chalmers's extended mind hypothesis, this much is clear: 'human cognitive capacities both depend on and have been transformed by environmental resources' (Sterelny 2010, 472). This is true both from an evolutionary perspective, according to which human cognitive competences emerge as part of a process of 'niche construction' and 'depend intimately on the environment being scaffolded to support adaptive decision making' (ibid, 466), and also at the level of individual development and cultural evolution.

Unlike the extended mind hypothesis, which is primarily about how external artefacts can be constitutive of human cognition, Sterelny's definition of cognitive scaffolding also includes natural factors and social practices (which cannot easily be 'individualised'). Sterelny explicitly mentions 'language and arithmetical notation', which 'enhance our capacity to think, even when we do not have external resources to hand' (ibid, 471). It is not by chance that these examples—language and arithmetical notation—are paradigmatic instances of collective symbol systems.

Whether we are dealing with natural language, simple arithmetical notation, or with the elaborate and complex formalisms of science, symbol systems constitute powerful representational resources, which are resources not just for an individual cognizer considered in isolation, but are entrenched and sustained through collective social practices.

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Acknowledgments: I am grateful to Eric Kerr for helpful comments on an earlier draft of this paper.

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